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Abstract This paper analyses the dynamics of a two-dimensional overlapping generations model with young and old age labour supply. It is shown that the public provision of health investments, which, in turn, affects the demand for material consumption, may represent a source of local indeterminacy, nonlinear dynamics and multiplicity of equilibria. Furthermore, global indeterminacy may also occur because of the co-existence of two attractors with tangled basins of attraction.

Keywords Chaos; Labour supply; OLG model; Public health expenditure

JEL Classification C62; C68; I18; J22; O41

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1. Introduction

The economic growth literature based on models with overlapping generations (OLG) has represented a benchmark for the study of fluctuations in macroeconomic variables, especially within the neoclassical growth context (Diamond, 1965), as their existence can be explained through periodic as well as aperiodic, but deterministic, orbits that resemble random ones[1] (e.g., Grandmont, 1985; Farmer, 1986; Reichlin, 1986; de Vilder, 1996; Cazzavillan et al., 1998), and it has therefore contrasted explanations of cycles grounded on a stochastic origin of it (e.g., Kydland and Prescott 1982; Long and Plosser, 1983).

While several authors consider one-dimensional or two-dimensional general equilibrium OLG models with either inelastic labour (e.g., Yokoo, 2000; Duranton, 2001; Wendner 2003) or endogenous labour supplied only by the young (e.g., Nourry, 2001; Nourry and Venditti, 2006),[2] scarce attention has been paid to the study of the local or global dynamics in models where agents choose how much time to devote to labour/leisure activities in both periods of life (i.e. when young and old).

Another important burgeoning strand of (theoretical) literature deals with the effects of public and private health spending on the individual length of life and labour productivity in model of economic growth (e.g., Chakraborty, 2004; Bhattacharya and Qiao, 2007; Leung and Wang, 2010; Fanti and Gori, 2011). While the first and the third authors concentrate on how the long-run demo-economic outcomes (i.e., savings, capital accumulation and longevity) are affected, respectively, by

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[1] Two reference textbooks are Azariadis (1993) and de la Croix and Michel (2002).

[2] Two important paper that analyse the equilibrium dynamics of an economy with endogenous labour-leisure choice, in a context different than the OLG model, are Ladrón-de-Guevara et al. (1999) and Ortigueira (2000). The former uses an infinite horizon continuous-time endogenous growth model with physical and human capital accumulation in which leisure enters the utility function. The latter characterises the transitional and long-run outcomes of an infinite horizon continuous-time economy by considering a measure of the effective amount of leisure through human capital accumulation and education.

private and public health spending, finding that through the longevity-enhancing channel, the provision of health services represents a stimulus to economic growth and welfare, the second authors show that if the private health system is accompanied by complementary tax-financed health services, the economy can be exposed to endogenous fluctuations and even chaos (however, this holds only under the rather stringent assumption that the public health expenditure is a convex function of the tax rate). Different from the previous cited papers, the forth authors study the dynamical features of an OLG economy where the public provision of health services affects the supply of efficient labour of the old-aged, showing that period-doubling and bubbling phenomena can occur even when individuals are perfectly foresighted.

From an empirical point of view, the medical and economic literature has emphasised how the rapid growth in public and private health spending over the GDP, observed especially in developed countries (see, e.g., World Health Statistics, 2010), can have important positive effect on the survival rates and health status across population (see, e.g., Evans and Pritchard, 2000). Moreover, the increase in the public health expenditure rises concerns about the sustainability of the government health plans over time (e.g., Hartwig, 2008), as well as whether health is a necessity or a luxury good (e.g., Baltagi and Moscone, 2010) and if government ideology is an important determinant of the growth in health budgets (e.g., Potrafke, 2010).

This paper is framed in these two strands of literature and aims at studying the dynamical properties of a simple general equilibrium OLG economy where agents work in both the first and second period of their life, by assuming, following a literature pioneered by Viscusi and Evans (1990), that the health status affects consumption possibilities: i.e., the healthier an individual, the more she can consume. It is shown that the financing a public health plan can generate multiplicity of equilibria as well as complex dynamics in our two-dimensional model.

Our results also represents a policy warning about the destabilising effects of the financing of the public health programme, as our numerical experiments based on common values of parameters assumed in the economic literature reveal.

The rest of the paper is organised as follows. Section 2 builds on the model. Section 3 presents the local and global dynamic analyses. Section 4 concludes.

2. The model

2.1. Individuals

Consider an OLG closed economy populated by one individual per generation who lives for two periods: youth and old age (Diamond, 1965). The typical agent born at t is endowed with two units of labour in every period and supplies the fraction $0 < \ell_t < 2$ when young and the fraction $0 < L_{t+1} < 2$ when old to firms, while receiving the wage w_t and the expected wage $w^e{}_{t+1}$, respectively, per unit of labour, as in Gaumont and Leonard (2010). The income of the young is taxed away by the government at the constant rate $0 < \tau < 1$, which uses the revenues so collected to finance a balanced-budget public health programme (e.g., hospitals, vaccines, scientific research and so on). Individuals consume only when old (see, e.g., Galor and Weil, 1996; Grandmont et al., 1998; Antoci and Sodini, 2009). Therefore, the budget constraint of the young at t simply reads as

$$s_t = (1-\tau) w_t \ell_t ,$$

that is, the disposable income is entirely saved (s_t) for consumption purposes in the second period of life (C_{t+1}). Indeed, old-age consumption is constrained by the amount of resources saved when young plus the expected interest accrued from t to $t+1$ at rate $r^e{}_{t+1}$ as well as by the labour income earned in such a period, i.e.

$$C_{t+1} = \left(1 + r^e{}_{t+1}\right) s_t + w^e{}_{t+1} L_{t+1} .$$

Therefore, the lifetime budget constraint of an individual born at t can easily by expressed as

$$C_{t+1} = (1-\tau) w_t \ell_t \left(1 + r^e{}_{t+1}\right) + w^e{}_{t+1} L_{t+1} . \tag{1}$$

Individuals have preferences towards material consumption and leisure when young and old. For simplicity, we assume the lifetime utility index of generation t (U_t) is described by the following logarithmic function:

$$U_t = \ln(2-\ell_t) + \beta[\ln(2-L_{t+1}) + \eta_t \ln(C_{t+1})], \tag{2}$$

where $0<\beta<1$ is the psychological subjective discount factor and $\eta_t > 0$ is an index that measures the elasticity of material consumption with respect to leisure time when old. It is assumed that such an index positively depends on the individual health status which is, in turn, augmented by the public provision of health investments, h_t.[3] In particular, this relationship is described by a non-decreasing function

$$\eta_t = \eta(h_t),$$

to capture the idea that the healthier an individual at the end of youth, the higher the demand for consumption goods when old, i.e. consumption at older ages becomes more attractive in such a case (see, e.g., Viscusi and Evans, 1990; Sloan et al., 1998; Domeij and Johannesson, 2006).

