Learning, Generalization and the Perception of Information: an Experimental Study

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ABSTRACT. This article experimentally explores the way in which human agents learn how to process and manage new information. In an abstract setting, players should perform an everyday task: selecting information, making generalizations, distinguishing contexts. The tendency to generalize is common to all participants, but in a different way. Best players have a stringer tendency to generalise rules. A high score is, in fact, associated with low entropy for mistakes, that is with a tendency to repeat the same mistakes over and over. Though the repetition of mistakes might be considered a failure to properly employ feedback or a bias, it may instead turn out as a viable and successful procedure. This result is connected to the literature on learning.

KEY WORDS: behavioural entropy, cognitive economics, complexity, experiments, feedback, heuristics, learning

JEL CLASSIFICATION: A12, C91, D83

Sensorial perception, information processing, mental representation, and learning do not sound like the typical economics jargon. Microeconomic mainstream does not entertain with these concepts because its approach abstracts from real psychological properties and actual decision processes. According to the standard microeconomic approach, individual behaviour is rational – in a *substantive* sense – when it achieves the given goals of an agent within the exogenous limits of the choice environment. Individual preferences and meta-preferences, for instance egoism and altruism, are external to this approach and must be posited *a priori*. In order to realise given (egoistic or altruistic) goals, an agent must possess complete knowledge of the choice environment and be capable of perfectly computing all this information in an optimal fashion. Both conditions are hardly ever realised and the capacity of microeconomic models to explain individual behaviour are very scarce. Within the ranks of economics, however, on several occasions different scholars have called for an expansion of economic analysis to include more nuanced and plausible accounts of human agency.

For instance, Herbert Simon (1976), suggested a concept of rationality which is bounded – i.e. with limited available information and limited capacity to process it – and which is based on *procedures* instead of substantive goals. His research, therefore, had a positive focus on the uncovering of actual decision processes, but inevitably took a normative lean in the definition of what are the best procedures available to real economic agents. Uncovering the way people think, decide, and learn affords a better understanding of the social world, but it also empowers the development of better choice aids and teaching methods.

This article falls within this approach, which may be called Cognitive Economics, and it experimentally explores the way in which the participants learn how to process and manage new information. Our experimental setting is abstract so that the participants cannot rely on any knowledge they already have and must instead learn everything from scratch. In such an abstract setting, our players should perform an everyday task: selecting information, making generalizations, distinguishing contexts. Can they learn how to consistently make the best choice in a new complex environment?

LITERATURE REVIEW

In standard economics, the pressure of competition (Alchian 1950) ensures that agents who do not make the best choices are forced out of the market in a fashion akin to natural selection (Vromen 1995). Individual agents are therefore routinely modelled in such a way that they always make the best choices: this means that they are assumed to possess perfect information and unlimited computational skills, and to pursue their narrow material advantage. Although it is implausible that individuals are (or even can be) as microeconomic models represent them, it may be enough for economists to show that people become (or tend to become) such. Agents capable of improving their performance over time and of progressing towards ever more efficient decisions may uphold, and justify the recourse to, the assumption of perfect individual rationality. This requires the modelling of some individual capacity to learn.

Some examples of how this has been attempted are the Bayesian and the Least Square Learning (e.g. Marcet and Sargent 1989). Both describe the optimal processing of available empirical data by individual agents. These data are then employed in subsequent decision making in a way that approximates the assumption of complete information. Though also the assumption of perfect processing of information is implausible, even psychological models which assume an imperfect processing of the information suggest that people can learn how to make the best choices. Reinforcement Learning models (e.g. Erev and Roth 1998), for instance, suggest that agents repeat choices which allowed positive results in the past and consequently adjust their behaviour to empirical evidence in a way that makes it increasingly likely to observe a repetition of the same behaviour (although a, smaller and smaller, probability of making a different choice remains). In standard and stable contexts, reinforcement learning easily results in consistently optimal behaviour just like microeconomic models require.

Learning, however, should not be considered as a black-box mechanism that prompts automatic choices, but rather as a process of assigning specific meanings to different states of the world. Brian Arthur (1992), for instance, observed learning cannot be reduced to the acquisition of new data, but it requires the construction of semantic categories that categorise the data. Moreover, individuals build mental models that organise large chunks of empirical evidence. Starting from observation, individuals generate hypotheses about causality and develop models that allow prediction and decision-making. These hypotheses and models are neither static nor unique. Choices are thus repeatedly tested against real world phenomena, associated with their observed outcome, and eventually reinforced or abandoned. The world presents traceable patterns and Arthur believes that the skill to detect these patterns is both a necessary and advantageous human cognitive skill.

