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ABSTRACT

The growing presence of financial operators in the oil market has brought about the diffusion of techniques - such as feedback trading - which lead to departures of prices from their fundamental values and increase their variability. Oil price changes are here associated with changes in stocks, bonds and effective USD exchange rate. The feedback trading mechanism is combined with an ICAPM scheme. This original model is estimated in a four asset CCC GARCH non linear framework, where the risk premium and the feedback trading components of the conditional means are multiplicative functions of the system's conditional variances and covariances. The empirical analysis, which encompasses the 2008-2009 financial crisis, identifies a structural change in the year 2000. From then on oil returns tend to become more reactive to the remaining assets of the model and feedback trading more pervasive. A comparison is drawn between three and four asset minimum variance portfolios in the two sub-periods, 1992-1999 and 2000-2009. Indeed, the trade-off between risk and returns – measured here by the average return per unit of risk index – indicates that in the last decade oil diversifies away the empirical risk of our portfolio.

Keywords: oil price dynamics; feedback trading; multivariate GARCH; portfolio allocation.

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1. Introduction

Systematic deviations from the tenets of the efficient markets hypothesis are commonly accepted in the financial literature and are often attributed to trading techniques based on extrapolative expectations. This kind of market behavior is conducive to feedback trading: "positive" if investors buy when prices rise and sell when they fall and "negative" if investors buy when prices fall and sell when they rise.

Positive feedback trading is considered irrational, since it moves prices away from their equilibrium values and raises market risk. Among many others, Lakonishok et al. (1992), Nofsinger and Sias (1999) and, more recently, Boyer and Zheng (2009) attribute this trading behavior to specific groups of market operators, such as foreign institutional investors. It was detected in the US stock market by Cutler et al. (1991) and Sentana and Wadhwani (1992) in two classic articles and in later studies by Koutmos (1997) and Koutmos and Saidi (2001) in, respectively, European and emerging equity markets. The growing number of financial operators entering the oil market suggests that this paradigm be extended to the modeling of oil price behavior.

Shiller (1984) and Sentana and Wadhwani (1992) analyze feedback trading in the context of a behavioral CAPM, a single factor model which fails to capture the risk return components due to cross asset linkages. We adopt, therefore, Merton's (1973) multifactor ICAPM parameterization, which introduces additional measures of risk and allows the covariance between the assets under investigation and the variables that enter the investment opportunity set to influence the behavior of returns over time. This framework is used here to assess the role of oil in financial portfolio hedging decisions.

Oil price dynamics is often associated with stock and bond markets and exchange rate behavior. Several studies ascertain a negative linkage between oil, bond, and stock prices, i.e. a negative covariance risk between oil and a diversified portfolio of financial assets.[1]

Alternatively, it is claimed that there is a positive real sector linkage between the value of financial assets and oil via production and business cycle,

[1] See, among others, Sadorsky (1999) and Bhar and Nikolova (2009).

expansionary periods (related to asset price increases) being associated with oil price rises.

The dollar exchange rate too is strongly interlinked with oil prices. From a macroeconomic point of view, higher oil prices raise US trade deficits, weaken the dollar, and bring about compensatory price increase policies by oil exporting countries. From a financial point of view, the correlation between oil and financial asset prices is likely to be negative. As noted by Roache (2008), commodities (such as oil) behave differently from stocks and bonds and provide risk diversification opportunities. Traders that expect a dollar depreciation will sell dollar denominated financial assets and buy oil (and vice-versa if they are bullish on the dollar) in order to diversify their portfolio. Indeed, crude oil seems to have attracted funds away from financial markets in periods of stress.

The nonlinear behavior of the oil returns has recently been ascribed to the presence of noise traders and heterogeneous arbitrageurs in crude oil markets by Lee et al. (2008) and Cifarelli and Paladino (2009). The role of speculative pressures on oil price dynamics in recent times is also confirmed by the analysis of Kaufman (2010).

Our study follows these lines and analyzes the behavior of weekly changes in the WTI crude oil price over a time period spanning the last seventeen years, providing estimates of the financial interrelation between oil, US stocks, bonds, and dollar effective exchange rate changes. We check for the presence of speculative components in oil pricing using long and homogeneous time series which encompass large shifts in market sentiment. The multivariate investigation builds on the parameterization of feedback trading by Sentana and Wadhwani (1992) and on the two factor ICAPM of Scruggs (1998). The main goal is to assess if (and how) the different behavior of oil brings about a reduction of the unpriced risk of a financial portfolio.

The paper has several innovative features with respect to the existing literature.

(i) The short run dynamics of four asset returns is parameterized with the help of a highly non linear simultaneous GARCH multivariate model documenting both the interaction between noise and informed trading and

between markets covariances. An estimation of this kind is technically challenging and, to the best of our knowledge has, not yet been performed.

(ii) The changing nature of oil from a physical commodity to a financial asset is empirically detected both in the mean and variance equations. It is reflected in the enhancing effect of the introduction of oil in a financial portfolio in terms of overall return and risk.

(iii) The analysis encompasses the 2008-2009 financial crisis, providing an homogeneous interpretative framework of the oil dynamics in recent years.

The remainder of this paper is structured as follows. After briefly introducing the theoretical model mentioned above, the empirical results are set forth. The multivariate GARCH analysis - performed over the 1992-1999 and 2000-2009 sample periods - reveals that feedback trading mechanisms gain momentum in the crude oil market from 2000 to 2009. The potential diversification effect of oil is then analyzed through a comparison of modified Sharpe's ratios (average return per unit of risk indexes) obtained from multi asset-class portfolios which provides support for our hypotheses.