The individual born at t chooses ℓ_t and L_{t+1} to maximise Eq. (2) subject to Eq. (1), $0<\ell_t<2$ and $0<L_{t+1}<2$, by taking factor prices and the index η_t as given. Therefore, the young and old age labour supply are determined as:

$$\ell_t = \frac{2\beta(1+\eta_t)}{1+\beta(1+\eta_t)} - \frac{2}{(1-\tau)w_t} \cdot \frac{w^e{}_{t+1}}{1+r^e{}_{t+1}}, \tag{3}$$

$$L_{t+1} = \frac{2(1+\beta\eta_t)}{1+\beta(1+\eta_t)} - \frac{2\beta(1-\tau)w_t}{1+\beta(1+\eta_t)} \cdot \frac{1+r^e{}_{t+1}}{w^e{}_{t+1}}, \tag{4}$$

[3] In a one-dimensional OLG context with inelastic labour supply, Chakraborty (2004) analyses a model where the public provision of health services affects longevity, while Fanti and Gori (2011) consider the case of the relationship between public health spending and old-age efficient labour.

that is, the supply of labour when young positively depends on the current wage and negatively on the present value of the future wage, while the labour supply when old positively depends on the future wage and negatively on the capitalised current wage.

2.2. Government

The per capita government expenditure on health at t (h_t) is constrained by the following budget:

$$h_t = \tau w_t \ell_t , \tag{5}$$

where $0 < \tau < 1$ is tax rate levied on the young workers' wage (see Chakraborty, 2004; Bhattacharya and Qiao, 2007).

2.3. Firms

At time t identical competitive firms produce a homogeneous good, Y_t, by combining capital and labour, K_t and Z_t, respectively, through the constant returns to scale Cobb-Douglas technology

$$Y_t = AK_t^{\alpha} Z_t^{1-\alpha} ,$$

where $A > 0$ and $0 < \alpha < 1$. The supply of labour at t (which is equal to the demand in equilibrium) is

$$Z_t = \ell_t + L_t .$$

Assuming that capital fully depreciates at the end of each period and output is sold at unit price, profit maximisation implies that factor inputs are paid their marginal product, i.e.:

$$r_t = \alpha A k_t^{\alpha-1} - 1 , \tag{6}$$

$$w_t = (1-\alpha) A k_t^{\alpha} , \tag{7}$$

where $k_t := K_t / Z_t$ is capital *per efficient worker*.

2.4. Equilibrium dynamics

Given the government budget Eq. (5), market-clearing in goods and capital market is expressed by the equality between investments and savings, which can also be expressed as:

$$k_{t+1}(\ell_{t+1} + L_{t+1}) = (1-\tau)w_t \ell_t . \tag{8}$$

Taking into account the first order conditions for the agent's problem Eqs. (3) and (4), the competitive equilibrium conditions Eqs. (6) and (7), and the capital accumulation function Eq. (8), the two-dimensional system that describes the dynamics of the economy is given by the following equations:

$$k_{t+1} = \frac{1}{2}\alpha A k_t^{\alpha}(1-\tau)[\beta(1+\eta_t)(2-\ell_t) - \ell_t], \tag{9}$$

$$\ell_{t+1} = \frac{2[\ell_t - \alpha\eta_t(2-\ell_t)]}{\alpha[2(1-\ell_t) + \eta_t(2-\ell_t)]} . \tag{10}$$

In the rest of the paper, we assume, in particular, that the health technology η_t takes the form:

$$\eta_t = \eta(h_t) := \frac{1}{B}(\overline{\eta} + h_t)^{\sigma} , \tag{11}$$

where $\overline{\eta} > 0$ is the biological health status, i.e. in the absence of any health spending, $\sigma > 0$ measures the efficiency of public health investments as an inducement to better health and, hence, higher consumption, $B > 0$ is a scale parameter and $\eta'_h > 0$. If $0 < \sigma < 1$ ($\sigma > 1$), then Eq. (11) is a concave (convex) function. Therefore, raising health expenditure determines a less (more) than proportional increase in the individual health status and, hence, in the consumption possibilities when old. In the former case, $0 < \sigma < 1$, we may think about some medical advances in the treatment of diseases (that can, for instance, already be treated efficiently) helping to smoothly induce a healthy life. The latter, instead, $\sigma > 1$, can be viewed as the case of, e.g., some new programmes of vaccines or discoveries that, due to the accumulated knowledge, permits to treat efficiently some critical diseases, thus raising the wellness across population more than proportionally.

3. Dynamic analysis

3.1. Existence of stationary equilibria

The dynamic system characterised by Eqs. (9) and (10) defines the couple k_{t+1} and ℓ_{t+1} as functions of k_t and ℓ_t. In this section, we study the stability of fixed points of such discrete dynamical system. We use the geometrical-graphical method developed by Grandmont et al. (1998), that allows us to characterise the stability properties of the fixed points. We impose some conditions on the parameters A, B and η under which the "normalised" fixed point (ℓ^*, k^*), with $\ell^* = k^* = 1$, exists. This allows us to analyse the effects on stability due to changes in some parameter values while being sure that the fixed point does not disappear. Moreover, for the sake of simplicity, we set $\beta = 1$ without loss of generality.

Setting $k^* = k_{t+1} = k_t = 1$ and $\ell^* = \ell_{t+1} = \ell_t = 1$ in Eqs. (9) and (10), we obtain:

$$A = A^* := \frac{3}{1-\tau}, \quad B = B^* := \frac{3\alpha}{2}, \quad \overline{\eta} = \frac{1-\tau+3\tau(\alpha-1)}{1-\tau}, \tag{12}$$

with $\tau < \dfrac{1}{4-3\alpha}$.

We note that the model admits a multiplicity of fixed points. Then, the following proposition holds.

