An empirical study of corporate bond pricing with unobserved capital structure dynamics.

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Abstract

This work empirically examines six structural models of the term structure of credit risk spreads: Merton (1974), Longstaff & Schwartz (1995) (with and without stochastic interest rates), Leland & Toft (1996), Collin-Dufresne & Goldstein (2001), and a constant elasticity of variance model. The conventional approach to testing structural models has involved the use of observable data to proxy the latent capital structure process, which may introduce additional specification error. This study extends Jones, Mason & Rosenfeld (1983) and Eom, Helwege & Huang (2004) by using implicit estimation of key model parameters resulting in an improved level of model fit. Unlike prior studies, the models are fitted from the observed dynamic term structure of firm-specific credit spreads, thereby providing a pure test of model specification. The models are implemented by adapting the method of Duffee (1999) to structural credit models, thereby treating the capital structure process is truly latent, and simultaneously enforcing cross-sectional and time-series model constraints. Quasi-maximum likelihood parameter estimates of the capital structure process are obtained via the extended Kalman filter applied to actual market trade prices on 32 firms and 200 bonds for the period 1994 to 2000.

We find that including an allowance for time-variation in the market liquidity premium improves model specification. A simple extension of the Merton (1974) model is found to have the greatest prediction accuracy, although all models performed with similar prediction errors. At between 28.8 to 34.4 percent, the root mean squared error of the credit spread prediction is comparable with reduced-form models. Unlike Eom, Helwege & Huang (2004) we do not find a wide dispersion in model prediction errors, as evidenced by an across model average mean absolute percentage error of 22 percent. However, in support of prior studies we find an overall tendency for slight underprediction, with the mean percentage prediction error of between -6.2 and -8.7 percent. Underprediction is greatest with short remaining bond tenor and low rating. Credit spread prediction errors across all models are non-normal, and fatter tailed than expected, with autocorrelation evident in their time series.

More complex models did not outperform the extended Merton (1974) model; in particular stochastic interest-rate and early default accompanied by an exogenous write-down rate appear to add little to model accuracy. However, the inclusion of solvency ratio mean-reversion in the Collin-Dufresne & Goldstein (2001) model results in the most realistic latent solvency dynamics as measured by its implied levels of asset volatility, default boundary level, and mean-reversion rate. The extended Merton (1974) is found to imply asset volatility levels that are too high on average when compared to observed firm equity volatility.

We find that the extended Merton (1974) and the Collin-Dufresne & Goldstein (2001) models account for approximately 43 percent of the credit spread on average. For BB rated trades, the explained proportion rises to 55 to 60 percent. For investment grade trades, our results suggest that the amount of the credit spread that is default related is approximately double the previous estimate of Huang & Huang (2003).

Finally, we find evidence that the prediction errors are related to market-wide factors exogenous to the models. The percentage prediction errors are positively related to the VIX and change in GDP, and negatively related to the Refcorp-Treasury spread.

Certificate

This is to certify that:

(i) the thesis comprises only my original work,

(ii) due acknowledgment has been made in the text to all other material used,

(iii) the thesis is less than 100,000 words in length, exclusive of tables, maps, bibliographies and appendices.

Iain Campbell Maclachlan

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Chapter 1

Introduction

Understanding credit risk is central to the smooth operation of capital markets and stability of the banking system. Credit risk is the risk of unexpected value changes of debt instruments due to changes in perceived default risk of the issuer, that is, the firm's inability to meet promised debt payments in a timely manner. Financial claims that represent a promise to pay include a risk premium to compensate the holder for lost value in the event of default by the obligor. Structural credit models describe default risk in terms of unexpected movements in total firm value, where total firm value comprises the market value of debt and equity instruments issued by the firm.

This study presents empirical tests of alternative structural models of the defaultable bond credit spread, in which we adopt a novel approach of estimating the models directly from observed credit spreads. We incorporate a test of the Merton (1974, hereafter Merton) model, and compare its predictive accuracy against a representative sample of recent theoretical extensions that have relaxed the strong assumptions of the original Merton model.

Robert Merton first proposed a structural approach in 1974 by drawing on the seminal option valuation model of Black & Scholes (1973). Since then, there has been a plethora of theoretical extensions that can be jointly classified as structural models. The essential characteristic of a structural model is that the default risk premium is a function of the potential for the latent firm value process to reach a lower default boundary. In this respect, the problem of defaultable debt valuation is akin to the problem of valuing an option in which the unobserved firm value is the underlying variable, and an unobserved lower value of the firm is the strike price.

Structural credit models are central to financial theory and credit risk-management practice. For example, because structural models relate observable debt prices to assumptions about the firm's asset risk, leverage, and dynamic behaviour of management, they offer a link between capital structure theory and asset valuation theory. Testing alternative structural models is an indirect test of capital structure theory to the extent that the market anticipates and prices into the term structure of credit spreads, expected management behaviour.

For finance practitioners, the recent growth in traded credit derivative markets and development of portfolio credit models has been supported by use of variations of the Merton model. For example, one of the most widely used commercial structural models is Moody's-KMV Credit Monitor, which is used by banks and traders to guide risk-debt valuation and the risk management of debt portfolios. As observed in the financial press, '... Merton models are now so frequently used that they are actually driving the credit market', (Ferry 2003).

Importantly, structural models also hold a central position in modelling the adequacy of bank capital and estimation of bank solvency. Crouhy, Galai & Mark (2000) describe a common method for constructing economic capital models for credit risk based on particular assumptions about multiple obligor default consistent with the Merton model. Recently, this method of capital modelling has been embedded in global bank regulation issued by the Bank for International Settlements, commonly known as Basel II. Under recent changes, the minimum capital formula to be applied by all internationally active advanced banks is based on a result obtained by Gordy (2002). He derives a closed-form solution for the risk weighting of each loan in a bank's portfolio under the assumption that joint defaults are explained by a structural model incorporating a single source of common firm-asset return.

The competing method of defaultable debt valuation is termed 'reduced-form' and is exemplified by Jarrow & Turnbull (1995), Madan & Unal (1999), Duffie & Singleton (1999), Duffee (1999), and Bakshi, Madan & Zhang (2001). Reduced-form models value defaultable debt instruments by assuming that the firm may default at any instant of time. The rate of instantaneous default is assumed to evolve stochastically through time by an exogenously specified stochastic process. There is no relationship assumed between the instantaneous default rate process and firm value. Reduced-form models offer mathematical tractability, and have achieved success at valuing defaultable debt. For example, Duffee (1999) reports an ability to predict bond yields reasonably well using a translated two-factor square-root diffusion model of the instantaneous default rate. A similar result is reported by Bakshi et al. (2001) who consider alternative specifications and conclude that reduced-form models, which include leverage and book-to-market ratio firm-specific information, offer the best fit.

However, reduced-form models do not attempt to explain credit spreads by firm capital structure theory and are therefore less rich in their implications. We therefore focus attention on empirical testing of structural models in preference to reduced-form models.

Despite the central role of Merton-style structural models in financial theory, market practice, and global bank regulation, the models have to date been unable to accurately predict observed levels of credit spreads. The empirical weakness of the model at asset valuation is well established in a series of studies. Commencing with Jones, Mason &

Rosenfeld (1983, hereafter JMR), who find that a Merton-type model is unable to price investment-grade corporate bonds better than a naive model that assumes no risk of default. The Merton model is found to generally overvalue bonds and underestimate credit spreads. Pricing errors are related to equity variance, leverage, maturity, and time period. A subsequent study by Lyden & Saraniti (2000, hereafter LYS) tests whether the Longstaff & Schwartz (1995, hereafter LS) model, which includes a more realistic specification of the default process and includes a stochastic risk-free rate, improves on the performance of the Merton model. Their study is the first to explore the broader class of structural models in a systematic manner. Like JMR they find an overall underprediction of credit spreads with prediction errors related to coupon and remaining time to maturity of the bond. Significantly, the LS model was found to not improve prediction accuracy. Finally, the most comprehensive test of structural models was most recently conducted by Eom, Helwege & Huang (2004, hereafter EHH). Using a similar method to JMR and LYS, EHH test a version of the Merton model and four subsequent theoretical extensions. Confirming the prior studies, the Merton model underpredicts credit spreads on average, however, they also find that the other models tend to overpredict credits spreads on average. Valuation prediction accuracy is found to be very poor with the newer structural models tending to severely overstate credit spreads on firms with high leverage or high asset volatility, yet underpredict credit spreads on safe bonds. Thus, it is well founded in the extant literature that the Merton model underpredicts credit spreads, and more so for short-tenor debt, and debt issued by default-remote 'safe' firms.

We therefore face the problem that the fundamental economic model for valuing default risk, which has immense practical and social value in its application, has failed to pass its most basic test; to explain real world credit spreads. Recently, a plethora of theoretical extensions have been proposed to address the underprediction problem. A line of inquiry pursued by the extant theoretical literature has been to relax the simplistic assumptions of the Merton model. Notwithstanding, these theoretical enhancements, the empirical literature has conspicuously lagged and failed to provide conclusive guidance on which model is superior. Further, LYS and EHH's results suggest that some of these enhancements have not increased valuation accuracy over and above the original Merton model.

Despite performing poorly at debt valuation, structural model predictions have been found to be highly correlated with other measures of firm default risk. Ogden (1987) uses probit analysis to regress firm credit ratings against firm volatility and leverage as an indirect test of the Merton model. He finds that their two variable model explains 79 percent of the cross-sectional variation in issuer ratings and the predicted credit spread explains nearly 60 percent of the variation in observed market spreads. Delianedis & Geske (1998) extract risk-neutral probabilities of default from the Merton and Geske (1977) structural models and perform an event study with rating migrations. They find first that these implied probabilities of default, from both structural models, possess significant and very early information about subsequent credit rating migrations. Tudela & Young (2003) further show that the implied probabilities of default from a Merton-style model are successful in discriminating between failing and non-failing rms. Their implementation of the Merton approach outperforms a reduced-form model based solely on company account data. Thus, practitioner use of structural models is supported by these studies when used as an early indicator of default risk and rating migration, or for assessing relative credit spreads between firms.

A possible reason for the contradictory performance of structural models may be due to the common use of proxy variables to represent latent variables, thereby introducing a source of estimation error. A proxy variable is a physical representation of a latent variable that is used in place of indirect estimation of the latent variable. A proxy for firm asset value is usually calculated as the sum of market value of equity and book value of debt, with the default boundary arbitrarily set to be variously: the book value of liabilities; the book value of debt; or, some proportion of either. The proxy method has been extensively used in the extant empirical literature by JMR, LYS, EHH, and partially by Huang & Huang (2003, hereafter HH). Since the total market value of the firm is rarely traded, it is not possible to directly estimate the stochastic process of the firm through historical observation of the firm's return. Similarly, it is even less clear where the default boundary of the firm may be as this is not observable unless ex post after the firm defaults.

In contrast, the reduced-form literature, has followed a different estimation approach. With no theoretical relationship posited between the stochastic default process and firm capital structure process, researchers have relied on implicit estimation of parameters from observed bond prices. For example, estimation methods such as quasi maximum likelihood using the extended Kalman filter (Duffee 1999), or maximum likelihood (Duffie, Pederson & Singleton 2000). These methods ensure that the models are calibrated with minimal average prediction bias. An obvious advantage of such an approach is that the models are fitted as well as they can be to the data. The residual errors are therefore related to model specification and not introduced by choice of proxy variable.

Maximum likelihood estimation methods have been applied in a limited way to structural credit models, and suggest that prediction errors can be controlled to levels found in reduced-form model implementations. Ericsson, Reneby & Wang (2003) fit a structural model on the time-series of firm equity prices using a maximum a likelihood method from Duan (2004). They test only one structural model specification but find prediction errors to be smaller and distinctly less variable than those found in previous implementations of structural and reduced-form models. Bruche (2005) repeats Ericsson et al. (2003) and achieves a similar result with a non-linear simulated maximum likelihood method. He shows that a maximum likelihood method, using non-linear ltering, is superior to the exact maximum likelihood method of Ericsson et al. (2003) because measurement error can be explicitly included, thus avoiding serious bias in structural model parameter estimates. Given the presence of market micro-structure, taxes, and price recording errors, it is unreasonable to assume zero measurement error. We adopt a similar method to Bruche (2005) and include specific allowance for measurement error, but use the more widely practiced estimation method of the extended Kalman filter (Cumby & Evans 1995, Claessens & Pennacchi 1996, Duffee 1999, Keswani 2005).

Secondly, the proxy method treats the capital structure of the firm as observable, however, the firm's observed debt-ratio is unlikely to be a sufficiently precise measure of firm solvency to be accurate in debt valuation. In an examination of defaulted firms, Davydenko (2005) reports that firms default, on average, when their assets are 72 percent of the value of the face value of debt. However, as many as one-third default when asset values are above this point, and an equal number of firms below it avoid default for at least a year. He concludes that, 'even if boundary-based models can be calibrated to predict the average probability of default, they are still likely to lack accuracy in the cross-section.' Further, in the presence of recapitalisation costs, Fischer, Heinkel & Zechner (1989) show that firms will be unwilling to adjust their debt-ratios unless there is a sufficiently large shock away from their desired target. The term structure of a firm's credit spreads contains information about expected changes to the firm's future debt levels, but in the presence of costly capital reorganisation costs, we can expect a large cross-sectional variation in the credit spreads of longer dated bonds not explained by current gearing ratios. A better method of estimation is to cast each model in state-space form, in which the transitional density of the firm's observed time-series of credit spreads is represented by a measurement equation with error, that is dependent upon the transition of the firm's latent capital structure process. The latter is determined by the theoretical form of the structural model and is implied by the observed pattern of credit spreads.

Finally, another weakness of the extant empirical literature is the failure to control for non-default related premiums. It is widely accepted that the credit spread contains premiums for more than default risk, for example, tax and liquidity (Delianedis & Geske 2001, Elton, Gruber, Agrawal & Mann 2001). Without control for the these components of the credit spread, the proxy estimation method will result in underpredicted credit spreads, and a maximum likelihood method will overstate asset volatility and understate solvency. To understand the influence of non-default components of the spread on model estimates and performance, we fit the models with three different empirical equations: no premium assumed as a base case; a constant premium per bond; and finally a constant, and time-varying premium combined, where time-variation is controlled by inclusion of the Refcorp ten year constant maturity spread in the measurement equation.

While Ericsson et al. (2003) and Bruche (2005) demonstrate the advantages of maximum likelihood methods over the proxy estimation method, neither study systematically tests a range of structural models. The question of whether theoretical developments have improved the Merton model, remains open to question. There is therefore a gap in the literature between the extant discussion of corporate structural model specification by EHH, and the advantages of improved estimation techniques suggested by Ericsson et al. (2003), Bruche (2005), and as implemented in the reduced-form literature by (Duffee 1999).

Our hypothesis is that the apparent poor prediction accuracy observed in the extant empirical literature is the result of estimation methods that assume that the firm asset process is observable, or is otherwise closely correlated with an observable proxy. Furthermore, it is hypothesised that the prediction errors evident from structural credit models are related to omitted factors that can be inferred from the extant theoretical and empirical evidence concerning the dynamic behaviour of firm capital structure.

To test the first part of our hypothesis, we fit five different structural models similar to EHH, but on firm specific term structure of credit spreads. Unlike the extant literature, we infer model parameters from the term structures directly, thereby avoiding the potential errors introduced in model estimation from proxy variables. We then compare our prediction errors with those of EHH and HH. The second part of our hypothesis is tested by regressing independent variables against the model prediction errors where the selection of variables is guided by stylised facts gathered from a review of the capital structure literature.

This study extends EHH with the introduction of alternative estimation methods and data. We therefore continue the line of enquiry that commenced with JMR and LYS. Our estimation method is a non-linear extended Kalman filter adapted from Duffee (1999) for the use with structural models. A unique feature of our study is that we estimate the models directly from firm-specific credit spreads measured across the term structure, with constant and time-varying controls for the liquidity premium. Specifically, our study introduces the following enhancements to the EHH method:

1. the use of high frequency real trade data. Frequent data observations are more appropriate when estimating the specifications of the firm's continuous latent asset process;

2. the use of more than one bond on issue by the firm. We fit the models to the term structure of credit spreads thereby enforcing both cross-sectional valuation and transitional distribution model constraints;

3. models are estimated via quasi maximum likelihood, with controls for unexplained components of the credit spread, thereby ensuring the best possible fit to the data. Any biases that remain are therefore attributable to model specification and not ad hoc proxy variable selection;

4. missing data and measurement error is explicitly allowed for by the transformation of the models into state space form and subsequent filtering.

There is strong evidence that the credit spread is unlikely to contain compensation for default risk alone; liquidity, tax, and other non-credit related factors have been found to be embedded in credit spreads (Elton et al. 2001, Delianedis & Geske 2001, Longstaff 2002). When referring to 'credit' spreads we refer to the difference in yield between the yield to maturity of corporate bonds and the yield to maturity of same maturity Treasury bonds, and acknowledge that there may exist some compensation for other factors.

The models chosen for examination are similar to EHH and HH and enable comparison with their results. We select models where tractable solutions are available. Firstly, we implement the extended Merton model of EHH. This serves a useful comparison between our results and EHH. Secondly, we implement two versions of the LS model. This model represents an exogenous boundary model, and is the first major extension of the Merton model which includes many features subsequently adopted in the theoretical literature. It has also been studied by EHH and LYS. The first version, hereafter referred to as the LS1 model, holds the risk-free rate constant, and is comparable to the base case model of HH. The second version, hereafter referred to as the LS2 model, incorporates a stochastic risk-free rate. An endogenous boundary model is represented by the Leland & Toft (1996, hereafter LT). This model has also been fitted by HH and EHH. Next we consider the most commonly studied dynamic boundary model of Collin-Dufresne & Goldstein (2001, hereafter CDG). Finally, an alternative to the usual asset distribution assumption of geometric Brownian motion is tested by way of a constant elasticity of variance model (hereafter CEV). This model has the property that asset variance increases as default is approached, thus consistent with hypothesised management behaviour under the agency theory of (Jensen & Meckling 1976). A structural CEV model of capital structure was first proposed by Barone-Adesi & Colwell (1999), and has been independently suggested for valuing equity default swaps (Albanese & Chen 2005, Campi & Sbuelz 2005).

This study is the first to systematically compare a range of structural models for miss-specification using quasi maximum likelihood methods, thus limiting the influence of estimation errors introduced by the use of proxy variables. We ask which of the theoretical extensions tested, if any, have improved model specification. Secondly, we ask whether the prediction error biases are consistent with the prior literature, and what theoretical developments can be inferred from their presence after consideration of the biases and the related capital structure literature. Further, we provide a comparison to HH by asking how much of the credit spread is explained by structural models, when the structural models are fitted directly from credit spreads. Finally, we determine where the implied default boundary is relative to the firm's debt level.

We find that all models underpredict short-term spreads on low leveraged firms and on short-term debt. Recent theoretical extensions to include stochastic interest rates and firm target debt management behaviour do not appear to have significantly improved model performance. Prediction errors across all models are non-normal and fatter tailed than expected with autocorrelation evident in their time series. Inclusion of dynamic recovery risk, linked to an observable business cycle proxy, and a firm asset jump, appear to be necessary theoretical enhancements.

Chapter 2 sets out a review of the theoretical and empirical literature, related capital structure theory and evidence, and a summary of extant research methods and findings. In Chapter 3, the data and econometric estimation method is discussed including an explanation of the empirical forms of the models to be tested. Findings are detailed in Chapter 4, including a discussion of diagnostic results, comparison of implied and observed solvency, a quantification of the extent to which credit spread levels are explained by default risk models, and testing for potential missing variables. Chapter 5 concludes.

Chapter 2

Literature Review

2.1 Introduction

Since the seminal work of Black & Scholes (1973) and Merton (1974) the theoretical credit literature has grown rapidly, however, empirical studies have been relatively few, particularly in the comparison of competing models. The aim of the chapter is to summarise past empirical work, showing where this study contributes to the extant empirical literature, and to describe the choice of models and potential sources of miss-specification that may exist in the extant models.

In this chapter we review of the extant credit risk models placing particular focus on assumptions made about firm asset process and capital structure dynamics. The central difference between a reduced-form approach to bond valuation, versus the structural model approach, is that the latter posits that credit spread dynamics are a function of management decisions concerning the firm's physical capital structure. Underpinning all structural models of credit risk is a hypothesised relationship between the dynamic process of the firm's solvency and its credit spreads; ceteris paribus, short-term credit spreads are a function of present debt levels, long-term spreads are also a function of the firm's expected future debt-ratio, and the change in credit spreads is a function of the firm's expected rate of change in its debt-ratio. For the sole purpose of bond valuation, it is not necessary that the structural model be based on an accepted theory of the firm; reduced-form models are an example of a valuation approach that works without seeking theoretical justification. However, a theoretically well-founded specification is more likely to prove a more robust valuation model.

We therefore review the related capital structure theory and evidence, in order to place the structural models in perspective of the theory of the firm. We classify the theoretical literature by the manner in which the firm's default boundary is assumed to evolve; distinguishing between those where the default boundary level is assumed to be exogenous to the model, and those where it is determined by management reacting endogenously within the model. We further distinguish between the assumption of static and dynamic variation of the default boundary.

We show how the theoretical models of capital structure and empirical evidence of capital structure dynamics have influenced the design of extant structural credit models. We summarise the capital structure evidence into a set of stylised facts. Gaps and inconsistencies in the application of capital structure theory are identified as possible sources of structural model miss-specification suitable for further testing.

The remainder of the chapter is set out as follows. The difference between reduced-form and structural credit models is explained in Section 2.2. Section 2.3 describes the Black-Scholes-Merton model, its assumptions, and their limitations. In Section 2.4 we review a sample of main structural models, classifying models by the assumed properties of the default boundary. Section 2.5 briefly reviews the two main theories of capital structure, Pecking-Order and Trade-Off theory, describing the static and dynamic forms. A set of stylised facts are gathered and their implication on debt valuation and structural model design is discussed. Finally, in Section 2.6 the extant empirical literature on structural model testing is reviewed noting the contribution of this study.

2.2 Reduced-Form versus Structural Models

There are two theoretical approaches to the valuation of default-risky bonds. The first, termed 'structural modelling', characterises default as the result of the firm's asset value failing to exceed a future critical value, termed the default boundary, at which point default is triggered. Black & Scholes (1973) and Merton are the earliest examples. Credit spreads result from a theoretical model describing the firm value process and its lower default boundary. Equivalently, a structural model can be described as having an underlying ratio of firm value to the default boundary that represents a measure of economic solvency.

The second approach does not impose a theoretical structure to the specification of default. In the reduced-form literature, default is assumed to be a random event with a probability governed by a known intensity, or hazard rate, process. Default is therefore always an unexpected surprise. There is no attempt to parameterise default intensity from any underlying theory of the firm and the dynamics of the firm's underlying solvency, instead the intensity is derived directly from credit spreads or other observable data such as credit ratings. Examples of this approach include: Jarrow & Turnbull (1995), Jarrow (1997), Duffie & Singleton (1999), and Madan & Unal (1996).[1] In contrast, the default event in structural modelling depends upon the first passage of a continuously diffusive state variable to a fixed boundary. In continuous time, default is instantaneously predictable and therefore never a surprise.

[1]Nandi (1998) gives an overview of earlier reduced-form models. Duffie & Singleton (2003) and Schönbucher (2003) discuss more recent developments including methods of estimation.

Where pricing data is readily available, the reduced-form approach is able to replicate observed credit spreads relatively simply and in a manner consistent with no arbitrage. Thus, it fits neatly into the financial engineer's toolkit for relative debt pricing. Of course if price data is limited (the market is not complete), reduced-form models are of limited assistance. This is a particular problem for credit modelling where there are often insufficiently similar assets available to fully span the market on default risk for an individual issuer. Litterman & Iben (1991) demonstrate how different firms can be grouped into cohorts of similar rating agency grades and industry, under a strong assumption of homogeneity, in order to achieve sufficient data points to construct a term structure of hazard rates.

On the other hand, structural models are well known for being difficult to estimate since the underlying stochastic process driving the risk of default is unobserved. Despite this, structural models are worthy of study for two reasons. Firstly, they are valuable in their own right as tools for theoretical development. Structural models relate debt valuation to an underlying theory of the firm directly, or to stylised facts that obtain from our understanding of how the firm manages default risk and bankruptcy costs. Credit spread term structures are the market's perception of how management will balance the costs and benefits of debt, over time, responding to shocks in the firm's operating cash flows, and the value of its assets. Direct measurement of the firm's debt management policy is difficult since the firm's capital structure target, if it exists, is not directly observable and may be masked by the influence of transaction costs. Robust structural models, estimated from market prices, could bring additional information to the problem of understanding dynamic capital management, and whether firms do target a debt-ratio or not, as perceived and encapsulated in prices by the debt markets. Secondly, structural models are finding uses in risk management for the estimation of firm-specific default risk, pricing of credit, and portfolio modelling of joint default risk including being the basis for the setting of regulatory capital for the largest international banks operating under the Basel II supervisory accord.

We focus our attention on structural models of credit risk, and in particular, the empirical estimation of several competing models, using a latent variable approach that minimises the potential confounding error present when the firm's solvency is incorrectly assumed to be observable, and well approximated, by accounting and stock information.

2.3 The Black-Scholes-Merton Model

Black & Scholes (1973) first proposed that debt and equity are contingent claims on the assets of the firm. Their argument is elegantly simple. In a leveraged firm, shareholders have a claim to the residual value of the firm with a payoff analogous to a call option. Assume the firm is financed by equity and a single zero-coupon bond and there are no transaction costs, taxes, and other market imperfections. At maturity of the bond, if firm value is insufficient to pay the bondholders, the shareholders will rationally default to floor their loss at zero, and allow ownership of the firm to transfer to the bondholders. If firm value exceeds the face value of debt, shareholders maximise their wealth by paying bondholders and receiving the value of the firm in return. Therefore, a share represents the right to buy the firm's assets from bondholders contingent on the future value of the firm relative to the face value of debt. The value of the default-risky debt is established by put-call parity arguments to be equivalent to the value of a riskless bond and a short put option on the firm's assets. Credit risk is a function of the capital structure and dynamics of the firm. This simple insight represents the classic economic model of credit risk that links debt and equity valuation to the rational behaviour of shareholders, the volatility of the firm's assets (its operational risk), and the level of firm gearing. For different bond maturities, a term structure of bond values, or credit spreads, can be fully determined.

A shortcoming of the model is the simplicity of its assumptions. While the model can show how bond value will change in response to leverage and therefore the influence of gearing on borrowing costs, it does not explain what the optimal choice of debt for the firm should be, or why the firm has the present level of debt. Without the presence of taxes and bankruptcy costs, there is no benefit of debt to offset the increased costs of borrowing. The model is therefore unable to satisfactorily link asset value to capital structure theory. In Section 2.3.1 we discuss the model's assumptions, and limitations, more fully.

2.3.1 Theory

Merton applies Black & Scholes (1973) directly to the valuation of default-risky bonds. To keep the debt valuation solution tractable, he makes the following simplifying assumptions:

[A1] Markets are complete and frictionless; no transaction costs, taxes, or other market imperfections including bankruptcy costs. Trading of the firm's assets takes place in continuous-time;

[A2] The value of the firm is independent of the capital structure of the firm. This is the Modigliani & Miller (1958) theorem. It follows from [A1] that, in the absence of taxes and bankruptcy costs, firm value is independent of the level of debt;

[A3] The risk-free rate, r, is the same for all maturities over time;

[A4] Firm value return is stochastic described by geometric Brownian motion with drift

$$\frac{dV(t)}{V(t)} = (r - \delta)dt + \sigma_v dW(t)^Q \tag{2.1}$$

where $V(t)$ is the stochastic market value of the firm, r is the risk-free rate, δ is the net payout to other claimants (for example, dividend payments), σ_v is the time-invariant volatility of the firm's assets, and $W(t)^Q$ is a standard Weiner process under the risk-neutral measure;

[A5] The firm is leveraged by a single-zero coupon bond liability with face value F, payable at maturity T.

The assumptions imply a simplistic theory of capital structure. The firm's market value leverage will evolve stochastically up to the point of maturity of the debt. Shareholders are indifferent to the intermediate levels of gearing, and passively wait for the outcome of whether firm assets exceed the face value of debt or not. Likewise, bondholders are indifferent to the firm's intermediate levels of gearing, and hold no solvency covenants that may have otherwise enabled them to trigger early default.

At maturity of the bond, $t = T$, if assets fail to exceed the required repayment on debt, F, it is rational for value maximising shareholders to not pay, thereby limiting their payoff to zero. Otherwise, if $V(T) > F$, it is optimal for them to pay F and receive $V(T) - F$. The payoff at maturity to shareholders is therefore given by, $E(T) = \max[0, V(T) - F]$. Conversely, the value at maturity of the bond is obtained from put call parity as $P(T) = F - \max[0, F - V(T)]$. Thus, the payoff to bondholders is equivalent to a portfolio comprising a riskless bond paying face value, F, and a short 'put-to-default' option on the firm. From (Black & Scholes 1973), Merton shows the value of the default-risky bond, at time $t = 0$, to be

$$P(0,T;V(0),F,\Theta) = Fe^{-rT}\left(N(d_2) + \frac{V(0)}{F}e^{(r-\delta)T}N(-d_1)\right) \tag{2.2}$$

where

$$d_1 = \frac{\ln\frac{V(0)}{F} + (r - \delta + \frac{\sigma_v^2}{2})T}{\sigma_v\sqrt{T}} \tag{2.3}$$

$$d_2 = d_1 - \sigma_v\sqrt{T} \tag{2.4}$$

and $N(\cdot)$ denotes the area under the standard cumulative normal distribution and the conditioning parameters are $\Theta = \{r, \sigma_v, \delta\}$. The bond's credit spread is defined as the difference between the risky yield to maturity and the risk-free spot rate,

$$s(0,T;V(0),F,\Theta) = -\frac{1}{T}\ln\left(\frac{P(0,T;V(0),F,\Theta)}{F}\right) - r\,. \tag{2.5}$$

It should be noted that the underlying capital structure process is risk-neutral. That is, under the assumption of no-arbitrage, debt valuation can be performed as if investors do not require a premium for the uncertainty of their investment, and require only the risk-free rate of return. The probability of the firm's assets crossing the default boundary (i.e. the firm defaulting), and the expected payoff in default, do not include any compensation for a risk premium. For debt valuation purposes it is not necessary to distinguish between risk neutral and real-world probabilities, however, the latter is strictly lower due to the additional premium for risk taking by risk-averse investors. It follows that parameters of the asset process obtained implicitly from asset prices will overestimate the true real-world (or physical) probability of default.

2.3.2 Firm solvency dynamics

LS first noted that for the purpose of valuing debt, it is not knowledge of the stochastic process followed by the firm's assets per se, but rather knowledge of the process followed by the ratio of the firm's value of assets, V, to the default boundary, K. We define the firm's log-solvency ratio to be $x(t) = \ln V(t) - \ln K(t)$, and following LS, a description of the stochastic differential process for $x(t)$ is then sufficient to value debt without knowledge of the firm's asset value or the default boundary level. The log-solvency ratio, $x(t)$, can be interpreted as a continuous measure of the firm's solvency. The larger its value, the greater the buffer between the firm's present value and its default boundary, hence the increased ability of the firm to absorb unexpected shocks and remain solvent. A similar measure, popularised by Moody's-KMV for the purpose of default prediction, is the 'distance-to-default', or $DD(t)$ (Crosbie & Bohn 2002). This latter measure is the number of standard deviations the firm's assets are from the default boundary, when measured from time-t to the maturity of the bond at time-T given by the Merton model as

$$DD(t) = \frac{\ln V(t) - \ln K + (\mu - \sigma_v^2/2)T}{\sigma_v \sqrt{T}} \,, \tag{2.6}$$

where μ is the expected rate of return on the firm's assets. The firm's risk-neutral probability of default, measured from time-t to time-T, is given by $N(-DD(t))$.

The distance-to-default nests the log-solvency ratio. Thus, log-solvency ratio is economically equivalent to an unstandardised distance-to-default, measured instantaneously relative to immediate asset values, and not projected to the maturity of the bond, or other future date. Compared to the distance-to-default statistic, the log-solvency ratio is independent of bond maturity and does not require estimation of the firm's expected rate of return. However, unlike $DD(t)$, the log-solvency ratio is not a sufficient measure of default risk without knowledge of the firm's asset volatility. So whilst theoretically correct in use as a state variable for our purpose of debt valuation, it is not a complete a measure of firm default risk for the purpose of default prediction.[2]

[2]The empirical performance of the distance-to-default measure as a predictor of future default events has been assessed by Bharath & Shumway (2004), as contributory factor explaining forward risk-neutral hazard rates by Duffie, Saita & Wang (n.d.), and on firm equity return and prediction of default by Vassalou & Xing (2004). Since the liabilities of the firm vary over time, and bond maturities vary between firms, it is

Merton assumes the default boundary is equal to the face value of debt, which is constant for all time. Therefore, $K(t) = K = F \, \forall t \, \varepsilon [0,T]$ and $x(t) = \ln V(t) - \ln K$.

A simplifying assumption of Merton is that default can only occur at T, if $x(T) \leq 0$. From Ito's lemma and equation (2.1) it follows that the stochastic differential equation of $x(t)$ is arithmetic Brownian motion with constant drift

$$dx(t) = (r - \delta - \sigma_v^2/2)dt + \sigma_v dW(t)^Q \,. \tag{2.7}$$

With no loss of generality, we let $K = F = 1$ so that equation (2.2) is more compactly expressed as being conditional on the firm's initial log-solvency ratio, $x(0)$. The present, time t=0, value of a zero-coupon bond, with a one dollar face value payable at time t=T, is then

$$P(0,T;x(0),\Theta) = e^{-rT}N(d_2) + e^{x(0)-\delta T}N(-d_1) \tag{2.8}$$

where

$$d_1 = \frac{x(0) + (r - \delta + \frac{\sigma_v^2}{2})T}{\sigma_v \sqrt{T}} \,,$$
$$d_2 = d_1 - \sigma_v \sqrt{T} \,.$$

2.3.3 Default and Recovery

The Merton model defines a functional relationship between the risk-neutral probability of default and the risk-neutral expected payoff to bondholders in the event of default. This is illustrated in this section by expressing the Merton model into the notation of an equivalent reduced-form model. Let $Q(0,T;x(0),\Theta)$ be the risk-neutral probability of default at time T, and (1-ω)K be the risk-neutral expected payoff to bondholders in the event of default. Using the terminology of LS, the fraction of bond face value lost in bankruptcy, ω, is termed the 'writedown rate' and $1-\omega$ the 'recovery rate'.

Given the firm's asset return is assumed to be normally distributed in the Merton model, the risk-neutral probability of the firm defaulting at time-T, measured from t=0, is the area under the standard normal distribution where $V(T) < K$. The risk-neutral probability of default is therefore given by

$$Q(0,T;x(0),\Theta) = N(-d_2) \,. \tag{2.9}$$

The risk-neutral expected recovery rate is the expected value of $V(T)$, given that the firm

usual to set T=1, and replace K with $K(t)$, measured as the rms book measure of short-term debt, plus one half of its long-term debt, based on its quarterly balance sheet.

has defaulted and $V(T) < K$, expressed per dollar of K

$$(1-\omega) = \frac{E[V(T)|V(T)<K]}{K} = \frac{V(0)}{K} e^{(r-\delta)T} \frac{N(-d_1)}{N(-d_2)} \, . \tag{2.10}$$

Once again, we generalise with $F = K = 1$, and recognising that $x(0) = \ln V(0) - \ln K$, the risk-neutral expected recovery rate, per dollar of face value, is compactly expressed in terms of the log-solvency ratio as

$$(1-\omega) = e^{x(0)+(r-\delta)T} \frac{N(-d_1)}{N(-d_2)} \, . \tag{2.11}$$

The fair value of the default-risky bond is the present value of the risk-neutral expectation of the bond's payoff at time-T. The bond will pay either its one-dollar face value if the firm is solvent at time T, or the expected recovery, $(1-\omega)$ if in default. The fair value is therefore the given by [3]

$$P(0,T;x(0),\Theta) = e^{-rT}\big(Q(0,T;x(0),\Theta)(1-\omega) + (1 - Q(0,T;x(0),\Theta))\big) \, . \tag{2.12}$$

Thus, we show that the Merton model implies that a firm's risk-neutral default probability and its risk-neutral expected recovery rate are endogenous and are negatively correlated. A rise in default risk caused by an increase in asset volatility, or reduction in solvency, or lower asset growth, will also result in reduced expected bond recovery. An increase in leverage increases the likelihood of default and decreases the expected recovery for bondholders. An increase in volatility increases the value of the put-to-default option and therefore increases the credit spread by similarly increasing the probability of default and reducing the recovery for bondholders. The short rate and net payout jointly influence the drift of the firm's assets; the greater the positive drift the lower the likelihood of default and better recovery for bondholders.

Real-world, or physical probabilities, can be obtained from the Merton model with an adjusted drift rate. All previous results hold with the difference that the drift rate of the firm is adjusted upwards by the addition of an asset risk premium, π, so that the log-solvency process of the firm evolves as

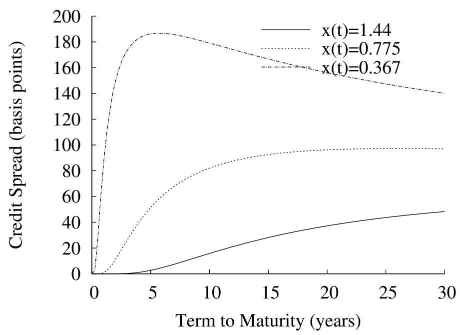
$$dx(t) = (r + \pi - \delta - \sigma_v^2/2)dt + \sigma_v dW(t)^P, \tag{2.13}$$

where P denotes a physical probability distribution. This result is used by HH to calibrate the Merton model parameters to observed historical default rates.

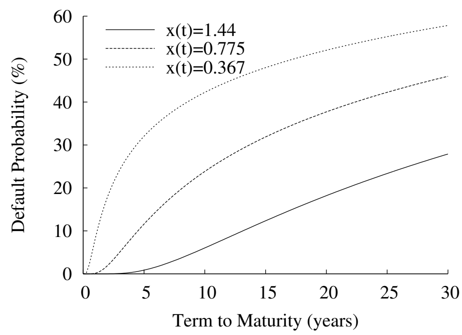
[3]Note that equation (2.8) can be obtained by substituting into equation (2.12), the risk-neutral expected default (equation (2.9)) and risk-neutral expected recovery (equation (2.11)).

2.3.4 The Predicted Term Structure of Credit Spreads

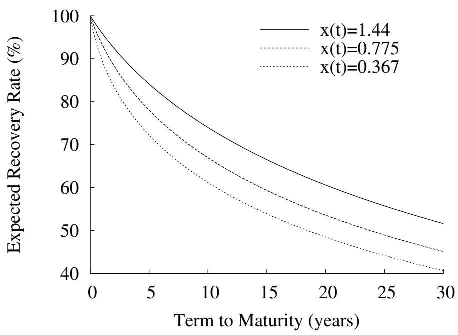
A firm's term structure of credit spreads is the set of credit spreads, observed jointly at time-t, for bonds issued by the same firm with different remaining maturities. The Merton model predicts that a bond's credit spread at time-t, is a function of its term to maturity, $T-t$, the log-solvency ratio of the issuing firm, $x(t)$, the firm's asset return volatility, σ_v, the risk-free rate, r, and the net payout to other claimants, δ. and By varying the remaining maturity, holding all other parameters constant, the Merton model predicts a theoretical term structure for the firm. In Figure 2.1(a), we plot the predicted



(a) Credit spreads



(b) Risk-neutral default probability



(c) Risk-neutral expected recovery rate

Figure 2.1: Shown in figure (a) is the credit spread term structure predicted by the Merton model at different initial levels of log-solvency, $x(t) = \ln V(t)/K$. The risk-neutral default probability is shown in (b), and the risk-neutral expected recovery rate is shown in (c). Initial log-solvency levels of 1.44 (low risk), 0.775 (medium risk), and 0.367 (high risk), are equal to the sample quartile levels of the log of the market solvency ratio. The market solvency ratio is calculated from CRSP and COMPUSTAT data as the sum of the firm equity value and book debt over book debt. Other parameters are σ_v = 25 percent, r=6 percent, and δ=5 percent.

credit spread term structures, assuming different levels of initial solvency, for a representative firm. Figure 2.1(b) shows the implied risk-neutral probabilities of default, and Figure 2.1(c) shows the risk-neutral expected recovery rates, corresponding to the model parameters and solvency levels. The initial level of unobserved log-solvency, $x(t)$, is set equal to the quartiles of our sample of observed log-solvency ratios, measured across trade dates and firms. For illustration only, we use the observed log-solvency ratio as a proxy for the unobserved latent firm solvency, $x(t)$. Table 3.5 reports the market credit spreads observed in our sample, pooled across time and firms, by quartiles of the market log-solvency ratio.

The observed log-solvency ratio is calculated as the firm's market value of equity plus book value of debt, divided by the book value of debt. Computation of the observed solvency ratio is explained further in Section 3.2.6 where we use it to drive our initial firm-specific estimate of the latent log-solvency ratio, and to derive a firm-specific initial estimate of asset volatility. Other parameters are chosen for illustration.

As shown in Figure 2.1, the shape of the term structure of credit spreads is a function of the term structures of the default probability, expected recovery rates, and the time value of money. For the safest firms, at the upper quartile of solvency at $x(t)$=1.44, credit spreads are low and increase monotonically with maturity from zero at time-t to approximately 50 basis points at $(T-t)$=30 years. The default probability is the lowest for the safest firms and rises approximately linearly with respect to maturity from zero at time-0, to approximately 25 percent at $(T-t)$=30 years . The expected recovery rate for the safest firms is the highest, beginning at 100 percent at time-t, and decreases with maturity to approximately 50 percent at $(T-t)$=30 years. Thus, the combined effect of increasing default probabilities and decreasing recovery rates, causes the credit spread to increase with maturity. The negative correlation between default and recovery with maturity, is caused by the increasing volatility of the firm's future asset value over time. As maturity is lengthened, the range of possible asset values that may result at maturity increases, causing a greater chance of asset values ending below the default boundary. If the firm defaults, the expected asset value available to pay bondholders in recovery is also lower.

For the highest risk firms, illustrated in Figure 2.1(a) by the lower quartile of solvency ratios, the credit spread term structure initially increases rapidly caused by the very steep rise in default rate as shown in Figure 2.1(b). The default probabilities are the highest for this quartile, rising from zero at time-t to approximately 60 percent at $(T-t)$=30 years, and the recovery rates, as shown in Figure 2.1(c), are the lowest falling to approximately 40 percent at $(T-t)$=30 years. Higher default risk and lower recovery is due to the initial firm value starting close to the default boundary. The default probability rises with maturity but at a decreasing rate. The resulting flattening of the default probability results in a falling credit spread for medium and long-term bonds due to the influence of discounting; the present value cost of default outweighs the marginal increase in default probability. Figure 2.1(a) shows the peak credit spread for the highest risk quartile of firms to be at approximately $(T-t)$=5 years. Thus, the Merton model, and structural models generally, predict a characteristic 'hump-shape' to credit spread term structures of high risk firms. Empirically there is mixed support for this prediction. Sarig & Warga (1989) average credit spreads across firms and time and report downward sloping term structures for firms rated B and C. However, Helwege & Turner (1999) show that this does not hold when term structure is measured from bonds issued by the same firm on the same date. They conclude that most speculative grade firms exhibit positively sloped credit spread term structures.

The Merton model, like all structural models, is a dynamic model of the credit spread term structure. The term structure, at a given time-t, is a result of the model's prediction at time-t by equation (2.8) for for different remaining maturities as shown in Figure 2.1(a). The term structure will also vary in time as a function of the path taken by the firm's log-solvency ratio as modelled by equation (2.7). For a given maturity, the credit spread is expected to change over a finite time step as a function of the firm's expected future log-solvency ratio. Knowledge of a firm's model parameters, therefore facilitates prediction of a credit spread term structure and the likely transition of the credit spreads between time periods. We refer to the former model property as a cross-sectional constraint and the latter as a time-series constraint. The distinction is important when implicitly estimating parameters from term structure models. If we were to average spreads over time, or find the best fitting parameters to match a firm's term structure at a single point in time, then parameters are fitted using only cross-sectional information without regard to how well the time-series of observed credit spread movements is explained by the model. If we were to fit the time-variation of constant maturity bond spreads then we would select parameters that are maximised to only explain the time series transition of credit spreads without regard to how well the model jointly fits bonds of different maturity. Geyer & Pichler (1999) show that in the context of risk-free term structure modelling, the state-space estimation method, 'simultaneously integrates time series and cross-sectional aspects of the model. Since this approach is consistent with the underlying economic model, and can utilise all available information, it provides a powerful test', (Geyer & Pichler 1999, p.1). Therefore, in order to provide the strongest test of model specification, we transform the credit models into state-space form as described further in Section 3.2, and estimate the models on panel data.

2.3.5 Limitations of the Black-Scholes-Merton Model

In this section we describe the main simplifying assumptions of the Merton model. In particular, the Merton model assumes an unrealistically simple capital structure, and ignores dynamic capital management. As discussed further in Section 2.4, efforts to relax these unrealistic assumptions has motivated the development of the broader structural modelling literature.

2.3.5.1 Complex Debt Structures

Firms are usually funded by many individual coupon-paying bonds, together with bank finance, trade credit, leasing, and other forms of credit. Unlike the Merton model, the aggregate payment pattern for the firm requires almost continuous partial liquidation of firm assets. Consequently, neither F nor T is unambiguously observable. In practice, researchers have approached estimating the Merton model by several methods.

Firstly, samples may be restricted to include only firms with relatively simple capital structures that are close to the single-payment zero-coupon bond ideal of the Merton model. This approach is widely adopted in the empirical literature, for example JMR select firms with 'a small number of debt issues', LS include firms with only one publicly traded bond, and EHH select firms with no more than two publicly traded bonds on issue. An obvious problem with restricting the sample is that the sample is biased towards firms that are less actively traded in the market and are less representative of firms that access the bond market.

Secondly, the level of F may be estimated by a proxy variable that attempts to include a weighting across maturities. The proxy adopted by Moody's-KMV, for the purpose of default probability prediction, is to set F to be one-half of total long-term debt plus all debt due in one year (Crosbie & Bohn 2002, Vassalou & Xing 2004). The rationale is that the short dated liabilities contribute more to default risk, however, Bharath & Shumway (2004) show that default prediction accuracy is relatively unaffected by whether F is the face value of debt or a more complex proxy.

Unlike default prediction modelling, where average calibration error is mitigated by mapping the measured $DD(t)$ to historical default rate data, the predicted credit spread obtained from the Merton model is sensitive to the choice of default boundary. The difficulty in choosing a default boundary is reflected in the various approaches followed in the literature with no single method dominating. EHH assume the firm's default boundary to be equal to the book value of total liabilities. They observe that the Moody's-KMV rule of thumb for weighting debt by maturity results in a lower default boundary compared to using total liabilities, and therefore a much lower predicted credit spread that compounds the underprediction bias of the Merton model. LS tested alternative measures of the default boundary and found the 'Equal All' approach gives the lowest price prediction errors. This assumes that all debt is retired at the maturity of the bond, with equal priority given to all creditors in the event of default. JMR assume that the default boundary is equal to the face value of the bond being valued.

Finally, the Merton model has been extended to value coupon-paying debt in an ad hoc manner by EHH. They sum the default-risky present value of the bonds promised payments assuming independence between the payments. The default boundary is not the face value of the promised payment, but rather the total of the firms liabilities. The approach is simple in practice to apply and is shown by EHH to perform relatively well compared to more complex structural models. We adopt a similar extension to the Merton model as EHH, but unlike all prior studies, we do not assume the default boundary to be an accounting value, but rather let the log of the ratio of the firm value to default boundary be an unobserved state variable that is recovered implicitly from the observed term structure of credit spreads. The state variable and our extension to the Merton model is explained more fully in Section 3.2.6.1.

The advantage of our method of implying the default boundary is threefold. Firstly, we do not impose a bias due to ad hoc choice of the default boundary. In the absence of any single proxy dominating in prior studies, it is far from clear what an appropriate proxy should be. Secondly, accounting leverage is updated only quarterly whereas our credit spread data is observed on a daily basis with approximately a week on average between observations per bond. Our method enables an estimate of the underlying leverage of the firm on each trade date. Finally, the firm may not default when market value hits the face value of debt. HH assume that the firm's default boundary is equal to 60 percent of total liabilities when calibrating their base case LS model to observed average default rates. HH argue that the default boundary must be below the level of liabilities, on average, based on prior studies of bond recovery rates that show assets available for distribution to bondholders are less than can be reasonably explained by the presence of bankruptcy costs alone. Recently, Davydenko (2005) provides evidence that default occurs, on average, when firm value is 72 percent below the face value of total debt. However, since as many as one third of defaults happen above this boundary, while 54 percent of rms with market assets below the face value of debt do not default for at least a year, and 38 percent never default throughout his sample period. For defaulted firms, the mean ratio of the firm market value to the face value of debt is 65.1 percent with a 26.6 percent standard deviation.

Davydenko's (2005) empirical observations cast considerable doubt on the accuracy of empirical studies of structural model performance in which the default boundary is assumed to be a simple observation of accounting debt. He poses the question,

> How can models of credit risk be advanced, given this evidence regarding empirical default triggers? One approach is to try to model more accurately the boundary empirically, in the hope that its value can be explained if a sufficient number of firm characteristics are included as explanatory variables. An alternative is to abandon hope of predicting default as a deterministic 'cause-and-effect event, and assume that either the boundary or the true value of assets is unobservable. (Davydenko 2005, p.25)

Our method of implicitly estimating the unobserved log-solvency ratio is consistent with the suggestion of Davydenko (2005), and avoids adding unnecessary bias and noise into the structural model estimation process that is likely to be prevalent in the extant empirical studies of JMR, LYS, EHH, and HH.

2.3.5.2 Default Only at Maturity

In the Merton model, default can only occur at maturity of the bond, T, regardless of the path followed by the firm's asset value prior to maturity. Thus, firm net worth may be negative prior to bond maturity, but since repayment of the bond is fixed at time-T, the firm will not default if the firm is otherwise solvent at time-T. However, in practice, it is common that bonds include covenants designed to reduce the risk of wealth transfer from bondholders to shareholders by specifying that the bond's face value is to be repaid immediately should the firm fail to maintain minimum solvency levels at all times (Smith & Warner 1979).

Black & Cox (1976) extend the Merton model default process to include a minimum net worth 'safety' covenant. They assume that should firm value fall below the level $K = F \exp -r(T-t)$ prior to time-T, the firm is defaulted by bondholders. Unlike the Merton model, default in the presence of safety covenants becomes a first-passage distribution of asset value to default boundary K. The default boundary is assumed to be exogenously known and its dependency on time is purely arbitrary.

LS suggest that an exogenous default boundary may be used to model default under alternative assumptions of the default process. In the simplest net worth insolvency case, K may represent total liabilities. Alternatively, K may be the result of contractual covenants agreed with bondholders as per Black & Cox (1976), or it may be the level of firm assets associated with default by lack of working capital. The latter may arise where the firm is net worth positive but has an insufficient excess of current assets over current liabilities to meet liabilities falling due. For bond valuation purposes, it is sufficient that K represent the lower level of firm value at which it is exogenously known that default will occur. Unlike Black & Cox (1976), LS assume the default boundary is constant through time.

Davydenko (2005) finds that the ratio of the market value of assets to the face value of debt is the single most important predictor of default therefore providing empirical support for LS's contention. Unfortunately, it is not straight forward to exogenously know what the appropriate level of default boundary, K, relative to firm value, V, is firm-by-firm.

Rather than assuming an exogenous, continuous default boundary, an alternative method for extending the Meton model to include early default is offered by Geske (1977) (hereafter Geske). He models default-risky debt as a compound option. At each promised payment date shareholders choose to pay a strike price for the right to continue ownership of the firm, or otherwise allow the firm to default. However, the more complex the firm's debt structure is, and the greater the number and frequency of coupon payments to be made are, the less tractable the compound option solution becomes. The model has therefore experienced little empirical implementation (EHH being a notable exception).

2.3.5.3 No Bankruptcy Costs

The Merton model assumes that there is no loss of firm value as a consequence of default, that is, there are no bankruptcy costs associated with default. Since firm asset ownership passes to the bondholders at default, any loss in value results in a reduced payoff to bondholders. By ignoring bankruptcy costs, that are otherwise rationally expected by bondholders, the Merton model may overprice debt and underpredict credit spreads.

Bankruptcy costs arise for a number of reasons. These can be either direct costs (legal and other professional fees), or indirect costs (reduced management focus and consequent loss in firm competitiveness, loss of key suppliers and markets and increased operating costs). Direct costs are estimated to be five percent, three percent and four percent of firm assets by Warner (1977), Weiss (1990), and Altman (1984) respectively. While direct costs appear relatively minor, especially in a present value context, indirect costs could be very substantial. This is particularly so if we consider the underinvestment agency problem raised by Myers (1977). Underinvestment occurs when shareholders in a high default risk firm choose rationally not to invest in positive NPV projects, if the future benefits are likely to pass primarily to bondholders. Altman (1984) estimates total bankruptcy costs, including indirect costs, on large industrial firms to be 24 percent. Alderson & Betker (1995) measures the difference between the going concern value, immediately prior to default, and firm liquidation value. They estimate average total bankruptcy costs to be 36.5 percent of the prior-to-default going concern value. Focussing only on highly-leveraged transactions, Andrade & Kaplan (1998) estimate the total net cost of financial distress to be 10 to 20 percent of non-distressed firm value.[4]

2.3.5.4 No Breach of Absolute Priority Rule

The Absolute Priority Rule (APR) describes an ideal outcome in bankruptcy law where no claimant can receive payment from firm assets until more senior claimants have been satisfied in full. A breach occurs where there is effectively an ex post change in priorities of creditors resulting from the bankruptcy process (Franks & Torous 1989). The Merton model assumes bondholders receive the value of the firm in the event of default. A breach of APR decreases the amount of firm assets receivable by the bondholders in the event of default and increases the required credit spread, if expected ex ante by bondholders, in a similar manner to bankruptcy costs.

Under U.S. bankruptcy law there are two possible methods of corporate bankruptcy: Chapter 7 provides for the orderly liquidation of a firm's assets by a court-appointed trustee, and payment to claimants in order of priority is always maintained; Chapter 11 provides for reorganisation of the firm by which a plan of reorganisation must be

[4]Qualitatively, they concluded bankruptcy costs were mainly attributable to: curtailed capital expenditure, asset disposals at depressed prices, and delay in restructuring.

negotiated among the various parties and agreed with the court (Weiss 1990, p.291). Importantly Chapter 11 provides protection from creditors for management acting on behalf of shareholders, while a reorganisation plan is developed. Costs accrued during this period are ultimately borne by creditors. Chapter 11 is the predominate method of bankruptcy filing. Because of the need for agreement amongst claimants, in Chapter 11 junior creditors and residual claimants have increased bargaining power and may threaten to delay the final resolution and to force the firm to incur additional costs. For a reorganisation plan to be agreed, a majority in number and at least two-thirds by amount owed to the creditors who vote in each class of impaired creditors must approve the plan before it can be confirmed by the bankruptcy court. Importantly, even equity holders must also approve the plan by a two-thirds majority, giving them some bargaining control over creditors. In order to avoid costly reorganisation, creditors may therefore rationally agree to violate the APR in order to speed agreement.

Weiss (1990) examines 37 listed U.S. firms that filed for bankruptcy between 1979 and 1986. He finds that the priority of claims is violated for 29 of the 37 firms studied and that the breakdown of priority occurs primarily between the unsecured creditors and equity holders and among the unsecured creditors. The common occurrence of breaches in APR is reported by Franks & Torous (1989) who examined the terms of reorganisation of 30 firms that emerged from Chapter 11. Of their sample, 27 firms exhibited breaches of APR, and in 18 cases shareholders received some consideration.

Eberhart, Moore & Roenfeldt (1990) examines the return to shareholders resulting from 30 U.S. bankruptcy filings over the period 1979 to 1984. The average value received by shareholders in breach of APR was found to be 7.6 percent of firm value. Analysis of the share price before and after the filing showed that the equity market anticipated the breach of APR beforehand. A similar result is obtained by Betker (1995) who examines 75 Chapter 11 U.S. corporate bankruptcies from 1982-1990. He finds that the average value of APR gained by shareholders to be 2.86 percent of firm value, measured at the firm's emergence from bankruptcy. Eberhart & Sweeney (1992) show that in the bankruptcy filing month, bond prices incorporate the subsequent breaches of APR in an unbiased manner.

The effect of breaches of APR and bankruptcy costs act to reduce the recovery to bondholders in the event of default. The historical recovery rate on senior unsecured bonds is estimated by Altman & Kishore (1996) to be, on average, 47.65 percent of the face value of the bond, and includes average bankruptcy costs and breaches of APR. A simple way to include the combined effect in structural model was suggested by LS who specify an exogenously determined recovery rate, informed from bond recovery studies such as Altman & Kishore (1996). Consequently, their model incorporates but does not attempt to separately estimate the effect of bankruptcy and APR breach costs. EHH also extend the Merton model in a similar manner, and assume the bond recovery is a constant

51.31 percent of the face value of total liabilities.

2.3.5.5 Static Capital Management

In the Merton model the level of debt is fixed regardless of the path taken by the firm's solvency. This has been criticised as being an unrealistically passive description of management behaviour by Taurén (1999) and Collin-Dufresne & Goldstein (2001) (hereafter CDG). Observation of physical debt-ratio dynamics suggests management adjust their issuance of debt versus equity, and dividend policy, in order to revert their firm's debt-ratios towards a preferred target debt-ratio over time (Taggart 1977, Marsh 1982, Jalilvand & Harris 1984, Hovakimian, Opler & Titman 2001).

By implication the risk-neutral log-solvency ratio, $x(t)$, is also likely to be mean-reverting to a target level to the extent that market bond prices factor in mean-reversion of the firm's future debt-ratios. With reversion to a debt target, management is unlikely to let debt-ratios fall too far, should firm value rise unexpectedly, thereby acting to increase debt and increase future default risk. Similarly, the firm value is less likely to progress towards insolvency without management effort to reduce the level of the default boundary by adjusting the firm's operating plan and funding strategy to reduce its future debt-ratio. CDG show that the effect on credit spreads of assuming mean-reversion in the log-solvency ratio is to reduce long-term credit spreads levels and volatilities relative to the Merton model. The credit spread term structure flattens, and by controlling the target-debt-ratio, offers more degrees of freedom to match observed credit spread term structures.

By relaxing Merton's assumption of passive capital structure management, an important new field of capital structure model theory is revealed by CDG. We therefore discuss more fully the literature related to capital structure management in Section 2.5.

2.3.5.6 Constant Risk-Free Rate

The Merton model assumes a constant risk-free rate. JMR suggest that this assumption could be the cause of the Merton model's underprediction of credit spreads on investment grade bonds. They infer this conclusion by regressing residual price prediction errors against dummies for the year amongst other variables, and note the year of observation is a significant explanatory variable. A time-varying asset volatility or a time-varying risk-free rate is suggested as possible sources of error.

LS describe more fully the influence of a stochastic risk-free rate on predicted credit spreads. They show that default probability and credit spreads are positively related to correlation between firm asset return and the risk-free rate. The risk-neutral future distribution of firm asset value depends on the risk-free rate. If the correlation is positive, changes in the risk-free rate tend to be in the same direction of the firm's asset changes.

For example, an increase in the risk-free rate is associated with a positive innovation in firm value. Introducing additional future volatility by way of stochastic interest rates tends to increase overall firm value volatility, thereby increasing the probability that future firm asset value will reach the default boundary and the firm will default. If correlation is negative, changes in the risk-free rate tend to dampen changes in the firm asset values and the probability of default reduces. Consequently, firm asset volatility is higher, in the presence of negative interest rate correlation, for the same predicted level of credit spread.

Empirical evidence is conflicting as to whether interest rate correlation has a material effect on explaining default risk and credit spreads. Clouding the evidence is the presence of call provisions in some bond indentures, and the influence of the historical interest rate environment in the sample period. JMR's sample covered the period of high interest rate uncertainty from 1975 to 1981. Also in the same period, most corporate bonds were callable or subject to sinking fund provisions, thereby making the valuation problem complex and incomplete within the Merton model. For example, Ogden (1987) fits the Merton model to bonds traded between 1973 and 1985 and finds that the risk-free rate and yield curve slope partially explain credit spread prediction errors from the Merton model. However, all bonds in his sample are fully callable or partially callable via sinking fund provisions.

The conclusions of JMR and Ogden (1987) may have been unduly influenced by the high proportion of callable bonds in their samples since corporates issued few noncallable bonds prior to the mid-1980s. Duffee (1998) reports that in 1984 only 271 bonds, out of a population of 5,497 bonds issued by corporates, were noncallable for life. The proportion increased dramatically post 1985, and by 1995, 2,814 from 5,291 were noncallable. In the more recent studies of LYS and EHH, their samples have excluded bonds that are callable and subject to sinking fund provisions. Both studies conclude that adding correlated stochastic interest rates to structural models has not improved credit spread prediction accuracy.

In contrast to JMR, EHH find no evidence that interest rate correlation is a significant omission from the Merton model since neither interest rate volatility, nor correlation with firm asset return, differs between those bonds that underpredict and overpredict credit spreads. However, the LS and CDG models, which include stochastic and correlated interest rates, exhibit a systematic prediction error. Firms with very negative (positive) estimated correlations have greater underprediction (overprediction) of credit spreads relative to the Merton model. A similar result is offered by LYS who tests the LS model with and without stochastic interest rates. With a constant interest rate they report a mean credit spread underprediction error of 8.78 basis points. The error worsens to an underprediction of 25.37 basis points when stochastic interest rates are introduced with firm-wise observed correlations. Further, absolute spread prediction errors are found to be insensitive to the choice of interest rate correlation assumption, suggesting that adding firm-wise correlation to the model has not improved cross-sectional spread prediction accuracy.

Lastly, researchers have attempted to measure the correlation between firm asset value and interest rates using a proxy for the unobservable firm asset return. For example, using the industrial stock index returns, as a proxy for firm asset return, LS report the correlation in first differences with 30-year Treasuries to be -27 percent. A similar value was assumed by HH when fitting the LS model to historical default rates. EHH also assume stock return is a proxy for asset return, but their method is closer to the spirit of the Merton model; they measure stock return at a firm and not index level, and use a risk-free rate proxied by the 3 month T-bill as their risk-free rate. Using a 5 year window of firm equity return they report a much lower average correlation of only -2 percent. Similarly, LYS measure the correlation between monthly stock prices and the yield of the on-the-run 10 year Treasury for the period January 1990 to June 1999. The average interest rate correlation is found to be -7.2 percent.

It therefore remains unclear as to whether the absence of stochastic interest rates in the Merton model is a serious omission. Attempts at measuring the level of correlation directly suggests the correlation is small on average. Tests of predictive accuracy with the LS and CDG models suggest that including firm-wise interest rate correlation has not improved accuracy and may have decreased it. The theoretical models that have included interest rate correlations, such as LS and CDG, have used a simple Vasicek (1977) single-factor model, and perhaps this is not sufficiently descriptive of the risk-free yield curve, since accuracy of credit spread predictions have not improved over the single-factor structural credit models.

2.3.6 A Summary of the Merton Model

In this section we introduced the Merton model describing its theoretical foundation as an option theoretic model of corporate bond valuation. The Merton model is parsimonious with few variables to be estimated, however, it has unrealistically simple assumptions. We discussed the main theoretical limitations of the model and the empirical evidence against the simplifying assumptions. In the next section we discuss the main theoretical extensions to the Merton model that attempt to relax its assumptions.

2.4 A Survey of Structural Models

The structural model literature consists of extensions to the Merton model addressing the aforementioned limitations. The plethora of models with minor variations is extensive and so we restrict our discussion to main theoretical developments, which in turn, guides a representative selection of models for testing.

Following a well accepted taxonomy (see Uhrig-Homburg (2002) and Leland (2004)), we first distinguish between models that assume that the default boundary is determined from an optimal decision made within the debt valuation problem (*endogenous* boundary models), or the default boundary is assumed to be exogenously known and is a given input into the debt valuation problem (*exogenous* boundary models). We propose a further sub-classification in which we distinguish whether the default boundary is predicted to change stochastically through time (*endogenous-dynamic, exogenous-dynamic* models), or whether the firm's default boundary is assumed to be a non-stochastic function of time, either constant or a deterministic function (*endogenous-static, exogenous-static* models).

Set out in table 2.1 is a summary of the models discussed in this section classified according to the aforementioned scheme. Abbreviations in bold refer to models that we later fit and test comparatively.

Table 2.1: Tabled is a classification of extant theoretical structural models.

Barrier	Default Barrier	
Behaviour	Endogenous	Exogenous
Static	Leland (1994) Leland & Toft (1996)(**LT**) Acharya & Carpenter (2002)	Merton (1974)(**EM**) Black & Cox (1976) Geske (1977) Kim, Ramaswamy & Sundaresan (1993) Longstaff & Schwartz (1995)(**LS1**,**LS2**) Briys & de Varenne (1997) Zhou (1997) Barone-Adesi & Colwell (1999)(**CEV**)
Dynamic	Fischer et al. (1989) Anderson, Sundaresan & Tychon (1996) Mella-Barral & Perraudin (1997) Leland (1998) Fan & Sundaresan (2000) Goldstein, Ju & Leland (2001) Moraux (2002) Dangl & Zechner (2004) Francois & Morellec (2004) Galai, Raviv & Wiener (2005)	Nielsen, Saá-Requejo & Santa-Clara (1993) Saá-Requejo & Santa-Clara (1999) Taurén (1999) Mueller (2000) Collin-Dufresne & Goldstein (2001)(**CDG**) Demchuk & Gibson (2006)

2.4.1 Endogenous Boundary Models

Endogenous boundary models are a group of models in which the default boundary is the outcome of an optimal decision made by shareholders within the bond valuation model. Unlike the exogenous boundary model, such as Merton, shareholders not only decide whether to default the firm, but also the level of the default boundary. The first group considered here is the simplest static boundary structural models in which the boundary is assumed to be a deterministic function of time. We then discuss the second subgroup of more complex dynamic boundary models where the boundary is assumed to be stochastically time-varying.

2.4.1.1 Endogenous-Static

In an early extension of the Merton model, Black & Cox (1976) consider the valuation of debt when management choose the timing of default, acting to optimise the value of shareholder equity. In doing so the early default assumption of Merton is relaxed and the level of the default barrier is determined endogenously. It is assumed that the firm is financed by equity and a single consol (infinite maturity) bond paying a continuous coupon. For large firms with many debt issues, this choice of bond payment is a more realistic representation of the firm's aggregate going-concern debt financing requirements than Merton's single zero-coupon bond assumption. At each point in time, management choose whether to pay coupons or otherwise default and pass the assets of the firm to bondholders. Default will be avoided provided that the value of equity, after the coupon payment, is not less than the coupon payment. Thus, default occurs if the value of the firm falls to a point where new equity cannot be raised to service debt; in continuous time equivalent to the value of equity equal to zero. An important result achieved by Black & Cox (1976) is to identify the shareholder wealth maximising default boundary as

$$K = \frac{c}{r + \frac{\sigma_V^2}{2}} \tag{2.14}$$

where c is the coupon rate, and r is the risk-free rate. Equation (2.14) shows that the default barrier is independent of firm value, and decreases as asset volatility and the risk-free rate increase.

Leland (1994) extends Black & Cox (1976) to include the effects of tax and bankruptcy costs on the default boundary. Taxation presents management with an opportunity to increase firm value by utilising tax savings on interest payments. An optimum value maximising level of debt exists, at which point the marginal benefit of the tax shield is equalled by the marginal cost of increased bankruptcy risk. Like Black & Cox (1976), Leland (1994) assume that management seek to maximise the value of the shareholder's claim, and not the value of the firm. They find that the default boundary of the firm is

$$K = \frac{(1-\tau)c}{r + \frac{\sigma_V^2}{2}}, \tag{2.15}$$

where τ is the tax rate.

Equation (2.15) shows that the default barrier is positively related to the after-tax coupon rate, and is negatively related to the risk-free rate and firm asset volatility. The default boundary is unaffected by bankruptcy costs, which are borne by the bondholders in the event of default, and not by the shareholders, and therefore do not enter into consideration by management when setting the firm's debt-ratio. Bankruptcy costs reduce the overall value of the firm, but leave the value of equity unchanged with the cost passed to bondholders in a reduced value of the bond.