Richard Nelson (2007) suggests that the search for better ways of doing something is both oriented and constrained by what agents currently know. Current knowledge suggests some behaviour consistent with an agent's goal. The received feedback results either in a more efficient behaviour or in an improved understanding of the specific decision-making context. The agent "either needs to learn how to identify different contexts, as well as a set of context specific guides of action, or find a broad guide to action that works reasonably well in all or most contexts he will face" (Nelson 2007, p. 6). Therefore, problem solving requires both trial-and-error learning and abstract theorizing.

The study of the capacity to manage information in a complex environment is also central to Ronald Heiner's (1983, 1985) model of behavioural entropy. According to Heiner, individuals more or less consciously make a choice between very few of the many different actions which are possible on each occasion. This subset consisting of 'reliable' actions, or actions which typically afford satisfactory results, is a result of uncertainty – which can be defined as a lack of knowledge of (or lack of the skill to define) the link between contexts and optimal decisions. A reduction in the number of potential options may be a consequence of reacting only to some information, ignoring the rest, of disregarding the distinction among certain pieces of information, or of individual failures in the processing of information, resulting in somewhat generic rules of behaviour that disregard some context-specific variables.

In the presence of uncertainty, it can be expected that agents try out several alternative choices until they figure which ones are reliable. Therefore we observe high variability of behaviour and it is very hard to predict which option will an actor choose next. Over time, as agents learn to react to selected information, their behaviour should become less erratic and therefore more predictable. Heiner employs behavioural entropy as a measurement of the variability of behaviour. It can be computed as follows (see also the Appendix):

$$E^B = -\sum h_a \log h_a \qquad (1)$$

where *a* is an element in the set of possible actions *A*, and *ha=p(a)* is the probability (relative frequency) of choosing a given action. The higher the number of different actions attempted in the same choice-context and the more uniform their frequency (for instance when an agent gives random answers), the higher an agent's behavioural entropy is and the harder it is to predict this agent's choices. Conversely, if an agent's behaviour is stable (because he always makes the same choice), entropy is zero.

Though he does not directly explores learning, from Heiner's reflections, and consistently with Arthur's and Nelson's above, learning may be considered a capacity to discover ever better reliable actions, which means to better use information and to better interpret decision contexts. This immediately translates in the abandonment of any concept of perfect rationality, which is instead replaced by a definition of bounded rationality (à la Simon, 1983) as the capacity to manage only some subsets of useful information. As people learn, they use larger and larger amounts of important information and they react in more specialised ways to subtle changes in environmental conditions. The overall variability of their behaviour therefore scales up, while its predictability is diminished. Within narrowly defined choice contexts, however, variability shrinks.

Since behaviour reflects individual cognitions, learning ultimately affects an agent's behaviour through a change in the type or in the amount of his processed information – e.g. concerning (un-)attainable or (un-)desirable outcomes; (un-)feasible, (in-)effective or (in-)efficient actions. In this sense, all learning modifies the knowledge agent possesses about the task he is facing (Novarese, 2012). The two main vehicles of learning (Bandura 1977, Rizzello and Turvani 2002, Witt 2000) are vicarious learning, which occurs via observation or imitation of the behaviour of others, and direct learning, which takes place when actors obtain information from the outcome of their own actions.

This idea of learning can be understood looking at an experiment which analyses (direct) learning in a complex, but stable, choice environment with strong monetary incentives and where full feedback is immediately available.

THE EXPERIMENT

The experiment took place at the Centre for Cognitive Economics at the University of Eastern Piedmont in Alessandria (Italy) on the 5 July 2000.

Participants and Experimental Design

The participants were twenty-three undergraduate students of Law, enrolled in the first-year optional Seminar of Economics. Each sat in a cubicle with a computer and was not allowed to take notes or to communicate with the others. After reading the written instructions (see the Appendix), the participants started the experiment, which lasted about one hour. The students were compensated with 40 ITL (€ 0,02) per point scored in the present experiment. They were told that the participation would have no impact on their academic career outside the Seminar[1].

Task

The experiment was constructed around a fictional association, whose members fall within one of five age categories: *Children*, *Adolescents*, *Young*, *Adults*, and *Elderly*. The information about members is reported on a set of cards located on either of two shelves (*Right* and *Left*). Each card presents two features: one of four animals (*Cow*, *Horse*, *Goose*, and *Chicken*) and one of four shapes (*Square*, *Rectangle*, *Circle*, and *Oval*)[2], as in FIG. 1.