2. The behavioral ICAPM

Merton's (1973) dynamic Intertemporal Capital Asset Pricing Model, in spite of its sophistication, does not account for the serial correlation of the returns, a standard stylized characteristic of asset and commodity pricing. We follow therefore Dean and Faff (2008) and insert the feedback trading paradigm of Cutler et al. (1991), among others, into the ICAPM.

Two types of agents enter our model, as in Sentana and Wadhwani (1992), feedback traders or trend chasers, and smart money investors. The former react to past price changes only while the latter respond to expected risk-return considerations using an ICAPM framework.

According to Merton investors price an asset in relation not only to the expected systematic risk, but also in relation to the expected future change in the investment opportunity set, proxied by n state variables. The analysis is set in a continuous time framework, where the returns and the state variables

follow standard diffusion processes. Risk averse investors maximize the utility of wealth function $J(W(t),\overline{F}(t),t)$ where $W(t)$ is wealth and $\overline{F}(t)$ is a $n \times 1$ vector of state variables ($F_1, F_2, ..., F_n$) that represent the behavior over time of the investment opportunity set.

In equilibrium the expected market risk premium for asset M is given by [2]

$$E_{t-1}[r_{M,t} - \alpha] = \left[\frac{-J_{WW}W}{J_W}\right]\sigma^2_{M,t} + \left[\frac{-J_{WF_1}}{J_W}\right]\sigma_{MF_1,t} + ... + \left[\frac{-J_{WF_n}}{J_W}\right]\sigma_{MF_n,t} \quad (1)$$

where α is the risk free rate $E_{t-1}[.]$ is the expectation operator, $r_{M,t}$ is the return of asset M, $\sigma^2_{M,t}$ and $\sigma_{MF_i,t}$ are the corresponding conditional variance and covariance with the state variable F_i, where $i = 1,...,n$. The first coefficient $\left[\frac{-J_{WW}W}{J_W}\right]$ quantifies the degree of relative risk aversion.[3] It is always positive since $J_W > 0$ and $J_{WW} < 0$, which suggests a positive relationship between risk premium and conditional variance. The sign of the impact on excess returns of the ith state variable will depend upon the interaction of the signs of J_{WF_i} and $\sigma_{MF_i,t}$, which are both a priori indeterminate. If J_{WF_i} and $\sigma_{MF_i,t}$ are of the same sign, i.e. either both positive or both negative, $J_{WF_i}\sigma_{MF_i,t}$ is positive and investors will demand a lower risk premium. If J_{WF_i} and $\sigma_{MF_i,t}$ are of the opposite sign, $J_{WF_i}\sigma_{MF_i,t}$ is negative and investors will demand a higher risk premium.

In the empirical analysis it will be assumed that the risk premium is a linear function of market variance and of the covariances between the returns and the state variables. Equation (1) can then be rewritten as follows

$$E_{t-1}[r_{M,t} - \alpha] = \Phi_t \quad (2)$$

where

[2] Equation (1) is derived from Merton's first order conditions. See Merton (1973, equation (15), page 876).

$$\Phi_t = \Phi_1(\sigma^2_{M,t}) + \Phi_2(\sigma_{MF_1,t}) + ... + \Phi_{n+1}(\sigma_{MF_n,t}) \tag{3}$$

The proportionate demand for asset M by smart money traders, DS_t, is governed by standard mean-variance considerations:

$$DS_t = \frac{E_{t-1}[r_{M,t} - \alpha]}{\Phi_t} \tag{4}$$

The demand of risky asset M rises with the expected excess return and declines when its riskiness Φ_t increases.

If $DS_t = 1$ equation (4) reverts to the standard ICAPM equilibrium equation (2).

The relative asset demand by feedback traders, DF_t, is formulated as

$$DF_t = \gamma r_{M,t-1} \tag{5}$$

If $\gamma > 0$ we have positive feedback trading. Agents buy (sell) when the rate of change of the price of the previous period is positive (negative) and may destabilize the market if asset prices overshoot their equilibrium values based on fundamentals. When $\gamma < 0$, with negative feedback trading, agents sell (buy) when prices are rising (falling) in the previous period and tend to stabilize the market.

Equilibrium requires that the two investor groups clear the market and $DS_t + DF_t = 1$. Adding equations (4) and (5) and replacing Φ_t by its determinants according to equation (3), we obtain the following feedback trading equation

$$\begin{aligned} E_{t-1}[r_{M,t} - \alpha] &= \Phi_1(\sigma^2_{M,t}) + \Phi_2(\sigma_{MF_1,t}) + ... + \Phi_{n+1}(\sigma_{MF_n,t}) \\ &- \gamma[\Phi_1(\sigma^2_{M,t}) + \Phi_2(\sigma_{MF_1,t}) + ... + \Phi_{n+1}(\sigma_{MF_n,t})]r_{M,t-1} \end{aligned} \tag{6}$$

Equation (6) is the behavioral ICAPM relationship that shall be used to parameterize the dynamics of the assets analyzed in the paper. The sign of

[3] Low case letters indicate partial derivatives.

the coefficient of the lagged rate of return $r_{M,t-1}$ will depend upon the nature of the feedback trading behavior, either positive of negative, and upon the signs and absolute values of the conditional covariances with the state variables $\sigma_{MF_{i,t}}$, $i = 1,...,n$ and of the corresponding $\Phi_2,...,\Phi_{n+1}$ risk loadings.

3. Empirical results

The empirical evidence relies on the multivariate CCC GARCH parameterization of the ICAPM model. Feedback trading mechanisms are accounted for in a four asset portfolio context.