LT extends Leland (1994) and Black & Cox (1976) with the more realistic assumption that the firm issues finite maturity debt. To find a tractable solution for the value of the firm's debt, they assume debt is 'rolled over', i.e. refinanced, in perpetuity at a constant maturity, T maintaining a constant level of principal. The firm's capital structure is assumed to be time-homogeneous and management choose the optimal debt level only initially leaving the aggregate level of debt static thereafter. In addition to the explanatory variables in equation (2.15), the default boundary is found to be an increasing function of the debt-ratio, and bankruptcy costs, and a decreasing function of debt maturity (refer equation (3.35)). Default only occurs when new equity cannot be raised, which will generally occur when debt service costs equal the expected equity return.[5]

The LT model provides a plausible theoretical basis for Davydenko's (2005) observation that firms default with negative net equity and not immediately at a zero equity default boundary. The reason is related to the maturity of debt and expected equity return. With long-term debt, the default boundary will typically be less that the debt principal due to the potential for equity to appreciate before the debt is rolled over. The longer the maturity of the debt and the higher the expected equity return, the greater the opportunity for the firm to attract additional equity and avoid bankruptcy despite immediate negative net worth. However, as maturity approaches zero, new equity will only be attracted if the value of the firm, after bankruptcy costs, exceeds the par value of debt. Thus, the default boundary is predicted to approach $K = P/(1-\alpha)$ as $T \to 0$, where P is the debt principal and $\alpha \geq 0$ is the bankruptcy cost. Thus, default is predicted to occur when the firm has positive net worth if bankruptcy costs are non-zero.

[5]Debt service is defined as the intertemporal change in firm value due to leveraging; includes after tax cost of coupons and principal repayments at par, less funds from new debt issued at market value and cash available for payout to shareholders generated from operations.

2.4.1.2 Endogenous-Dynamic

The models of Black & Cox (1976), Leland (1994), and LT assume that management make a single initial capital structure decision, with the level of debt principal remaining time homogeneous thereafter for the remaining maturity of the bond being valued. In practice, firms can be expected to adjust their level of debt over time, in particular to preserve the value of tax shields should as assets grow over time as expected. The absence of time-varying management adjustment of firm gearing is a weakness in the endogenous-static structural models. In this section we consider a group of structural models that permit the debt-ratio of the firm to adjust over time in response to asset growth and exogenous shocks to the firm's asset value. We refer to these as endogenous-dynamic models since these models permit dynamic adjustment of the default boundary.

Endogenous-dynamic capital structure models retain the same basic assumptions of Leland (1994), but assume management adjusts the firm's debt-ratio in response to changing firm value. Depending upon the importance of transaction costs associated with adjusting the firm's debt-ratio, the literature predicts different relationships between current firm leverage and the term structure of credit spreads. Initially we discuss models that ignore any bargaining between bondholders and equityholders.

Fischer et al. (1989) recognise that changing the firm's capital structure is costly and will not occur until there is sufficient movement in the debt-ratio. Thus, it is hypothesised that firm's have a 'region of no recapitalisation' bounded by an upper and lower solvency barrier. The firm capital structure policy follows a simple rule; gearing is increased if the firm's ratio of asset value to debt reaches an upper boundary, but default occurs if the asset value of the firm reaches the lower default boundary. Compared with Leland (1994), the option to recapitalise at an upper boundary causes an initially lower optimal debt level and higher default risk. Importantly, Fischer et al. (1989) show that firms may allow their debt-ratios to vary over time within a set of optimal boundaries. Therefore, firms with similar recapitalisation preferences and similar default risks may exhibit different observed debt-ratios on any given balance date. The cross-sectional observed term structure of credit spreads may therefore be poorly explained by the use of current balance sheet debt if used to proxy for the firm's default point in the presence of recapitalisation costs. This may be an explanation for the large firm-wise prediction error variance reported by EHH who use balance sheet debt to estimate the firm's default boundary.

Using similar capital management rules as Fischer et al. (1989), Dangl & Zechner (2004) show that default risk is monotonically decreasing with respect to rising solvency under static analysis when incremental adjustments to capital structure are made. However, with significant fixed costs of recapitalisation, the firm is assumed to leverage back to its initial optimum in the event that the upper boundary of firm value to debt-ratio is hit. Such an adjustment is only warranted if bond indentures prohibit the issuing of new debt, therefore forcing the firm to retire existing debt and re-issuing new debt, or if raising debt has significant fixed costs that encourage large scale debt issuance in preference to smaller incremental debt raisings. It follows that Dangl & Zechner (2004) predict that the probability of default, at the upper solvency boundary, is equal to the probability of default at the initial optimum solvency point. Thus, default risk is predicted to be 'U'-shaped with respect to solvency; initially it falls with increasing asset value relative to debt, but then rises as the re-leveraging solvency boundary is approached. This hypothesis does not appear to be well supported empirically. For example, structural models that map a predicted distance-to-default to observed default rates, show monotonically increasing default risk with respect to increasing leverage (Sobehart & Stein 2000, Crosbie & Bohn 2002).

Goldstein et al. (2001) provide a variation to Fischer et al. (1989) in which the state variable is not the value of the firm, but rather the firm's earnings before interest and tax expenses. The EBIT state variable is therefore unaffected by the firm's choice of leverage, whereas the firm value state variable in other endogenous-dynamic models must be interpreted as the pre-leverage firm value.

Like LT, endogenous-dynamic models also predict that default will occur below the face value of debt. Unless debt is immediately due, shareholders will continue to service debt until the expected value of the firm is not sufficient to warrant paying coupons. For bankruptcy costs of 5 percent, the equivalent of default boundary to face value of debt, K/F, is reported to be: Fischer et al. (1989), 57 percent; Goldstein et al. (2001, Table 2), 51 percent; Dangl & Zechner (2004), 69 percent.[6] Empirical support is provided by Davydenko (2005) who finds that on average K/F is 65 percent, but varies widely in the cross-section, depending on balance sheet liquidity, asset volatility, and asset tangibility.

An important theoretical result of the aforementioned endogenous-dynamic models is the prediction of asymmetric debt-ratio adjustments. The result arises from assuming that shareholders follow a second-best capital management policy in which they maximise their own wealth. Fischer et al. (1989) argue under a second-best policy it is never optimal for debt to be repurchased when firm value declines. This is because the rising cost of bankruptcy is fully borne by the bondholders. The implication is that capital adjustments are asymmetric and negative value shocks are not matched by reducing debt. Under an alternative first-best policy, shareholders maximise total firm value and may seek to reduce some debt rather than default, thereby introducing some debt reduction near the default boundary. This is only likely if there is a precommitment in the bond indenture to maintain a minimum level of solvency.

Some support for the hypothesised asymmetric recapitalisation behaviour is given by

[6] Independently Huang & Huang (2003) estimate the default boundary to be 60 percent of the face value of debt using the reasonable 'back-of-the-envelope' assumption that bond recovery is 50 percent of debt face value with 10 percent total bankruptcy costs.

Gilson (1997) who examines the behaviour of leverage ratios before and after bankruptcy proceedings. He finds that firms that proceed through Chapter 11 do experience a decrease in leverage after recontracting with creditors, but that the leverage ratios remain well above the industry mean, and substantially above the levels five years prior to entering Chapter 11. He concludes that the optimal target debt-ratio has most likely increased. One possible reason cited is that restructured firms benefit from the additional operational discipline imposed by debt (Jensen 1986).

The predicted asymmetric capital adjustment behaviour is a consequence of assuming strict second-best behaviour, in the presence of bankruptcy costs, with no voluntary debt reduction agreed by bondholders. We now discuss a second group of endogenous-dynamic models that consider the potential for negotiation between stockholders and bondholders at the default boundary.

Working in discrete-time, Anderson & Sundaresan (1996) estimates the default boundary as an outcome from a non-cooperative game; shareholders make take-it-or-leave-it offers of debt service to bondholders. In the presence of bankruptcy costs, bondholders may accept to renegotiate a lower than contracted payment to preserve the value of their claim on the firm and avoid the costs of bankruptcy. In other words, reduction in leverage occurs at a reorganisation boundary that is higher than the insolvency boundary in the absence of negotiation. The resultant reduction in debt, due to the threat of default, is termed strategic default. Shareholders have an incentive to offer below contracted debt payments, but not sufficiently low to force rejection by bondholders and subsequent liquidation. Bondholders will rationally accept a lower debt service payment up until the point at which the concessions offered equal the expected bankruptcy costs. The greater the potential bankruptcy costs the greater the concessions that will be accepted. Thus, the assumption of negotiation in the presence of bankruptcy costs implies firms contractually default at a higher asset value than they otherwise would in the absence of the opportunity to negotiate.

For a given level of gearing, the potential for future strategic debt service implies higher credit spreads than predicted by Merton. Mella-Barral & Perraudin (1997) and Anderson et al. (1996) work in continuous-time and solve the value of a consol bond analytically. Fan & Sundaresan (2000) introduce taxes and assume equal bargaining power between shareholders and bondholders. They find that because shareholders have some power to exploit bondholders, the default boundary is always higher than predicted by Leland (1994) resulting in a greater probability of default.

Default may also arise from a breach of minimum cash flow covenants. Fan & Sundaresan (2000) examine non-negotiable cash flow covenants and their influence on the bargaining process. They suggest that in the presence of cash flow covenants, shareholders would rather sacrifice dividends to reinvest and avoid a premature liquidation of the firm. They reinvest the minimum amount such that the strategic default point is reached before the cash flow covenant becomes binding.

In summary, the endogenous-dynamic models provide a rich theoretical field for predicting the relationship between dynamic capital structure choice and credit spread term structures. These are as follows:

1. In the presence of bankruptcy costs and taxes, shareholders will not allow the firm's debt-ratio to deviate to below a desired level of gearing necessary to protect the value of tax shields. The option to increase leverage in the future results in higher predicted future credit spreads relative to the endogenous static models;

2. Where there are costs of recapitalisation, the firm will resist increasing leverage until the benefit exceeds the cost, resulting in a region of no recapitalisation, making inference of the firm's target debt-ratio impossible from simply observing its current debt-ratio. If the costs are largely fixed, or bond indentures prevent incremental debt changes, the firm will increase is predicted to change to increase its leverage sharply from an upper solvency boundary to its optimal level. The speed of mean-reversion in debt-ratios depends upon the size of recapitalisation costs, and whether it is a smooth adjustment depends upon the proportion that is variable as opposed to fixed. Slower rates of mean-reversion will result in lower future credit spread term structures more closely represented by the static models;

3. In the absence of bargaining between bondholders and equityholders, debt-ratios may not revert as a result of downward shocks in firm value since the increased potential cost of bankruptcy is passed to bondholders, when a second-best capital management strategy is followed by shareholders;

4. If bargaining is permitted, then the theory of strategic debt service suggests that default will occur at higher firm asset values, when bankruptcy costs are high and impediments to renegotiation are low. Shareholders are able to extract a negotiated debt service reduction or reduction in the amount of debt, by threatening to default. This mechanism may cause the future debt-ratio to reduce in response to negative shocks in firm value as a consequence of partial debt forgiveness by bondholders. Credit spreads are predicted to be higher than in the absence of bargaining, a consequence of the potential to default earlier, and the expected increased loss associated with debt renegotiation.

Relative to the Merton model, the theoretical extensions of the endogenous model literature suggest a greater likelihood of future default and higher future credit spread. For empirical estimation, the implication is that the default boundary is not simply proxied by the current balance sheet level of debt:

1. The presence of longer tenor debt encourages shareholders to maintain debt service payments despite the firm value falling below the face value of debt;

2. If bankruptcy costs are high, the owners have bargaining strength, and the debt can be easily negotiated, shareholders will seek to offer below the contracted debt payment. Default, under the original terms of the debt contract, occurs earlier at a higher solvency threshold;

3. Default can be triggered by breach of minimum cash flow covenants, however, the influence of liquidity default is contingent upon how binding the liquidity covenant is relative to the strategic default threshold.

2.4.2 Exogenous Boundary Models

Exogenous boundary models abstract from the complexity of the endogenous boundary models. Rather than specifying the default boundary level to be the outcome of shareholder wealth maximisation, the default boundary is assumed to be known exogenously. The default boundary may be assumed to be constant, or a deterministic function of time, which we term exogenous-static. Alternatively, the default boundary may be assumed to vary stochastically through time, in which case we define the model as exogenous-dynamic. The difference between endogenous and exogenous dynamic forms of structural models is that the former specifies the default boundary's time variation as the consequence of shareholders re-evaluating the firm's capital structure at each point in time, whereas the latter assumes that the default boundary varies through time in a stochastic manner independent of shareholder choice.

The Merton model is an example of an exogenous-static model that assumes the default boundary to be equal to the book value of the firm's debt. By construction, the default boundary is time-invariant. In some more recent exogenous boundary models, the default boundary is stochastic. However, this leads us to two particular challenges facing exogenous models. Firstly, there is a wide variety of potential underlying default boundary processes that could be selected. The exogenous literature, does not in itself provide a theoretical basis for one preference over another. A robust choice of default boundary process should therefore be one that is consistent with the well established capital structure literature. Secondly, as discussed in the previous section, due to the complex firm-specific factors affecting the timing of default, the default boundary is not readily proxied by observable variables. This raises the difficult question of how the models should be parameterised. We suggest this is best solved by latent estimation of the models and an examination of the resultant prediction errors for miss-specification.

In this section we review the extant exogenous boundary structural credit models distinguishing between static and dynamic approaches. We show how the firm's latent solvency ratio is determined by the specification of the default boundary and compare the behaviour of the firm's solvency ratio with the extant capital structure literature.

2.4.2.1 Exogenous-Static

We use the term exogenous-static to group models in which the firm's default boundary is determined independently of shareholder preferences and is a constant function of time.

The simplest version of this model is to assume that the firm is funded by a single bond and its default boundary is determined by the bond's indenture. For example, the Merton model assumes that the default boundary is equal to the face value of debt payable only at maturity. Black & Cox (1976) permit early default where they let the default barrier be equal to a minimum solvency level, as contained in the bond indenture, which is assumed to grow over time at the risk-free rate. A more common method to modelling the triggering of early default was first suggested by LS. Their model assumes the default barrier is constant through time but are silent on the level of the barrier. Importantly, they show that the ratio of firm value over the default barrier is a sufficient state variable to value bonds with coupons and across multiple bonds issued from the same firm. The spanning of multiple securities by a predefined underlying state process, is the defining feature that makes the exogenous-boundary model form useful in finance, and is the property we exploit further in Section 3.2 to to derive estimates of the state process. To illustrate this property we first consider the LS model in detail.

LS define default as the first passage of firm asset value $V(t)$ across a constant default boundary, K. Unexpected shocks in V are correlated by $\rho_{r,V}$ with a stochastic risk-free rate $r(t)$ that follows a mean-reverting stochastic process as per Vasicek (1977), thereby characterising LS as a two-factor model of the log-solvency ratio $x(t) = \ln V(t)/K$

$$dx(t) = (r(t) - \delta - \frac{\sigma_v^2}{2})dt + \sigma_v dW_{V,t}^Q \tag{2.16}$$

$$dr(t) = \kappa_r(\theta - r(t))dt + \sigma_r dW_{r,t}^Q, \tag{2.17}$$

where, $W_{V,t}^Q$ and $W_{r,t}^Q$ are Weiner processes, κ_r is the speed of mean-reversion for the instantaneous short rate, θ is the long-run level of $r(t)$, and σ_r is the short rate volatility.

Defining the first passage stopping time by $\tau = \inf\{t \geq 0 : x(t) = 0\}$ then the probability of default between $t = 0$ and T is

$$Q(0,T;x(0),r(0),\Theta) = Pr(\tau \leq T | \tau \geq t = 0)\,. \tag{2.18}$$

With no explicit modelling of the firm's cash flows and debt covenants, there is no specific bankruptcy cause ascribed to K; it is simply the value of the firm at which default is triggered. This simplification enables LS to value complex debt structures. At $t = \tau$ all debt is assumed to default under cross-collateralisation rules. Different priority levels between debtors is accommodated by varying the writedown rate ω. Valuation of a zero-coupon bond then proceeds as the risk neutral expected payoffs in default and non-default states. The assumption that payment to bondholders occurs only at the original bond maturity facilitates the valuation of coupon paying bonds by valuing each contracted payment as a zero-coupon bond and summing together as a 'portfolio of zeros'.

The division of assets in the event of default, is also exogenously specified. A proportion $(1-\omega)$ of F is assumed to be paid to bondholders at the original maturity of the debt.[7] The writedown rate, ω, represents K/F, being the expected outcome from strategic default and negotiation, or bankruptcy and liquidation including expected breaches of APR. However, as noted by Briys & de Varenne (1997), there is nothing to limit the payment to bondholders to be no greater than the value of the firm nor to ensure that the value of the firm is sufficient to cover the payment of the bond at maturity. They suggest a more structured barrier equal to the present value of the firm's single-zero coupon liability adjusted for expected APR breaches. Unfortunately, the adjustment by Briys & de Varenne (1997) prohibits valuation of complex debt structures since the boundary is made a function of the face value of debt.[8]

A further weakness of the LS model is evident from examination of the latent log-solvency process in equation (2.17). If $r(t)$ is on average greater than $(\delta - \sigma_v^2/2)$, then the firm is assumed to deleverage ad infinitum. Such behaviour is not expected in the presence of tax shield benefits (nor observed empirically).

The LS model has been extended to consider alternative asset processes. Zhou (1997) extends LS to a jump diffusion model. The motivation to consider downward jumps in firm value comes from the empirical observation that structural models understate short term credit spreads. The potential for the firm value at default, to jump below the face value of debt, gives an endogenous variation in recovery rates. The first-passage crossing time is solved by Monte Carlo. HH describe an alternative jump-diffusion model with the assumption of a constant risk-free rate. They adopt the same exogenous default boundary and recovery assumptions of LS but let the asset value evolve with a double-exponential distribution such that a semi-analytic solution for the crossing time is known.[9] The calibration of the jump component is difficult considering that the firm asset process is unobserved. HH and Delianedis & Geske (2001) demonstrate that by adjusting jump parameters the predicted credit spreads on short tenor bonds can be made close to those observed, but the resultant jump parameters are found to be unrealistic. Further, jump parameters have most effect at short tenors and without mean-reversion in the leverage level, the previous criticism of the LS model remains. A simple comparison of Merton against a jump-diffusion equivalent derived from Merton (1976) by Hull, Nelken & White (2004) showed that in all cases the non-jump Merton model provided

[7] Termed a 'Treasury at Default' assumption. Alternative writedown specifications are 'Recovery of Par at Default' and 'Recovery of Market Value'. The differences are explained further in Guha (2003).

[8] Formally, $K = \alpha \exp(-rT)F$ where $0 < \alpha < 1$ is a scalar to accommodate expected breaches of APR.

[9] The probability of default requires numeric methods to solve for a Laplace inversion. Refer to Huang & Huang (2003, Appendix B2).

significantly better predictions of default probabilities and credit spreads.

Drawing from agency theory, Barone-Adesi & Colwell (1999) propose a model for valuing zero-coupon bonds where the volatility of the firm's assets increases as the firm's value approaches the default boundary. This is consistent with Jensen & Meckling's (1976) theory of asset-substitution that proposes that it is in the interests of shareholders to take greater business risks, thus increasing the volatility of the firm, the closer to the firm is to default. Using a constant barrier and risk-free rate, a closed-form value for a zero-coupon bond is obtained under the assumption that the return on the firm follows a constant elasticity of variance (CEV) process as first described by Cox (1975). Under the CEV model the firm's asset value has a local volatility that is a deterministic function of solvency.

Barone-Adesi & Colwell (1999) propose the firm follows the CEV process

$$dX(t) = (r-\delta)X(t)dt + \bar{\sigma}_v X(t)^\rho dW(v,t)^Q, \tag{2.19}$$

where $X(t) = V(t) - K$ is the firm's net worth, as measured by its equity value, and $\bar{\sigma}_v$ is a constant scale factor for the instantaneous volatility. The local asset return volatility is given by $\bar{\sigma}_v X(t)^{(\rho-1)}$ and is therefore time-varying with $X(t)$. Default occurs at the first passage of $X(t)$ to zero; $\tau = \inf\{t \geq 0 : X(t) = 0\}$. For the case where $(\rho - 1) < 0$, volatility increases with default risk and declines with solvency, which is the agency theory predicted relationship. Because the firm's level of solvency is time-varying, it follows that the firm's asset return volatility is also time-varying. However, the functional relationship is constant through time, fixed by the elasticity parameter ρ, and so the firm's volatility is assumed to have a rigid volatility skew.

Usefully, the probability of the first passage time is known analytically for a 100 percent drop in value to zero. This result has been used by Campi & Sbuelz (2005) and Albanese & Chen (2005) to value equity default swap contracts, and by Campi, Polbennikov & Sbuelz (2005) to value bonds and credit derivatives.[10]

A weakness of the extant CEV models is that default occurs only when the firm is market-value insolvent. This precludes the possibility of strategic default or default with the firm having positive net worth. A more general approach is achieved in our estimation method by letting K be the earliest unobservable default threshold whether triggered by insolvency or strategic default. This implies that volatility increases as the point of default is reached, but permits the value of equity to be strictly positive.

The aforementioned exogenous-boundary models assume the writedown rate, ω, is exogenously determined and unrelated to the firm's asset value. Intuitively, it would

[10]Equity default swaps are a recent financial innovation used as an alternative to credit default swaps. Normally these instruments pay 50 percent of notional value if the firm's equity price drops by 30 percent. Campi & Sbuelz (2005) value the special case of a benchmark equity default swap that pays nothing in the event of a 100 percent value drop. The instruments are described in more detail in Medova & Smith (2004).

seem reasonable that the firm's post-default recovery prospects would be tied to the stochastic process that led to default. However, the LS model and subsequent exogenous-boundary literature, define the default barrier as an absorbing state for the firm value process; the expected value of bond recovery ceases to be informed from the ongoing dynamics of the firm post default, even though bankruptcy proceedings may take several years to complete and the firm may not be liquidated. The distinction between immediate liquidation, and continuation under Chapter 11 with court imposed renegotiation, was first suggested by Francois & Morellec (2004) in the context of extending the Leland (1994) model. Under U.S. Bankruptcy Code firms can either liquidate assets immediately under Chapter 7, or renegotiate with their creditors under Chapter 11. The latter is the predominate option chosen. Upon entering Chapter 11, the court grants the firm a period of observation, protected from the actions of bondholders to liquidate assets, during which time the firm renegotiates its debt. Consequently, liquidation does not arise at the moment of first passage under Chapter 11 bankruptcy. At the end of this period, the court decides whether the firm continues as a going concern or not (Francois & Morellec 2004, page 390).

Following Francois & Morellec (2004), Moraux (2002) separates default timing from liquidation. The former remains as specified under LS so that default remains exogenous and is triggered by the first passage of firm value to a constant boundary. Unlike LS, once the default boundary is hit the firm remains trading and the state process continues for the duration of the period the firm remains in administration under Chapter 11. Liquidation is then a separate uncertain event that occurs if the cumulative time spent below the reorganisation barrier exceeds a given fixed period of time. Francois & Morellec (2004) assume liquidation occurs when the unbroken period of time spent in default exceeds a fixed period. Moraux (2002) defines liquidation by the total cumulative time the firm value is below the reorganisation boundary. He shows that the effect of delayed liquidation is bounded between the results of two well known models that have analytic solutions. With infinite delay, there is no early liquidation of the firm, and the model approaches Merton; with coincident default and liquidation, the model approaches Black & Cox (1976). A further refinement is suggested by Galai et al. (2005) to trigger liquidation after the weighted cumulative time in default exceeds a maximum time, where the weight is the distance of the firm value from the reorganisation boundary. The model therefore weights the severity of the financial distress. However, debt can only be valued numerically. While promising in the suggestion that the writedown rate should be endogenously related to the stochastic state process, no empirical tests of these models appears to have been attempted. The models are limited to simple capital structures and involve time-intensive numeric solutions that discourage econometric estimation of parameters.

2.4.2.2 Exogenous-Dynamic

Lastly, we consider a group of structural models in which the default boundary is assumed to vary stochastically through time, but unlike the endogenous models, time-variation in the default boundary is governed by an exogenously known stochastic process and not by shareholders seeking to maximise their wealth at each point in time.

The first group of related models that fits this description belong to models that share the assumption that the default boundary is a traded financial instrument. Nielsen et al. (1993) and Saá-Requejo & Santa-Clara (1999) let the default boundary be the exogenously determined market value of the firm's liabilities. The default boundary value is specified as a geometric Brownian motion under risk neutrality with correlated innovations with the firm asset return and risk-free rate. The state variable, for pricing purposes, is the log of the ratio of firm value to the default boundary, $x(t)$, which is shown to be an arithmetic Brownian motion with constant drift μ_x (Saá-Requejo & Santa-Clara 1999, Equation 5)

$$dx(t) = \mu_x dt + \sigma_x dW_{x,t} \ , \tag{2.20}$$

where σ_x is the volatility of the change in the solvency ratio, $x(t)$, and $dW_{x,t}$ is a Brownian motion correlated with the risk-free rate innovations. Default is triggered on the first passage of $x(t)$ to zero. The constants in equation (2.20) summarise a larger number of parameters that we do not reproduce here.[11] Since we only need to estimate the joint process in equation (2.20), the model has been criticised for its unwarranted complexity by Uhrig-Homburg (2002, p. 54). Not surprisingly, the model shares some similarity with a constant rate version of the LS model. The key difference between the two models is that the drift rate does not vary with the risk-free rate, which it does in the LS model.[12] A further difficulty with the model is that the market value of the firm's liabilities must approach the value of the firm in default; at some stage the two market assets V and K become one. This does not appear to have been treated in the model. Recently, Hsu, Saa-Requejo & Santa-Clara (2003) have redefined the meaning of V to be the continuation value of the firm in a non-default state and K to be the value of the firm in bankruptcy. However, it still appears problematic to treat V and K as separate financial assets when at default they are the same asset, and it is conceptually problematic to correlate the return on the same underlying asset in two mutually exclusive states of the world; V and K cannot trade simultaneously. In contrast, K is usually treated as a threshold value of V in the exogenous boundary literature, or alternatively, within the endogenous boundary literature the value of the firm's liability is valued coherently with firm assets and equity. In neither case does the default boundary suffer from definitional problems.

[11] Refer Saá-Requejo & Santa-Clara (1999, Equations (6)-(8)).

[12] If we were to fit the two models implicitly from market prices, we would find it impossible to distinguish between Saá-Requejo & Santa-Clara (1999), with its correlated stochastic boundary and constant drift μ_x, and a constant short rate LS model with its constant boundary and drift $(r - \delta_v)$.

The other type of exogenous-dynamic boundary model assumes that the default boundary varies in a stochastic manner that mirrors the stylised facts of observed capital structure dynamics. Thus, the models permit expected management driven capital structure changes, but the expected debt-ratio behaviour must be exogenously known before debt can be valued.

As discussed in Section 2.4.1.2, there is good theoretical reason to suggest that management (acting on behalf of shareholders) dynamically adjust their firm's debt-ratio over time. Rather than attempting to fully explain debt-ratio dynamics as a consequence of shareholder wealth maximisation, the exogenous-boundary models take an assumed debt-ratio behaviour as given. The theoretical complexity is reduced, but the disadvantage is that we must a priori form an opinion as to the most appropriate underlying stochastic process for the firm's capital structure process.

Further, the process for the latent log-solvency ratio (the state variable, $x(t)$) must be parameterised. To date, the exogenous-dynamic empirical literature has parameterised expected capital structure dynamics from observed changes in book debt-ratios. However, since we have shown that the firm's book value of debt to be potentially unreliable as a measure of the default boundary, using capital structure parameters sourced from debt-ratio movements may also be flawed. Alternatively, we propose to parameterise the capital structure process implicitly from the credit spreads and avoid the potential measurement error introduced by use of proxy variables.

Taurén (1999) propose a structural model in which the state variable is the firm's ratio of book liabilities to its market value of assets that follows a mean reverting stochastic process. Thus, the default boundary, represented by the firm's liabilities, is assumed to be stochastic and governed by a known process that represents, in a reduced manner, the dynamic behaviour of the firm's management. Like the endogenous-dynamic literature, permitting mean-reversion in firm-leverage captures the additional risk to bondholders of management's option to re-leverage the firm in the future. The effect of assuming mean-reversion in firm leverage results in higher forward default rates, and a flatter credit spread term structure relative to the Merton and LS models. Longer-term credit spreads are an increasing function of the firm's target leverage and decreasing with respect to the speed of reversion. In addition, short-term spreads are more sensitive to the current level of leverage. It is argued by Taurén that capital structure mean-reversion results in more realistic credit spread term structures.

Independently, Collin-Dufresne & Goldstein (2001) (hereafter CDG) suggested a similar model, which has become the most widely known and empirically studied target-leverage structural model, empirically fitted by EHH and HH. CDG differ from Taurén in their choice of state variable and boundary dynamics. The default boundary is assumed to change dynamically over time. As in Taurén the firm adjusts its level of debt, mean reverting to a long-run target level. Secondly, the firm will issue debt opportunistically to time the debt market, issuing more (less) debt when the current risk-free rate is below (above) the long-run risk-free rate level and therefore expected to increase (decrease) in future. The default boundary is assumed to evolve as

$$\begin{aligned} d\ln K(t) &= \kappa_v\big(\ln V(t) - \ln K(t) - \nu - \phi(r(t) - \theta)\big)dt \\ &= \kappa_v\big(x(t) - \nu - \phi(r(t) - \theta)\big)dt\ . \end{aligned} \tag{2.21}$$

Thus, the default boundary is assumed to be a function of the current level of the log-solvency ratio, $x(t)$, a long-run target level of the log-solvency ratio, ν, the speed of mean-reversion to the target in the absence of debt market timing, $\kappa_v \geq 0$, the sensitivity of the firm's debt issuance policy to the expected change in the risk-free risk-free rate, ϕ, and the trend in risk-free rate expectations given by the difference in the current short rate, $r(t)$, and its expected long-run level, θ, as per Vasicek's (1977) interest rate model. The LS model is nested within CDG. For estimation purposes we restate the models to be dependent on their log-solvency ratios $x(t)$ defined as the log of the ratio of the firm's market value to its default boundary (refer Appendix A for derivation). The dynamic process for the log-solvency ratio in the LS model is given by

$$dx(t) = (r(t) - \delta - \sigma_v^2/2)dt + \sigma_v dW_{v,t}^Q, \tag{2.22}$$

and for the CDG model it is

$$dx(t) = \kappa_v\left[\left(\frac{r(t) - \delta - \sigma_v^2/2}{\kappa_v} + \nu + \phi(r(t) - \theta)\right) - x(t)\right]dt + \sigma_v dW_{v,t}^Q, \tag{2.23}$$

where $dW_{v,t}$ is correlated with the short rate under the Vasicek (1977) model.

From equation (2.22), it is evident that the LS model has no mean-reversion; shareholders do not invoke the option to re-leverage nor attempt to reduce bankruptcy costs. In the other models, there is an assumed continuous adjustment towards a long run solvency ratio target, that is symmetrical above and below the target. Implicit in this specification is that capital structure adjustments are continuous and that shareholders actively adjust debt-ratios downwards in response to rising bankruptcy costs. We can also see that the log-solvency ratio drift is time-varying and positively related to the risk-free rate. An increase in the short rate, ceteris paribus, decreases default risk and credit spreads. From equation (2.23), the CDG model's log-solvency drift rate is shown also to be positively influenced by the level of target log-solvency, ν, and to the size of the risk-free term-structure slope; the latter is intended to capture management debt-timing behaviour.

Mueller (2000) shows that the drift rate in equation (2.21) can be expanded to include multivariate factors influencing the direction of capital structure decisions. The resultant model has the disadvantage of a significant increase in numeric processing needed to solve the expected first passage crossing time. More recently, Demchuk &

Gibson (2006) adapt the CDG model without a substantial increase in numeric complexity. They assume that management adjusts debt levels continuously toward a target as per CDG, but with the target a stochastic function of by recent past equity returns.

2.5 Capital Structure Theory and Evidence

As discussed in Sections 2.4.1 and 2.4.2, recent extensions of the Merton model have either evolved to have endogenous default boundaries or exogenous default boundaries. The former group of models is motivated by the hypothesis that shareholders adjust capital structure to maximise the value of their claim, balancing the benefits of tax shields against bankruptcy costs. The latter group takes the dynamic process for capital structure as given.

A possible cause of the credit spread prediction biases associated with structural models, may be due to the model's implied capital structure dynamic process specification being inconsistent with actual firm capital structure behaviour. In this section we compare the underlying state variable dynamics assumed in the main structural credit models with the parallel theoretical and empirical literature on capital structure behaviour. Specifically, we examine the main theories of capital structure; the trade-off, pecking-order, and market timing theories. From this review we identify potential sources of miss-specification for further examination in our model residual prediction error testing.

2.5.1 Trade-Off Theory

In perfect and frictionless markets Modigliani & Miller (1958) prove that the choice of capital financing between debt and equity has no affect on the value of the firm. However, in the presence of taxes, firm value can be increased by borrowing and reducing the tax burden by claiming a tax deduction on interest expenses (the tax shield). Borrowing also introduces default and potential bankruptcy costs that can decrease firm value. The static trade-off theory states that the firm chooses an optimal debt level that balances the benefit of tax shield with the added cost of potential bankruptcy. In other words, an optimal firm maximising debt-ratio is predicted to exist.

The dynamic version of the trade-off theory states that firms will adjust their debt-ratios towards an optimal debt-ratio target, but in the presence of recapitalisation costs, the adjustment process may be slow. Firms are not likely to be currently at their target debt-ratio, and are expected to mean-revert to it over time.

For the endogenous boundary structural models to be consistent with the trade-off theory, the firm's debt-ratio should be set by shareholders with regard to the effect of tax shields and bankruptcy costs, and the default boundary of the firm chosen as consequence of optimising the debt-ratio. The endogenous boundary models shown in Table 2.1 are all consistent with the trade-off theory's prediction of the existence of an optimal debt-ratio. However, the focus of the decision maker in the endogenous boundary models is shareholder wealth maximisation and not maximisation of the firm's value.

For the exogenous boundary models to be consistent with the trade-off theory, the firm's capital structure must be specified as a stochastic process with mean-reversion to a target level. Most exogenous-dynamic structural credit models make this assumption, with variations on whether the target is fixed or time-varying. Taurén (1999) assumes the firm's debt-ratio reverts to a fixed target debt-ratio, Mueller (2000) permits the firm's target log-leverage ratio to vary with business conditions contingent on macroeconomic factors such as GDP, Collin-Dufresne & Goldstein (2001) vary the target log-leverage ratio with the slope of the risk-free yield curve, Demchuk & Gibson (2006) assume stochastic mean-reversion of the firm's log-leverage ratio to a target that is negatively related to the historic return on the equity market index.

2.5.2 Pecking Order Theory

The static pecking order theory states that a firm will finance its investment needs in a preferential order of funding sources using those with the lowest information cost first (Myers 1984, Myers & Majluf 1984). The theory is based on the assumption that informational asymmetry costs typically dominate other agency and bankruptcy costs associated with debt. Management will therefore seek to finance investments in order of decreasing information asymmetry. Retained earnings has no information asymmetry and is preferred first followed by collateralised debt, unsecured debt and lastly equity. Unlike the trade-off theory, the choice between debt or equity is made in a predetermined order according to the amount of external funding required. The lowest information cost source is internal cash flows, followed by debt, then followed lastly by equity. Thus a firm will borrow, rather than issue equity, when internal cash is not sufficient to fund capital expenditures. No target debt level exists and dividends are assumed to be 'sticky'. Thus, the firm's observed debt-ratio is predicted to be simply the outcome of the cumulative need for external funds. The main difference between the trade-off model and the pecking order model is that the former predicts the firm to have an optimal debt-ratio, and the latter predicts that there is no optimal target debt-ratio.

In the dynamic version of the pecking order theory, also offered by Myers (1984), management are concerned with future as well as current information costs. Firms with large expected future investments are predicted to maintain low default risk levels of debt (i.e. maintain their debt capacity), to avoid the future cost of financing with higher premium debt or forgoing the investment. Thus, management are predicted to adjust leverage to maintain a minimum level of expected future debt capacity dependent upon their expected future investments and current debt capacity. When the firm is not debt constrained, or its investment opportunity set is low, the firm is predicted to not actively manage its debt-ratio. If the firm is approaching its debt capacity threshold, reversion to a safe region is predicted, even though there is no explicit debt-ratio target. Therefore, in practice the firm may appear to have a 'soft' debt-ratio target resulting from anticipating future investment requirements subject to also maintaining sufficient debt capacity to provide financial flexibility (Fama & French 2002). Importantly, Shyam-Sunder & Myers (1999) demonstrate that cyclical changes in operating earnings and capital expenditure can lead to autocorrelated behaviour in the net external funding deficit, and therefore mean-reverting behaviour of the firm's debt-ratio even when the pecking-order theory holds. Consequently, mean-reversion of the debt-ratio is a consistent prediction from both the trade-off and dynamic pecking order theories.

There is no endogenous boundary structural model explicitly based on the pecking-order theory. If we were to construct such a model, we would have to specify the firm's stochastic net external funding requirement, with knowledge of the firm's cash flows and investment plans, in order to endogenously determine a projected path for the changes in debt and equity funding over time. Clearly this is an daunting task for outside parties to the firm to undertake.[13]

More simply we can value debt using an exogenous boundary model that specifies mean-reversion of the capital structure, whether by the trade-off or pecking order theory, without resolving which theory holds. For example, Taurén (1999) specify the firm's debt-ratio to be mean-reverting under an Ornstein-Ulenbeck stochastic differential process.

2.5.3 Market Timing Theory

Market timing theory proposes that managers, when deciding to issue debt or equity, are predominately influenced by current economic conditions in the debt and equity markets, and attempt to lower their average cost of capital by timing the raising of capital. As discussed further in Section 2.5.4, there is increasing empirical support for this behaviour.

The theory of equity market timing is contingent upon there being information asymmetry between management and investors. Management must believe that equity is underpriced relative to the firm's value based on their own inside information, and the market must react slowly to the information released from the equity issue announcement. If these conditions hold, and management prove to be correct on average, then extra value can be gained for the benefit of existing long-term shareholders. Baker & Wurgler (2002) propose that equity market timing is the predominant behavioural char-

[13]Goldstein et al. (2001) use the firm's cash flow as the state variable but assume shareholder's maximise the value of their claim. A pecking-order based structural model could be developed from the same state variable but replacing the assumption that the debt-ratio is the outcome of wealth maximisation behaviour undertaken by shareholders, with the assumption that it is the cumulative result of the firm's investment strategy, net external funding requirements, and preferential access to funding sources.

acteristic of management to such an extent that a firm's present capital structure is the consequence of past efforts to time the equity market. Their view is supported by survey evidence gathered in an anonymous survey of 392 chief financial officers by Graham & Harvey (2001). They find that one of the most important determinants of equity issuance is management's belief as to whether the equity market has, in their view, mispriced the firm's stock.

Debt market timing is based on the assumption that management attempt to exploit expected changes in the level of interest rates. More debt is predicted to be issued when the risk-free rate is low and expected to rise in the future. Following the expectations theory of interest rates, this is expected when the slope of the yield curve is positive and present risk-free rate is low. The survey of Graham & Harvey (2001) also finds qualitative support for debt market timing, particularly larger firms, with more sophisticated treasury functions, who cite the level of interest rates as an important consideration when making debt issues.

There is no endogenous boundary structural model consistent with the marketing timing theory. However, the theory is readily implemented via an exogenous boundary model because the target level of solvency is easily conditioned on external market factors. For example, debt market timing behaviour is assumed by CDG who specify the risk-neutral target leverage ratio to be a decreasing function of the stochastic spot rate and an increasing function of the stochastic slope of the yield curve. In a variation of the CDG model, Demchuk & Gibson (2006) construct am exogenous boundary model with equity timing capital structure behaviour. They assume that the firm continuously adjusts its capital structure, i.e. issues either equity or debt, in response to the observed past returns of the stock market index and to changes in its leverage ratio. They create a stock market index variable that, in the spirit of Baker & Wurgler (2002), measures recent aggregate equity market performance. It is a time weighted geometric average of the historical stock market index return with a higher weighting placed on recent performance, and is introduced into the CDG framework in replacement of the stochastic risk-free rate.

Similar to Demchuk & Gibson (2006), Baker & Wurgler (2002) construct an index of past equity returns designed to measure the extent to which the firm has historically raised external funds when its equity return was high. Their 'external finance weighted-average' market-to-book ratio is a weighted average of the firm's past market-to-book ratios which, for example, takes high values for firms that raised debt or equity when their market-to-book ratio was high. The variable is found to be explain present leverage ratios; firms with current low leverage are those that raised funds when their market valuations were high, as measured by the market-to-book ratio, while high leverage firms are those that raised funds when their market valuations were low.

The structural modelling literature has the capacity to include market timing behaviour in the valuation of risky debt. The CDG model includes debt timing behaviour and Demchuk & Gibson (2006) includes equity timing behaviour. In a repeat the of the HH base case exercise, the Demchuk & Gibson (2006) show that their model can potentially explain more of the credit spread than existing models, for example, 40 percent of the credit spread on Aaa bonds can be explained by their model compared to only 18 percent for the CDG model.

2.5.4 Empirical Evidence

That firms mean-revert their debt-ratios over time is widely supported empirically (Taggart 1977, Jalilvand & Harris 1984). More recently, Fama & French (2002) show firm-specific debt-ratios to be very slowly mean-reverting at a rate between 0.07 and 0.18 per annum. Frank & Goyal (2003) find the average firm debt-ratio mean-reverts at 0.124 per annum on average. For small firms the rate was faster at 0.115 and slower for large firms slower at 0.104.

What is unclear is whether leverage mean-reversion evidence supports the trade-off theory or dynamic pecking order theory. If it is the latter, then the endogenous boundary model literature is misspecified since it relies upon the existence of an optimal debt level, whether it be to maximise firm value or shareholder wealth. Shyam-Sunder & Myers (1999) show that such slow rates of debt-ratio mean-reversion can also be explained by the dynamic pecking order theory. They are unable to reject the dynamic pecking order model because such a slow rate may be due to autocorrelated net cash flows or from firms reserving debt capacity for future financial flexibility as posited in the dynamic pecking order theory. The slowness of reversion is attributed to management giving low priority to maintaining a target debt-ratio; an opinion supported by a qualitative survey of financial controllers undertaken by Graham & Harvey (2001).

Baker & Wurgler (2002) reject both the pecking order and trade-off theories. They argue that if the trade-off theory holds, temporary fluctuations in the market-to-book ratio, or any other variable, should have temporary effects. However, their finding of statistically robust persistence in past equity returns is not consistent with dynamic debt management under the trade-off theory. Only an exogenous boundary model, such as CDG and Demchuk & Gibson (2006), is consistent with the market timing theory. That Graham & Harvey (2001) find that two-thirds of surveyed Chief Financial Officers claim market timing influences their sourcing of funding, supports market timing behaviour as the predominant explanatory model of the firm's dynamic capital structure. If market timing is the main capital structure behaviour, then only an exogenous boundary structural credit model conditioned on past equity returns would be consistent with the empirical literature supporting market timing behaviour.

That past stock prices influence firm leverage is supported by Welch (2004). He finds that firm leverage ratios are are strongly negatively related to past stock returns, which is inconsistent with the trade-off theory's prediction that firms will increase leverage to revert to a static debt-ratio in response to decreasing market leverage. Using a variation of Baker & Wurgler's (2002) market timing variable, Kayhan & Titman (2007) confirm that historical stock price changes and external funding deficits have a significant effect on capital ratios over the short to medium term, but in the longer term there is evidence of partial reversion back to target ratios based on traditional trade-off variables. Firms are found to raise equity capital when their stock prices are high and tend to reduce their debt-ratios subsequently. The persistence of the equity market influence partially persists for up to 10 years.

Frank & Goyal (2003) and Fama & French (2002) argue that firms with more asymmetric information should follow the pecking order of funding sources more closely. However, small high growth firms issue significant amounts of equity, in contradiction of the pecking order theory. In contrast, the pecking order theory explains well the equity issuance of large mature firms. The result is explainable if capital constraints are considered in the context of a dynamic pecking order model. Lemmon & Zender (2002) find that small high growth firms carry additional equity due to the presence of debt capacity constraints. Similarly, Dissanaike, Lambrecht & Saragga (2001) show, on a sample of UK firms, that those that did not target a debt-ratio were on average larger, more profitable, have higher market-to-book ratios, and carry more tangible assets, than firms that did target a debt-ratio. Debt targeting may therefore be a consequence of concern over managing future debt capacity.

Instead of firm size, Chang, Dasgupta & Hilary (2006) use equity analyst coverage as a proxy for information asymmetry. They find that firms with low coverage and high information asymmetry, are more affected by Baker & Wurgler's (2002) external finance weighted market-to-book ratio. In other words, these firms seek external funds when their share price is high relative to recent history. The implication is that firms with high information asymmetry, including small firms, will have term structures of credit spreads that reflect the potential to re-leverage based on the expected evolution of market equity returns.

Korajczyk & Levy (2003) consider the influence of macroeconomic conditions on firm leverage and show that it affects firm behaviour via market timing subject to capital constraints. Empirically they find that unconstrained firms are more sensitive to market conditions and are more likely to issue equity when the recent average stock price is high. A constrained firm is defined as not having sufficient cash to undertake investment opportunities and faces severe agency costs when accessing financial markets. Constrained firms are more influenced by the deviation from firm-specific target, exhibiting faster reversion, and were only marginally affected by macroeconomic conditions.

Hovakimian et al. (2001) find that deviation from target is an important but not dominant factor in explaining capital structure adjustments. The likelihood of issuing equity is positively related to the firm's current and immediate past stock return, and firms with a low market-to-book ratio tend to issue debt rather than equity.

In summary, the most recent literature emphasises the role of information asymmetry, equity market timing, past external funding deficits, and current debt capacity in explaining changes and the level of firm capital structure. Where firms have high information asymmetry, they access the external markets less frequently and equity market conditions are more persistent. Firms with immediate debt capacity constraints are less concerned with market timing and exhibit relatively faster debt-ratio reversion rates. Large, mature firms are more likely to conform to the pecking order theory and be less influenced by equity market timing. Finally, concern for maintaining future debt capacity may result in slow levels of mean-reversion to an apparent target even though an optimal debt target does not exist.

Finally, the trade-off theory has limited empirical support in explaining short term changes in capital structure, although evidence by Kayhan & Titman (2007) suggests that the trade-off theory may hold in part over the longer term.

2.5.5 Implications for Structural Credit Models

The earliest structural credit models assume passive capital structure management. Based on the evidence of the capital structure literature, such a specification is not supported. The future path of thr firm's capital structure is more likely to be the result of external funding requirements, subject to capital constraints and information asymmetries.

The endogenous boundary literature is premised on the existence of an optimal trade-off between bankruptcy costs and tax shields, yet managing the firm's deviation from an optimal target is not supported as an imperative of management. On the other hand, the exogenous boundary literature has been extended to accommodate market timing behaviour, and can accommodate differences in capital constraints and information asymmetry across firms.

The essential characteristics of a model consistent with the empirical literature is one that specifies:

Mean-reversion of leverage to target: The speed of mean-reversion depends upon the level of the firm's debt capacity. For large and well rated firms the rate of mean-reversion is likely to be very small, and higher for smaller or poorly rated firms;

Leverage depends on past equity performance: High historical equity returns are associated with lower leverage with the effect on leverage persisting well into the future. Equity market timing is greater when the firm has higher information asymmetries, for example, small firms.

Therefore, of the extant structural credit models, the dynamic exogenous boundary models with mean-reversion and market timing are the most likely to correctly incorporate the firm's expected future leverage path, for example, CDG and Demchuk & Gibson (2006). To the extent that existing models do not include such behaviour, but the bond market expects it, credit spread prediction errors are likely to be related to these missing characteristics. Specifically, the capital structure literature suggests the following errors may exist in some structural models where they are absent:

Mean-reversion of firm solvency to a target: When the firm is debt constrained, in the absence of mean-reversion, the credit model will overpredict long term spreads due to the absence of leverage reversion to a less constrained region. For unconstrained firms (high credit rating), long term credit spreads are expected to be understated due to the absence of mean-reversion to a soft target arising from autocorrelated external funding requirements. Thus, a negative relationship between credit spread prediction errors and the bond's term to maturity is expected;

Firm solvency as a function of equity market timing: When the equity market has performed strongly, future leverage and default risk is expected to be low due to firms favouring equity over debt issuance. In the absence of equity market timing, credit models can be expected to overestimate long term credit spreads when recent equity performance is strong. Thus, a positive relationship between spread prediction error and recent equity market performance is expected. The error is expected to be higher for small firms since information asymmetry is higher.

We later test for the influence of these stylised facts on model prediction error in Section 4.2.2.

2.6 Review of Prior Credit Spread Studies

While the theoretical field has seen considerable research effort, the empirical testing of structural credit models has not kept pace. The number of studies that directly assess the performance of structural model predictions remains quite small in comparison. As noted by EHH,

> ...the empirical testing of these models is quite limited. Indeed, only a few articles implement a structural model to evaluate its ability to predict prices or spreads. (Eom et al. 2004, p.200)

In this section we review the findings from prior empirical research, summarise methods and limitations, and place our work in the context of addressing some of these shortcomings.

2.6.1 Past Findings on Predictive Accuracy

Until recently, the most comprehensive empirical studies of a structural model's pricing accuracy was conducted on the Merton model by JMR. Their study has since been widely cited as supporting the view that the Merton model underestimates credit spreads. Their method of estimation pioneered fitting of structural models and so we first examine this aspect of their work.

JMR collected Standard & Poors price data on 350 bonds, issued by 27 firms, sampled over the period 1977 to 1981. They carefully selected firms to include only those displaying a simple capital structure, close to the Merton ideal; with one class of stock and mostly public, long dated bonds with limited numbers of issues outstanding. The sample unavoidably included callable bonds due to the high prevalence on issue in the sample period, and so the Merton model was modified to include the influence of call provisions on bond value. As an additional control, a naive model was constructed in which the bond's promised cash flows were discounted at the maturity matched relevant risk-free rate plus the effects of call provisions.

To fit the Merton model, JMR used two different methods to estimate the firm's asset volatility. The first relies on constructing a monthly time series of the approximate value of the firm. Firm value is estimated as the sum of the market value of equity, the market value of traded debt, and the market value of nontraded debt. The latter was estimated by assuming that the ratio of book to market values was the same for traded and nontraded debt, and was then applied to the book value of the nontraded debt. Asset volatility was then calculated as standard deviation of monthly firm value return measured over the prior 24 months. This method is general and does not impose any model specific assumptions.

The second method relies on inverting the implied asset volatility from the Merton model. Since equity is a derivative of the firm's assets we know from Ito's Lemma that the volatility of equity is related to asset volatility by

$$\sigma_e = \sigma_v \frac{V(t)}{E(t)} \frac{\partial E(t)}{\partial V(t)}, \tag{2.24}$$

where σ_e is the equity volatility, E is the market value of equity, V is the market value of firm assets, and σ_v is the volatility of the firm's assets. For the Merton model the partial derivative of equity value with respect to asset value is given by $N(d_1)$ where d_1 is defined in equation (2.3) and $N(\cdot)$ is the cumulative density function of the standard normal distribution.

JMR use the time series method to obtain initial estimate of $V(t), E(t)$. The standard deviation of equity is measured directly as the sample standard deviation of the firm's daily equity return observed over the last three months. An estimate of asset volatility is then obtained by inputting the sample equity volatility into equation (2.24) and solving for asset volatility.

The findings of JMR give mixed support for the Merton model. For investment grade bonds the model was unable to perform better than their naive model. For sub-investment grade bonds, the Merton model provided some incremental explanatory power over the naive model. The mean percentage pricing error for the whole sample was 4.5 percent and 10 percent on the sub-investment grade bonds indicating a tendency to overvalue and therefore underestimate credit spreads. An examination of the residuals showed that, across all rating levels, the higher (lower) the estimated asset volatility the more likely the bond price was underestimated (overestimated). A year effect was noted as possibly caused by time variation in the risk-free rate that a stochastic rate model may have improved. Bonds paying larger coupons tended to be overvalued in the sub-investment grade, which may indicate a possible missing tax effect related to default risk. JMR conclude that introducing uncertain interest rates and tax would improve the valuation of corporate bonds. However, their results may have been influenced by the large number of callable bonds in their sample and the high prevailing interest rate volatility caused by a shift in Federal Reserve monetary policy during the sample period (EHH).

A further problem with the JMR sample concerns the lack of control over matrix prices in the Standard & Poors bond price data set. Warga & Welch (1993) report that the data contains mostly matrix prices and not actual trade prices. A matrix price is a hypothetical price used where no trade was observed. It is calculated from, 'rules that specify the addition of a fixed spread over either an actively traded benchmark issue of the same company, another company's issue with similar rating, maturity, and coupon, or a U.S. Treasury', (Warga & Welch 1993, p.963).

The JMR study was followed Ogden (1987), who selects 57 newly issued bonds with maturities greater than 10 years, issued between 1973 and 1985. On the date of issue, he finds credit spreads to be underpredicted, by an average of 104 basis points. A similar call for the inclusion of stochastic interest rate was made, but is also potentially influenced by the prevailing volatile interest rate environment.

A particular feature of structural credit models is the upward sloping term structure of low-default risk bonds and the "hump-shaped" rise and fall of high-default risk bonds (refer Figure 2.1). Sarig & Warga (1989) examined the Merton model's predicted credit spread term structure shape with observed data. Their data comprised trader quotes for corporate zero-coupon bonds collected over the period 1985 to 1987. Matrix price data points were removed. Over half the bonds in the sample were callable and a further filter applied to remove bonds where the call option was likely to be economically valuable. Treasury rates were then deducted from observed bond yields and the resulting yield spread data averaged over time and across issuers grouping by similar ratings. The resulting credit spread term structures showed a similar pattern to Merton. Their finding that high-risk credit spreads are downward sloping was subsequently questioned by Helwege & Turner (1999), whose data was the primary issue yields, on matched pairs of bonds of different maturities, issued on the same day over the period 1977 to 1994. For speculative grade borrowers, Helwege & Turner (1999) find that the term structure on the day of issue was, on average, upwardly sloping by 14 basis points. However, five out of fourteen cases of non-callable bond term structures were downward sloping as predicted by Merton, thereby demonstrating that the credit risk term structure is influenced by firm-specific factors.

Wei & Guo (1997) compared the accuracy of the Merton and LS models to predict the credit spread term structure using weekly observations of Eurodollar yields sampled during 1992. Five data points along the Eurodollar term structure, spanning seven days to one year, were used to invert the models for the unknown parameters using maximum likelihood. The models were refitted each week cross-sectionally adopting a similar approach to that pioneered by Brown & Dybvig (1986) in fitting a single factor bond pricing model to the risk-free term structure. The ratio of firm value to default boundary $V(t)/K$ was permitted to vary and re-estimated each week. Their paper is therefore similar in spirit to our proposed latent estimation method except that we propose fitting parameters constrained intertemporally and cross-sectionally. They find that the Merton and LS models provide similar explanatory power, despite the additional number of parameters in the LS model. When the volatility estimate in Merton is allowed to be re-estimated each week, the model clearly outperformed LS, thus suggesting that implied volatility is time-varying.

A difficulty with Wei & Guo (1997) is that they only consider very short-term debt of one year or less.[14] For tests of model predictive accuracy we are interested in comparing performance over longer time periods typical of corporate bond tenors. Because structural models rely on diffusion of the firm value across a default boundary, very short tenors do not provide a sufficient period of time for differences in stochastic process to be robustly tested. We would expect larger differences between models to appear over longer term structure periods. The estimated parameters reported appear to be adversely affected as a consequence. At a mean of 1.025, the estimated ratio of $V(t)/K$ implies that firms are very close to their default boundaries and will almost certainly default. The mean variance is correspondingly very low at a reported level of only 0.9 percent (refer (Wei & Guo 1997, Exhibit 9)). To match the short term credit spreads in the sample, with a mean of 45 basis points for a 7 day tenor, the firm is implied to be very close to default in order to invoke a sharp non-linear rise in credit spreads as illustrated in the upper plot for the highly leveraged firm in Figure 2.1. When estimating credit model parameters by inversion, it is important to control for non default risk related pricing factors. For example, in estimating a reduced-form model from bond spreads, Duffee (1999) controls for the unexplained, non-stochastic components of the credit spread, by

[14] Specifically, 7 days, 1, 3, 6, and 12 months.

the use of an intercept term that implies a constant non-zero probability of default at all tenors. It would have been informative to see what additional explanatory power the credit models could provide over and above the assumption of a constant credit spread. Unfortunately, Wei & Guo's (1997) method of cross-sectional estimation precluded this; without the additional time-series restrictions on the model, it would be impossible to estimate a non-stochastic component of the credit spread.

The testing of credit models has been hampered by 'poor quality and limited availability of corporate bond price data' (LYS. p.3). This situation was partly remedied by the introduction of the Bridge Information System's bond database, first applied to testing the Merton and LS model by LYS. Their database comprised daily bid quotes shared in a consortium by major U.S. investment banks (Bridge Information Systems). The data is not without some potential error however, with the authors noting that, 'Bridge quotes reflect the informed judgment of Bridge personnel rather than results of specific transactions', (LYS, p.3). Firms were selected that had only a single bullet bond issued, including banks, in the final sample of 56 issuers. Model parameters were estimated from observable proxies. Estimates of asset volatility are the annualised historical observed quarterly value of outstanding stock plus book value of non-bond liabilities and the market value of the bond. Various tests of different default boundaries for the Merton model were attempted with the best performing found to be K equal to total book liabilities. The LS model was estimated in two steps: firstly, just the introduction of early default was incrementally tested relative to the Merton model by fitting a single-factor constant interest rate version of LS; secondly, the full LS model with correlated stochastic risk-free rate model was fitted. The asset-rate correlation was proxied by the sample correlation between the monthly stock price and yield of on-the-run 10 year Treasury bonds. Firm level correlations fluctuated widely ranging from -44.7 percent to 28.9 percent with an average of -7.2 percent. Recovery rates were tested using a single rate of 47.7 percent and varying by industry as per historical rates found by Altman & Kishore (1996).

The results of LYS supported earlier problems with the Merton model and was damning of the performance of the LS model. Specifically, the introduction of early default while, 'certainly more realistic, does not seem to improve model accuracy', (LYS, p.13). The median spread prediction error for the Merton model was found to be 58.5 basis points, and LS was 62.7 basis points using individual firm correlations and only 52.0 basis points with either a constant interest rate or assuming a stochastic rate with zero correlation. The theoretical improvements made by LS over Merton were not supported empirically. Introducing stochastic interest rates into the LS model did not improve price prediction relative to the Merton model. The LS model prediction error biases were found to be directionally the same as Merton but greater. Overall both models were found to underestimate credit spreads, more so with longer dated tenors and with higher coupon rates. Possible reasons suggested by the authors were the presence of a liquidity premium, not controlled for in their experiment, and concerns about the appropriateness of their brute force volatility estimation procedure. They state that,

> The classical model could still be correct if, perhaps among other possibilities, we have greatly mis-estimated future asset volatility. At the end of the day, we are unable to distinguish two hypotheses: Either the model is rejected or expected future asset volatility differs significantly from past volatility. (LYS, p.8)

Their firm selection procedure also limits the conclusions that can be drawn. Only firms with single public bond issues were included. This precludes larger firms from their sample where we would expect greater information to be contained in public bond and equity markets, and their results to be more representative of the broader bond market. Similar to preceding papers, their test is cross-sectional only, and with only a single bond outstanding, the model's ability to replicate the firm's term structure of credit spreads remains untested.

In the most comprehensive study to date, EHH extend the scope of LYS and undertake a 'horse race' amongst five structural models: Merton, Geske, LT, LS, and CDG. Their data source is the Fixed Income Database (FID) containing month-end firm-specific bid quotes made by Lehman Brothers bond traders over the period 1986 to 1997.[15] The sample was restricted to senior noncallable bonds issued by non-financial and non-utility companies that had a simple capital debt structure with only one or two public bonds on issue and were listed.[16] To avoid biases due to infrequent trading, bonds of remaining maturity of less than one year were omitted. The resultant sample comprised 182 bonds, from issuers mostly in the manufacturing industry (68 percent of sample) that tended to be large and of low risk.

The five models were implemented using observable equity and accounting data. Thus, their study shares a similar method to JMR and LYS. The exogenous default boundary was assumed equal to the firm's total book liabilities, and firm asset value equal to the sum of total liabilities and the market value of equity. A 10 year time series of observed monthly leverage ratios (total liabilities over firm value) was used to estimate the parameters of the firm's mean-reverting leverage process. For the LS and CDG models, the mean-reversion of the latent leverage ratio, the sensitivity of leverage to the risk-free term structure, and target leverage ratios, were all estimated by regressing leverage on lagged leverage and interest rates. The sample correlation coefficient

[15] Lehman Brothers maintain a comprehensive database of price quotes to support their widely published bond indexes (refer Hong & Warga (2000) for description)

[16] Gas and electricity utilities were excluded on the basis that their return on equity and therefore default risk is influenced by regulation. We see no reason to exclude the firms from our sample given that they represent an important source of debt in the bond market and market prices for bonds should fully encapsulate the issuer's default risk. These firms may have targets set by government but they are not guaranteed.

between asset and interest rate movement was estimated to be only -2 percent. Asset volatility was estimated by equation (2.24) where the known Merton model analytic solutions for the partial derivatives was applied across all models, although not strictly correct, was considered a tractable approximation.

EHH's main finding is that the five structural bond pricing models tested all had difficulty in accurately predicting credit spreads, demonstrating wide dispersion in prediction errors. The single-factor models of Merton and Geske, on average underpredicted credit spreads, but the other three more advanced two-factor models overpredicted. At the extremes, the Merton model was the worst at underpredicting and the CDG model was the worst at overpredicting with mean percentage prediction errors of -50.4 percent and 269.8 percent respectively.[17] The main point made by EHH is that the mean prediction errors mask the true extent of their poor performances. Referring instead to the absolute prediction errors the Merton and Geske models are fairly similar in having the lowest errors (78.0 percent and 66.9 percent). Once again, the poorest performer was clearly the CDG model at 319.3 percent. The LS model was found to overpredict with a mean prediction error of 42.9 percent outperforming LT at 115.7 percent.

The results of EHH are surprising. Their conclusion that the LS model overpredicts credit spreads is in contradiction to LYS. The absolute spread prediction error was nearly double that of the Merton model indicating much wider prediction error variance. The LS model showed a tendency to predict either very high or very low credit spreads with most dispersion occurring with shorter maturities. The LS model is nested in the CDG model and it is to be expected that many of the same error patterns were also evident in the CDG model. With a mean percentage spread prediction error of 270 percent, the CDG model overpredicted credit spreads by more than the LS model. However, with its additional parameter control over long term credit spreads, and consistency with the capital structure stylised facts, we would have expect the model to perform better than the LS model. It is therefore important to understand whether this finding obtains in other samples.

As these models contain a second stochastic factor for the risk-free rate, regression testing of the errors against interest rate volatility was performed by EHH and found to be significant. When the interest rate volatility is high, the two-factor models tend to overpredict credit spreads. Further, the correlation between asset return and interest rates was found to have minimal influence on predicted spreads. Taken together, the results suggest that introducing a stochastic interest rate factor has led to no greater prediction accuracy and has introduced additional over-sensitivity to interest rates.

Because the CDG model nests LS, but contains more variables that are difficult to estimate, the worsening of performance may be due to measurement error. We can identify two possible causes. Unlike the LS model, CDG requires estimates of the target

[17] Refer Eom et al. (2004, Table 3).

leverage ratio and speed of the leverage ratio reversion to the target, estimated by a univariate regression on the time series of observed leverage ratios. In small samples, the estimate of the speed of mean-reversion may be biased downwards when the process is close to being a unit root (refer Ball & Torous (1996) and references therein). The second problem may reside with their method for estimating the target log-solvency ratio, ν. In general, the expected first passage time is very sensitive to the level of the drift rate. Their estimation method involves solving for the target ratio under the physical measure using a regression on first differences of the observed market leverage ratio, then estimating the real expected growth of the firm's assets to back out ν, which is defined under the risk-neutral measure.[18] The expected growth rate is taken as the past 10 years of value growth. Since realised growth may differ from the expected an alternative is also tested using a longer-term view of market-wide equity returns (15 percent return and an equity risk premium of 6 percent is applied). Under the alternative estimation method, the mean spread prediction error decreases to 79 percent, but still remains higher than the LS model. Their results indicate that errors in the CDG model are very sensitive to the target leverage ratio assumption, which of course, is unobservable. Consequently, EHH suggest that,

> An alternative to using estimates of mean asset returns and speed of adjustment would be to simply estimate the implied risk neutral long-term mean of leverage, which not only fits the data to the spreads, but avoids estimation of the mean asset return in the model. (Eom et al. 2004, p.523)

EHH find the LT model consistently overpredicts credit spreads. The mean percentage spread prediction error is 115 percent. The problem is worse on high-coupon bonds, short maturities, and for sub-investment grade issuers. Its behaviour is different to the other models considered, tending to overpredict across all leverage levels and was not sensitive to parameter estimates.

In summary, EHH find that all models except LT, exhibit a similar pattern of error. Prediction errors are positively related to leverage; overprediction occurred with higher leverage and under prediction with lower leverage firms. The results confirm a leverage bias consistent with the view of JMR that the Merton model overvalues safe bonds. A similar result is found when spread prediction errors by bond rating are examined; spreads are overpredicted for poorer rated issuers and underpredicted for investment grade, with the exception of LT. This model exhibited the lowest leverage and no significant rating bias. No reason is apparent why the LT model should not exhibit the same leverage bias as the other structural models, although it is interesting to note that it is the only endogenous-barrier model tested. The other key latent variable is asset volatility. Here, a positive bias is generally reported; when asset volatility is estimated to be

[18] Refer Eom et al. (2004, Appendix B.2) for details.

high from the use of 150 day historical window on stock prices, the credit spread is over estimated and vice versa. The exception is again the LT model.

Structural models are sensitive to the choice of default boundary level and asset volatility, both of which are estimated from observable proxy data. The spread underprediction problem with investment grade debt was found to be typically less severe when the book value of leverage was close to the market value of leverage. Conversely, the underprediction problem was worst for firms with high market-to-book ratios (except for the LT model). EHH suggest debt markets are less sanguine about the future prospects of high market-to-book firms; the debt markets imply a higher default boundary or a lower leverage target to accommodate the risk of reversion. Since the default boundary is unobservable, there is no empirical reason to consider the book value of liabilities to be an accurate proxy for the default boundary. Recently, Davydenko (2005) confirmed in a study of defaulted companies, that the default boundary is typically less than the face value of the firm's total debt.

HH assess the ability of a variety of structural models to explain the observed credit spread levels. The models examined included LS (with and without stochastic interest rates), LT, CDG, Anderson & Sundaresan (1996), Anderson et al. (1996), Mella-Barral & Perraudin (1997), and a jump-diffusion extension of LS. Using a mixture of implied and observed variables, a novel calibration approach is performed. Rather than model individual firms and their quoted credit spreads, a representative generic firm was created for different levels of default risk based on issuer ratings. The average 10 year cumulative default rates for each rating class was obtained from Moody's (data compiled for the period 1973-1998), and a representative bond was assumed to be ranked senior unsecured with a constant recovery rate of 51.31 percent of face value. Typical leverage ratios and associated market equity premiums were matched to the rating agency levels. For the exogenous-boundary LS, CDG, and jump-diffusion models, the default boundary was assumed to occur at 60 percent of the value of liabilities. Corresponding mean credit spreads on 10 year bonds were then obtained from Lehman Bond indexes and other studies. To fit each model, model specific variables such as the asset risk premium, asset volatility, and firm asset value are solved in order to match the known target variables of leverage ratio, equity premium, and cumulative real default probability. The resultant predicted credit spread for each generic firm was then compared with the long-run average observed credit spread.

To compare the results of HH with EHH we calculate an equivalent credit spread predicted error for HH by comparing the model predicted spread with the market average level as reported by HH. Table 2.2 presents the difference between the extant studies. EHH find a wide disparity in percentage spread prediction errors across models. The EHH find that the CDG model excessively overpredicts compared to the other models. In contrast, HH find a relatively smaller overprediction. EHH report that the LT and

Table 2.2: Shown is the mean percentage credit spread prediction error reported by Eom et al. (2004) (EHH) and Huang & Huang (2003) (HH). Results for HH are the author's calculations and are computed by deducting the reported model predicted credit spreads from the market average divided by the market average. HH results are sourced from HH's Tables 3, 4, and 6 for a single A rated bond. EHH mean percentage error is sourced from EHH's Table 3. EHH only report an overall sample average error, but from their Table 1, the sample average firm rating is comparable at approximately A-. All numbers are in percentage.