On each turn the participants were presented with a sequence of animal, shape, and shelf, and were asked to guess the corresponding membership category within ten seconds[3]. The logical relationship between the card features, the shelves, and the membership was based on a specified criterion (i.e. it was not random) and it remained constant throughout the 231 turns of the experiment, but it was not related to any real world fact and it explicitly did not require any academic knowledge (TAB. 1). The connection could and should be learned during the experiment in order to fulfil the ultimate goal of scoring as many points as possible.

Table 1 - Solution

<table>
<tr><th>Animal</th><th>Shape</th><th>Shelf</th><th>Membership Group</th></tr>
<tr><td>Mammal
(cow, horse)</td><td>With angles
(square, rectangle)</td><td>Right</td><td>Children</td></tr>
<tr><td>Mammal
(cow, horse)</td><td>With angles
(square, rectangle)</td><td>Left</td><td>Adolescents</td></tr>
<tr><td>Mammal
(cow, horse)</td><td>Without angles
(circle, oval)</td><td>Right</td><td rowspan="4">Young</td></tr>
<tr><td>Mammal
(cow, horse)</td><td>Without angles
(circle, oval)</td><td>Left</td></tr>
<tr><td>Bird
(chicken, goose)</td><td>With angles
(square, rectangle)</td><td>Right</td></tr>
<tr><td>Bird
(chicken, goose)</td><td>With angles
(square, rectangle)</td><td>Left</td></tr>
<tr><td>Bird
(chicken, goose)</td><td>Without angles
(circle, oval)</td><td>Right</td><td>Adults</td></tr>
<tr><td>Bird
(chicken, goose)</td><td>Without angles
(circle, oval)</td><td>Left</td><td>Elderly</td></tr>
</table>

[1] The best performer thus earned € 25.52, the worst performer earned € 14.32. The average and median compensation were € 18.62 and € 18.08, respectively.

[2] The features needed be as neutral as possible. In a previous experiment (Novarese and Rizzello 2006), the employment of bright/dark colours and large/small sizes may explain why the subjects associated certain features with value judgements (i.e. insufficient to excellent). Here we also tested the features to ensure neutrality.

[3] The main results are similar to those in Novarese and Rizzello (2006), where there was no time constraint.

Fig. 1

At the end of each turn, subjects were given full feedback (FIG. 2). The score was calculated with respect to the distance between the answer given an the correct answer: the distance is 0 when the answer is correct and in this case the score is 6, the distance is 1 when the answer given is one membership category above or below the correct answer (e.g. *Children/Adolescent*, *Elderly/Adults*) and in this case the score is 4, etc.

Fig. 2

RESULTS

Earlier articles that investigated experiments such as the present one (Novarese and Rizzello 2007, Lanteri and Novarese 2007) reveal a clear result: memorization does not explain individual performance (nor did we expect this to be the case on the basis of our background literature). There are so many sequences, which change with such frequency, that memorization is not cognitively speaking an option for the participants. It is both more natural and more efficient to develop actual theories about the experimental world that result in the repetition of choices consistent with these theories, both when they are correct and when they are wrong (because the revision of some theory is time consuming).

Participants progress from random choices at the beginning towards more stable (and therefore predictable) ones, based on a limited number of elements of the sequence – i.e. only Animal and Shelf, disregarding Shape (see Lanteri and Novarese 2007) – and then on to a more complete and sophisticated representation of the experimental environment. The responses of each participant thus become ever more predictable, so that, given a sequence, we may forecast his responses with increasing accuracy. This is because, on the one hand, the number of correct answers increases, but so do the number of repeated mistakes. In the coming paragraphs we explain these trends.

The development of theories

When a participant gives several times the correct answer to a sequence, she must have understood the exact working of that portion of the experiment. If she often gives wrong answers, perhaps she has not yet uncovered the principle of that sequence. However, if the wrong answer is consistently the same, it is very likely that the participant has developed a mistaken theory.

In this section we study this phenomenon. Since it is possible that theories change over the experiment, as participants learn, we require that the repetitions occur at least for a period of time, and specifically for a third of the overall experiment, which gives us three phases: turns 1-77, 78-154, and 155-231. We only focus on the stable (which allows us to plausibly assume that it is principled, too) association of an answer with a sequence, therefore we only consider responses given 75% of the times. Any answer given to a sequence which only appears once would be given 100% of the times. This, however, does not seem enough to assume stability of behaviour. Instead, we require that a sequence has appeared at least four times during the experiment (but on average they appear ten times) and at least three times in each phase.

Participants indeed develop stable associations between sequences and responses, just like we expected. The number of theories that qualify for our analysis increases from 130 in the first phase to 235 in the third and they also become increasingly accurate going from a 56% rate of correct answers in the first phase, up to 67% in the third. Although participants get better and better, the number of stably mistaken theories is astonishing: 31% in the first phase and 42% in the third. Note that, though our condition was that answers were given 75% of the time, we have numerous observations with a 100% frequency. But only 57% of these were correct, while 43% were wrong.

