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**Abstract:**

We construct one triple-threshold GARCH model to analyze the asymmetric response of mean and conditional volatility. In parameter estimation, we apply Griddy-Gibbs sampling method, which require less work in selection of starting values and pre-run. As we apply this model in Chinese stock market, we find that 12-days-average return plays an important role in defining different regimes. While the down regime is characterized by negative 12-days-average return, the up regime has positive 12-days-average return. The conditional mean responds differently between down and up regime. In down regime, the return at date *t* is affected negatively by lag 2 negative return, while in up regime the return responds significantly to both positive and negative lag 1 past return. Moreover, our model shows that volatility reacts asymmetrically to positive and negative innovations, and this asymmetric reaction varies between down and up regimes. In down regime, volatility becomes more volatile when negative innovation impacts the market than when positive one does, while in up regime positive innovation leads to more volatile market than negative one.



Keywords: Threshold; Griddy-Gibbs sampling; MCMC method; GARCH

## 1. Introduction

GARCH models are widely accepted in modeling changing volatility in financial time series. One limitation of GARCH models is the implied symmetric effect of positive and negative innovation on conditional volatility. The GJR model of Glosten, Jagannathan, and Runkle (1993) introduced asymmetric GARCH, and Zakoian (1994) considered one similar model - TGARCH model, in which conditional variance was replaced by conditional standard deviation in volatility equation. Yet, these two asymmetric models are limited in that the threshold variables are assumed to be past innovations, and that the threshold values are set to be 0.

Li and Lam (1995) applied the SETAR model of Tong (1978, 1983) to model the daily return series of the Hong Kong Hang Seng index. The GARCH parameters in their model were estimated using maximum likelihood method, and the delay parameters via AIC criterion. Li and Li (1996) proposed one double-threshold ARCH models, and Brooks (2001) extended to double-threshold GARCH model to capture asymmetry in both mean and volatility simultaneously.

Chen et. al. (2003) employed one similar model to study the asymmetric reaction of both stock returns and volatility in six major stock markets, in response to good and bad news from the US market. Yang and Chang (2008) explored asymmetric response of stock return to foreign exchange market using a double-threshold GARCH. And Hwang et. al. (2010) found evidence of "explosive volatility" in Korean foreign exchange market with a threshold-asymmetric GARCH model.

The above mentioned double-threshold GARCH model differs in the choice of threshold variable. While Chen et. al. (2003) takes the past return of US market as threshold variable, Yang and Chang (2008) proposes past return in foreign exchange market, and Chen and So (2006) suggests a weighted average of auxiliary variables to be the threshold variable.

In Chinese stock market, moving average trading strategies are widely implemented in practice. Thus, we have reason to believe that moving average return will have a significant impact on regime switching. Based on this argument, we propose a new threshold variable $z_t$ :

$$z_t = \frac{1}{m}\sum_{i=1}^{m} y_{t-i} \ .$$

In the above equation, $y_i$ is the stock market return at time *t*. Similar as Chen and So (2006), this threshold variable is only partially determined, for lag parameter *m* should be estimated in statistic inference. In special case, if $m = 1$, then $z_t = y_{t-1}$. So our definition of threshold variable incorporates the trivial case in which the lag one past return are used to be the threshold variable.

With the goal of understanding more asymmetric reaction in Chinese stock market, we construct one triple-threshold GARCH model. Besides the threshold variable defined above, we also let the conditional mean react differently to past return, and the conditional volatility to past innovations.

As to parameter estimation, we propose Griddy-Gibbs sampling, one of the Markov Chain Monte Carlo methods. In classical methods, the thresholds in TAR are determined via grid search, and the threshold variables through AIC criterions, which are widely assumed to be imprecise.

MCMC methods provide one way to estimate normal parameters, thresholds and delay parameters simultaneously (see, e.g., Geweke and Terui (1993), Chen and Lee (1995), Phann et. al. (1996), etc.).