	Study	
Model	EHH	HH
LT	115.7	-68.7
LS	42.9	-88.2
CDG	269.8	71.1

LS models also overpredict, yet HH find underprediction of market credit spreads. In contradiction to EHH, HH do not report wide variance of credit spreads predictions as a problem,

> We show a large class of structural models - both existing and new ones - that incorporate many realistic economic considerations can indeed generate very consistent credit yield spreads if each of the models is calibrated to match historical loss experience data. (Huang & Huang 2003, p.3)

Therefore, it appears that by calibrating each model to match observed default rate levels, some of the firm-specific discrepancies that may have been measurement errors under EHH's proxy method, appear to have been removed. If this result arises when models are calibrated to loss rates, then calibrating to observed credit spreads could potentially result in a similar finding.

However, the HH method is not without its own shortcomings. Firstly, calibrating to real default rates necessitates a change in measure of the drift rate to include the firm's expected rate of return under the real measure as was discussed in Section 2.3.1. The task of estimating the unobserved risk premium on firm assets is not a trivial exercise and potentially introduces further estimation errors. Secondly, real transaction price data is not used; the credit spreads are for hypothetical bonds only issued by a generic representative firm. Thirdly, the representative firm and its associated default rate require averaging of characteristics across firms and across time (for example default rates are measured over a 25 year period and credit spreads are pooled by rating grade over time). Finally, we note that this is not a test of a structural model's prediction of the dynamic term structure of credit risk. The full extent of a structural model's predictions remain untested.

Anderson & Sundaresan (2000) explore the relative ability of alternative endogenous boundary models to explain the time-series variation in 30-year maturity, on-the-run bond yields for industrial corporations as reported in the Salomon Brothers Book of Analytical Yields. Models considered include Merton, Leland, and Anderson & Sundaresan (1996) at an economy-wide level for a generic firm with characteristics constructed from aggregate data; annual aggregate balance sheets of non-financial corporations reported in the US Federal Reserve's Flow of Funds Accounts, and monthly aggregate index data from 1970 to 1996 for S&P ratings of AAA, A, and BBB. Financing by way of a single consol bond is assumed so that analytic solutions are obtained for the various models considered. In their regressions, an intercept term is used to capture a premium for the illiquidity of corporate bond markets relative to Treasuries caused by a tax differential between corporate and Treasury bond income, and any other systematic bias that may be prevalent in the credit model. Their study is the first to control for non-credit related factors in the fitting of structural models and the estimation of standard errors around implied parameter estimates.

Anderson & Sundaresan (2000) find that a consol version of the Merton model contributes very little incremental spread prediction over and above the intercept term, however the models do explain intertemporal variations in credit spreads related to leverage changes. The implied asset volatility was close to the S&P 500 and considered therefore unrealistically high. Their study is useful in highlighting that econometric estimation of structural model parameters can give superior information about relative model performance compared to the cross-sectional methods employed by earlier studies. However, the study does acknowledge limitations. The measurement of firm characteristics and yield spreads is extremely coarse and limited only to investment grade default risk, maturity is limited to 30 year bonds to match against consol bond predictions, no term structure restrictions are enforced on the models (given the infinite maturity assumption) therefore dynamic term structure predictions of finite maturity structural models remain untested.

2.6.2 Non-Default Components of the Credit Spread

There is evidence that the credit spread is unlikely to contain compensation for default risk alone; liquidity, tax, and other non-credit related factors have been found to be embedded in credit spreads (Elton et al. 2001, Delianedis & Geske 2001, Longstaff 2002).

As shown by Anderson & Sundaresan (2000), controlling for any non-credit related components of the credit spread is important for unbiased estimation of structural models. Similarly, Duffee (1999) also estimates an intercept term when estimating implied parameters for a reduced-form model. The difference between the two studies is that Duffee (1999) assumes the presence of a constant non-zero instantaneous default risk, whereas, Anderson & Sundaresan (2000) assume a constant non-zero credit spread premium without implying that the effect is the result of default risk. The latter specification is more consistent with the presence of liquidity and tax premiums and is adopted in our empirical testing discussed further in Chapter 3.

How much of the credit spread is related to default risk is an unresolved question and estimates vary widely between studies. HH calibrate structural credit models to the long-term average default rate for a representative firm, and suggest that structural models account for only a small fraction of the observed corporate-Treasury yield. They conclude that the LS model explains only 16 percent of observed AAA credit spreads, increasing to 29 percent for BBB, and 83 percent for B rated firms. The LT model achieves a better performance explaining 59 percent for AAA, 31 percent for Baa, and 87 percent for B rated firms. The CDG model exhibits explanatory behaviour similar to the LS model.

Independently, Delianedis & Geske (2001) suggest that only 5 percent of the AAA credit spread is related to default risk and 22 percent of the BBB credit spread, which is of a similar magnitude to the LS base-case model of HH for mid-rated firms. The remaining components are hypothesised to be related to taxes, liquidity, jumps in asset values, and market risk factors.

Elton et al. (2001) find that the level of credit spreads can be explained by three components: i) compensation for default risk, ii) compensation for state taxes charged on corporate bond income that is exempt on government bonds, iii) compensation for additional systematic risk over and above that incurred by government bonds. They conclude that on average, taxes and default risk account for at most 20 percent of the credit spread.

How much of the credit spread is related to default risk requires an estimate of the credit spread due to default risk. Elton et al. (2001) use historical real probabilities of default obtained from rating agency data. HH calibrate various structural models to similar rating agency default data, then obtain the predicted credit spreads from the calibrated models. Delianedis & Geske (2001) use the predicted spreads from a Merton model with parameters fitted from observable equity market and accounting proxy data. In all these studies, reliance is made on the predicted spreads from structural models fitted by proxy variables.

2.6.3 Contribution to the Literature

We extend EHH with improved estimation methods and data, thereby building upon the extant literature of JMR, LYS, and EHH. We are also able to answer how much of the credit spread is explained by the structural models, therefore providing a comparison with HH using actual trade data and models calibrated with minimum average prediction error.

As an alternative to the traditional 'observed-proxy' methods employed in the extant literature, we adapt an estimation technique from the interest rate term structure modelling field. Using an extended Kalman filter (EKF) quasi-maximum likelihood method, model parameters are estimated with asymptotic standard errors. A strong advantage of this technique is that full use is made of cross-sectional and time-series predictions of the models, thereby using more information and constraining the parameter solutions to be truly consistent with the model's stochastic asset value assumptions. Latent estimation should provide the best opportunity for these models to perform in an unbiased manner, and the resulting credit spread prediction errors, a more accurate guide to future research direction. To the best of our knowledge this is the first test between competing corporate structural credit models where the credit model is fitted as a latent process in a state-space framework.

The use of Kalman filtering to parameterise term structure models is common in term structure model fitting (Chen & Scott 1995, Geyer & Pichler 1999, Duan & Simonato 1999). In an approach related to that used in this study, Duffee (1999) applies EKF to estimate a reduced-form hazard rate model of the term credit spread term structure. Our empirical method follows closely, but is applied to structural credit models. We also use actual trade data, instead of bid-quotes, and therefore have an additional complication of dealing with unequal time steps between trades. Other researchers have applied EKF on panel data in different contexts. Cumby & Evans (1995), Claessens & Pennacchi (1996), Keswani (2005), and Duffie et al. (2000) apply EKF to estimate models of sovereign bond prices. Despite a long-history of application to sovereign debt modelling, state-space methods have seen little application in corporate debt modelling. The notable exception in corporate debt modelling is the fitting of a single reduced-form model by Duffee (1999). A similar, but unrelated method of maximum likelihood estimation is presented by Ericsson & Reneby (2002). They derive a likelihood function for the firm's equity, which is then fitted to a time series of stock price data from which teh firm asset parameters are estimated. The credit spread is then predicted from the resulting firm asset parameters. Their method only applies a time-series restriction on the likelihood function whereas we estimate model parameters directly from the history of credit spread term structures. The EKF method also has the advantage of handling missing data and measurement error that an exact likelihood method cannot.

2.7 Selection of Models for Testing

To extend and contrast EHH, we parameterise several alternative models treating the log-solvency ratio as truly unobserved. The models considered are an extended Merton model (EM), a single factor LS model (LS 1) and two-factor LS model (LS 2), the LT and CDG models, and lastly a single-factor CEV model.

The models are chosen to be comparable with earlier empirical studies, where tractable solutions are available, that represent the range of structural modelling literature. The EM closely mirrors a similar ad hoc implementation of the Merton model by EHH.

The LS1 model is a simple single-factor implementation of the LS model with a constant interest rate. It is comparable to the LS base case model in HH. A two-factor LS model allows comparison against the LS1 model to gauge the incremental benefit of the stochastic interest rate process to the models. The LT model is a parsimonious endogenous boundary model previously tested by EHH and HH. The CDG model is the most widely analysed model with debt targeting and debt market timing behaviours drawn from the capital structure literature. Finally the CEV model offers a closed-form solution with a time-varying local asset volatility.

Chapter 3

Method

In this section we describe our method for estimating structural models. Unlike the proxy variable methods of JMR and EHH, and loss calibration method of HH, we show how the firm's asset process can be implied from the observed time series of firm-specific credit spreads. The models are therefore fitted to observed term structures of credit spreads with minimum prediction error. We treat the firm's asset process as truly unobserved and estimate its stochastic properties, and its path through time, implicitly as a latent process. By doing so, we allow each model to be estimated as well as it can within the limitations of its theoretical construction. No errors are introduced through the use of inappropriate observable proxy variables. We can therefore compare the relative performance of models, on a level playing field, where each model is permitted to perform with a minimum of bias. Any errors remaining are likely to represent theoretical specification errors only, which are then subject to further tests for model robustness. Our method provides a sharper instrument to test the performance of structural models than has hitherto been used.

Our model estimation method follows closely (Duffee 1999), which we adapt for estimation of structural, as opposed to reduced-form, credit models. However, unlike his study, we use actual trade data instead of month-end broker quotes, and we control for the effect of non-default related premiums directly as a component of the credit spread. We refer to these components of the credit spread loosely as the 'liquidity' premium. Three methods of controlling for the liquidity premium are conducted: no premium for comparison with prior studies; a constant premium component of the bond spread; a constant premium component and a time-varying premium that is a linear factor of the Refcorp 10 year maturity bond spread. A selection of firms is made choosing firms that have bonds that actively trade across a broad range of maturities.

The credit models are fitted with respect to minimising the error in predicted versus observed credit spreads, as opposed to predicting bond values. The latter is likely to contain error related to estimation of the risk-free yield curve, and our main focus is on isolating the errors related to credit risk valuation, we prefer to optimise model fit to observed credit spreads. Observed credit spreads are calculated by deducting from corporate bond yields, the equivalent maturity Treasury bond yields, with the same maturity on the date of the trade. Since the credit data may contain some recording errors, we explain a method of cleansing extreme outliers. In Section 3.1, the source, sampling, and preparation of the corporate bond data is described.

Section 3.2 describes the method of fitting the credit models to the bond data. In particular, we extend (Duffee 1999) to deal with unequally spaced observations. The credit models are estimated by first converting them into state-space form comprising two equations: a dynamic measurement equation inclusive of a measurement error, and a latent log-solvency ratio that is related to the observed measurement error. So while we cannot observe the firm's log-solvency process directly, we can infer with confidence its path and parameters from the observed path taken by the firm's term structure of credit spreads, and with knowledge of the theoretical model that determines the form of the log-solvency process. By use of an EKF, quasi maximum likelihood estimates of model parameters are obtained from the time series of observed credit spread term structures. A similar method is shown in Section 3.3 to estimate the risk-free rates for use in the two-factor credit models.

3.1 Data

In Section 3.1.1, the source of traded bond data is discussed. The selection criteria of firms and bond issues is described in Section 3.1.2, the method of converting bond prices to credit spreads is explained in Section 3.1.3, and the treatment of outlier observations is discussed in Section 3.1.4. The resultant cleaned sample data set of credit spreads is described in Section 3.1.5. The data and method used for calculating the Refcorp bond yield spreads is described in Section 3.1.6.

3.1.1 Credit data sources

Secondary market price data, on North American corporate bonds, is sourced from the Fixed Income Securities Database for Academia (FISD). The FISD data contains trade prices reported by the National Association of Insurance Commissioners (NAIC), supplemented with additional issue and firm characteristics supplied by LJS Global Services, Inc. Warga (2000) provides a further description of the NAIC data.[1] The sample period is from 1 January 1994 to 31 December 2000.

Prior to the availability of the NAIC data, the extant empirical literature for reduced-form, and structural credit modelling, has predominately used month-end bid quotes

[1] A product of the Fixed Income Research Program at the University of Houston, College of Business Administration.

originated by Lehman Brothers traders, quoted for the purpose of regular construction of the Lehman Brother's bond indexes, as reported in the Fixed Income Database (FID), and described further by Hong & Warga (2000). Much of our empirical knowledge comes from this single data source, for example, Duffee (1999), Eom et al. (2004), Mueller (2000), Bakshi et al. (2001), Ericsson & Reneby (2002), Elton et al. (2001), and Collin-Dufresne, Goldstein & Martin (2001).

Insurance companies are required by the National Association of Insurance Companies to provide a record of all bond transactions via quarterly submission of Schedule D reports. Included in the report is the date of the transaction, the par amount traded and the total market value of the transaction. Prices are recorded for the day of the trade. We can be confident that the data is representative of a broader bond-trading market. Hong & Warga (2000) report that insurance companies account for roughly 25 percent of the market for non-investment grade debt, while their share of trading in the investment-grade debt market is around 40 percent.

For estimating continuous time models we ideally want to use data that is close to be continuously observed, in other words, the data has a small interval between observations. In particular, the EKF used by Duffee (1999) and by this study, linearises the non-linear relationship between the model pricing function (the measurement equation) and the firm's log-solvency transition (the transition equation). The smaller the observation interval, the better the approximation. In contrast to Duffee (1999), we use NAIC sample data, which comprises actual corporate bond prices trade prices recorded for the date of trade. We therefore have, on average, a shorter observation period more suited for calibrating continuous-time credit models.

3.1.2 Sample Selection

Unlike the extant empirical literature of EHH, Lyden & Saraniti (2000), and JMR, we do not have the objective of selecting only firms with simple capital structures and few bonds on issue. Instead we select firms with a broad range of maturities traded so that we obtain the most information possible across bonds and across the term structure. That tends to direct our sampling toward more frequently traded firms.

The NAIC database comprises all fixed interest trades by North American insurance companies including Treasury and corporate debt trades over the period commencing 1 January 1994 to 31 December 2000. Before combining bought and sold trades, the data comprises over 734,000 trades. For our purposes we require panel data from a smaller subset of corporate firms that exhibit relatively frequent trading, and have several outstanding bonds with remaining maturities that span a term structure, with a history of trading over an extended period of time. Unlike early empirical studies we do not restrict ourselves to firms with debt structures that approximate the zero-coupon Merton ideal (see for example JMR and EHH). This enables us to consider the broad population of corporate issuers, tempered only by the limitations of data availability and accuracy.

We exclude government entities (FISD industry code 04), banks (FISD industry code 20), and savings and loan institutions (FISD industry code 25). Deposit-taking institutions have a special role in the economy and bond pricing may be influenced by the implicit government guarantee arising from the moral hazard associated with their failure. Since financial firms are a major source of bond issues, we have chosen to include them in the sample.

Where issuing firms are non-listed subsidiaries of a listed parent, the parent's market capital and balance sheet data is used. Where there is more than one subsidiary within a corporate group with a unique CUSIP (for example, Ford Motor Company and Ford Motor Credit Company), only one subsidiary is included in the sample.

The second level of filtering removes issues that may be subject to embedded option features, or credit enhancements, not included in the theoretical models under consideration. From issue-level data sourced from FISD we:

1. exclude bonds that are convertible, or redeemable (via call, IPO clawback, maintenance and replacement call or sinking fund), subject to puts, or are credit enhanced, for example by financial guarantees; and,

2. include only fixed-interest coupon bonds, and corporate debentures with semi-annual compounding with 30/360 day convention, where there are no planned future variation in coupons.

Further issue-level filters are then applied to minimise potential data errors. To understand what data errors may be present in the NAIC data we reviewed prior studies that have utilised schedule D submissions from insurance companies. Hong & Warga (2000) match the recorded bond prices between New York Stock Exchange's Automated Bond System (ABS), and Schedule D sourced NAIC price data supplied by Capital Access Inc. (CAI), and compare with the closest-in-time bid quotes from Lehman Brothers as reported in the FID. They find that the transaction-based prices from the ABS and NAIC sources are broadly in agreement with each other and with the month-end dealer quotes given by Lehman Brothers dealers. A source of bias was identified in the recording practices of NAIC, in which total transaction costs were rounded upward to the nearest $1,000. Hong & Warga (2000) minimised the bias by restricting their sample to trades with costs of $500,000 or more.[2] Bedendo, Cathcart & El-Jahel (1994) exclude bonds with transaction prices below $80 and above $135 as well as bonds with negative credit spreads.

[2]For example, a sale with the value of $1,500,900 would be reported as $1,501,000. To limit the potential upward bias in reported prices we adopt the same filter rule and exclude trades of less than $500,000 in total cost which limits the maximum percentage error in reported price to be no more than 0.20 percent of the total transaction cost.

Chakravarty & Sarkar (1999) use CAI sourced NAIC data to compare spreads between government, corporate and municipal debt. In order to minimize incidences of data entry error, they remove all observations where the transaction price is outside the range \$500,000 to \$1,500,000. Some entries were observed on non-trading days and were removed, and trades occurring on June 30, 1995, June 30, 1996, and December 31, 1997 are removed. Anecdotal evidence suggests insurance companies may have used these dates for recording transactions that they had failed to report in a timely manner.

Consideration is made of the prior findings of errors in the NAIC data. We therefore exclude observations if:

1. there are missing or invalid trade dates;
2. the credit spread is negative (due to measurement error or estimation error for the matched Treasury rate);
3. the remaining maturity of the bond, at the time of the trade, is less than 12 months. We exclude very short-dated bonds due to their sensitivity to small measurement errors as suggested by Cooper & Davydenko (2004);
4. the remaining maturity is greater than 30 years because this is the maximum constant maturity Treasury (CMT) risk-free maturity available;
5. total cost of the trade is less than \$500,000 in total cost as per Hong & Warga (2000); and,
6. trades occur on 30 June 1995, 30 June 1996 and 31 December 1997 as in Chakravarty & Sarkar (1999).

Where more than one trade occurs on a single day, we use a single representative observation calculated as the weighted average price, where the relative total transaction costs form the weight. This removes duplicate records caused when insurance companies are on both sides of the same transaction.

After these adjustments the data set comprises 1,373 issuers, with 8,799 issues and 96,472 trades. We then apply issuer-level filters to select firms with patterns of trading suitable for robust panel estimation. We select firms with:

1. at least 3 bonds, where each averages at least 6 trades per annum, for a minimum period of 12 months;
2. a broad range of remaining terms to maturity; and,
3. a complete balance sheet history on COMPUSTAT and stock prices from CRSP over the sample period.This information is used for comparison with the firm's observable solvency ratio, and for deriving initial parameter estimates for the EKF.

To reduce the dimension of the model estimation, no more than ten bonds are included per issuer. Where more than ten bonds have met the filter rules, the most frequently traded are chosen. Finally, the beginning, and end dates, of the issuer's sample period are chosen as the earliest, and latest dates respectively, that a cross-section of maturities is evident. Consequently, the beginning and end dates of each issuer's sample vary slightly. Our selection criteria is intended to result in a sample of observations that are closely spaced in time and well represented across the dynamic term structure of the issuing firm. The overall density of the observations provides cross-sectional and time-series information best suited to fitting continuous-time structural credit models. Figure 3.1 shows an example of the trading density for Northrop Grumman Corporation (Northrop). Each point represents an observed trade from one of the four bonds in the sample. Each bond has a different maturity causing the distribution of trades into a pattern of distinct parallel lines. Two features in Figure 3.1 are noteworthy. Firstly, our selection criteria results in trades with a broad distribution across the term structure represented by the vertical spacing of observations. Secondly, the horizontal axis shows the passage of time and an overall shortening of the remaining maturities. The final sample comprises 32 issuers, 200 bonds, and 8,953 trades. Table 3.1 shows the final sample of firms and numbers of bonds and observations per issuer. Table C.1 details each bond's characteristics and descriptive statistics of the bond's credit spreads. The relatively small sample size is due mainly to merger and acquisition activity causing incomplete COMPUSTAT and CRSP histories, which are not strictly required for successful model fitting, but are necessary to compare implied solvency ratios with observed market leverage ratios.

3.1.3 Calculating the Observed Credit Spreads

Similar to EHH, the observed credit spread is calculated as the difference between the yield to maturity of the corporate bond and the par yield of an on-the-run Treasury bond of the same remaining maturity. The corporate yield to maturity is calculated as the rate that equates the present value of the contractual cash flows (obtained from the FISD database) with the trade price assuming by convention a semi-annual compounding and a 30/360 day count convention. The Treasury yields are constant maturity Treasury yield rates (CMT) sourced from the Federal Reserve H15 report and interpolated to the remaining maturity matched to the corporate bond. We use the Nelson & Siegel (1985) model for inteprolation and construct 1,753 risk-free term structures corresponding to the number of unique trading days in the sample.

Adopting the notation of EHH, the risk-free constant maturity yield at time t for a

Table 3.1: Shown are the final sample of issuers, number of bonds per issuer and number of observations per issuer. CUSIP identifies the issuer, and SIC is the Standard Industry Code (SIC) of the issuer, as reported in the Fixed Income Securities Database. The Ticker is the stock exchange unique identifier of the issuer, if listed, and of its parent if unlisted. Where a financial services issuer is a wholly owned subsidiary of a non-financial firm, the industry sector refers to the predominate activity of the corporate group.

Issuer	CUSIP	Ticker	SIC	Sector	No. Issues	No. Trades
Aetna Inc.	8117	AET	6324	Finance	3	88
Associates Corp.	46003	C	6141	Finance	10	316
Atlantic Richfield Co.	48825	ARC	2911	Industrial	3	122
A T & T Corp.	1957	T	4813	Utility	3	160
Bear Stearns Companies Inc.	73902	BSC	6211	Finance	10	349
Black & Decker Corp.	91797	BDK	3540	Industrial	3	180
Boeing Co.	97023	BA	3721	Industrial	3	151
Dayton Hudson Corp.	239753	TGT	5331	Industrial	10	328
Commonwealth Edison Co.	202795	UCM	4911	Utility	8	306
Enron Corp.	293561	ENE	5172	Industrial	6	186
Federated Department Stores	31410H	FD	5311	Industrial	4	182
Ford Motor Co.	345370	F	3711	Industrial	9	365
General Motors	370442	GM	3711	Industrial	9	501
Georgia Pacific Corp.	373298	GP	2600	Industrial	3	120
HCA Healthcare Corp.	19767Q	HCA	8062	Industrial	4	135
IBM Corp.	459200	IBM	7370	Industrial	3	285
International Paper Co.	460146	IP	2600	Industrial	5	209
Lehman Brothers Holdings Inc.	524908	LEH	6211	Finance	10	405
Merrill Lynch & Co.	590188	MER	6211	Finance	10	513
Motorola Inc.	620076	MOT	3663	Industrial	4	125
Nabisco Group Holdings Corp.	629527	NGH	2052	Industrial	6	381
Niagara Mohawk Power Corp.	653522	NMK	4931	Utility	4	206
Northrop Grumman Corp.	666807	NOC	3812	Industrial	4	214
Paine Webber Group Inc.	695629	PWJ	6211	Finance	9	326
Penney J C Co. Inc.	708160	JCP	5311	Industrial	6	263
Philip Morris Companies Inc.	718154	MO	2111	Industrial	10	529
Seagram Co. Ltd.	811850	VO	3652	Industrial	4	188
Sears Roebuck Acceptance Corp.	812404	S	5311	Industrial	9	397
Service Corp. International	817565	SRV	7200	Industrial	8	250
Union Pacific Corp.	907818	UNP	4011	Utility	8	321
Viacom Inc.	925524	VIA.B	4841	Industrial	3	259
Wal-Mart Stores Inc.	931142	WMT	5331	Industrial	9	593
Total					200	8,953

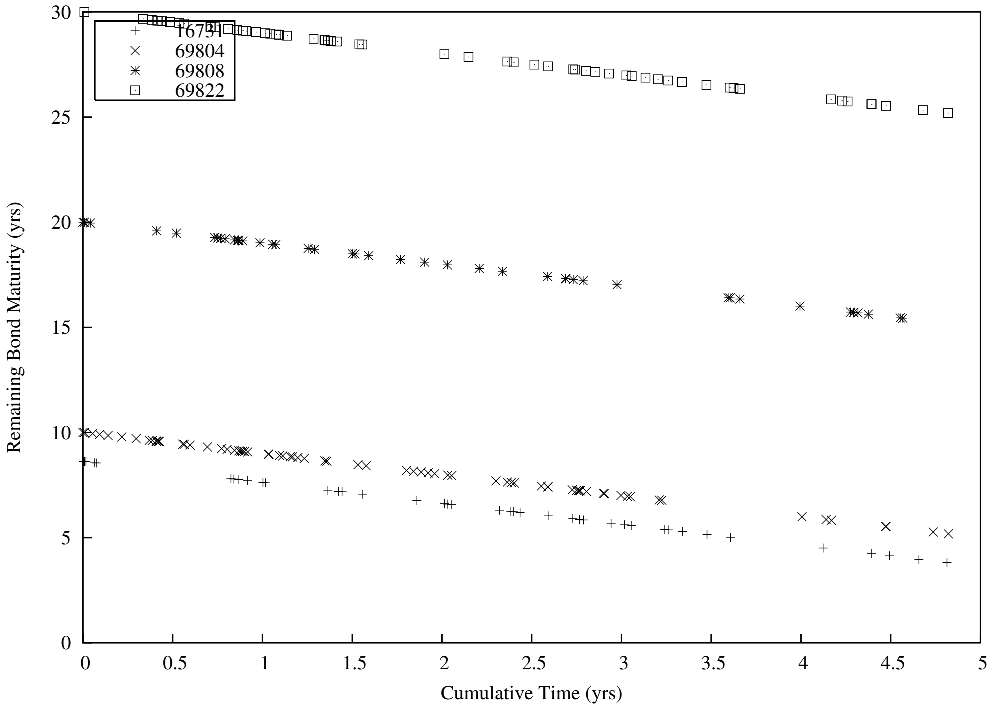


Figure 3.1: Presented is a plot the structure of the Northrop Grumman Corporation's sample data. Each observation is an observed trade plotted by remaining maturity on the vertical axis, and the passage of time along the horizontal axis. Evident is the irregular spacing of observations coinciding with actual trades, and the.banding of observations into four parallel lines corresponding to the bonds on issue. The bonds are identified by their Fixed Income Securities Database identifier. Further details are shown in Table C.1. Sample average remaining maturities are 6.4 yrs (16731), 8.2 years (69804), 18.0 years (69808), and 28.0 years (69822). The downward slope to the right is a result of the remaining maturity decreasing with time. The firm sample period is 27 February 1996 to 22 December 2000.

Table 3.2: This table shows a summary of the distribution of bond characteristics per issuer. Further details by issuer and issue are shown in Table C.1. Number of issues is the number of bonds sampled per issuer. Sample period refers to the cumulative trading periods per issuer. Number of trades is the number of price observations per issuer. The mean interval is the average time between observed trades. Mean maturity refers to the average remaining maturity at each trade per issuer.

	Across 32 Issuers		
Issuer-level statistic	Min.	Median	Max.
Number of issues	3	6	10
Sample period (yrs)	3.30	6.22	6.97
No. of trades	88	261	593
No. of trading days	84	233	486
Mean interval (yrs)	0.0141	0.0262	0.0550
Mean maturity (yrs)	5.54	8.12	21.43
Min. maturity (yrs)	1.00	1.12	12.00
Max. maturity (yrs)	10.01	19.71	29.99
Mean coupon (%)	6.41	7.21	9.92

Treasury bond maturing at time T is give by the Nelson & Siegel (1985) model as

$$y(t,T;\Theta_r(t)) = \beta_0 + \delta_1(\beta_1+\beta_2)\frac{(1-e^{-(T-t)/\delta_1})}{T-t} - \beta_2 e^{-(T-t)/\delta_1}, \qquad (3.1)$$

where, the parameter set is $\Theta_r(t) = (\beta_0, \beta_1, \beta_2, \delta_1)$, and $\beta_0 > 0$ and $\delta_1 > 0$. For every date of bond trading in our sample, new parameters are estimated by minimising the sum of squared errors, between the model predictions and observed yield spreads, simultaneously across ten equally weighted maturities (3 and 6 months, 1, 2, 3, 5, 7, 10, 20, and 30 years). The sample average parameter value set is $\bar{\Theta}_r$ =(0.0630, -0.2612, 0.2528, 3.8128). The average RMSE is 8.5 basis points, which is comparable to the level of error reported by Nelson & Siegel (1985).

3.1.4 Treatment of Outliers

A disadvantage of using actual traded bond prices is the potential for additional data handling errors. This has not been a consideration in the extant empirical literature of structural models due to the predominate use of month-end bid quotes. The greater frequency of data recording, and large volumes of data handling, leaves the NAIC data more at risk of containing recording errors. The filter rules described above help to remove known problems, but idiosyncratic errors may still influence individual trade prices, and therefore, observed credit spreads.

A review of the literature found little consistency in dealing with idiosyncratic outliers in the NAIC bond data, but did confirm that data errors may be present. For example, Bedendo et al. (1994), when fitting spline curves to interpolate across the term structure of credit spreads, found evidence of extreme observations that are, ‘suggestive of obvious pricing errors’. They judgmentally remove outliers without applying an objective cleaning rule. Campbell & Taksler (2002) addressed the problem in the NAIC data by excluding the top and bottom one percent of credit spreads. Motivated by these studies we developed an objective cleaning method that resulted in minimal modification of the data.

We identify potential data errors by recognising that the deviation in credit spreads expected across bonds of different maturities is likely to be much lower that the differences arising from an incorrect data entry. We start by constructing issuer specific, time-series of credit spreads, pooling all maturities together. Where more than one maturity is traded on the same day, we use a simple average as a single observation for the trade date. The first differences in the resulting time-series are then standardised, and absolute deviations greater than four, replaced by the mean of the adjacent observations for the bond with the outlier. Consequently, outliers are cleaned only when large, in first difference terms, and directionally inconsistent, relative to adjacent trades across all bonds. Our method of outlier data replacement improves on Bedendo et al. (1994) and

Campbell & Taksler (2002) since we use all the available price information on the firm's other bond prices. In total, only 70 out of 8,953 data points were cleaned representing only 0.78 percent of the final sample.

An example of the filtered and cleaned data set for Northrop is shown in Figure 3.2. Evident is an increase in spreads through the sample period, consistent with the market trend as evidenced by a similar increase in the generic Baa Moody's corporate spread shown in Figure 3.3.

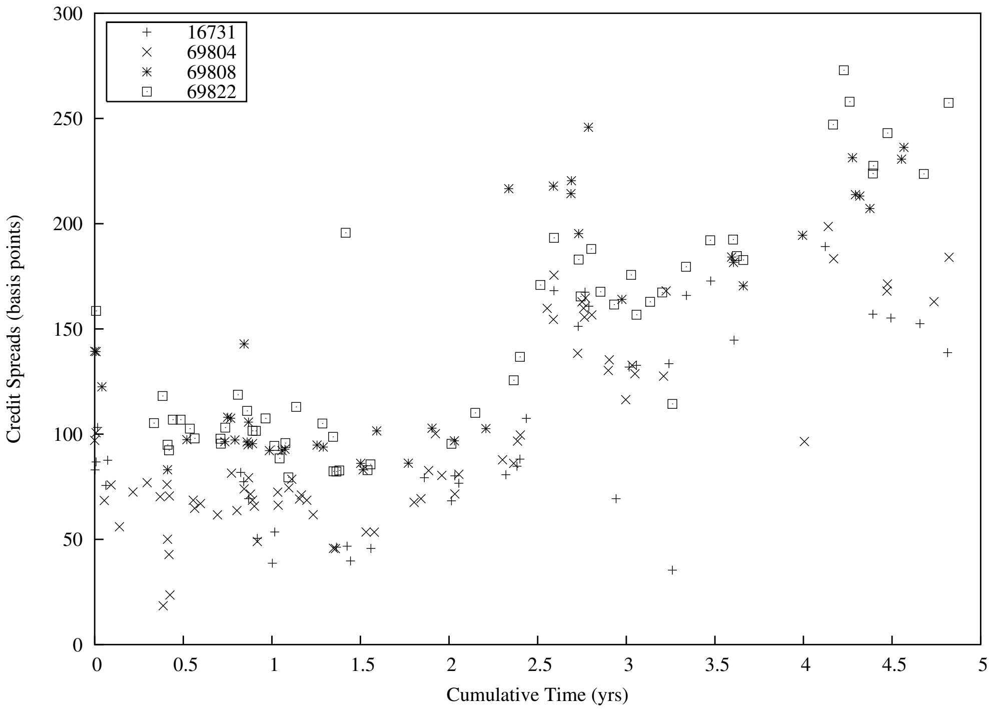


Figure 3.2: Presented is a plot of the Northrop Grumman Corporation's bond credit spreads, by time from the beginning of the firm's sample. The bonds are identified by their Fixed Income Securities Database identifier. Further details are shown in Table C.1. Average remaining maturities are 6.4 years (16731), 8.2 years (69804), 18.0 years (69808), and 28.0 years (69822). The firm sample period is 27 February 1996 to 22 December 2000.

3.1.5 Sample Description

As shown in Table 3.2, the median firm has 6 issues observed at various times over a period of 6.22 years, with an average time step between trading days of 0.0262 years, or approximately 10 calendar days. For the median firm, the term structure spans from a minimum remaining term to maturity of 1.12 years to a maximum of 19.71 years. Note that because more than one of the issuer's bonds may trade on the same day, the median firm's number of trades of 261, exceeds the median firm's number of trading days of 233.

Table 3.3: This table shows the frequency distribution of credit spread observations within the sample. Industry refers to the industrial sector of the issuing firm, Issuers is the count of issuing firms, and Trades are the counts of bond trades.

	Issuers		Issues		Trades	
Industry	No.	(%)	No.	(%)	No.	(%)
Industrial	22	68.8	125	62.5	5,963	66.6
Finance	6	18.8	52	26.0	1,997	22.3
Utility	4	12.5	23	11.5	993	11.1
Totals	32	100.0	200	100.0	8,953	100.0

Duffee (1999, Table 1) applies a similar estimation method on 161 issuers reported in the FID database. He uses month-end quotes and monthly time partitions compared to our use of actual trades with a 10 day average interval between trades. His median firm has 92 month-ends over 7.6 years, which equates to 227 bid quotes compared to our 261 actual trades. Thus, we have slightly more data per issuer, with information on the firm's default risk observed approximately three times as frequently.

Table 3.3 shows that industrial firms comprise 62.5 percent of our sample by number of issues, followed by finance at 26.0 percent, and utilities at 11.5 percent. EHH exclude financial firms and their sample includes only 5.5 percent in transport and utilities. Table 3.4 shows the sample credit spreads by rating and remaining maturity at the time of their trade. As expected the median credit spread is higher for poorer ratings and higher for longer dated maturities. Davydenko & Strebulaev (2004, Table II) report similar results for credit spread levels and sample standard deviations across a larger sample (43,402 trades in total compared to our 8,953) confirming that our sample is representative of the larger NAIC universe. Our median credit spread is 81 basis points, increasing from 50 basis points for AA trades, 78 basis points for A, 97 basis points for BBB, and up to 157 basis points for BB trades.[3] Table 3.4 shows that spreads are on average, monotonic with remaining maturity; spreads for AA and BBB are lower for the medium tenors than the short maturities (4 basis points for AA and 13 for BBB medians) and only one basis point higher for A rated issuers. The expected monotonic increase is present in long maturities where we find all spreads are greater than shorter maturities.

Table 3.5 shows the credit spread descriptive statistics, after controlling for gearing, matching solvency and credit spreads on the date of trade. It is convenient to express solvency in a manner that is consistent with structural credit modelling. We therefore use an observable proxy that hereafter is referred to as the observed log-solvency ratio (S), defined as the natural log of the inverse of the observed leverage ratio, $S = \ln((D+E)/D)$, where D is the total book value of debt, and E is the market value of equity. Book val-

[3] The comparable median spreads from Davydenko & Strebulaev (2004, Table II) are 51, 71, 103 and 197 basis points respectively.

Table 3.4: This table reports the descriptive statistics of the sample credit spreads by issuer rating and remaining maturity, coincident with the trade date. All credit spreads are reported in basis points.

	All	AA	A	BBB	BB
Panel A: All					
Mean	102	62	91	115	312
Median	81	50	78	97	157
Std. Dev.	124	36	51	109	486
5% quantile	31	23	34	45	86
95% quantile	201	132	189	218	1,465
n	8,953	1,691	3,704	3,263	295
Panel B: Remaining Maturity $\leq$7 years					
Mean	102	61	89	117	378
Median	73	51	70	96	159
Std. Dev.	158	36	58	145	628
5% quantile	28	24	28	39	64
95% quantile	207	132	201	212	1,729
n	4,107	875	1,687	1,409	136
Panel C: Remaining Maturity 7$-$15 years					
Mean	93	60	84	100	270
Median	75	47	71	83	155
Std. Dev.	93	36	45	68	328
5% quantile	32	20	40	48	97
95% quantile	182	129	175	185	1,285
n	3,407	727	1,266	1,276	138
Panel D: Remaining Maturity $>$15 years					
Mean	120	77	108	142	166
Median	109	65	100	121	163
Std. Dev.	57	35	37	72	32
5% quantile	60	44	60	83	122
95% quantile	217	159	181	246	209
n	1,439	89	751	578	21

ues of debt are quarterly values reported by COMPUSTAT (items 45 and 51), and the market value of equity is obtained from CRSP for the day of the trade. Lower values of log-solvency measure greater levels of debt relative to firm value. As expected, Table 3.5 shows that, on average, higher credit spreads are associated with lower levels of solvency as expected. The median spread in the upper quartile of log-solvency is 65 basis points and 99 basis points in the lowest quartile. Also observable is a higher median spread associated with longer remaining maturity for all quartiles of log-solvency except the upper quartile (lowest default risk). In Figure 3.3 the time series of the median monthly sample credit spread and Moody's generic corporate spread indexes are shown. The median spread has a 90.4 percent correlation with the Moody's Aaa index levels and 41.4 percent in first differences with the index showing that our sample of firms exhibits similar time series behaviour relative to the wider universe of publicly traded debt. The level of our median credit spreads aligns most closely with the Moody's Aaa index although our most frequently observed sample corporate rating is single A. This is possibly due to our sample selection method that favors mature listed firms with frequently traded, and therefore, relatively more liquid bonds.

Across our sample and the Moody's indexes, there is a general upward shift in credit spreads during the sample period. Most noticeable is the sharp increase in credit spreads in August 1998 when the Russian default and LTCM bail-out triggered a rise in secondary market yields (hereafter we refer collectively as the LTCM crises). The effect of the LTCM crises on market-wide bond credit spreads is shown in Table 3.6. Before 1 August 1998, the sample average credit spread is 70 basis points. After 1 August 1998, the average increases to 159 basis points. Controlling for rating, the rise in spreads post the LTCM crises is just over double the pre-crises values for ratings AA, A, and BBB. The increase, is however much larger for the highest default grade BB, being an increase of around six and a half times.

3.1.6 Refcorp Yield Spread

Motivated by the observation that our sample contains a potential systemic shift in credit spreads due to the LTCM liquidity crises, we incorporate issue-specific and market-wide liquidity components in the predicted credit spread. In addition to a constant yield spread intercept term, we also wish to control for a time-varying liquidity premium. As a direct measure of market-wide liquidity we follow Longstaff (2002) and construct a 10 year constant maturity spread between bonds issued default free from the U.S. government agency Refcorp, and on-the-run Treasury bonds.

Refcorp was established in 1989 to raise funds for the Resolution Trust Corporation, in turn created to resolve insolvent savings and loans deposit-taking institutions in the late 1980s. Refcorp issued USD 30 billion of debt from 1989 to 1991 with various long-dated maturities, of which about 90 percent is traded in strip form (Reinhart &

Table 3.5: This table shows descriptive statistics of the sample credit spreads reported by remaining maturity, and quartiles of observed log-solvency of the issuing firm, as measured on the trade date. We define log-solvency as: $S = \ln((D+E)/D)$, where D is the total book value of debt, and E is the market value of equity. Book values of debt are quarterly values reported by COMPUSTAT (items 45 and 51), and the market value of equity is obtained from CRSP for the day of the trade. All credit spreads are reported in basis points.

		Quartiles of observed log-solvency (S)			
	All	100-75%	75-50%	50-25%	0-25%
		Panel A: All maturities			
Mean	102	74	92	106	135
Median	81	65	75	90	99
Std. Dev.	124	44	54	82	218
5% quantile	31	24	34	38	41
95% quantile	201	157	193	211	240
n	8,953	2,239	2,237	2,238	2,239
		Panel B: Maturity 1-7 years			
Mean	102	78	87	101	139
Median	73	67	64	76	89
Std. Dev.	158	49	57	102	270
5% quantile	28	24	67	32	37
95% quantile	207	173	189	218	243
n	4,107	1,121	883	945	1,158
		Panel C: Maturity 7-15 years			
Mean	93	63	80	100	131
Median	75	56	67	85	96
Std. Dev.	97	34	47	60	166
5% quantile	32	24	34	45	48
95% quantile	182	119	167	190	254
n	3,407	879	854	902	772
		Panel D: Maturity 15-30 years			
Mean	120	97	120	129	128
Median	109	88	104	111	125
Std. Dev.	57	41	49	65	65
5% quantile	60	45	63	77	77
95% quantile	217	166	218	240	195
n	1,439	239	500	391	309
Quartile ranges of S		> 1.440	1.440– 0.775	0.775– 0.367	< 0.367

Table 3.6: This table shows credit spreads by rating before and after 1 August 1998, approximately the date of the LTCM crises on bond credit spreads. Credit spreads and issuer ratings are matched on the date of trade. All credit spreads are reported in basis points.

	All	AA	A	BBB	BB
Panel A: Spreads pre 1 August 1998					
Mean	70	42	65	81	155
Median	63	41	62	77	139
Std. Dev.	37	14	26	33	59
5% quantile	28	21	30	42	83
95% quantile	133	66	106	135	271
n	5,797	1,107	2,361	2,087	242
Panel B: Spreads post 1 August 1998					
Mean	159	100	138	174	1,030
Median	166	96	127	148	1,249
Std. Dev.	189	34	51	160	823
5% quantile	69	54	74	91	144
95% quantile	257	166	228	287	2,870
n	3,156	584	1,343	1,176	53

Sack 2002). Refcorp bonds are taxed at the same rate as Treasury bonds and Refcorp is considered to be a default-free issuer; coupon payments are ultimately backed by the U.S. Treasury, and principal payments are backed by pledged non-marketable zero-coupon Treasury bonds. Despite their default-free status, the bonds are not as liquid as Treasury bonds making their spread relative to on-the-run Treasury bonds suitable for quantifying the change in general market preference for liquidity.

Our choice of Refcorp bonds as a measure of market-wide liquidity is motivated by Longstaff (2002) and Reinhart & Sack (2002) who report a significant increase in the Refcorp spread post the LTCM crises in August 1998, which we also note to be a feature of corporate credit spreads in Figure 3.3. Reinhart & Sack (2002) estimate the total liquidity premium in AAA 10 year yield spreads to be between 14 and 34 basis points, and in BBB 10 year yield spreads between 31 to 78 basis points and concluded that a heightened preference for liquidity during the hedge fund crises contributed at least as much as credit risk to the widening of corporate credit spreads. Longstaff (2002) estimates the Refcorp spread to range between 10 and 16 basis points, rising to 90 basis points at the end of 2000. He shows that changes in the Refcorp spread are related to consumer confidence, flows into money market and stock mutual funds, and foreign holdings of Treasury bonds, and consequently interprets the Refcorp spread as a flight-to-liquidity premium. Recently, Longstaff, Mithal & Neis (2005) directly estimate the non-default component of corporate credit spreads by comparing credit default swap premiums to bond spreads. They estimate the non-default component to be time-varying and strongly related to measures of individual bond liquidity, such as issue size and bid-ask spreads.

Table 3.7: This table reports descriptive statistics of the daily term structure strip yields.

Statistic	Minimum	Maximum	No. obs.
Mean	3.6	29.5	26.5
Min	1.0	29.0	23
Max	7.5	30.0	29
n	1,753	1,753	1,753

Changes in the non-default component are found to be related to similar market-wide liquidity factors that Longstaff (2002) also found as significant regressors on changes in Refcorp bond spreads. We therefore consider the observable Refcorp spread as a reasonable observable measure of the time variation in a market-wide liquidity premium. We enter the Refcorp spread into the predicted spread, scaled per issuer to accommodate different firm sensitivities to changes in market-wide liquidity preferences.

To measure the Refcorp spreads we first obtain bid yields from a sample of traded bond strips sourced from Bloomberg (PXRS screen). Table 3.7 shows a summary of the strips. The strip data is trimmed to include maturities between one and 30 years as at the quotation date; in total, we have 46,416 prices for 34 Refcorp strips for the period beginning 3 January 1994 to 29 December 2000. Table 3.7 shows the distribution of yields available each trading day. On average, there are 26.5 yields available per day. The average minimum maturity is 3.6 years and the maximum of the minimum maturities observed for all days is 7.5 years. The maximum sample maturity averages 29.5 years with a minimum of 29 years. Consequently, we have insufficient data to construct a complete daily yield curve but we are able to interpolate the Refcorp yield between maturities of 7.5 years and 29 years. We choose a 10 year constant maturity Refcorp spread as our instrumental variable due to similarity with the median remaining maturity of our sample data. Refcorp strip yields are quoted on a semi-annual compounding convention as per Treasury bonds but with zero coupon payments. In contrast, the CMT series is quoted as a par yield with coupons. We therefore convert the CMT rate to an equivalent Refcorp basis before calculating the Refcorp spread. The CMT series is initially boostrapped to spot rates and expressed as a semi-annual zero-coupon yield. The Refcorp premium is then calculated as the difference in the linearly interpolated 10 year Refcorp bid yield and the 10 year zero-coupon adjusted CMT yield.

In Figure 3.3 the monthly mean 10 year Refcorp spread is plotted against our sample of monthly median credit spreads measured over all firms and trades. The correlation between Refcorp liquidity spreads and corporate bond spreads is high (85.4 percent correlation in levels and 29.2 percent in first differences). From 1995 to the beginning of 1998, the spread on our sample of corporate bonds generally declined in a manner matching the fall in Refcorp spreads; by the third quarter of 1998 the LTCM crises had pushed spreads higher, from where they generally tended to increase for the remainder

Table 3.8: This table shows descriptive statistics of the Refcorp yield spread estimated daily for the period 3 January 1994 to 29 December 2000. The spread is the difference between the 10 year constant maturity zero-coupon Refcorp strip bid yields and the 10 year zero-coupon constant maturity on-the-run Treasury bond. First differences of the spread is denoted by Δ10yr. AR(1) refers to the coefficient on the autoregressive lag. All spreads are reported in basis points and standard errors are shown in parentheses.

Statistic	10 yr	Δ10 yr
Mean	24.28	0.01
Median	20.76	-0.01
Std Dev	12.37	4.25
Min	-1.68	-37.23
Max	71.06	45.64
AR(1)	0.94	-0.35
	(0.01)	(0.02)
n	1,753	1,752

of our sample. Interestingly, the LTCM crises did not impact the Refcorp spreads as much as it did corporate credit spreads; the flight-to-liquidity was coincidentally associated with an increase in perceived credit risk (flight-to-quality). Thus, the average rise in corporate credit risk post August 1998 cannot be fully accounted for by a change in the market's increased preference for liquidity.

Table 3.8 reports descriptive statistics for the daily 10 year Refcorp yield spread level and first differences. The mean 10 year Refcorp spread is 24 basis points, reaching a maximum of 71 basis points and a minimum of -1.7 basis points. The presence of negative spreads is also reported by Longstaff (2002) but is not frequent and the mean spread is likewise found to be significantly different from zero. Table 3.8 also shows mean-reversion in Refcorp spreads as evidenced by the negative coefficient on the first difference time-series.

Given the magnitude and time-variation in the Refcorp spread, we have evidence that a component of the corporate-Treasury credit spread is likely to be unrelated to changes in idiosyncratic firm default risk; Refcorp and Treasury bonds have equivalent default risk yet we see a time-varying price differential that Longstaff (2002) has shown to be related to 'flight-to-liquidity' measures, and in our own sample we see a similar increase in Refcorp spreads associated with the LTCM crises.

3.2 Fitting the Credit Models

The credit models are fitted to the observed firm-specific term structure of credit spreads with key parameters estimated implicitly from bond price data. Section 3.2.1 introduces the state-space framework. In Section 3.2.3 the empirical forms of the measurement equations are described and the generic form of the underlying state process is described in Section 3.2.4. The extended Kalman filter (EKF) method of estimation is explained in

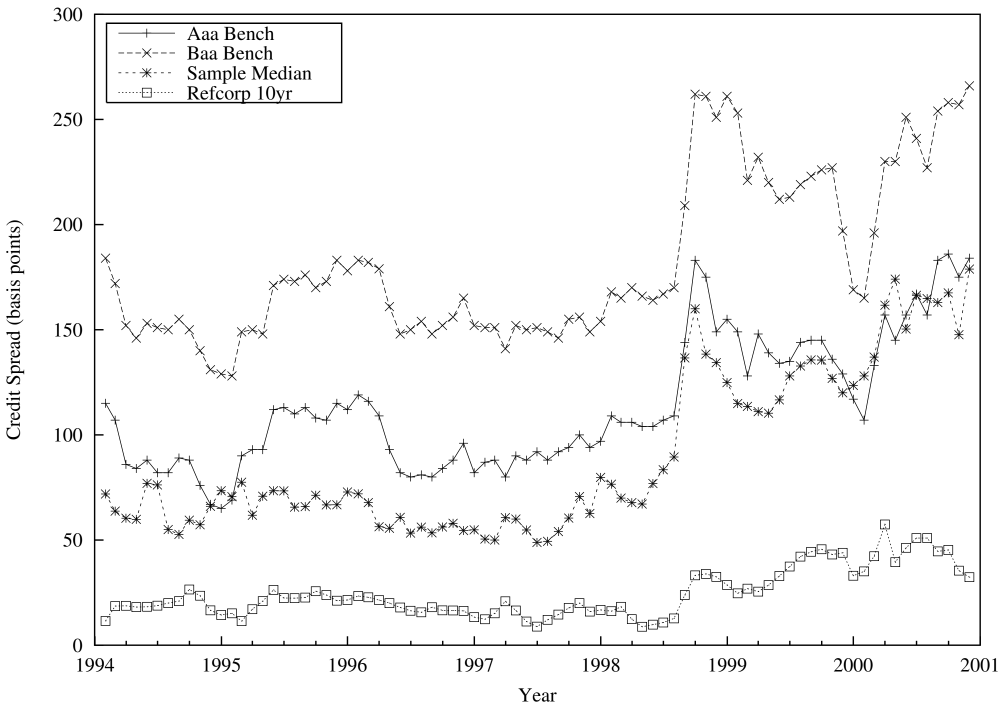


Figure 3.3: This is a plot of the monthly median credit spread sampled, against the monthly Moody's generic credit spreads, and monthly Refcorp 10 year constant maturity Treasury spread.

Section 3.2.5. Model specific details including the parameters, bond valuation functions, and model-specific state-space equations are explained in 3.2.6.

3.2.1 Introduction to Model Estimation Method

Our estimation method follows Claessens & Pennacchi (1996), Cumby & Evans (1995), and Duffee (1999), by use of an EKF to estimate an underlying stochastic model of the observed term structure of credit yield spreads. Unlike these earlier studies, our method uses corporate bond trade data and is applied to corporate structural models of credit risk. A Kalman filter is a maximum-likelihood method widely used in finance and economics for estimating term structure models, in which the observable data from the market is used to infer the values for an underlying unobserved state process. The EKF is a category of filter where the measurement equation is non-linear with respect to the state process. The EKF has been applied to yield curve modelling by Lund (1997) and in pricing sovereign bonds by Claessens & Pennacchi (1996), Cumby & Evans (1995), Keswani (2005), and Duffie et al. (2000). Its application to corporate debt credit modelling has been limited to estimation of a two-factor reduced form model on monthly bid-quotes by Duffee (1999). Further examples of the EKF's use in financial time-series modelling is provided by Harvey (1989), and Durbin & Koopman (2001).

In our modelling, the state variable is the firm's solvency. For all models except the CEV model, the state variable is the natural log of the ratio of firm value to the default boundary, $x(t) = \ln V(t)/K(t)$. For the CEV model, the state process is defined as the distance of the firm from the default boundary scaled by the default boundary, $X(t) = (V(t) - K)/K$. The passage of the state variable to zero is the trigger for default. We cannot observe the state variable directly, however we can observe the credit spreads of the firm's bonds as traded in the market, and from their time series behaviour, infer the path taken by the firm's solvency, and the most likely parameters governing the solvency process under the respective credit model.

Before estimating the credit models by EKF, it is first necessary to express them in terms of a state-space framework comprising a measurement and transition equation. Here we introduce the generic state-space framework into which all models fit. Sections 3.2.3 and 3.2.4 further elaborate by incorporating bond and issue dependencies.

Let $\boldsymbol{y}(t) = (y_1(t), y_2(t), \ldots, y_n(t))'$ be a vector of time-t observed credit spreads for i=(1 to n) bonds and let $\alpha(t)$ be the value of the state variable at time-t. Suppressing dependence on other model parameters for clarity, we have the measurement equation

$$\boldsymbol{y}(t) = \boldsymbol{d}(t) + \boldsymbol{G}(\alpha(t)) + \varepsilon(t), \qquad t = 1, \ldots, \tau \tag{3.2}$$

where $\boldsymbol{d}(t)$ is an n by one vector and $\varepsilon(t)$ is an n by one vector of serially uncorrelated disturbances with mean zero and n by n covariance matrix $\boldsymbol{H}(t)$, that is, $E(\varepsilon(t)) = 0$ and $Var(\varepsilon(t)) = \boldsymbol{H}(t)$. The function $\boldsymbol{G}(\cdot)$ maps the state vector to the observed credit spread.

The error term, ε, in equation (3.2) is referred to as the measurement error. A feature of the state-space set up is the inclusion of an error term separate from the variability of the underlying state variable itself. The latter is governed by the variance of the state process. The measurement error, on the other hand, recognises that the observation may be imperfectly measured and hence an additional source of error is introduced into the system via the measurement equation alone. In finance, measurement errors pertain to incorrect recording of deals, rounding errors, etc.

The values of $\boldsymbol{y}(t)$ depend upon the path taken by the unobserved state variable. The state variable transition equation is specified by the specific credit model but in all cases follows a univariate first-order Markov process of the general form

$$\alpha(t) = \bar{T}(t)\alpha(t-1) + \bar{c}(t) + R(t)\eta(t), \qquad t = 1, \ldots, \tau \tag{3.3}$$

where $\bar{T}(t)$ is an autoregressive coefficient, $\bar{c}(t)$ and $R(t)$ are constants, and $\eta(t)$ is an independent and identically distributed error term with mean zero and variance $Q(t)$, that is, $E(\eta(t) = 0)$ and $Var(\eta(t)) = Q(t)$. The disturbances of the measurement and transition equations are assumed to be uncorrelated with each other in all time periods and with the initial starting value of the state variable, $\alpha(0)$.

In Table 3.9 the parameters of the credit models and state variables are summarised. The parameter set, ψ, is model specific and is comprised of implied model parameters (Panel A) and implied state-space parameters (Panel B) that are inferred from maximum-likelihood estimation. The latter are termed hyperparameters and the set is denoted ψ^h. Other model parameters include those that are exogenously set or otherwise known such as bond cash flow characteristics (Panel C).

In the remainder of this section we elaborate on the measurement and transition equations, the state-space form of the models, and the method of quasi maximum-likelihood estimation by EKF.

3.2.2 Calculating the Predicted Credit Spread

The predicted credit spread is calculated as the difference between the yield to maturity of the risky corporate bond and an equivalent risk-free bond yield to maturity. The equivalent risk-free bond yield is found by first valuing the corporate bond as if it was a Treasury bond. A risk-free value for the same promised cash flows of the corporate bond is calculated by discounting the corporate bond's promised cash flows at the relevant Treasury spot rates. The Treasury spot rates are in turn taken from the Vasicek (hereafter Vasicek) interest rate model, estimated at the trade date, for the term of the promised cash flow holding interest rate model parameters constant through the sample. In other words, the interest rate model is not updated through time, but the forward estimates of future interest rates update as we step through the sample.

We chose the Vasicek model for valuing an equivalent risk-free Treasury bond to minimise error from the yield curve fitting in the two-factor credit models, which use the Vasicek model internally for discounting expected payoffs. For the sake of comparability between models, we retain the same risk-free term structure for all models. The fitting of the Vasicek interest rate model is discussed further in Section 3.3.

Denote $p(t,T)$ as the time-t value of a default-risky bond, maturing at time T with face value of one-dollar, and $B(t,T)$ as the risk-free equivalent Treasury bond value. The bond has $z = 1,2,\ldots m$ promised payments at time $t(z)$ with maturity at $t(m) = T$. The continuous yields-to-maturity for the default-risky bond y^c, and the equivalent risk-free bond r^c, are inverted, using the bisection numeric search method, from

$$p(t,T) = e^{-y^c(T-t)} + c\sum_{z=1}^{m} e^{-y^c(t(z)-t)}, \tag{3.4}$$

and,

$$B(t,T) = e^{-r^c(T-t)} + c\sum_{z=1}^{m} e^{-r^c(t(z)-t)}, \tag{3.5}$$

where c is the coupon, and the value of the default-risky bond is evaluated from the relevant credit model and the value of the default-free bond equivalent value obtained from the Vasicek term structure model.

To compare the model results with observed yield spreads, the continuous yield-to-maturity estimates are converted to semi-annual compounding equivalent rates by

$$\begin{aligned} y^s &= 2(e^{y^c/2} - 1) \\ r^s &= 2(e^{r^c/2} - 1). \end{aligned} \tag{3.6}$$

The predicted credit spread, before allowance for any liquidity premiums, is then calculated as the difference between the default-risky yield-to-maturity and the riskless equivalent yield-to maturity, conditional on all firm parameters and bond characteristics

$$g(t; \psi_i) = y^s - r^s. \tag{3.7}$$

Promised cash flows are sourced from the FISD file.

3.2.3 Generic Measurement Equations

We are motivated by the empirical findings by Delianedis & Geske (2001), Elton et al. (2001), and HH, that corporate bond credit spreads contain significant non-credit related premiums. We therefore construct three alternative empirical specifications to control for their potential influence on model fit and relative performance. The first assumes no liquidity and tax components in the credit spread as per the extant empirical literature of JMR, Lyden & Saraniti (2000), EHH, and HH. The second includes a constant premium for each bond in the measurement equation, thereby capturing differential tax, stationary liquidity components, and other unexplained components of the credit spread. This allows a comparison of spread errors with Duffee (1999), who likewise assumes a constant unexplained spread component. Finally, the third method controls for additional time-varying flight-to-liquidity components of the credit spread that are related to changes in the general market preference for liquidity as measured by the Refcorp ten year constant maturity credit spread. The use of the yield spread differential between Refcorp bonds and Treasury bonds has been shown by Longstaff (2002) and Longstaff et al. (2005) to be an observable measure of the time-varying liquidity premium embedded in the corporate-Treasury yield spread.

Henceforth, for ease of exposition, the empirical equations are referred to as having: 1) no liquidity premium, 2) a constant liquidity premium, or 3) a time-varying liquidity premium, acknowledging that not all of the unexplained premium is necessarily liquidity related.[4]

Expanding on equation (3.2), the elements of the generic measurement equation are:

[4]For example, it would also include model misspecification

1. **No Liquidity Premium**

$$y_{i,j}(t) = g(\alpha_j(t); \psi_{i,j}) + \varepsilon_{i,j}(t), \tag{3.8}$$

where, $y_{i,j}$ is the time-t observed credit spread for bond i issued by firm j, $\varepsilon_{i,j}(t)$ is normal i.i.d error, $g(\cdot)$ is the model predicted credit spread, conditional on bond and firm specific parameters $\psi_{i,j}$, and the value of the state process, $\alpha_j(t)$. This specification assumes that the theoretical model factors of firm leverage, asset volatility, and bond cash flow factors are sufficient to estimate the Treasury bond spread without bias,

2. **Constant Liquidity Premium**

$$y_{i,j}(t) = d_{i,j} + g(\alpha_j(t); \psi_{i,j}) + \varepsilon_{i,j}(t), \tag{3.9}$$

where, in addition to the no-liquidity case, $d_{i,j} \geq 0$ controls for issue-specific idiosyncratic tax and liquidity, and other unexplained components of the credit spread,

3. **Time-Varying Liquidity Premium**

$$y_{i,j}(t) = d_{i,j} + \beta_j^R Ref(t) + g(\alpha_j(t); \psi_{i,j}) + \varepsilon_{i,j}(t), \tag{3.10}$$

where, in addition to the constant liquidity case, $\beta_j^R > 0$ is the sensitivity of the firm's average spread to market-wide liquidity measured relative to a 10 year constant maturity Refcorp spread, $Ref(t)$. This specification follows the observation that market liquidity, as measured by the Refcorp spread, is time-varying, and individual issuer's bond spreads may have different sensitivity to changes market liquidity.

At time-t we observe $i = (1, 2, \cdots, n(t))$ yield-to-maturity credit spreads for firm j, stacked into the n(t) by one vector $\boldsymbol{y}_j(t)$. Likewise, we observe n(t) predictions from the credit model stacked into the n(t) by one vector $\boldsymbol{G}(\alpha_j(t)$ due to each bond having different remaining terms to maturity and coupon rates. The empirical equations (3.8),(3.9), and (3.10) can therefore be expressed more succinctly as the firm-specific generic measurement equation

$$\boldsymbol{y}_j(t) = \boldsymbol{d}_j(t) + \boldsymbol{G}(\alpha_j(t); \psi_j) + \boldsymbol{\varepsilon}_j(t), \qquad t = 1, \ldots, \tau \tag{3.11}$$

where, for ease of notation, we let $\boldsymbol{d}_j(t)$ be the n(t) by one stack of liquidity premiums with each i-th element equal to $d_{i,j} + \beta_j^R Ref(t)$.

The mapping vector, $\boldsymbol{G}(\cdot)$, depends on the specification of the structural model. As shown in Table 3.9 we have 6 different functional specifications corresponding to the Merton, LT, CEV, CDG, and two forms of LS credit models.

Because we estimate model parameters using actual trade data, and not equally spaced bid quotes as in Duffee (1999),the time between trades is variable. Let $t(z)$ be the time of the z-th observation for $z = (1,2,\ldots,m)$ where $\tau = t(m)$. For notational clarity, we hereafter refer to $t(z)$ by $t = z$ so that the sequence of trade dates $(t(1),t(2),\ldots,t(m))$ is represented by the above notation of $(1,2,\ldots,\tau)$.

An additional consequence of using actual trade data is that the number of bonds in the observation vector is variable with $0 < n(t) \leq 10$. The EKF procedure is robust to missing data. Where partial data is missing from the observation vector, information from the available bonds is used to predict the next set of observations. The method we adopt for handling missing data is from Harvey (1989, p. 143) and Durbin & Koopman (2001, p. 92). The size of $\boldsymbol{Y}_j(t)$ is redimensioned at time-t according to the number of bonds that traded on the day. Consequently, the size of the measurement error vector, $\varepsilon_j(t)$ and its covariance $\boldsymbol{H}_j(t)$ is redimensioned to the size $n(t)$. To facilitate the redimensioning of the covariance matrix, we follow Duffee (1999) and assume that the measurement error variance is the same for all of firm-j's bonds. The assumption is reasonable since measurement errors are likely to be random in nature and not related to the specific characteristics of the bond. It follows that $\boldsymbol{H}_j(t) = \boldsymbol{I}(t)\sigma^2_{\varepsilon,j}$ where $\boldsymbol{I}(t)$ is an $n(t)$ by $n(t)$ identity matrix, the dimension of which varies with time, and $\sigma_{\varepsilon,j}$ is a firm-specific, time-inhomogeneous, standard deviation of the measurement error.

3.2.4 Generic State Transition Equation

In the state-space set up, the transition equation is a first-order Markov process that links the observed discrete pricing process to the unobserved capital structure process.

All models have the generic univariate transition equation

$$\alpha_j(t) = \bar{c}_j(t) + \bar{T}_j\alpha_j(t-1) + R_j(t)\eta(t), \qquad t = 1,\ldots,\tau \tag{3.12}$$

where α_j is the latent state variable. From the theoretical foundations of the structural credit models, the unobserved state variable process can be interpreted as a dynamic solvency ratio of the firm.

3.2.5 Applying the Extended Kalman Filter

In this section, the EKF as applied to the credit models explained. We follow closely the notation of Harvey (1989) and Duffee (1999). The theory and use of Kalman filters in economics is further explained in Harvey (1989), Hamilton (1994), and Durbin & Koopman (2001).

The Kalman filter is a recursive procedure for inferring the optimal estimator, $a(t)$, of the true latent state-process, $\alpha(t)$, when the measurement equation is functionally linear with respect to the state variable. For a linear Gaussian model, the Kalman filter is directly equivalent solving the likelihood function (refer [p.126]Harvey (1989)). Where the measurement equation is functionally non-linear, as is the case with structural credit models, linearisation of the prediction equation provides only approximately optimal estimates, as the errors are no longer multivariate Gaussian, and the EKF produces an approximate quasi-likelihood function. Duan & Simonato (1999) demonstrates that the quasi-likelihood properties remain reasonably reliable for non-linear Kalman filters in small samples.

Let $a(t-1)$ be the optimal estimator of $\alpha(t-1)$ given all observations up to and including $\boldsymbol{y}(t-1)$, then the optimal prediction of $\alpha(t)$ is

$$a(t|t-1) = T(t)a(t-1) + \bar{c}(t), \tag{3.13}$$

and the variance, or mean square error (MSE), of the state prediction error is

$$P(t|t-1) = \bar{T}(t)P(t-1)\bar{T}(t)' + R(t)Q(t)R(t)'. \tag{3.14}$$

Moving through time from first to last observations, once a new vector of observed spreads, $\boldsymbol{y}(t)$, becomes available the prediction, $a(t|t-1)$, is updated to give the best inference of the unobserved state value using all information up to, and including, time-t. The updating equation for the state vector is

$$a(t) = a(t|t-1) + P(t|t-1)\boldsymbol{G}(t)'\boldsymbol{F}^{-1}(t)\big(\boldsymbol{y}(t) - \boldsymbol{G}(t) - \boldsymbol{d}(t)\big), \tag{3.15}$$

where,

$$\boldsymbol{F}(t) = \boldsymbol{G}(t)P(t|t-1)\boldsymbol{G}(t)' + \boldsymbol{H}(t). \tag{3.16}$$

The EKF is distinguished from a linear filter by the linearisation of the measurement equation using a Taylor series expression around the conditional mean of the state vector, $a(t|t-1)$. The vector $\boldsymbol{G}$ is an $n(t)$ by one vector comprising

$$\boldsymbol{G}(t) = \begin{bmatrix} \frac{\partial g(\alpha(t);\psi_1)}{\partial \alpha(t)} \\ \frac{\partial g(\alpha(t);\psi_2)}{\partial \alpha(t)} \\ \cdot \\ \cdot \\ \frac{\partial g(\alpha(t);\psi_{n(t)})}{\partial \alpha(t)} \end{bmatrix}_{\alpha(t)=a(t|t-1)} \tag{3.17}$$

where $g(\alpha(t);\psi_i)$ is the predicted credit spread from the credit model for the i-th bond at time-t conditional on the model parameters. The partial derivatives are evaluated numerically at the prior period prediction of the state value. The MSE of the updated state variable is then

$$P(t) = P(t|t-1) - P(t|t-1)\boldsymbol{G}(t)^{'}\boldsymbol{F}(t)^{-1}(t)\boldsymbol{G}(t)P(t|t-1). \tag{3.18}$$

The step-ahead prediction errors, or innovations, are

$$\boldsymbol{v}(t) = \boldsymbol{y}(t) - \boldsymbol{G}(t) - \boldsymbol{d}(t). \tag{3.19}$$

The EKF is defined as the system of prediction and updating equations as shown above. Given starting values, for the initial state vector, $a(0)$, and variance of the state vector, $P(0)$, the EKF predictive and updating equations are applied on each trade date to compute the time-series of step-ahead prediction errors and their variances. From a single pass through the EKF, the log-likelihood, conditional on the hyperparameter set is calculated by

$$\ln L(\psi^h) = -\frac{N\tau}{2}\ln 2\pi - \frac{1}{2}\sum_{t=1+d}^{\tau}\left(\ln|\boldsymbol{F}(t)| + \boldsymbol{v}(t)^{'}\boldsymbol{F}^{-1}(t)\boldsymbol{v}(t)\right) \tag{3.20}$$

where the total number of non-missing observations is given by $N = \sum_{t=1+d}^{\tau} n(t)$ given that the size, $n(t)$, of the observation vector varies with each trade date and d represents a diffuse prior and is equal to one where used.