Proposition 1. *Consider the dynamic system described by Eqs. (9) and (10). It generically admits an odd number of fixed points.*

Proof. The fixed points of the system Eqs. (9) and (10) represent constant values of k and ℓ such that the following equations are fulfilled:

$$k = \frac{1}{2}\alpha A k^{\alpha}(1-\tau)\{\beta(1+\eta(\ell,k))(2-\ell)-\ell\}, \tag{13}$$

$$\ell = \frac{2[\ell-\alpha\eta(\ell,k)(2-\ell)]}{\alpha[2(1-\ell)+\eta(\ell,k)(2-\ell)]}. \tag{14}$$

where $\eta(\ell,k)=\frac{1}{B^{*}}\left[\overline{\eta}+\tau(1-\alpha)Ak^{\alpha}\ell\right]^{\sigma}$ is Eq. (11) at the steady state under the conditions stated in (12). Solving Eq. (14) for k and replacing it in Eq. (13) we may conclude, after some algebra, that the steady-state values of ℓ are represented by the intersection points between the function

$$q = g(\ell) := \frac{3^{\frac{1}{\alpha}}[2\alpha(1-\ell)+\ell]\cdot[\ell\tau(1-\alpha)]^{\frac{1-\alpha}{\alpha}}}{\left\{(1-\tau)\left[\frac{3\ell(1-\alpha+\ell\alpha)}{(2-\ell)(2+\ell)}\right]^{\frac{1}{\sigma}}-(1-4\tau+3\alpha\tau)\right\}^{\frac{1-\alpha}{\alpha}}(2+\ell)}. \tag{14}$$

and the horizontal line with ordinate 1. Proposition 1 therefore follows as an asymptote at $\ell=\overline{\ell}$, $0<\overline{\ell}<1$, exists such that $\lim_{\ell\to\overline{\ell}} g(\ell)=+\infty$, $\lim_{\ell\to 2} g(\ell)=0$ and, hence, at least one equilibrium exists after the normalisation. **Q.E.D.**

Even if it cannot be proved analytically, several numerical experiments have revealed that the stationary equilibria of our model are either one or three. We denote the (possible) second and third steady states as (ℓ^{**},k^{**}) and (ℓ^{***},k^{***}), where $\ell^{**}>\ell^{***}$.

3.2. Local dynamics and indeterminacy

In this model, physical capital K_t represents a state variable, so its initial value K_0 is given. Different from k_t, the variables ℓ_t and L_t are "jumping" variables, as they represent the labour input of the typical agent when young and old, respectively, and they are chosen by taking into account the key parameters of the model, the public health spending and the expectations on future factor prices. As a consequence, individuals choose the initial value of ℓ_0 and L_1 (and thus the initial value[4] of capital per efficient worker). If the normalised fixed point is a saddle and K_0 is close enough to 1, then there exists a unique initial value of ℓ_t, namely ℓ_0, such that the orbit that passes through (ℓ_0, k_0) approaches the fixed point. When the fixed point is a sink, given the initial value K_0, then there exists a continuum of initial values ℓ_0 such that the orbit that passes through (ℓ_0, k_0) approaches the fixed point; as a consequence, the orbit the economy will follow is "indeterminate"[5] as it depends on the choice on ℓ_0.

The Jacobian matrix of (9) and (10), evaluated at the normalised fixed point (ℓ^*, k^*), is:

$$J(\ell^*, k^*) = \begin{pmatrix} \alpha + \dfrac{3\sigma\tau(1-\alpha)}{1-\tau} & \dfrac{3\sigma\tau(1-\alpha)}{1-\tau} - (3\alpha+1) \\ \dfrac{9\sigma\alpha\tau(\alpha-1)}{1-\tau} & 3(2+\alpha) + \dfrac{9\sigma\tau(\alpha-1)}{1-\tau} \end{pmatrix},$$

with

$$Det(J^*) = 3\alpha(2+\alpha) - \frac{18\alpha^2\sigma\tau(1-\alpha)}{1-\tau}, \tag{15}$$

$$Tr(J^*) = 4\alpha + 6 - \frac{3\sigma\tau(1-\alpha)(3-\alpha)}{1-\tau}. \tag{16}$$

Note that, changing (ceteris paribus) the parameter σ, the point (see Eqs. 15 and 16):

$$(P_1, P_2) := \left(4\alpha + 6 - \frac{3\sigma\tau(1-\alpha)(3-\alpha)}{1-\tau}, 3\alpha(2+\alpha) - \frac{18\alpha^2\sigma\tau(1-\alpha)}{1-\tau} \right),$$

describes a half-line T_1 with slope $\dfrac{6\alpha^2}{3-\alpha} \in (0,3)$ (see Figure 1) in the plane $(Tr(J), Det(J))$, starting from the point (obtained when $\sigma = 0$ in Eqs. 15 and 16):

[4] Notice that form Eqs (4), (5) and (11) it follows that the dynamics of L_t is completely determined by the dynamics of ℓ_t and k_t. Indeed, in the following analysis we concentrate on the study of the system in the variables ℓ_t and k_t.

[5] See Cazzavillan (2001).

$$\left(\overline{P}_1,\overline{P}_2\right):=\left(4\alpha+6, 3\alpha(2+\alpha)\right). \tag{17}$$

The point $\left(\overline{P}_1,\overline{P}_2\right)$, when α varies, describes a curve T_2 starting from the point $(6{,}0)$, in the plane $(Tr(J), Det(J))$. Notice that $\left(\overline{P}_1,\overline{P}_2\right)\to(10{,}9)$ for $\alpha\to 1$ and $(P_1,P_2)\to(-\infty,-\infty)$ for $\sigma\to+\infty$.

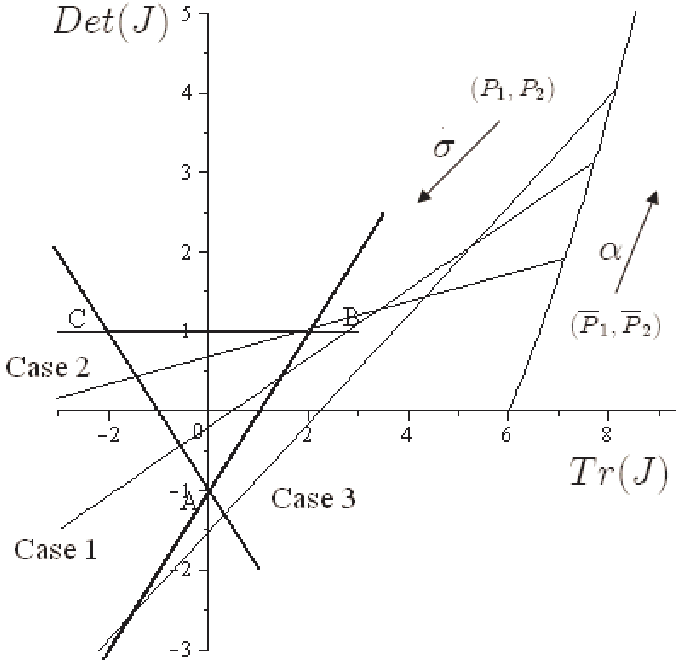

Figure 1. The half-line T_1 and the curve T_2 in the plane (T,D).