Chen and So (2006) used a Metropolis-Hasting algorithm for parameter estimation in threshold GARCH model. Goldman and Agbeyegbe (2007) introduced an efficient jump Metropolis-Hasting sampling method, which was claimed to be better at controlling the acceptance rate, thus resulting enhanced sampling efficiency. Griddy-Gibbs method may require more time in sampling process, but it save much time in predetermining proposed mean and standard deviations of parameters, which is indispensable with Metropolis-Hasting method. Considering that improper proposal distribution in M-H method may take long time to converge, sometimes even don't converge at all, the Griddy-Gibbs sampling has its own advantage in that parameter convergence is less sensitive to initial parameters. Thus, Griddy-Gibbs sampling do not need much pre-run work before sampling process, and this save much calculation time, although the parameter simulation process may take longer time than M-H sampling.

The structure of the paper is as follows. Section 2 defines the triple-threshold GARCH which aims to analyze the asymmetric reaction in conditional mean and volatility. Section 3 shows the Griddy-Gibbs sampling method for parameter estimation. Application of the model to Chinese stock market is presented in section 4. Section 5 concludes.

## 2. A triple-threshold GARCH model

Our triple-threshold GARCH model is defined as follows:

$$y_t = \mu^{(j)} + \sum_{i=1}^{P} \phi_i^{(j)} \cdot y_{t-i} \cdot I(y_{t-i} \geq 0) + \sum_{l=1}^{P} \varphi_l^{(j)} \cdot y_{t-l} \cdot I(y_{t-l} < 0) + \varepsilon_t \text{, if } r_{j-1} \leq z_t < r_j$$

$$\varepsilon_t = \sqrt{h_t^{(j)}} \cdot e_t^{(j)},\ \ h_t^{(j)} = \omega^{(j)} + \alpha^{(j)} \varepsilon_{t-1}^2 \cdot I(\varepsilon_{t-1} \geq 0) + \theta^{(j)} \varepsilon_{t-1}^2 \cdot I(\varepsilon_{t-1} < 0) + \beta^{(j)} h_{t-1}^{(j)} \qquad (1)$$

where $z_t = \dfrac{1}{m}\sum_{i=1}^{m} y_{t-i}$ , $j = 1,..., J$ , $m$ is a positive integer which determines the threshold varia

-ble, and { $e_t$ } is a sequence of iid random variables, which are assumed to follow standardized

Student-*t* distribution with degree of freedom parameter $v^{(j)}$ . $I$ is an indicator function. The

threshold values $r_j$ satisfy $-\infty < r_1 < r_2 < ... < r_{J-1} < +\infty$ , $r_0 = -\infty$ and $r_J = +\infty$ .

The main threshold variable in our model is a moving average process of the past returns. This definition is actually one special case in Chen and So (2006). Yet we argument that this is suitable for analyzing the Chinese stock market, because moving average trading strategies are frequently used in investment practice in China, and the optimal trade timing of these strategies are easily demonstrated using plots which can be understood by most of the people in China. The popularity of moving average trading strategies leads to its success itself. According to the fact that the moving average trading strategies are intensively implemented in Chinese stock market, we have reason to believe that moving average return may have a great impact on the market, thus our definition of threshold variable is appropriate.

With threshold variable $Z_t$ and threshold $\{r_j\}$, $J$ regimes in the model is defined. Within each regime, we implement two additional threshold effects in mean equation and volatility equation, respectively. In mean equation, we impose one asymmetric reaction of mean in response to positive or negative past return in each regime. And in volatility equation, one similar asymmetry as GJR model is imposed.

Thus our model incorporates totally three threshold processes: one is defined by threshold variable $z_t$, which separate the return series into $J$ different regimes; the other defined by residual $\varepsilon_t$, which incorporate the asymmetric reaction of volatility to positive or negative shocks in each regime; the third threshold process is demonstrated by past return, as the mean equation shows.

Among these three threshold processes, the first one is dominant, for the regimes in equation 1 are solely determined by threshold variable $z_t$. The other two threshold processes present asymmetry in conditional mean and volatility in correspondent regime respectively.