The log-likelihood function is conditional on the hyperparameters. The optimal set of hyperparameters, $\hat{\psi}^h$, is found by numeric search performed to maximise the log-likelihood function shown in equation (3.20). The optimisation is achieved by an initial search using the simplex search method, followed by the well known Broyden-Fletcher-Goldfarb-Shanno (BFGS) gradient descent search method that we apply using numeric gradients. All code is implemented in OX software calling the MaxBFGS routine.[5] The simplex method has the advantage of not requiring gradients and reduces the risk of locating a local minimum early in the search process. Transformations are made to the hyperparameters to enforce economic restrictions on the range of permissible values. Table 3.9, Panels A and B summarise the model parameter restrictions.

Asymptotic standard errors on the hyperparameters are estimated numerically via inversion of the Hessian matrix of the log-likelihood function. The EKF provides only approximate standard errors because of the non-linearity of the measurement equation and hence error terms.

A pass through of the data for each firm, at the optimal hyperparameter set, generates a time series of state estimates, $\hat{a(t)}$, termed the filtered estimates. It is the best estimate of the state vector given all information up to time-t. However, an improved

[5]OX is a C based matrix language designed for econometric use. Further details are available at http://www.doornik.com/products.html#Ox.

inference of the state vector can be made with the additional information available after time-t. Following Duffee (1999), a process termed smoothing is applied to the filtered estimate to give a better inference of the true underlying state process. The smoothing method applied uses the full data set and is termed a fixed-interval smoother and is further described in Harvey (1989, p.154), and Hamilton (1994, Section 13.6).

Recall that the transition equation is autoregressive, the process of smoothing works backwards from the final prediction of the state vector, $a(T)$, and its MSE, $P(T)$. Let $a(t|T)$ denote the *smoothed* estimate of the state vector and $P(t|T)$ its MSE, then the smoothed estimates are given by

$$a(t|T) = a(t) + P^*(t)(a(t+1|T) - T(t+1)a(t) - \bar{c}(t+1)), \tag{3.21}$$

and

$$P(t|T) = P(t) + P^*(t)(P(t+1|T) - P(t+1|t))P^{*'}(t), \tag{3.22}$$

where

$$P^*(t) = P(t)T^{'}(t+1)P^{-1}(t+1|t), \tag{3.23}$$

with the initial values $a(T|T) = a(T)$, and $P(T|T) = P(T)$.

3.2.6 Model-Specific Implementation

In this section, the model-specific parameters and state-space equations for the credit models are presented: the EM model (Section 3.2.6.1), the single factor LS1 model and two-factor LS2 model (Section 3.2.6.2), the LT model (Section 3.2.6.3), the CEV model (Section 3.2.6.4), and the mean-reverting two-factor CDG model (Section 3.2.6.5). A summary of model parameters is shown in Table 3.9. The resultant smoothed estimates are the best estimates of the underlying log-solvency process given all information in the complete time series of credit spread term structures. Hereafter, when discussing the estimated path of the firm's log-solvency, we refer to the smoothed estimates.

Table 3.9: This table shows the credit model parameters. Panel A shows parameters inferred from credit spreads with their restrictions. Panel B shows the state variable and other implied parameters with restrictions, and Panel C shows other exogenously set parameters with their source or assumption. COMP refers to COMPUSTAT, AK is Altman & Kishore (1996), Roberts is Roberts (2002), and Vasicek refers to estimates sourced from fitting the Vasicek term-structure model to CMT data sourced from the Federal Reserve Board H15 report. FISD is the Fixed Income Securities Database.

Parameter	Models:	Merton	LS1	LS2	LT	CEV	CDG
Panel A: Implied Model Parameters							
Firm volatility	σ_v	>0	>0	>0	>0	n/a	>0
Firm vol. scalar	$\bar{\sigma}_v$	n/a	n/a	n/a	n/a	>0	n/a
Firm vol. elasticity	ρ	n/a	n/a	n/a	n/a	<1	n/a
Firm payout rate	δ_v	0-1	0-1	0-1	0-1	0-1	n/a
Bankruptcy costs	α_v	n/a	n/a	n/a	0-1	n/a	n/a
Mean reversion rate	κ_v	n/a	n/a	n/a	n/a	n/a	>0
Debt timing	ϕ	n/a	n/a	n/a	n/a	n/a	>0
Panel B: Implied State-Space Parameters and State Variable							
Measurement vol.	σ_m	>0	>0	>0	>0	>0	>0
Refcorp slope	β^R	>0	>0	>0	>0	>0	>0
Spread constant	d	>0	>0	>0	>0	>0	>0
Latent state	$\alpha(t)$	$\ln V(t)/K$	$\ln V(t)/K$	$\ln V(t)/K$	$\ln V(t)/K$	$(V(t)-K)/K$	$\ln(V/K)(t)$
Panel C: Known Model Parameters							
Initial state	$x(0)$	COMP, CRSP	COMP, CRSP	COMP, CRSP	endogenous	COMP, CRSP	COMP, CRSP
Writedown rate	ω	n/a	AK	AK	AK	AK	AK
Short-rate	$r(t)$	Vasicek	Vasicek	Vasicek	Vasicek	Vasicek	Vasicek
Long-run rate	θ	n/a	n/a	Vasicek	n/a	n/a	Vasicek
Mean-reversion of $r(t)$	κ_r	n/a	n/a	Vasicek	n/a	n/a	Vasicek
Asset-rate correl.	$\rho_{v,r}$	n/a	n/a	0	n/a	n/a	0
Effective tax rate	τ	n/a	n/a	n/a	0.155	n/a	n/a
Average debt ratio	P_{LT}	n/a	n/a	n/a	COMP, CRSP	n/a	n/a
New bond maturity	T_{LT}	n/a	n/a	n/a	FISD	n/a	n/a
Target debt ratio	ν	n/a	n/a	n/a	n/a	n/a	Roberts
Bond promised cash	c, T	FISD	FISD	FISD	FISD	FISD	FISD

3.2.6.1 Extended Merton Model

The firm follows the continuous-time capital structure process

$$dx(t) = (r - \delta_v - \sigma_v^2/2)dt + \sigma_v dW^Q(t). \tag{3.24}$$

where $x(t) = \ln(V(t)/K)$.

To estimate the model on discretely observed data, equation (3.24) is discretised recognising that the time intervals between trades is variable, and permitting the risk-free rate to vary deterministically through time

$$x(t) = x(t-1) + (r(t-1) - \delta_v - \sigma_v^2/2)\Delta t + \sigma_v\sqrt{\Delta t}\eta(t), \qquad \eta(t) \sim N(0,1). \tag{3.25}$$

It follows that the transition equation for the Merton model is the generic transition equation (3.12) with the elements

$$\begin{aligned} \alpha(t) &= x(t), & \bar{c}(t) &= (r(t-1) - \delta_v - \sigma_v^2/2)\Delta t, \\ \bar{T} &= 1, & R(t) &= \sigma_v\sqrt{\Delta t}, \end{aligned} \tag{3.26}$$

where Δt is the length of the time step, $t(z) - t(z-1)$.

To start the EKF procedure we require an initial estimate of the firm's implied solvency level, $\alpha(0) \equiv x(0)$. Because the value of the firm and the firm's default boundary are not observable, we use an approximation from observable accounting and market data. At the first trade date in the sample, we take the last quarterly reported book value of short-term and long-term debt (D) from COMPUSTAT (items 45 and 51 respectively), and the market value of equity (E) sourced from CRSP. The observed log-solvency ratio is then calculated as

$$S(t) = \ln\frac{D(t) + E(t)}{D(t)}, \text{ and } x(0) = S(0). \tag{3.27}$$

The initial estimate for the variance of the state vector $P(0)$ is

$$P(0) = \hat{\sigma}_v{}^2(t(1) - t(0)) \times 1000, \tag{3.28}$$

where, $\hat{\sigma}_v$ is an initial estimate of the firm's asset volatility, and $t(1) - t(0)$ is the first time interval in the firm's sample. For the implementation of all models we use a diffuse prior and scale the initial state variance by 1000. The purpose of the diffuse prior is to recognise that we do not have complete a priori knowledge of the true latent log-solvency level, and therefore increase the initial state variance to allow for uncertainty in our initial estimate (Harvey 1989, p. 121). In doing so, subsequent values of the state process are less influenced by our initial choice. The spread prediction errors from the first prediction step are excluded from the log-likelihood function (Harvey 1989, p.

127), resulting in 8,953 spread observations excluding the initial observations. For ease of comparison, all descriptive statistics for actual and predicted spreads, exclude the initial observations.

The initial estimate of asset volatility is the firm-specific standard deviation of the historical daily firm asset value return, measured over the sample period, and annualised over 250 trading days. Observed market value is defined as the sum of the quarterly book value of debt D, and the daily market value of equity, E. The initial asset payout rate, $\delta_v(0)$, is the mean estimate of 4.83 percent reported by EHH. The initial measurement volatility $\sigma_m(0)$ is set at 0.245 percent for all firms. The initial spread intercept term $d_i(0)$ is the average sample spread per firm, pooled across time and bonds of the issuer.

The Merton model, as originally specified, values only zero-coupon debt. We therefore adapt its use for application to coupon-paying bonds via the extension method of EHH. Our results are therefore broadly comparable with their proxy variable method, with the exception that, in keeping with the original model, the Merton writedown rate is endogenously implied, whereas EHH force the writedown rate to be exogenously specified. Under the sum-of-zeros method, the bond is valued by summing the present value of each promised cash flow, treating the default-risky value of the coupon as if it was a zero-coupon bond. The model permits default only at each coupon date and excludes the possibility of default between coupon dates. The resultant model, incorporating the sum-of-zeros and deterministic risk-free rate, is referred to as the extended Merton (EM) model.

Let $p(t,T;c)$ be the time-t value of a bond paying coupons of c at dates $t(z)$, where $z=(1,2,\dots m)$, with maturity of $t(m)=T$. Suppressing the dependency of bond value on other model parameters, the value of the coupon bond is given by

$$p(t,T;c) = c\sum_{z=1}^{m} p(t,t(z)) + p(t,T). \tag{3.29}$$

The zero-coupon default-risky bond values, $p(t,t(z))$, are calculated from the log-solvency specification of the Merton model shown in equation (2.8). The bond value is then converted to an equivalent yield-to-maturity credit spread as shown in equation (3.7).

The EM model is fitted to observed credit spreads with three different hyperparameter sets corresponding to different liquidity treatments. Suppressing dependency on firm j gives:

$$\begin{aligned}
&\text{No liquidity:}\\
&\psi^h(1) = \{\ln\sigma_v^2, \ln\sigma_m^2, \ln(\delta_v/(1-\delta_v))\},\\
&\text{Constant liquidity:}\\
&\psi^h(2) = \{\ln\sigma_v^2, \ln\sigma_m^2, \ln(\delta_v/(1-\delta_v)), \ln d_i\},\\
&\text{Time-varying liquidity:}\\
&\psi^h(3) = \{\ln\sigma_v^2, \ln\sigma_m^2, \ln(\delta_v/(1-\delta_v)), \ln\beta^R, \ln d_i\}.
\end{aligned} \tag{3.30}$$

The hyperparameters are transformed to ensure estimates sit within economically reasonable values. Asset and measurement volatility are restricted to be greater than zero, asset payout is controlled to be between zero and one using the logit transformation, spread constants are assumed greater than one, and the sensitivity of credit spread to the Refcorp yield is assumed to be greater than zero. The risk-free rate is not part of the hyperparameter set as it is treated as a known state variable.

3.2.6.2 Longstaff and Schwartz 1 and 2

Two forms of LS model are tested. The first, LS1 model, is a single-factor implementation where the risk free rate is treated as non-stochastic; similar to HH's base case model. The second, LS2 model, assumes the risk-free rate is stochastic. The LS1 model is of interest due to its analytic tractability having a closed-form solution to the first passage crossing time. The LS2 model must be solved numerically at considerable additional computing cost and added approximation error. To aid identification, and to facilitate a comparison with the LS1 analytic solution, zero correlation between asset values and the risk-free rate is assumed.

The LS1 model shares the same firm asset process as the EM model. Consequently, the state-space equations are given by equation (3.11), and equation (3.26). The LS1 model differs from the EM in the specification of the default boundary, recovery level, and the interest rate process. The LS1 model permits early default, and assumes an exogenous proportion of the face value of debt, equal to $(1-\omega)$, is recovered at maturity. The Merton model permits default only at maturity with an endogenous recovery (refer equation (2.11)). In our extended implementation of the Merton model, the writedown rate implicitly varies at each coupon date as a negative function of the log-solvency level. In other words, the further the at maturity value of the log-solvency level, $x(T)$, falls below the default boundary, the greater the expected writedown rate. Holding all else constant, the closer the firm is to default, the higher the expected writedown rate on all the remaining coupons and face value in the event of default. Thus, our sum of zeros extension to the Merton model permits some time-variation in the writedown rate, such that it varies over time negatively correlated with the log-solvency ratio.

Industry-specific exogenous writedown rates are applied per firm using the prior results of Altman & Kishore (1996), (refer Table 3.10). Our across-firm sample average writedown rate is 54.49 percent, with a minimum of 22.26 percent for utilities Commonwealth Edison Corp. and Niagara Mohawk Power Corp., and a maximum of 79.50 percent for hospital operator HCA Healthcare Corp. In comparison, EHH and HH apply a writedown rate of 48.69 percent of face value across all firms. Lyden & Saraniti (2000) also rely on Altman & Kishore (1996), but apply the senior unsecured average of 52.3 percent across all firms.

The default probability for the LS1 model is calculated analytically using the known solution for a single-factor first crossing time problem (Harrison 1985). Suppressing dependency on model parameters the risk-neutral cumulative probability of default at any time between times t and T is given by

$$Q(t,T) = N\left(\frac{-x(t)-\mu(t)}{\sigma_v\sqrt{(T-t)}}\right) + \exp\left(\frac{-2x(t)\mu(t)}{\sigma_v^2}\right) N\left(\frac{-x(t)+\mu(t)}{\sigma_v\sqrt{(T-t)}}\right), \tag{3.31}$$

where the drift rate is $\mu(t) = (r(t,T) - \delta_v - \sigma_v^2/2)$ and $N(\cdot)$ represents the cumulative standard normal function. The risk-free rate $r(t,T)$ is the time-t spot rate for payment to be received in the future at time T. The drift rate is therefore time-dependent and assumed to evolve deterministically. The value of a zero-coupon bond per dollar of face value is

$$p(t,T) = e^{-r(t,T)(T-t)}\big(Q(t,T)(1-\omega) + (1-Q(t,T))\big), \tag{3.32}$$

and the time-t value of a bond paying coupons of c, at times $t(z)$ for $z = (1,2,\cdots m)$, with a face value of one-dollar payable at $t(m) = T$ is given by equation 3.29 using the sum of zeros approach previously described for the Merton model. The bond value is then converted to an equivalent yield-to-maturity credit spread by equation (3.7).

The LS2 model introduces a stochastic risk-free rate and shares the same state-space equations as the LS1 and EM models. It also shares the same writedown assumption as the LS1 model. However, additional complexity is involved in the computation of the crossing time, even though there is no correlation assumed between the risk-free rate and firm return. Volatility in the risk-free rate and dependence of the firm's solvency drift rate on the risk-free rate, require that the crossing time be calculated on the path taken by $x(t)$ as a function of two stochastic processes. The numeric solution we use follows the numeric grid method of CDG. Details are shown in Appendix B.1.

For the LS1 and LS2 models, the hyperparameters and initial values, are the same as the EM model.

3.2.6.3 Leland and Toft

The LT model assumes the firm issues and retires debt continuously. At each point in time, shareholders consider whether to continue to service the debt, which they do provided that the value of the firm exceeds debt servicing costs. Leland & Toft (1996) show that the equilibrium default boundary, K_{LT}, is therefore stationary through time, and Leland (2004) exploits this stationarity to demonstrate that valuation under the LT model is equivalent to implementing a single-factor LS model with an endogenous writedown rate. We therefore implement the LT model as a special case of the LS1 model.

The two key parameter differences from the LS1 model is that the starting value of the log-solvency process, and writedown rate, are functionally related and endogenous

Table 3.10: Shown are the firm specific parameter assumptions used in model estimation. The writedown rate, ω, is sourced from Altman & Kishore (1996) matched to each firm on the basis of the firm's SIC code. The initial value of the firm's asset volatility, $\sigma_v(0)$ is the sample standard deviation of the daily time series of the market log-solvency ratio. The initial value of the firm's log-solvency ratio, denoted $x(0)$, is the first observation in the sample of the firm's market log-solvency ratio. The initial estimate of the constant liquidity component of the credit spread is common across all firm's bonds and is denoted $d(0)$. It is equal to the pooled sample mean credit spread measured across all the firm's bonds. The volatility scalar is computed from $x(0)$ and $\sigma_v(0)$ by equation (3.44).

Issuer	SIC	ω	$\sigma_v(0)$	$x(0)$	$d(0)$	$\bar{\sigma}_v(0)$
Aetna Inc.	6324	0.6132	0.2935	2.4007	0.0029	0.0498
Associates Corp.	6141	0.6132	0.5851	0.2819	0.0018	0.4008
Atlantic Richfield Co.	2911	0.2809	0.1860	1.0831	0.0021	0.0135
A T & Tcorp.	4813	0.6543	0.3364	1.4409	0.0017	1.0742
Bear Stearns Companies Inc.	6211	0.6132	0.2025	0.0477	0.0027	0.0172
Black & Decker Corp.	3540	0.5245	0.2061	0.5126	0.0024	0.0181
Boeing Co.	3721	0.6917	0.3572	1.9513	0.0020	0.0885
Dayton Hudson Corp.	5331	0.5545	0.3664	0.7136	0.0027	0.0955
Commonwealth Edison Co.	4911	0.2226	0.2313	0.4848	0.0027	0.0251
Enron Corp.	5172	0.6100	0.2696	1.3540	0.0028	0.0389
Federated Department Stores	5311	0.5545	0.2426	0.6941	0.0038	0.0288
Ford Motor Co.	3711	0.5245	0.1369	0.1866	0.0030	0.0058
General Motors	3711	0.5245	0.1378	0.4885	0.0027	0.0059
Georgia Pacific Corp.	2600	0.5267	0.2274	0.8526	0.0036	0.0239
HCA Healthcare Corp.	8062	0.7950	0.2758	1.6425	0.0034	0.0416
IBM Corp.	7370	0.5245	0.2812	0.8089	0.0016	0.0440
International Paper Co.	2600	0.5267	0.2370	0.9467	0.0024	0.0269
Lehman Brothers Holdings Inc.	6211	0.6132	0.2409	0.0259	0.0039	0.0282
Merrill Lynch & Co.	6211	0.6132	0.1745	0.0731	0.0022	0.0113
Motorola Inc.	3663	0.5245	0.3643	2.5651	0.0023	0.0938
Nabisco Group Holdings Corp.	2052	0.4558	0.2703	0.4609	0.0029	0.0392
Niagara Mohawk Power Corp.	4931	0.2226	0.2132	0.5322	0.0044	0.0199
Northrop Grumman Corp.	3812	0.5245	0.3189	1.1845	0.0036	0.0634
Paine Webber Group Inc.	6211	0.6132	0.1449	0.0452	0.0037	0.0067
Penney J C Co. Inc.	5311	0.5545	0.2178	1.2930	0.0043	0.0212
Philip Morris Companies Inc.	2111	0.4558	0.2887	1.4258	0.0031	0.0474
Seagram Co. Ltd.	3652	0.5245	0.2812	1.1444	0.0032	0.0440
Sears Roebuck Acceptance Corp.	5311	0.5545	0.1766	0.5681	0.0027	0.0117
Service Corp. International	7200	0.5245	0.2902	0.9851	0.0104	0.0482
Union Pacific Corp.	4011	0.6917	0.2037	1.1596	0.0035	0.0175
Viacom Inc.	4841	0.6543	0.4488	0.7170	0.0041	0.1756
Wal-Mart Stores Inc.	5331	0.5545	0.3132	2.0136	0.0014	0.0601
Mean		0.5449	0.2663	0.9401	0.0031	0.0527
SD		0.1218	0.0929	0.6696	0.0015	0.0338
Min		0.2226	0.1369	0.0259	0.0014	0.0058
Max		0.795	0.5851	2.5651	0.0104	0.4008

to the model. We choose to implement a constant writedown rate, consistent with the assumption of stationary default boundary. Following Leland (2004), the recovery rate per firm is calculated as the fraction of firm assets available at default to bondholders, K_{LT}, per average dollar of face value of bonds outstanding, $\bar{D}$,

$$1 - \omega = \max\left(1 - \frac{(1-\alpha_v)K_{LT}}{\bar{D}}\right), \tag{3.33}$$

where $0 < \alpha_v < 1$ is the sum of direct and indirect bankruptcy costs associated with default. The writedown rate is constrained to be non-negative during the optimisation in the event that $K_{LT} > \bar{D}$ and α_v is insufficiently large. With no loss of generality, K_{LT} and $\bar{D}$ are expressed per dollar of firm value, therefore $\bar{D}$ is equivalent to the firm's average debt ratio and is inverted from the time-series of the observed log-solvency ratios by

$$\bar{D} = \sum_{t=1}^{\tau} \big(\exp S(t)\big)^{-1}/\tau. \tag{3.34}$$

The default boundary is then found as a function of the risk-free rate r, asset volatility σ_v, asset payout rate δ_v, maturity of new issue debt T_m, the effective tax rate τ_x, bankruptcy costs α_v, and debt level $\bar{D}$

$$K_{LT}(c, r, \sigma_v, \delta_v, T_m, \tau_x, \alpha_v, \bar{D}) = \frac{c/r\big(A/rT - B\big) - A\bar{D}/rT - \tau_x c(a+z)/r}{1 + \alpha_v(a+z) - (1-\alpha_v)B}, \tag{3.35}$$

where,

$$\begin{aligned}
A &= 2ae^{-rT}N(a\sigma_v\sqrt{T_m}) - 2zN(z\sigma_v\sqrt{T_m}) - \frac{2}{\sigma_v\sqrt{T_m}}n(z\sigma_v\sqrt{T_m}) \\
&\quad + \frac{2e^{-rT}}{\sigma_v\sqrt{T_m}}n(a\sigma_v\sqrt{T_m}) + (z-a), \\
B &= -\left(2z + \frac{2}{z\sigma_v^2 T_m}\right)N(z\sigma_v\sqrt{T_m}) - \frac{2}{\sigma_v\sqrt{T_m}}n(z\sigma_v\sqrt{T_m}) \\
&\quad + (z-a) + \frac{1}{z\sigma_v^2 T_m}, \\
a &= \frac{r - \delta_v - \sigma_v^2/2}{\sigma_v^2}, z = \frac{\sqrt{(a\sigma_v^2)^2 + 2r\sigma_v^2}}{\sigma_v^2},
\end{aligned}$$

and the standard normal density function is denoted $n(\cdot)$, and the standard cumulative distribution function is denoted $N(\cdot)$. The initial value of log-solvency at t=0 is $x(0) = \ln(1/K_{LT})$ and is endogenously determined from the initial parameter estimates.

Because the default boundary is constant, the firm's log-solvency evolves as per LS1, and the cumulative risk-neutral probability of default is given by the same first-passage solution as LS1 (equation (3.31)), after a change in the initial starting value of $x(0)$ from $\ln(V(0)/K)$ to $\ln(V(0)/K_{LT})$. Valuation of the finite maturity coupon-paying bond is the same as for the LS1 model as shown in equations (3.7), (3.32), and (3.29).

Equation (3.35) requires knowledge of the average maturity of new debt issued. Inspection of the data shows a systematic difference in the average new issue maturity. Therefore, the remaining maturity is firm-specific and is calculated as the average original contractual maturity of bonds on issue during the sample period. Bond maturity and issue date is sourced from FISD; new issue maturity is measured as the difference between the contractual maturity date and the issue date. The overall average new issue maturity is 15.25 years, with the mean for industrial firms at 16.74 years, financial firms at 7.42 years and utilities at 18.80 years. The all-firm average is comparable with other studies of debt maturity. For example, Guedes & Opler (1996, Table II) report a mean new issue maturity, across all U.S.-based firms, of 12.2 years. Table 3.11 shows new maturity averages per sample firm.

The effective corporate tax rate is assumed equal to 13.3 percent across all firms as per Leland (2004). His rate includes an estimate of the effect on firm value after personal shareholder tax. The coupon rate varies by firm and is equal to issue-average of the filtered bond rates. The overall average coupon rate is 7.20 percent per annum (refer Table C). The initial level of bankruptcy costs is assumed to be a relatively high 75 percent. Experimentation with lower levels such as 30 percent as assumed by Leland (2004) sometimes failed to result in convergence. A higher initial value led to rapid convergence in all cases.

Unlike the exogenous boundary models, the initial log-solvency level is endogenously determined from the hyperparameter set, and includes the inferred level of bankruptcy costs amongst other parameters. We therefore do not assume a diffuse prior, which would be inconsistent with the theoretical model, but rather let it be endogenously determined from the hyperparameter set, varying with each pass through the filter as the implied level of bankruptcy costs is iterated.

The state-space equations are the same as the EM, LS1 and LS2 models. There are three hyperparameter sets estimated. Suppressing dependency on the firm we have:

$$\begin{aligned}
&\text{No liquidity:}\\
&\psi^h(1) = \{\ln \bar{\sigma}_v^2, \ln \sigma_m^2, \ln(\delta_v/(1-\delta_v)), \ln(\alpha_v/(1-\alpha_v)),\\
&\text{Constant liquidity:}\\
&\psi^h(2) = \{\ln \bar{\sigma}_v^2, \ln \sigma_m^2, \ln(\delta_v/(1-\delta_v)), \ln(\alpha_v/(1-\alpha_v)), \ln d_i\},\\
&\text{Time-varying liquidity:}\\
&\psi^h(3) = \{\ln \bar{\sigma}_v^2, \ln \sigma_m^2, \ln(\delta_v/(1-\delta_v)), \ln(\alpha_v/(1-\alpha_v)), \ln \beta^R, \ln d_i\}.
\end{aligned} \tag{3.36}$$

In addition to the LS1 model, bankruptcy costs are included and constrained to lie between zero and one using the logit transformation.

Table 3.11: In the upper panel, shown is the descriptive statistics of the original contractual maturity of all bonds on issue during the sample period from 5 January 1994 to 27 December 2000. In the lower panel, descriptive statistics for the original maturity is shown by firms by industry sector. Means are issue-weighted simple averages.

	New Issue Maturity (years)			
Name	Mean	Min	Max	No of Obs
Aetna Inc.	19.62	5.01	40.03	10
Associates Corp.	6.64	0.48	40.03	653
Atlantic Richfield Co.	16.08	3.99	40.06	57
A T & T Corp.	20.97	2.02	60.07	33
Bear Stearns Companies Inc.	3.48	0.81	29.97	1,118
Black & Decker Corp.	6.79	2.00	12.05	18
Boeing Co.	33.65	10.01	50.06	13
Dayton Hudson Corp.	19.67	2.01	40.01	48
Commonwealth Edison Co.	21.91	1.02	50.26	99
Enron Corp.	10.30	1.51	40.05	44
Federated Department Stores	16.90	5.03	30.06	26
Ford Motor Co.	26.76	5.02	100.09	30
Georgia Pacific Corp.	17.42	3.03	40.07	33
General Motors	22.25	3.05	30.05	31
HCA Healthcare Corp.	20.95	3.02	100.05	26
IBM Corp.	8.38	1.00	100.06	88
International Paper Co.	12.86	1.63	30.05	30
Lehman Brothers Holdings Inc.	5.27	0.80	39.92	410
Merrill Lynch & Co.	3.69	1.00	30.07	970
Motorola Inc.	26.09	5.02	100.05	14
Nabisco Group Holdings Corp.	17.78	3.97	37.07	13
Niagara Mohawk Power Corp.	18.63	1.04	31.02	59
Northrop Grumman Corp.	19.89	9.71	30.03	12
Paine Webber Group Inc.	5.82	0.98	20.24	197
Penney J C Co. Inc.	19.08	2.98	100.09	43
Philip Morris Companies Inc.	8.10	1.01	30.04	51
Seagram Co. Ltd.	18.43	7.04	30.04	5
Sears Roebuck Acceptance Corp.	6.32	0.93	40.05	293
Service Corp. International	12.49	4.99	25.05	17
Union Pacific Corp.	13.69	2.05	30.06	36
Viacom Inc.	15.87	4.73	50.05	13
Wal-Mart Stores Inc.	12.11	1.08	30.06	33
All issues	15.25	3.06	45.53	4,523
Industrials	16.74	3.76	49.33	22
Financial	7.42	1.51	33.38	6
Utilities	18.80	1.53	42.85	4
All firms	15.25	3.06	45.53	32

3.2.6.4 Constant Elasticity of Variance

The firm's asset value is assumed to transition in continuous time by

$$dV(t) = (r-\delta_v)V(t)dt + \bar{\sigma}_v V(t)^{\rho} dW(v,t)^{Q}, \tag{3.37}$$

where r is the risk-free rate, δ_v is the firm asset payout rate, and $\bar{\sigma}_v$ is a volatility scalar. The instantaneous variance of the firm is $\bar{\sigma}_v{}^2 V(t)^{2\rho}$, and the variance of the firm's return is $\bar{\sigma}_v{}^2 V(t)^{2(\rho-1)}$.

The default boundary is assumed constant. Let the latent state variable be $X(t) = (V(t)-K)/K$. Default occurs on the first passage of $X(t)$ to zero. For comparison with the other structural models, the equivalent log-solvency ratio of $x(t) = \ln(V(t)/K)$, is recovered from $X(t)$, by the relationship $x(t) = \ln(X(t)+1)$. From Ito's lemma and equation (3.37), it follows that $x(t)$ is also a CEV process

$$dX(t) = (r-\delta_v)X(t)dt + \bar{\sigma}_v X(t)^{\rho} dW(v,t)^{Q}. \tag{3.38}$$

When ρ approaches one, the CEV model approaches the single-factor LS1 model, however the two models are not strictly nested because, as discussed further below, ρ is restricted to be less than one.

The CEV model has the convenient property that a closed-form solution for the crossing-time to a zero-value boundary is known when $\rho < 1$. Cox (1975) shows that

$$\begin{aligned} Q(t,T) &= \frac{\Gamma(\nu_Q, H_Q)}{\Gamma(\nu_Q)} \\ \nu_Q &= \frac{1}{2-\beta} \\ H_Q &= kX(t)^{(2-\beta)} \exp\big(\mu(t)(2-\beta)(T-t)\big) \\ k &= \frac{2\mu(t)}{\sigma_v^2(2-\beta)\big(\exp(\mu(t)(2-\beta)(T-t))-1\big)} \end{aligned} \tag{3.39}$$

where $\beta = 2\rho$, $\mu(t) = (r-\delta_v)$, ν_Q is a shape parameter and H_Q is the evaluation point in the standard complementary gamma function. A necessary restriction is that $\rho < 1$ in order for the boundary to be an absorbing state and therefore for the closed-form crossing-time solution to be equivalent to the cumulative default probability. This is because, when $\rho < 1$, the local volatility of the proportional change in solvency becomes infinite as the firm approaches insolvency. Consequently, the CEV model is not strictly nested within the LS model.

The time-t value of a one-dollar face value, default-risky, zero-coupon bond is given by

$$p(t,T) = e^{(-r(t,T)(T-t))}\big(1-\omega Q(t,T)\big), \tag{3.40}$$

where the risk-free rate is allowed to vary deterministically with time and is maturity matched with the timing of the promised payment, $Q(t,T)$ is the cumulative risk-neutral probability of default at any time between times t and T, and ω is the exogenously determined writedown rate.

The valuation of a coupon bond follows the method used for the LS1 model, and is solved for by the same sum of zeros approach, as shown in equation (3.29). The CEV and LS1 models share common valuation assumptions conditional on the firm's level of solvency, but the potential paths are different. The CEV model's firm volatility increases as solvency reduces, whereas firm asset return volatility is independent of solvency level in the LS1 model.

The elements of the transition equation are found by firstly expressing the continuous state process shown in equation (3.38) into discrete time using the Eueler approximation and time dependency on the risk-free rate

$$\begin{aligned} X(t) = X(t-1) + (r(t-1) - \delta_v)\Delta t \\ + \bar{\sigma}_v X(t-1)^{\rho} \sqrt{\Delta t}\eta(t), \quad \eta(t) \sim N(0,q(t)). \end{aligned} \tag{3.41}$$

It follows that the transition equation for the CEV model is the generic transition equation (equation (3.12)) with the elements:

$$\begin{aligned} &\alpha(t) = X(t), && \bar{c}(t) = 0, \\ &\bar{T} = 1 + (r(t-1) - \delta_v)\Delta t, && R = \bar{\sigma}_v X(t-1)^{\rho} \sqrt{\Delta t}. \end{aligned} \tag{3.42}$$

The initial value of the state variable, for each firm, is obtained from the observed market capitalisation and book debt by

$$X(0) = \left(\frac{V(0) - K}{K}\right) \equiv \exp(x(0)) - 1, \tag{3.43}$$

where, $x(0)$ is the initial sample value of the firm's observed log-solvency ratio, shown in Table 3.10.

No constraint is placed on the path that X(t) may take other than zero being an absorbing boundary. Due to the discretisation, it is possible that X(t) may be projected below zero during the filtering procedure. Any prediction of X(t) below zero in the EKF is trimmed to zero.

To initialise the variance of the state vector, the volatility scalar is first estimated by equation (3.44) using the observed sample asset return volatility, $\sigma_v(0)$, and an assumed initial elasticity parameter, $\rho(0)$, taken from a prior result by Albanese & Chen (2005). They find that equity default swap prices can be explained, on average, by a CEV equity diffusion model with ρ equal to -0.65. Since our state variable is defined as the scaled net worth of the firm, it is reasonable to consider a similar result may hold in our sample.

The initial estimate of the volatility scalar is then given by

$$\bar{\sigma}_v(0) = \sigma_v(0)X(0)^{(1-\rho(0))} = \sigma_v(0)X(0)^{1.65}. \tag{3.44}$$

Having estimated $\bar{\sigma}_v(0)$, the initial diffuse variance of firm solvency is calculated as

$$P(0) = \bar{\sigma}_v(0)^2 X(0)^{2\rho(0)}(t(1) - t(0)) \cdot 1000, \tag{3.45}$$

where $(t(1) - t(0))$ is the length of the first observation time interval.

There are three hyperparameter sets conditional on the treatment of the non-credit component of the credit spread. Suppressing dependence on the firm for notational clarity, we have:

$$\begin{aligned}
&\text{No liquidity:}\\
&\psi^h(1) = \{\ln \bar{\sigma}_v^2, \ln \sigma_m^2, \ln(\delta_v/(1-\delta_v)), \ln(1-\rho)\},\\
&\text{Constant liquidity:}\\
&\psi^h(2) = \{\ln \bar{\sigma}_v^2, \ln \sigma_m^2, \ln(\delta_v/(1-\delta_v)), \ln(1-\rho), \ln d_i\},\\
&\text{Time-varying liquidity:}\\
&\psi^h(3) = \{\ln \bar{\sigma}_v^2, \ln \sigma_m^2, \ln(\delta_v/(1-\delta_v)), \ln(1-\rho), \ln \beta^R, \ln d_i\}.
\end{aligned} \tag{3.46}$$

As shown in equation (3.46), the volatility scalar and measurement errors are transformed to be non-zero, and the elasticity parameter is transformed to constrain the optimal estimate to be less than one. The writedown rate is assumed to be exogenously known, applying the same industry-specific values as applied to the LS1 and LS2 models, as shown in Table 3.10.

3.2.6.5 Collin-Dufresne and Goldstein

In the CDG model, the dynamic term structure of credit spreads is assumed to be a function of a bivariate stochastic differential process for the mean-reverting capital structure of the firm and the risk-free risk-free rate

$$\begin{aligned}
dx(t) &= \kappa_v\left[\left(\frac{r(t) - \delta - \sigma_v^2/2}{\kappa_v} + \nu + \phi(r(t) - \theta)\right) - x(t)\right]dt + \sigma_v dW_{v,t}^Q\\
dr(t) &= \kappa_r(\theta - r(t))dt + \sigma_r dW_{r,t}^Q.
\end{aligned}$$

As per the implementation of the LS2 model, we follow the two-step estimation method of Duffee (1999) and fit the risk-free rate process separately from the log-solvency process, thereby ensuring that the same risk-free rate model parameters are applied equally to each firm. The risk-free rate is then assumed to be exogenously known and the log-solvency process implied from the observed credit spreads using the optimal estimates of the risk-free rate, fitted from a prior filtration. Consequently, the state-space framework simplifies to be univariate and the transition equation is the generic transition equation (equation (3.12)) with the elements:

$$\begin{aligned} \alpha(t) &= x(t), & \bar{c}(t) &= \kappa_v \bar{x}(t)\Delta t, \\ \bar{T} &= 1 - \kappa_v \Delta t, & R(t) &= \sigma_v \sqrt{\Delta t}, \end{aligned} \tag{3.47}$$

where $\bar{x}(t)$ is the risk-neutral target log-solvency ratio

$$\bar{x}(t) = \left(\frac{r(t) - \delta - \sigma_v^2/2}{\kappa_v} + \nu + \phi(r(t) - \theta) \right). \tag{3.48}$$

The first-passage crossing time of the log-solvency state variable must be numerically solved due to the CDG model having two stochastic processes. The numeric grid method of CDG is used, details of which are described in Appendix B.1. The time involved in searching for an implicit solution for the CDG model is substantial with processing times, for a single firm solution, varying from 5 hours for the no-liquidity case, to 24 hours for the time-varying liquidity case.[6] An important numerical simplification made is that we value only the principal cash flows and not the coupons of each bond. The simplification proved to be reasonable because of the panel nature of the data, there are sufficient bonds to span the term structure for each firm. Further, we seek to minimise the prediction error of the credit spread, valuing both the risky and risk-free values of the bond using the same set of promised cash flows. Thus, the simplification is present in the value of both sides of the credit spread calculation.[7]

To aid estimation, further simplifying assumptions can reasonably be made. Firstly, the asset payout rate is assumed to be zero following the finding of EHH, that the effect of the payout ratio on debt pricing in the CDG model is exactly cancelled by the inclusion of a target debt-ratio. The amount of payout in interest, dividends, capital raisings and share repurchases are subsumed into management's choice of the speed of adjustment toward a target debt-ratio and level of the target.

Secondly, the asset-interest rate correlation is assumed to be zero. EHH report a correlation of -2 percent and find that CDG model's credit spread prediction errors vary little with respect to correlation. Table 3.14 shows the correlation between daily changes in the 3-month CMT rate and daily changes in the observed log-solvency ratio to be 0.38 percent.

Finally we consider the problem of identifying the solvency-ratio dynamic param-

[6]In comparison, the LS1 model times range from 4 minutes to 38 minutes. All computations were performed in OX software using a desktop PC running Windows XP on a Pentium 4, 2.53 GHz processor.

[7]Testing showed no material difference in the estimates of the LS1 model when shifting from valuing all cash flows to just the principal cash flows. We can draw further comfort that no significant errors have been introduced by referring to the results shown in Table 4.7. The standardised step-ahead prediction errors are of comparable magnitude between the analytically calculated LS1 model with no cash flow simplification, and the numerically solved LS2 model with only the principal amounts valued.

eters, ν, ϕ, and κ_v. The extant empirical estimation method is exemplified by EHH and Suo & Wang (2006). It involves estimating the parameters of the CDG asset process from a regression of observed firm-specific leverage ratios and interest rates. In the remainder of this section we explore this regression method as a potential means of identifying initial parameter estimates.

The regression equation for the CDG model is obtained by first expressing the log-leverage process under the physical measure with a constant firm asset growth of μ

$$dx(t) = \kappa_v \left[\left(\frac{\mu - \delta - \sigma_v^2/2}{\kappa_v} + \nu + \phi(r(t) - \theta) \right) - x(t) \right] dt + \sigma_v dW_{v,t}^P. \tag{3.49}$$

Let $\alpha_x = \mu - \delta - \sigma_v^2/2 + \kappa_v(\nu - \phi\theta)$ then,

$$dx(t) = (\alpha_x + \kappa_v \phi r(t) - \kappa_v x(t))dt + \sigma_v dW_{v,t}^P. \tag{3.50}$$

Equation (3.50) is then discretised by the Euler approximation

$$x(t) - x(t-1) = (\alpha_x + \kappa_v \phi r(t-1) - \kappa_v x(t-1))\Delta t + \sigma_v \sqrt{\Delta t}\eta(t), \tag{3.51}$$

where $\eta(t) \sim N(0,1)$. For estimation purposes, equation (3.51) is then expressed as a linear equation on the risk-free rate and the lagged observed log solvency ratio, $S(t)$, with a normal i.i.d. error term $\varepsilon(t)$

$$\frac{S(t) - S(t-1)}{\Delta t} = a + bS(t-1) + cr(t-1) + \varepsilon(t) \tag{3.52}$$

where $a \equiv \alpha_x$, $b \equiv -\kappa_v$, and $c \equiv \kappa_v \phi$. Parameter estimates are then obtained from the slope coefficients: $\hat{\kappa}_v = -b$, and $\hat{\phi} = -c/b$. To find an estimate of ν from the regression method it is also necessary to know the firm's expected asset return. Let

$$\hat{\nu} = \hat{\phi}\,\theta + \frac{a - \mu + \delta_v + \sigma_v^2/2}{\hat{\kappa}_v}. \tag{3.53}$$

Then, μ can be further expressed as the sum of the risk-free rate and a market risk premium on the firm's assets.

EHH regress 10 years of monthly data and find an average mean-reversion rate of 0.1. The expected asset return, μ, is obtained from the monthly 10 year historical firm value return and on average is 24 percent. They recognise that this is an ex-post measure not necessarily the market's required return and find that their model prediction errors are sensitive to the choice of μ. EHH suggest that the high absolute spread prediction errors they find with the CDG model may be a result of their estimation method. Alternatively, they suggest fitting an implied level of ν from credit spreads. Unfortunately, no regression statistics or sample estimate of ν is reported to confirm the significance of their result. Suo & Wang (2006) find a mean reversion rate close to zero, and consequently, their implementation of the CDG model is little different to the LS model.

As an alternative to regression, HH simply assume the long-run risk-neutral log-solvency level, $\bar{x}$, to be 0.38, which gives an estimate for v of 0.55 based on other assumed parameters including an asset risk premium.[8] CDG similarly assume v to be between 0.5 and 0.6 which equates to a target debt-ratio of similar magnitudes. The target debt level chosen by CDG for illustrative purposes appears very conservative and may result in an over-estimation of long-term credit spreads. Opler & Titman (1994) report similar debt levels for firms in the top 20 percent of population gearing levels. For other firms in normal industry conditions the debt-ratio is 0.193 giving a log-solvency level of 1.65. More recently, using the same data sources and time period as our study, Davydenko & Strebulaev (2004) report a higher mean debt-ratio of 0.322, which is equivalent to mean log-solvency ratio of 1.13.

The result of applying equation (3.52) to our sample of firms is shown in Table 3.13. Data is observed quarterly, over the sample period, to match the release of COMPUSTAT balance sheet debt figures. The rate of mean-reversion is found, on average, to be 0.79 per quarter across all firms, with estimates ranging from -0.54 to 3.56, with a median of 0.70. In most cases the estimate of mean reversion is not significantly different from zero. The average mean-reversion rate is much higher than the capital structure empirical literature has found for the debt-ratio dynamics in recent years. For example, Roberts (2002, Table 3) reports 0.16 using a similar definition of leverage. In comparison, CDG and HH assume the average firm mean-reversion rate to be 0.18.

Table 3.13 also shows that the observed sensitivity of the firm's log-solvency to the slope of the yield curve is negative as often as it is positive. This observation contradicts the debt market timing hypothesis of CDG, which assumes that changes in the firm's solvency are positively related to the slope of the risk-free yield curve slope. However, in no case is the estimated parameter, $\hat{\phi}$, significant at the 5 percent level. The across-firm mean value of $\hat{\phi}$ is -12.67 with a very high standard deviation of 60.8 and a median of 0.64. EHH do not report their regression estimate. HH adopted an implicit assumption that ϕ is zero by omitting the parameter as evident in their specification of the CDG model (Huang & Huang 2003, Appendix A, equations (23) and (24)). For illustrative purposes CDG assume ϕ to be 2.8. We initialise ϕ at the firm-wide median regression estimate of 0.64 and restrict its value to be positive consistent with the theoretical restriction in the CDG model. However, given these results, it is not expected that ϕ will be significant in the filtered model estimates.

Faced with limited results from the regression model we have little choice but to consider alternatives to the standard empirical estimation methods for the other asset pa-

[8] Estimate is based on HH assumptions of: r=8 percent, $\delta_v = 6$ percent, $\kappa_v = 0.18$, $\sigma_v = 26.5$ percent, and an asset risk premium of 4.9 percent.

Table 3.12: This table shows the average target log-solvency levels assumed in the CDG model implementations. Data is sourced from Roberts (2002) and is matched to firms by second level SIC. The target log-solvency level is denoted ν and the initial starting value of log-solvency is denoted $x(0)$ and is the first observed log-solvency ratio in the sample period. Observed log-solvency is the sum of total book debt (COMPUSTAT items 45 and 51) and market equity capital (from CRSP), divided by total book debt.

Sector	No. of Firms	ν	$x(0)$
Industrial	21	1.42	1.12
Finance	6	0.46	0.48
Utility	4	0.19	0.90
Total	31	1.36	0.94

rameters. Two alternatives were considered. The first uses the property that the LS model is nested in the CDG model. The mean filtered estimate of $x(t)$ from the LS2 model may be used as a reliable estimate of the average level of latent log-solvency, which we know will match the CDG model, as mean-reversion rate approaches zero. The advantage of this method is that the target log-solvency level is treated as a latent firm-specific variable. The disadvantage is that it is a measure of the sample firm-specific mean only, and not necessarily a target that is pursued by management. Only over a longer sample period, and across more firms, can we abstract sufficiently from idiosyncratic factors. The second method, which we adopt, uses an estimate of ν under the mild assumption that debt timing is not material; an assumption well supported by our earlier regression of the observed changes in log-solvency as shown in Table 3.13. From equation (3.54) it can be seen that ν has a directly observable physical interpretation. Ignoring debt-timing behaviour for simplicity, CDG posits that management changes the level of debt so that the log-solvency ratio mean-reverts to ν where ν is a target level of log-solvency. To see this consider the dynamics of the default boundary in the absence of debt timing behaviour. The natural log of the default boundary then follows the process

$$d\ln K(t) = \kappa_\nu(x(t) - \nu)dt + \sigma dW_t^P. \tag{3.54}$$

To use the sample specific mean level of $x(t)$ from the LS model would, for example, underestimate the ex-ante target level of solvency of a firm that ex-post suffered a deterioration in value. Rather, an industry-level observed log-solvency level is a better proxy for management's desired target. The industry mean is preferred over the firm-specific sample averages since it is less influenced by idiosyncratic shocks from target, and when averaged over many firms and time, is more likely to represent a long-run equilibrium level. The use of industry is a natural conditioning variable since Opler & Titman (1994) find that target debt-ratio levels vary systematically with industry-specific business conditions. Each firm's physical target level is set equal to the industry-specific 1980 to 1998 average of the observed log-solvency ratio as reported by Roberts (2002, Table

1). His study of debt-ratio dynamics is useful for our purposes because it draws on a similar sample of firms over the same time period. Firms are mapped to industries by their second level SIC codes. The rate of mean-reversion is then included in the hyperparameter set using the Roberts's (2002) industry-wide average to initialise, and fixing ν to the industry average observed log-solvency ratio. Attempts at searching implicitly for both parameters was found to be infeasible due to multicollinearity. Our sample average value of ν is 1.36 implying a mean target debt-ratio of 0.27. In Table 3.12 the variation by industry sector is shown along with the initial values of log-solvency. For industrial firms, our initial parameterisation suggests a decrease in leverage over time, little change for finance companies, and an increase over time for utilities.

Conditional on the treatment of the non-credit component of the credit spread, the three hyperparameter sets are as follows after suppressing dependence on the firm for notational clarity:

$$\begin{aligned}
&\text{No liquidity:}\\
&\psi^h(1) = \{\ln\sigma_v^2, \ln\sigma_m^2, \ln\kappa_v, \phi\},\\
&\text{Constant liquidity:}\\
&\psi^h(2) = \{\ln\sigma_v^2, \ln\sigma_m^2, \ln\kappa_v, \phi, \ln d_i\},\\
&\text{Time-varying liquidity:}\\
&\psi^h(3) = \{\ln\sigma_v^2, \ln\sigma_m^2, \ln\kappa_v, \phi, \ln\beta_R, \ln d_i\}.
\end{aligned} \tag{3.55}$$

Further parameters are: $\delta_v = 0$, $\rho_{v,r} = 0$.

3.2.7 An Example of the Method Applied to Northrop

We now provide an example of the outcomes from applying the estimation method.

An example of the observed, and step-ahead predicted, credit spreads is plotted for Northrop in Figure 3.4 using predictions obtained from the EM model fitted without liquidity premiums. The predicted credit spread is the optimal prediction based upon the Taylor series projection of the latent log-solvency ratio, given all information about the underlying state process up to that point in time including its variance. Our starting value $x(0)$ was initially the observed log-solvency ratio, but as new predictive error information is added, the forecast for the latent log-solvency ratio is updated incorporating the direction of the past predictive error and the degree of confidence that the new observation contains information in excess of the expected level of variation in the state variable. As new information is received from the subsequent observation, expectations are updated for the next step-ahead prediction. The EKF procedure therefore reflects the behaviour of market participants who rationally update their knowledge of the credit worthiness of the firm using all available trade data across the term structure of credit spreads. The path followed by the latent log-solvency ratio process is shown in Fig-

Table 3.13: This table shows the results of estimating the CDG firm process by ordinary least squares regression of $\frac{\Delta S(t)}{\Delta t} = a + bS(t-1) + cr(t-1) + \varepsilon(t)$, where a, b, and c are constants, Δt is 0.25 years, $S(t-1)$ is the observed log-solvency lagged one-quarter, and $r(t-1)$ is the 3 month CMT rate lagged one-quarter. Standard errors are shown in parenthesis. The estimate of the log-solvency mean-reversion rate is given by $\hat{\kappa}_v = -b$, and the estimate of the sensitivity of changes in log-solvency with the term structure slope is $\hat{\phi} = -c/b$. Data is sourced from CRSP and COMPUSTAT for the period from 1994:Q2 to 2000:Q4. Observed log-solvency is the sum of total book debt (COMPUSTAT items 45 and 51) and market equity capital divided by total book debt.

Issuer	a		b		c		$\hat{\kappa}$	$\hat{\phi}$
Aetna Inc	-0.60	(1.93)	-0.87	(0.51)	37.76	(31.85)	0.87	43.31
Associates Corp	0.08	(0.79)	-1.17	(0.61)	7.01	(15.43)	1.17	6.02
Atlantic Richfield Co	0.72	(0.93)	-0.51	(0.38)	1.04	(16.24)	0.51	2.05
A T & T Corp	-0.20	(0.38)	0.54	(0.56)	2.95	(7.96)	-0.54	-5.46
Bear Stearns Comp Inc	0.17	(0.09)	-3.56	(0.81)	-0.15	(1.26)	3.56	-0.04
Black & Decker Corp	-0.07	(1.19)	-0.70	(0.38)	17.66	(22.19)	0.70	25.22
Boeing Co	0.31	(1.18)	-0.68	(0.41)	22.23	(20.08)	0.68	32.63
Dayton Hudson Corp	2.18	(1.07)	-0.21	(0.23)	-35.06	(20.29)	0.21	-163.63
Comm Edison Co	-0.04	(0.52)	-0.72	(0.50)	9.09	(8.36)	0.72	12.58
Enron Corp	0.04	(1.20)	-0.70	(0.67)	19.25	(23.81)	0.70	27.47
Fed Dept Stores	0.67	(1.12)	-0.59	(0.43)	-3.59	(18.45)	0.59	-6.13
Ford Mtr Co	0.21	(0.39)	-0.72	(0.46)	-0.30	(6.68)	0.72	-0.42
General Mtrs	0.54	(0.55)	-0.92	(0.63)	-4.83	(7.79)	0.92	-5.25
Georgia Pacific Corp	1.59	(0.99)	-0.68	(0.67)	-21.49	(14.61)	0.68	-31.47
Columbia / Hca Corp	0.12	(1.11)	-0.83	(0.48)	18.99	(17.02)	0.83	22.77
IBM Corp	1.72	(0.90)	-0.32	(0.23)	-20.86	(17.82)	0.32	-64.65
Int Paper Co	3.09	(1.28)	-1.84	(0.65)	-29.38	(18.73)	1.84	-15.98
Lehman Bros Hldgs Inc	-0.23	(0.12)	-0.14	(0.77)	4.65	(2.25)	0.14	33.76
Merrill Lynch & Co	-0.05	(0.15)	-0.12	(0.40)	1.56	(3.11)	0.12	13.25
Motorola Inc	5.97	(2.32)	-1.20	(0.70)	-63.14	(34.34)	1.20	-52.76
Nabisco Grp Hldgs Corp	-0.43	(1.04)	-1.32	(0.82)	26.17	(22.42)	1.32	19.85
Niagara Mohawk Corp	0.38	(0.42)	-1.33	(0.48)	1.77	(6.53)	1.33	1.33
Northrop Grumman Corp	-0.76	(1.26)	-1.41	(0.65)	43.61	(27.08)	1.41	31.03
Paine Webber Grp Inc	-0.18	(0.17)	0.17	(0.41)	3.68	(3.33)	-0.17	-21.88
Penney J C Co Inc	0.42	(1.27)	-0.44	(0.46)	-2.80	(19.64)	0.44	-6.36
Philip Morris Comp Inc	1.17	(1.38)	-0.83	(0.43)	6.42	(22.34)	0.83	7.75
Seagram Co Ltd	0.35	(1.59)	-1.72	(0.67)	38.05	(25.28)	1.72	22.13
Sears Roebuck Acc Corp	0.65	(0.78)	-1.34	(0.65)	3.87	(11.17)	1.34	2.88
Service Corp Intl	-2.23	(1.48)	0.14	(0.33)	37.62	(25.92)	-0.14	-268.43
Union Pacific Corp	0.45	(0.91)	-0.50	(0.38)	-0.26	(14.43)	0.50	-0.52
Viacom Inc	1.60	(1.64)	-0.32	(0.33)	-19.40	(33.01)	0.32	-61.24
Wal-Mart Stores Inc	1.12	(1.85)	-0.45	(0.38)	-2.33	(29.98)	0.45	-5.23
Mean	0.59		-0.79		3.12		0.79	-12.67
SD	1.37		0.74		22.53		0.74	60.80
Median	0.33		-0.70		2.36		0.70	0.64
Min	-2.23		-3.56		-63.14		-0.54	-268.43
Max	5.97		0.54		43.61		3.56	43.31

Table 3.14: Shown is the firm-specific within sample correlation coefficient between the first differences in the observed log-solvency ratio, S(t), and the first differences in the 3-month constant maturity Treasury risk-free rate. Observed log-solvency is the sum of the quarterly total book debt (COMPUSTAT items 45 and 51) and daily market equity capital (from CRSP) divided by total book debt.

Issuer	Correlation Coefficient
Aetna Inc.	0.0021
Associates Corp.	-0.0294
Atlantic Richfield Co.	-0.0179
A T & T Corp.	-0.0009
Bear Stearns Companies Inc.	0.0406
Black & Decker Corp.	-0.0180
Boeing Co.	0.0190
Dayton Hudson Corp.	0.0673
Commonwealth Edison Co.	-0.0103
Enron Corp.	-0.0225
Federated Department Stores	0.0508
Ford Motor Co.	0.0068
General Motors	-0.0008
Georgia Pacific Corp.	-0.0229
HCA Healthcare Corp.	0.0342
IBM Corp.	0.0184
International Paper Co.	-0.0063
Lehman Brothers Holdings Inc.	0.0068
Merrill Lynch & Co.	-0.0122
Motorola Inc.	0.0775
Nabisco Group Holdings Corp.	0.0671
Niagara Mohawk Power Corp.	-0.0519
Northrop Grumman Corp.	0.0047
Paine Webber Group Inc.	0.0150
Penney J C Co. Inc.	-0.0056
Philip Morris Companies Inc.	0.0039
Seagram Co. Ltd.	0.0101
Sears Roebuck Acceptance Corp.	-0.0006
Service Corp. International	0.0166
Union Pacific Corp.	-0.0029
Viacom Inc.	-0.1143
Wal-Mart Stores Inc.	-0.0020
Mean	0.0038
SD	0.0359
Median	0.0008
Min	-0.1143
Max	0.0775

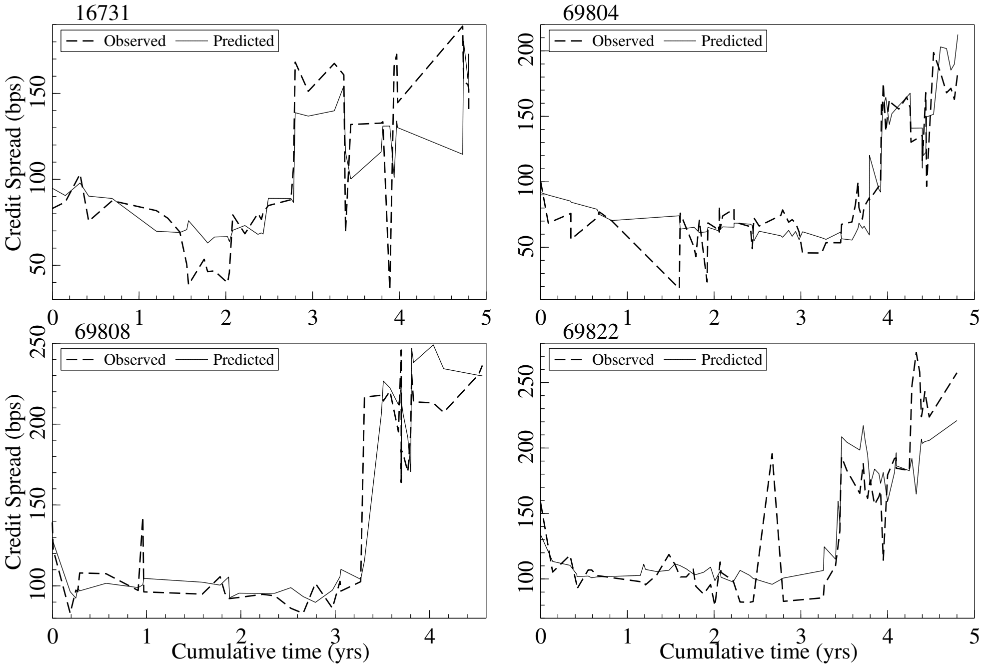


Figure 3.4: Predicted and observed credit spreads for Northrop Grumman Corporation. The model implemented is the EM model with time-varying non-default components of the credit spread. Each panel shows the path of predicted and observed credit spreads by bond on issue.

ure 3.5. We compare this with the observed log-solvency ratio. Evident is the decline in both ratios midway through the sample period. However, while the observed solvency ratio corrected towards the end of the sample period, the latent solvency ratio continued to decline suggesting the market was pricing in more default risk than the observed capital structure changes of the firm would have predicted.

3.3 Fitting the Vasicek Risk-Free Model

In this section we describe the method used to estimate the Vasicek model using a linear Kalman filter. Our method follows James & Webber (2001) and Babbs & Nowman (1999).

Under the physical measure of actuarial probability densities, the risk-free rate is assumed to follow the stochastic differential process

$$dr(t) = \alpha_r(\theta - r(t))dt + \sigma_r dW_{r,t}^P. \tag{3.56}$$

The equivalent martingale process under risk-neutrality is

$$dr(t) = (\alpha_r(\theta - r(t)) - \lambda_r\sigma_r)dt + \sigma_r dW_{r,t}^Q, \tag{3.57}$$

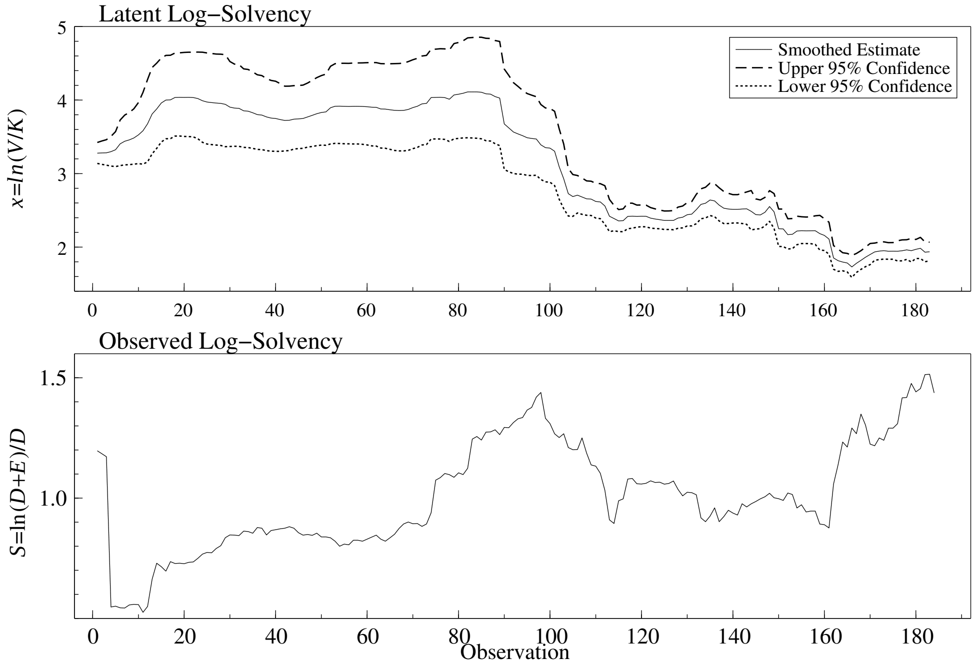


Figure 3.5: This shows the estimated latent versus observed ratios of solvency. for Northrop Grumman Corporation. The top panel shows the smoothed estimate of the latent log-solvency ratio $x = \ln(V/K)$, implied from the EM model, assuming time-varying non-default components of the credit spread. The lower panel shows the observed log-solvency ratio, $S = \ln(D+E)/D$, where D is the book value of debt and E is the market value of equity.

Table 3.15: This table shows the summary statistics of the zero-coupon Treasury yields. Data is bootstrapped CMT Treasury yields, extracted on each Wednesday, from the U.S. Federal Reserve H15 data series for the period 3 January 1994 to 29 December 2000.

Maturity (yrs)	Mean	Std Dev	Min	Max	Skewness	Kurtosis
0.25	0.0516	0.00611	0.0301	0.0636	- 0.797	1.585
0.5	0.0526	0.00609	0.0317	0.0648	- 0.630	1.144
1	0.0545	0.00624	0.0347	0.0715	- 0.303	0.969
2	0.0570	0.00664	0.0397	0.0748	- 0.106	0.559
3	0.0585	0.00691	0.0402	0.0759	- 0.084	0.325
5	0.0604	0.00715	0.0414	0.0770	- 0.042	0.164
7	0.0615	0.00719	0.0426	0.0781	0.022	0.052
10	0.0625	0.00714	0.0446	0.0791	0.114	- 0.129
20	0.0641	0.00703	0.0492	0.0808	0.263	- 0.549
30	0.0648	0.00710	0.0511	0.0816	0.305	- 0.718

where λ_r is a constant market price of interest rate risk, $\sigma_r > 0$ is the volatility of the interest rate, $\theta \geq 0$ is the long-run risk-free rate level, and $\alpha_r > 0$ is the speed of mean-reversion towards θ. The market price of risk represents the equilibrium compensation required by risk-averse investors to hold interest rate risk.

Term structure data is sourced from the Federal Reserve Bank of New York Constant Maturity Treasury (CMT) H15 data series.[9] These yields are interpolated by the U.S. Treasury from the daily yield curve based on the closing market bid yields on actively traded Treasury securities in the over-the-counter market. We select weekly data (Wednesdays) to avoid weekend effects and observed 10 yields with maturities of: 3, 6 months, and; 1, 2, 3, 5, 7, 10, 20, and 30 years. The sample period is 3 January 1994 to 29 December 2000 giving 361 observed yield curves. The 3 and 6 month rates represent zero-coupon bond yields, and the longer rates are expressed in terms of semi-annual coupon-paying Treasury bonds. The latter are bootstrapped and then added to the risk-free rates to produce an equivalent riskless term structure of spot rates. Summary statistics are reported in Table 3.15. The term structure is plotted in the first panel of Figure 3.6 and the term-spread, defined as the difference between the 30 year yield and the 3 month yield, is plotted in the second panel. From the second panel, and table 3.15, it can be seen that the term structure has been very flat, with the term-spread gradually decreasing before turning negative. The term-spread compensating investors for 30 year risk has averaged only 132 bp for the sample period.

In the estimation of the credit risk model parameters, it is feasible within the state space framework to estimate the parameters of the risk-free model and the credit model jointly. This can be achieved by expanding the measurement equation to include the observed credit spread and risk-free rate term structures.

[9] CMT yields are sourced from the Federal Reserve and are available at http://www.federalreserve.gov/releases/h15/data.htm

However, as noted by Duffee (1999), this unfortunately leads to the problem of having as many different estimates of the risk-free rate as there are firms in the sample. We therefore follow Duffee (1999) and Duffie et al. (2000) and estimate the risk-free rate independently from the credit model. Having obtained the underlying smoothed estimates of the unobserved risk-free rate, $r(t)$, and the optimal interest rate parameter set $\hat{\psi}_r$, we treat the risk-free rate process as known and true in the filter of the credit process. All discounting is performed using the Vasicek model, discounting to time-t, cash flows promised after time-t. As per EHH, we also relax the theoretical constraint of the constant interest rate models that the risk-free rate is time-invariant, and allow the discounting to be dependent on the predicted term structure at time-t, and let $r(t)$ be updated at each trade date, however, the other model parameters are held constant. The same risk-free rates are used in all models, and consistent Vasicek model parameters are applied in the two-factor models. Since our main focus is the performance of the credit models, allowing some ad hoc updating reduces error introduced by the performance of the risk-free model and enables a more even comparison between the one and two-factor credit models.

Our estimation method follows recent convention with maximum likelihood estimation of parameters obtained from linear Kalman filtering on panel data (Pennacchi 1991). It has the advantages that: the risk-free rate and other factors are treated as truly unobservable without proxy error, all bonds in the term structure can be assumed to have measurement error, serial and cross-sectional constraints implied by the theoretical model are enforced, many more bonds than underlying factors can be fitted. The Kalman filtering method has shown to be particularly robust for term structure estimation. Duan & Simonato (1999) perform Monte Carlo simulation of the Vasicek and CIR models and find the method to be reasonably reliable for sample sizes as small as 150.

At time t, we observe the vector of CMT Treasury yields, $Y_t = y(t,\tau)$ for $\tau = (0.25, 0.5, 1, 2, 3, 5, 7, 10, 20, 30)$ where $\tau = T - t$ is the maturity of the bond. The 3 and 6 month yields are expressed as annualised Treasury bills and the longer dated yields represent par coupon paying Treasury notes and bonds. The longer dated CMT yields can be compactly expressed in terms of discount functions, as follows;

$$y(t,\tau) = 2\left[\frac{1 - D(t,\tau)}{\sum_{i=1}^{2T} D(t,i/2)}\right], \tag{3.58}$$

where $D(t,\tau)$ is the time t value of a riskless zero-coupon bond maturing at time $t+\tau$. To find the time t term structure, Treasury yields are bootstrapped beginning with the first period (6 month) discount rate,

$$D(t,0.5) = \frac{1}{1 + \frac{1}{2}y(t,0.5)}. \tag{3.59}$$

Subsequent semi-annual yields are interpolated from the yield curve using the polynomial function of Nelson & Siegel (1985) fitted on Y_t by ordinary least squares regression. We then solve recursively for the semi-annual discount rates by,

$$D(t,\tau) = \frac{1 - \frac{1}{2}y(t,\tau)\sum_{i=1}^{2\tau-1} D(t,i/2)}{1 + \frac{1}{2}y(t,\tau)}. \tag{3.60}$$

The term structure at time t is then described by the vector R_t where $R(t,\tau) = -\frac{1}{\tau}\ln D(t,\tau)$.