Using simple geometrical considerations, the following proposition holds (see Figure 1).

Proposition 2. *Consider the normalised steady state $\left(\ell^*,k^*\right)$. Under the hypotheses (12), there exist*

$$\sigma_{tc}=\frac{(1-\tau)(3\alpha+5)}{3\tau\left(3-\alpha-6\alpha^2\right)},\quad \sigma_{fl}=\frac{(3\alpha+7)(1+\alpha)(1-\tau)}{3\tau(1-\alpha)\left(6\alpha^2-\alpha+3\right)},\quad \sigma_{ns}=\frac{\left(3\alpha^2+6\alpha-1\right)(1-\tau)}{18\alpha^2\tau(1-\alpha)}\quad \textit{and}\quad \alpha^*\cong 0.4965$$

such that the following results generically hold:

1. If $\alpha\in\left(0,\dfrac{-5+2\sqrt{13}}{9}\right)$ or $\alpha\in\left(\dfrac{1}{3},\alpha^*\right)$, then the starting point $\left(\overline{P}_1,\overline{P}_2\right)$ of the half-line T_1 lies in the region "saddle" and consequently the normalised fixed point $\left(\ell^*,k^*\right)$ is saddle-point stable;

when σ rises, the point (P_1, P_2) moves along T_1 and (ℓ^*, k^*) becomes a sink via a transcritical bifurcation (occurring for $\sigma = \sigma_{tc}$) that gives rise to a change in the stability properties between the normalised steady state and (ℓ^{**}, k^{**}). If σ continues to increase, then the point (P_1, P_2) leaves the region "sink" and enters the region "saddle" giving rise to a supercritical flip bifurcation (occurring for $\sigma = \sigma_{fl}$) which generates a periodic orbit of period 2; increasing σ further on leads to chaotic behaviour via period doubling bifurcations.

2. If $\alpha \in \left(\frac{-5 + 2\sqrt{13}}{9}, \frac{1}{3} \right)$, then the starting point $(\overline{P}_1, \overline{P}_2)$ of the half-line T_1 lies in the region "saddle" and consequently the normalised fixed point (ℓ^*, k^*) is saddle-point stable; when σ rises, the point (P_1, P_2) moves along T_1 and (ℓ^*, k^*) becomes first a source (via a transcritical bifurcation) and successively a sink (via a subcritical Neimark-Sacker bifurcation). If σ continues to increase, then the point (P_1, P_2) leaves the region "sink" and enters the region "saddle" giving rise to a supercritical flip bifurcation (occurring for $\sigma = \sigma_{fl}$) which generates a periodic orbit of period 2; further increases in σ leads to chaotic behaviour via period doubling bifurcations, as Figure 2.b shows.

3. If $\alpha \in (\alpha^*, 1)$, then the starting point $(\overline{P}_1, \overline{P}_2)$ of the half-line T_1 lies in the region "saddle" and consequently the normalised fixed point (ℓ^*, k^*) is saddle-point stable; when σ rises, the point (P_1, P_2) moves along T_1 and (ℓ^*, k^*) becomes a source via a transcritical bifurcation (occurring for $\sigma = \sigma_{tc}$). This gives rise to a change in the stability properties between the normalised steady state and (ℓ^{**}, k^{**}). If σ continues to increase, then the point becomes again a saddle.

The results on the stability of cycles as well as those of the invariant closed curves given in Proposition 2, are obtained through several numerical experiments (not reported here for economy of space). We also note that the health tax rate τ does non discriminate between Cases 1-3 of Proposition 2, while sensibly affecting the bifurcation values of σ.

3.3. Global analysis: some numerical exercises

In this section we show that a rise in the parameter σ, which measures the elasticity of material consumption with respect to leisure when old, may generate global indeterminacy and chaos. We recall that global indeterminacy[6] occurs when starting from the same initial value K_0 of the state variable K, different fixed points or other ω-limit sets can be reached according to the initial choice ℓ_0 of the jumping variable ℓ. The scenario of global indeterminacy becomes more complex if one of the reachable ω-limit set is a chaotic attractor. In such a case, the long-run behaviour of the orbits that approach the same (chaotic) attractor depends on ℓ_0 as well.

We obtain some insights about the dynamics of our economy by means of some numerical simulations. In first experiment (which, with respect to the local analysis of the previous section, refers to Case 2 of Figure 1), we set $\alpha = 0.3$ and $\tau = 0.2$. The choice on the value of the output elasticity of capital α is in line with empirical estimates that refer to developed countries (see, e.g., Gollin, 2002; Kraay and Raddatz, 2007), while as regards the value of the health tax rate τ, it generates an health expenditure over the GDP (per efficient worker) ratio of about 14 per cent (which is in line with data on health spending in countries such as France and Germany, where the public, as part of general, health spending is fairly high, and by taking the government health care expenditure as a proxy of total expenditure on health, see, e.g., Hartwig, 2008; World Health Statistics, 2010).

For fairly low values of σ, a unique (saddle) fixed point exists. An increase in σ gives birth to two fixed points ($\sigma \cong 3.44$) far from the normalized one: one of which, (ℓ^{***}, k^{***}), is a saddle, the

[6] See Antoci et al. (2010).

other one, (ℓ^{**}, k^{**}), is an attractor. During this phase, the normalised fixed point (ℓ^{*}, k^{*}) continues to be a saddle. For $\sigma \cong 3.59$ the intermediate fixed point (ℓ^{**}, k^{**}) loses stability through a subcritical Neimark-Sacker bifurcation, and no attractor exists for $\sigma \in (3.59, 3.77)$. Note that, within this interval of the parameter σ, the normalised fixed point undergoes a transcritical bifurcation when $\sigma = 3.6419$ and then becomes a source. With a small increase of σ ($\sigma = 3.7742$), the normalised fixed point undergoes a subcritical Neimark-Sacker bifurcation and an unique attractor exists. If we let σ increase further on, the basin of attraction undergoes important changes and several holes appear. Moreover, a flip bifurcation is obtained when $\sigma = 6.0376$, which gives rise to the birth of attractors with increasing period and, hence, chaotic, around the normalised steady state (see Figure 3.b). When $\sigma > 6.53$, only saddles and repellors exist (as can be ascertained from Figure 2.b).