As to parameter constraints, we restrict AR parameters by

$$\sum_{i=1}^{P}\phi_i^{(j)}+\sum_{l=1}^{P}\varphi_l^{(j)}<1.$$

Constraints on parameters of volatility equation are as follows.

$$0<\omega^{(j)},\alpha^{(j)},\theta^{(j)},\beta^{(j)}<1.$$

These constraints do not explicitly require the addition of ARCH and GARCH parameters to be less than 1, thus the stationarity of volatility is not guaranteed, and it's possible that some regimes may show explosive volatility process. Yet the regimes switch themselves, and this mechanism has a counter effect on possible explosive volatility, leading to stationarity in the model as a whole. Markov Chain Monte Carlo simulation has one explicit advantage that we can get correct parameter estimate even if we do not impose the stationary restriction before sampling, only we need to check whether the resulted parameters satisfy the normal stationary restriction of GARCH model. On the contrary, imposing stationarity constraints in sampling process may miss some interesting feature in empirical application, as our analysis in Chinese stock market show.

Bauwens et. al. (2010) provides stationarity condition for Markov Switching GARCH (1,1) mode, which states

$$\sum_{j=1}^{J}\pi_j E[\log(\alpha^{(j)}\varepsilon_t^2+\beta^{(j)})]<0 \qquad (2)$$

where $\pi_j$ is the probability of regime $j$. Of course, this stationary constraint can be implemented in triple-threshold GARCH model, if $\pi_j$ of regime $j$ is calculated using its duration over time.

## 3. Griddy-Gibbs sampling of the model

Unlike the existing literature, we apply Griddy-Gibbs sampling in estimating threshold

GARCH model. Comparing the other methods, such as Metropolis-Hasting algorithm, our method is relatively easy to handle, in that it do not need specified values in sampling process. Although Griddy-Gibbs sampling need to specify the maximum and minimum values of grid interval, it's relative easy to set, and it's easy to check whether these values are specified correctly by plotting posterior distributions using grid points.

The $P$ value is determined according to Bayesian information criterion. The threshold value $r$, is so constrained that the observations in each regime exceed one minimum number, which is normally set to be 5% of the number of total observations.

We describe the sampling process for the case of two regimes. For high number of regimes, the process is similar. The parameters we estimate include: (1) parameters in mean equation $\mu$ , $\phi$ and $\varphi$; (2) parameters in volatility equation $\omega$ , $\alpha$ , $\theta$ and $\beta$; (3) threshold value $r$; (4) threshold parameter $m$; (5) degree of freedom $v$.

Given threshold variable $\{z_t\}$ and threshold value $r$, the return series $\{y_t\}$ is split into different regimes, which we denote $\{y_\tau^{(j)}\}$. The posterior distribution in regime $j$ is given by

$$p(y\,|\,\mu,\phi,\varphi,\omega,\alpha,\theta,\beta,r,m) = p(\mu,\phi,\varphi,\omega,\alpha,\theta,\beta,r,m)\cdot\prod_{t=P+1}^{T_j}[\frac{1}{\sqrt{2\pi h_t^{(j)}}}\exp(\frac{(y_t^{(j)}-\mu^{(j)})^2}{h_t^{(j)}})]$$

where $p$ denotes the prior distribution, and $T_j$ is the maximum number of observations in regime j. For simplicity, we assume a uniform prior for all parameters, thus the prior distribution can be ignored in sampling process. The Griddy-Gibbs sampling process is as follows.