The Vasicek model endogenously predicts the term structure based upon a set of parameter values, $\Psi_r = (\sigma_r, \theta, \alpha, \lambda_r)$, and the stochastic path followed by r_t. Since we do not observe continuous trading it is necessary to approximate the transition process in discrete time which we do by simple Euler approximation,[10]

$$r_{t+\Delta t} = \alpha_r(\theta_r - \lambda_r\sigma_r)\Delta t + (1 - \alpha_r\Delta t)r_t + \sigma_r\sqrt{\Delta t}\,\eta_t,\ \eta_t \sim N(0,1). \tag{3.61}$$

The measurement equation is linear with respect to the transition equation, making Kalman filtering of the time series straightforward, and with Guassian errors, the Kalman filter yields an exact maximum likelihood estimation of the model parameters (Harvey 1989, page 104)

Duffie & Kan (1996) show that the Vasicek model belongs to the class of one-factor exponential affine models in which the value of a zero-coupon bond is conveniently given by the following linear function,

$$D(t,\tau) = \exp\left(A(\tau) + B(\tau)r_t\right), \tag{3.62}$$

where

$$\begin{aligned}
A(\tau) &= -R(\infty)\big(\tau + B(\tau)\big) - \frac{\sigma^2}{4\alpha_r}B(\tau)^2 \\
B(\tau) &= \frac{e^{-\alpha_r\tau} - 1}{\alpha_r} \\
R(\infty) &= \theta - \frac{\lambda_r\sigma_r}{\alpha_r} - \frac{1}{2}\left(\frac{\sigma_r}{\alpha_r}\right)^2
\end{aligned} \tag{3.63}$$

The spot rate at time t for a zero-coupon bond with maturity τ is then,

$$R(t,\tau) = -\frac{1}{\tau}[A(\tau) + B(\tau)r_t]. \tag{3.64}$$

[10] Because the state variable transition equation is linear Gaussian, an exact transition density is known (refer (Lund 1997) and (Babbs & Nowman 1999)). The exact transition equation is $dr_t = \theta(1 - \exp(-\alpha\Delta t)) + \exp(-\alpha\Delta t)r_{t-\Delta t} + \sqrt{\frac{\sigma^2}{2\alpha}(1 - \exp(-2\alpha\Delta t))}\eta_t$. Testing (unreported) provided no material difference from the Euler approximation results presented in tables 3.16 and 3.17.

At time t, the term structure of spot yields is observed with measurement error,

$$\mathbf{R}(t,\tau) = -\frac{1}{\tau}\mathbf{A}(\tau) - \frac{1}{\tau}\mathbf{B}(\tau)r_t + \varepsilon_t(\tau), \quad \varepsilon_t(\tau) \sim N(0,\mathbf{H}),$$
$$\mathbf{R}(t,\tau) = \begin{pmatrix} R(t,0.25) \\ . \\ . \\ R(t,30) \end{pmatrix}, \mathbf{A}(\tau) = \begin{pmatrix} \frac{1}{0.25}A(0.25) \\ . \\ . \\ \frac{1}{30}A(30) \end{pmatrix}, \tag{3.65}$$

$$\mathbf{B}(\tau) = \begin{pmatrix} \frac{1}{0.25}B(0.25) \\ . \\ . \\ \frac{1}{30}B(30) \end{pmatrix}, \varepsilon(t,\tau) = \begin{pmatrix} \varepsilon(t,0.25) \\ . \\ . \\ \varepsilon(t,30) \end{pmatrix}$$

We follow accepted convention and assume that the measurement errors are cross-sectionally independent so that H is a 10 x 10 diagonal matrix with elements h_i ($i = 1 \ldots 10$) that vary by maturity to account for potential differences in trading activity and associated bid-ask spread (Duffee 1999, Duan & Simonato 1999, Geyer & Pichler 1999).[11]

A further necessary assumption for the Kalman filter is that the measurement errors are serially uncorrelated, $\varepsilon(t,\tau)\varepsilon(t+i,\tau) = 0, \nabla i = 1 \ldots n-1$. We also assume homoskedasticity.

Starting values for the filtering process are estimated from fitting equation (3.61) on the 3 month rate as a proxy for the risk-free rate using a naive method suggested by James & Webber (2001, page 122). Estimates of a, b, and the variance of the prediction error, $\mathrm{Var}(e_t)$, are estimated by ordinary least squares regression of the AR(1) model,

$$R(t+\Delta t, 0.25) = a + bR(t,0.25) + e_t,$$
$$\hat{\alpha}_r = \frac{1-b}{\Delta t}, \quad \hat{\theta} = \frac{a}{1-b} + \lambda_r\sigma_r, \quad \hat{\sigma}_r = \sqrt{\frac{\mathrm{Var}(e_t)}{\Delta t}} \tag{3.66}$$

The parameters $a = 0.001390$ and $b = 0.9744$ are significant with standard errors of 0.0004075 and 0.007849 respectively. The estimate of b is indicative of the 3 month rate being close to a random walk and implies $\hat{\alpha}_r = 1.33$. The error variance is low, $\mathrm{Var}(e_t) = 8.230 \cdot 10^{-7}$ giving $\hat{\sigma}_r = 0.0065$. The long-run risk-free rate level is obtained by assuming the initial market price of risk to be zero so that, $\hat{\theta} = a/(1-b) = 0.0544$. The measurement error is usually found to be small in other studies applying Kalman filtering (for example, Duan & Simonato 1999, Babbs & Nowman 1999, De-Jong 2000),

[11] As discussed in Geyer & Pichler (1999), the alternative assumption of a single measurement error is convenient when the number of observed bonds and their maturities vary over time. We utilize this alternative specification in the estimation of the default-risky model parameters due to missing data resulting from unequal trading dates.

and the starting assumption is 10 bp for all maturities.

These estimates serve only as a starting point for maximum likelihood estimation. Limitations of the orindary least squares procedure for unbiased and consistent estimation include: lack of theoretical cross-sectional restrictions on α and θ and a potential upward bias in the estimate of α when equation (3.66) is close to having a unit root (Ball & Torous 1996); and the failure of orindary least squares regression to ensure e_t is standard normal and independent and identically distributed (James & Webber 2001, p.122).

Parameter estimates for the filtered Vasicek model are shown in Table 3.16 and the latent path followed by the smoothed estimate of the risk-free rate, during the sample period, is plotted in Figure 3.7. All parameters except λ_r and θ_r are found to be significant. The speed of mean-reversion is low at 0.02324, implying a half-life of 29.8 years, defined as the expected time for the risk-free rate to return halfway from its long-run mean. The mean reversion rate is similar to extant studies; 0.0222 in De-Jong (2000, table 2C), 0.1908 in Babbs & Nowman (1999, table 2), and 0.0463 in Duan & Simonato (1999). The Vasicek model predicts the risk-free rate to converge to θ_r, but it is evident from Table 3.15 that the sample standard deviation of weekly rates is similar across all maturities. Thus, a small $\alpha(t)$ is necessary to match the observed variance in 30 year yields. The estimated half-life for the risk-free rate of 29.8 years is much longer compared to the high degree of persistence observed in the serial data for the 3 month rate.

The insignificant value for λ_r is a consequence of the very flat, and partly inverted yield curves, observed during our sample period. The market price of risk can only be identified from bond prices measured across the yield curve, but at an average term spread of only 132 basis points for a 30 year investment period, the implied price of risk is necessarily small. Finally, λ_r and θ_r strongly interact via their influence on the risk-neutral drift of the risk-free rate, and without greater cross-sectional restriction, neither is found to be significant.

Table 3.17 shows the step-ahead prediction errors of the zero-coupon spot rates for each maturity. The prediction errors exhibit a U-shaped pattern; highest at 3 months at 75.5 basis points, decreasing to 14.3 basis points at 3 years, then increasing to 51.8 basis points at 30 years. A similar pattern and magnitude of errors is reported by (Babbs & Nowman 1999, Table 1).

Table 3.16: This table reports estimated parameters from the one-factor Vasicek term-structure model. The instantaneous risk-free rate is assumed to follow the risk-neutral process: $dr(t) = \alpha_r(\theta - r(t) - \lambda_r\sigma_r)dt + \sigma_r dW^Q_{r,t}$. Estimation method is linear Kalman filter applied to weekly panel data over the period 5 January 1994 to 27 December 2000. Data is zero-coupon yields bootstrapped from US Treasury constant maturities from U.S. Federal Reserve H15 data series. $H(r(t))$ is the half-life of the risk-free rate where $H(r(t)) = \ln(2)/\alpha_r$. The entry (*) signifies that a standard error could not be calculated.

Parameters	Estimate	Standard Error
α_r	0.0232	6.6x10^{-4}
θ	0.1605	0.1347
λ_r	0.0043	0.3885
σ_r	0.0147	9.3x10^{-5}
$\sigma_\varepsilon(0.25)$	6.0x10^{-5}	4.6x10^{-7}
$\sigma_\varepsilon(0.5)$	4.0x10^{-5}	3.0x10^{-7}
$\sigma_\varepsilon(1)$	1.6x10^{-5}	1.1x10^{-7}
$\sigma_\varepsilon(2)$	2.1x10^{-6}	1.2x10^{-8}
$\sigma_\varepsilon(3)$	0	*
$\sigma_\varepsilon(5)$	2.4x10^{-6}	1.4x10^{-8}
$\sigma_\varepsilon(7)$	5.8x10^{-6}	3.6x10^{-8}
$\sigma_\varepsilon(10)$	1.1x10^{-5}	7.1x10^{-8}
$\sigma_\varepsilon(20)$	2.1x10^{-5}	1.5x10^{-8}
$\sigma_\varepsilon(30)$	2.4x10^{-5}	1.7x10^{-7}
Log-likelihood	18,821.7	
n	361	
H(r_t) years	29.82	

Table 3.17: This table reports the weekly step-ahead yield prediction errors from fitting the one-factor Vasicek model. Errors are the predicted yields from a linear Kalman filter less the actual zero-coupon spot yields. RMSE is the root-mean-squared error and MAE is the mean-absolute error. All figures are reported in basis points. Yields are estimated from zero-coupon yields bootstrapped using US Treasury constant maturity data sourced from H15 data series, for the period 5 January 1994 to 27 December 2000.

Maturity (yrs)	Mean	RMSE	MAE
0.25	-40.48	75.34	58.13
0.5	-33.42	61.48	47.40
1	-19.99	39.58	29.94
2	-5.718	17.74	13.32
3	6.4x10^{-6}	14.30	10.38
5	1.238	23.13	16.92
7	-2.156	30.20	22.28
10	-8.392	37.68	37.68
20	-13.89	48.65	48.65
30	6.712	51.83	51.83

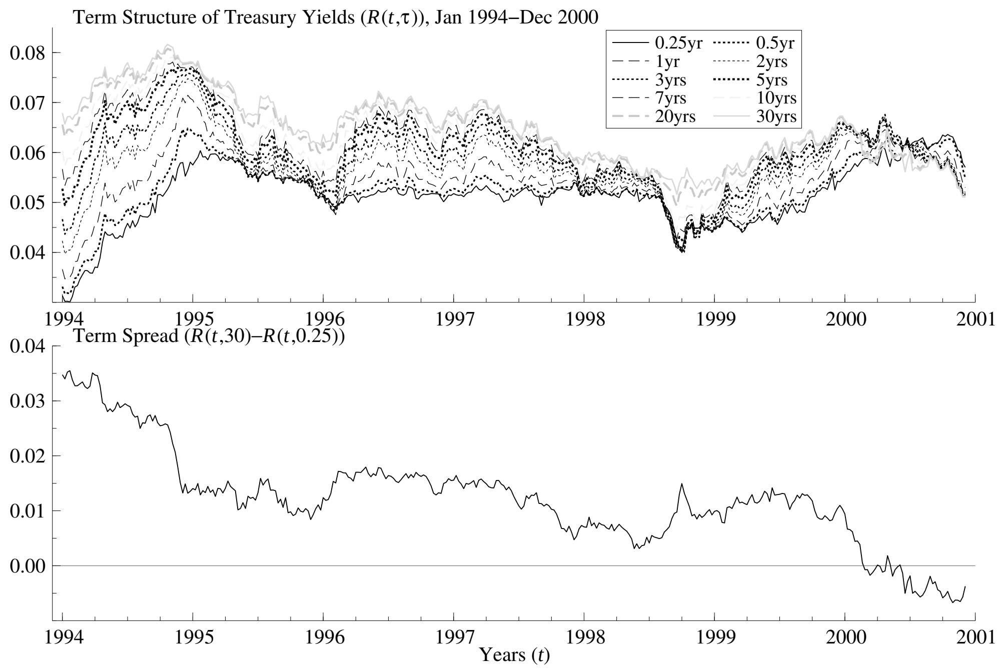


Figure 3.6: Plotted is the zero coupon risk-free term structures over time of U.S. Treasuries for the period 5 January 1994 to 27 December 2000. Source data is author's calculations derived from zero-coupon yields bootstrapped from US Treasury constant maturity rates, in turn sourced from the Federal Reserve H15 data report. Sample period is 5 January 1994 to 27 December 2000.

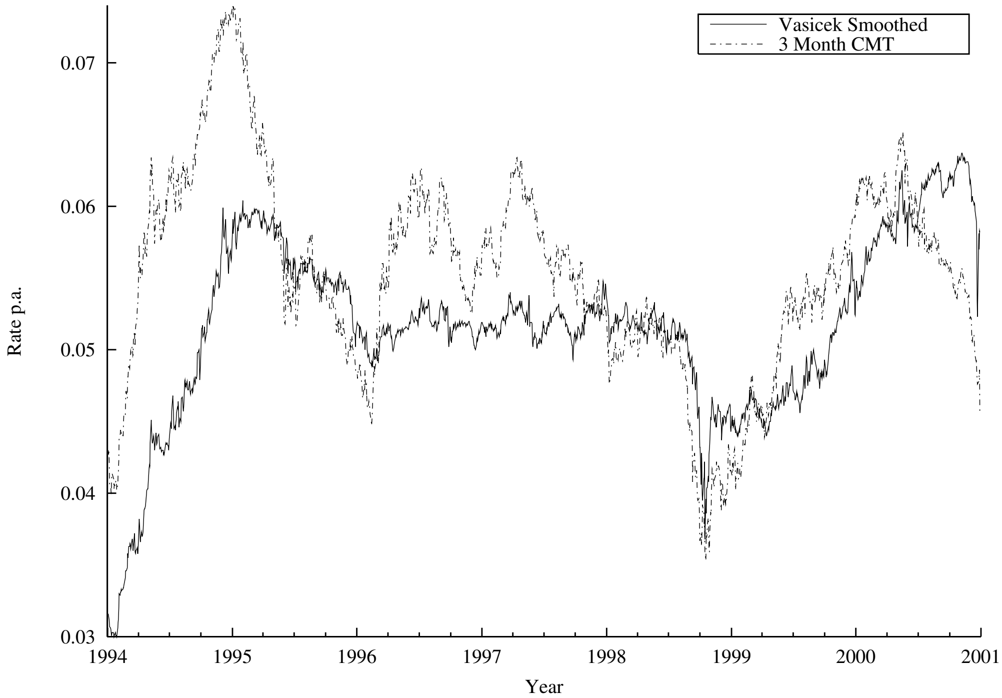


Figure 3.7: Plotted is the daily smoothed estimate of the risk-free rate (Vasicek Smoothed), computed from a linear Kalman filtering of the Vasicek model. Also plotted is the observed constant maturity 3 month Treasury rate (3 Month CMT). Source data is author's calculations derived from zero-coupon yields bootstrapped from US Treasury constant maturity rates, in turn sourced from the Federal Reserve H15 data report. Sample period is 5 January 1994 to 27 December 2000.

Chapter 4

Results

In this chapter we review and discuss our findings. The aims of our research is to firstly establish whether, through latent estimation of capital structure dynamics, we can reduce the high variability in prediction errors noted by EHH, thus providing a clearer picture of the systematic biases present in structural model specifications. Secondly, we test our hypothesis that the prediction errors evident in structural models are related to omitted factors identifiable from stylised facts selected from the extant capital structure literature. Finally, we examine the estimated form asset parameters for economic reasonableness using benchmarks for the firm asset volatility, the implied default boundary, and the rate of solvency mean-reversion.

In Section 4.1, the average credit spread prediction errors of the competing structural models are compared with each other, and with the prior empirical literature of LYS, EHH, and HH. To aid comparison with these studies, we report goodness of fit statistics; Mean Percentage Error (MPE) and Mean Absolute Percentage Error (MAPE). Further, to assess the relative impact of controlling for liquidity, and to provide a direct comparison with the findings of EHH, we report model performance separately for the three alternative measurement equations.

Our testing of model performance is guided firstly by EHH. Given the similarity of our study, but competing estimation method, it is important to show our results in a comparative manner. However, due to our state-space estimation method we are able to apply additional specification tests not available to EHH. In Section 4.2 we follow diagnostic methods described in Harvey (1989, p.256), Durbin & Koopman (2001, p.33), and as applied to interest rate term structure modelling by Geyer & Pichler (1999), to test the standardised residual prediction errors for consistency with the cross-sectional and time-series assumptions of the competing credit models. In other words, we test whether the credit spread prediction errors, including the standardised error as recommended by Harvey (1989), are unbiased with respect to firm and bond characteristics, are serially uncorrelated, and are normally distributed. We are not aware that similar performance test have been applied to structural credit models previously.

Lastly, because we are inferring model parameters from credit spreads, we may find that we achieve a high level of fit but with parameter values that are economically unrealistic. To confirm whether this is the case, the implied model parameters are checked against observable benchmarks in Section 4.3. Using our fitted measurement equations, in Section 4.4 we estimate the extent to which the observed level of credit spread is explained by the various structural models and estimate how much is due to non-default related liquidity premiums.

4.1 Credit Spread Prediction Accuracy

In this section, we consider the ability of the models to predict step-ahead market credit spreads is discussed controlling for: no liquidity premium (Section 4.1.1), constant liquidity premium (Section 4.1.2), and time-varying liquidity premium (Section 4.1.3). Step-ahead prediction errors are reported in Table 4.3, percentage spread step-ahead prediction errors in Table 4.4, absolute percentage prediction errors in Table 4.5, and standardised prediction errors in Table 4.6.

4.1.1 With No Liquidity Premium

In this section we report the overall credit spread prediction accuracy of the tested models, with no control for a liquidity premium, so that we can directly compare our results with the results of extant studies. Our method of implicit parameter estimation involves optimisation of model parameters to maximise model fit to observed credit spreads. It is therefore expected that the average prediction error will be close to zero. We are therefore interested in determining, whether as a consequence of our implicit estimation method, whether we find a lower level of error than the extant studies of EHH and HH.

Our results are shown in Panel A of Tables 4.3, 4.4, 4.5, and 4.6. A comparison of our results with EHH and HH is shown in Table 4.1.

In Panel A of Table 4.3 we report the unstandardised prediction error, measured in basis points, calculated simply as the predicted credit spread less the observed credit spread. Standard deviations of the prediction errors are shown in parentheses and are also reported in basis points. Pooled mean prediction errors are shown by model, external rating, and remaining maturity of the bond at the time of the trade. From the second column, it is evident that, as expected, the average prediction error for all models is small. The extended Merton model exhibits the smallest error of only -0.29 basis points, and the CEV model the most at 4.02 basis points. In comparison, LYS report an average spread prediction error for the Merton model of -61.15 basis points, and -8.78 basis points for a single-factor LS model with constant risk-free rate (equivalent in specification to our LS1 model), and -25.37 basis points for the two-factor LS model with stochastic interest rates (equivalent in specification to our LS2 model). HH also report underprediction of credit spreads for the one-factor LS model with an estimated error of -100 basis points for an A-rated bond with a 10 year maturity, and -108.5 basis points with a stochastic interest rate. HH show the LT model to have a spread prediction error of -84.5 basis points. For the CDG mean-reverting leverage model, HH report a spread prediction error of -100.5 basis points. Ericsson, Reneby & Wang (2005) apply a maximum-likelihood approach using rm-specic balance sheet and market data on stock prices to fit the Leland (1994), Fan & Sundaresan (2000), and LT models. They find average credit spread prediction errors of -108, -91, and -56 basis points respectively. It therefore appears that our approach of estimating structural models directly from firm-specific credit spread term structures achieves a less biased fitting of the models, on average, than achieved by: estimation of the firm process from equity prices alone (Ericsson et al. (2005)), proxy variables with no optimisation (LYS and EHH), or firm process parameters calibrated to match predicted with observed historical default rates (HH).

An alternative measure of the prediction accuracy is the mean percentage prediction error (MPE), which we calculate by deducting the observed credit spread from the predicted, and then scaling by the observed credit spread. The MPE has the property of being invariant to the size of the observed credit spread and is therefore a comparable measure of error across different levels of observed credit spread.

Turning to column two of Panel A of Table 4.4, we show that the EM model underpredicts credit spreads, with a mean percentage error of -6.66 percent. The LS1 and LT models show slightly less average bias at -6.25 percent and -6.09 percent respectively, the CEV model shows very slight average underprediction at -2.23 percent, followed by the LS2 model at -1.71 percent. The most biased model, on average, is found to be the CDG model, which overpredicts with a MPE of 11.46 percent. However, the standard deviations of the MPE, which are shown in parentheses, indicate that our MPE estimates are not significantly different from zero.

In Table 4.1 we compare our MPE results against the extant literature of EHH and HH. The EM model is reported by EH to have a MPE of -50.42 percent. EHH find that the LT model overpredicts with a MPE of 115.69 percent, however, HH report the model to underpredict with a MPE of -68.70 percent. Directionally, we agree with HH, with our absolute level of error is less than either study. A similar result is obtained for the two factor LS2 model. EHH report overprediction with a MPE of 42.93 percent and HH a MPE of -88.21 percent. We find little difference in prediction bias between the one factor (LS1) and two factor (LS2) versions of the LS model. Finally, we see overprediction in the CDG model but less than HH at 71.11 percent and an order of magnitude less than EHH who report a MPE of 269.78 percent An important feature of Table 4.1 is the relative consistency in MPE that we find between models. The mean and standard deviation of the MPEs show a much higher degree of similarity than reported by EHH. Our standard deviations of the MPE range between the lowest for the CDG model at 63.95 percent to the highest being the LS2 model at 82.80 percent. In contrast, EHH report standard deviations of MPE ranging between a low of 71.84 percent for the EM model, and a high of 490.19 percent for the LT model. In terms of the mean spread prediction error, HH report errors in the range of -68 percent to -88 percent for the models comparable to our LT, LS1, and LS2 model implementations. Our percentage errors are approximately ten times lower than HH, but otherwise show a similar level of relative error between models. The improved accuracy is most likely due to our more direct method of fitting the models to credit spreads whereas HH fit a representative average rated company to historical default rates.

The similarity in error levels between models is also evident in the mean absolute percentage errors (MAPE) of the credit spreads shown in Panel A, column two of Table 4.5. The MAPE is defined as the absolute value of the difference between the predicted and observed credit spread, divided by the observed credit spread. Compared to the MPE, the MAPE penalises variance in the prediction error. A smaller value of MAPE indicates a more consistently accurate prediction independent of the level of the observed credit spread. The lowest MAPE belongs to the EM model at 25.51 percent and the highest MAPE is shown by the CDG model at 34.88 percent. Once again, EHH report considerably greater MAPE for the same models as shown in Table 4.1. For the extended Merton model, EHH report MAPE of 78.02 percent and 319.31 percent for the CDG model; approximately nine times greater than our result.

Given our results, the variance in prediction errors shown by EHH are surprising. Since we have calibrated to market spreads, and HH calibrated to their models to historical average default rates, it is likely that we should achieve similar consistency in results to HH notwithstanding our higher level of prediction accuracy. In contrast, EHH do not attempt to calibrate model parameters and rely upon the adequacy of their proxy variables to match firm-specific model parameters. Our result suggests that in failing to optimise model fit to either default rates or credit spreads, additional error may have been introduced into their spread predictions. On the other hand, we are able to demonstrate from Tables 4.3 to 4.1 that our estimation method provides the least biased, and most consistently estimated set of structural models achieved to date. The advantage of achieving consistently low average prediction biases, is that the remaining biases are not obscured by the excessive levels of error, seen for example by the CDG and LT results in the EHH study. We have therefore provided each model with its best opportunity to perform accurately with as little average error as possible.

We now turn to the MPE related to the issuer's rating. As shown in Panel A of Table 4.4, a negative relationship between MPE and rating is evident amongst the single-factor models; higher ratings are associated with relatively greater underprediction of the yield spread. The EM model exhibits underprediction for BB trades of -3.07 percent, but -10.55 percent for AA rated trades. The LS1, LT, and CEV models also have a negative

Table 4.1: This table shows a comparison of model accuracy between the no-liquidity premium model implementation, with the results of Eom et al. (2004) (EHH) and Huang & Huang (2003) (HH) for equivalently specified models. MAPE is the mean absolute percentage credit spread preedcition error. MPE is the mean percentage credit spread prediction error. All numbers are in percentages.

				Comparative Studies		
	Comparative	No Liquidity		EHH		HH
Model	Model	MPE	MAPE	MPE	MAPE	MPE
EM	EM	-6.66	25.51	-50.42	78.02	-
		(69.12)	(64.58)	(71.84)	(39.96)	-
LT	LT	-6.09	31.38	115.69	146.05	-68.70
		(80.52)	(74.40)	(490.19)	(481.97)	-
LS1	LS (Base Case)	-6.25	28.36	-	-	-81.06
		(74.97)	(69.68)	-	-	-
LS2	LS (1-day CMT)	-1.71	31.24	42.93	124.83	-88.21
		(82.80)	(76.70)	(171.63)	(125.07)	-
CDG	CDG (Baseline)	11.46	34.88	269.78	319.31	71.11
		(63.95)	(54.81)	(370.41)	(328.42)	-

Table 4.2: This table shows the mean percentage spread prediction errors by rating reported by (Huang & Huang 2003, Tables 2, 3, 4, and 6) for a 10 year maturity bond. Calculations are the author's and all numbers are in percent. LS Base refers to a non-stochastic interest rate LS model and LS (CMT 1-day) refers to a two-factor LS model with daily updating of the risk-free rate.

Rating	LS Base	LS (CMT 1-day)	LT	CDG
AA	-84.40	-90.55	-62.13	-83.63
A	-81.06	-88.21	-68.70	-81.71
BBB	-70.88	-80.10	-69.35	-73.04
BB	-39.91	-51.91	-48.22	-42.91

rating bias, although less pronounced. The CEV model has the least rating bias amongst the single factor models, varying from -8.55 percent for AA bonds to 1.06 percent for BB bonds. Compared to the one-factor models, the two-factor models exhibit an opposite rating bias. The LS2 mostly underpredicts the BB rated trades with -5.04 percent error, and underpredicts the AA rated trades by -0.36 percent. An examination of the prediction errors shows that the model did not converge as well as the LS models in the absence of a liquidity premium in the measurement equation. The rating bias of the LS2 model is similar to the LS1 model when a liquidity premium is included (refer Panels B and C), therefore, the n Panel A result for the LS2 model should be viewed with caution. The CDG model is found to overpredict for all ratings, but mostly for the AA rated trades where MPE is 15.83 percent and by 5.10 percent for the BB trades.

The MPE rating bias is also present in the estimates by HH, although the level of our error is lower. For ease of reference, HH's spread prediction MPE by rating is shown in Table 4.2. It is generally acknowledged that structural models underpredict credit spreads on bonds with short maturities. In Panel A of Table 4.4 we show the MPE by remaining maturity, as measured from the date of the trade, to the contractual maturity, for the respective bond being priced. For the shortest tenor range of less than 7 years, the CEV model exhibits the least bias with a small mean percentage underprediction of -1.13 percent. The CDG model has the largest short tenor percentage error of 31.31 percent, which shows that much of the model's average overprediction is due to excessively high predictions of short-term credit spreads. All models exhibit a common pattern of prediction error in which the mid-maturity bonds, with 7 to 15 years of remaining maturity, exhibit the largest relative underprediction of credit spreads. The LT and LS1 models at -11.46 and -11.29 percent respectively are the most biased in this respect, showing the greatest levels of underprediction for this maturity. For the longer maturity bonds, with remaining maturities in excess of 15 years, most models report relatively small absolute mean percentage errors. The exception is the LT model, which has a tendency for relatively large overprediction with an average mean percentage error of 12.56 percent.

Finally, in Panel A of Table 4.5 we show the absolute percentage spread prediction errors (MAPE) by rating and remaining maturity. The most accurate model is also the simplest EM model with a MPE of 25.51 percent, and the least accurate is the CDG model with a MAPE of 34.88 percent. By rating, it is evident that prediction accuracy is best for the lowest rating; at the BB rating the lowest MAPE is 15.04 percent for the EM model and the highest MAPE is the LS2 model 22.38 percent. Prediction accuracy is therefore greatest when default risk is highest and the credit spread levels are the greatest. In the maturity dimension, it is clear that accuracy is greatest for the long-dated maturities for all models; when remaining maturity exceeds 15 years,the most accurate model is the EM model with a MAPE of 14.95 percent, and the least accurate is the LT model with a MAPE of 24.18 percent. Consequently, we find that structural models all share the characteristic that their greatest accuracy occurs at low ratings and long maturities, i.e., when it can be expected that default risk is at its greatest.

An important view put forward by EHH is that the introduction of stochastic interest rates into the structural modelling literature has decreased prediction bias (by raising the level of the predicted credit spread), but in doing so, has substantially reduced prediction accuracy. In Table 4.1 we compare the MAPE from Panel A of Table 4.5 with the EHH results for the models where direct comparison is possible. While we do find the EM model has the most, and the CDG model the least accuracy as represented by their relative values of MAPE, we do not find the same degree of substantial inaccuracy found by EHH. Our observed deterioration in MAPE between the EM and CDG models is 25.51 percent to 34.88 percent, compared with EHH who report 78.02 percent to 319.31 percent respectively. Consequently, we are unable to conclude that the two-factor stochastic interest rate models are *substantially* less accurate, but we can state that they do not improve accuracy relative to the EM model, and therefore have failed to achieve their aim of improving structural model prediction accuracy.

In the next section we review whether these results hold when a liquidity premium is introduced into the measurement equation.

4.1.2 With Constant Liquidity Premium

In this section we examine the prediction errors with a constant liquidity premium included in the measurement equation. By doing so, the average level of credit spread is explained by the liquidity premium variable and not by the structural model. Thus, the structural model parameters are optimised to explain the changes in the level, and the slope of the firm's credit spread term structure over time.

In Panel B of Table 4.3, the step-ahead credit spread prediction errors, reported in basis points, are shown per model by rating and remaining bond maturity. The EM model has the smallest error of 0.26 basis points, and the CDG model has the largest average error equal to 13.30 basis points. Given the levels of standard deviation, as shown in parentheses, the errors are not significantly different from zero.

A better measure of relative prediction bias is given by the MPE as shown in Panel B of Table 4.4. Compared to the no-liquidity premium case shown in Panel A, the MPE is found to be negative for every model with an average underprediction bias that varies from -6.09 percent for the CDG model to -8.78 percent for the LS1 model. The overprediction bias of the CDG model as reported above for the no-liquidity premium case is no longer evident suggesting that it was caused by the model parameters overemphasising short term term default risk in order to compensate for the lack of a liquidity premium. Similarly, including a liquidity premium changes the pattern of the maturity bias so that it is generally monotonically increasing in underprediction as the maturity shortens. The pattern is consistent for all models except the CDG model where the 1 to 15 year maturity band is the least biased. Thus, compared to Panel A, we can see that including a liquidity premium results in greater consistency in the levels and directions of underprediction biases between the models. This result shows the importance of fitting the structural models to only that part of the credit spread that is likely to be related to firm default risk.

Finally, in Panel B of Table 4.5 we show the MAPE errors. Compared with Panel A it is evident that by including a liquidity premium in the measurement equation, prediction accuracy improves in all cases, and that the lowest ratings and longest maturities have the greatest prediction accuracy. On average, the most accurate model is the EM model with a MAPE of 21.74 percent, and the least accurate is the CDG model with a MAPE of 26.61 percent.

4.1.3 With Time-Varying Liquidity Premium

Panel C of Table 4.4 shows the MPE for the third empirical equation that includes the time-varying Refcorp spread in addition to a constant premium. Compared to Panel B of Table 4.4, we find the level of underprediction to be less, on average, for most models. The improvement in prediction bias is not large, being in the order of less than half of one percent. The exception is the CEV model, which is little improved by the addition of the Refcorp spread within the measurement equation, and the CDG model, which in contrast to the other models, increases its tendency to underpredict.

The lack of improved fit for the CEV model suggests that the model is already able to explain much of the time variation in the average level of credit spreads without the need for an additional time dependent explanatory variable. The model's ability to better capture average spread variation over time, relative to the other models, is most likely due to its specification that allows the local volatility to vary with the level of log-solvency.

The pattern of relative error across ratings and term-to-maturity dimensions follows closely the pattern reported in Panel B for the constant liquidity empirical equation demonstrating that the omission of time-variation in the liquidity premium is not the likely cause for the maturity and rating biases present in the structural models tested.

Panel C of Table 4.5 summarises the MAPE by model. The absolute errors are found to be very similar between models and reduced slightly by the inclusion of the Refcorp spread relative to the constant liquidity premium models. The model with greatest prediction accuracy is the EM model with a MAPE of 21.42 percent, and the highest MAPE is the CDG model at 25.86 percent. Thus, the simplest model, with the lowest number of parameters, is found to have the highest relative prediction accuracy, and the model with the highest number of parameters, exhibits the lowest relative prediction accuracy. Unlike EHH, the variation between model accuracy, is nonetheless, relatively low. Given the standard deviations of MAPE (shown in parentheses), we find no significant difference in prediction accuracy between structural models despite wide differences in underlying theory and specification. The relatively large standard deviation in MAPE demonstrates that improvement in forecast accuracy remains to be achieved, and the theoretical developments, embodied in the structural models tested in this paper, have not yet been able to realise the potential gains in accuracy.

Shown in Figures 4.1 to 4.6 are plots of predicted versus actual credit spreads by rating and term-to-maturity. These figures are presented in the same style as EHH to facilitate comparison. The vertical axis is a log scale of the credit spread shown in basis points, and the horizontal scale is the remaining term to maturity of the bond. The top panels show predicted and actual spreads for combined A and AA issuer rated bonds measured at the time of the trade. The middle panels show predicted and actual credit spreads for BBB rated bond results, and the bottom panels show the same for BB rated bonds.

Table 4.3: This table shows the **mean credit spread prediction error in basis points** by model. Error is defined as the predicted credit spread less the observed spread. Means are reported on the pooled sample, categorised by the issuer's rating and remaining term-to-maturity, measured coincident with the observed trade. Standard deviations are shown in parentheses.

Model	All	AA	A	BBB	BB	≤ 7 yrs	$7-15$ yrs	>15 yrs
A: No Liquidity Premium								
EM	-0.29	0.46	-0.08	0.68	-17.84	-1.33	-1.49	5.52
	(67.66)	(22.38)	(30.08)	(43.84)	(321.67)	(95.23)	(28.12)	(26.26)
LS1	-1.75	1.60	-0.09	-0.33	-57.49	-4.80	-2.57	8.89
	(131.45)	(26.50)	(49.85)	(53.16)	(675.41)	(186.92)	(51.57)	(36.85)
LT	0.94	1.00	1.96	1.90	-22.95	-3.02	-1.85	18.83
	(68.24)	(25.74)	(45.10)	(54.24)	(281.27)	(91.37)	(37.53)	(37.84)
CEV	4.02	1.81	3.01	6.32	4.03	3.75	2.13	9.29
	(63.30)	(26.91)	(40.74)	(59.23)	(240.70)	(86.62)	(32.69)	(30.87)
LS2	3.08	4.36	3.30	1.60	9.35	6.93	-1.69	3.37
	(57.76)	(27.91)	(38.43)	(62.79)	(186.24)	(69.86)	(45.89)	(41.83)
CDG	3.72	4.36	2.87	3.97	8.10	5.35	2.93	0.95
	(1.76)	(2.51)	(1.58)	(1.66)	-(0.55)	(0.95)	(3.59)	-(0.66)
B: Constant Liquidity Premium								
EM	0.26	0.40	-0.38	-0.02	10.55	-0.20	0.95	-0.09
	(38.61)	(18.70)	(26.42)	(43.66)	(115.46)	(49.81)	(26.30)	(23.55)
LS1	-2.32	0.59	-1.65	0.07	-53.79	-6.06	0.89	0.78
	(113.56)	(20.72)	(51.46)	(41.33)	(578.88)	(161.41)	(45.52)	(30.15)
LT	-1.34	0.44	-1.05	-0.48	-24.86	-4.41	1.66	0.28
	(63.43)	(19.52)	(38.40)	(45.75)	(279.17)	(87.70)	(30.52)	(28.78)
CEV	1.35	1.50	0.82	2.15	-1.74	-0.99	3.52	2.87
	(68.70)	(21.75)	(27.61)	(98.32)	(155.24)	(94.57)	(32.95)	(35.26)
LS2	-0.91	1.59	1.05	0.87	-59.66	-2.79	0.13	1.98
	(199.09)	(21.62)	(31.92)	(52.60)	(1075.7)	(291.59)	(36.60)	(27.93)
CDG	13.30	13.65	13.16	12.83	18.47	26.90	2.42	0.26
	(51.83)	(29.45)	(37.36)	(51.16)	(173.65)	(64.05)	(33.87)	(36.24)
C: Time-Varying Liquidity Premium								
EM	-0.59	0.73	-0.22	-0.08	-18.52	-1.82	0.62	0.03
	(62.36)	(18.21)	(25.43)	(38.16)	(303.08)	(88.13)	(25.57)	(21.72)
LS1	-1.59	0.81	-0.97	0.00	-40.62	-4.11	0.47	0.73
	(101.77)	(19.09)	(39.35)	(37.52)	(525.86)	(145.53)	(37.85)	(23.86)
LT	-1.10	0.61	-0.82	-0.22	-24.05	-3.78	1.53	0.35
	(61.45)	(18.88)	(34.86)	(41.22)	(279.67)	(85.00)	(29.88)	(26.91)
CEV	1.27	1.08	0.69	2.56	-4.47	-0.17	2.73	1.95
	(57.75)	(21.45)	(26.55)	(51.57)	(245.91)	(80.53)	(27.07)	(22.38)
LS2	1.37	1.18	1.17	0.93	9.89	1.24	1.39	1.72
	(42.99)	(20.25)	(27.41)	(38.66)	(166.75)	(55.95)	(28.14)	(26.32)
CDG	4.22	4.62	4.08	3.80	8.30	5.58	3.82	1.30
	(46.76)	(21.84)	(32.06)	(42.08)	(176.64)	(59.92)	(31.99)	(30.32)
n	8,953	1,691	3,704	3,263	295	4,107	3,407	1,439

Table 4.4: This table shows the **mean percentage credit spread prediction error** by model. Error is defined as the predicted credit spread less the observed spread, divided by the observed spread. Means are reported on the pooled sample, categorised by the issuer's rating and remaining term-to-maturity, measured coincident with the observed trade. All numbers are in percentages and standard deviations are shown in parentheses.

Model	All	AA	A	BBB	BB	≤ 7 yrs	$7-15$ yrs	> 15 yrs
A: No Liquidity Premium								
EM	-6.66	-10.55	-6.18	-5.53	-3.07	-7.38	-9.59	2.32
	(69.12)	(77.49)	(62.83)	(73.82)	(25.30)	(87.59)	(55.08)	(23.35)
LS1	-6.25	-8.29	-5.68	-5.89	-5.74	-5.96	-11.29	4.84
	(74.97)	(80.24)	(73.32)	(76.28)	(42.96)	(88.36)	(70.05)	(29.94)
LT	-6.09	-10.71	-5.16	-4.90	-4.49	-8.18	-11.46	12.56
	(80.52)	(75.44)	(87.41)	(77.78)	(34.16)	(103.56)	(59.30)	(31.29)
CEV	-2.23	-8.55	-1.74	0.18	1.06	-1.13	-6.27	4.19
	(77.18)	(79.54)	(81.10)	(74.29)	(25.27)	(99.56)	(58.46)	(24.63)
LS2	-1.71	-0.36	-0.11	-3.93	-5.04	2.92	-8.86	1.99
	(82.80)	(98.09)	(60.26)	(96.10)	(71.49)	(95.12)	(81.08)	(32.83)
CDG	11.46	15.83	13.39	7.57	5.10	31.31	-5.43	-5.24
	(63.95)	(88.93)	(54.00)	(61.08)	(24.19)	(64.74)	(66.37)	(31.24)
B: Constant Liquidity Premium								
EM	-7.96	-10.23	-7.45	-7.81	-3.14	-11.28	-5.92	-3.35
	(62.29)	(68.59)	(60.25)	(63.67)	(18.39)	(77.85)	(51.26)	(23.74)
LS1	-8.78	-11.26	-9.24	-7.22	-5.89	-12.65	-6.50	-3.11
	(68.50)	(71.45)	(73.16)	(63.58)	(34.38)	(82.81)	(61.33)	(25.15)
LT	-8.30	-10.37	-8.37	-7.36	-6.10	-11.81	-5.48	-5.01
	(70.94)	(70.37)	(79.71)	(62.47)	(37.56)	(91.44)	(52.87)	(27.76)
CEV	-7.04	-8.12	-6.23	-7.79	-2.70	-11.82	-3.57	-1.62
	(67.38)	(72.81)	(60.86)	(73.88)	(22.15)	(84.71)	(54.48)	(24.89)
LS2	-8.89	-11.98	-8.36	-8.08	-6.62	-14.79	-5.94	0.98
	(67.81)	(73.46)	(67.56)	(64.63)	(71.34)	(86.30)	(52.77)	(23.42)
CDG	-6.09	-7.98	-5.80	-5.88	-1.20	-9.88	-2.75	-3.17
	(65.37)	(75.74)	(67.51)	(59.46)	(21.74)	(79.24)	(57.77)	(26.37)
C: Time-Varying Liquidity Premium								
EM	-7.58	-10.00	-6.88	-7.44	-4.02	-10.51	-6.13	-2.65
	(62.93)	(72.80)	(58.97)	(64.15)	(24.48)	(77.97)	(53.51)	(21.40)
LS1	-8.26	-10.36	-8.09	-7.72	-4.23	-11.15	-7.35	-2.14
	(67.85)	(75.44)	(69.39)	(64.16)	(32.75)	(82.94)	(59.76)	(22.20)
LT	-7.97	-10.15	-7.91	-7.16	-5.03	-11.15	-5.72	-4.18
	(70.36)	(74.80)	(74.45)	(65.23)	(39.00)	(89.18)	(55.71)	(26.52)
CEV	-7.05	-10.49	-7.11	-5.72	-1.29	-11.52	-4.45	-0.47
	(67.73)	(77.98)	(66.75)	(65.79)	(21.59)	(84.52)	(56.50)	(21.95)
LS2	-8.65	-12.28	-8.09	-8.10	-0.91	-13.38	-6.70	0.24
	(69.86)	(83.22)	(64.72)	(70.74)	(19.05)	(86.44)	(59.28)	(23.93)
CDG	-6.22	-8.13	-6.38	-5.52	-1.00	-8.83	-4.34	-3.24
	(68.99)	(85.66)	(66.54)	(64.70)	(21.33)	(80.81)	(65.32)	(28.97)
n	8,953	1,691	3,704	3,263	295	4,107	3,407	1,439

Table 4.5: This table shows the **mean absolute percentage credit spread prediction error** by model. Error is defined as the absolute of the predicted credit spread less the observed spread, divided by the observed spread. Means are reported on the pooled sample, categorised by the issuer's rating and remaining term-to-maturity, measured coincident with the observed trade. All numbers are in percentages and standard deviations are shown in parentheses.

Model	All	AA	A	BBB	BB	≤ 7 yrs	$7-15$ yrs	>15 yrs
				A: No Liquidity Premium				
EM	25.51	32.60	24.57	23.84	15.04	30.89	23.47	14.95
	(64.58)	(71.07)	(58.16)	(70.08)	(20.56)	(82.29)	(50.75)	(18.08)
LS1	28.36	35.49	28.03	25.78	20.09	34.01	26.47	16.70
	(69.68)	(72.44)	(67.99)	(72.03)	(38.38)	(81.76)	(65.83)	(25.31)
LT	31.38	34.95	31.92	29.76	22.05	36.97	27.68	24.18
	(74.40)	(67.70)	(81.54)	(72.03)	(26.44)	(97.08)	(53.68)	(23.49)
CEV	28.62	34.72	28.23	26.97	17.00	36.56	24.20	16.42
	(71.72)	(72.06)	(76.05)	(69.23)	(18.71)	(92.61)	(53.58)	(18.83)
LS2	31.24	39.91	28.96	30.14	22.38	35.70	29.72	22.08
	(76.70)	(89.60)	(52.85)	(91.34)	(68.07)	(88.21)	(75.95)	(24.37)
CDG	34.88	48.35	32.41	32.25	17.68	44.86	28.78	20.80
	(54.81)	(76.30)	(45.22)	(52.41)	(17.25)	(56.21)	(60.05)	(23.89)
				B: Constant Liquidity Premium				
EM	21.74	27.28	20.92	20.64	12.59	26.13	20.36	12.49
	(58.91)	(63.76)	(56.99)	(60.74)	(13.75)	(74.20)	(47.41)	(20.46)
LS1	24.13	29.57	24.42	21.64	16.89	29.19	22.52	13.53
	(64.70)	(66.00)	(69.58)	(60.22)	(30.51)	(78.52)	(57.41)	(21.43)
LT	23.05	27.83	22.69	21.57	16.69	27.44	21.22	14.89
	(67.60)	(65.46)	(76.87)	(59.09)	(34.18)	(88.02)	(48.73)	(23.96)
CEV	23.37	28.51	21.53	23.71	13.16	28.60	21.49	12.88
	(63.59)	(67.48)	(57.27)	(70.41)	(18.00)	(80.60)	(74.00)	(21.35)
LS2	26.00	33.06	25.47	23.72	17.33	32.04	23.27	15.21
	(63.26)	(66.68)	(63.13)	(60.67)	(69.51)	(81.48)	(75.00)	(17.83)
CDG	26.61	33.20	27.39	23.42	14.34	31.78	24.82	16.10
	(60.02)	(68.54)	(61.97)	(54.97)	(16.36)	(73.25)	(52.24)	(21.12)
				C: Time-Varying Liquidity Premium				
EM	21.42	27.69	20.49	19.97	13.23	25.70	20.34	11.78
	(59.65)	(68.07)	(55.72)	(61.42)	(20.98)	(74.36)	(49.88)	(18.06)
LS1	22.89	28.54	22.33	21.33	14.76	27.17	22.29	12.08
	(64.40)	(70.60)	(66.19)	(61.00)	(29.52)	(79.15)	(55.93)	(18.74)
LT	22.69	27.90	22.43	20.87	16.24	27.05	21.23	13.69
	(67.08)	(70.14)	(71.43)	(62.21)	(35.80)	(85.71)	(51.82)	(23.09)
CEV	22.99	29.75	22.15	21.35	12.76	28.00	21.70	11.70
	(64.10)	(72.84)	(63.36)	(62.50)	(17.45)	(80.58)	(52.35)	(18.57)
LS2	25.18	32.25	24.41	23.50	12.78	30.11	23.66	14.69
	(65.73)	(77.70)	(60.48)	(67.21)	(14.14)	(82.12)	(54.76)	(18.89)
CDG	25.86	32.90	26.10	22.99	14.25	29.84	25.63	15.03
	(64.26)	(79.50)	(61.54)	(60.73)	(15.89)	(75.61)	(60.24)	(24.98)
n	8,953	1,691	3,704	3,263	295	4,107	3,407	1,439

Comparing between models, it can be seen that there is a generally close match between predicted and actual spreads, with a similar pattern of observations exhibited by all models. Like EHH, we find that there is greater dispersion of predicted spreads at shorter maturities, and likewise, there is a much wider dispersion of actual spreads at shorter tenors too.

Turning to the EM model, EHH find evidence of extreme underprediction and extreme overprediction of spreads with the underprediction cases more prevalent (EHH, Fig. 1). Their frequency of underprediction is greatest at the short maturities where ratings are BBB or better. Referring to the top panel of Figure 4.1, we also find that there is a higher occurrence of underprediction when the rating is high and the maturity less than 10 years. In such cases, the firm's leverage is likely to be low and with a short term to maturity, it is apparent that the assumption of geometric Brownian motion of the asset process specified under the EM model, fails to predict a sufficiently large probability of default when the time to diffuse is small. Our result adds weight to EHH's finding of prediction bias since we have controlled for liquidity premiums and allowed the model to fit the data by quasi maximum likelihood, thereby removing a potential source of contributory bias to EHH's results.

Our results, however, differ from EHH in the degree of under and over prediction found. Unlike EHH, there is no evidence of systematic extreme prediction errors. Referring to the middle and lower panels of Figure 4.1, we can see that actual credit spreads increase, as is to be expected due to the lower ratings, but the predicted spreads generally align well, whereas EHH report high levels of underprediction for BBB rated bonds, and extensive over and underprediction for BBB and lower rated bonds. At lower ratings, the EM model is able to match market spreads well even at short maturities. Thus, our estimation method provides a clearer view of the inherent bias in the structural models compared to the proxy empirical fitting method. By fitting the implied default boundary and asset volatility from panel data, we impose time-series restrictions on the change in credit spreads implied from the underlying asset process, which limits the extent to which the asset volatility can be increased to match cross-sectional short-term spreads. Interestingly, for higher default risk firms, the closer match of predicted to actual, as shown in Figure 4.1, suggests that the EM model is able to match both cross-sectional and time-series predictions of credit spreads with little bias.

A similar pattern of spread prediction dispersion is evident in the other structural models as shown in Figures 4.2 to 4.6. There is a common tendency for underprediction of spreads at short maturities for well rated bonds, with the bias disappearing as the rating worsens, or maturity lengthens.

The prediction biases we have identified, related to rating and maturity, confirm EHH and suggest a specification problem common to structural models. The models commonly assume that default risk arises from a continuous stochastic diffusion of firm asset value to a default boundary, and it appears that in the absence of asset value jumps to default, or excessive asset volatility, that not all cross-sectional short term market spreads can be explained fully. When the firm is well rated and the distance to default is high, there is insufficient asset volatility necessary to bridge the gap between predicted and actual. The result is underprediction of the credit spread even though we have controlled for a potential liquidity premium in the spread. Our finding is potentially more robust than prior studies since we have allowed the models to fit the data to the best of their abilities in accordance with model specifications.

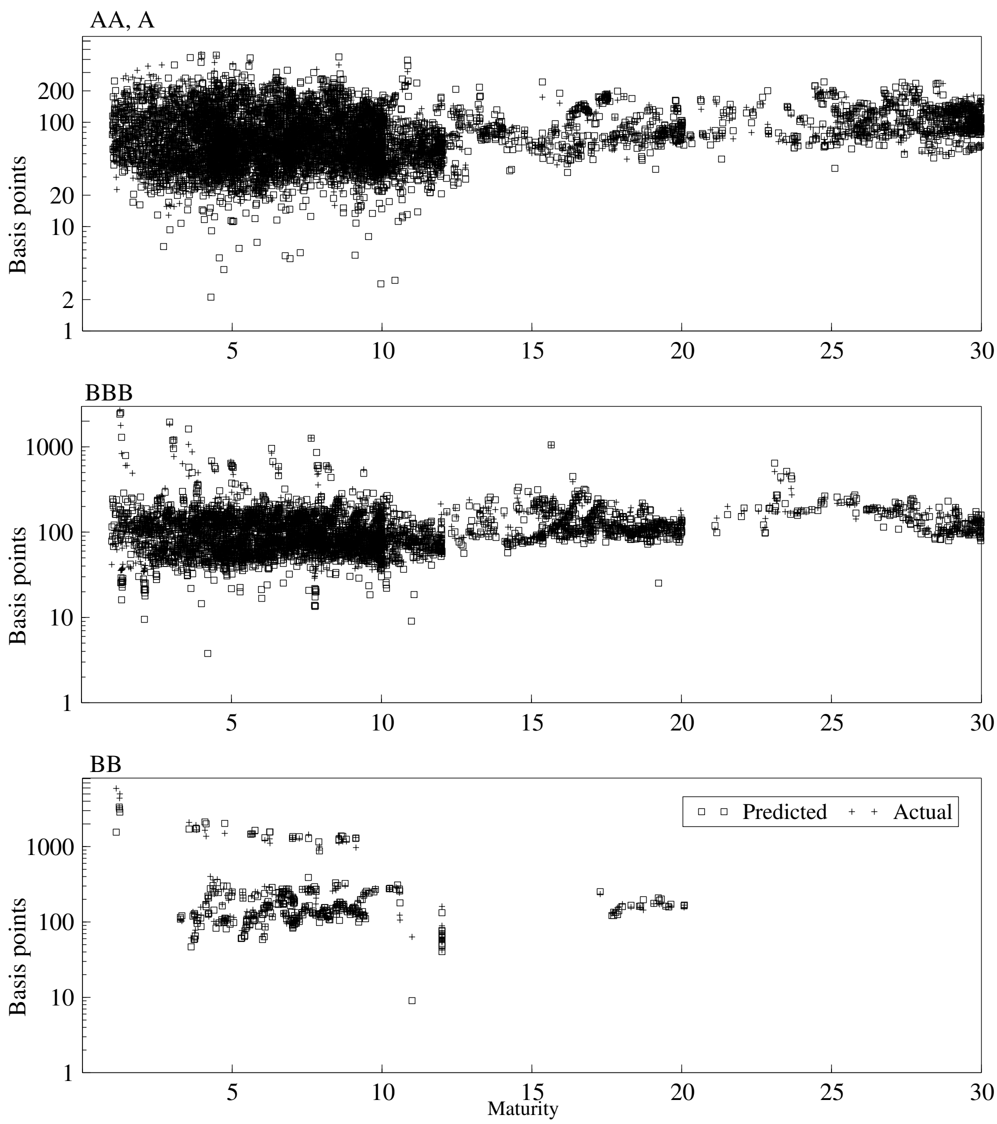


Figure 4.1: EM predicted versus actual credit spreads by remaining term-to-maturity and rating. Predicted credit spreads are estimated with a time-varying liquidity premium using the Refcorp spread as a control.

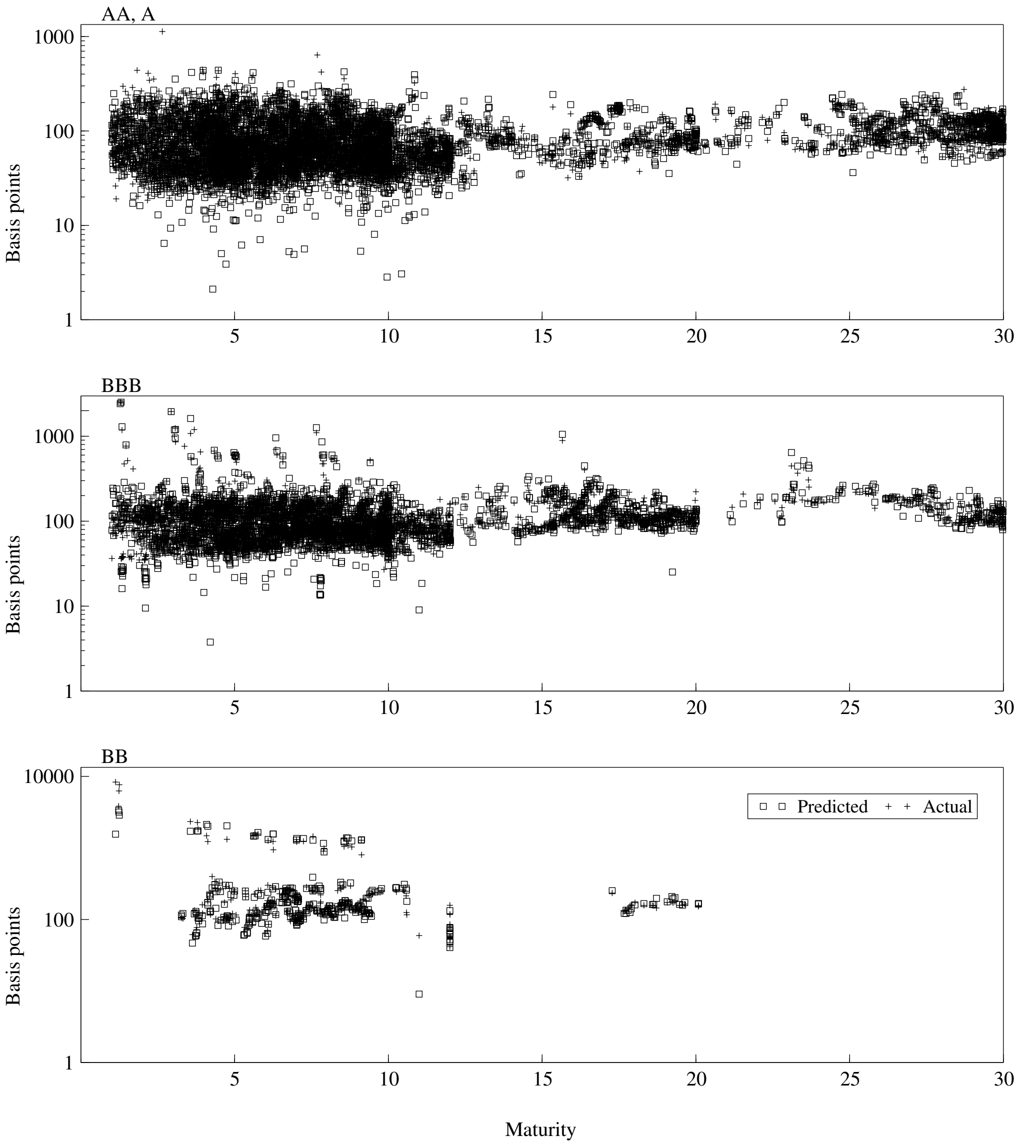


Figure 4.2: LS1 predicted versus actual credit spreads by remaining term-to-maturity and rating. Predicted credit spreads are estimated with a time-varying liquidity premium using the Refcorp spread as a control.

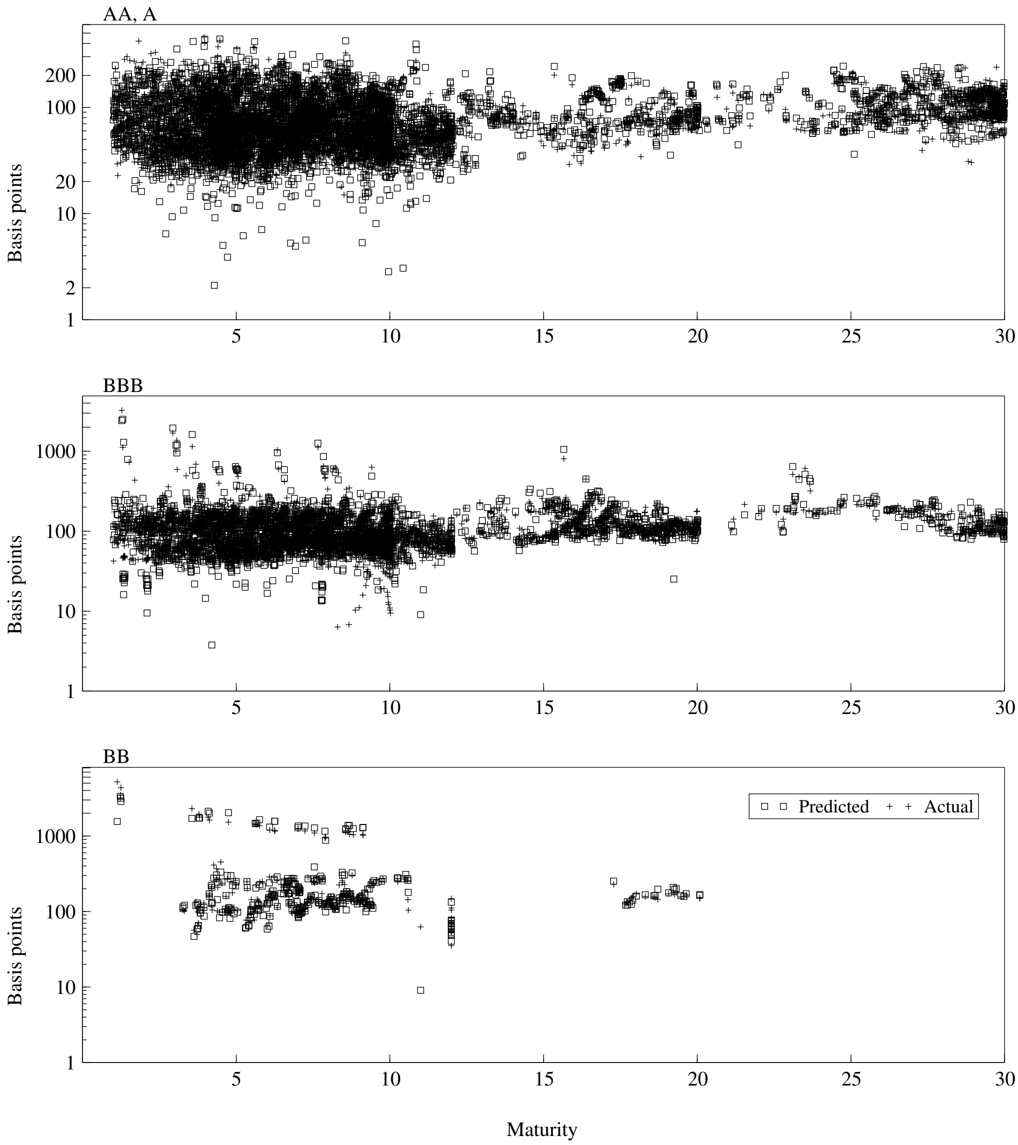


Figure 4.3: CEV predicted versus actual credit spreads by remaining term-to-maturity and rating. Predicted credit spreads are estimated with a time-varying liquidity premium using the Refcorp spread as a control.

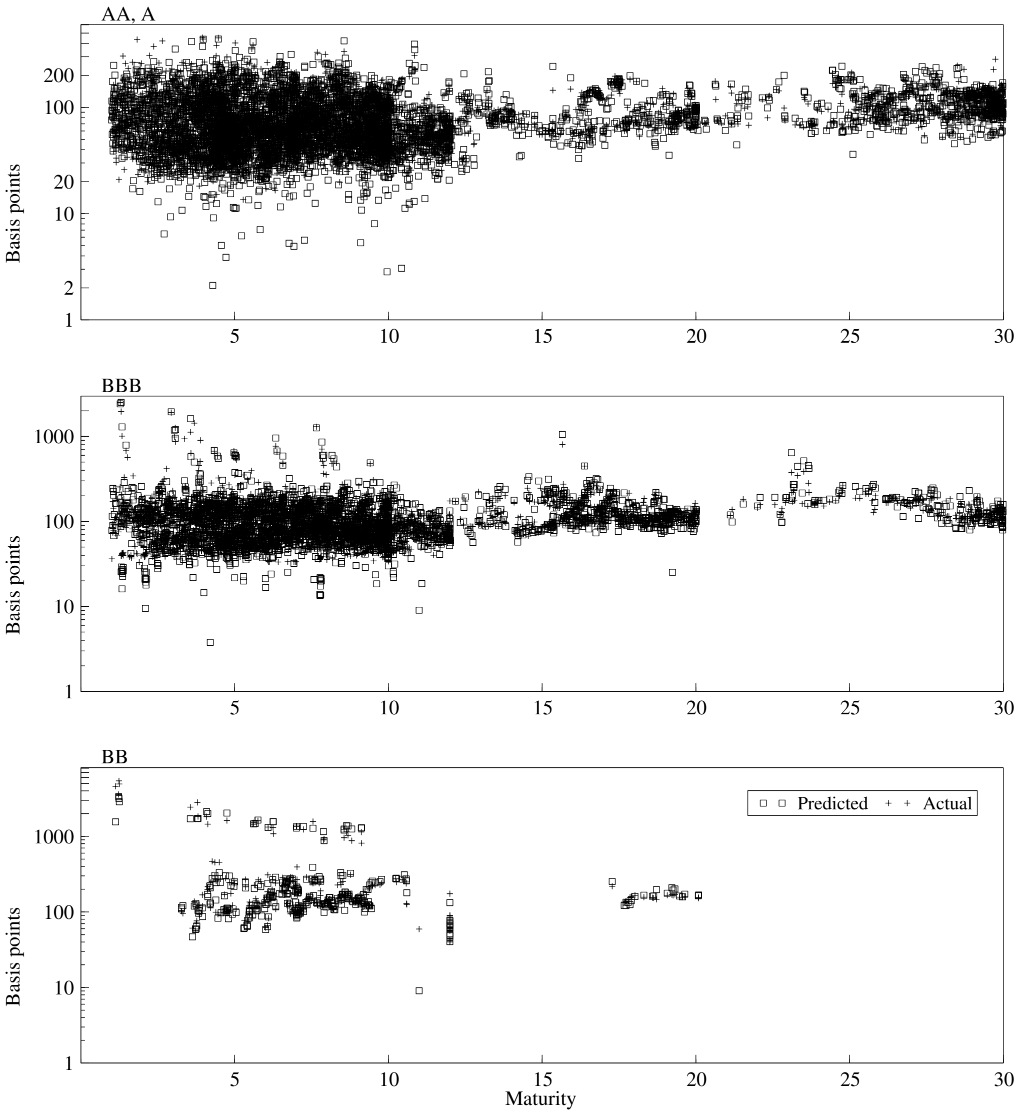


Figure 4.4: LT predicted versus actual credit spreads by remaining term-to-maturity and rating. Predicted credit spreads are estimated with a time-varying liquidity premium using the Refcorp spread as a control.

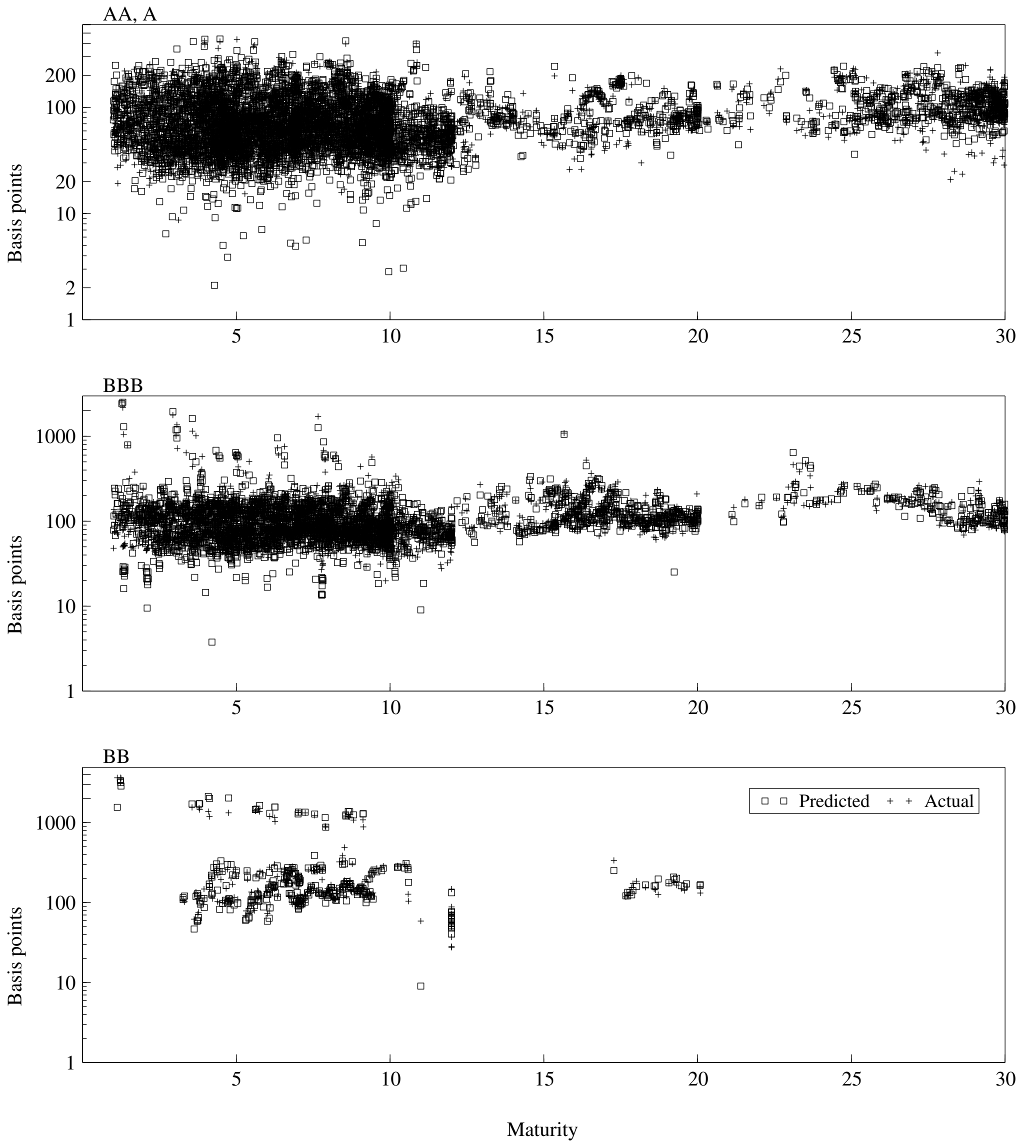


Figure 4.5: LS2 predicted versus actual credit spreads by remaining term-to-maturity and rating. Predicted credit spreads are estimated with a time-varying liquidity premium using the Refcorp spread as a control.