3.1 Description of the series

The weekly observations used in this study span the 6 October 1992 – 27 October 2009 time period. The data set includes oil spot prices (S_t, the WTI Spot Price fob expressed in US dollars per barrel) and futures oil prices (F_t, the contract 1 price) which are provided by the EIA database. The Dow Jones Industrial index (J_t), the US dollar nominal effective exchange rate (Z_t) and the US All Lives Government Bond Total Return index (K_t) are taken from Bloomberg, Fred Database, and Datastream International respectively.

According to the Andrews (1993) Wald tests for parameter stability with unknown switch point, the time series do not show any sign of regime shifts. The null hypothesis of no break point - with the usual trimming of 5% of the data at the endpoints – cannot be rejected.[4]

On the contrary the correlation between the time series does not seem to be constant over the whole sample. A standard Jenrich (1970) χ^2 stability test detects unequivocally a structural break in the correlation matrix of returns at the end of the year 1999.[5] We split therefore the data in two sub-samples;

[4] The tests are based on a first order autoregression with a constant in the case of oil and equity returns and on a regression on a constant term for the remaining time series. The statistics are available from the authors upon request.

[5] The maximum value of the test is 86.72 under the alternative of a breakpoint on 28 December 1999. It strongly rejects the null hypothesis (that two 4-variate normal populations have correlation matrices that have a common non-singular value), the $\chi^2(6)$ 5% critical value being 12.6. In order to deal with

the first goes from 6 October 1992 to 28 December 1999 (378 observations) and the second from 4 January 2000 to 27 October 2009 (513 observations). The descriptive statistics for each sub-sample are reported in Table 1.[6]

[Insert Table 1]

The relation between oil and stock returns undergoes a drastic change in the two time periods, stock returns being considerably larger on average than oil returns in the first period and smaller in the second. The standard deviation of the oil price rate of change is always greater than that of the returns of the remaining assets. All the series are mildly skewed and leptokurtic, and the Jarque Bera test statistics reject the normality of distribution hypothesis. Their stationarity, tested with the ADF procedure, stands out clearly. Inter-temporal dependency of weekly returns is assessed using first order autoregressions of the times series where as usual the standard errors are corrected for conditional heteroskedasticity.[7] With the exception of the effective exchange rate and of the US bond returns all the remaining series display autocorrelation. The significance of the Ljung Box Q-statistics shows that volatility clustering affects all the time series, while asymmetries are present, in the first sub-sample, only in the case of the equity and bond returns.

In Table 2 are set out the robust correlation matrices between the oil spot return and the returns of the stock price index, the effective exchange rate, and the US Government bond index over the two sub-samples.[8] They provide an informative preliminary measure of interdependence. The two matrices support our a priori hypothesis on a change in the role of oil. The interrelation between oil returns and the remaining assets rises in the second subsample. The negative sign of the correlation coefficients suggests an idiosyncratic pricing that is useful for portfolio diversification.

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potential distortions due to non-normality, we repeated the test using the standardized residuals of a full sample estimation of our CCC GARCH behavioral ICAPM system and obtained qualitatively similar results.

[6] Percentage rates of changes are used in the empirical analysis.

[7] The estimates are available from the authors upon request.

[8] The robust (to oulier distortions) correlation coefficients are obtained with the Donoho-Stahel procedure described in Maronna and Yohai (1995).

[Insert Table 2]

3.2 First period results - The role of oil in the nineties

We estimate simultaneously four ICAPM asset pricing relationships, one for each asset, over the 6 October 1992 – 28 December 1999 time period. A multivariate GARCH is used to parameterize the conditional second moments since the time series are conditionally heteroskedastic. The following operational version of equation (6) is introduced in order to model the conditional means

$$\begin{aligned}\Delta x_{1t} &= b_{0x_1} + b_{1x_1} h^2_{x_1,t} + b_{2x_1} h_{x_1x_2,t} + b_{3x_1} h_{x_1x_3,t} + b_{4x_1} h_{x_1x_4,t} \\ &+ (b_{5x_1} + b_{6x_1} h^2_{x_1,t} + b_{7x_1} h_{x_1x_2,t} + b_{8x_1} h_{x_1x_3,t} + b_{9x_1} h_{x_1x_4,t})\Delta x_{1t-1} + u_{x_1,t}\end{aligned} \tag{7}$$

where $\Delta x_{1t},...,\Delta x_{4t}$ are the rates of return of the four assets analyzed in the paper and $h^2_{x_{1,t}}$ and $h_{x_1x_i,t}$, i = 2,3,4, are, respectively, the conditional variance and covariances obtained with the GARCH model. $b_{1x_1} h^2_{x_1,t} + b_{2x_1} h_{x_1x_2,t} + b_{3x_1} h_{x_1x_3,t} + b_{4x_1} h_{x_1x_4,t}$ corresponds to $\Phi_1(\sigma^2_{M,t}) + \Phi_2(\sigma_{MF_1,t}) + \Phi_3(\sigma_{MF_2,t}) + \Phi_4(\sigma_{MF_3,t})$ in equation (6), while $b_{5x_1} + b_{6x_1} h^2_{x_1,t} + b_{7x_1} h_{x_1x_2,t} + b_{8x_1} h_{x_1x_3,t} + b_{9x_1} h_{x_1x_4,t}$ corresponds to $-\gamma[\Phi_1(\sigma^2_{M,t}) + \Phi_2(\sigma_{MF_1,t}) + \Phi_3(\sigma_{MF_2,t}) + \Phi_4(\sigma_{MF_3,t})]$.