How can this happen? Don't participants see their answers are wrong and change them accordingly? They do, but the reception, processing, and implementation of the feedback is imperfect.

The limited effect of feedback

TAB. 2 reports the answers a typical participant gave to the sequence Chicken-Oval-Left, whose correct answer is Elderly.

Table 2 – Response to Chicken-Oval-Left by one Participant

Turn	Answer given	Score
12	NA	0
26	Elderly*	6
56	Elderly*	6
60	Elderly*	6
109	Adolescents	-1
122	Adolescents	-1
135	Adults	4
144	Adults	4
171	Elderly*	6
181	Adults	4
199	Adults	4
209	Adults	4

* correct answer

On turn 12, the participant skips the answer, she observes feedback and on turn 26 he or she responds correctly. Also, she probably does so with some reason and not at random, provided that she repeats the correct answer on turns 56, and 60. One would then imagine that this participant grasped the criterion and is going to consistently give the correct answer from then on. Wrong. On turn 109, the participant switched to a mistaken response, and then repeats it on turn 122. Her theory was probably undergoing some revisions. But the feedback warned her against that response. Indeed, she abandons the mistake and... makes a different one! Although this mistake is less severe score-wise, she repeats it a few turns later and, after a single correct response on turn 171, from turn 181 until the end of the game she keeps repeating the mistake.

For the sequence under investigation, this participant incurs in a total of eight mistakes (including the missing answer on turn 12). Seven of these mistakes could be repeated (on turn 209 the mistake cannot be repetad because it's the last turn with this sequence). We consider repeated an error if the same wrong answer is given in two following appearance of the given sequence. This player repeats four times the same error . We can compute a mistake confirmation rate for this sequence (four on seven, that is 57%) and a mean overall value, for a given player during all the game. The mean value of this index for all player is 33%[4].

[4] For 20 out of 23 participants, we can reject the hypothesis that this happened by chance with a 90% confidence.

The trend of this phenomenon, moreover, is counterintuitive: the number of confirmed mistakes increases, instead of decreasing: it is 27% on average in the first part of the experiment and 37% in the last part. It is possible to demonstrate that this results can hardly be the effect of random choices. Players are building theories and representation of this world. These theories are often based on simplification and on reduced use of available information, as Table 3 shows.

Table 3 – Distribution of Responses, Turns 154-231

animal	shape	shelf	elderly	adults	young	adolescent	children
horse	circle	right	0%	4%	**57%**	13%	26%
horse	circle	left	4%	10%	**54%**	17%	14%
horse	oval	right	1%	9%	**45%**	24%	22%
horse	oval	left	4%	7%	**59%**	17%	12%
horse	square	right	2%	8%	20%	14%	**55%**
horse	square	left	0%	3%	43%	**38%**	16%
horse	rectangle	right	0%	3%	19%	14%	**64%**
horse	rectangle	left	0%	4%	48%	**35%**	13%
chicken	circle	right	**17%**	43%	26%	9%	4%
chicken	circle	left	57%	**20%**	18%	4%	0%
chicken	oval	right	**18%**	42%	28%	4%	7%
chicken	oval	left	70%	**12%**	17%	0%	1%
chicken	square	right	15%	33%	**39%**	7%	7%
chicken	square	left	30%	26%	**26%**	9%	9%
chicken	rectangle	right	17%	17%	**48%**	13%	4%
chicken	rectangle	left	32%	17%	**33%**	12%	6%
cow	circle	right	2%	7%	**50%**	23%	17%
cow	circle	left	0%	22%	**48%**	17%	13%
cow	oval	right	0%	13%	**52%**	17%	17%
cow	oval	left	8%	8%	**52%**	22%	10%
cow	square	right	0%	0%	17%	26%	**57%**
cow	square	left	0%	4%	17%	**65%**	13%
cow	rectangle	right	0%	4%	26%	22%	**48%**
cow	rectangle	left	1%	6%	30%	**52%**	10%
goose	circle	right	**21%**	48%	18%	12%	1%
goose	circle	left	51%	**25%**	19%	6%	0%
goose	oval	right	**7%**	55%	20%	14%	1%
goose	oval	left	50%	**28%**	15%	7%	0%
goose	square	right	17%	20%	**48%**	11%	4%
goose	square	left	26%	30%	**35%**	9%	0%
goose	rectangle	right	13%	39%	**30%**	9%	9%
goose	rectangle	left	23%	22%	**43%**	7%	4%

Even mistaken answers are not given at random. Participants indeed employ (imperfect, incomplete, shifting) theories of the experimental world, so that even their mistakes become predictable.