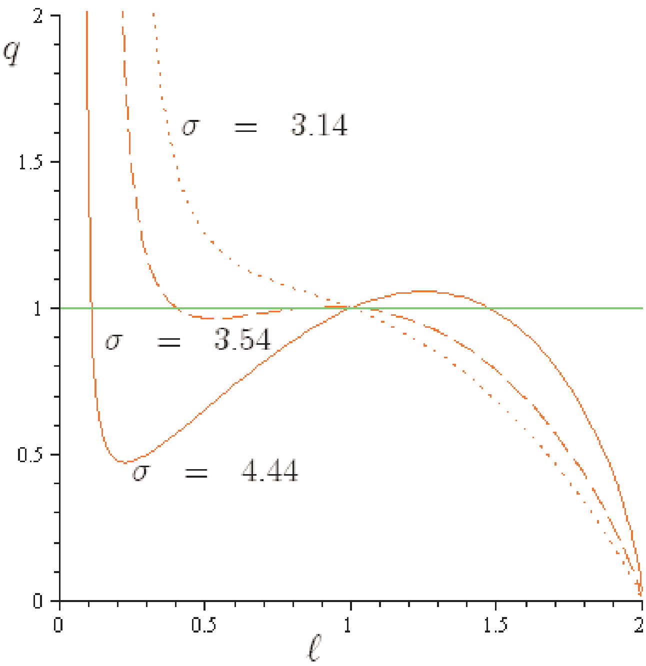

Figure 2.a. Existence of stationary states.

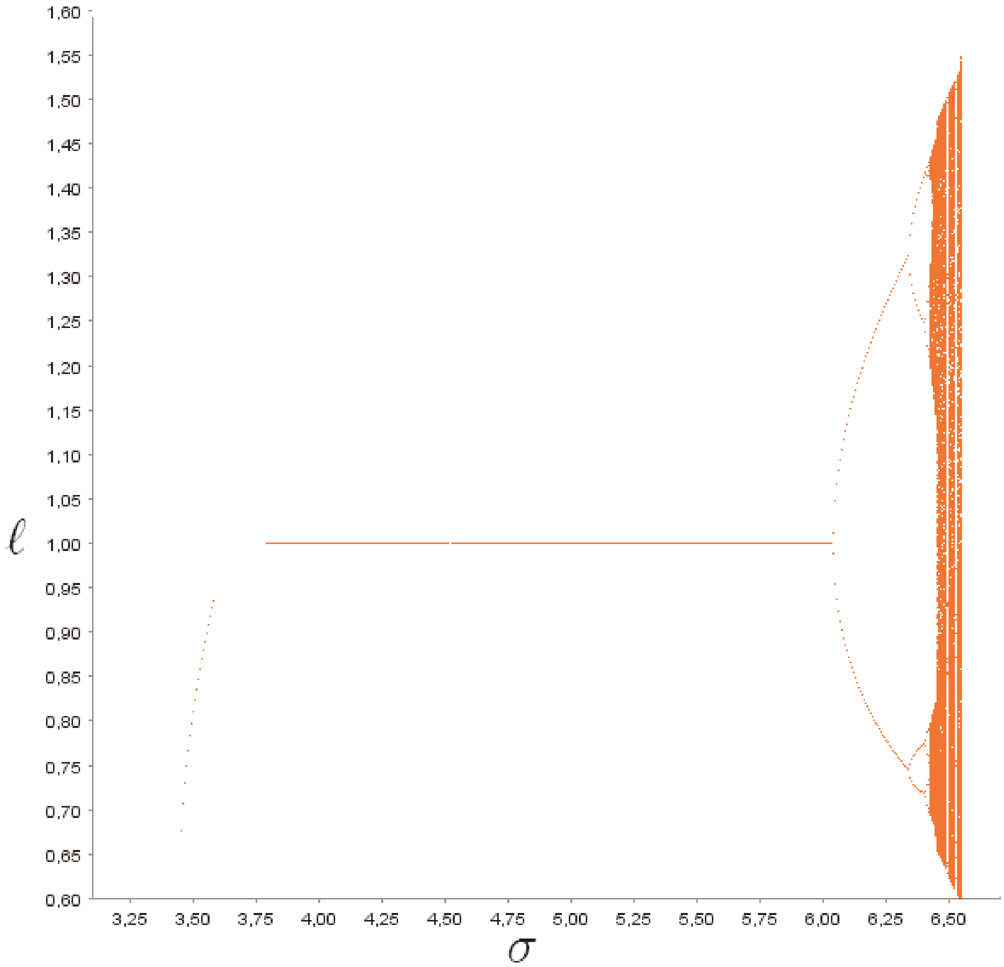

Figure 2.b. Bifurcation diagram for σ ($\alpha = 0.3, \tau = 0.2$, $\ell_0 = 1.01$ and $k_0 = 1.01$).