The relevance of the feedback trading component and the number of factors affecting the pricing of each asset are determined empirically. If there is no evidence of serial correlation, as is the case of bond and exchange rate returns, the feedback trading component is dropped from the corresponding conditional mean parameterization. In the same way we remove the variables with insignificant coefficients at the standard 5 percent level or that correspond to insignificant conditional covariances.[9] The conditional second moments are parameterized using a CCC GARCH(1,1) model. The behavior

[9] This parsimonious approach is motivated by need to reduce the large number of parameters entering our nonlinear system.

of the rate of change of the spot oil prices (Δs_t), the Dow Jones stock index (Δj_t), the US dollar effective exchange rate (Δz_t), and the US Government bond total return index (Δk_t) are then modelled using the system (A). For expositional simplicity we always report the conditional variance as the first regressor.

$$
\begin{aligned}
\Delta s_t &= b_{0s} + b_{1s}h_{s,t}^2 + b_{2s}h_{sj,t} + (b_{6s}h_{s,t}^2 + b_{7s}h_{sj,t})\Delta s_{t-1} + u_{s,t} \\
\Delta j_t &= b_{0j} + b_{1j}h_{j,t}^2 + b_{2j}h_{sj,t} + b_{3j}h_{jz,t} + b_{4j}h_{jk,t} \\
&\quad + (b_{5j} + b_{6j}h_{j,t}^2 + b_{7j}h_{sj,t} + b_{8j}h_{jz,t} + b_{9j}h_{jk,t})\Delta j_{t-1} + u_{j,t} \\
\Delta z_t &= b_{0z} + b_{1z}h_{z,t}^2 + b_{3z}h_{jz,t} + u_{z,t} \\
\Delta k_t &= b_{0k} + b_{1k}h_{k,t}^2 + b_{4k}h_{jk,t} + u_{k,t}
\end{aligned}
$$

$$
u_t = \begin{bmatrix} u_{s,t} \\ u_{j,t} \\ u_{z,t} \\ u_{k,t} \end{bmatrix}
$$

$$
u_t | \Omega_{t-1} N(0, H_t) \qquad \text{(A)}
$$

$$
H_t = \Delta_t R \Delta_t
$$

$$
R = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & 1 & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & 1 & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & 1 \end{bmatrix} \qquad \Delta_t = \begin{bmatrix} h_{s,t} & 0 & 0 & 0 \\ 0 & h_{j,t} & 0 & 0 \\ 0 & 0 & h_{z,t} & 0 \\ 0 & 0 & 0 & h_{k,t} \end{bmatrix}
$$

$$
h_{s,t}^2 = \varpi_s + \beta_s u_{s,t-1}^2 + \alpha_s h_{s,t-1}^2;\ h_{j,t}^2 = \varpi_j + \beta_j u_{j,t-1}^2 + \alpha_j h_{j,t-1}^2; h_{z,t}^2 = \varpi_z + \beta_z u_{z,t-1}^2 + \alpha_z h_{z,t-1}^2
$$

$$
h_{k,t}^2 = \varpi_k + \beta_k u_{k,t-1}^2 + \alpha_k h_{k,t-1}^2
$$

The QML estimates are set out in Table 3. The conditional mean determinants that are associated with the conditional covariances between oil returns and exchange rate changes, $h_{sz,t}$, between oil and US bond returns, $h_{sk,t}$, and between exchange rate changes and bond returns, $h_{zk,t}$, are removed since the corresponding conditional correlation coefficients estimates $\rho_{13}, \rho_{14}, \rho_{34}$ do not significantly differ from zero.

The quality of fit is satisfactory. Almost all coefficients are statistically significant and the usual tests for misspecification suggest that the standardized residuals ν_t are well behaved. For each equation we find that $E[\nu_t]=0$ and $E[\nu_t^2]=1$, and that ν_t is serially uncorrelated and homoskedastic. The sign bias tests by Engle and Ng (1993) support the choice of a symmetric conditional variance model. Asymmetry, a stylized characteristic of stock return volatility, is filtered out by the feedback trading conditional mean parameterization.

For the sake of notational simplicity let $\lambda_{i,t}$, where $i=s,j,z,k$, be the CAPM component - i.e. $\lambda_{s,t}=b_{1s}h_{s,t}^2+b_{2s}h_{sj,t}$, $\lambda_{j,t}=b_{1j}h_{j,t}^2+b_{2j}h_{sj,t}+b_{3j}h_{jz,t}+b_{4j}h_{jk,t}$, $\lambda_{z,t}=b_{1z}h_{z,t}^2+b_{3z}h_{jz,t}$, and $\lambda_{k,t}=b_{1k}h_{k,t}^2+b_{4k}h_{jk,t}$ - and $\phi_{i,t}$ be the feedback trading coefficient - i.e. $\phi_{s,t}=b_{6s}h_{s,t}^2+b_{7s}h_{sj,t}$, and $\phi_{j,t}=b_{5j}+b_{6j}h_{j,t}^2+b_{7j}h_{sj,t}+b_{8j}h_{jz,t}+b_{9j}h_{jk,t}$.

In both oil and stock returns conditional mean equations the overall CAPM component $\lambda_{i,t}$ and the feedback trading coefficient $\phi_{i,t}$ – computed with historical simulations which use the values of the conditional second moments - turn out to be, respectively, positive and negative on average. (Their behavior over time is set out in Graph 1 and their unconditional average values can be found in Table 4.) The negative sign of the feedback trading coefficient is due to the presence of destabilizing speculation, which tends to raise the volatility of the returns of the asset.