Because of the score system, when the correct answer is Young, responding Adults and Adolescents is indifferent and the same happens for the answers Elderly and Children: both mistakes have the same distance from the correct answer and therefore results in the same score. There should thus be no specific reason to expect that, when the correct answer is Young, mistakes be not random. However, we observe that the mistakes are strongly clustered in an 'almost-correct' direction: when the Animal is a Mammal, the mistakes group around the two youngest membership categories, while the vice versa is true for Birds[5]. This tendency may be explained by the fact that at least some of the participants disregard the second piece of information, i.e. Shape, for at least some of the sequences.

These observations serve as starting point for the analysis of the relationship between entropy and performance.

Performance and predictability

It is our goal in this article to analyse not performance itself, or best strategies, but learning[6]. In order to do so we divided the game in periods. Each period has a duration of 58 turns. The first period goes from turn 1 to turn 58, the second from turn 2 to 59, and so on. This way we obtain 175 periods, largely overlapping.

For each period, we compute:

(i) the score of each player; and

(ii) the behavioural entropy for each animal-shape-shelf sequence which appears at least 3 times.

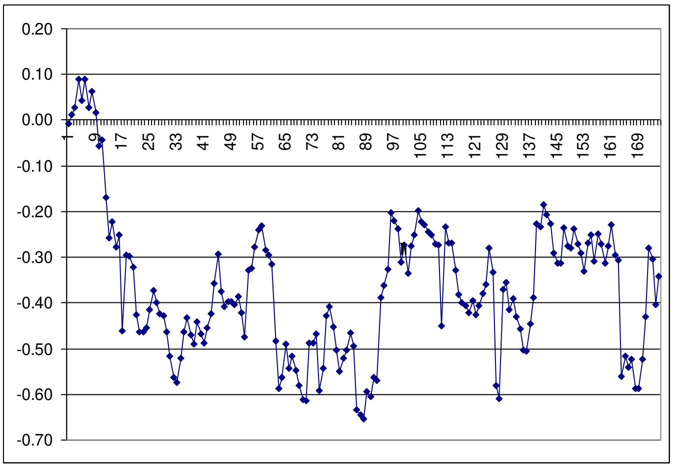
(i) is an indication of performance, which we could substitute e.g. with the number of correct answers, while (ii) measures the stability of mistaken responses. We then calculate an average for each participant. This way, we obtain two figures per player for each period. In each period, on the data of all our subjects, we can measure how these two values are related. The simpler measure of linear relation is the correlation coefficient which indicate if there is a linear relation between two variables[7]. FIG. 3 shows the evolution of this value.

[5] This is especially puzzling because, in reason of the score system, the most reliable option is always Young, which cannot be farther than two steps from any membership category and therefore always afford positive score.

[6] For deeper analysis of this experimental dataset, see Lanteri and Novarese (2007).

[7] The correlation coefficient can assume all values in the range between -1 and 1. It has negative values when the two variables move in different direction and therefoe a big value of one of them implies a low of the other. It has a value around zero when there is no linear relation among the two variables. Statistical tests are used to verify when there is evidence to support the idea of non zero correlation.

Fig. 3



Except for a brief time at the beginning, correlations are systematically negative (in 97 out of 175 cases the values are significantly different from 0), which means that the participants who perform best in that part of the game also have lower entropy. This result, on which we shall comment further below, is not trivial.

The irregular trend depends in some measure on technical factors: the sequences appearing on each period differ. During some periods there are several sequences which were clearly understood by the participants. This, obviously, reduces the number of mistakes and makes the correlation lower and less significant[8]. Since the values remain negative despite this problem, on the other hand, the results are especially robust. This can also be confirmed by means of a different analysis.

Table 4 – *Best*'s low entropy, By Period Cluster

1	24
2	35
3	22
4	16

Table 5 – *Best*'s low entropy, By Period Cluster

1	15
2	31
3	32
4	26

[8] Like other indexes, ours is most meaningful when computed on a sufficiently varied sample. If all participants yield similar results, the index cannot unearth very meaningful relations.

Consider now the participants who performed best in the last period and who, presumably, best understood the working of the experimental world. Call *Best* those who scored above the median and *Worst* the others. The *Best* have lower entropy in 159 out of 175 periods, of these 104 were statistically significant with the Kruskal Wallis test and were distributed across the clusters as in TAB. 4. Here, again, some of the periods do not yield meaningful results, because the *Best* perform so well that few mistakes in a single sequence crucially alter the index.