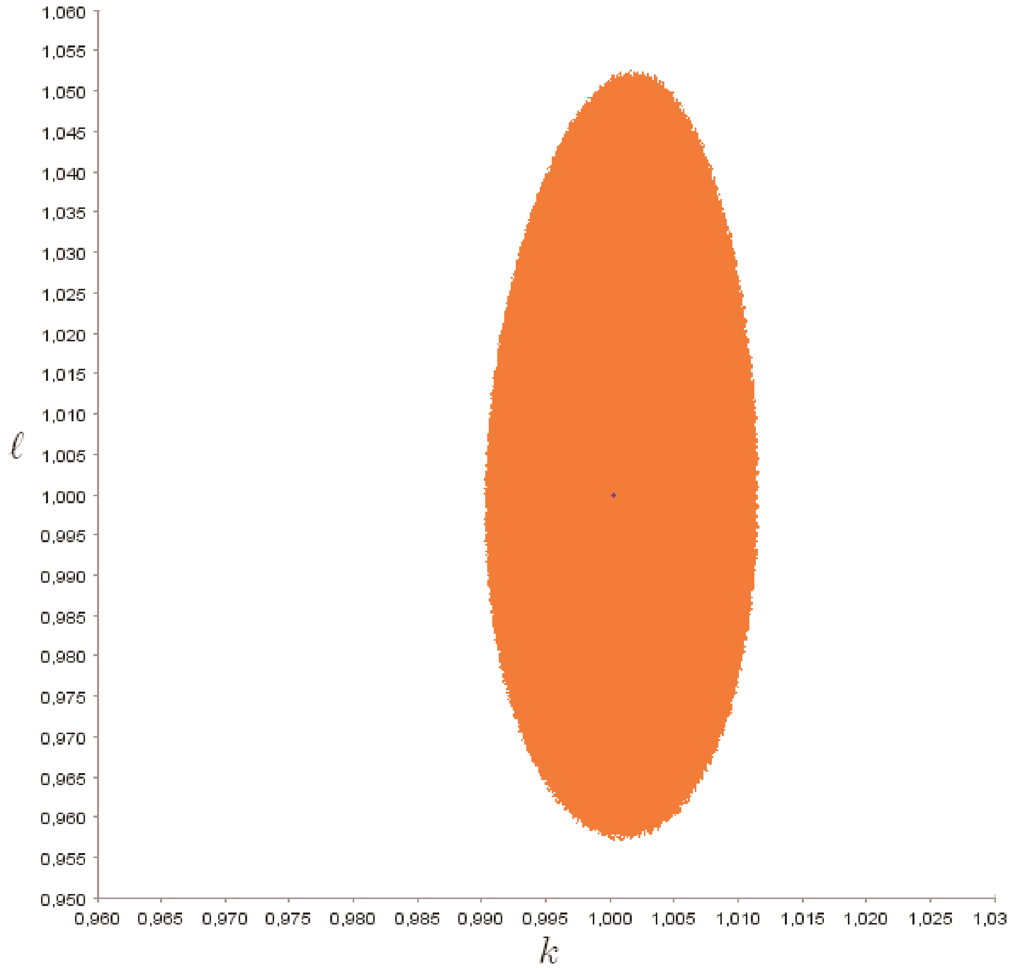

Figure 3.a. Subcritical Neimark-Sacker bifurcation: an invariant curve which defines the basin of attraction of the normalised steady state ($\alpha = 0.3$, $\tau = 0.2$ and $\sigma = 6.0376$).

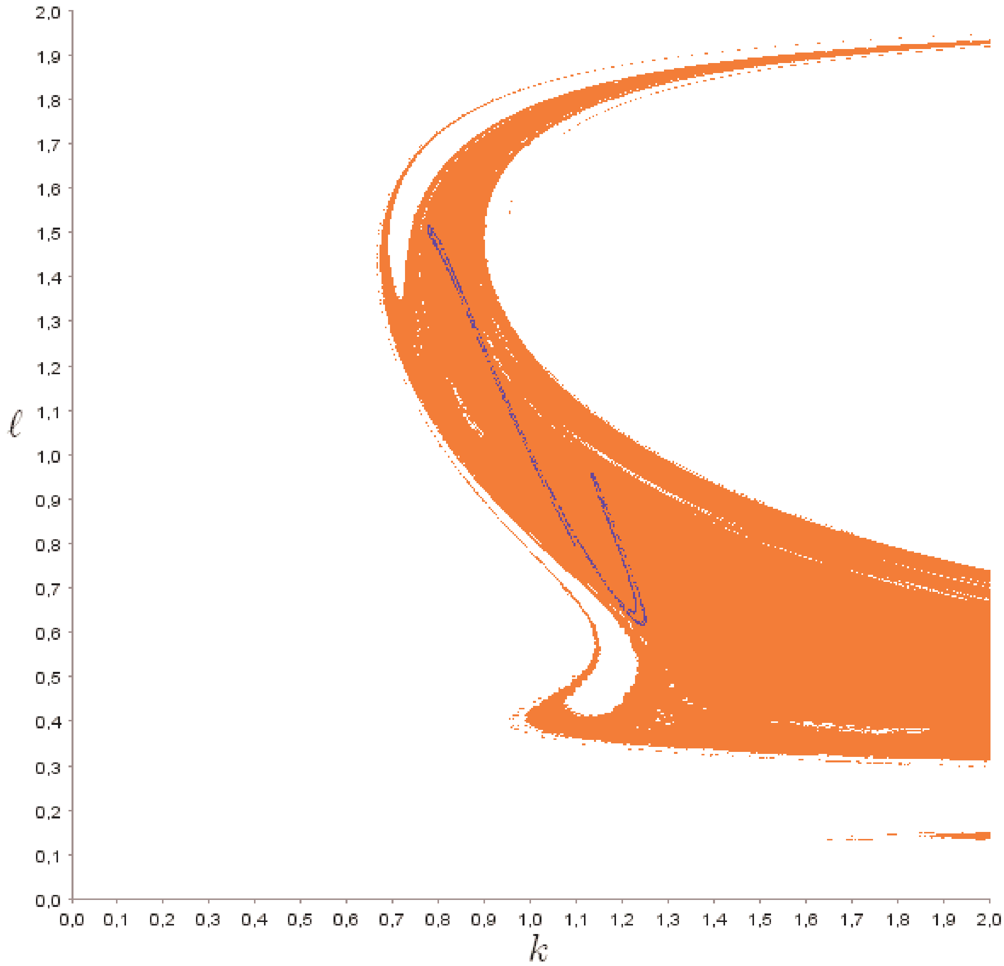

Figure 3.b. Multiply connected basins of attraction (see Mira et al., 1996) of the chaotic attractor just before its disappearance ($\alpha = 0.3, \tau = 0.2$ and $\sigma = 6.52$).

We now change the output elasticity of capital ($\alpha = 0.35$), while keeping the health tax rate τ unchanged (from the point of view of the local analysis of previous section, we now refer to Case 1 of Figure 1). Note that the health expenditure over the GPD ratio slightly reduces to 13 per cent. A small rise in the parameter α causes some important changes in the global properties of the map which indeed deserve attention. Focusing on the case in which three fixed point exist, we note that (*i*) for $\sigma = 4.1$ the basin of attraction of (ℓ^{**}, k^{**}) is bounded by the stable manifolds of the saddles (ℓ^{***}, k^{***}), (ℓ^{*}, k^{*}), the last one is the normalised steady state, as Figure 4.a shows; (*ii*) a global bifurcation occurs at $\sigma = 4.15$. In this case we note a sharp change in the basin of attraction (see

Figure 4.b): indeed its boundary now only involves the stable manifold of the normalised steady state; (*iii*) for higher values of the parameter σ different attractors co-exist (see Figures 5). The bifurcation diagram portrayed in Figure 5.a shows several discontinuities due to the existence of tangled basins of attractions: i.e. the same initial condition is captured by different attractors when σ varies (see Figure 5.b).