As for the rate of change of the US dollar effective exchange rate and the US bond returns, the overall CAPM component is negative. The negative sign of $\lambda_{z,t}$ implies that an increase in the conditional variance of the rate of change of the effective exchange rate $h_{z,t}^2$, and of its conditional covariance with the stock returns $h_{jz,t}$, will bring about a depreciation of the US effective exchange rate as traders sell dollars (see Graph 1). Similarly the negative value of $\lambda_{k,t}$ means that an increase in the bond return conditional variance $h_{k,t}^2$, possibly due to a rise in inflation risk and/or in general economic uncertainty, will lead

to a decline in bond returns as traders sell bonds which are losing their safe asset characteristics.[10]

[Insert Table 3]

[Insert Graph 1]

[Insert Table 4]

During the nineties, the link between oil prices and the other assets investigated in the paper is limited to a positive interaction between oil and stock returns, which can be attributed to a real (macroeconomic) channel. A rise in stock returns during the expansionary phase of the business cycle is associated with an increase in the demand for oil and a corresponding upward pressure on oil returns. The two spikes detected in the CAPM component and in the feedback trading coefficient of the oil return equation (see Graph 1) are caused by sharp increases in oil price variability. The first price shock in 1996 is idiosyncratic and can be attributed to a mismatch between actual and expected oil demand. It affects only the oil return equation by raising the risk premium and magnifying the feedback trading effects as the traders' uncertainty rises. The second shock is mainly connected to the Asian crisis and affects all of the remaining assets' conditional mean equations by increasing the risk premium that is required to price both oil and stock returns.

3.3 Second period results - Oil as a financial asset

Here too all time series turn out to be conditionally heteroskedastic - as shown in Table 1 above – and the multivariate CAPM is estimated according to the following CCC GARCH(1,1) non linear parameterization.

[10] Viceira (2007) finds that bond return volatility is positively related to the level and the slope of the yield curve, factors that proxy for inflation risk and overall economic uncertainty.

$$\Delta s_t = b_{0s} + b_{1s}h_{s,t}^2 + b_{2s}h_{sj,t} + b_{3s}h_{sz,t} + b_{4s}h_{sk,t} + (b_{5s} + b_{6s}h_{s,t}^2 + b_{7s}h_{sj,t} + b_{8s}h_{sz,t})\Delta s_{t-1}$$
$$+ b_{10s}\Delta f_{t-1} + b_{11s}D_1 + b_{12}D_2 + u_{s,t}$$
$$\Delta j_t = b_{0j} + b_{1j}h_{j,t}^2 + b_{2j}h_{sj,t} + b_{3j}h_{jz,t} + b_{4j}h_{jk,t} + (b_{5j} + b_{6j}h_{j,t}^2 + b_{7j}h_{sj,t} + b_{8j}h_{jz,t} + b_{9j}h_{jk,t})\Delta j_{t-1}$$
$$+ b_{11j}D_1 + u_{j,t}$$
$$\Delta z_t = b_{0z} + b_{1z}h_{z,t}^2 + b_{3z}h_{jz,t} + b_{4z}h_{zk,t} + u_{z,t}$$
$$\Delta k_t = b_{0k} + b_{1k}h_{k,t}^2 + b_{2k}h_{sk,t} + b_{3k}h_{jk,t} + u_{k,t}$$
$$u_t = \begin{bmatrix} u_{s,t} \\ u_{j,t} \\ u_{z,t} \\ u_{k,t} \end{bmatrix}$$
$$u_t | \Omega_{t-1} N(0, H_t) \qquad \text{(B)}$$
$$H_t = \Delta_t R \Delta_t$$

$$R = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & 1 & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & 1 & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & 1 \end{bmatrix} \qquad \Delta_t = \begin{bmatrix} h_{s,t} & 0 & 0 & 0 \\ 0 & h_{j,t} & 0 & 0 \\ 0 & 0 & h_{z,t} & 0 \\ 0 & 0 & 0 & h_{k,t} \end{bmatrix}$$
$$h_{s,t}^2 = \varpi_s + \beta_s u_{s,t-1}^2 + \alpha_s h_{s,t-1}^2; \quad h_{j,t}^2 = \varpi_j + \beta_j u_{j,t-1}^2 + \alpha_j h_{j,t-1}^2; \quad h_{z,t}^2 = \varpi_z + \beta_z u_{z,t-1}^2 + \alpha_z h_{z,t-1}^2$$
$$h_{k,t}^2 = \varpi_k + \beta_k u_{k,t-1}^2 + \alpha_k h_{k,t-1}^2$$

D_1 is a dummy accounting for the Lehman crisis that takes value 1 from 16 September 2008 to 13 January 2009 and D_2 is a dummy representing the oil upswing and takes value 1 from 23 January 2007 to 29 July 2008.

The estimates of system (B) are set out in Table 5. The second period variance covariance matrix points to a very intricate interrelation pattern. The conditional correlation coefficients are all significant and negative. This suggests that all the assets of the paper can be used for portfolio risk diversification and that the parameterization of both the feedback trading coefficients and the CAPM components has to account for a complex asset return interaction. No feedback trading component appears in the conditional mean equations of the rate of change of the US effective exchange rate and of the US bond returns, as these time series turn out to be serially uncorrelated also in the second time period.

The shifts over time of the CAPM component and feedback trading coefficient time series, computed using historical simulations, are set forth in Graph 2. Their respective average values are collected in Table 6.

The graphical analysis detects three major shocks to the CAPM component of the oil conditional mean equation which are mostly synchronized but opposite in sign to the shocks in the CAPM component of the stock equation.

The first shock is associated with the financial turmoil caused by the military operations against Iraqi oil infrastructures of 2001; the second is a direct consequence of the stock market collapse of 2002. The third and largest one is directly linked to the banking crisis of September–October 2008.

The positive sign of the oil CAPM component reflects the impact of the large oil price volatility. The feedback trading coefficient is strongly negative with the exception of the Fall 2008 crisis when oil speculation temporary dries up. An inspection of Tables 4 and 6 shows that positive feedback trading is, on average, more relevant in the second than in the first time period. Destabilizing speculation becomes a major driver of oil price movements until the inception of the crisis.