The same comparison for the overall experiment (which is a more relevant and robust index) reported in TAB. 5 reveals that the *Best* have an average mistake entropy of .23, versus .33 for the *Worst* (statistically significant both with the t-test and the Kruskal Wallis). The overall correlation between entropy and score is - .83 (significantly different from 0 at the 99% confidence interval).

We can show what does this pattern represent in a very intuitive fashion, by means of TAB. 6, which compares the behaviour of one of the *Worst* and one of the *Best* players. For each sequence in which the players made at least two mistakes in the period between turn 59 and 117, we calculate the frequency distribution of responses. It is quite evident – even without sophisticated indexes – that the worse player tends to have more heterogeneous mistakes, with many wrong answers only given once. The better player, on the other hand, tends to concentrate her mistakes on few sequences. Not only the mistakes are less numerous, but they are also more regular. The same results can be found throughout the experiment and for all players. In the very last turns, however, the *Best* players have so few mistakes that the comparison is meaningless.

Generally speaking, therefore, it seems that the capacity to give correct answers is associated with stable behaviour even with respect to mistakes, which is an indication of a tendency to apply rules even if these rules are wrong. On the other hand, the direction of causality is not clear, because both can in principle explain each other. Indeed, since this phenomenon can be observed very early in the game, but it is stronger in the central part of the experiment, and since it is larger for the *Best* group, suggests two interpretations.

* The participants who develop the most correct rules tend to apply rules even when they are not correct. In this case we imagine that people employ analogical reasoning and apply some 'default' or 'reliable' rule when they lack a context-specific rule.

* It is also plausible that the individual capacity or tendency to focus on some variables and the disregard of other variables (which produces steady behaviour and little entropy) facilitates the understanding of the solution and consequently results in a higher score.

Table 6 – Players' Comparison

			Worst						*Best*					
			Child.	Adol.	You.	Ad.	Eld.	TOT.	Child.	Adol.	You.	Ad.	Eld.	TOT.
Cow	Square	Right	-	-	-	-	-	-	-	1	1	-	-	2
	Rectangle	Right	-	1	-	1	1	3	-	-	-	-	-	-
		Left	-	-	1	-	1	2	-	-	-	-	-	-
	Circle	Right	-	-	-	-	-	-	1	2	-	-	-	3
		Left	1	1	-	-	-	2	-	2	-	-	-	2
	Oval	Left	-	1	-	2	1	4	-	2	-	-	-	2
Horse	Square	Right	-	-	2	-	-	2	-	2	-	-	-	2
	Circle	Left	-	1	-	3	-	4	-	5	-	-	-	5
	Oval	Right	-	2	-	-	-	2	-	1	-	3	-	4
		Left	-	1	-	1	-	2	-	2	-	-	-	2
Chicken	Circle	Left	-	1	1	-	-	2	-	-	-	-	-	-
	Oval	Right	-	2	-	-	-	2	-	-	-	-	-	-
		Left	-	1	-	1	-	2	-	-	-	-	-	-
Goose	Square	Right	-	1	-	-	1	2	-	-	-	-	-	-
	Rectangle	Left	-	1	-	-	1	2	-	-	-	-	-	-
	Circle	Right	1	-	-	-	2	3	-	-	3	-	-	3
		Left	1	-	-	1	-	2	-	-	2	-	-	2
	Oval	Right	-	-	1	-	1	2	-	-	3	-	-	3
		Left	-	-	1	1	-	2	-	-	-	-	-	-

It is not straightforward to understand whether the repetition of mistakes is caused by or is responsible for the high score. The observation that the participants who perform best at the end of (but not necessarily throughout) the experiment also have low entropy all along the game (and even at the beginning) suggests that it is the low entropy that favours a superior understanding of the experimental world. A deeper understanding of this issue is central to uncovering actual learning processes. Moreover, it may prove an important element towards defining better training and teaching techniques.

We may test this idea as follows. Consider the correlation between entropy in a period and score in an earlier period (e.g. 25 periods earlier). If low entropy is responsible for high score, which would mean that participants employ whatever rules they have learnt when lacking a better rule, this time-lagged correlation will be stronger than the normal correlation. Before they may apply a rule, participants ought to develop and work it out. Therefore a low observed entropy in a given turn should be a consequence of the correct answers given earlier in the experiment. In other words, under this hypothesis, if people do export to similar contexts the rules they have learnt in some decision contexts, there should be a slight delay in the correlation between high score and low entropy.