(1) Chose the initial values for parameters, which are denoted by $\mu^{(0,j)},\phi^{(0,j)},\varphi^{(0,j)},\omega^{(0,j)}$ $\alpha^{(0,j)},\theta^{(0,j)},\beta^{(0,j)},r^{(0)},m^{(0)}$ , $j=1,2$ . For parameters in mean and volatility equations are estimated in the same way by applying Griddy-Gibbs sampling. Thus we denote

$$\Theta^{(0,j)}=\{\mu^{(0,j)},\phi^{(0,j)},\varphi^{(0,j)},\omega^{(0,j)},\alpha^{(0,j)},\theta^{(0,j)},\beta^{(0,j)}\}$$

for simplicity in written. Given $r^{(0)},m^{(0)}$, the return series $\{y_t\}$ is separated into 2 regimes. Then each parameter in $\Theta^{(1,j)}$, is Griddy-Gibbs sampled using $\{y_t^{(0,j)}\}$, $j=1,2$ one after another. The sampling of parameter in different regime proceeds separately, because the posterior distribution in regime $j$ is solely determined by $\Theta^{(1,j)}$.

Given $r^{(i)},m^{(i)}$, the $i$-th Griddy-Gibbs sampling of $\Theta^{(i,j)}$ proceeds in the same way. Of course, the parameters within $\Theta^{(i,j)}$ can be updated, when another parameter in $\Theta^{(i,j)}$ is sampled.

(2) Threshold value sampling: The posterior distribution for threshold value sampling is as

follows.

$$p(y \mid \mu,\phi,\varphi,\omega,\alpha,\theta,\beta,r,m)=\prod_{j=1}^{2}\prod_{t=P+1}^{T_j}\left[\frac{1}{\sqrt{2\pi h_t^{(j)}}}\exp\left(\frac{(y_t^{(j)}-\mu^{(j)})^2}{h_t^{(j)}}\right)\right]$$

$$\text{If } z_t \le r\text{, } j\text{=1; and if } z_t > r\text{, } j\text{=2} \tag{3}$$

Grid interval can be subject to $[min(z_t), max(z_t)]$, with additional constraint that $T_j > 5\% \cdot T$. In case of multiple regime, the posterior distribution of $r_j$ depends on neighboring regimes $j$ and $j$+1. Thus only the parameters in these two regimes enter into the posterior distribution of $r_j$. The threshold values $r_j$, $j = 1,...,J$, are drawn one by one.

(3) The sampling of threshold parameter *m*. Similar as equation (3), the posterior of *m* is given as:

$$p(y \mid \mu,\phi,\varphi,\omega,\alpha,\theta,\beta,r,m)=\prod_{j=1}^{J}\prod_{t=P+1}^{T_j}\left[\frac{1}{\sqrt{2\pi h_t^{(j)}}}\exp\left(\frac{(y_t^{(j)}-\mu^{(j)})^2}{h_t^{(j)}}\right)\right] \tag{4}$$

with $y_t^{(j)}$ separated according to $r_{j-1} \le z_t < r_j$. Unlike threshold value, *m* depends on the parameters in all regimes. On the other side, the estimate of *m* affects all other parameters, in that it changes threshold variable $z_t$, which again result in change of $y_t^{(j)}$.

Unlike the other parameters, *m* is a positive integer. But this feature makes no change to the Griddy-Gibbs sampling itself. Only, we need to set the grid interval of m to be positive integer in [0, *M*]. *M* is the maximum choice of m, and in this paper we choose *M* = 30.

As with any standard MCMC procedure, we draw *N* times for each parameter, and delete the first *n* draws. The remaining *N* - *n* draws are used to calculate the mean and standard deviation of the parameters.

## 4. Empirical application in Chinese stock market

In this section, we apply the threshold GARCH model to Chinese stock market. We choose $y_t$ to be daily Shanghai Exchange Index from January 1997 to June 2010, which consist totally 3251 data. We collect the data from 1997, because before 1997 there was no 10% restriction to daily up and down of individual stock price, and after the introduction of this policy in January 1997 the volatility of Chinese stock market reduced significantly. Our data is collected from Wind Finance[1].

[1] One financial data provider in China with homepage: www.wind.com.cn.