From 2003 to 2008 the variability of the stock return CAPM component declines. The latter remains negative because of the strong impact of the negative covariation between stocks and the remaining assets of the model. This result corroborates the relevance of asset substitutability in the second period of the sample.

In the US dollar effective exchange rate conditional mean, b_{1z} is negative; an increase in volatility brings about a depreciation of the US effective exchange rate as traders sell dollars. The average negativeness of $\lambda_{z,t}$ is only partially mitigated by the positive impact of the covariance between bond returns and the US dollar, $b_{4z}h_{zk,t}$.

The switch in sign of the CAPM component of the conditional mean equation of the US bond return can be attributed to the influence of two major factors. The oil channel, $b_{2k}h_{sk,t}$, which identifies a joint nature of bonds and oil as safe assets, and the smaller financial channel, $b_{3k}h_{jk,t}$, which accounts for the substitution effect of stock price volatility spillovers.

[Insert Table 5]

[Insert Graph 2]

[Insert Table 6]

4. Portfolio analysis

If the hypothesis that in recent years oil behaved more and more as a financial asset is correct, its inclusion in a portfolio should have a beneficial effect on the corresponding risk/return trade-off.

We assess this proposition using a straightforward Markowitz procedure, with no short-selling restrictions, no borrowing and no lending, and base the portfolio composition on risk minimization criteria.

If $w = (w_1, ..., w_N)'$ is a Nx1 vector of portfolio weights and Σ is the NxN variance-covariance matrix of the returns, the portfolio variance is then $w'\Sigma w$. The global minimum variance portfolio is the solution of the minimization problem $min_w \; w'\Sigma w \quad s.t. \quad w'\underline{1} = 1$, where $\underline{1}$ is a Nx1 column vector of ones. The weights $w_{MV} = (w_{MV,1}, ..., w_{MV,N})'$ of the global minimum variance portfolio take the value $w_{MV} = \Sigma^{-1}\underline{1} / \underline{1}'\Sigma^{-1}\underline{1}$.

The expected return μ_{MV} and the variance σ^2_{MV} of the global minimum variance portfolio read as

$$\mu_{MV} = \mu' w_{MV} = \frac{\mu'\Sigma^{-1}\underline{1}}{\underline{1}'\Sigma^{-1}\underline{1}} \tag{8}$$

and

$$\sigma^2_{MV} = w'_{MV}\Sigma w_{MV} = \frac{1}{\underline{1}'\Sigma^{-1}\underline{1}} \tag{9}$$

where μ is a Nx1 column vector of asset returns. The corresponding expected return per unit of risk index is then computed as $\left(\mu_{MV} / \sqrt{\sigma^2_{MV}}\right)$.

The lower variance bound (9) can be attained only if the variance-covariance matrix of the asset returns is known. Typically, historical return observations are used for this estimation. We construct the portfolios either keeping the weights constant over each sub-sample or rebalancing them every week, mimicking a tactical asset allocation behavior. (Weekly portfolio rebalancing is also meant to account for the volatility clustering of the time series.) Every week the constrained variance minimization described above is performed over a predetermined data interval *j* and the corresponding global minimum variance weights, (expected) portfolio returns $\mu_{MV,j}$, and portfolio return variance $\sigma^2_{MV,j}$ are computed. The following week the same procedure is repeated over a sample interval shifted forward by one time period (i.e. one week). This iterative process continues until the end of the sub-period. A set of two time series for each portfolio holding period is obtained in this way. We selected here a 12 month and a 6 month holding period. In Table 7 are set out the unconditional means of these time series along with the differentials between the three and four asset portfolio average return per unit of risk indexes.

The entries suggest that, over the last decade, the introduction of oil into a multi asset-class portfolio improves the risk/return performance.

In the first sub-period the three asset portfolio (without oil) outperforms the four asset one, which includes oil, as pointed out by the value of the unconditional mean returns with and without portfolio rebalancing. The (positive) differential between average returns per unit of risk, computed following equation (2) in Ledoit and Wolf (2008)[11], provides analogous results, even if the null hypothesis that the differential is nil – tested using an autocorrelation and heteroskedasticity consistent z statistic - cannot be rejected at the standard levels of significance.

[Insert Table 7]

In the second period, when oil progressively acquires financial characteristics, we obtain the opposite results, which is coherent with the change in the

[11] Returns and squared returns in the formula are the averages over the holding period of the portfolio returns and squared returns obtained at each iterations.

correlation structure found in Table 2. The unconditional portfolio mean and variance and the (negative) average return per unit of risk differential detect a clear-cut dominance of the four asset portfolio. The z-test statistics strongly reject the null that the three and four asset portfolios have the same performance. The analysis is then repeated replacing WTI spot prices with the corresponding one month to expiration (contract 1) futures prices and provides similar results, a finding which further corroborates the hypothesis on oil spot pricing mentioned above.[12]

5. Conclusion

At the beginning of the year 2000 a regime shift is detected within a highly nonlinear behavioral ICAPM assumed to describe the interconnection between crude oil contracts, US stocks, bonds and effective dollar exchange rate. Indeed, the parsimonious estimates of the model over the 1992-1999 and 2000-2009 time periods differ considerably. The conditional correlations change in sign, absolute value, and statistical significance. The oil return conditional mean acquires a complex feedback trading component in the second sub-period and becomes similar in structure to the conditional mean of the stock returns. Oil contracts seem to behave as financial assets, which interact with stocks, bonds, and exchange rates.

In order to further investigate this hypothesis we construct global minimum variance portfolios containing standard financial assets along with WTI crude oil contracts. It stands out clearly – comparing return per unit of risk measures – that the introduction of oil has been of help in diversifying away the unpriced risk of the portfolios.

The paper thus suggests that, in the second sub-period, traders hedge their portfolios considering oil as a component of their wealth allocation.