To study this phenomenon, we investigate to what extent does entropy in a given period depends on the score of the same period and how much does it depend on that of an earlier period. We confine the most technical parts of our analysis to the Appendix for the readers willing to dig deeper in the matter. Suffice it to say here that the correlation is highest between entropy and current score than with score of periods which started 12, 25, or 35 turns earlier. In fact, the effect of time-lagged score is opposite to what we expect: those who scored the most in previous turns have higher entropy. One plausible explanation for this pattern is that the participants who have found some simple strategies to respond, then try to elaborate on those by means of trial-and-error, therefore their behaviour is less stable (see also Lanteri and Novarese 2007).

CONCLUDING REMARKS

The conclusions above, though perhaps not final, reinforce the two-headed interpretation. If the lower entropy of the best performing players depended on the application of past rules in the present, the effect of time-lagged score should be stronger. The fact that it is weaker suggests instead that best players tend to generalise rules, therefore reducing the information complexity of the environment. The tendency to generalize is common to all participants, but a decision context like that of our experiment certainly favours those for which the tendency is strongest.

The two procedures: <export rules beyond their context> (or analogical decision-making or learning spillover) and <reduce the amount of information employed> are not mutually exclusive. It is nonetheless better to keep them separated because they are conceptually distinct and both may prove either useful or dangerous.

Analogical problem-solving, on the other hand, amounts to a selective use of the information because it amounts to treating as identical or similar, two situations which differ in a number of respects. It is natural way of reasoning (Novarese, 2012), but also a strategy suggested by George Polya in *How to Solve It*, which can be described as perfectly rational. 'Do you know a related problem?' is one of the first questions the Polya lists in his strategy to solve any problem, elaborating on a method derived from mathematical theory. The first step to devise a plan is to "ask these questions: 'Have you seen it before? Or have you seen the same problem in a slightly different form?'" (p. xvi).

On the other hand, the reduction of employed variables is part and parcel of theorisation: it is the very core of *ceteris paribus*. In order to investigate the effect of one variable, every other variable is excluded from the analysis by being held (or assumed) constant. Our best chances at understanding the effect of a variable is to investigate it in isolation.

More generally we may advance the following suggestion. Though we may not yet say for what specific reason, a high score is associated with low entropy for mistakes and therefore with a tendency to repeat mistakes over and over. Though the repetition of mistakes might be considered a failure to properly employ feedback or a bias, it may instead turn out as a viable and successful procedure.

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Appendix A - Instructions

In what follows we report the instructions of the game you will take part in. You will be compensated with real money (40 ITL per point).

The Game

An association has different membership categories which pay different fees and have access to different services. The categories are:

- children
- adolescents
- young
- adults
- elderly

Children and adolescents do not pay. Young pay a reduced fee. Adults and Elderly pay the full fee. Members' information are recorded on a set of cards, stored on different shelves.

Each card is characterized by:

- the drawing of an animal
- the drawing of a shape
- a shelf

(e.g. A card might have the drawing of a cow and a square, and be placed on the right shelf.)

You do not know the classification system and thus which cards corresponds to which category. The goal of the game is to understand this correspondence.

The game lasts 231 turns. In each turn you will be shown the information from one card, so you will see information about:

- animal
- shape
- shelf

Based on this information you shall indicate the correct membership category, keeping in mind that:

- there is a logical relationship between the information and membership categories
- the relationship is constant throughout the game
- the relationship is completely artificial (therefore it is neither necessary nor useful to have experience of actual filing systems or any other specialised knowledge)

Obviously, the earliest answers will be given at random.

Each turn, therefore, the game will take place in the following way:

1- You see the information;

2- You give your answer (note: you must choose within 10 seconds, after this time the system proceeds to the next turn);

3- You are told the correct answer and your score in the last turn.

4- You move on to the next turn and you start again.

During the game you are not allowed to talk, nor to take notes

The Score

In order to calculate the score we define the distance between the answer you gave and the correct answer, as follows:

- if the answer is correct, the distance is 0 and you score +6;
- if the answer given is children and the correct answer is elderly (or vice versa), the distance is highest: 4 and you score -4;
- if the answer given is children and the correct answer is adult (or vice versa), or if the answer given is adolescents and the correct answer is elderly (or vice versa), the distance is 3 and you score -1;
- if the answer given is children and the correct answer is young (or vice versa), if the answer given is adolescent and the correct answer is adult (or vice versa), or if the answer given is young and the correct answer is elderly (or vice versa), the distance is 2 and you score +1;
- if the answer given is children and the correct answer is adolescents (or vice versa), if the answer given is adolescent and the correct answer is young (or vice versa), if the answer given is young and the correct answer is adults (or vice versa), or if the

answer given is adults and the correct answer is elderly (or vice versa), the distance is 1 and you score +4;

- if you do not answer, you score 0.