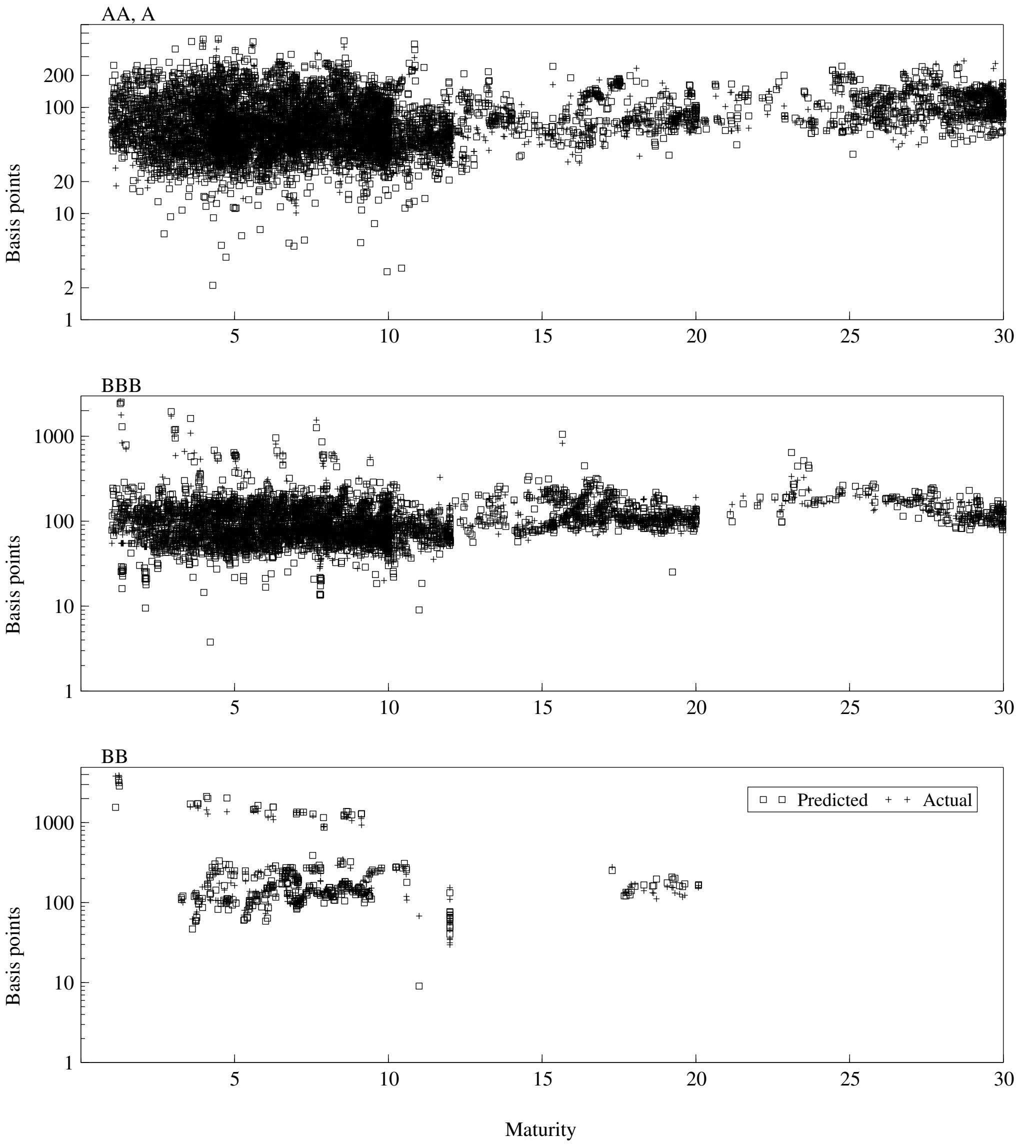


Figure 4.6: CDG predicted versus actual credit spreads by remaining term-to-maturity and rating. Predicted credit spreads are estimated with a time-varying liquidity premium using the Refcorp spread as a control.

4.2 Credit Model Specification Tests

In this section we discuss formal tests for model miss-specification. If the structural models are properly specified, then the prediction errors from the EKF should be independent and normally distributed. To examine this property, Harvey (1989, p.257) recommends construction of the standardised prediction error defined as

$$\tilde{\boldsymbol{v}}(t) = \boldsymbol{F}(t)^{-1/2}\boldsymbol{v}(t),\ t = 1, \ldots n. \tag{4.1}$$

The size of the prediction error is potentially influenced by the presence of measurement errors in the observed data, and by the mean level of the credit spread since higher credit spread errors are associated with higher credit spreads. For these reasons, diagnostic testing of model specification is preferable using the standardised prediction error. The average standardised prediction errors are reported in Table 4.6.

If the models are correctly specified, the standardised prediction error should be normally distributed, with mean zero and standard deviation of one, and be serially independent with constant variance. In Table 4.7 descriptive statistics are reported along with the Bowman & Shenton (1975) omnibus test for normality given by

$$BS = n\left(\frac{Sk^2}{6} + \frac{(Ku-3)^2}{24}\right), \tag{4.2}$$

where n is the number of observations, Sk denotes the sample skewness, and Ku denotes the sample kurtosis. The BS statistic is asymptotically χ^2 distributed with two degrees of freedom with a null hypothesis that the distribution is normal (Durbin & Koopman 2001, p.34).

Table 4.7 shows that all BS tests exceed a one percent critical level confirming that the standardised errors are not normally distributed. The means and standard deviations of the standardised errors are close to expected values, but there is a positive bias and skewness in the errors.

The skewness of the standardised error distribution is expected to be zero, however, in all models tested, and in all forms of measurement equation, the standardised errors exhibit positive skewness; there is a tendency for larger overprediction on a standardised basis than there is underprediction, resulting in a longer tailed distribution to the right. The tendency for positive skewness is also revealed in Figure 4.7 in which the histograms of the standardised prediction errors by model are plotted with a comparison against an equivalent normal density plot with the same sample mean and standard deviation.

The skewness levels for the models ranges from 0.56 for the LT model with no liquidity (Panel A), to 1.09 for the CEV model with constant liquidity (Panel B). There appears to be no systematic influence as to whether the model includes a liquidity premium or not, with the LT model generally exhibiting the least skewness.

Table 4.6: This table shows the **mean standardised credit spread prediction error** by model. Error is defined as the predicted credit spread, divided by the filtered standard deviation of the prediction error. Means are reported on the pooled sample, categorised by the issuer's rating and remaining term-to-maturity, measured coincident with the observed trade. Sample standard deviations are shown in parentheses.

Model	All	AA	A	BBB	BB	≤ 7 yrs	$7-15$ yrs	> 15 yrs
				A: No Liquidity Premium				
EM	0.04	0.07	0.04	0.03	0.08	0.04	-0.02	0.23
	(0.98)	(0.98)	(0.99)	(0.97)	(1.16)	(1.05)	(0.90)	(0.96)
LS1	0.09	0.13	0.09	0.07	0.14	0.10	-0.01	0.30
	(0.97)	(0.96)	(0.96)	(0.96)	(1.19)	(1.07)	(0.84)	(0.90)
LT	0.10	0.10	0.11	0.09	0.14	0.03	0.01	0.51
	(0.99)	(0.99)	(0.98)	(0.98)	(1.09)	(1.02)	(0.89)	(1.02)
CEV	0.14	0.12	0.14	0.14	0.30	0.13	0.09	0.31
	(0.95)	(0.96)	(0.94)	(0.93)	(1.10)	(1.03)	(0.84)	(0.93)
LS2	0.17	0.22	0.17	0.13	0.21	0.28	0.03	0.18
	(0.94)	(0.93)	(0.93)	(0.95)	(1.06)	(1.00)	(0.81)	(1.04)
CDG	0.38	0.47	0.42	0.30	0.41	0.74	0.11	0.03
	(1.02)	(0.98)	(1.00)	(1.05)	(1.11)	(1.07)	(0.82)	(0.97)
				B: Constant Liquidity Premium				
EM	0.03	0.05	0.02	0.01	0.09	0.01	0.06	-0.01
	(1.03)	(1.01)	(1.02)	(1.03)	(1.20)	(1.07)	(0.97)	(1.02)
LS1	0.05	0.07	0.05	0.04	0.07	0.02	0.09	0.04
	(1.02)	(1.01)	(1.00)	(1.01)	(1.31)	(1.08)	(0.93)	(1.03)
LT	0.03	0.06	0.03	0.02	0.04	0.00	0.09	-0.01
	(1.02)	(1.01)	(1.02)	(1.01)	(1.16)	(1.03)	(0.96)	(1.11)
CEV	0.09	0.13	0.07	0.08	0.09	0.04	0.15	0.09
	(1.00)	(0.99)	(0.98)	(1.03)	(0.98)	(1.03)	(0.94)	(1.01)
LS2	0.08	0.11	0.08	0.06	0.02	0.06	0.08	0.13
	(1.02)	(1.02)	(1.00)	(1.03)	(1.09)	(1.11)	(0.91)	(1.00)
CDG	0.16	0.21	0.18	0.11	0.21	0.19	0.17	0.06
	(1.06)	(1.04)	(1.05)	(1.07)	(1.28)	(1.17)	(0.91)	(1.07)
				C: Time-Varying Liquidity Premium				
EM	0.03	0.07	0.03	0.00	0.04	0.01	0.05	0.00
	(1.03)	(1.01)	(1.01)	(1.03)	(1.29)	(1.09)	(0.97)	(0.99)
LS1	0.05	0.08	0.04	0.03	0.09	0.03	0.06	0.04
	(1.02)	(1.00)	(1.00)	(1.03)	(1.29)	(1.09)	(0.96)	(0.97)
LT	0.03	0.07	0.02	0.02	0.08	0.00	0.08	0.00
	(1.02)	(1.02)	(1.02)	(1.02)	(1.13)	(1.04)	(0.97)	(1.08)
CEV	0.08	0.10	0.07	0.08	0.16	0.05	0.11	0.10
	(1.00)	(1.00)	(0.98)	(1.01)	(1.14)	(1.08)	(0.92)	(0.95)
LS2	0.07	0.08	0.07	0.04	0.14	0.05	0.06	0.11
	(1.03)	(1.01)	(1.02)	(1.04)	(1.12)	(1.11)	(0.92)	(1.00)
CDG	0.14	0.20	0.14	0.11	0.21	0.19	0.13	0.05
	(1.08)	(1.04)	(1.06)	(1.09)	(1.30)	(1.17)	(0.95)	(1.07)
n	8,953	1,691	3,704	3,263	295	4,107	3,407	1,439

From Figure 4.7 we can see that the standardised prediction errors have an empirical density that is more peaked than the normal distribution. The prediction errors exhibit fatter tails than expected under the normal distribution. Table 4.7 confirms the tendency to fat tailed errors with an excess kurtosis that is very pronounced for each model. The excess kurtosis ranges from a low of 4.08 for the CDG model without a liquidity premium (Panel A), to 8.26 for the CEV model with constant liquidity (Panel B), relative to an expected value of zero.

Thus, we find that the standardised prediction errors exhibit fatter, more positive tails, than expected. This result suggests that, even after limiting the effect of noisy data via the standardisation of prediction errors, the structural models tested tend to under and overestimate spreads to a higher degree than expected under their theoretical specification.

The relative effectiveness of controlling for liquidity in model specification can be judged by comparing results across the panels of Table 4.7. The highest mean error bias is associated with Panel A. Introducing a control for liquidity premiums in Panels B and C result in lower mean errors providing support for their inclusion. However, skewness and kurtosis levels are of similar magnitude regardless of liquidity premium treatment, with all models evidencing non-normality.

The skewness evident in the standardised error is opposite in sign to the unstandardised error. As shown in Table 4.8, the prediction errors exhibit a negative skew consistent with our finding of underprediction of spreads at short maturities and high ratings. The positive skewness of the standardised errors is most likely a result of our sample data. Where there are sudden rises in market credit spreads, the step-ahead prediction error is negative, but because we find that these movements are associated with periods of high volatility in spreads, the prediction errors are scaled relatively more when standardised compared to sudden falls in market spreads. Quantile-Quantile (QQ) plots of the standardised errors are shown in Figure 4.8. The straight line in the QQ plot maps the expected standardised data under the normal cumulative frequency distribution against the observed cumulative distribution of errors. The upward curvature of the observed data in the upper right of the plots shows that positive prediction errors that would be expected to be three standard deviations from the mean are, for all models, approximately five standard deviations from the mean. However, other than the tails being fatter than expected, Figure 4.8 shows that the errors for all models are reasonably similar to the expected normal distribution. The behavior of the standardised prediction errors across maturity and rating dimensions is illustrated in Figures 4.9 to 4.14. In the top panels the standardised prediction errors are plotted against the remaining maturity at the date of trade for bonds rated AA and A. The middle panels includes BBB rated bonds, and the bottom panels comprise BB rated bonds. The vertical axes are measured in standard deviations. The pattern exhibited by all models is remarkably similar with all bonds

Table 4.7: This table shows the descriptive statistics of the standardised step-ahead prediction errors pooled across all trades by model. Sample means, standard deviations (SD), minimums, and maximums are reported in basis points. BS refers to the Bowman & Shenton (1975) sample test statistic for normality.

	EM	LS1	LT	CEV	LS2	CDG
			Panel A: No Liquidity			
Mean	0.04	0.09	0.10	0.14	0.17	0.38
SD	0.98	0.97	0.99	0.95	0.94	1.02
Skewness	0.86	0.99	0.56	0.90	0.98	0.85
Exc. Kurtosis	5.33	5.99	4.57	6.87	5.26	4.08
Min	-5.69	-5.48	-6.83	-9.82	-4.36	-4.48
Max	7.78	7.48	7.35	8.22	8.54	8.61
BS	11,703	14,866	8,252	18,806	11,749	7,287
			Panel B: Constant Liquidity			
Mean	0.03	0.05	0.03	0.09	0.08	0.16
SD	1.03	1.02	1.02	1.00	1.02	1.06
Skewness	0.94	1.05	0.88	1.09	0.98	1.02
Exc. Kurtosis	6.08	5.79	5.62	8.26	5.21	4.96
Min	-5.53	-4.83	-6.82	-7.16	-5.07	-5.21
Max	8.58	8.46	8.23	11.24	9.18	8.89
BS	15,118	14,164	12,923	27,215	11,568	10,727
			Panel C: Time-Varying Liquidity			
Mean	0.03	0.05	0.03	0.08	0.07	0.14
SD	1.03	1.02	1.02	1.00	1.03	1.08
Skewness	0.89	0.94	0.79	0.95	0.96	0.87
Exc. Kurtosis	6.09	6.05	5.43	6.40	5.60	5.17
Min	-5.68	-5.02	-5.94	-5.22	-5.41	-5.55
Max	8.69	8.62	8.44	8.43	9.23	8.87
BS	15,029	14,978	11,933	16,631	13,057	11,090
n	8,953	8,953	8,953	8,953	8,953	8,953

Table 4.8: This table shows the descriptive statistics of the step-ahead prediction errors pooled across all trades by model. Sample means, standard deviations (SD), minimums, and maximums are reported in basis points. BS refers to the Bowman & Shenton (1975) sample test statistic for normality.

	EM	LS1	LT	CEV	LS2	CDG
			Panel A: No liquidity			
Mean	-0.20	-1.08	1.50	4.47	3.01	13.65
SD	67.54	129.31	65.79	61.21	57.24	50.95
Skewness	-39.83	-47.51	-19.74	-20.14	-8.47	-7.21
Exc. Kurt.	2,526.40	2,674.10	752.32	1,326.50	274.08	426.40
Min	-4,566.30	-7,849.30	-2871.10	-3,521.50	-1,996.90	-2,182.60
Max	709.47	904.44	628.09	1,408.60	867.59	830.47
BS	$2 \cdot 10^9$	$3 \cdot 10^9$	$2 \cdot 10^8$	$7 \cdot 10^8$	$3 \cdot 10^7$	$7 \cdot 10^7$
			Panel B: Constant Liquidity			
Mean	0.50	-1.83	-0.90	1.67	-0.84	4.64
SD	37.85	111.84	60.61	68.35	199.43	45.67
Skewness	-0.51	-41.96	-26.01	-23.32	-87.38	-12.47
Exc. Kurt.	313.07	2,085.80	1,110.40	1,694.60	8,040.60	746.74
Min	-1,378.60	-6,101.40	-3,037.80	-4,051.30	-1,8354.00	-2,266.50
Max	1,062.40	773.19	557.18	2,182.70	1390.00	734.81
BS	$4 \cdot 10^7$	$2 \cdot 10^9$	$5 \cdot 10^8$	$1 \cdot 10^9$	$2 \cdot 10^{10}$	$2 \cdot 10^8$
			Panel C: Time-Varying Liquidity			
Mean	-0.38	-1.25	-0.70	1.48	1.39	4.09
SD	62.14	100.75	59.07	57.57	42.99	45.32
Skewness	-43.50	-48.92	-27.35	-28.19	-11.00	-12.75
Exc. Kurt.	2,890.10	2,925.70	1,217.40	1,938.20	680.30	768.93
Min	-4,358.20	-6,753.90	-3,037.70	-3,654.20	-2,085.30	-2,266.20
Max	728.87	785.09	557.17	1,396.10	817.35	733.92
BS	$3 \cdot 10^9$	$3 \cdot 10^9$	$6 \cdot 10^8$	$1 \cdot 10^9$	$2 \cdot 10^8$	$2 \cdot 10^8$
n	8,953	8,953	8,953	8,953	8,953	8,953

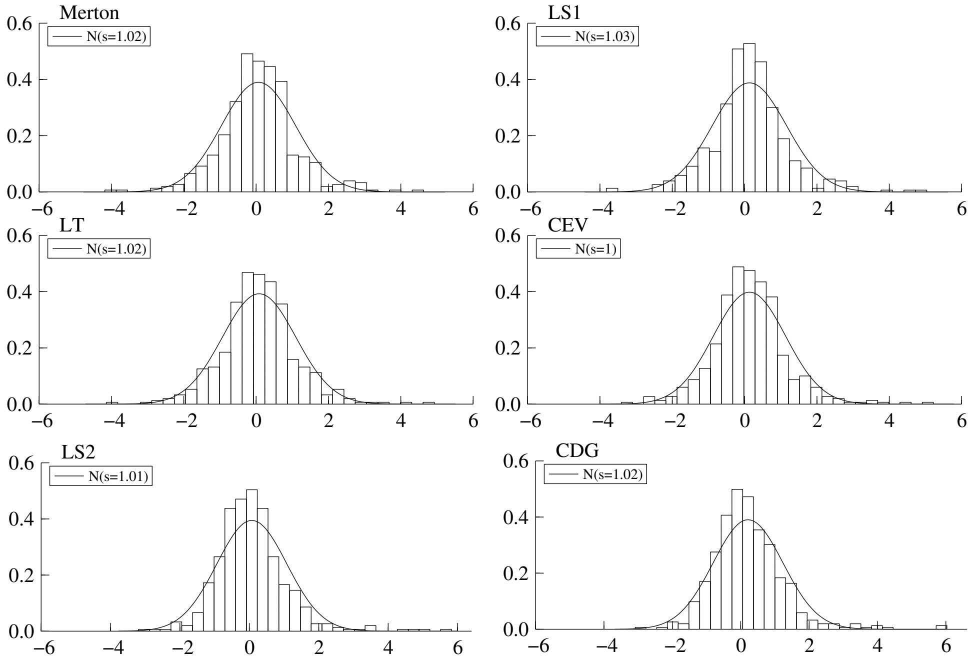


Figure 4.7: Histogram of the pooled sample standardised step-ahead spread prediction error by model with time-varying liquidity premium. Comparative density is normal with the same mean and variance.

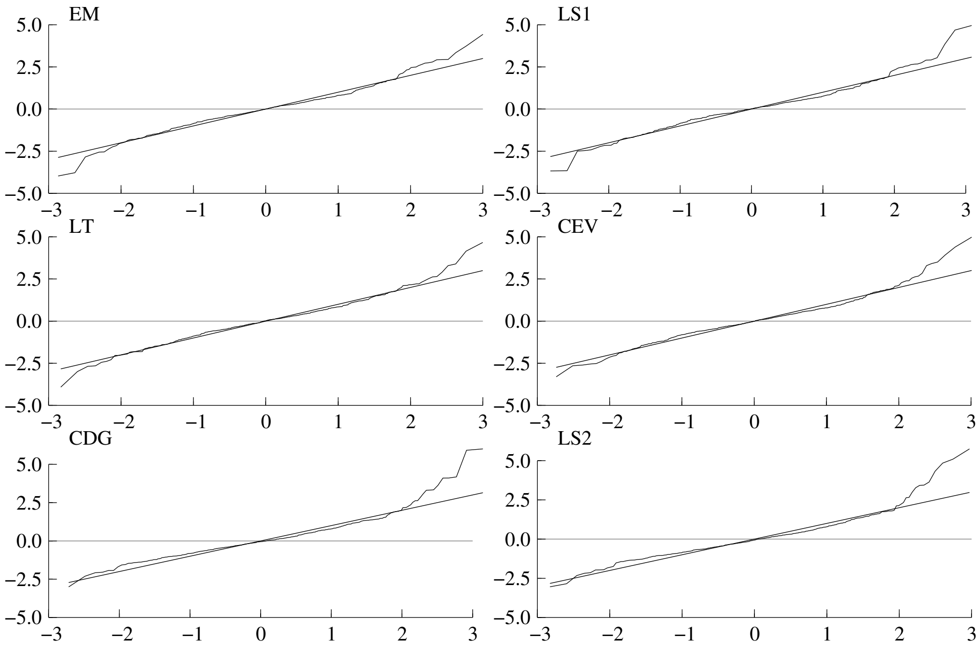


Figure 4.8: Shown is a Normal Quantile-Quantile plot of the pooled sample standardised step-ahead spread prediction error by model with time-varying liquidity premium. Horizontal axis is the expected sample quantile assuming a normal distribution and vertical axis is the quantiles of the observed sample.

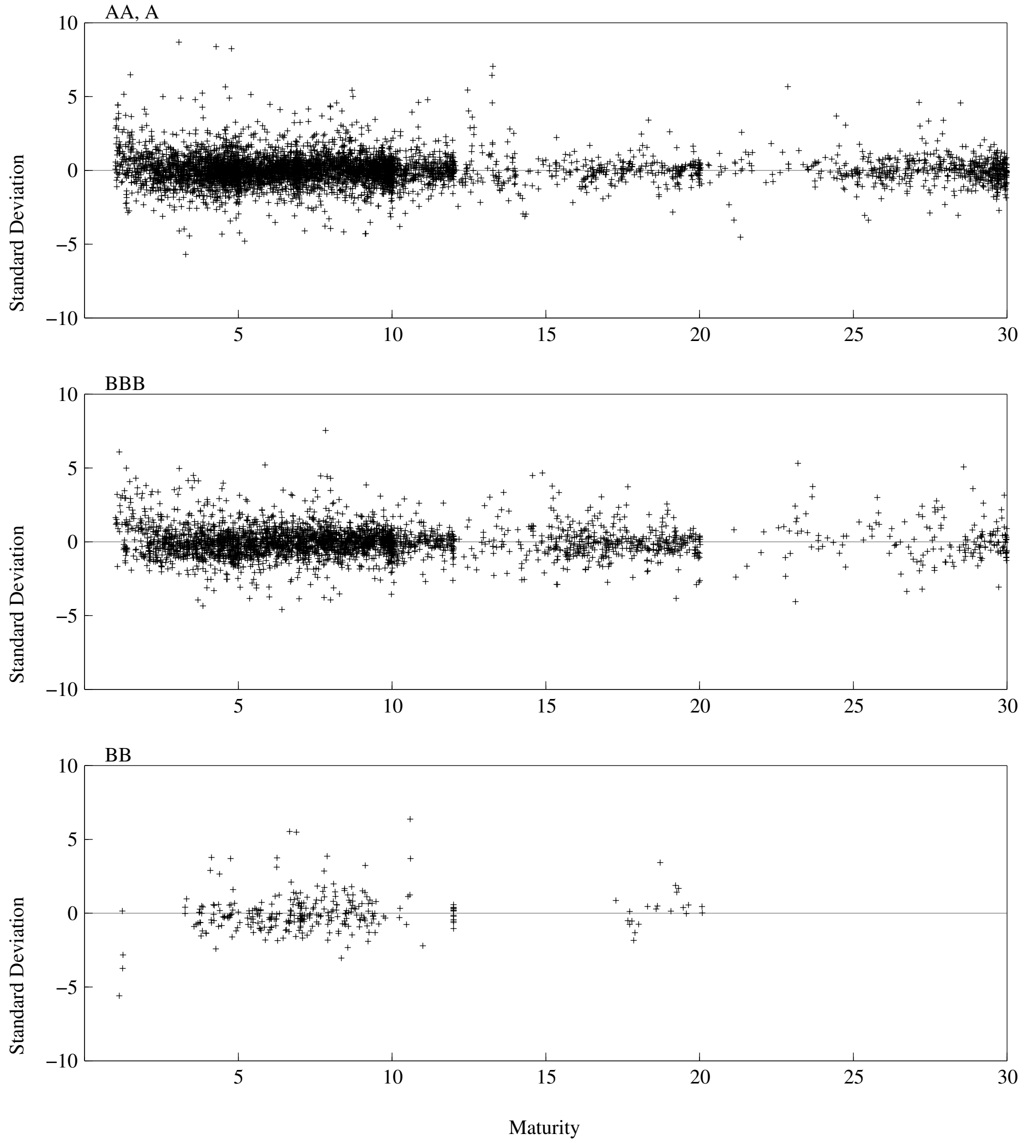


Figure 4.9: EM standardised spread prediction errors by rating and remaining term-to-maturity with time-varying liquidity premium. Prediction error is shown in standard deviations of basis points and maturity is in years.

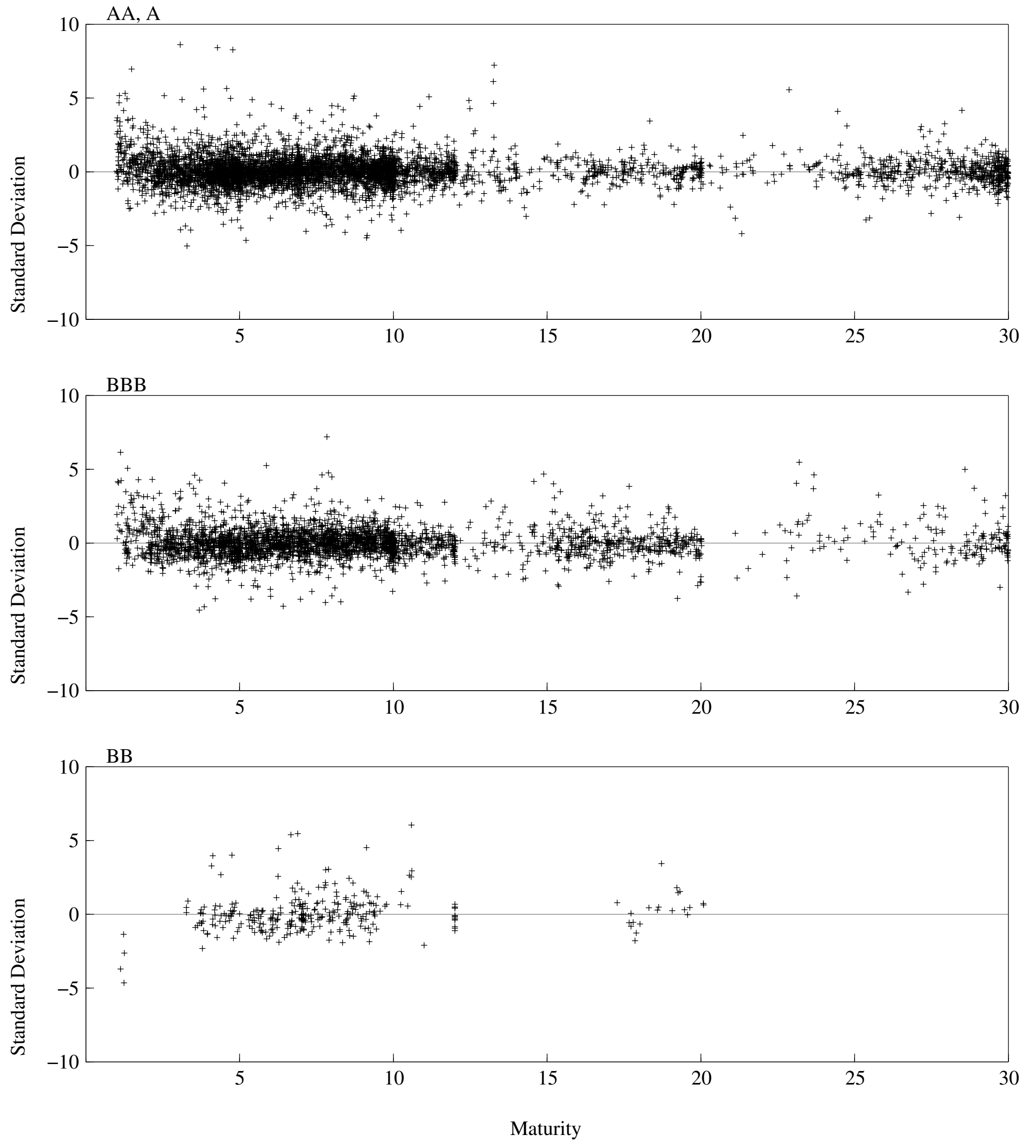


Figure 4.10: LS1 standardised spread prediction errors by rating and remaining term-to-maturity with time-varying liquidity premium. Prediction error is shown in standard deviations of basis points and maturity is in years.

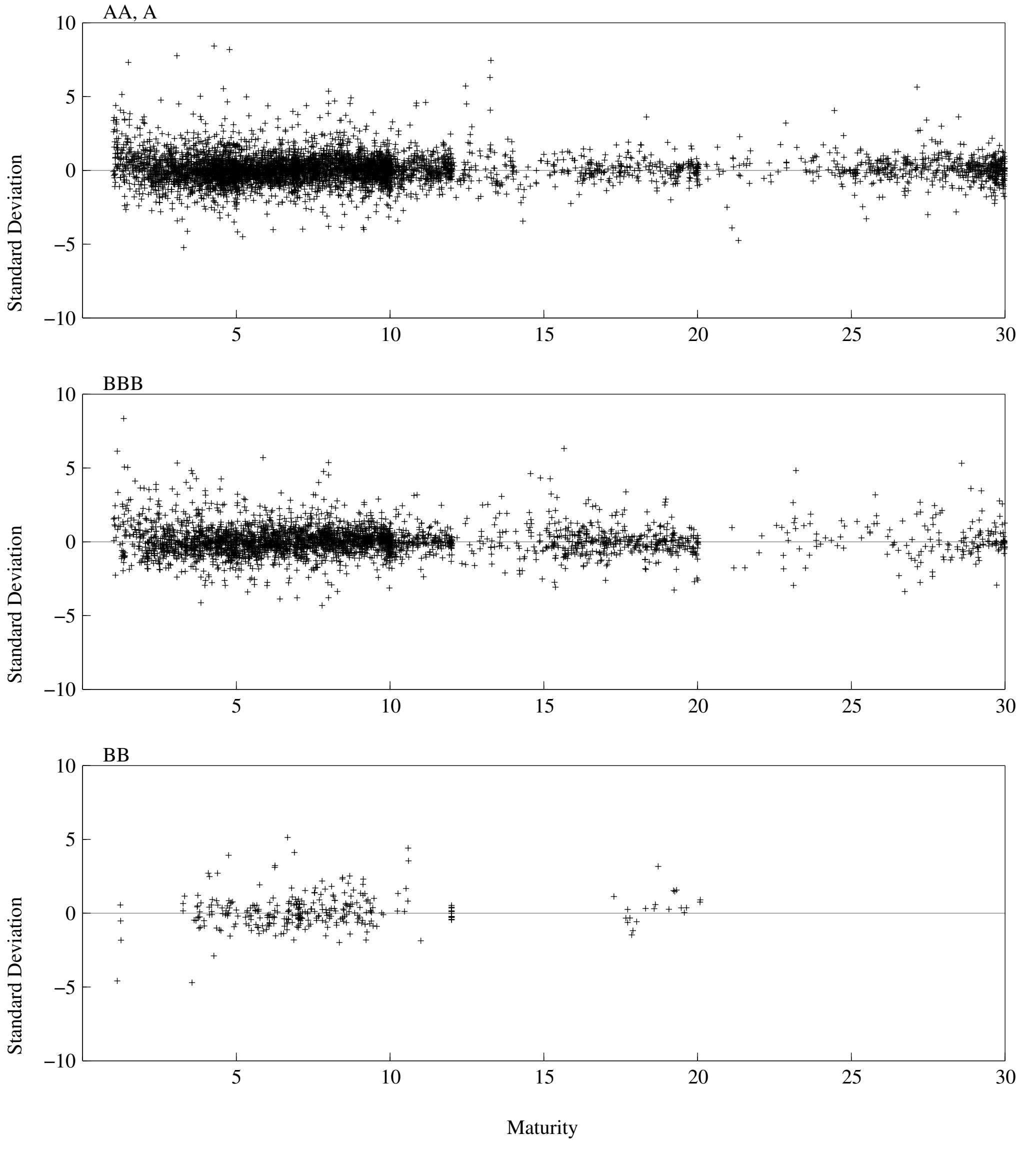


Figure 4.11: CEV standardised spread prediction errors by rating and remaining term-to-maturity with time-varying liquidity premium. Prediction error is shown in standard deviations of basis points and maturity is in years.

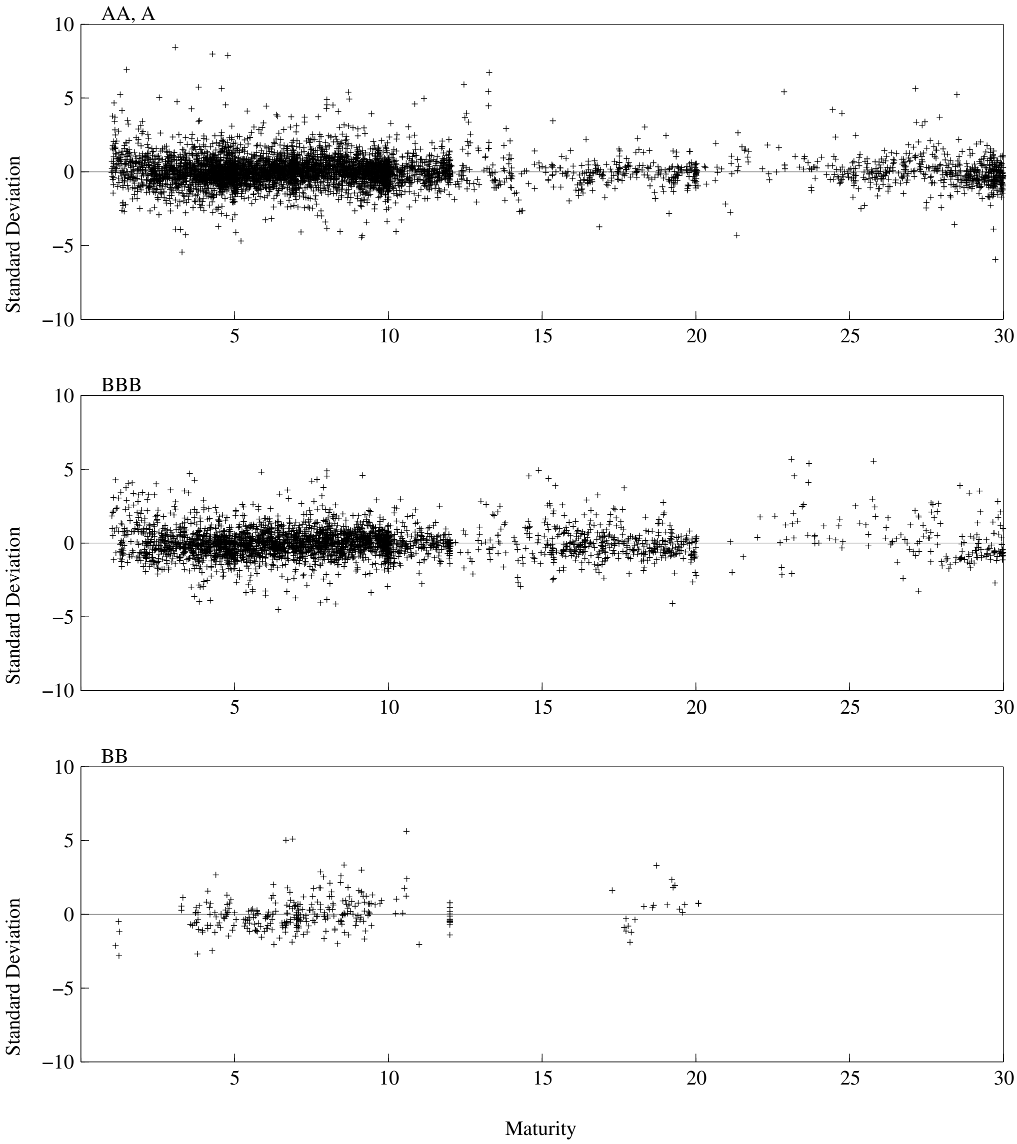


Figure 4.12: LT standardised spread prediction errors by rating and remaining term-to-maturity with time-varying liquidity premium. Prediction error is shown in standard deviations of basis points and maturity is in years.

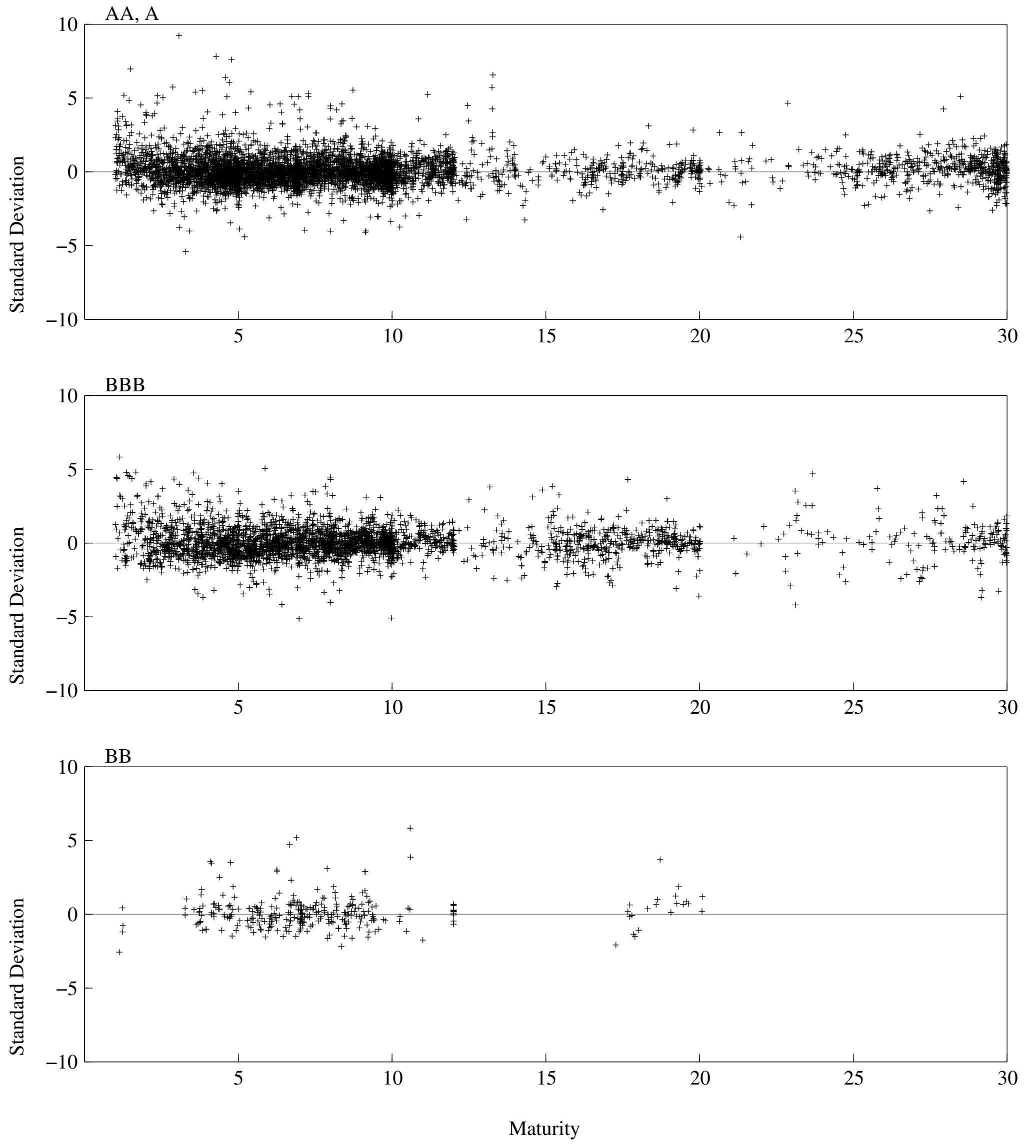


Figure 4.13: LS2 standardised spread prediction errors by rating and remaining term-to-maturity with time-varying liquidity premium. Prediction error is shown in standard deviations of basis points and maturity is in years.

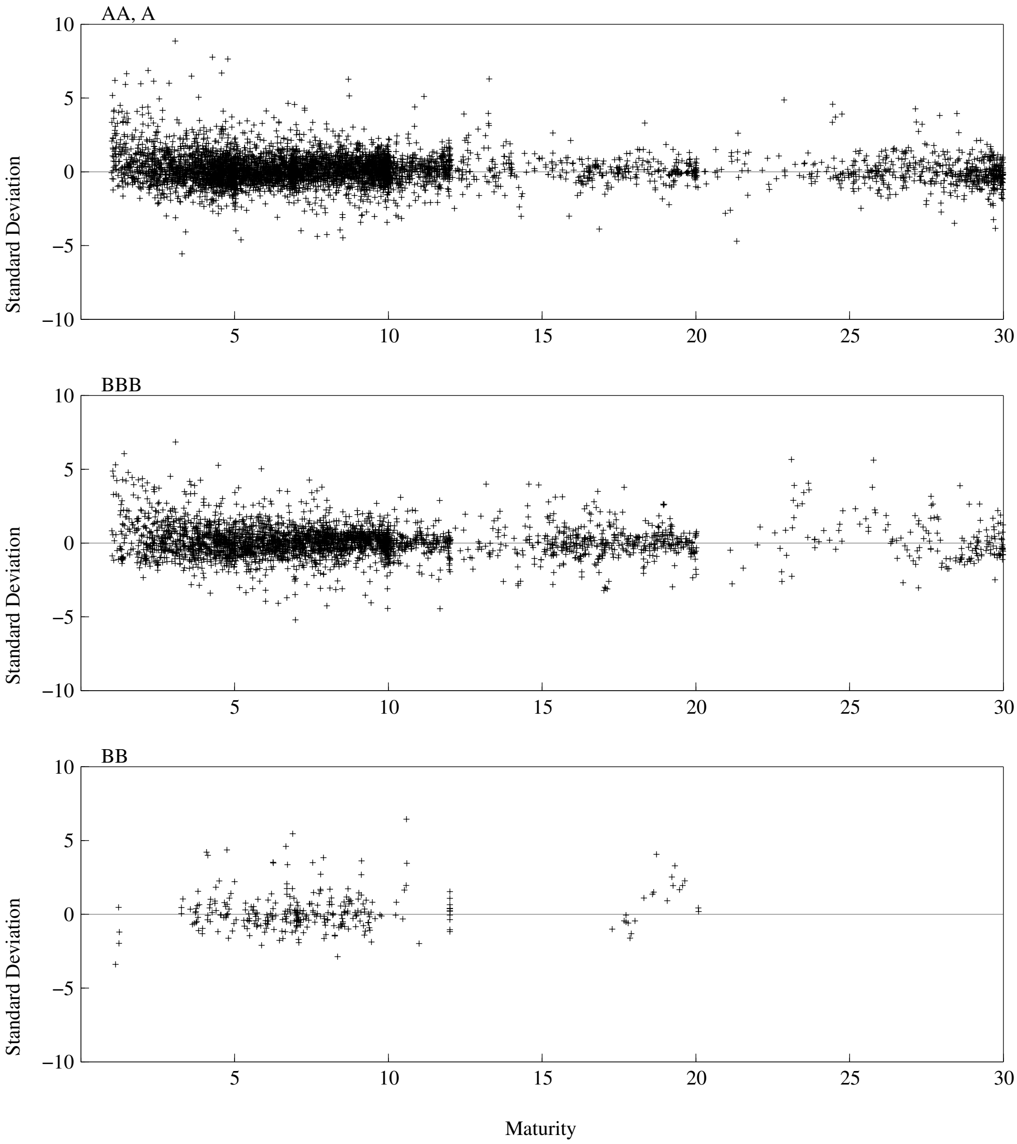


Figure 4.14: CDG standardised spread prediction errors by rating and remaining term-to-maturity with time-varying liquidity premium. Prediction error is shown in standard deviations of basis points and maturity is in years.

Table 4.9: This table shows the mean autocorrelation function of the standardised prediction errors for lags one to three, averaged across two hundred bonds in the sample.

Lags	EM	LS1	LT	CEV	LS2	CDG
Panel A: No liquidity						
1	0.158	0.195	0.270	0.216	0.231	0.230
2	0.084	0.106	0.161	0.113	0.147	0.126
3	0.081	0.090	0.133	0.099	0.119	0.107
Panel B: Constant Liquidity						
1	0.144	0.216	0.210	0.176	0.278	0.270
2	0.065	0.116	0.115	0.090	0.185	0.162
3	0.054	0.081	0.090	0.068	0.142	0.136
Panel C: Time-varying Liquidity						
1	0.137	0.164	0.181	0.156	0.264	0.159
2	0.062	0.084	0.094	0.079	0.175	0.078
3	0.045	0.058	0.066	0.059	0.131	0.056

showing a wide dispersion centered around zero. The errors reach approximately plus or minus five standard deviations consistent with the finding of excess kurtosis, and is only slightly greater for short maturities.

Next we consider whether the longitudinal specification of the structural models is correct by examining whether the standardised errors are serially independent. Serial correlation in the standardised errors would indicate that a systematic movement in the observed credit spreads remains unexplained by the model's dynamic specification implying the presence of a missing factor influencing credit spread inter-temporal behaviour. There are 200 separate bonds in the sample on which the autocorrelation functions (ACFs) are computed for the first, second, and third lags. To summarise the results across the large number of bonds, the bond specific sample ACF values for one to three lags are compared with their respective 95 percent confidence levels computed as $2/\sqrt{n}$ for n trades per bond. The number of bonds with significant ACFs are then counted. Because the time steps between trades are unequally spaced, the ACF significance tests are approximate only. However, since each model is fitted to the same data, counting the number of issues with significant ACFs is suitable for comparing the *relative* extent of autocorrelation between models and measurement equation specifications. The mean ACFs are reported in Table 4.9 and the counts of significant ACFs is shown in Table 4.10. Table 4.9 shows that all models exhibit some degree of serial correlation in their standardised errors. The highest levels are associated with models fitted without liquidity premiums, as shown in Panel A. The LT model has the largest mean first lag correlation at 27.0 percent, and the lowest is the EM model at 15.8 percent. The level of ACFs drop as the lags are increased demonstrating that, on average, the errors are stationary and likely to be an AR(1) process. Turning to Panel A of Table 4.10, the cor-

Table 4.10: This table shows the count of bonds with autocorrelation functions of the standardised prediction errors that exceed 95 percent confidence. The relative sample frequency of bonds with significant lags is shown in parentheses. Total number of sample issues is two hundred.

Lags	EM	LS1	LT	CEV	LS2	CDG
			Panel A: No liquidity			
1	48	56	84	67	74	67
	(24.0)	(28.0)	(42.0)	(33.5)	(37.0)	(33.5)
2	30	36	48	40	43	36
	(15.0)	(18.0)	(24.0)	(20.0)	(21.5)	(18.0)
3	27	28	38	27	34	29
	(13.5)	(14.0)	(19.0)	(13.5)	(17.0)	(14.5)
			Panel B: Constant Liquidity			
1	38	59	57	47	85	80
	(19.0)	(29.5)	(28.5)	(23.5)	(42.5)	(40.0)
2	28	35	37	22	53	47
	(14.0)	(17.5)	(18.5)	(11.0)	(26.5)	(23.5)
3	12	23	21	16	37	36
	(6.0)	(11.5)	(10.5)	(8.0)	(18.5)	(18.0)
			Panel C: Time-varying Liquidity			
1	36	44	46	41	85	41
	(18.0)	(22.0)	(23.0)	(20.5)	(42.5)	(20.5)
2	22	30	28	21	45	19
	(11.0)	(15.0)	(14.0)	(10.5)	(22.5)	(9.5)
3	10	15	15	16	31	12
	(5.0)	(7.5)	(7.5)	(8.0)	(15.5)	(6.0)

responding counts of significant ACFs shows that the LT model has 84 from 200 bonds where the first lag ACF is significant at 95 percent confidence. This count represents 42 percent of the sample of bonds and is displayed in parentheses. In contrast the EM model has only 48 bonds that have significant ACFs, which represents 24 percent of the sample. By the third lag, the EM model mean ACF declines to 8.1 percent and the LT model mean ACF to 13.3 percent. In terms of counts, the EM model has 13.5 percent of the sample with significant ACFs, and the LT model 19.0 percent.

With the introduction of a constant liquidity premium, the overall average level of ACFs drops as show in Panel B of Table 4.9, indicating that structural models fit the time-series of observed credit spreads better when the allowance is made for a liquidity premium in the credit spread. At first order lags, the EM model and CEV models have the lowest ACFs at 14.4 percent and 17.6 percent respectively. The LS2 model has the highest ACF at 27.8 percent.

The introduction of a time-varying liquidity premium is expected to further decrease serial correlation since it should control for common firm-level time variation in credit spreads caused by exogenous flight-to-liquidity events such as the Russian bond-LTCM crises. Panel C of Table 4.9 confirms a further reduction in serial correlation is achieved as expected, indicating a further improvement in model fit. However, some autocorrelation remains. The best fitting model is the EM model with a first lag ACF of 13.7 percent, which corresponds to 36 bonds with significant ACFs representing 18 percent of the sample. By the third lag the EM model's mean ACF has fallen to 4.5 percent, or 10 bonds representing 5 percent of the sample. Thus, the EM model exhibits modest autocorrelation in errors. The next best fitting model is the CEV model, followed closely by the CDG model. The worst performing model is the LS2 model suggesting that the introduction of stochastic interest rates has not improved model performance relative to a single factor LS1 model.

The relatively good time-series behaviour of the EM model's errors is surprising. The model is in all respects the same as the LS1 model, except that the recovery rate is endogenous, being a function of the firm's distance from the default boundary, asset volatility, and term. Furthermore, by applying the sum of zeros method of valuation, we allow the recovery rate to vary across the term structure with each coupon valued, and with time as the latent solvency of the firm changes. This additional flexibility appears to be the main reason for the Merton model's relatively better time-series behaviour of its prediction errors.

4.2.1 Goodness of Fit

In this section we consider which model fits the data best. Because we have models with different numbers of parameters it is important when comparing the relative goodness of fit between models that we control for the different number of parameters used in each model. Simple measures of fit, such as the mean percentage error (MPE) and root mean squared error (RMSE), tend to overstate the goodness of fit of models that have the largest number of parameters. For this reason we also quote the widely used Akaike Information Criterion (AIC) to give a better relative comparison of fit between models. The AIC is defined as

$$\text{AIC} = 2(\text{LogLikelihood}) + 2K,$$

where K is the number of estimated parameters included in the model. A lower value represents a better fit.

In Table 4.11 we show the mean RMSE and mean AIC per model, measured across the 32 sample firms, assuming no liquidity premium. We find that the EM model is the best fitting having both the smallest RMSE of 33.97 basis points and lowest AIC of -10.51. The next best fitting model is the LS1 model with a RMSE of 43.66 basis points and AIC of -10.08. The worst performing model is the CDG model which achieves a reasonable RMSE of 40.98 basis points, but does so using more parameters, therefore increasing its AIC to -9.52.

In Table 4.12 we show the mean RMSE and mean AIC where the measurement equation includes a constant liquidity premium. Compared to the no liquidity case, overall fit is found to improve. The EM model remains the best fitting with a RMSE of 28.76 basis points and AIC of -10.76. The second best model is now the LT model with an RMSE of 38.66 basis points and AIC of -10.50. The worst performing model remains the CDG model despite having the second lowest RMSE with an AIC of -10.15, showing that the range of disparity between models has decreased with the addition of controls for non-default related components of the spread. Lastly, Table 4.13 shows results for the models estimated with both a constant term and Refcorp-Treaury spread as controls for time varying liquidity. Across the board, all models improve in fit, in particular the CDG model improves to be third ranked behind the EM and LS1 models based on AIC. The EM model remains the best fitting mode with a RMSE of 28.76 basis points and AIC of -10.76. The worst fitting model is the LS2 model with a RMSE of 31.57 basis points and an AIC of -10.43.

A criticism raised against structural models is that they cannot match real world spreads. For example, Lyden & Saraniti (2000, Table III) report a mean RMSE of 107.44 basis points for the Merton model, 199.34 basis points for an LS1 equivalent model, and 200.64 basis points for an LS2 equivalent model. It is therefore important to compare our results against another study that fits the alternative reduced-form model approach with a similar implicit estimation. Duffee (1999), also use an EKF on similar data and reports a median RMSE of 9.38 basis points for a two-factor square-root reduced-form model. In comparison, the median RMSE we achieve for the EM model ranges from 14.06 basis points (constant liquidity) to 20.13 basis points (time-varying liquidity) (refer Tables 4.12 and 4.13). Obviously, direct comparisons are not possible due to different samples and periods, in particular, our data set includes additional spread volatility associated with the credit downturn of 2000 which may bias our model error upwards. On balance, our results therefore appear reasonable with respect to Duffee (1999), thereby confirming that, at least, some of the criticism of structural models is a consequence of the difficulty in fitting these models.

Table 4.11: This table presents the cross-sectional means and medians of the estimated model parameters, fitted with no liquidity premium. All parameters are implicit estimations from application of quasi-maximum likelihood via EKF (i.e. hyperparameters). Mean RMSE refers to cross-sectional average root mean squared error of the credit spread prediction. Mean Log-Lik. is the cross-sectional average of the maximised log-likelihood function value under the hyperparameter set. Mean AIC is the cross-sectional average of the firm-specific Akaike Information Criterion.

	EM		LS1		LT		CEV		LS2		CDG	
Parameter	Med.	Mean	Med.	Mean	Med.	Mean	Med.	Mean	Med.	Mean	Med.	Mean
σ_v	0.2305	0.2668	0.2393	0.2505	0.1774	0.2079	-	-	0.1386	0.1411	0.1161	0.1326
σ_m	0.0018	0.0020	0.0022	0.0022	0.0026	0.0028	0.0024	0.0028	0.0024	0.0026	0.0029	0.0032
δ	0.0502	0.0425	0.0000	0.0173	0.0587	0.0558	0.0399	0.0341	0.0000	0.0006	-	-
α	-	-	-	-	0.4804	0.4236	-	-	-	-	-	-
ρ	-	-	-	-	-	-	- 0.8866	- 1.2727	-	-	-	-
κ_v	-	-	-	-	-	-	-	-	-	-	0.1004	0.1221
ϕ	-	-	-	-	-	-	-	-	-	-	0.0000	0.0000
Mean RMSE	17.68	33.97	20.21	43.66	22.28	44.83	20.30	37.22	36.56	45.37	35.47	40.98
Mean Log-Lik.	1,108.6	1,267.1	1,079.2	1,228.4	1,037.3	1,202.5	1,053.7	1,212.4	1,043.1	1,200.0	1,005.2	1,164.2
Mean AIC	-10.57	-10.51	-10.16	-10.08	-9.95	-9.74	-10.19	-9.90	-9.84	-9.77	-9.47	-9.52

Table 4.12: This table presents the cross-sectional means and medians of the estimated model parameters, fitted with constant liquidity premium. All parameters are implicit estimations from application of quasi-maximum likelihood via EKF (i.e. hyperparameters). Mean RMSE refers to cross-sectional average root mean squared error of the credit spread prediction. Mean Log-Lik. is the cross-sectional average of the maximised log-likelihood function value under the hyperparameter set. Mean AIC is the cross-sectional average of the firm-specific Akaike Information Criterion.

	EM		LS1		LT		CEV		LS2		CDG	
Parameter	Med.	Mean	Med.	Mean	Med.	Mean	Med.	Mean	Med.	Mean	Med.	Mean
σ_v	0.2477	0.2807	0.1711	0.1773	0.1449	0.1672	-	-	0.1541	0.1538	0.1480	0.1641
σ_m	0.0015	0.0018	0.0017	0.0020	0.0017	0.0019	0.0016	0.0023	0.0018	0.0023	0.0021	0.0024
δ	0.0154	0.0238	0.0000	0.0201	0.0276	0.0251	0.0135	0.0181	0.0000	0.0016	-	-
α	-	-	-	-	0.5014	0.5010	-	-	-	-	-	-
$\bar{\sigma}_v$	-	-	-	-	-	-	0.3443	1389.1906	-	-	-	-
ρ	-	-	-	-	-	-	-1.4361	-1.9605	-	-	-	-
κ_v	-	-	-	-	-	-	-	-	-	-	0.1375	0.1318
ϕ	-	-	-	-	-	-	-	-	-	-	0.0000	0.0000
c(1)	0.0045	0.0044	0.0052	0.0053	0.0048	0.0050	0.0055	0.0054	0.0061	0.0059	0.0039	0.0039
c(2)	0.0041	0.0040	0.0050	0.0050	0.0046	0.0049	0.0051	0.0053	0.0061	0.0058	0.0031	0.0036
c(3)	0.0043	0.0046	0.0055	0.0060	0.0052	0.0061	0.0057	0.0062	0.0055	0.0054	0.0034	0.0030
c(4)	0.0040	0.0046	0.0057	0.0057	0.0058	0.0058	0.0052	0.0064	0.0055	0.0060	0.0030	0.0035
c(5)	0.0037	0.0039	0.0045	0.0049	0.0042	0.0046	0.0052	0.0052	0.0054	0.0057	0.0041	0.0036
c(6)	0.0040	0.0039	0.0050	0.0051	0.0048	0.0050	0.0050	0.0053	0.0044	0.0054	0.0013	0.0022
c(7)	0.0039	0.0043	0.0052	0.0059	0.0050	0.0058	0.0056	0.0058	0.0049	0.0054	0.0029	0.0037
c(8)	0.0034	0.0040	0.0043	0.0053	0.0046	0.0051	0.0045	0.0052	0.0046	0.0057	0.0033	0.0038
c(9)	0.0049	0.0050	0.0060	0.0063	0.0060	0.0065	0.0061	0.0061	0.0049	0.0050	0.0030	0.0031
c(10)	0.0037	0.0037	0.0040	0.0042	0.0049	0.0046	0.0054	0.0048	0.0050	0.0050	0.0000	0.0005
Mean RMSE	14.06	27.10	16.36	38.66	15.97	35.77	15.21	32.73	27.59	44.48	26.93	32.29
Mean Log-Lik.	1,151.4	1,318.9	1,125.4	1,293.8	1,132.6	1,309.1	1,133.0	1,296.5	1,160.8	1,280.7	1,065.9	1,252.1
Mean AIC	-10.84	-10.73	-10.64	-10.50	-10.79	-10.57	-10.80	-10.52	-10.48	-10.36	-10.28	-10.15

Table 4.13: This table presents the cross-sectional means and medians of the estimated model parameters, fitted with time-varying liquidity premium. All parameters are implicit estimations from application of quasi-maximum likelihood via EKF (i.e. hyperparameters). Mean RMSE refers to cross-sectional average root mean squared error of the credit spread prediction. Mean Log-Lik. is the cross-sectional average of the maximised log-likelihood function value under the hyperparameter set. Mean AIC is the cross-sectional average of the firm-specific Akaike Information Criterion.

	EM		LS1		LT		CEV		LS2		CDG	
Parameter	Med.	Mean	Med.	Mean	Med.	Mean	Med.	Mean	Med.	Mean	Med.	Mean
σ_v	0.2657	0.2827	0.1886	0.1830	0.1386	0.1662	-	-	0.1559	0.1418	0.1375	0.1498
σ_m	0.0015	0.0017	0.0016	0.0018	0.0017	0.0019	0.0016	0.0020	0.0020	0.0022	0.0020	0.0023
δ	0.0063	0.0212	0.0000	0.0161	0.0254	0.0240	0.0000	0.0142	0.0000	0.0028	-	-
α	-	-	-	-	0.5628	0.4905	-	-	-	-	-	-
$\bar{\sigma}_v$	-	-	-	-	-	-	0.3073	9.5517	-	-	-	-
ρ	-	-	-	-	-	-	-1.0333	-1.6268	-	-	-	-
κ_v	-	-	-	-	-	-	-	-	-	-	0.1457	0.1422
ϕ	-	-	-	-	-	-	-	-	-	-	0.0000	0.0031
Ref	0.3875	0.4450	0.4672	0.5818	0.5822	0.7607	0.6119	0.6397	1.0911	1.1175	0.8529	0.8061
c(1)	0.0042	0.0038	0.0043	0.0041	0.0036	0.0038	0.0040	0.0043	0.0043	0.0041	0.0024	0.0026
c(2)	0.0037	0.0035	0.0038	0.0040	0.0034	0.0037	0.0041	0.0043	0.0037	0.0041	0.0021	0.0026
c(3)	0.0039	0.0041	0.0045	0.0047	0.0044	0.0049	0.0045	0.0050	0.0043	0.0039	0.0024	0.0026
c(4)	0.0036	0.0038	0.0044	0.0045	0.0049	0.0046	0.0041	0.0051	0.0037	0.0043	0.0026	0.0030
c(5)	0.0032	0.0034	0.0035	0.0041	0.0035	0.0035	0.0037	0.0039	0.0040	0.0041	0.0024	0.0027
c(6)	0.0037	0.0034	0.0045	0.0043	0.0037	0.0040	0.0040	0.0043	0.0029	0.0034	0.0009	0.0019
c(7)	0.0036	0.0038	0.0049	0.0053	0.0045	0.0049	0.0046	0.0049	0.0038	0.0041	0.0017	0.0027
c(8)	0.0032	0.0034	0.0040	0.0047	0.0037	0.0041	0.0032	0.0043	0.0033	0.0038	0.0026	0.0032
c(9)	0.0044	0.0044	0.0051	0.0051	0.0051	0.0052	0.0057	0.0051	0.0031	0.0036	0.0024	0.0033
c(10)	0.0034	0.0034	0.0038	0.0040	0.0046	0.0041	0.0047	0.0041	0.0039	0.0038	0.0047	0.0041
Mean RMSE	20.13	28.76	21.61	34.40	23.18	33.80	22.07	29.49	24.85	30.90	26.52	31.57
Mean Log-Lik.	1,157.3	1,323.5	1,159.7	1,313.7	1,140.7	1,317.8	1,142.8	1,306.5	1,165.9	1,288.3	1,067.9	1,266.1
Mean AIC	-10.86	-10.76	-10.76	-10.67	-10.79	-10.64	-10.77	-10.61	-10.43	-10.43	-10.80	-10.66

4.2.2 Regression of Prediction Errors

In the preceding section it was noted that the models exhibit cross-sectional prediction biases related to rating and maturity, and autocorrelation in the time-series of prediction errors. In this section we use multivariate regression analysis on the pooled spread prediction errors to identify what plausible factors are related to the biases.

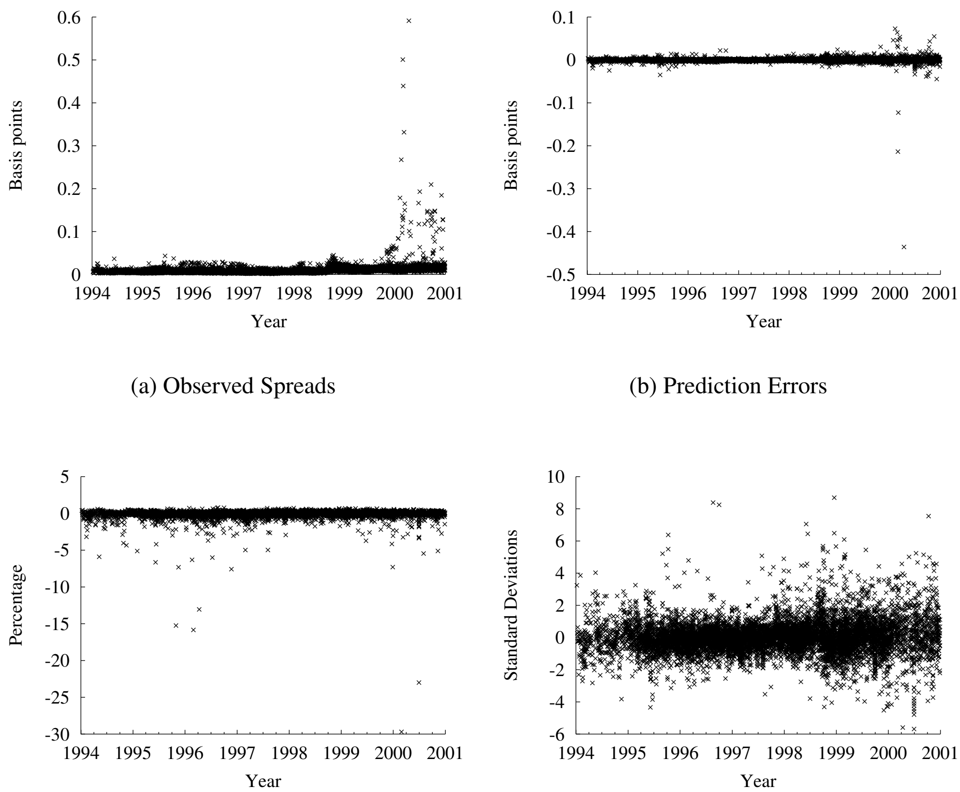
Our analysis considers three measures of error as the dependent variable: the unstandardised prediction error, the percentage prediction error, and the standardised prediction error. The unstandardised error is the simplest measure of prediction error, however, it is dependent on the size of the predicted spread, since the same relative error when spreads are large gives rise to larger errors. Within the sample, it is therefore weighted to the high default-risk bonds and periods of financial stress. The percentage spread prediction error scales for the relative size of the error, and is therefore more representative of all bonds in the sample, however, it is also strongly influenced by extreme jumps in observed spreads. The standardised prediction error is the most robust test of model specification within the Kalman filter framework, since each error is scaled by its standard deviation, which dampens the influence of measurement error and extreme outlier observations in the data.

The behaviour of the different error measures can be seen from Fig. 4.15 where the errors from the EM model are plotted across time. Shown in Fig. 4.15(a) is the full sample of observed credit spreads in sequential order of the trade date. The maximum sample credit spread observed within a year, increased during 1995-1996, at the end of 1998, and finally in late 2000. These dates coincide with a period of economic slowing in 1995-1996 (as seen in the fall in GDP growth in Fig. 4.16(a)), the LTCM and Russian bond crises of 1998, and the lead up to the recession of 2001.[1] The sample variance of prediction errors also increase in these periods as shown by the increased scatter of observations around zero in Fig. 4.15(b), but was reasonably symmetrical with under and overpredictions occurring. The percentage error, on the other hand, falls significantly in 1995-1996 and again in 2000 (refer Fig. 4.15(c)). This highlights that the greatest relative errors occurred during periods of economic downturn and were not influenced by the liquidity crises of late 1998. The large negative percentage error observations are associated with firm-specific downgrading and rise in default risk, and is more prevalent, in our data set, during contractionary periods. The distribution of the percentage error is not symmetric; there are no sudden relative decreases in our sample to balance the sudden increases. Finally, the standardised error shown in Fig. 4.15(d), has a similar intertemporal behaviour to the unstandardised error, and shows that the range of observed errors is outside that expected if the prediction errors were normal. The sample variance of the standardised error increases in the stress periods, responding to liquidity and

[1] U.S. recessions as dated by the NBER occurred from July 1990 to March 1991 and from March 2001 to November 2001.

economic conditions indicating heteroskedasticity in the pooled sample.