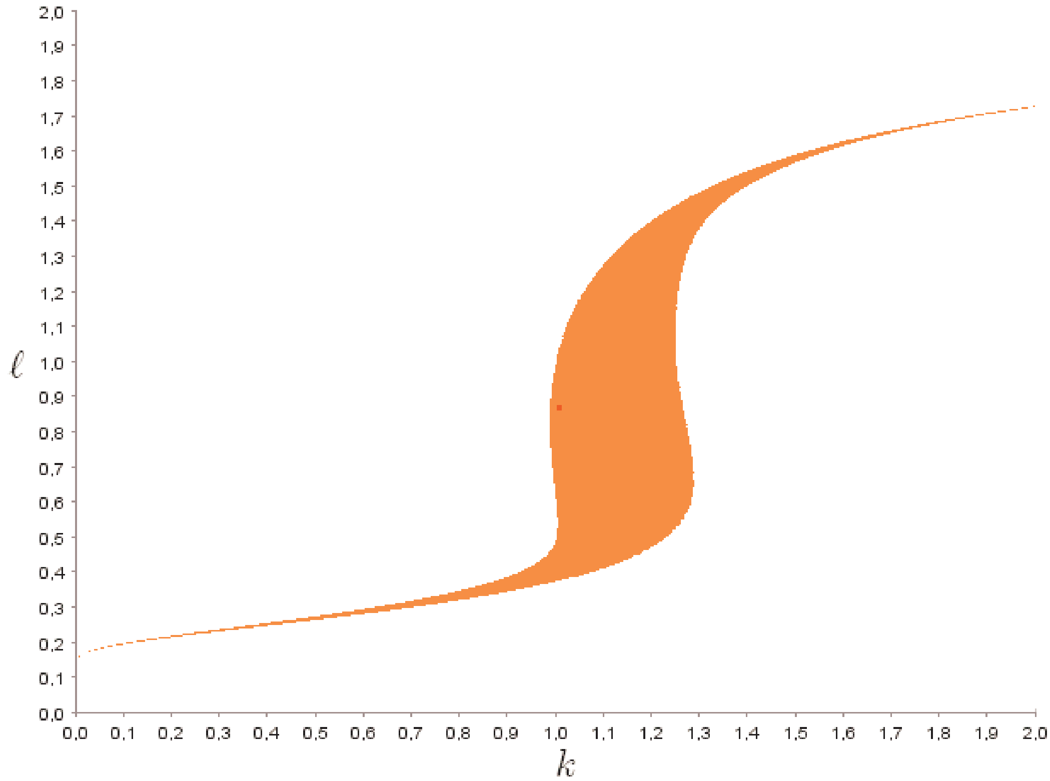

Figure 4.a. Basin of attraction of $\left(\ell^{**},k^{**}\right)$ for $\sigma=4.1$ ($\alpha=0.35$ and $\tau=0.2$).

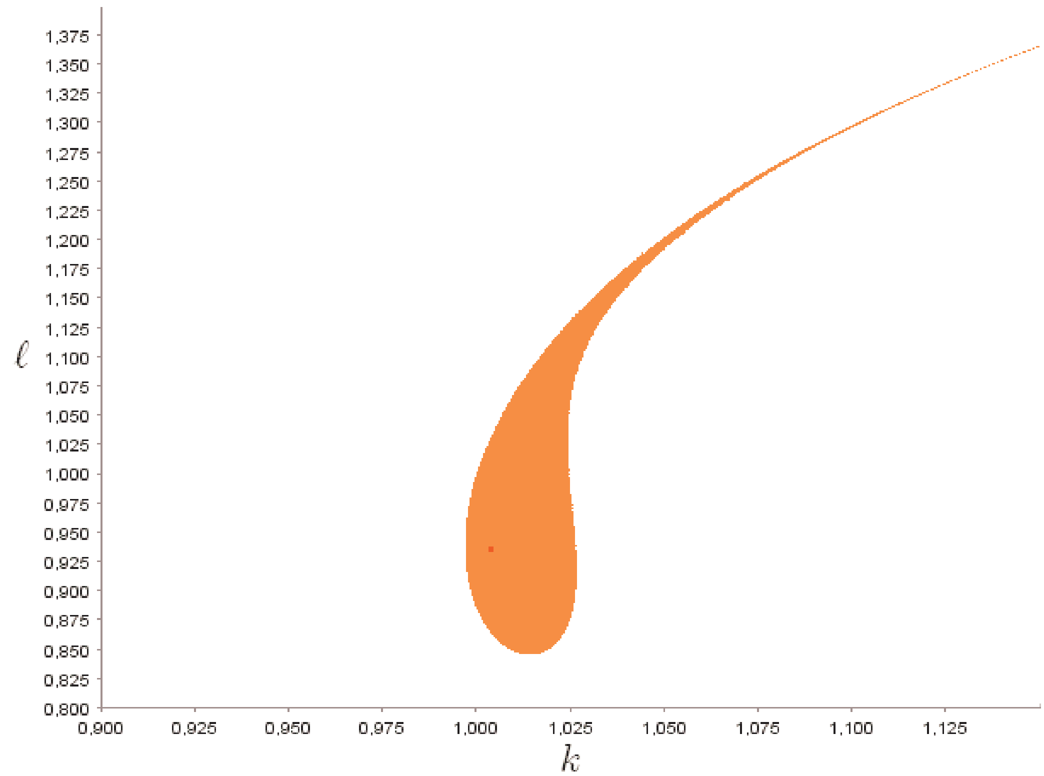

Figure 4.b. Basin of attraction of (ℓ^{**}, k^{**}) for $\sigma = 4.15$ ($\alpha = 0.35$ and $\tau = 0.2$).