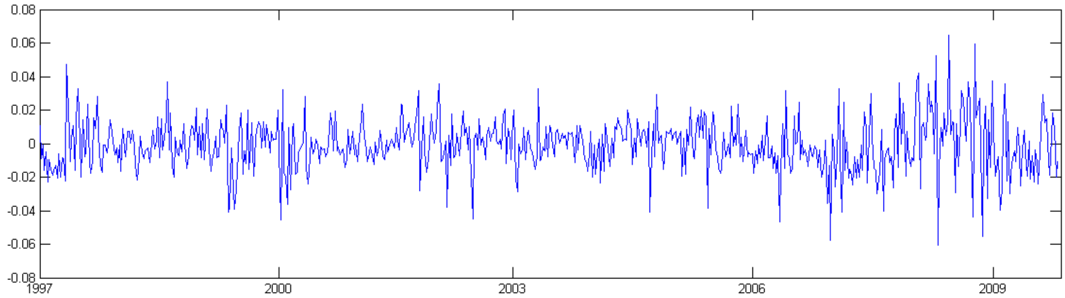


Figure 1. Return of Shanghai Exchange Index from January 1997-June 2009.

4.1. Structural break test of volatility

We use *SupF* tests proposed by Andrews (1993), and Andrews and Ploberger (1994) to detect the volatility breaks in absolute value of $\{|y_t|\}$. The testing equation is as follows.

$$|y_t| = \omega_1 + \sum_{i=1}^{P} \varphi_{1,i} |y_{t-i}| + (\omega_2 + \sum_{j=1}^{P} \varphi_{2,j} |y_{t-j}|) \cdot I(t > c) + \varepsilon_t \tag{5}$$

Maximum lag $P$ = 8, which is determined through Bayesian information criterion. As testing result, *SupF* = 29.24 with the break date on Dec. 20, 2006. Clearly, we reject the null hypothesis of no volatility break.

This volatility break was obviously connected with some events that happened in Chinese stock market. As figure 2 shows, after Dec. 2006, the Chinese market stepped into one phenomenal bull market, which enhanced the market volatility significantly. Why this most spectacular bull market in Chinese history began in 2006? We though, that the stock market reform in 2006 which aimed to make the all of the stocks exchangeable[2] was responsible for the subsequent euphoria in the market. This reform seemed to promise the market one bright future, and evoked the dormant passion of most of the Chinese citizen for stock speculation. And indirectly, this reform leaded to the bull market from 2006 to 2007 and the subsequent burst on Oct. 2007.

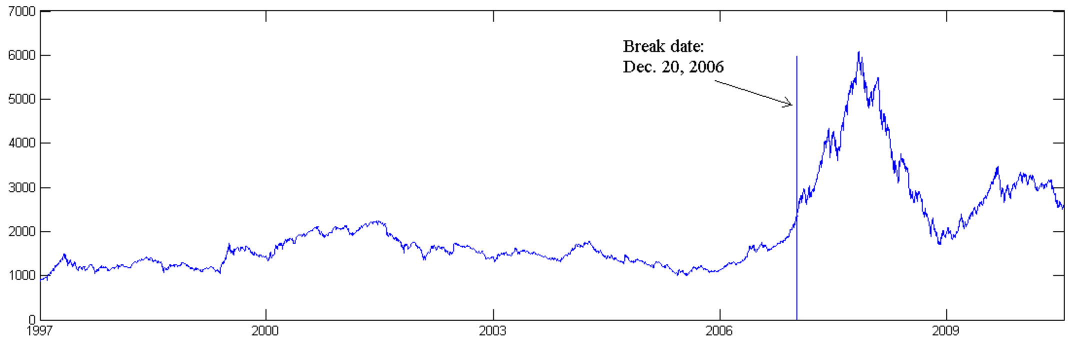


Figure 2. Shanghai Exchange Index with the volatility break

As the structural break test and analysis shows, the Chinese stock market after Dec. 2006 was greatly affected by the reform in 2006. Thus, we separate the return series into two periods: before and after Dec. 20, 2006. We apply the threshold GARCH to the first period, in order to study the

[2] As historic reason, a large stock of shares in most of the state-owned companies in Chinese stock market is not exchangeable in the market. Thus the dominant shareholder, which usually control the companies too, have no incentive to enhance the share price, which again results in variety of frauds.

asymmetric response of conditional mean and volatility to different threshold regimes, positive or negative shocks, and positive or negative past returns.