[12] Also Geman and Kharoubi (2008) find that WTI crude oil futures contracts can be used to efficiently diversify equity portfolios.

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Table 1: Descriptive statistics

10/6/1992-12/28/1999

	Oil spot price return	Stock price index return	Effective exchange rate rate of change	Oil futures price return	US Govt bond return
Mean	0.054	0.341	0.033	0.054	0.224
Std. Dev.	4.386	1.870	0.823	4.506	1.587
Skewness	0.199	-0.656	-0.134	0.360	-0.298
Kurtosis	4.372	5.370	4.500	4.823	5.847
Jarque Bera	32.10*	115.2*	36.5*	60.4*	133.0*
ADF	-21.30*	-20.98*	-19.65*	-21.73*	-21.50*
$Q_x^2(12)$	66.93*	55.10*	43.02*	81.92*	37.63*
J.T.A.	1.369	60.70*	2.589	1.388	27.697*

1/4/2000 –10/27/2009

	Oil spot price return	Stock price index return	Effective exchange rate rate of change	Oil futures price return	US Govt bond return
Mean	0.212	-0.031	-0.050	0.212	0.349
Std. Dev.	5.944	2.596	1.045	5.587	1.958
Skewness	-0.875	-0.372	-0.112	-0.619	-0.243
Kurtosis	7.419	7.280	4.493	4.655	6.260
Jarque Bera	482.8*	403.4*	48.7*	91.4*	232.3*
ADF	-25.09*	-24.38*	-22.35*	-24.55*	-22.50*
$Q_x^2(12)$	208.4*	97.1*	257.0*	110.3*	320.2*
J.T.A.	63.18*	28.16*	27.31*	18.75*	26.80*

Notes. * significant at the 5 percent level; ADF: Augmented Dickey Fuller unit root test statistic; $Q_x^2(k)$: Ljung Box Q-statistic for k^{th} order serial correlation of the squared variable x^2; J.T.A.: Joint Wald test of the null hypothesis of no asymmetry distributed as χ^2 with 3 degrees of freedom (Engle and Ng, 1993). The data have a weekly frequency.

Table 2: Robust Correlations

10/6/1992-12/28/1999

	Oil spot price return	Stock price index return	Effective exchange rate rate of change	US Govt bond return
Oil spot price return	1			
Stock price index return	0.067	1		
Effective exchange rate rate of change	-0.146	0.089	1	
US Govt bond return	-0.041	0.434	0.018	1

1/4/2000 –10/27/2009

	Oil spot price return	Stock price index return	Effective exchange rate rate of change	US Govt bond return
Oil spot price return	1			
Stock price index return	-0.034	1		
Effective exchange rate rate of change	-0.316	-0.120	1	
US Govt bond return	-0.099	-0.298	-0.213	1

Table 3: Multivariate ICAPM, October 1992-December 1999

System (A) Conditional mean equations

	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	$E[\nu_t]$	$E[\nu_t^2]$	Sk.	Kurt.	LQ(1)	LQ(2)	LM(1)	J.T.A.
Δs_t	-1.40	0.04	1.57				-0.014	0.50			-0.008	1.00	-0.004	0.63	0.015	2.28	0.005	2.70
	(-7.37)	(3.49)	(1.41)				(-2.61)	(1.86)										
Δj_t	0.22	0.08	-1.98	-9.37	2.93	-0.44	-0.16	0.45	6.61	-0.54	-0.012	1.00	-0.64	1.48	0.171	0.81	1.46	4.69
	(0.97)	(2.11)	(-2.51)	(-29.15)	(18.70)	(-1.93)	(-3.53)	(2.10)	(5.82)	(-14.73)								
Δz_t	1.86	-2.52		-1.00							-0.001	1.00	-0.09	1.55	0.65	0.84	2.04	3.44
	(30.22)	(-19.50)		(-2.99)														
Δk_t	0.79	-0.40			0.49						0.007	1.00	0.06	0.77	3.18	3.35	2.15	3.64
	(8.77)	(-16.02)			(3.50)													

Conditional variance equations

	ϖ	α	β	ρ_{12}	ρ_{13}	ρ_{14}	ρ_{23}	ρ_{24}	ρ_{34}	LLF
$h_{s,t}^2$	1.02	0.87	0.08	0.06	-0.04	-0.06	0.12	0.30	0.04	
	(4.34)	(39.37)	(3.63)	(2.54)	(-0.91)	(-1.36)	(7.24)	(7.76)	(1.01)	2941.60
$h_{j,t}^2$	0.17	0.88	0.07							
	(5.95)	(113.86)	(7.83)							
$h_{z,t}^2$	0.16	0.74	0.01							
	(9.89)	(35.53)	(1.81)							
$h_{k,t}^2$	0.82	0.55	0.12							
	(13.82)	(24.80)	(5.09)							

Notes. $\nu_t = u_t / \sqrt{h_t^2}$; Sk.: Skewness; Kurt.: Kurtosis; LM(k): Lagrange Multiplier test for kth order ARCH; LQ(k) Ljung Box Q-statitic for kth order serial correlation; J.T.A.: Joint Wald test of the null hypothesis of no asymmetry, distributed as χ^2 with 3 degrees of freedom (Engle and Ng, 1993); t-statistics are in parentheses; the t-ratios are based on robust standard errors computed with the Bollerslev and Wooldridge (1992) procedure. These notes apply also to Table 5.

Table 4: Average values of the conditional mean CAPM components (CAPM comp.) **and feedback trading coefficients** (Fbt coef.)