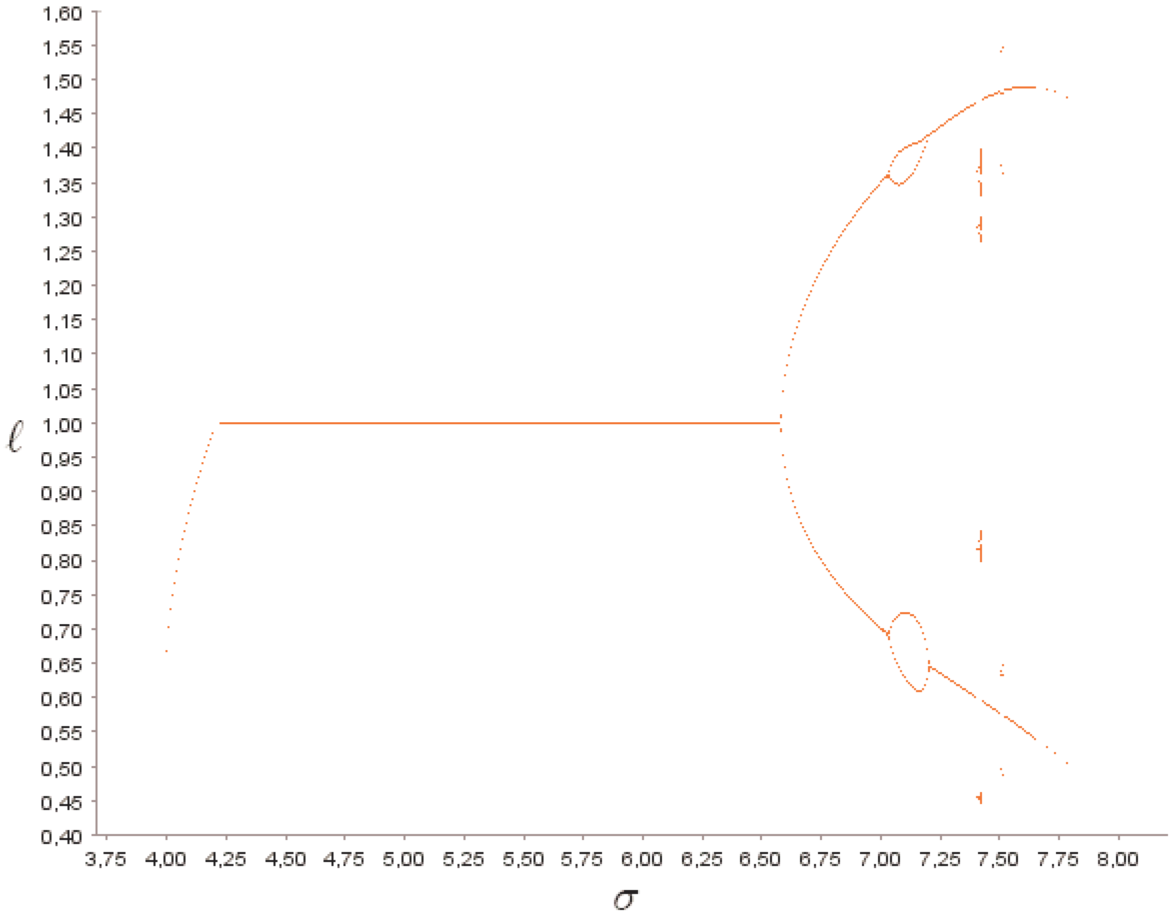

Figure 5.a. Bifurcation diagram for σ ($\alpha = 0.35$, $\tau = 0.2$, $\ell_0 = 0.99$ and $k_0 = 1.01$).

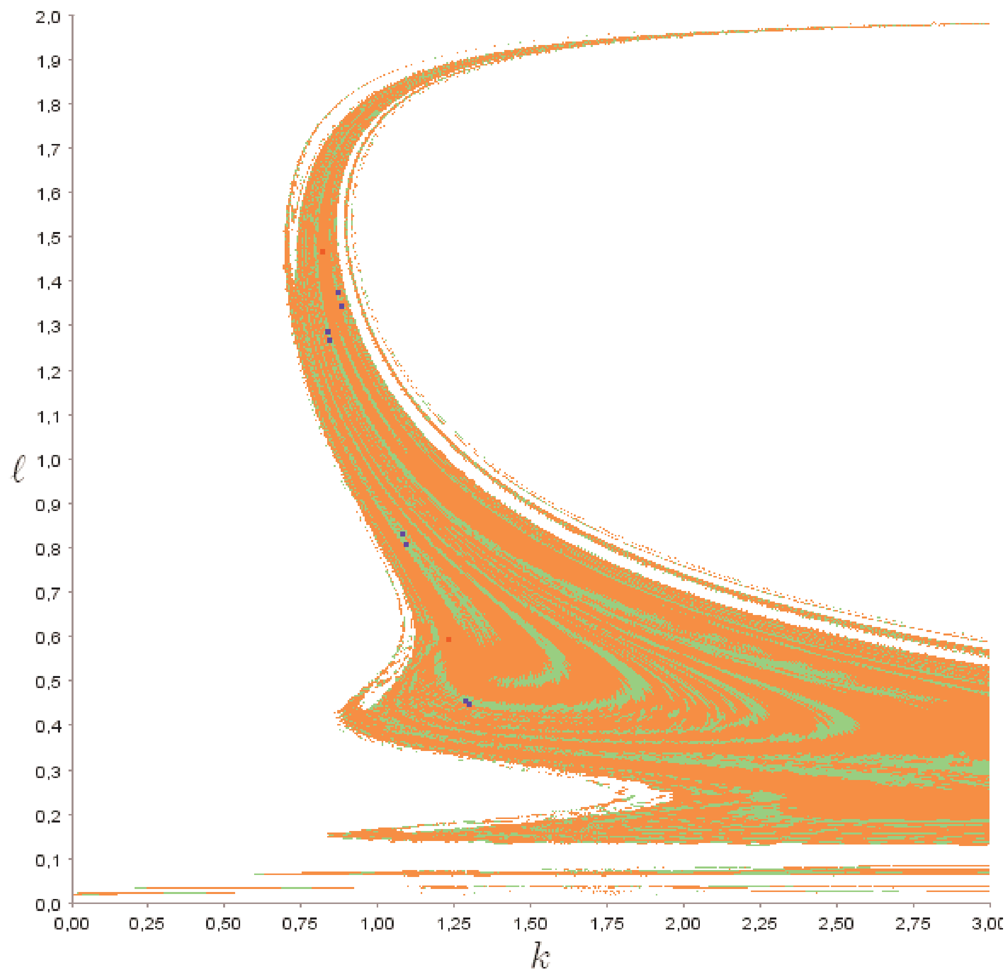

Figure 5.b. Bistable regime ($\alpha = 0.35$, $\tau = 0.2$ and $\sigma = 7.53$).

4. Conclusions

We studied the dynamical properties of an overlapping generations model with both young- and old-age labour supply, and where the public provision of health services positively affects the individual health status which, in turn, increases the consumption of goods and services in the second period of life. We found that the financing of public health care services can generate multiplicity of equilibria as well as complex dynamics. In particular, given the size of government expenditure, a small change in the efficiency of the health technology as an inducement to higher consumption may or may not be responsible to dramatic changes in the long-run outcomes of the economy, because the dynamics can be captured by a different attractor and, hence, a sort of indeterminacy of the government intervention can be found out.

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