4.2. Threshold GARCH model before Dec. 2006

This section presents the results of estimation of the threshold GARCH model. We assume 2 regimes. And we calculate $P = 3$ according to BIC. Totally 10000 parameter simulations are drawn, with the first 2000 simulations taken as burn-in data. Table 1 presents the mean and standard deviation of the parameter.

| | Regime 1: $z_t \le r$ | | Regime 2: $z_t > r$ | |
|---|---|---|---|---|
| | mean | std | mean | Std |
| $\mu$ | $-9.18\times10^{-4**}$ | $3.41\times10^{-4}$ | $-3.19\times10^{-4}$ | $2.63\times10^{-4}$ |
| $\phi_1$ | 0.0478 | 0.0532 | $0.1765^{**}$ | 0.0531 |
| $\phi_2$ | 0.0211 | 0.0530 | 0.0470 | 0.0502 |
| $\phi_3$ | -0.0266 | 0.0624 | 0.0563 | 0.0527 |
| $\varphi_1$ | -0.0871 | 0.0639 | $-0.1463^{**}$ | 0.0546 |
| $\varphi_2$ | $-0.1676^{**}$ | 0.0599 | -0.0365 | 0.0598 |
| $\varphi_3$ | 0.0397 | 0.0605 | 0.0623 | 0.0529 |
| $\omega$ | $3.29\times10^{-6\,**}$ | $1.14\times10^{-6}$ | $1.47\times10^{-6**}$ | $0.57\times10^{-6}$ |
| $\alpha$ | 0.0295 | 0.017 | $0.0998^{**}$ | 0.025 |
| $\theta$ | $0.176^{**}$ | 0.042 | $0.058^{**}$ | 0.028 |
| $\beta$ | $0.72^{**}$ | 0.06 | $0.80^{**}$ | 0.05 |
| $v$ | $4.77^{**}$ | 0.74 | $4.44^{**}$ | 0.6061 |
| $r$ | $-1.88\times10^{-4}$ | $1.38\times10^{-4}$ | | |
| $m$ | $12.33^{**}$ | 3.09 | | |

Given $r$ and $m$, the returns can be separated into 2 regimes: the returns in regime 1 (down regime) satisfy $z_t \le r$, and regime 2 (up regime) $z_t > r$. Obviously, the average return in up regime is larger than that in down regime. The parameter estimate in table 1 shows that $r$ is indifferent to 0, which means whether 12 days average return is positive or not, is an important criterion to differentiate the regimes. With another interpretation, in case positive 12-days-average return people may have different speculation strategies, perhaps more aggressively.

Next, we explain the asymmetric response of mean and volatility to positive and negative past returns, positive and negative innovations, in down regime and up regimes respectively. In down regime, we may reject the null hypothesis that $\varphi_2 = 0$ with 5% significant level, which means one negative $y_{t-2}$ may significantly reduce the return at date $t$. Yet, in up regime, $\phi_1$ and $\varphi_1$ show that both positive and negative $y_{t-1}$ have significant effect on $y_t$, which corresponds to one well known speculation strategy "buy when index rises, and sell when it downs".

We think its one evidence of irrationality in Chinese stock market. In mature market, the similar pattern may not be seen. This irrationality may partly be explained when we look into the psychology of a speculator. In down regime, because of average negative returns, the speculator has no confidence on the market, thus do not responds to the positive returns. However, in this bad time he is afraid of losing more, so he may reduces his portfolio heavily as $y_{t-2}$ is significantly negative. The reason that $\varphi_1$ is not significantly differentiated from 0, is that there are always someone who hope to get some profit in case of rebound, and these buys let the sells insignificant in $\varphi_1$.