Oil returns		**US dollar changes**	
CAPM comp.	1.52 (52.57)*	CAPM comp.	-1.83 (-256.51)*
Fbt coef.	-0.043 (-12.62)*		
US stock returns		**US bond returns**	
CAPM comp.	0.16 (6.30)*	CAPM comp.	-0.58 (-44.85)*
Fbt coef.	-0.035 (-6.23)*		

Notes. t-statistics (H_o: average = 0) are in parentheses; *: significant at the 5% level.

Table 5: Multivariate ICAPM, January 2000-October 2009

System (B) Conditional mean equations

	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}	$E[v_t]$	$E[v_t^2]$	Sk.	Kurt.	LQ(1)	LQ(2)	LM(1)	J.T.A.
Δs_t	-1.08 (-3.19)	0.11 (6.92)	3.15 (4.06)	-0.22 (-0.98)	-0.01 (-0.13)	-0.54 (-6.25)	0.01 (3.84)	0.28 (2.94)	-0.13 (-2.97)		0.09 (1.22)	-6.33 (-4.81)	0.67 (1.08)	0.02	1.00	-0.34	1.02	0.72	4.87	0.09	2.87
Δj_t	0.67 (6.51)	0.14 (4.47)	0.93 (7.39)	2.12 (4.65)	0.23 (2.63)	-0.31 (-7.09)	-0.02 (-5.56)	0.29 (2.55)	-4.41 (-7.29)	0.05 (1.77)		-8.28 (-12.5)		-0.09	0.99	-0.34	1.82	3.86*	3.95	0.88	5.93
Δz_t	0.36 (6.99)	-0.76 (-26.4)		-0.49 (-1.40)	-0.83 (-8.85)									-0.01	1.00	0.05	0.40	0.19	0.25	1.22	2.26
Δk_t	0.15 (-1.07)	-0.10 (-1.96)	-0.53 (-2.41)	-0.15 (-1.91)										0.05	1.00	-0.21	0.96	0.10	0.18	3.79	6.00

Conditional variance equations

	ϖ	α	β	ρ_{12}	ρ_{13}	ρ_{14}	ρ_{23}	ρ_{24}	ρ_{34}	LLF
$h_{s,t}^2$	17.60 (8.80)	0.25 (4.23)	0.15 (3.59)	-0.05 (-4.43)	-0.24 (-8.60)	-0.10 (-9.56)	-0.06 (-9.09)	-0.33 (-11.12)	-0.19 (-6.81)	-4366.29
$h_{j,t}^2$	0.71 (6.70)	0.61 (32.50)	0.29 (9.97)							
$h_{z,t}^2$	0.06 (12.36)	0.91 (107.8)	0.03 (5.39)							
$h_{k,t}^2$	0.15 (9.26)	0.79 (117.9)	0.17 (14.15)							

Note: * significant at the 5% level.

Table 6: Average values of the conditional mean CAPM components (CAPM comp.) **and feedback trading coefficients** (Fbt coef.)

Oil returns		**US dollar changes**	
CAPM comp.	1.42 (29.50)*	CAPM comp.	-0.40 (-61.79)*
Fbt coef.	-0.25 (-50.81)*		
US stock returns		**US bond returns**	
CAPM comp.	-0.36(-16.72)*	CAPM comp.	0.41 (19.61)*
Fbt coef.	-0.09 (-18.14)*		

Notes. t-statistics (H_o: average = 0) are in parentheses; *: significant at the 5% level.

Table 7. Portfolio analysis

	No Portfolio Rebalancing			Weekly Portfolio Rebalancing 12 Month Holding Period			Weekly Portfolio Rebalancing 6 Month Holding Period		
	Without Oil	With Oil Spot Price	With oil Futures Price	Without Oil	With Oil Spot Price	With oil Futures Price	Without Oil	With Oil Spot Price	With oil Futures Price
Unconditional Mean									
First Sub-sample 10/6/92 12/28/99	0.0937	0.0909	0.0908	0.1010	0.0924	0.0940	0.1050	0.0966	0.0978
Second Sub-sample 1/4/00 10/27/09	0.0477	0.0564	0.0584	0.0527	0.0627	0.0606	0.0648	0.0753	0.0726
Unconditional Variance									
First Sub-sample 10/6/92 12/28/99	0.5270	0.5016	0.5037	0.4953	0.4631	0.4679	0.4440	0.4042	0.4082
Second Sub-sample 1/4/00 10/27/09	0.5076	0.4354	0.4207	0.3762	0.3248	0.3065	0.3582	0.2905	0.2787

Average Risk Return Differential

	No Portfolio Rebalancing		Weekly Portfolio Rebalancing 12 Month Holding Period		Weekly Portfolio Rebalancing 6 Month Holding Period	
	Δ_s	Δ_F	Δ_s	Δ_F	Δ_s	Δ_F
First Sub-sample 10/6/92 12/28/99	0.0009	0.0013	0.0041 [0.833]	0.0004 [0.984]	0.0372 [0.111]	0.0311 [0.177]
Second Sub-sample 1/4/00 10/27/09	-0.0186	-0.0233	-0.2078 [0.000]	-0.1453 [0.000]	-0.1454 [0.000]	-0.0965 [0.037]

Notes. Δ_s : average return per unit of risk differential, computed subtracting the average return per unit of risk of the four asset portfolio (with oil priced on the spot market) from the average return per unit of risk of the three asset portfolio (without oil). Δ_F : average return per unit of risk differential, computed subtracting the average return per unit of risk of the four asset portfolio (with oil priced on the futures market) from the average return per unit of risk of the three asset portfolio (without oil). The probability values of the z-test statistics of the null hypothesis $H_0 : \Delta_{s,F} = 0$ are set out in square brackets. Following Ledoit and Wolf (2008), they are corrected for heteroskedasticity and serial correlation at one lag using the Newey and West (1987) procedure.