Game dynamics

Each turn of the game can be divided into two parts.

The first part requires that you choose one of the five alternatives offered, by means of selecting the corresponding button and then “Enter”.

It is important that you complete these operations within 10 seconds because, when such time has elapsed, the system moves on with the test and it records “No Answer” corresponding to zero points.

After you made your choice, or after 10 seconds, you move on to the second part of the turn.

The screen will report the outcome of the present turn. It reminds you the choice you made, the correct answer, and your score for this turn. The window stays open for 6 seconds, after which the system starts the next turn.

Appendix B - Time-lagged Score and Entropy

FIG. 4 compares the correlations (after period 26). The values do not differ too much. Generally speaking, the correlation between entropy and time-lagged score has values closer to 0 and these values are less significant than the correlation between entropy and simultaneous score (79 vs. 91 values are significantly different from 0).

To assess more precisely the influence of simultaneous score compared with the time-lagged one, we shall employ a linear regression analysis, in which entropy is the dependent variable. The two scores are used as independent variables so to measure their joint effect and compare their relative strength.

Fig. 4

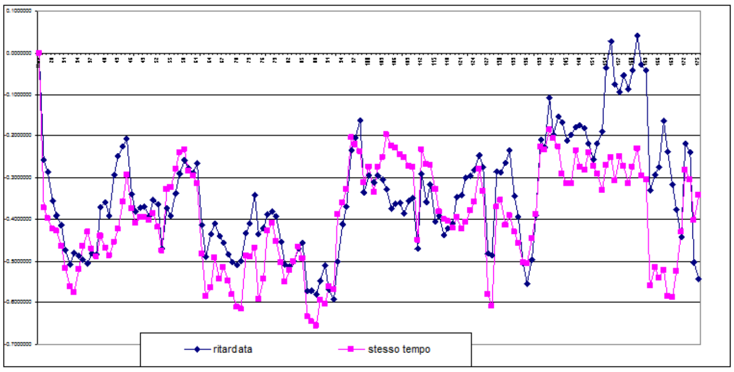

Table 7 – Score and Entropy (Time-lag 25)

			TIME-LAG SCORE 25				
			not significant		significant		Tot.
			-	+	-	+	
SCORE	not significant	-	56	47			*103*
		+	21		1		*22*
	significant	-	1	16		8	*25*
		+	0	0	0	0	*0*
	Tot.		*78*	*63*	*1*	*8*	150

In most periods the correlation is not significant. It is sometimes significant, but neither independent variable is individually significant (this is also because of the correlation between time-lagged and simultaneous score). The sign of the variables SCORE and TIME-LAG SCORE are nonetheless noteworthy. TAB. 7 does this and it also distinguishes the cases in which the two independent variables are significant at the 90% level. The SCORE variable is negative in 128 cases out of 150 and it is significant in 25 of these. The TIME-LAG SCORE variable is significantly different from 0 in just 9 cases, in 8 of which it has positive sign. Out of 150 repetitions, it is positive 71 times.

We can thus make some inferences.

* The effect of simultaneous score is stronger and it is negative.

* The effect of time-lagged score is less strong. Perhaps during some periods it reinforces the other effect, but with an opposite sign: for a given score in a period, a higher the time-lagged score is associated with higher entropy for mistaken answers. During such periods, the players who previously performed better have higher entropy, possibly because they are confident with some portions of the solution and are more inclined to try and understand the remaining portions.

Table 8 – Score and Entropy (Time-lag 12)

			TIME-LAG SCORE 12				
			not significant		significant		
			-	+	-	+	
SCORE	not significant	-	63	50			113
		+	32		2		34
	significant	-		6		10	16
		+					0
	Tot.		95	56	2	10	163

Table 9 – Score and Entropy (Time-lag 35)

			TIME-LAG SCORE 35				
			not significant		significant		
			-	+	-	+	
SCORE	not significant	-	54	41			95
		+	15		1		16
	significant	-	1	23		5	29
		+					
	Tot.		70	64	1	5	140

The time-lag we use is obviously arbitrary, but we employ two more time-lags, 12 and 35 periods, to test the robustness of our inferences. In this former case (TAB. 8), there are fewer significant cases, but the other conclusions hold. In the latter case (TAB. 9), despite a smaller number of estimates (because we ought to skip the first 35 turns), the significant cases grow. The interpretation does not change and TIME-LAG SCORE is ever less significant and with a positive sign.

The larger the time-lag, therefore, the less significant is time-lagged score (and with a positive sign), but the more significant the simultaneous score (and with a negative sign) also because the correlation between the two is reduced and the estimates are more reliable.