From an economic perspective, financial market participants are most concerned with a model's percentage prediction error, however, if the models are well specified, no exogenous factors or bond characteristics should be systematically related to any of the three dependent variables. In total, we perform 18 separate regressions, comprising



(a) Observed Spreads

(b) Prediction Errors

(c) Percentage Prediction Errors

(d) Standardised Prediction Errors

Figure 4.15: Shown in panel (a) is the actual credit spread by year for the pooled sample of all bonds fitted with time-varying liquidity parameterisation of the EM model. Panels (b) through to (d) are measures of error by year. Prediction error is the observed credit spread, less the step-ahead prediction, expressed in basis points. Percentage prediction error is prediction error scaled by the observed spread. Standardised prediction error is the prediction error scaled by the standard deviation of the error, expressed in standard deviations.

the three dependent variables and six structural models. For brevity, we restrict the analysis to the errors arising from the time-varying liquidity premium parameterisation of the models. Each regression is performed on pooled data of 8,953 prediction error observations, comprising 200 bonds issued by 32 firms.

Our choice of explanatory variables is informed from several sources. Firstly, we include bond characteristic and variables that are observable proxies for the firm asset process. It is not expected that these variables will be significant because their latent counterparts are included in the model specifications, however, previous research has identified related biases (JMR, EHH). We are observing whether these biases persist after the change from observable proxy to implicit estimation method. These variables are:

TENOR The remaining term to contractual maturity of the bond measured in years from the date of the trade. JMR, LS, and EHH report underprediction of credit spreads at short tenors, indicating a likely positive correlation with spread prediction error.

TEN5YR A dummy variable that takes the value one if the tenor is less than five years, or zero otherwise. As observed previously, spread underprediction is most prevalent in the very short maturity bonds where we also observe a number of extreme percentage errors. The absence of a jump component limits the ability for a structural model to match severe real-world changes in spreads at short maturities. We therefore control for this known weakness to separately quantify the very short term from any longer term tenor bias. It is expected that the coefficient on regressions against either spread levels or spread prediction errors will be negative, in other words, spreads will be lower and errors will be more negative, than average, when the variable takes the value of one.

SOLV The observed log-solvency ratio is defined previously in equation (3.27), and is calculated as the log of the sum of book debt (sourced from COMPUSTAT), plus stock market capitalisation (sourced from CRSP), divided by book debt. Book debt is updated quarterly and the stock data is the daily value as at the trade date. Based on the structural model arguments, the higher the solvency the lower the default risk and credit spread, implying a negative relationship with spread levels. JMR and EHH find greater underprediction is associated with very safe bonds issued by firms with low leverage. If consistent with prior findings, prediction errors will be negatively related to solvency.

COUPON The annualised coupon rate of the bond as sourced from the FISD database. Elton et al. (2001) note that bonds with higher coupons are taxed more through their life than bonds with lower coupons, thus increasing their required market spread in compensation. A negative relationship with prediction error is therefore implied. EHH confirm a negative relationship with percentage spread error in all models except their implementation of the LT model.

VOL This variable is the firm-specific, 150-day moving average annualised equity return standard deviation. Data is obtained from CRSP. Campbell & Taksler (2002) show that idiosyncratic firm equity volatility explains as much cross-sectional variation in corporate bond yields as ratings; the higher the firm's equity volatility the higher the required spread due to associated higher firm asset volatility and increased default risk. JMR and EHH find underprediction of credit spreads with firms of low equity volatility. All models, except the CEV model, assume asset return volatility to be time-invariant and this may also be a source of additional error. Higher firm-specific asset volatility, and the presence of stochastic asset volatility, is likely to be associated with increased asset risk, higher default risk and credit spreads, implying a negative relationship with spread prediction errors. Similarly, firms with high historical equity volatility are likely to find the equity markets expensive and delay issuing equity, thereby increasing leverage and default risk.

CMT3 The daily constant maturity 3 month Treasury spot rate sourced from the Federal Reserve H15 report. Consistent with structural models, a higher risk-free rate increases firm asset growth and decreases leverage, thereby reducing expected default risk and credit spreads, implying a negative relationship with spread levels and a positive relationship with prediction errors if omitted from a structural credit model.

VOLR The moving average standard deviation, measured over the prior 150 days, of the CMT 3 month spot rate. The single factor models assume no interest rate volatility, and the two-factor models (LS2 and CDG) assume a constant volatility. As shown by (Longstaff & Schwartz 1995), interest rate volatility is likely to increase the level of credit spreads. Insufficient control for this effect, or its complete absence in the case of the single-factor models, is likely to cause a negative relationship with prediction error. Any prediction error should be less negative, or insignificant, in the LS2 and CDG models.

Secondly, we include possible missing explanatory variables not included in the models.

TERM The 3 month less the 30 year constant maturity Treasury spot rates, measured on the trade date. Data is sourced from the Federal Reserve H15 report. Estrella & Mishkin (1998) demonstrate that the yield curve slope is predictive of future recessions when looking more than one-quarter into the future. They find a fall in long-term yields, relative to short-term yields, is an accurate predictor of slowing economic growth. Thus, we expect that credit spreads will *rise* in association with the yield curve flattening and the TERM variable increasing. The only model we test that includes a Treasury yield curve slope variable is the CDG model. For the other models, we expect the prediction error to be negatively correlated with TERM to the extent that the bond market uses the slope of the yield curve to condition their expectation of future default risk, and hence their pricing of bonds.

However, Jalilvand & Harris (1984) report that firms adjust their capital structure in response to the interest rate outlook. They propose that expectations of lower long term interest rates in the future (a rise in TERM) causes postponement of the issuance of long term debt, with firms increasing short term debt and equity financing, thereby decreasing leverage. Therefore, if debt timing is a significant influencing factor on management, and the bond market anticipates this behaviour, then credit spreads should *fall* as TERM increases because the bond market anticipates future decreasing leverage, thereby implying a positive correlation with prediction errors for all models except the CDG model. The CDG model embodies the stylised debt-timing behaviour of Jalilvand & Harris (1984) with the assumption that TERM and credit spreads are negatively correlated. We do not expect TERM to be related to CDG prediction errors if debt timing behaviour is factored into bond market expectations.

Thus, we are faced with two contradictory relationships between TERM and prediction errors. If the business cycle information content of the yield curve dominates bond market prices, the spread prediction errors are likely to be negatively related to TERM on all models. On the other hand, if the debt-timing hypothesis dominates, then positive coefficients against TERM can be expected on all models, except the CDG model, where the coefficient should be insignificant from zero.

RATING A variable that takes a numeric value depending upon the external rating of the issuer. For example, our sample is over the set AA+=1, AA=2, AA-=3, A+=4, ..., BB-=13. A rating represents an independent view of the creditworthiness of the issuer and is a major determinant of observed credit spreads. Clearly, credit spreads levels are expected to be positively related. We include it in the error regressions for any default-risk related influences not explained by the models. JMR and EHH find that underprediction of credit spreads is more common with well rated firms implying a positive error correlation.

MTB The market-to-book ratio at the trade date calculated as the daily equity capitalisation (sourced from CRSP) divided by the most recent quarterly reported total assets (sourced from COMPUSTAT). There are two possible mechanisms by which MTB influences credit spreads. Firstly, Hovakimian et al. (2001) find that firms with high MTB tend to issue more equity and decrease leverage. Firms with high MTB ratios may therefore be expected to decrease future leverage implying a reduction in future default risk and lower long-term credit spreads. The absence of MTB in structural models implies that the prediction error is expected to be positively related to MTB.

Secondly, Varma & Cantor (2004) report a positive relationship between firm MTB and bond recovery rates; firms with higher than average MTB one-year prior to default were found to have higher recovery rates, perhaps because the market believed these firms had continued growth potential, or persistent franchise value post default. If the market factors a firm's present MTB into debt pricing, an increase in MTB would be associated with a fall in market credit spreads, but if not included in the credit model specification, a positive prediction error results.

Via both mechanisms, credit spread levels are expected to be negatively related to MTB and prediction errors are expected to be positively related.

NYU The Altman-NYU Defaulted Bond Index return. It is the monthly 90 day cumulative return interpolated within the month to the trade date. Data is source from Altman & Pompeii (2003, Table A1). Since our models assume a constant recovery rate (excluding the EM model), realised cyclical variation may be related to systematic prediction error. A rise in the index should be associated with a fall in credit spreads as the market anticipates higher expected recovery values for the firm in default. This implies a positive spread prediction error across all models, except possibly the Merton model, where the recovery rate is endogenously linked to firm value.

VIX The daily value of the Chicago Board of Exchange VIX index, divided by 100. The VIX index is the implied volatility of a synthetic at-the-money option on the S&P 500 index.[2] It is a widely used benchmark measure of the market's expectation of risk. It is possible that the VIX index may influence credit spreads through two channels. The first is through expected recovery values. Trück, Harpaintner & Rachev (2005) find evidence that the VIX index can explain up to 80 percent of future aggregate yearly recovery rates on defaulted bonds. They find that low recoveries are historically anticipated by high implied volatility in historical stock options. Using principal components analysis on recovery rates extracted from credit default swap prices, Das & Hanouna (2006) give support, finding that the level of interest rates, which we proxy by CMT3, and VIX together, explain 87 percent of variation in implied recovery rates as estimated from credit default swap prices. The second channel is via changes in market-wide expectations of contagious defaults in response to unexpected market-wide credit events. Bierens, Huang & Kong (2003) determine that jumps play an important role in explaining the dynamics of Merrill Lynch daily series of option-adjusted credit spreads, and that the jump intensity depends on the lagged level of the VIX index. Collin-Dufresne et al. (2001) show changes in VIX to be highly significant in explaining the changes in firm-specific credit spreads, particularly at short tenors. A possible reason for the importance of the VIX index in explaining credit spreads is provided by Collin-Dufresne, Goldstein & Helwege (2003) who find evidence of

[2] Refer http://www.cboe.com/micro/vix/vixwhite.pdf

a jump-to-default risk component in the credit spread that varies over time in response to unexpected credit events. Therefore, an increase in credit event risk may reasonably be expected to be associated with a rise in the VIX, thereby causing the index to be positively related to credit spreads, and negatively related to prediction errors due to the absence of a time-varying measure of event risk in the structural models. We expecte that firms that attempt to time the equity market will not do so when the VIX is high due to the expense of raising equity.

CGDP The seasonally adjusted quarterly time series of real Gross Domestic Product (GDP), expressed in chained 2000 dollars, sourced from the Federal Reserve.[3] The change in GDP is a measure of concurrent and recent past business conditions. We measure the quarterly change in GDP by calculating the percentage change in quarter by quarter GDP. Linear interpolation is used to fit the data to each trade date between quarters. Expansionary economic conditions can be expected to increase firm asset worth and recovery rates leading to reduced credit spreads. For example, Frye (2000) shows that in a recession, aggregate recovery is approximately one-third lower than during an expansion, however, Altman, Brady, Resti & Sironi (2005) find GDP growth to be only significant in explaining aggregate defaulted bond recovery rates when annual GDP growth was less than 1.5 percent p.a. Acharya, Bharath & Srinivasan (2003) confirm GDP growth effects are quite small, but default rates and equity returns are economically materially associated with average recovery. Models that omit CGDP can be expected to carry positive spread prediction errors.

REF The 10 year constant maturity spread between Refcorp bonds and Treasury yields. It is a measure of time-varying market premium for liquidity risk. A rise in the Refcorp spread can be expected to directly increase corporate credit spreads, since credit spreads are calculated using Treasury yields as the reference rate. The absence of market liquidity premiums implies a negative correlation with corporate credit spread prediction errors.

Descriptive statistics of the independent variables are shown in Table 4.14 and sample correlations in Table 4.15.

Reasonably, firm-specific historical equity volatility and the VIX are positively correlated at 47.5 percent. However, the VIX is -33.5 percent correlated with interest rate volatility. The cause is evident to different behaviour at the beginning of the sample period. In Figure 4.17(d), volatility in the 3-month rate shows a spike in 1994-1995, when Figure 4.16(c) shows that the VIX was at its lowest. In late 1998, both time series evidence an increase in response to the LTCM crises.

[3]The chained dollar measure is an average of the prices of goods and services in successive pairs of year, and is therefore subject to less distortion over time than a single year constant dollar measure.

Table 4.14: Shown is the sample descriptive statistics of the independent variables used in the regressions of the prediction errors.

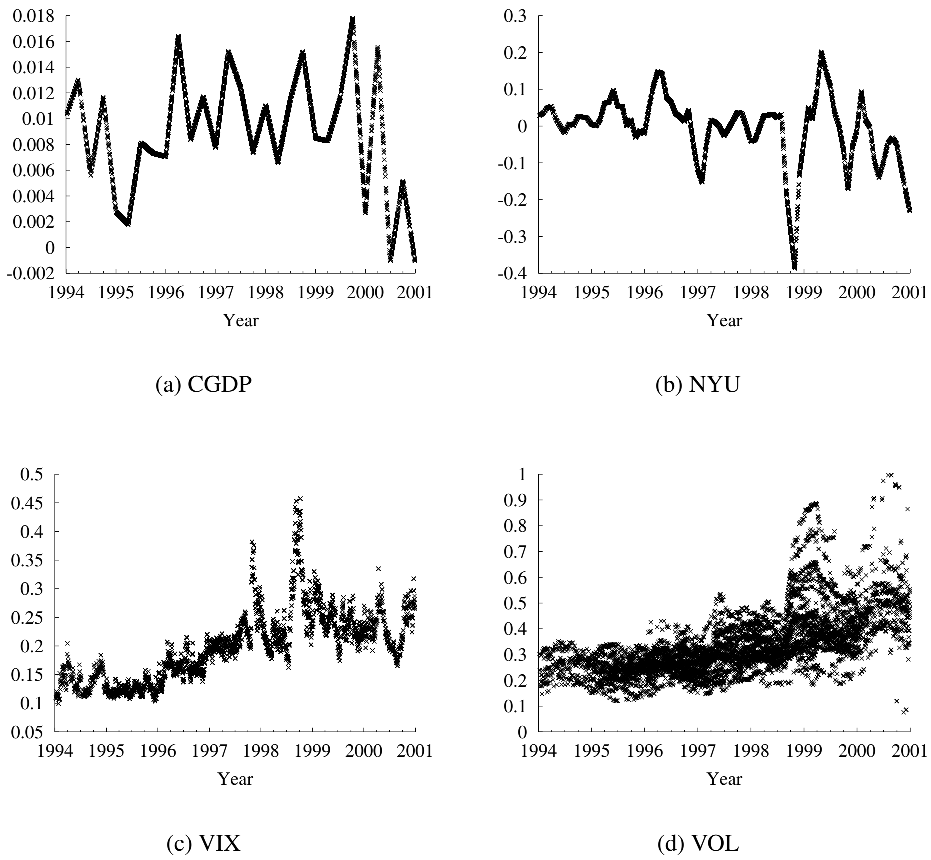
Variable	Mean	SD	Median	Min	Max
TENOR	9.348	6.830	7.470	1.000	29.990
TEN5YR	0.270	0.444	0.000	0.000	1.000
SOLV	0.922	0.690	0.775	0.015	3.110
COUPON	0.073	0.009	0.072	0.053	0.109
VOL	0.340	0.123	0.311	0.076	0.997
CMT3	0.051	0.005	0.052	0.030	0.064
TERM	-0.012	0.007	-0.012	-0.038	0.008
VOLR	0.002	0.001	0.001	0.001	0.005
RATING	6.808	2.286	6.000	2.000	13.000
PTB	2.561	1.771	2.059	0.163	13.061
NYU	-0.008	0.096	0.011	-0.386	0.201
VIX	0.208	0.062	0.205	0.099	0.457
CGDP	0.010	0.003	0.010	-0.001	0.018
REF	0.002	0.001	0.002	0.000	0.007

The firm-specific market-to-book ratio and log-solvency ratio are found to be highly positively correlated at 68.9 percent due to covariation in firm capital values. Higher values of both are associated with firms with higher net worth and better debt ratings.

We find a small negative relationship of -6.9 percent between defaulted bond return and economic growth, with the two series only showing a parallel decline from late 1999 onwards (refer Figures 4.16(a) and 4.16(b)). Expected recovery rates are more strongly influenced by market risk aversion as shown by the negative correlation of -45.9 percent between NYU and VIX. Both time series show a strong reaction to the LTCM liquidity crises.

Table 4.15: Shown is the sample correlations between independent variables used in the prediction error regressions.

	TENOR	SOLV	COUPON	VOL	CMT3	TERM	VOLR	RATING	MTB	NYU	VIX	CGDP	REF
TENOR	1.000	-0.028	0.093	-0.160	-0.059	-0.089	0.006	0.043	-0.098	0.015	-0.043	0.036	-0.047
SOLV		1.000	0.100	-0.192	0.025	-0.005	-0.017	-0.272	0.689	0.016	-0.030	-0.022	-0.021
COUPON			1.000	-0.180	0.068	-0.129	0.033	0.103	-0.012	0.057	-0.139	-0.007	-0.070
VOL				1.000	-0.103	0.399	0.197	-0.060	0.005	-0.198	0.475	-0.053	0.464
CMT3					1.000	0.454	-0.004	0.025	-0.058	0.062	-0.335	-0.529	0.087
TERM						1.000	-0.098	0.032	0.071	-0.272	0.408	-0.323	0.334
VOLR							1.000	-0.025	-0.025	-0.098	-0.027	-0.325	0.317
RATING								1.000	-0.436	-0.022	0.009	0.011	0.020
MTB									1.000	0.014	0.090	0.003	0.027
NYU										1.000	-0.459	-0.069	-0.231
VIX											1.000	0.294	0.253
CGDP												1.000	-0.067
REF													1.000



(a) CGDP

(b) NYU

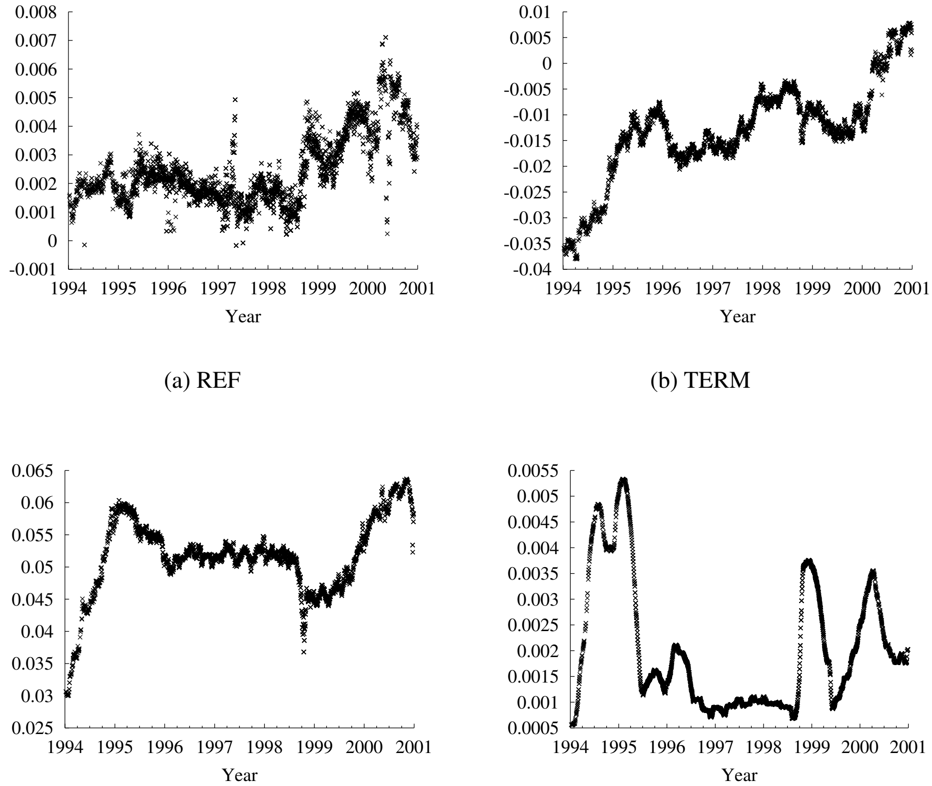
(c) VIX

(d) VOL

Figure 4.16: Shown are the time series plots of firm structure related independent variables used in regressing prediction errors. CGDP refers to the linearly interpolated, quarterly, seasonally adjusted, real change in Gross Domestic Product. NYU refers to the linearly interpolated, 3-month cumulative return on the Altman-NYU defaulted bond index. VIX refers to the daily Chicago Board of Exchange volatility index divided by 100. VOL is the 150-day historical firm-specific equity return volatility, using stock return data sourced from the Center for Research in Security Prices.

To confirm that the candidate independent variables are related to credit risk as expected, we conduct univariate regressions in which the dependent variable is the observed credit spread, pooled across firms and over time, and the independent variables as defined above. Two types of relationship are shown in Tables 4.16 and 4.17. The first is the coefficient ‘Beta’ defined as the percentage change in the regression prediction for a one standard deviation increase from the mean of the independent variable. It measures the sensitivity of the credit spread to change in the independent variable. The second measure is the R-squared of the regression, which is the percentage of the variation in the sample of credit spreads explained by variation in the independent variable.

Table 4.16 presents results for firm specific and bond characteristic variables. As shown by the Beta of the VOL variable, a one standard deviation increase in firm-specific equity volatility is, on average, associated with a 51 percent increase in credit spread. In



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(a) REF

(b) TERM

(c) CMT3

(d) VOLR

Figure 4.17: Shown are the time series plots of yield related independent variables used in regressing prediction errors. REF is the daily spread between 10-year constant maturity bonds issued by Refcorp and Treasury 10-year constant maturity rates. Refcorp data is sourced from Bloomberg. TERM is the daily annualised 3-month constant maturity yield less the 30 years constant maturity yield. CMT3 is the daily constant maturity annualised 3-month yield. VOLCMT is the average 150-day historical standard deviation of the 3-month constant maturity yield. All constant maturity Treasury yields are sourced from the Federal Reserve H15 report.

comparison, credit rating (RATING) has a lower Beta of 34 percent. In confirmation of the importance of leverage and firm volatility in the structural model theory, firm solvency (SOLV) is also strongly related with a Beta of -20 percent. The firm's market-to-book ratio (MTB) is similar in direction with a Beta of -23 percent. Apart from remaining tenor (TENOR) and COUPON, which show little explanatory power, all coefficient signs are as expected. The remaining tenor of the bond is significant only when the bond is less than 5 years (TEN5YR).

Regression results of market-wide factors are shown in Table 4.17. All variables are significant. The coefficient on the slope of the yield curve, TERM, is positive supporting the argument that it is viewed by market participants as a leading indicator of future economic activity. An increase in TERM is a consequence of the term structure flattening, accompanied by an increase in credit spreads. Note that this is directly *opposite* to the CDG model's specification which assumes a negative relationship between credit spread levels and TERM via the assumption of reduced debt issuance and decreasing leverage when the short rate rises relative to the long term interest rate. Thus, within our sample period, the debt-timing hypothesis of the yield curve is dominated by expectations of future economic conditions contained within the yield curve slope. We find that the Refcorp spread (REF) is the most closely related to observed credit spreads with a Beta of 41 percent, followed by TERM as the next most explanatory variable with a Beta of 28 percent. The least explanatory variables is the change in GDP (CGDP) and the level of the 3-month constant maturity interest rate (CMT3). All market-wide variables carry correct expected signs except the interest rate (CMT3). Figure 4.17(c) illustrates that, on average over the sample period, credit spreads increased as interest rates increased. In particular, the rapid rise in spreads at the end of the sample period, was accompanied by an increase in the short rate and a flattening of the yield curve. Next we perform multivariate regressions on the prediction errors. The results are discussed as follows by model.

4.2.3 Multivariate Error Regression

In the previous section we confirmed that the candidate independent variables are generally related to the sample levels of credit spreads as expected. In this section we examine whether the same variables are related to the residual step-ahead prediction errors. If the structural models have fully explained credit spreads, we do not expect that the variables will be related to the prediction errors. To test this hypothesis, we construct three multivariate equations using the same independent explanatory variables, and three different specifications of the prediction error as the dependent variable. The dependent variables are: the step-ahead credit spread prediction error (refer equation (3.19)), the step-ahead credit spread prediction percentage error, which is the prediction error expressed as a fraction of the observed credit spread, and the standardised step-ahead credit spread pre-

Table 4.16: Shown is the estimated coefficients from univariate OLS regressions of firm-specific and bond characteristic. The dependent variable is the observed credit spread, and t-stats are shown in parentheses. Results are from sample data pooled across 32 firms, 200 bonds, and 8,953 observations. Coefficient Beta is the change in credit spread associated with a one standard deviation change in the independent variable.

Variable	Coefficient Expected Sign	Constant	Coefficient	Coefficient Beta (%)	R^2 (%)
TENOR	+	0.0099	0.0000	2.56	0.03
		(38.98)	(1.75)		
TEN5YR	-	0.0100	0.0008	3.49	0.06
		(57.27)	(2.39)		
SOLV	-	0.0130	-0.0030	-20.05	2.10
		(52.56)	(-13.87)		
COUPON	+	0.0110	-0.0112	-1.02	0.01
		(9.45)	(-0.70)		
VOL	+	-0.0042	0.0423	50.91	13.57
		(-10.21)	(37.49)		
RATING	+	-0.0004	0.0016	34.84	6.36
		(-0.84)	(24.65)		
MTB	-	0.0136	-0.0013	-22.7	2.70
		(52.44)	(-15.76)		

Table 4.17: Shown is the estimated coefficients of univariate OLS regressions of financial market and economic independent variables. The dependent variable is the observed credit spread, and t-stats are shown in parentheses. Results are from sample data is pooled across 32 firms, 200 bonds, and 8,953 observations. Coefficient Beta is the change in credit spread associated with a one standard deviation change in the independent variable.

Variable	Coefficient Expected Sign	Constant	Coefficient	Coefficient Beta (%)	R^2 (%)
TERM	+/-	0.0153	0.4272	28.07	4.13
		(51.57)	(19.63)		
CMT3	-	0.0004	0.1916	8.77	0.40
		(0.22)	(6.02)		
VOLR	+	0.0069	1.871	19.77	2.05
		(24.66)	(13.67)		
NYU	-	0.0101	-0.0189	-17.73	1.65
		(67.76)	(-12.24)		
VIX	+	0.0032	0.0336	20.22	2.14
		(6.19)	(14.00)		
CGDP	-	0.0126	-0.2471	-8.46	0.38
		(28.94)	(-5.80)		
REF	+	0.0018	3.601	40.54	8.61
		(5.41)	(29.03)		

diction error (refer equation (4.1)). We discuss the prediction errors for the regressions by independent variable in turn, and for the sake of brevity, limit the discussion to the case where all models include a time-varying Refcorp liquidity premium in the measurement equation.

4.2.3.1 Simple Spread Prediction Error

The coefficients and t-stats (in parentheses) obtained from performing multivariate OLS of the credit spread prediction error are displayed in columns two and three in Tables 4.18 to 4.23. The credit spread prediction error is defined as the model estimate less the observed credit spread.

For the single factor models that ignore stochastic interest rates, the prediction errors are commonly related to the Refcorp spread (REF) and implied equity market volatility (VIX). The significant negative coefficient on the REF variable indicates that despite the inclusion of a the 10 year Refcorp spread into the measurement equation we have not managed to fully control for the influence of time-varying liquidity on credit spreads. Examination of the time series of Refcorp spreads in Figure 4.17(a) shows that the liquidity premium jumped strongly in the first half of 2000, coinciding with a market-wide rise in credit spreads, and associated with underprediction of market credit spreads by the EM model. Our specification places a time-invariant coefficient against the 10 year Refcorp spread that appears to have underestimated the sensitivity of credit spreads to the rise in the market liquidity premium observed during this period.

On the other hand, the rise in market spreads in 2000 was also associated with a decline in the VIX index from its post-LTCM high in late 1998. Our expectation under a structural credit model is that a rise in credit spreads would be associated with a rise in equity volatility, since the firm's equity volatility is directly a result of the firm's underlying asset value volatility, however, our sample includes a period of sharply rising credit spreads as the recession of 2001 neared, but occurring commensurate with declining equity market volatility. The result is an unexpected positive coefficient on the VIX index caused by the opposite movements in market pricing seen between the bullish equity markets and the more bearish view of average firm value held by the credit markets. This effect is present in all our regression results for all structural models. In addition to the above systematic errors, the single-factor LT and LS1 models also exhibit a significant negative prediction bias related to remaining maturity (TEN5YR).

The two-factor structural models of LS2 and CDG exhibit additional sytematic errors that are related to the risk-free interest rate process. The errors of the CDG model are positively related to the level of the 3-month constant maturity interest rate (CMT3), and its volatility (VOLR), and to the slope of the yield curve (TERM). Further, the LS2 and CDG models prediction errors exhibit a positive relationship with asset volatility that was not present in the single-factor models.

4.2.3.2 Percentage Spread Prediction Error

Results of the regression of independent variables against the percentage spread prediction error of the EM model is shown in columns four and five of Table 4.18. Unlike the simple spread prediction error, the percentage error is invariant to the size of the observed spread, however, it is more sensitive to any common component jump in credit spreads, such as a liquidity premium, when the observed credit spread is low.

Like the spread prediction regression, we find the Refcorp spread is significant, however, the VIX is not. A wider range of variables now enter the regression significantly. For the single-factor models we find that generally across all single-factor models that the prediction error is positively related to remaining maturity (TENOR), positively related to the slope of the yield curve (TERM), and positively related to the firm's proxied asset volatility (VOL). For the two-factor models of LS2 and CDG, additional systematic errors are found to be positively related to the market-to-book ratio (MTB), positively to the rating (RATING), and positively to interest rate volatility (VOLR).

The first interesting observation from this finding is that the contention of JMR that the Merton model's accuracy can be improved by the inclusion of a stochastic interest rate process is not strongly supported. Rather, we find the models that include a second stochastic interest rate process result in additional systematic errors on the factors that they were meant to correct. Secondly, we expected that the CDG model's prediction errors would not be related to the yield curve slope, yet column five of Table 4.23 shows that the TERM has the most significant t-stat of all independent variables tested. Thus, we find that the CDG model does not fully control for the influence of the yield curve slope on credit spreads, despite its specific inclusion in the firm's solvency process specification. Finally, it is useful to compare our results with EHH, who also conduct multivariate analysis on the percentage prediction error (refer Table 5, EHH). Across models, we agree with EHH that percentage prediction errors are positively related to proxied asset volatility, but only find a negative relationship with solvency for the two-factor models and not for the single-factor models. In contrast, EHH find leverage to be a strong explanatory variable of prediction error. Unlike EHH we find no relationship with coupon, but do find a significant positive relationship with remaining tenor for all models except the LT and CDG models. The additional parameter controlling mean-reversion in the CDG model appears to be corrected this error. In both our results and EHH's results, the LT model has the lowest number of systematic errors identified by regression of the percentage prediction errors. EHH did not test for liquidity or slope of the term structure, however, we find the Refcorp spread to be significantly negatively related to prediction error in all models. Likewise, the slope of the term structure is very strongly positively related to prediction error in all our models. Both effects appear to a result of our sample period that includes a credit cycle downturn coinciding with an inversion of the yield curve.

From our review of the capital structure literature, the firm's market-to-book ratio is hypothesised to be positively related to the spread predication error. We find some supportive evidence with the MTB variable significantly positively related to prediction error in the prediction error regressions for the LS1, CEV, LS2, and CDG models, but not significant for the EM or LT models.

4.2.3.3 Standardised Spread Prediction Error

In this section analyse the standardised error to test our hypothesis that missing factors identified from the capital structure literturte are related to model misspecification.

The standardised prediction error scales the prediction error relative to the size of the observed spread and standard deviation of the predicted credit spread. It is therefore less sensitive to large changes in the observed credit spread, since large increases in prediction errors results in higher estimates of the standard deviation of the step-ahead predicted credit spread. Results of regressing the standardised spread prediction error against our independent variables are shown in columns six and seven of Tables 4.18 to 4.23. The standardised prediction error regressions show that the models are well fitted for the firm and bond parameters that are specified in the models. For example, firm solvency, firm asset volatility, firm rating, and bond coupon rate are mostly unrelated to the standardised error. Only the LS1 model shows some mild positive relationship with asset volatility.

For all one-factor models, except the LT model, the remaining maturity is unrelated to the standardised error. This suggests that much of the earlier documented prediction bias is related to the structural model's inability to predict sudden *relative* increases in credit spreads at short maturities. The standardisation of the prediction estimates, within the EKF, places lower confidence on these estimates, which decreases the size of the standardised error relative to the percentage prediction error. Visually we can see the greater variance of observed credit spreads at short maturities displayed in the plots of sample credit spreads by remaining maturity as shown in Figures 4.1 to 4.6. We are not able to identify why there are large changes in relative credit spreads at short maturities, however, changes in a fixed component of the credit spread, related to the Refcorp spread, is a possible cause. Our regression results indicate that an increase in the Refcorp spread is associated with periods of underprediction as shown by the negative relationship of REF with the standardised prediction error.

We identifed that market timing behaviour as important features of capital structure management. We expect that the omission of debt market timing from a structural model would result in a positive coefficient against TERM, which we confirm to be the case across all models. The CDG model also shows this error despite including debt timing in its specification. For equity market timing we expect that the coefficient against VOL and VIX are negative since an increase in either would tend to increase the cost of equity issuance and delay equity raising. In contradiction to equity timing behaviour, we find the opposite signs. The firm's market-to-book ratio is also found to be unrelated to standardised errors despite the theoretical support that management of the firm's capital structure is influenced by the firm's relative equity value. The significance of VOL, VIX, and TERM is our multivariate regressions suggests that the signs our results are possibly influenced by multicollinearity.

We also find that the one and two-factor model standardised errors are related to the interest rate process, whereas this is not evident in our percentage error regressions of the one-factor models. Thus, the standardised error reveals that the structural models are miss-specified without inclusion of a stochastic interest rate process, yet the LS2 and CDG models that include a stochastic short-rate model, fail to remove the systematic error bias.

The positive and significant coefficient on the CGDP variable across all models shows that positive pricing errors occur when both values are high as expected when the variables are interpreted as decreasing future default risk or increasing expected future recovery in default. The present market value of recovery in default is proxied by the secondary market recovery rate on defaulted bonds measured by the NYU variable. Unlike CGDP, there is weaker role for defaulted bond return with only the LS1 and EM models showing significance. The general lack of relationship with secondary market bond prices, but strong positive relationship with the change in CGDP, suggests that the market is placing greater informational value on the general business climate when assessing firm asset value.

4.2.3.4 Model Error Regressions Compared to Merton

As measured by the simple and percentage prediction errors regressions, the EM model exhibits the lowest level of systematic error. When standardised errors are regressed, the CEV, LT, and LS1 models perform equally as well. We find that the structural models share many of the same prediction biases despite being derived from fundamentally different theoretical underpinnings. The original Merton model in its extended form performs remarkably well in comparison to newer structural models designed to overcome its theoretical shortcomings.

Compared to the EM model, the LS1 model exhibits a very similar pattern of regression errors (refer Table 4.19). Therefore, the relaxation of default at maturity to early default in the LS1 model appears to add little to specification improvement in practice over the Merton model. We find that the LT model has no tendency to overpredict longer maturities, but it does consistently underpredict short-term maturity debt spreads (refer Table 4.20). EHH report that the LT model percentage errors increase with market leverage, but find no evidence of a leverage related prediction bias. The CEV model exhibits greater percentage errors related to firm leverage than the EM model (refer Table 4.21).

Table 4.18: Shown are the results of three multivariate regressions on the prediction errors for the extended **Merton (1974) (EM)** model fitted with a time-varying liquidity premium. Results are from multivariate OLS on sample data pooled across 32 firms and 200 bonds. Equation (i) has the dependent variable Error, which is the step-ahead yield prediction error; equation (ii) has the dependent variable Percentage Error, which is the prediction error expressed as a fraction of the observed credit spread; equation (iii) has the Standardised Error, which is the step-ahead standardised prediction error calculated by scaling the prediction error by the standard deviation of the prediction. Independent variables are shown in the first column and the regressions include a constant term. Coeff. is the estimated regression coefficient on the independent variables. T-stats on the coefficients are shown in parentheses and are White (1980) adjusted robust estimates. Significance at the 5% level is denoted by '*', and significance at the 1% level by '**'.

	Dependent Variable								
	(i) Error x 100			(ii) Percentage Error			(iii) Standardised Error		
Independent Variable	Coeff.	t-stat		Coeff.	t-stat		Coeff.	t-stat	
const	-0.077	(-0.203)		0.113	(0.592)		-1.125	(-3.824)	**
TENOR	0.000	(0.534)		0.002	(3.876)	**	-0.001	(-0.585)	
TEN5YR	-0.035	(-1.803)		-0.065	(-3.104)	**	-0.038	(-1.272)	
SOLV	0.003	(0.347)		-0.008	(-0.579)		0.027	(1.200)	
COUPON	0.488	(1.098)		-0.073	(-0.091)		-0.240	(-0.189)	
VOL	-0.005	(-0.023)		0.140	(2.789)	**	0.215	(1.530)	
CMT3	1.424	(0.317)		-3.576	(-1.348)		17.201	(4.283)	**
TERM	1.927	(1.673)		9.041	(6.545)	**	11.035	(3.857)	**
VOLR	-2.156	(-0.169)		14.593	(1.513)		39.516	(2.892)	**
RATING	-0.007	(-1.297)		0.006	(1.891)		-0.009	(-1.736)	
MTB	-0.002	(-0.503)		0.005	(1.031)		-0.005	(-0.532)	
NYU	0.050	(0.843)		0.040	(0.525)		0.277	(1.994)	*
VIX	0.509	(4.360)	**	0.206	(1.477)		1.831	(6.088)	**
CGDP	-0.367	(-0.053)		6.314	(1.445)		14.082	(2.756)	**
REF	-23.598	(-4.837)	**	-48.126	(-5.097)	**	-71.863	(-5.471)	**
Adj R^2 (%)	0.301			1.308			4.405		
F(14, 8,938)	10.255			13.486			12.338		
n	8,953			8,953			8,953		

Table 4.19: Shown are the results of three multivariate regressions on the prediction errors for the extended **Longstaff & Schwartz (1995) (LS1)** model fitted with a time-varying liquidity premium. Results are from multivariate OLS on sample data pooled across 32 firms and 200 bonds. Equation (i) has the dependent variable Error, which is the step-ahead yield prediction error; equation (ii) has the dependent variable Percentage Error, which is the prediction error expressed as a fraction of the observed credit spread; equation (iii) has the Standardised Error, which is the step-ahead standardised prediction error calculated by scaling the prediction error by the standard deviation of the prediction. Independent variables are shown in the first column and the regressions include a constant term. Coeff. is the estimated regression coefficient on the independent variables. T-stats on the coefficients are shown in parentheses and are White (1980) adjusted robust estimates. Significance at the 5% level is denoted by '*', and significance at the 1% level by '**'.

	Dependent Variable								
	(i) Error x 100			(ii) Percentage Error			(iii) Standardised Error		
Independent Variable	Coeff.	t-stat		Coeff.	t-stat		Coeff.	t-stat	
const	0.463	(0.722)		0.214	(1.070)		-0.732	(-2.523)	*
TENOR	0.001	(0.911)		0.003	(4.545)	**	-0.001	(-0.332)	
TEN5YR	-0.074	(-2.321)	*	-0.069	(-3.074)	**	-0.043	(-1.443)	
SOLV	-0.006	(-0.343)		-0.021	(-1.407)		-0.002	(-0.089)	
COUPON	1.065	(1.495)		0.267	(0.305)		0.590	(0.466)	
VOL	-0.239	(-0.713)		0.155	(3.127)	**	0.340	(2.456)	*
CMT3	-5.756	(-0.752)		-5.524	(-1.957)		9.201	(2.340)	*
TERM	5.395	(2.880)	**	12.579	(7.891)	**	16.565	(5.794)	**
VOLR	-9.965	(-0.483)		20.173	(2.011)	*	58.396	(4.301)	**
RATING	-0.012	(-1.2900		0.007	(2.301)	*	-0.003	(-0.637)	
MTB	0.005	(0.933)		0.009	(2.042)	*	0.010	(1.009)	
NYU	0.056	(0.619)		0.085	(1.038)		0.368	(2.651)	**
VIX	0.505	(2.898)	**	0.122	(0.824)		1.485	(4.952)	**
CGDP	-5.096	(-0.452)		7.180	(1.538)		14.890	(2.961)	**
REF	-28.147	(-3.822)	**	-52.949	(-5.461)	**	-81.593	(-6.299)	**
Adj R^2 (%)	0.365			1.624			2.554		
F(14, 8,938)	9.658			16.086			15.128		
n	8,953			8,953			8,953		

Table 4.20: Shown are the results of three multivariate regressions on the prediction errors for the extended **Leland & Toft (1996) (LT)** model fitted with a time-varying liquidity premium. Results are from multivariate OLS on sample data pooled across 32 firms and 200 bonds. Equation (i) has the dependent variable Error, which is the step-ahead yield prediction error; equation (ii) has the dependent variable Percentage Error, which is the prediction error expressed as a fraction of the observed credit spread; equation (iii) has the Standardised Error, which is the step-ahead standardised prediction error calculated by scaling the prediction error by the standard deviation of the prediction. Independent variables are shown in the first column and the regressions include a constant term. Coeff. is the estimated regression coefficient on the independent variables. T-stats on the coefficients are shown in parentheses and are White (1980) adjusted robust estimates. Significance at the 5% level is denoted by '*', and significance at the 1% level by '**'.

	Dependent Variable								
	(i) Error x 100			(ii) Percentage Error			(iii) Standardised Error		
Independent Variable	Coeff.	t-stat		Coeff.	t-stat		Coeff.	t-stat	
const	-0.068	(-0.189)		-0.108	(-0.297)		-0.836	(-2.790)	**
TENOR	0.000	(-0.409)		0.001	(0.932)		-0.004	(-1.888)	
TEN5YR	-0.076	(-3.635)	**	-0.095	(-3.614)	**	-0.111	(-3.799)	**
SOLV	-0.004	(-0.390)		-0.019	(-1.106)		0.030	(1.375)	
COUPON	0.510	(0.846)		0.428	(0.414)		0.236	(0.192)	
VOL	-0.112	(-0.544)		0.087	(1.574)		0.248	(1.885)	
CMT3	2.180	(0.498)		0.505	(0.104)		15.021	(3.615)	**
TERM	4.871	(3.511)	**	11.952	(6.601)	**	17.445	(6.088)	**
VOLR	9.293	(0.776)		25.739	(1.945)		58.139	(4.225)	**
RATING	-0.009	(-1.732)		0.005	(1.638)		-0.010	(-1.927)	
MTB	0.000	(0.040)		0.005	(0.887)		-0.019	(-1.905)	
NYU	0.086	(1.381)		0.060	(0.672)		0.078	(0.564)	
VIX	0.611	(3.925)	**	0.334	(1.571)		1.674	(5.547)	**
CGDP	2.925	(0.488)		10.398	(1.868)		14.136	(2.770)	**
REF	-33.120	(-5.609)	**	-60.468	(-6.096)	**	-98.676	(-7.603)	**
Adj R^2 (%)	0.882			1.639			3.217		
F(14, 8,938)	8.938			11.696			15.914		
n	8,953			8,953			8,935		

Table 4.21: Shown are the results of three multivariate regressions on the prediction errors for the extended **constant elasticity of variance (CEV)** model fitted with a time-varying liquidity premium. Results are from multivariate OLS on sample data pooled across 32 firms and 200 bonds. Equation (i) has the dependent variable Error, which is the step-ahead yield prediction error; equation (ii) has the dependent variable Percentage Error, which is the prediction error expressed as a fraction of the observed credit spread; equation (iii) has the Standardised Error, which is the step-ahead standardised prediction error calculated by scaling the prediction error by the standard deviation of the prediction. Independent variables are shown in the first column and the regressions include a constant term. Coeff. is the estimated regression coefficient on the independent variables. T-stats on the coefficients are shown in parentheses and are White (1980) adjusted robust estimates. Significance at the 5% level is denoted by '*', and significance at the 1% level by '**'.

	Dependent Variable								
	(i) Error x 100			(ii) Percentage Error			(iii) Standardised Error		
Independent Variable	Coeff.	t-stat		Coeff.	t-stat		Coeff.	t-stat	
const	-0.244	(-0.743)		0.217	(1.111)		-0.915	(-3.236)	**
TENOR	0.001	(1.333)		0.003	(4.9230	**	0.002	(0.902)	
TEN5YR	-0.032	(-1.737)		-0.087	(-3.772)	**	-0.039	(-1.338)	
SOLV	-0.002	(-0.191)		-0.040	(-2.528)	*	-0.014	(-0.655)	
COUPON	0.794	(1.832)		1.054	(1.215)		1.422	(1.142)	
VOL	0.240	(1.329)		0.174	(3.480)	**	0.252	(1.835)	
CMT3	2.457	(0.626)		-6.672	(-2.432)	*	11.738	(3.043)	**
TERM	1.919	(1.435)		10.594	(7.143)	**	13.176	(4.664)	**
VOLR	3.060	(0.279)		17.311	(1.665)		70.236	(5.258)	**
RATING	0.001	(0.313)		0.010	(3.282)	**	0.001	(0.110)	
MTB	0.000	(-0.014)		0.014	(3.106)	**	0.012	(1.198)	
NYU	0.006	(0.097)		0.013	(0.157)		0.099	(0.712)	
VIX	0.313	(2.594)	**	0.086	(0.587)		1.642	(5.650)	**
CGDP	-0.158	(-0.027)		3.879	(0.846)		8.858	(1.796)	
REF	-25.816	(-5.394)	**	-48.229	(-4.805)	**	-91.326	(-7.131)	**
Adj R^2 (%)	0.438			1.649			2.543		
F(14, 8,938)	9.361			14.528			13.967		
n	8,953			8,953			8,953		

Table 4.22: Shown are the results of three multivariate regressions on the prediction errors for the extended **Longstaff & Schwartz (1995) (LS2)** model fitted with a time-varying liquidity premium. Results are from multivariate OLS on sample data pooled across 32 firms and 200 bonds. Equation (i) has the dependent variable Error, which is the step-ahead yield prediction error; equation (ii) has the dependent variable Percentage Error, which is the prediction error expressed as a fraction of the observed credit spread; equation (iii) has the Standardised Error, which is the step-ahead standardised prediction error calculated by scaling the prediction error by the standard deviation of the prediction. Independent variables are shown in the first column and the regressions include a constant term. Coeff. is the estimated regression coefficient on the independent variables. T-stats on the coefficients are shown in parentheses and are White (1980) adjusted robust estimates. Significance at the 5% level is denoted by '*', and significance at the 1% level by '**'.

	Dependent Variable								
	(i) Error x 100			(ii) Percentage Error			(iii) Standardised Error		
Independent Variable	Coeff.	t-stat		Coeff.	t-stat		Coeff.	t-stat	
const	-0.197	(-0.892)		0.612	(2.92)	**	0.379	(1.319)	
TENOR	0.002	(3.282)	**	0.005	(8.77)	**	0.006	(2.986)	**
TEN5YR	0.016	(1.165)		-0.057	(-2.43)	*	0.060	(2.054)	*
SOLV	0.008	(1.093)		-0.036	(-2.24)	*	-0.003	(-0.117)	
COUPON	0.678	(1.889)		1.262	(1.46)		1.305	(1.053)	
VOL	0.380	(3.162)	**	0.201	(3.93)	**	0.552	(4.117)	**
CMT3	-0.239	(-0.090)		-14.061	(-4.85)	**	-13.158	(-3.300)	**
TERM	4.210	(4.590)	**	15.834	(10.23)	**	25.126	(8.901)	**
VOLR	34.410	(4.481)	**	48.349	(4.85)	**	148.504	(10.716)	**
RATING	0.005	(1.419)		0.011	(3.18)	**	0.004	(0.690)	
MTB	0.001	(0.437)		0.019	(3.60)	**	0.017	(1.813)	
NYU	0.053	(1.031)		-0.003	(-0.04)		0.283	(1.949)	
VIX	0.113	(1.121)		-0.160	(-1.03)		0.677	(2.292)	*
CGDP	4.436	(1.135)		2.643	(0.61)		12.798	(2.632)	**
REF	-39.908	(-8.558)	**	-62.625	(-7.00)	**	-119.123	(-9.452)	**
Adj R^2 (%)	1.955			2.592			4.544		
F(14, 8,938)	16.544			32.047			24.912		
n	8,953			8,953			8,953		

Table 4.23: Shown are the results of three multivariate regressions on the prediction errors for the extended **Collin-Dufresne & Goldstein (2001) (CDG)** model fitted with a time-varying liquidity premium. Results are from multivariate OLS on sample data pooled across 32 firms and 200 bonds. Equation (i) has the dependent variable Error, which is the step-ahead yield prediction error; equation (ii) has the dependent variable Percentage Error, which is the prediction error expressed as a fraction of the observed credit spread; equation (iii) has the Standardised Error, which is the step-ahead standardised prediction error calculated by scaling the prediction error by the standard deviation of the prediction. Independent variables are shown in the first column and the regressions include a constant term. Coeff. is the estimated regression coefficient on the independent variables. T-stats on the coefficients are shown in parentheses and are White (1980) adjusted robust estimates. Significance at the 5% level is denoted by '*', and significance at the 1% level by '**'.

	Dependent Variable								
	(i) Error x 100			(ii) Percentage Error			(iii) Standardised Error		
Independent Variable	Coeff.	t-stat		Coeff.	t-stat		Coeff.	t-stat	
const	-0.639	(-2.687)	**	-0.014	(-0.066)		-1.360	(-4.526)	**
TENOR	-0.001	(-2.092)	*	0.000	(0.735)		-0.006	(-3.030)	**
TEN5YR	0.003	(0.201)		-0.093	(-4.170)	**	0.006	(0.216)	
SOLV	-0.003	(-0.312)		-0.060	(-3.913)	**	-0.064	(-2.848)	**
COUPON	0.586	(1.422)		1.256	(1.452)		1.450	(1.167)	
VOL	0.357	(2.831)	**	0.203	(3.983)	**	0.583	(4.103)	**
CMT3	8.343	(2.882)	**	-2.346	(-0.829)		18.138	(4.367)	**
TERM	6.827	(6.394)	**	18.250	(11.253)	**	27.748	(9.623)	**
VOLR	19.490	(2.374)	*	29.803	(3.086)	**	98.122	(6.978)	**
RATING	0.001	(0.403)		0.007	(2.119)	*	-0.010	(-1.729)	
MTB	-0.003	(-0.873)		0.019	(3.828)	**	0.012	(1.183)	
NYU	-0.050	(-0.985)		-0.026	(-0.306)		-0.152	(-1.054)	
VIX	0.565	(5.540)	**	0.391	(2.571)	*	2.417	(8.094)	**
CGDP	8.642	(2.057)	*	10.850	(2.567)	*	28.777	(5.644)	**
REF	-25.896	(-5.473)	**	-56.482	(-6.574)	**	-89.710	(-6.856)	**
Adj R^2 (%)	4.139			3.547			8.823		
F(14, 8,938)	28.347			33.505			45.499		
n	8,953			8,953			8,953		

It tends to underpredict spreads on well rated firms, and on firms with high solvency. The market-to-book ratio is also significant due to high correlation with firm solvency and rating.

The percentage prediction error performance of the two-factor models is relatively poor compared to the EM model. The LS2 model, with stochastic interest rate, exhibits additional errors related to the level and volatility of the 3-month CMT rate (refer Table 4.22). The CDG model also suffers from additional errors related to the interest-rate process (refer Table 4.23). The differences between one and two-factor models is less evident when we consider the standardised errors. An innovation of the CDG model is the linkage of longer-term default risk with the slope of the yield curve. All measures of prediction error are found to be strongly positively related to the slope; implying that overprediction of spreads is greatest when the risk-free rate is low relative to the long-rate. The CDG model imposes a positive coefficient on the yield curve slope implying higher credit spreads when short-term rate is low, which we find in our data set to be counterfactual, as shown by regression results in Table 4.17 and time series plots in Figures 4.15(a) and 4.17(b). Part of the poor relative performance of the CDG model may therefore be due to the absence of debt-timing behaviour evident in our sample of data.

4.3 Estimated Model Parameters

In this section we discuss the parameters implied from our estimation procedure. Our maximum likelihood estimates of the latent firm process parameters are compared with independently estimated parameter values using observable proxies in the manner of JMR, LYS, HH, and EHH. If the observable proxy for the firm process is similar in level and dynamics, then the observable proxies may serve as useful means of deriving model estimates. If not, then debt pricing cannot be accurately performed without latent estimation of the structural model's parameters.

4.3.1 Firm Asset Volatility

In this section we compare the average firm implied asset volatility with an approximation of firm asset volatility. To construct an observable proxy for the asset volatility, we collect the daily equity capitalisation for each firm sourced from CRSP, and add the total book debt sourced from COMPUSTAT, which is updated quarterly. From the constructed time series of approximate firm market value, we calculate the daily log-return for each firm over the full sample period. The observed standard deviation per firm is then the annualised sample standard deviation of the daily log-returns. Shown in the last row of Table 4.24 is the descriptive statistics of the firm-specific volatility estimates pooled cross-sectionally over the sample of 32 firms. The average annualised firm asset return volatility, estimated from the observed proxy value of the firm, is 26.62 percent,

Table 4.24: This table shows the descriptive statistics of implied asset volatility and firm solvency ratio compared with a market value proxy for the firm. The implied asset volatility and sample average smoothed filtered estimate of the solvency ratio $(V(t)/K(t))$ are obtained from application of an EKF to the term structure of credit spreads. For comparison, descriptive statistics for the daily time series of an independently estimated proxy for the firm value is shown in 'Observed'. The observed time series of firm value is calculated as the sum of the firm's market capitalisation and total book debt, and the firm's solvency is the observed firm value divided by total book debt. Data for book debt is quarterly sourced from COMPUSTAT, and firm equity capital is daily frequency sourced from CRSP. The observed firm volatility is the sample standard deviation of daily log returns of observed firm value. SD is standard deviation. Correl refers to the correlation coefficent between the firm-specific sample average observed and implied estimates of asset volatility, and firm solvency. All numbers are in percent.

	Firm Asset Volatility σ_v				Mean Solvency Level $(V(t)/K(t))$			
	Implied				Implied			
Model	Mean	SD	Median	Correl	Mean	SD	Median	Correl
Panel A: No Liquidity								
EM	26.68	11.13	23.05	17.19	2.82	2.53	1.71	34.78
LS1	25.05	8.05	23.93	19.30	3.06	1.87	2.42	65.41
LT	20.79	10.14	17.74	1.34	2.53	0.89	2.23	41.18
CEV	1,278.84	7,042.15	20.57	9.32	5.29	9.06	2.39	51.72
LS2	14.11	4.10	13.86	-12.44	1.51	0.41	1.40	30.25
CDG	13.26	4.60	11.61	-1.33	3.80	3.35	2.75	45.41
Panel B: Constant Liquidity								
EM	28.07	12.73	24.77	10.29	4.66	6.91	2.49	18.24
LS1	17.73	6.11	17.11	27.10	3.02	2.09	2.45	78.14
LT	16.72	8.03	14.49	7.89	2.21	0.97	1.84	31.78
CEV	15.24	7.50	14.62	-16.40	4.21	6.64	2.48	71.04
LS2	15.38	4.80	15.41	23.03	2.29	1.20	1.93	56.79
CDG	16.41	5.45	14.80	-7.16	4.72	3.63	3.67	50.55
Panel C: Time-Varying Liquidity								
EM	28.27	10.48	26.57	22.21	3.72	3.23	2.82	47.53
LS1	18.30	6.75	18.86	27.18	2.88	1.99	2.31	76.70
LT	16.62	9.62	13.86	1.29	2.27	1.48	1.91	24.84
CEV	15.03	6.56	15.35	-23.26	4.16	6.65	2.49	72.05
LS2	14.18	5.89	15.59	33.21	2.47	1.41	1.94	64.94
CDG	14.98	5.53	13.75	-4.58	5.07	3.83	3.58	39.95
Observed	26.62	9.29	25.61	100.00	3.15	2.43	2.30	100.00

with a standard deviation of 9.21 percent, and a median of 25.61 percent. The descriptive statistics of the maximum likelihood estimates of the latent firm value volatility are shown in Panels A to C in Table 4.24. In Panel A results are shown excluding a liquidity premium in the measurement equation, Panel B includes a constant liquidity premium, and Panel C assumes a constant and a time-varying liquidity component of the credit spread that is directly proportional to the 10 year Refcorp spread.

Because of the influence of outlier observations associated with the local rise in asset volatility under the CEV model, we use the median as the most appropriate central tendency measure to compare volatilities across models. With no liquidity premium as shown in Panel A of Table 4.24, the EM model at 23.05 percent, and LS1 model at 23.93 percent, are quite close in value to the observed median proxy volatility of 25.61 percent. The medians of the LT and CEV models are lower at 17.74 percent and 20.57 percent respectively. The two-factor models are noticeably lower due to the additional spread volatility that is contributed by their stochastic interest rate process. The medians of the LS2 and CDG models are only 13.86 percent and 11.61 percent respectively. In Panel B of Table 4.24 we can see that the median of the EM model's volatility remains close to the observed proxy, but all the other models reduce the implied level of asset volatility. With the addition of time-varying liquidity premium in Panel C of Table 4.24 the EM model median asset volatility is clearly higher than the other models at 26.57 percent, with the next highest the LS1 model at 18.86 percent. From this result it is evident that using an observed proxy for the implied firm asset volatility is reasonably robust on average for pricing under an extended Merton model, but is excessive on average for other structural models, particularly for two-factor models.

Next we compare the sample firm-wise correlation between the implied asset volatility estimates obtained from our EKF, with estimates from the observed proxy method. Correlations are shown in column five of Table 4.24 where we can see that the correlations vary widely between models. With no liquidity premium, the LS1 model has the highest degree of across firm correlation at only 19.30 percent. The LS2 model records a negative correlation at -12.44 percent. The situation shows little improvement with the introduction of controls for liquidity premiums in the measurement equation. For example, with time-varying liquidity in Panel C of Table 4.24 the correlations vary from -23.26 percent for the CEV model to 33.21 percent for the LS2 model. Consequently, there is little consistent correlation evident between the latent estimated asset volatility and the observed proxy volatility, so that even though on average the measures appear similar, the proxy variable will give rise to significant errors when used to price individual firm bonds.

As a further check on the reasonableness of the level of the implied asset volatilities, we compare the firm-wise implied asset volatility estimates with the firm's historical equity volatility. As Merton (1974) demonstrated, a firm's equity value is equivalent to a call option on the firm's underlying assets, therefore its equity volatility should be strictly greater than the firm's asset volatility when the firm is leveraged. We therefore expect that our implied asset volatility estimates are less than the sample equity volatility. Table 4.25 presents results for sample of 32 firms for each of the six models tested. In the final two rows we show the number of times that a firm estimate of asset volatility exceeds its equity volatility and the relative frequency that this occurs in the sample. The first point to note is that the means and medians of the implied asset volatilities are below the corresponding central tendencies of the equity volatility as expected. However, if we consider how many times that the firm level estimate of asset volatility exceeds the firm's equity volatility, clear differences between models become evident. The EM model on eight occasions, or 25 percent of the sample, has implied firm asset volatilities that are excessive relative to the firm's observed equity volatility. In contrast, the LS1 and LS2 model firm estimates are never excessive. The LT model also shows three breaches, or 9.38 percent of the sample. The EM and LT models have much greater variability in firm estimates as evidenced by higher sample standard deviations than the other models. It is therefore evident that the EM model, and to a lesser extent the LT model, achieve calibration to market observed credit spreads by implying excessive asset volatility necessary generate sufficiently high credit spreads. A similar conclusion was reached by EHH, but they did not control for liquidity premiums that might have otherwise encouraged reliance on excessive asset volatility to match market credit spreads. We reach the same conclusion in respect of the EM model, in particular, even after controlling for market liquidity. Importantly, we do not find strong evidence that the implied asset volatilities of the LS, CEV, or CDG are excessive.

4.3.2 Level of the Firm's Default Boundary

Our estimation method provides an estimate of the path taken by the firm's state variable, which identifies implicitly when the firm is expected to default under different structural model specifications. In this section we consider whether the estimated average levels of firm solvency are well approximated by an observable proxy for market solvency. Further, we compare the reasonableness of our implied default boundary with the directly observed empirical estimates by Davydenko (2005).

The implied solvency ratio per firm, is the across time average of the transformed smoothed estimate of the state variable. For all models, except the CEV model, the state variable is defined as $x(t) = \ln(V(t)/K(t))$ for all trades from $t = 1$ to $t = T$. From the EKF we obtain a smoothed estimate of the state variable, $a(t|T) = x(t)$, as described in equation (3.21). Transforming the state estimate by $\exp(a(t|T))$ gives our best estimate of the path taken by the firm's solvency ratio, $V(t)/K(t)$, that underpins the predicted term structure of credit spreads. For the CEV model, the state variable is $X(t) = \ln(V(t) - K(t))/K(t)$, so we transform the smoothed state variables to an equiv-

Table 4.25: This table shows the implied asset volatility, per model, for the sample of 32 firms fitted with a time-varying liquidity premium. The final column shows the sample average historical annualised daily equity return volatility for the issuing firm. Summary descriptive statistics per model are shown in the second panel. The lower panel shows the count of firms where the asset volatility estimate exceeds the sample average equity volatility, and the associated relative frequency that this occurs in the sample. All numbers other than the counts are shown in percentage.

	Implied Asset Volatility						Obs
Issuer	EM	LS1	LT	CEV	LS2	CDG	Eq Vol
Aetna Inc	33.55	23.03	11.07	11.15	27.01	18.74	35.16
Associates Corp	21.81	9.37	18.10	7.72	11.69	10.10	37.08
Atlantic Richfield Co	24.09	20.26	11.74	13.96	17.02	21.28	24.21
A T & T Corp	31.03	23.77	15.32	7.17	19.98	16.53	35.69
Bear Stearns Comp Inc	8.92	8.84	13.79	14.09	0.71	11.55	41.05
Black & Decker Corp	28.14	13.46	31.01	10.35	2.51	14.42	30.96
Dayton Hudson Corp	17.86	14.64	2.30	6.22	12.13	1.91	36.67
Comm Edison Co	25.39	20.74	24.74	16.74	14.47	15.35	23.00
Enron Corp	17.96	19.35	5.52	6.87	17.79	12.27	32.36
Fed Dept Stores	43.28	26.26	22.41	18.45	16.80	12.13	37.56
Ford Mtr Co	22.82	8.37	10.47	15.90	11.80	11.95	30.98
General Mtrs	26.43	14.44	12.11	16.44	12.95	11.00	31.91
Georgia Pacific Corp	35.27	25.05	28.74	16.94	16.88	14.82	34.04
Hca Healthcare Corp	26.72	18.35	12.65	13.01	19.39	13.38	36.86
IBM Corp	15.68	20.88	53.42	8.08	13.16	14.69	35.04
Int Paper Co	12.11	11.08	10.69	14.80	14.37	14.11	33.60
Lehman Bros Hldgs Inc	23.33	11.22	19.53	17.89	7.31	12.71	49.24
Merrill Lynch & Co	21.81	8.45	13.53	9.59	6.27	9.87	42.39
Motorola Inc	48.45	36.76	16.02	18.65	21.12	21.82	40.20
Nabisco Group Hldgs Corp	37.00	18.32	10.95	21.02	15.52	12.88	37.85
Niagara Mohawk Corp	17.63	25.77	13.93	36.27	13.94	31.41	29.75
Northrop Grumman Corp	36.22	23.66	7.78	19.26	18.19	18.89	32.36
Paine Webber Grp Inc	19.47	6.94	9.27	10.88	0.00	13.10	45.72
Penney J C Co Inc	29.39	19.43	20.72	23.85	19.90	21.95	34.17
Philip Morris Comp Inc	31.69	16.82	25.12	16.08	17.61	13.10	33.91
Seagram Co Ltd	25.50	18.38	7.72	17.18	11.97	18.59	32.37
Sears Roebuck Acc Corp	32.15	16.91	19.38	16.60	10.61	9.07	38.50
Service Corp Intl	59.03	12.56	15.30	29.49	16.01	27.50	55.53
Union Pacific Corp	32.60	19.35	12.85	13.21	15.95	12.81	29.44
Viacom Inc	37.90	22.28	18.65	16.07	15.70	15.11	38.32
Wal-Mart Stores Inc	24.46	24.59	27.58	7.07	19.46	11.97	35.06
Mean	28.27	18.30	16.62	15.03	14.18	14.98	35.74
Std Dev	10.48	6.75	9.62	6.56	5.89	5.53	6.43
Median	26.57	18.86	13.86	15.35	15.59	13.75	35.05
Count Asset Vol > Equity Vol	8	0	3	1	0	1	
Rel Frequency	25.00	0.00	9.38	3.13	0.00	3.13	

alent firm solvency ratio by $\exp(a(t|T))+1$ for $t=1$ to $t=T$.

For comparative purposes an observable proxy of the firm solvency is constructed using the approximate market value of the firm divided by the book value of debt. The firm value is calculated as above for the estimation of the observed asset volatility, by the sum of the firm's market capitalisation and total book debt. The firm's observed solvency ratio is the observed firm value divided by total book debt. Book values of debt are quarterly values reported by COMPUSTAT (items 45 and 51), and the market value of equity is obtained from CRSP for the day of the trade.

For each firm we average the implied and observed estimates of firm solvency for across the trade dates. Descriptive statistics for the firm averages, measured across 32 firms, is shown in Table 4.24. For illustration, plots of the paths taken by $x(t)$ and $S(t)$ are shown in Appendix D for the EM model fitted with a time-varying liquidity premium.

Panels A to C of Table 4.24 show the descriptive statistics for the smoothed estimate of the average implied firm solvency, controlling for different liquidity treatment, and the last row of Table 4.24 shows the descriptive statistics of the observed proxy of the average firm solvency. The cross-sectional observed solvency mean is 3.15 with a standard deviation of 2.43 and median of 2.30. Therefore, on average, our sample of firms have asset values 3.15 times greater than the book value of debt, or conversely, book debt is on average $1/3.15$, or 31.7 percent, of firm value.

In comparison, Panel A shows the descriptive statistics of the average firm implied level of solvency in th absence of a liquidity premium. The median solvency ratios vary from 1.40 for the LS2 model to 2.42 for the LS1 model. The across firm correlation with the observed solvency ratio ranges from 30.25 percent for the LS2 model to 65.41 percent for the LS1 model. Unlike asset volatility, there is greater consistency between observable and implied levels of solvency. Panels B and C introduce liquidity premiums into the measurement equation. From Panel C we can see that the correlation with the observed solvency ratio improves to range between 39.95 percent for the CDG model to 76.60 percent for the LS2 model. The additional liquidity component in the credit spread also increases the average implied solvency level since less of the observed credit spread must be explained by default risk. Implied solvency levels now range from 2.27 percent for the LT model to 5.07 percent for the CDG model. From these results we can conclude that the use of an observable proxy, and assumption that the default barrier is well approximated by the book value of debt, is reasonable on average for pricing purposes, if no liquidity premium is included in the predicted credit spread. However, firm specific differences remain, and the use of a proxy solvency ratio will not be an accurate measure of the default boundary for any individual firm's bond pricing.

A further question we address is whether the implied solvency levels appear to be economically reasonable. The observed solvency ratio is a direct proxy for the firm's solvency ratio if default occurs when firm assets equal the face value of debt, i.e. $K=D$.

However, evidence suggests that firms do not default when the asset reach the book value of debt. For example, in order to match observed historical recovery rates and bankruptcy costs, Huang & Huang (2003) set the default boundary to be 60 percent of the face value of debt. Davydenko (2005) find that firms, on average, default when the market value of assets, measured using the same proxy measure of firm value we have employed, is 65 percent of the face value of debt. Our observed solvency ratio in Table 4.24 is therefore likely to be biased downward relative to the empirical default boundary. To adjust to a more appropriate benchmark we assume that on average $K = 0.65D$ and multiply our observed solvency ratios by $1/0.65$ or 1.54. The adjusted observed mean solvency ratio benchmark becomes 4.85 with a median of 3.54.