In up regime, the speculators are less risk-averse because the market promises positive return on average. They trade intensively, running after the profit and trying to avoid the loss. So they respond both to positive and negative $y_{t-1}$ quickly.

Parameters in volatility equation tell another interesting story about asymmetric reaction to innovations. We plot the conditional posterior pdf of $\omega$, $\alpha$, $\theta$ and $\beta$, so we can show the parameter difference vividly.

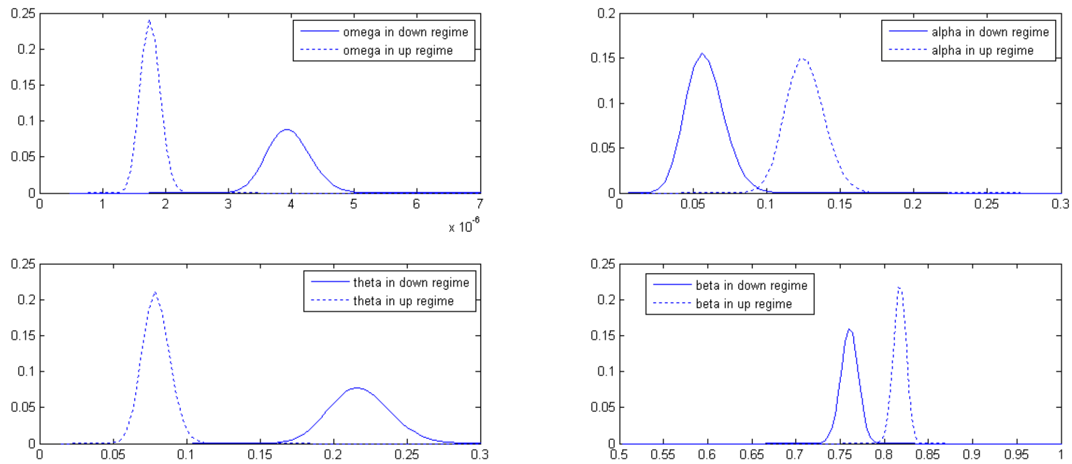


Figure 3. Conditional posterior pdfs of $\omega$, $\alpha$, $\theta$ and $\beta$ in down and up regimes

The GARCH persistence parameters $\gamma$, calculated as $\gamma = 0.5\cdot(\alpha+\theta)+\beta$ in case of symmetric distribution of innovations, are 0.823 in down regime and 0.879 in up regime. Combining with $\omega$, we can get that the conditional volatility in down regime is c.a. twice as that in up regime.

Moreover, the other parameters differentiate themselves significantly between these two regimes. Considering $\alpha$ and $\theta$, the volatility in down regime responds more intensively to negative innovations, while in up regime the positive innovation affects the volatility more significantly. It can be explained, if we study the psychology of the speculator in Chinese stock market. It's the fear for loss in "bad time", and the desire for profit in "good time" that drives the market to respond asymmetrically to different innovation.

Many paper pointed out that negative innovations tend to increase volatility more than positive ones in American stock market. However, in Chinese stock market, we need to analyze the case in different regimes, because the asymmetric response of volatility varies widely between down and up regimes.

## 5. Conclusions

We construct one triple-threshold GARCH model to analyze the asymmetric response of mean and conditional volatility. In parameter estimation, we apply Griddy-Gibbs sampling method, which require less work in selection of starting values and pre-run.

We apply our model in Chinese stock market, and find that 12-days-average return plays an important role in defining different regimes. While the down regime is characterized by negative 12-days-average return, the up regime has positive 12- days-average return. The conditional mean responds differently between down and up regime. In down regime, the return at date $t$ is affected negatively by lag 2 negative return, while in up regime the return responds significantly to both positive and negative lag 1 past return.

Moreover, our model shows that volatility reacts asymmetrically to positive and negative innovations, and this asymmetric reaction varies between down and up regimes. In down regime, volatility becomes more volatile when negative innovation impacts the market than when positive one does, while in up regime positive innovation leads to more volatile market than negative one does.

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