In Panel A of Table 4.24 we can see that the mean implied solvency ratio varies by model and are well below the adjusted benchmark. However, when a constant liquidity premium is introduced as shown in Panel B, the resultant average implied solvency levels are much closer to the adjusted benchmark. The closest implied levels to the benchmark of 4.85 are the EM and CDG models with means of 4.66 and 4.72 respectively. With a time-varying liquidity premium, the CDG model average increases to 5.07 with a median of 3.58. Thus, the implied level of the default boundary appears to be the most economically reasonable when a liquidity premium is included into the predicted credit spread. The models that most closely match a reasonable empirical benchmark are CDG model followed by thee CEV model. Other models such as the LS1, LS2, EM, and LT models, imply default boundary levels that are too large, on average, relative to empirical evidence.

4.3.3 Solvency mean-reversion Rate

In Section 2.5 we provided theoretical and empirical support for the stylised fact that firms debt-ratios can be expected to mean revert. This is supported under both the trade-off theory, which emphasises the role of management targeting a preferred debt-ratio, and the dynamic version of the pecking order theory, in which auto correlated net financing deficits result in mean-reversion in the level of debt without management specifically targeting a debt-ratio. The CDG model explicitly includes an assumption of capital structure mean-reversion. In this section we compare the implied CDG mean-reversion rate, obtained from fitting the CDG model to credit spread term structures, with the capital structure empirical evidence. For brevity, we restrict our discussion of results to the measurement equation that includes a time-varying liquifity premium.

As reported in Table 4.28, we find significant levels of log-solvency mean-reversion rates in the CDG model. The firm-specific parameter estimates of the mean-reversion rates, and equivalent half-life statistics, are shown with asymptotic t-stats reported in parentheses. In 31 out of 32 firms we find that the mean-reversion rate is highly significant. The average mean-reversion rate is 0.142 per annum with a half life of 5.392

Table 4.26: This table shows the descriptive statistics of the estimated firm-specific log-solvency mean-reversion rates by industry sector of the issuing firm. Panel A summarises the mean-reversion rate parameter (κ_V) of the log-solvency ratio, $x(t) = \ln(V(t)/K(t))$, estimated by fitting the CDG model to the credit spread term structures of $n = 32$ firms, assuming a time-varying liquidity premium. Panel B summarises the equivalent half life of the log-solvency ratio mean-reversion expressed in years. Half life is calculated as $\ln(2)/\kappa_V$.

	Finance	Industrial	Utility	All
Panel A: Implied Mean-Reversion Rate (κ_V)				
Mean	0.179	0.135	0.126	0.142
Std Dev	0.033	0.042	0.036	0.043
Median	0.175	0.142	0.114	0.146
Min	0.132	0.055	0.099	0.055
Max	0.215	0.230	0.178	0.230
Panel B: Half Life of Mean-Reversion (H(κ_V))(yrs)				
Mean	3.996	5.701	5.785	5.392
Std Dev	0.771	2.135	1.408	1.960
Median	3.965	4.881	6.138	4.756
Min	3.219	3.007	3.890	3.007
Max	5.259	12.691	6.976	12.691
n	6	22	4	32

years. The minimum reversion rate found is 0.055 per annum and the maximum is 0.230 per annum. Our results are very close, on average, to the results of Fama & French (2002) who find firm-specific debt-ratios to be mean-reverting at a rate of between 0.07 and 0.18 per annum. Similarly, Frank & Goyal (2003) report an average firm debt-ratio mean-reversion rate of 0.124 per annum with small firms averaging 0.115 and larger firms 0.104 per annum. It appears therefore, that the market implies levels of mean-reversion in firm solvency similar to levels that are historically observed by debt-ratio movements.

In Table 4.28 we also show that the log-solvency mean-reversion rates vary by firm, and in Table 4.26 we show that the average mean-reversion rates vary by industry. Faster mean-reversion, ceteris paribus, implies a flatter term structure of the firm's credit spreads. It is clear that financial firms have the fastest rate of implied mean-reversion with an average estimated half life of 4.0 years, followed by Industrial with an average half life of 5.7 years, with utilities showing the slowest mean-reversion rate with a half life of 5.8 years. To better understand the causes of the cross-sectional differences in the average mean-reversion rate between firms, we conduct a multivariate regression of the estimated mean-reversion rate with firm and bond characteristics. As discussed previously in Section 2.5.5, the capital structure theory suggests that firms that are more debt constrained have a greater concern with targeting a debt-ratio. Therefore, we consider whether the expected mean-reversion rate of the firm's solvency, as implied from the CDG model, is consistent with the capital structure literature.

We apply a linear multivariate regression across the pool of 32 firms. The independent variable is the natural log of the firm-specific estimated mean-reversion rate parameter, κ_v, and the independent variables are defined as follows:

1. The firm's average debt rating. The firm's rating is assigned a numeric score that takes the values AA+=1, AA=2, AA-=3, A+=4, ..., BB-=13. Because the firm's rating may change over time, a sample weighted average rating is calculated by averaging the numeric score coincident with each observed trade date. Firms with low (high) average rating scores are likely to be less (more) debt constrained and therefore can be expected to have lower (higher) implied mean-reversion rates.

2. The sample average annualised daily equity return volatility for the firm. Firms with higher (lower) in sample equity volatility may carry higher (lower) default risk and therefore be more (less) debt constrained and exhibit faster (slower) implied mean-reversion rates.

3. The sample average of the daily ratio of the firm's observed solvency ratio. This is calculated as the sum of the firm's equity capital (sourced from CRSP daily) and the firm's book debt (sourced from COMPUSTAT quarterly), divided by book debt. Firms with higher (lower) average solvency are expected to be less (more) debt constrained therefore exhibit lower (higher) implied mean-reversion rates.

4. The sample average of the firm's daily market-to-book ratio. Firms with a high (low) average MTB tend to target debt-ratios less (more). If the bond market factors this into debt valuation we can expect lower (higher) implied mean-reversion rates.

5. The natural log of the sample average of the firm's daily market capitalisation as sourced from CRSP. Larger (smaller) firms can be expected to have greater (less) access to capital markets and are therefore may be less (more) concerned with debt targeting and exhibit lower (higher) implied mean-reversion rates.

6. The sample average of the firm's remaining tenor of its bonds. Included as a control variable to test whether the implied rate of mean-reversion is robust to the firm's average length of bond maturity. It is not expected be significant if the CDG model is fitted without bias by our selection of bonds.

7. A dummy variable that takes the value of one if the issuer is a financial firm, otherwise zero. A control variable included to test whether industry related factors, in addition to firm solvency, are significant.

8. A dummy variable that takes the value of one if the issuer is a utility firm, otherwise zero.

Table 4.27: This table shows the results of multivariate linear regression. The dependent variable is the natural log of the implied mean-reversion rate, κ_v, obtained by fitting the CDG model by EKF to the credit spread term structures on a sample of 32 firms. Parameter estimation was performed assuming that the credit spread includes a time-varying liquidity premium related to the Refcorp spread. t-stats are shown in parentheses. Significance of the mean-reversion rate at the 5 percent level is signified by '*'.

Independent Variable	Coefficient	t-stat	
constant	-2.547	(-1.981)	
Weighted Average Debt Rating	-0.001	(-0.010)	
Average Daily Equity Volatility	0.035	(0.074)	
Average Daily Observed Solvency Ratio (V/K)	-0.065	(-2.076)	*
Average Daily MTB ratio	0.077	(1.110)	
Natural Log of the Average Daily Market Capital	0.040	(0.544)	
Average Remaining Bond Tenor	-0.009	(-0.504)	
Finance Industry Dummy	0.241	(1.457)	
Utility Industry Dummy	-0.006	(-0.034)	
Adj R^2 (%)	10.939		
F(8, 23)	4.344		
n	32		

As reported in Table 4.27 we find that the only significant independent variable is the firm's average solvency level. The negative coefficient suggests that as expected, less solvent firms are priced by the bond market with greater expected levels of mean-reversion in the firm's capital structure. The dummy variables for industry are not significant suggesting that the apparent difference in mean-reversion rates by industry, as shown in Table 4.26, is due to the average differences in firm solvency across industries. Therefore, our exogenous setting of the firm's target solvency level, made necessary for estimation purposes, using industry average debt-ratios, does not appear to have systematically biased our estimates of mean-reversion. Importantly, other measures of debt constraint, such as the firm's rating, are not significant. This suggests that allowing firm-specific estimation of solvency levels and mean-reversion rates, is sufficient to capture the effects of debt constraint as priced by the bond market. Importantly, we find that the expected mean-reversion rate of the firm's solvency is a significant factor in the market's pricing of debt across the term structure. Unlike the extant capital structure literature our estimates of capital structure mean-reversion represent a new insight into the debt market's expectation of firm behaviour. Our estimates are obtained by inverting the market's expected reversion rate implied in the credit spread term structure shape, and not by direct observation of the firm's debt-ratio. We find that the debt market, on average, anticipates and prices into a firm's term structure of credit spreads, anticipated changes in leverage in manner comparable with observed debt-ratio mean-reversion rates. The rate of mean-reversion is found to be greater for firms with lower solvency levels consistent with these firms behaving with more concern for debt-targeting. We cannot conclude that these firms are more capital constrained since other proxy measures of capital constraint were not found to be significant explanatory variables. The negative relationship between solvency and implied solvency mean-reversion rate is found to be robust to other measures of firm capital constraint and industry membership.

4.4 Composition of the Credit Spread

In this section we use the results of the time-varying model specification to address the question of how much of the observed credit spread is explained by structural credit models. In predicting credit spreads, we have introduced additional control parameters for liquidity premium components of the spread. A question therefore arises as to how much of the credit spread is explained by the firms capital structure and asset risk, and how much is not. A better fitting model can be expected to explain more of the credit spread in terms of default risk related parameters.

Using a method of calibrating to real default probabilities, HH concluded that credit risk only explained a small proportion of the observed spread; typically around 20 percent to 30 percent for investment grade bonds, and decreasing as maturity shortens. For sub-investment grade bonds, the proportion explained by structural credit models increases. The HH method has some shortcomings. The models were fitted to average realised default rates, reported by rating, however for investment grade debt the underlying default rate level will be downwardly biased due to the low-frequency of observed defaults. The second problem is that some of the model parameters are proxied by observable variables, and any poor fit that results from the choice of proxy, can lead to the mistaken conclusion that structural models do not sufficiently explain the credit spreads. A particularly difficult aspect of their method is the need to estimate the unobserved market price of risk. Since we calibrate to observed bond prices and not to historical average default rates, we avoid the empirical difficulties of having to estimate the market price of credit risk.

In contrast to HH, our estimation method ensures that the structural models are fitted in with minimal credit spread prediction bias. Thus, we are able to present an alternative method for decomposing credit spreads: a component that is related to a liquidity premium, which is exogenous to the firm's default risk; the amount that is related to the firm's default risk, as predicted by the structural models; and, the residual component that is not predicted by way of liquidity premium nor predicted default risk.

Recall the measurement equation with time-varying liquidity is

$$y_{i,j}(t) = d_{i,j} + \beta_j^R Ref(t) + g(\alpha_j(t); \psi_{i,j}) + \varepsilon_{i,j}(t). \tag{3.10}$$

From equation (3.10) the following components of predicted spread are estimated at

Table 4.28: This table shows the estimated firm-specific log-solvency mean-reversion rates. κ_v is the estimated mean-reversion rate parameter of the log-solvency ratio, $x(t) = \ln(V(t)/K(t))$, estimated by fitting the CDG model to the credit spread term structures of n=32 firms, assuming a time-varying liquidity premium. H(κ) is the equivalent half life of the log-solvency ratio mean-reversion expressed in years. Half life is calculated as $\ln(2)/\kappa$. Asymptotic t-stats of the mean-reversion rate are shown in parentheses.

Issuer	κ	t-stat	H(κ) (yrs)
Aetna Inc	0.132	(7.467)	5.259
Associates Corp	0.215	(0.320)	3.219
Atlantic Richfield Co	0.055	(21.790)	12.691
A T & T Corp	0.125	(11.554)	5.550
Bear Stearns Companies Inc	0.186	(21.410)	3.733
Black & Decker Corp	0.139	(22.800)	4.983
Boeing Co	0.155	(10.930)	4.466
Dayton Hudson Corp	0.178	(15.627)	3.890
Commonwealth Edison Co	0.103	(17.016)	6.725
Enron Corp	0.123	(10.892)	5.630
Federated Dept Stores	0.183	(12.564)	3.795
Ford Mtr Co	0.168	(25.780)	4.135
General Mtrs	0.161	(22.607)	4.316
Georgia Pacific Corp	0.152	(13.061)	4.554
Hca Healthcare Corp	0.160	(13.698)	4.339
IBM Corp	0.186	(9.555)	3.719
International Paper Co	0.113	(16.970)	6.150
Lehman Brothers Holdings Inc	0.165	(29.105)	4.197
Merrill Lynch & Co	0.213	(28.720)	3.249
Motorola Inc	0.091	(20.526)	7.646
Nabisco Group Hldgs Corp	0.145	(21.946)	4.779
Niagara Mohawk Pwr Corp	0.099	(16.191)	6.976
Northrop Grumman Corp	0.095	(16.578)	7.290
Paine Webber Group Inc	0.160	(18.917)	4.320
Penney J C Co Inc	0.087	(32.836)	7.941
Philip Morris Companies Inc	0.146	(13.015)	4.733
Seagram Co Ltd	0.093	(33.324)	7.479
Sears Roebuck Accep Corp	0.230	(14.997)	3.007
Service Corp Intl	0.100	(30.394)	6.954
Union Pacific Corp	0.178	(17.039)	3.890
Viacom Inc	0.107	(29.959)	6.465
Wal-Mart Stores Inc	0.107	(13.911)	6.464
Mean	0.142		5.392
Std Dev	0.043		1.960
Median	0.146		4.756
Min	0.055		3.007
Max	0.230		12.691
n	32		32

each trade date, t, for bond, i, and firm, j, scaling by the observed credit spread:

$$\begin{aligned}
&\text{(i) Constant Liquidity Premium} = d_{i,j}/y_{i,j}(t),\\
&\text{(ii) Time-Varying Liquidity Premium} = \beta_j^R Ref(t)/y_{i,j}(t),\\
&\text{(iii) Model} = g(\alpha_j(t);\psi_{i,j})/y_{i,j}(t),\\
&\text{(iv) Prediction Error} = \varepsilon_{i,j}(t)/y_{i,j}(t).
\end{aligned} \tag{4.3}$$

The sum of components (i) and (ii) is our estimate of the proportion of the observed credit spread at time-t attributable to a liquidity premium, component (ii) is the proportion of the observed credit spread attributed to default risk, and component (iii) is the unexplained proportion of the observed credit spread.

The time-t estimates of the four components described above in equation (4.3) are pooled across time, bonds, and firms. The mean and standard deviations (shown in parentheses) of the components are reported in Table 4.29.

Panel A shows the results for the total pooled sample, Panel B the results for bonds with remaining maturities of less than 7 years, Panel C the results for remaining maturities between 7 and 15, and Panel D the results for bonds with remaining maturities greater than 15 years. Table 4.31 compares our results with HH for the models where a direct comparison can be made.

The average proportion of credit spread explained by a structural model is found to vary by model. Beginning with Panel A of Table 4.29, the lowest proportion of the credit spread explained by a model is by the CEV model at only 17.85 percent, which increases to a maximum of 43.72 percent for the EM model. Across models, the average credit spread explained by structural models is 31.95 percent. Our estimates, therefore, are at the upper end of HH's estimates of 20 to 30 percent but gives support to their view that the only a small proportion is attributable to default risk. The amount of credit spread explained by the structural models increases as the remaining tenor is lengthened. For trades with remaining maturity of greater than 15 years, the average across all models is 36.10 percent with the lowest being the LT model at 22.48 percent and the highest being the CDG model at 53.59 percent. HH report that the default risk component of the spread decreases as remaining maturity decreases.

The proportion of the credit spread explained by structural models also increases as the rating declines. In Table 4.30, the means and standard deviations (shown in parentheses) of the credit spread components of equation (4.3) are reported by the issue rating attributed to the bond at the date of trade. Results for AA rated trades are shown in Panel A, A rated trades in Panel B, BBB rated trades in Panel C, and BB rated trades in Panel D. For AA rated trades, the across model average proportion of the credit spread explained by the models is 26.71 percent, with the LS2 model the lowest at 15.77 percent, and the EM model the highest at 33.03 percent. For BB rated trades, the across model average increases to 50.56 percent, with the CEV model the lowest at 40.20 percent, and the CDG model the highest at 59.63 percent.

Thus, the proportion of the observed credit spread that structural models explain to be related to default risk is on average 31.95 percent, but improves with lengthening maturity or declining rating. Both results are characteristic of the models unable to generate sufficiently high probabilities of default when the firm's assets have only short time to diffuse to the default boundary, or the firm is sufficiently solvent for firm assets to be distant from the default boundary. For higher default risky BB rated issues, the structural models performed noticeably better explaining on average 50.56 percent of the credit spread.

The time-varying spread component measures the percentage of observable spread that varies in proportion with the Refcorp spread, and is found to be, on average, nearly of similar magnitude as the model component, varying between 19.25 percent for the LS1 model, to 33.26 percent for the CEV model. Thus, we find that time-varying liquidity, is of almost equal magnitude to that explainable by firm default risk. The constant liquidity component ranges from the smallest average at 36.11 percent for the CDG model to the highest at 58.61 percent for the CEV model.

The amount of spread attributable to the constant term includes effects not sufficiently specified in the structural model. The fact that we see some variation between models shows that we cannot conclude that the model component is all due to default risk, rather it is the amount of spread explained by the model. HH, on the other hand, attribute the modelled component all to default risk, thus assuming that structural models fully explains default risk.

The LS1 and LT models show similar average explanatory behaviour, which is not surprising given their similarity in specification. The CEV model is particularly poor in comparison to the other models. The marginal effect on explanatory power from the added complexity of a stochastic interest rate process can be made by comparing the model component of the LS1 model and LS2 model. Panel A of Table 4.29 shows that the LS1 model, on average, predicts that default risk comprises 10 percent less of the spread than the LS1 model that has deterministic interest rates. HH report a similar result as shown in Table 4.31. We therefore, find that the introduction of stochastic interest rates into the LS model reduces the estimated default risk component. Rather than predicting more of the observed spread, the additional complexity of the model only lessens the proportion of the credit spread predicted by the model. In Table 4.30 we show the average spread components reported by the issuer rating extant at the date of trade. For the highest rating of AA, the model component ranges between 15.77 percent for the LS2 model up to 33.03 percent for the EM model. Our model component estimates compare with HH who report 15.6 percent for AA rated 10 year debt for their LS Base case model, 16.4 percent for the CDG model, and 37.9 percent for the LT model. Unlike HH, we find that, with the exception of the LS2 model, most of our models explain 25-30

Table 4.29: This table shows the average composition of the observed credit spreads by remaining contractual maturity. Component (i) is the mean predicted constant liquidity premium expressed as a percentage of the observed credit spread. Component (ii) is the mean predicted time-varying liquidity premium expressed as a percentage of the observed credit spread. Component (iii) is the mean structural model predicted credit spread (excluding liquidity premiums) expressed as a percentage of the observed credit spread. Component (iv) is the mean credit spread prediction error, measured as predicted less actual yield to maturity credit spreads, expressed as a percentage of the observed credit spread. Descriptive statistics are based on pooling across all firms, trades, and time. Remaining tenor is the difference in the contractual maturity of bond and the date of the trade. Sample standard deviations are shown in parentheses.

	(i) Constant		(ii) Time-varying		(iii) Model		(iv) Prediction Error	
Panel A: All								
EM	49.80	(35.49)	14.06	(25.71)	43.72	(43.92)	-7.58	(62.93)
LS1	56.69	(37.26)	19.25	(41.30)	32.32	(40.73)	-8.26	(67.85)
LT	53.80	(36.10)	21.53	(36.30)	32.65	(45.84)	-7.97	(70.36)
CEV	58.61	(46.75)	30.59	(84.55)	17.85	(88.63)	-7.05	(67.73)
LS2	53.11	(37.71)	33.26	(47.12)	22.29	(30.90)	-8.65	(69.86)
CDG	36.11	(39.73)	27.27	(47.57)	42.84	(44.06)	-6.22	(68.99)
Panel B: Remaining Maturity ≤ 7 years								
EM	52.31	(38.55)	16.51	(29.47)	41.69	(53.19)	-10.51	(77.97)
LS1	54.74	(39.89)	22.70	(52.97)	33.71	(46.90)	-11.15	(82.94)
LT	50.23	(39.36)	23.31	(45.95)	37.62	(60.54)	-11.15	(89.18)
CEV	59.47	(58.98)	32.66	(79.86)	19.40	(97.05)	-11.52	(84.52)
LS2	56.62	(42.38)	38.51	(57.02)	18.26	(31.86)	-13.38	(86.44)
CDG	49.43	(44.84)	32.87	(58.95)	26.53	(38.55)	-8.83	(80.81)
Panel C: Remaining Maturity $7-15$ years								
EM	48.62	(35.41)	12.62	(25.33)	44.89	(36.73)	-6.13	(53.51)
LS1	59.51	(39.13)	17.44	(31.78)	30.41	(38.62)	-7.35	(59.76)
LT	54.56	(35.87)	20.22	(28.95)	30.95	(28.99)	-5.72	(55.71)
CEV	56.72	(36.89)	34.48	(104.56)	13.25	(95.09)	-4.45	(56.50)
LS2	54.02	(34.98)	31.57	(41.79)	21.12	(29.73)	-6.70	(59.28)
CDG	23.23	(32.33)	23.13	(39.84)	57.98	(45.10)	-4.34	(65.32)
Panel D: Remaining Maturity > 15 years								
EM	45.43	(24.27)	10.47	(9.05)	46.75	(26.64)	-2.65	(21.40)
LS1	55.55	(21.01)	13.69	(11.80)	32.90	(22.33)	-2.14	(22.20)
LT	62.16	(23.10)	19.54	(13.03)	22.48	(21.04)	-4.18	(26.52)
CEV	60.66	(20.82)	15.46	(11.12)	24.34	(22.16)	-0.47	(21.95)
LS2	40.93	(25.35)	22.27	(14.09)	36.56	(26.44)	0.24	(23.93)
CDG	28.56	(25.54)	21.10	(15.38)	53.59	(40.08)	-3.24	(28.97)

percent of the AA spread, with the LT model proving similar in performance to the LS1 model. At the other end of the rating spectrum, we find the models explain relatively more of the credit spread. As seen in Table 4.31,we find that our estimates of the model component of the credit spread are very similar to HH for the lowest BB rating. HH estimate the default risk component on a 10 year maturity BB rated issuer for an LS1 equivalent model to be 60 percent, 51.8 percent for the LT model, and 57.1 percent for the CDG model. We find that the CDG model explains the most of the observed BB rated spreads with 59.63 percent, and the CEV the least at 40.20 percent. The LS1 and LT models explain approximately 50 percent of the observed BB rated spread. Thus, we are able to affirm the findings of HH, with the exception that we find the one and two factor LS models, and the CDG model, explain more of the spread at higher ratings. In summary we find less variation between models and ratings than estimated by HH, but agree with the relatively lower explanatory power of the LS models and with the presence of a positive relationship between percentage of spread explained and rating. Structural models vary in their ability to explain credit spreads, but for the best fitting EM and CDG models, between 30 percent to 60 percent of the spread is explained by firm-specic default risk.

Table 4.30: This table shows the composition of the observed credit spreads by issuer rating. Component (i) is the mean predicted constant liquidity premium expressed as a percentage of the observed credit spread. Component (ii) is the mean predicted time-varying liquidity premium expressed as a percentage of the observed credit spread. Component (iii) is the mean structural model predicted credit spread (excluding liquidity premiums) expressed as a percentage of the observed credit spread. Component (iv) is the mean credit spread prediction error, measured as predicted less actual yield to maturity credit spreads, expressed as a percentage of the observed credit spread. Descriptive statistics are based on pooling across all firms, trades, and time. Rating refers to the issuer's rating for the bond recorded on the date of the trade. Sample standard deviations are shown in parentheses.

	(i) Constant		(ii) Time-varying		(iii) Model		(iv) Error	
Panel A: AA								
EM	59.67	(45.12)	17.30	(37.65)	33.03	(33.07)	-10.00	(72.80)
LS1	61.72	(45.84)	23.30	(45.83)	25.33	(31.41)	-10.36	(75.44)
LT	59.28	(44.91)	19.51	(34.97)	31.36	(30.47)	-10.15	(74.80)
CEV	63.29	(46.02)	23.55	(42.32)	23.66	(38.20)	-10.49	(77.98)
LS2	51.25	(40.84)	45.26	(57.65)	15.77	(26.82)	-12.28	(83.22)
CDG	32.20	(40.06)	44.81	(63.30)	31.12	(46.76)	-8.13	(85.66)
Panel B: A								
EM	48.36	(30.71)	14.94	(22.96)	43.58	(43.53)	-6.88	(58.97)
LS1	56.13	(31.47)	19.38	(35.67)	32.58	(48.60)	-8.09	(69.39)
LT	52.31	(34.47)	22.21	(28.28)	33.39	(60.04)	-7.91	(74.45)
CEV	58.90	(31.57)	22.49	(37.27)	25.73	(41.68)	-7.11	(66.75)
LS2	54.04	(35.53)	32.69	(43.98)	21.37	(30.68)	-8.09	(64.72)
CDG	38.07	(40.61)	24.90	(41.40)	43.41	(45.19)	-6.38	(66.54)
Panel C: BBB								
EM	47.18	(34.73)	11.87	(21.42)	48.39	(48.79)	-7.44	(64.15)
LS1	55.82	(38.86)	17.94	(46.03)	33.96	(33.83)	-7.72	(64.16)
LT	53.52	(33.02)	22.78	(45.25)	30.86	(31.01)	-7.16	(65.23)
CEV	57.04	(60.54)	44.79	(129.49)	3.89	(3.89)	-5.72	(65.79)
LS2	53.95	(39.12)	29.66	(45.13)	24.49	(31.95)	-8.10	(70.74)
CDG	36.23	(39.31)	22.53	(44.21)	46.76	(41.05)	-5.52	(64.70)
Panel D: BB								
EM	40.13	(25.29)	8.76	(10.11)	55.13	(33.50)	-4.02	(24.48)
LS1	44.37	(25.58)	8.82	(9.88)	51.04	(41.43)	-4.23	(32.75)
LT	43.99	(26.84)	10.51	(10.70)	50.53	(46.76)	-5.03	(39.00)
CEV	45.68	(28.77)	15.42	(16.14)	40.20	(28.99)	-1.29	(21.59)
LS2	42.77	(26.31)	11.29	(10.01)	46.85	(29.24)	-0.91	(19.05)
CDG	32.50	(27.71)	8.87	(12.94)	59.63	(29.51)	-1.00	(21.33)

Table 4.31: This table compares the percentage of the credit spread explained by structural models, by issuer rating, against comparable findings reproduced from Huang & Huang (2003)(HH). All numbers are in percentage.

		Model Component (%)			
Model	HH Model	AA	A	BBB	BB
LT		31.4	33.4	30.9	50.5
	LT	37.9	31.3	30.6	51.8
LS1		25.3	32.6	34.0	51.0
	LS (Base Case)	15.6	19.0	29.1	60.0
LS2		15.8	21.4	24.5	46.9
	LS (1-day CMT)	9.4	11.8	19.9	48.1
CDG		31.1	43.4	46.8	59.6
	CDG (Baseline)	16.4	18.3	26.9	57.1

Chapter 5

Conclusion

Structural models of credit risk are well known to perform poorly at predicting observed credit spreads. In a series of studies by JMR, LYS, and recently, EHH, the Merton model has been found to generally underpredict credit spreads; more so for short tenor bonds and for default risk remote bonds issued by highly rated firms. The structural credit modelling literature has subsequently developed along the path of theoretical extensions to address these biases and to relax the strong assumptions made by Merton. The result is a plethora of theoretical models but relatively little empirical work to verify the contribution made by these developments. In an important recent study by EHH they conclude that newer structural credit models improve on the average credit spread underprediction problem, but do so at the cost of losing considerable predictive accuracy. The common view, therefore remains, that structural credit models cannot adequately explain market bond yield spreads and that recent theoretical developments have not improved their performance at predicting credit spreads.

In this study we tested the hypothesis that the apparent poor predictive accuracy apparent across a wide range of structural credit models is due to the assumption made in extant empirical models that the firm's log-solvency process can be adequately approximated by the use of observable proxy variables. In other words, the assumption is commonly made that the firm's solvency level is fully, or in part, observable. Secondly, we test the hypothesis that having fitted a range of structural models assuming more correctly that firm solvency is truly unobserved, that the remaining biases will be related to missing factors identifiable from the extant capital structure theory and empirical evidence of dynamic management behaviour.

We make several contributions to the extant literature of JMR, LYS, EHH, and HH. Firstly, this is the first study to apply a quasi maximum likelihood estimation technique to fit a broad range of structural credit models on actual corporate bond trade data where the firm's log-solvency is properly treated as truly unobservable. We improve on prior studies by avoiding potential errors and biases introduced by the ad hoc choice of proxy variables. Our improved method ensures that the cross-sectional and time-series restrictions implied by the models is fully included in the estimation of the models, and thus, provide the most extensive and robust test of structural credit models to date. We therefore provide new insight into the relative performance of the models. We also present, for the first time, model error specification tests in addition to the usual discussion of prediction error biases. We also introduce controls for liquidity and are able to show their impact on model miss-specification.

Secondly, as a consequence of our model fitting method, we present a new insight into the decomposition of the credit spread into explained and unexplained components. Our method differs from HH in that we decompose the credit spread using firm-specific information from the credit spread term structures and avoid the empirical difficulty that HH have of converting risk-neutral probabilities of default to physical probabilities. We thus avoid an important source of calibration error inherent in HH.

Finally, we demonstrate how the implied default boundary of the firm can be extracted from market information thus providing new insight into the implied default point for non-defaulted firms.

In the remainder of this chapter we presents our main findings of model accuracy in Section 5.0.1, specification robustness in Section 5.0.2, and potential missing factors in Section 5.0.3. Related findings and a suggested direction for future research is discussed in Section 5.0.5.

5.0.1 Predictive Accuracy

We asked the question as to whether the implicit estimation of firm solvency and model parameters improves the predictive accuracy, relative to the extant literature, across a range of structural credit models. We confirm this to be true using a number of measures.

We find that by using EKF we can achieve mean levels of prediction error that are comparable with the reduced-form literature as evidenced by a comparison with the results of Duffee (1999). We find that, after inclusion of a time-varying liquidity premium, the average level of prediction error across models is essentially zero, ranging from between -0.59 basis points and 4.22 basis points across models. The RMSE is likewise similarly small across models, ranging between 28.76 percent and 34.4 percent across models. Some evidence of average underprediction is confirmed with negative MPE reported for all models ranging between -6.22 percent to -8.65 percent. In general, these errors are very small with little variation between the models, unlike the findings of EHH and HH.

A concern raised by EHH is the apparent wide variance in model error for structural models generally. By using implicit estimation of firm-specific model parameters we have reduced the variance in prediction errors dramatically. We observe an across firm mean MAPE per model of around 22 percent with no significant difference between models. In contrast, EHH report a mean MAPE ranging from 78 percent to 319 percent across different models.

5.0.2 Specification Problems

The improved accuracy of our fitting enables us to examine more carefully the structure of prediction errors across models. Beginning with the MPE, we confirm that two generic biases are evident in all models tested, as has been previously noted in the extant literature. The models tend to underpredict credit spreads more on short-term debt than on long-term debt. Similarly, the models generally underpredict credit spreads more on bonds from well rated issuers than they do on lower rated issuers. Default under a structural model only occurs with the passage of the firm's asset values to a default boundary, and it appears that the assumption of a smooth diffusive process tends to understate the probability of default when the distance of the firm from default is great or the passage of time allowed for diffusion is small. Our result is robust for two important reasons. Firstly, asset volatility is estimated using jointly the level implicit in the model's debt valuation, across the full term structure, and in the time-series behaviour of credit spreads. Thus, we do not allow the asset volatility to artificially inflate to match short term market spreads. Secondly, the biases are evident after controlling for a liquidity premium in the credit spread.

In a novel examination of specification errors, we examined the standardised prediction errors for consistency with the usual assumption of normal errors in the theoretical models and state-space framework. We find that all the models exhibit non-normal prediction errors. Standardised errors are fatter tailed than expected under normality, and exhibit positive skewness. It appears that the models are unable to explain large positive deviations in credit spreads that occur rapidly, but are nonetheless prevalent in the data. The problem is evident even after controlling for a time-varying liquidity premium. An investigation of autocorrelation in the standardised errors reveals that all models exhibit significant autocorrelation that rapidly decays with time lags. Inclusion of a time-varying liquidity premium, based on the 10 year Refcorp spread, reduced the level of autocorrelation confirming its usefulness in improving model specification. The presence of autocorrelated prediction errors suggests that there is a missing factor related to time-variation in spreads yet to be properly specified in the models.

The evidence of excess-kurtosis in the standardised errors, when taken together with evidence of percentage prediction biases at short bond maturities and high credit ratings, suggests that the diffusive asset process should be augmented with a firm asset value jump necessary to explain sudden changes in credit spreads.

5.0.3 Potential Missing Factors

We asked whether structural credit model prediction errors are related to missing factors identifiable from the capital structure literature. From our review of the theory and evidence, important stylised facts concerning the expected evolution of firm solvency are mean-reverting debt behaviour and market timing. Most structural models ignore these behaviours, with only the CDG model including debt targeting and debt market timing behaviour.

To test our hypothesis we chose a set of independent variables to represent potentially missing variables, together with other factors that prior studies had found important, and regressed these against the prediction errors of the models. For the standardised prediction error, we found that the models are well fitted for the firm and bond parameters that are specified in the models. For example, firm solvency, firm asset volatility, firm rating, and bond coupon rate are mostly unrelated to the standardised error. Only the LS1 model shows some mild positive relationship with asset volatility.

Our hypothesis was that debt market timing is important was confirmed by a generally significant relationship with the risk-free term structure. Unexpectedly, the CDG model also carried this relationship. The equity market timing variables, VIX and firm equity volatility, were less conclusive. Whilst generally significant, they carried the wrong signs as we would have expected that an increase in both would have decreased equity issuance and resulted in higher credit spreads and underprediction. We find the opposite. However, we cannot dismiss equity timing as an important omitted feature. During our sample period, equity volatility increased but the equity market had a bull run. Perhaps the use of volatility is not a sufficient measure of equity value. As an alternative measure of relative equity value, we expected that the MTB would be significant but in most cases it was not.

We also test for explanatory values that may affect the speed of mean-reversion in log-solvency under the CDG model. We find that the implied speed of mean-reversion is related to debt constraint to the extent that more highly leveraged firms have faster implied levels of mean-reversion implied into their terms structures of credit spreads. Other measures of capital constraint were not found to be significant. We find the levels of mean-reversion to be on average comparable with those reported from direct measurement of capital structure by by Fama & French (2002).

A strong relationship was found with the change in GDP and spread errors across all models. The result suggests that the market anticipates an improvement in firm asset value with improved business conditions. A less reliable relationship was found with the secondary market return on defaulted assets. It is possible that the return in this market is also influenced by current supply and demand conditions which are less relevant for non-defaulted firm valuation.

Finally, we find that errors are related to the risk-free rate level and volatility. However, the two-factor models do not adequately address the specification problem and the errors prevail in the two-factor LS2 model and CDG model. It appears that stochastic interest rates are yet to be adequately addressed by structural credit models and is possibly due to the simplistic nature of the single-factor Vasicek model.

5.0.4 Related Findings

As an extension to HH we report a decomposition of the observed credit spread and ask how much of the credit spread is explained by structural models. We find that, on average, the component of the observed credit spread explained by structural credit models is 31.95 percent. The percentage explained varies by model, with the highest achieved by the EM model at 43.72 percent and the lowest from the CEV model at 17.85 percent. The amount of the spread explained is related to the predictive accuracy of the models, and improves with lower credit ratings and longer remaining maturities. For the lowest BB rating, our estimates of the default risk component are very similar to HH, but we find that the LS1, LS2 and CDG models explain considerably more of the credit spread for AA rated firms than previously reported by HH. Also of additional interest is our finding that time-varying liquidity, is of almost equal magnitude to that explainable by firm default risk. Taking the modelled time-varying liquidity premium and model estimate suggests that we can account for approximately half of the credit spread through time variation in firm default risk and time-varying liquidity premiums.

As a consequence of estimating the models by EKF we are able to estimate the most likely path taken by the firm's latent log-solvency ratio. This provides a measure of the firm value relative to the default boundary through time. The average across models of the ratio of firm value to default boundary is found to be 2.51 compared to the average ratio of observed market value (using market equity capitalisation and book debt) of 2.30. Thus, we can infer that the default boundary is implied to be below the level of book debt. There is considerable variation across firms and models with the correlation between the implied solvency level and the market-accounting proxy sufficiently low that by using a proxy variable for leverage, to input into a structural credit model, would result in estimation error.

Finally, we compared the firm's implied asset volatility with the firm's observed equity volatility. A simple test for an upper bound is that the firm's asset volatility should be below its equity volatility on average. The Merton model requires an asset volatility, sufficient to match market spreads, that is excessive under this test. While the EM model performs with lowest prediction errors, we find that the CDG model has perhaps the most realistic description of the firm's solvency dynamics, as evidenced by reasonable levels of implied asset volatility, level of the default boundary, and speed of mean reversion.

5.0.5 Summary

In performing our testing we selected a representative range of models that have been subject to prior empirical analysis and are tractable. Which model performed best? The answer depends on the intended use of the model with no single model dominating under all criteria.

Unexpectedly, the EM model has best prediction accuracy as evidenced by the lowest MPE, lowest MAPE, lowest RMSE, and has the best overall goodness of fit as measured by the AIC. It also has the lowest autocorrelation in prediction errors. This is an important finding since the only difference between our EM and LS1 implementations is that the EM model includes an endogenous writedown rate that is allowed some time variation. Thus, the reduction in autocorrelation of errors appears to be attributable to this feature of the model. Thus, for simple debt valuation purposes, we find that the EM model performs the best amonst the sample of structural models. However, to match bond spreads, the implied level of asset volatility is excessive relative to the firm's equity volatility. We therefore, do not consider that the EM model is the most realistic description of the firm.

The LS1 model is a simple extension of the Merton model but performs worse than the EM model in all specification tests. Our finding lends support to similar results by LYS and EHH. It has the highest RMSE of all models. It would therefore appear that assuming the write-off rate to be time-invariant, and exogenous to the firm asset process, has reduced the model's performance relative to the EM model. What appeared to be a simple and elegant extension to permit a more realistic description of firm default has decreased model performance.

The LS2 model introduces a stochastic interest rate to the LS1 model but it offers only a modest improvement in prediction accuracy over the LS1 model. We find the LT model has the worst RMSE but a relatively low MAPE. Its goodness of fit is quite similar to the LS1 model, which is not surprising given their common roots. The LT model does improve on the LS1 model in specification tests with errors that are more normally distributed. The CEV model introduces local asset volatility but proves to offer little advantage over the extant model time-homogenous asset volatility models.

Finally, we find that the CDG model, which is the most complex model, suffers from relatively low accuracy and has the worst AIC score of all the models tested. However, the implied asset parameters appear reasonable, with mean-reversion rates and implied default boundary close to expected values. Therefore, it appears that the CDG model achieves its aim of describing a more realistic capital structure process than the Merton model, but suffers from a large number of parameters that are difficult to estimate. Along with the LS2 model, it is not apparent that a stochastic interest rate process has added much improvement. However, the mean-reversion of the firm's solvency does enable realistic levels of the default boundary to be achieved.

Overall, we conclude that the more advanced theoretical extensions to the Merton model have not been able to improve model performance significantly beyond an ad hoc implementation of the original Merton model. The most promising extension is the CDG model, however, its improvements in performance are attributable to the mean-reversion of the firm's underlying solvency ratio. However, the assumption of exogenously fixed writedown rate and stochastic interest rate volatility do not appear in their own rights, to enhance the specification of the models despite being common features of the newer structural models.

All the models demonstrate non-normality and autocorrelation in errors, and a positive correlation in error with the business cycle. Relationships between market equity timing variables and prediction errors did not hold as expected although prediction errors are found to be related to market sentiment (VIX, Refcorp spread, and yield curve term structure) and change in GDP. We therefore conclude that the first part of our hypothesis is supported; structural credit models provide similar levels of performance and bias, when estimated implicitly, with significantly higher accuracy than achieved in prior studies that relied on proxy variables for model fitting. The second part of our hypothesis is not supported as we are unable to conclusively determine that behaviours identified from the capital structure theory and evidence, can satisfactorily explain our credit spread prediction errors. Much work therefore remains to be done in order to better understand the relationships found between the market and business variables and prediction errors.

This study can be extended in two further directions. Firstly, our data covers a period of time when credit spreads began to rise sharply, but together with a rising equity market. Before drawing conclusions on the importance of market timing on bond prices, it would be informative to lengthen the time series to more recent periods so that a fuller credit and equity market cycle is included. Further, since commencing this study, the credit default swap market has continued to deepen. A more recent sample of data could use name specific credit default swap prices as a more refined measure of the name-specific liquidity premium. Secondly, in terms of guiding theoretical development of future structural credit models, it is apparent that market and business conditions are important missing factors. Possibly, changes in GDP can be used to condition the firm's asset value and a time-varying conditional write-off rate. The additional complexity of stochastic interest rates does not appear to warrant further work without introducing additional interest factors, which would result in an impractical model to fit with maximum-likelihood methods. Rather, the computational burden of an additional stochastic factor is potentially better utilised in modelling external market asset value or business cycle conditions. Finally, the use of mean-reversion in firm solvency appears to be useful, resulting in realistic levels of the implied default boundary and additional control over longer term credit spreads. The challenge remains however, to improve goodness of fit relative to the simplest model of them all.

Appendix A

Derivation of Exogenous-Boundary Dynamics

In this appendix we derive the stochastic differential process for the continuous latent solvency of the firm. We define $x(t)$ to be the log-solvency ratio, $x(t) = \ln V(t) - \ln K$ where $V(t)$ is the market value of the firm's assets and K is the firm's re-organisation boundary. Default occurs at the first instance $x(t) = 0$. The boundary may be time-varying or constant depending on the model's assumptions.

A.1 Longstaff-Schwartz (1995)

The risk-neutral return on the firm follows the same process assumed by Merton (1974)

$$\frac{dV(t)}{V(t)} = (r(t) - \delta)dt + \sigma_v dW_{v,t}^Q \, . \tag{A.1}$$

Since the default boundary is constant, we know that the s.d.e. for $x(t)$ is the same as for $\ln V(t)$. From Ito's Lemma we can express the latter as

$$d(\ln V(t)) = \frac{1}{V(t)} dV(t) - \frac{1}{2V(t)^2}(dV(t))^2 \, . \tag{A.2}$$

Substituting (A.1) into (A.2) and simplifying gives

$$dx(t) = (r(t) - \delta - \frac{\sigma_v^2}{2})dt + \sigma_v dW_{v,t}^Q \, . \tag{A.3}$$

A.2 Collin-Dufresne and Goldstein (2001)

The firm's assets are assumed to evolve under the risk neutral measure in the same manner as Merton and LS (refer (A.1)). Default occurs if the firm's asset value equals the default boundary, $K(t)$. The key assumption of CDG is that the boundary is actively adjusted by management so that it evolves with a time-varying drift given by

$$\begin{aligned} d\ln K(t) &= \kappa_v\big(\ln V(t) - \ln K(t) - \nu - \phi(r(t) - \theta)\big)dt \\ &= \kappa_v\big(x(t) - \nu - \phi(r(t) - \theta)\big)dt\ , \end{aligned} \tag{A.4}$$

where, θ is the long-run risk-free rate as per (Vasicek 1977), ϕ is a parameter controlling sensitivity of the drift to the current risk-free rate and κ is the speed of management's adjustment of the boundary level, and ν is a fixed parameter representing the target level of the boundary.

The stochastic differential equation for the instantaneous log distance to default is then,[1]

$$\begin{aligned} dx(t) &= d\ln V(t) - d\ln K(t) \\ &= (r(t) - \delta - \frac{\sigma_v^2}{2})dt + \sigma_v dW_{v,t}^Q - \kappa(x(t) - \nu - \phi(r(t) - \theta))dt \\ &= \kappa_v\left[\left(\frac{r(t) - \delta - \sigma_v^2/2}{\kappa_v} + \nu + \phi(r(t) - \theta)\right) - x(t)\right]dt + \sigma_v dW_{v,t}^Q\ , \end{aligned} \tag{A.5}$$

where $\kappa_v \geq 0$, $\nu \geq 0$, $\phi \geq 0$ and $\theta \geq 0$.

More simply, we can express the log-solvency ratio as a mean reverting process to a time-varying target level

$$dx(t) = \kappa_v\big[\bar{x}(t) - x(t)\big]dt + \sigma_v dW_{v,t}^Q, \tag{A.6}$$

where the target solvency level is positively related to the level of the short-rate, implying that a rise in interest rates, results in a reduction of the firm's target debt ratio

$$\bar{x}(t) = \frac{-\delta - \sigma_v^2/2}{\kappa_v} + \nu - \phi\theta + r(t)\left(\frac{1}{\kappa} + \phi\right). \tag{A.7}$$

[1]CDG define the state process as the inverse of $x(t)$.

Appendix B

Solutions to the First-Passage Crossing Times

B.1 Collins-Dufresene-Goldstein Model

In this section we reproduce the solution for the first-passage crossing time for the two-factor CDG model from (Collin-Dufresne & Goldstein 2001) Appendices C2 and C4.

The CDG model is described by a bivariate stochastic differential process

$$dx(t) = \kappa_v \left[\left(\frac{r(t) - \delta - \sigma_v^2/2}{\kappa_v} + \nu + \phi(r(t) - \theta) \right) - x(t) \right] dt + \sigma_v dW_{v,t}^Q$$
$$dr(t) = \kappa_r(\theta - r(t))dt + \sigma_r dW_{r,t}^Q.$$

Time is discretised into n_T equal intervals of Δt, and date $t(j) = \frac{jT}{n_T} \equiv j\Delta t$ for $j \in (1, 2, \ldots, n_T)$. Similarly, the short rate space is discretised into n_r equal intervals of Δr bounded between an upper value of $\overline{r}$ and lower bound of $\underline{r}$. Define $r(i) = \underline{r} + i \cdot \Delta r$ for $i \in (1, 2, \ldots, n_r)$ and $\Delta r = \frac{\overline{r} - \underline{r}}{n_r}$.

The probability that default occurs at any time between time zero and time T is given by the probability of first passage of x to zero, approximated by CDG as

$$Q(0, T; r(0), l(0)) = \sum_{v=1}^{nT} \sum_{i=1}^{nr} q(r(i), t(j)), \tag{B.1}$$

where $r(t)$ is the short-rate and the log-leverage ratio is $l(t) = \ln(K(t)/V(t)) \equiv -x(t)$ and the target log-leverage ratio $\bar{l}(t) = -\bar{x}(t)$.

Given the values of the functions ψ and Ψ, $q(r(i), t(j))$ is found recursively by the scheme

$$q(r(i), t(1)) = \Delta r \Psi(r(i), t(1)) \qquad \forall i \in (1, 2, \ldots, n_r), \tag{B.2}$$

and

$$
\begin{aligned}
&q(r(i),t(j)) = \\
&\quad \Delta r\left[\Psi(r(i),t(j)) - \sum_{v=1}^{j-1}\sum_{u=1}^{n_r} q(r(u),t(v))\psi(r(i),t(j)|r(u),t(v))\right] \\
&\quad \forall i \in (1,2,\ldots,n_r), \forall j \in (2,3,\ldots,n_T).
\end{aligned}
\tag{B.3}
$$

Under the T-forward measure, the means, variances and covariance of the transition of the log-leverage and short-rate between time u and t are:

$$
\begin{aligned}
E^T(u)[l(t)] &= l(u)e^{-\kappa_v(t-u)} - (1+\kappa_v\phi)\left(r(u)+\frac{\sigma_r^2}{\kappa_r^2}-\theta\right)e^{-\kappa_r(t-u)}B^{(t-u)}_{(\kappa_v-\kappa_r)} \\
&- \left(\frac{\sigma_r\rho_{r,v}\sigma}{\kappa_r} + (1+\kappa_v\phi)\frac{\sigma_r^2}{2\kappa_r^2}\right)e^{-\kappa_r(T-t)}B^{(t-u)}_{(\kappa_v+\kappa_r)} \\
&+ (1+\kappa_v\phi)\frac{\sigma_r^2}{2\kappa_r^2}e^{-\kappa_r(T-t)}e^{-2\kappa_r(t-u)}B^{(t-u)}_{(\kappa_v-\kappa_r)} \\
&+ \left(\frac{\sigma_r\rho_{r,v}\sigma}{\kappa_r} + \kappa_v\bar{l} - (1+\kappa_v\phi)(\theta - \frac{\sigma_r^2}{\kappa_r^2})\right)B^{(t-u)}_{\kappa_v},
\end{aligned}
\tag{B.4}
$$

$$
E^T(u)[r(t)] = r(u)e^{-\kappa_r(t-u)} + \left(\phi\kappa_r - \frac{\sigma_r^2}{\kappa_r}\right)B^{(t-u)}_{\kappa_r} + \left(\frac{\sigma_r^2}{\kappa_r}\right)e^{-\kappa_r(T-t)}B^{(t-u)}_{2\kappa_r},
\tag{B.5}
$$

$$
\begin{aligned}
Var^T(u)[l(t)] &= \left(\frac{(1+\kappa_v\phi)\sigma_r}{\kappa_v-\kappa_r}\right)^2 B^{(t-u)}_{2\kappa_r} \\
&+ \left[\sigma^2 + \left(\frac{(1+\kappa_v\phi)\sigma_r}{\kappa_v-\kappa_r}\right)^2 - \left(2\frac{\rho_{r,v}\sigma(1+\kappa_v\phi)\sigma_r}{\kappa_v-\kappa_r}\right)\right]B^{(t-u)}_{2\kappa_v} \\
&+ 2\left[\left(\frac{\rho_{r,v}\sigma(1+\kappa_v\phi)\sigma_r}{\kappa_v-\kappa_r}\right) - \left(\frac{(1+\kappa_v\phi)\sigma_r}{\kappa_v-\kappa_r}\right)^2\right]B^{(t-u)}_{(\kappa_v+\kappa_r)},
\end{aligned}
\tag{B.6}
$$

$$
Var^T(u)[r(t)] = \sigma_r^2 B^{(t-u)}_{2\kappa_r},
\tag{B.7}
$$

$$
Cov^T(u)[l(t),r(t)] = -\frac{(1+\kappa_v\phi)\sigma_r^2}{\kappa_v-\kappa_r}B^{(t-u)}_{2\kappa_r} - \left(\sigma\sigma_r\rho_{r,v} - \frac{(1+\kappa_v\phi)\sigma_r^2}{\kappa_v-\kappa_r}\right)B^{(t-u)}_{(\kappa_v+\kappa_r)},
\tag{B.8}
$$

where $B_z^{(t-u)} = \frac{1}{z}(1-e^{-z(t-u)})$.

The functions Ψ and ψ are given by

$$
\Psi(r(t),t) \equiv \pi(r(t),t|r(0),0)N\left(\frac{\mu(r(t),t|l(0),r(0),0)}{\Sigma(r(t),t|l(0),r(0),0))}\right),
\tag{B.9}
$$

$$
\psi(r(t)|r(s),s) \equiv \pi(r(t),t|r(s),s)N\left(\frac{\mu(r(t),t|l(s)=\underline{l},r(s),s)}{\Sigma(r(t),t|l(s)=\underline{l},r(s),s)}\right),
\tag{B.10}
$$

where $N(\cdot)$ is the cumulative normal distribution function and $\pi(r(t),t|r(s),s)$ is the transition density function for the one-factor Markov Gaussian short-rate process

$$\pi(r(t),t|r(s),s) = N\left(\frac{r(t)-(\theta+(r(s)-\theta))e^{-\kappa_r(t-s)}}{\sqrt{\frac{\sigma_r^2}{2\kappa_r}(1-e^{-2\kappa_r(t-s)})}}\right), \tag{B.11}$$

, and

$$\begin{aligned}\mu(r(t),l(s),r(s)) &\equiv E^T(s)[l(t)|r(t)] \\ &= E^T(s)[l(t)] + \frac{Cov^T(s)[l(t),r(t)]}{Var^T(s)[r(t)]}(r(t)-E^T(s)[r(t)]),\end{aligned} \tag{B.12}$$

and

$$\begin{aligned}\sum(r(t),l(s),r(s))^2 &\equiv Var^T(s)[l(t)|r(t)] \\ &= Var^T(s)[l(t)] - \frac{Cov^T(s)[l(t),r(t)]^2}{Var^T(s)[r(t)]}.\end{aligned} \tag{B.13}$$

In the interest rate dimension, the minimum rate is set to three standard deviations below the long run level, of $\theta = 16.05\%$, and the maximum rate equal to three times above the long run level. The number of intervals in the short rate is six, resulting in $\Delta r = 2.68\%$. Disretisation in the time dimension depends upon the remaining maturity of the bond, $(T-t)$. If the term is less than two years, the number of equal intervals is $8(T-t)$, and if the remaining maturity is greater than 2 years, the numbers of equal intervals is $4(T-t)$.

B.2 Longstaff-Schwartz Model

In this section we reproduce the solution for the first-passage crossing time for the two-factor LS model as corrected and reported in (Collin-Dufresne & Goldstein 2001) Appendices C2 and C3.

The LS model has the bivariate form

$$\begin{aligned}dx(t) &= (r(t)-\delta-\sigma_v^2/2)dt+\sigma_v dW_{v,t}^Q \\ dr(t) &= \kappa_r(\theta-r(t))dt+\sigma_r dW_{r,t}^Q.\end{aligned}$$

The solution for the expected first-crossing time follows the above solution for the CDG model with the moments of the forward leverage process given by:

$$\begin{aligned}E^T(u)[l(t)] = l(u) &- \left(\theta - \frac{\sigma_r^2}{\kappa_r^2} - \delta_v - \frac{\sigma_v^2}{2} - \frac{\rho_{r,v}\sigma_v\sigma_r}{\kappa_r}\right)(t-u) \\ &- \left(\sigma_r - \theta + \frac{\sigma_r^2}{\kappa_r^2} + \frac{\rho_{r,v}\sigma_v\sigma_r}{\kappa_r}e^{-\kappa_r(T-t)}(B_{\kappa_r}^{(t-u)})^2,\right.\end{aligned} \tag{B.14}$$

$$
\begin{aligned}
Var^T(u)[l(t)] = &\left(\sigma_v^2 + 2\frac{\rho_{r,v}\sigma_v\sigma_r}{\kappa_r} + \frac{\sigma_r^2}{\kappa_r^2}\right)(t-u) \\
&- 2\left(\frac{\rho_{r,v}\sigma_v\sigma_r}{\kappa_r} + \frac{\sigma_r^2}{\kappa_r^2}\right)B_{\kappa_r}^{(t-u)} + \frac{\sigma_r^2}{\kappa_r^2}B_{2\kappa_r}^{(t-u)},
\end{aligned}
\tag{B.15}
$$

and

$$
Cov^T(u)[l(t), r(t)] = \frac{\sigma_r^2}{\kappa_r}B_{2\kappa_r}^{(t-u)} - \left(\frac{\sigma_r^2}{\kappa_r} + \rho_{r,v}\sigma_r\sigma_v\right)B_{\kappa_r}^{(t-u)}. \tag{B.16}
$$

Appendix C

Description of Bond Data

Table C.1: This table shows the bonds sampled. FISD ID refers to the unique identifier attached to each bond by LJS Global Information Services, as reported in the Fixed Investments Securities Database (FISD). Coupon is the annualised semi-annual coupon rate, Mean $(T-t)$ is the average remaining maturity measured in years, and Rank is the ranking of the debt in the capital structure of the firm. The priority ranking of the bonds are: senior unsecured (SEN), senior secured (SENS), and senior subordinated (SS). The number of observed trade prices is shown by n, and the sample period is the date between the First Trade and Last Trade. Mean Δt is the average time between observed trades measured in years. The mean, median, minimum and maximum credit spread observed over the sample period is reported in basis points. SD refers to the sample standard deviation of credit spreads and SD d(1) is the sample standard deviation of the first differences.

Bond Characteristics				Sample Characteristics				Credit Spread Descriptive Statistics (basis points)					
FISD ID	Coupon (%)	Mean $(T-t)$	Rank	n	First Trade	Last Trade	Mean Δt (yrs)	Mean	Median	SD	SD d(1)	Min	Max
Aetna Inc													
42328	6.750	3.64	SEN	26	19-Aug-96	29-Sep-99	0.17	73.59	37.84	63.76	59.09	3.88	265.42
42329	7.125	8.66	SEN	32	15-Aug-96	01-Dec-99	0.15	85.90	70.19	39.17	23.67	40.59	178.11
42330	7.625	28.54	SEN	30	15-Aug-96	12-Nov-99	0.16	132.51	124.82	50.14	25.19	74.04	236.21
Associates Corp													
1555	7.500	2.80	SEN	30	02-Feb-95	01-Apr-98	0.15	48.44	47.50	14.42	14.34	9.34	76.15
1563	6.000	3.17	SEN	27	28-Feb-95	11-Feb-99	0.21	54.09	48.66	25.76	27.36	21.15	136.86
1568	5.250	2.87	SEN	26	17-Oct-94	04-Mar-99	0.25	60.63	49.78	39.27	33.57	25.05	180.87
1569	5.750	6.46	SEN	27	22-Feb-95	30-Aug-99	0.24	62.96	54.25	37.58	34.22	11.58	184.69
1575	7.875	4.78	SEN	44	29-Sep-94	03-Jan-00	0.17	55.70	49.73	24.60	19.85	17.44	118.96
26127	6.625	8.69	SEN	38	05-Jun-95	18-Aug-99	0.16	57.54	49.74	24.56	21.50	21.77	134.65
31674	6.375	5.35	SEN	35	31-Oct-95	23-Nov-99	0.17	57.87	49.85	29.57	30.15	22.59	140.44
32803	6.000	5.52	SEN	32	04-Dec-95	16-Aug-99	0.17	58.85	52.48	25.66	15.50	31.17	127.65
45820	6.875	10.99	SENS	30	12-Nov-96	26-Feb-99	0.11	71.02	60.88	28.62	16.77	30.76	154.28
91982	5.800	4.73	SEN	27	14-Apr-99	17-Dec-99	0.04	83.91	80.91	17.98	14.80	54.37	134.29

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Bond Characteristics				Sample Characteristics				Credit Spread Descriptive Statistics (basis points)					
FISD ID	Coupon (%)	Mean $(T-t)$	Rank	n	First Trade	Last Trade	Mean Δt (yrs)	Mean	Median	SD	SD d(1)	Min	Max
Atlantic Richfield Co													
1719	10.875	8.88	SEN	45	02-Mar-94	14-Mar-00	0.20	59.29	54.40	25.12	14.94	20.66	123.78
1724	9.875	19.58	SEN	37	25-Jan-94	12-Apr-00	0.25	71.55	69.65	16.77	15.88	43.20	130.62
1726	9.000	24.49	SEN	40	06-May-94	12-Apr-00	0.22	79.28	75.30	18.48	16.60	44.43	131.28
A T & T Corp													
94	7.500	8.47	SEN	54	28-Feb-95	19-Dec-00	0.16	64.48	48.01	41.86	19.85	13.85	194.58
96	7.750	9.71	SEN	70	28-Feb-95	18-Dec-00	0.12	57.35	43.40	33.30	22.09	20.67	205.04
25597	7.000	8.13	SEN	36	08-May-95	04-Oct-00	0.22	50.94	39.55	32.81	25.11	14.83	177.86
Bear Stearns Companies Inc													
2297	6.750	6.13	SEN	36	03-May-95	19-Jul-99	0.17	81.87	72.92	39.57	29.56	44.40	235.42
2300	6.625	6.75	SEN	55	24-May-95	13-Dec-99	0.12	82.78	73.78	32.30	28.40	36.52	213.95
28017	6.750	3.61	SEN	36	01-Aug-95	04-Jun-99	0.16	71.51	61.21	46.88	38.43	21.36	247.54
31069	6.875	8.04	SEN	33	02-Oct-95	09-Nov-00	0.23	92.01	73.23	41.24	24.35	43.49	217.31
36450	5.750	3.41	SEN	28	09-Feb-96	01-Oct-99	0.19	78.83	53.42	56.03	59.88	32.40	269.82
44710	7.250	8.86	SEN	38	08-Oct-96	03-Jul-00	0.14	93.37	64.81	54.92	31.75	49.90	261.38
50334	7.000	8.78	SEN	35	24-Feb-97	15-Dec-99	0.12	111.36	91.75	55.13	31.90	39.10	239.60
62272	6.625	6.14	SEN	36	14-Oct-97	01-Jun-00	0.11	92.86	80.04	34.62	21.12	35.03	174.26
68479	6.125	4.47	SEN	28	04-Feb-98	24-Nov-99	0.09	88.94	67.54	38.66	27.53	53.97	167.52
71515	6.200	4.35	SEN	24	24-Mar-98	10-Nov-00	0.16	107.59	113.65	43.80	33.42	42.95	197.99
Black & Decker Corp													
2532	7.500	6.48	SEN	76	19-Jan-94	07-Jun-99	0.10	80.99	68.08	38.62	21.14	25.28	236.18
2533	6.625	4.04	SEN	48	01-Feb-94	30-Apr-99	0.16	74.32	63.75	37.97	24.64	14.51	226.38
2534	7.000	9.66	SEN	56	25-Jan-94	20-May-99	0.14	88.25	79.41	27.47	16.48	48.67	164.87

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Bond Characteristics				Sample Characteristics				Credit Spread Descriptive Statistics (basis points)					
FISD ID	Coupon (%)	Mean $(T-t)$	Rank	n	First Trade	Last Trade	Mean Δt (yrs)	Mean	Median	SD	SD d(1)	Min	Max
						Boeing Co							
2559	8.750	24.22	SEN	38	22-Mar-94	26-Dec-00	0.26	91.40	72.32	41.48	31.01	36.19	200.54
2561	8.100	9.32	SEN	55	02-Jun-94	06-Nov-00	0.17	58.02	44.92	32.78	20.65	11.23	137.95
2565	6.350	6.20	SEN	58	28-Feb-94	21-Dec-00	0.17	45.72	38.11	26.59	20.85	5.32	123.03
						Dayton Hudson Corp							
5613	9.750	5.75	SEN	32	22-Feb-94	03-Mar-00	0.28	68.42	64.41	22.56	18.10	37.86	126.51
5615	10.000	4.12	SEN	36	31-Mar-94	27-Oct-99	0.23	73.72	63.46	28.35	22.78	38.33	149.43
5621	8.600	15.24	SEN	38	24-May-94	26-Aug-99	0.20	99.63	96.30	25.54	26.58	57.24	206.99
5626	6.625	5.60	SEN	40	09-Jun-95	04-Jan-00	0.17	65.47	57.14	24.30	19.43	24.06	129.59
36560	6.400	5.32	SEN	52	09-Feb-96	05-Dec-00	0.14	82.04	66.03	41.23	24.56	36.93	234.59
41534	7.500	8.84	SEN	43	19-Jul-96	09-Aug-00	0.14	72.86	67.25	20.94	17.84	47.21	153.56
44611	6.800	3.65	SEN	33	04-Oct-96	04-Aug-99	0.13	66.80	60.19	24.39	14.78	35.22	121.34
66437	6.750	29.02	SEN	27	06-Jan-98	10-Nov-00	0.15	122.11	121.27	28.27	25.68	93.12	217.48
78300	6.650	29.47	SEN	16	06-Aug-98	06-Aug-99	0.09	133.96	131.22	18.75	20.92	105.56	173.95
83731	5.875	9.50	SEN	11	29-Oct-98	11-Oct-00	0.26	106.42	111.37	29.33	26.34	74.46	151.25
						Commonwealth Edison Co							
4989	8.000	11.74	SS	39	23-Feb-94	10-Apr-00	0.23	89.01	80.58	22.17	19.88	56.44	146.47
4994	7.375	5.49	SS	37	24-Aug-94	15-Nov-99	0.21	73.97	67.73	22.35	20.50	44.73	137.07
4998	6.500	3.75	SS	38	26-Jan-94	26-Feb-99	0.20	74.10	70.47	23.84	22.65	35.67	162.21
5002	7.000	8.72	SS	54	19-Jan-94	19-Nov-99	0.16	81.61	75.74	20.82	15.85	50.64	140.67
5003	7.500	16.47	SS	41	21-Jul-94	03-Feb-00	0.20	101.57	95.42	18.83	13.65	71.83	147.01
5004	6.375	4.09	SS	32	27-Jul-94	29-Jun-99	0.22	69.26	66.26	20.32	17.09	42.27	142.24
5007	6.400	8.94	SEN	38	03-Feb-94	11-Aug-99	0.21	108.03	105.75	18.71	19.59	82.03	171.56
48196	7.625	8.62	SEN	27	06-Feb-97	29-Nov-99	0.15	109.43	106.71	25.97	22.83	65.53	155.44

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Bond Characteristics				Sample Characteristics				Credit Spread Descriptive Statistics (basis points)					
FISD ID	Coupon (%)	Mean $(T-t)$	Rank	n	First Trade	Last Trade	Mean Δt (yrs)	Mean	Median	SD	SD d(1)	Min	Max
Enron Corp													
6408	9.125	5.48	SEN	39	08-Mar-94	04-Oct-00	0.25	96.43	78.19	38.94	23.42	37.88	169.36
6409	7.625	7.49	SEN	45	16-Feb-94	12-Jun-00	0.21	83.86	65.28	41.36	26.83	39.49	182.57
25750	7.125	10.31	SEN	40	17-May-95	12-Jun-00	0.19	61.12	57.42	27.19	19.58	18.54	162.91
59681	6.750	6.04	SEN	17	04-Dec-97	21-Dec-99	0.18	101.30	89.87	37.80	35.02	65.57	199.53
63932	6.450	2.88	SEN	23	13-Nov-97	08-Sep-00	0.18	101.24	98.59	42.70	43.04	51.70	205.00
64197	6.625	6.94	SEN	22	19-Nov-97	12-Apr-00	0.16	106.15	106.94	33.09	25.42	56.36	164.61
Federated Dept Stores													
9573	10.000	3.34	SEN	30	14-Aug-95	11-Jan-00	0.21	107.35	100.25	65.55	33.58	27.81	249.72
31136	8.125	4.40	SEN	70	03-Oct-95	25-Oct-00	0.11	118.68	108.63	51.93	25.79	37.90	258.23
39610	8.500	4.53	SEN	44	16-May-96	30-Nov-00	0.15	135.24	130.88	56.59	21.91	40.33	251.24
57170	7.450	18.70	SEN	38	09-Jul-97	29-Dec-00	0.13	151.98	135.16	64.41	22.47	76.48	318.35
Ford Mtr Co													
10164	9.500	14.55	SEN	40	13-Oct-94	25-May-99	0.17	88.35	75.65	34.88	25.26	54.68	216.46
10168	7.500	2.66	SEN	31	08-Dec-94	05-Nov-98	0.18	50.67	47.43	17.33	14.98	22.79	92.13
32037	7.125	28.46	SEN	35	20-Nov-95	05-Nov-99	0.17	94.14	83.78	26.02	13.61	57.61	162.93
41993	7.500	28.40	SEN	38	06-Aug-96	08-Nov-99	0.13	110.30	99.27	31.26	17.26	74.57	171.45
44330	7.250	10.69	SEN	73	27-Sep-96	09-Dec-99	0.06	71.94	68.54	23.01	14.44	38.04	126.39
69523	6.625	29.55	SEN	30	18-Feb-98	04-Nov-99	0.08	111.55	105.26	22.06	9.78	82.23	156.31
78374	6.500	19.48	SEN	25	22-Jul-98	03-Jul-00	0.11	118.25	116.20	25.07	19.83	80.36	176.09
81320	6.625	29.49	SEN	46	06-Oct-98	05-Apr-00	0.05	130.14	126.75	22.31	20.58	100.97	232.28
87654	6.375	29.57	SEN	47	02-Feb-99	02-Aug-00	0.05	128.43	120.50	25.82	14.71	90.40	236.85

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Bond Characteristics				Sample Characteristics				Credit Spread Descriptive Statistics (basis points)					
FISD ID	Coupon (%)	Mean $(T-t)$	Rank	n	First Trade	Last Trade	Mean Δt (yrs)	Mean	Median	SD	SD d(1)	Min	Max
General Mtrs													
11135	9.625	3.77	SEN	82	29-Sep-94	29-Nov-99	0.09	65.08	56.48	35.35	39.36	20.27	277.03
11138	9.125	4.38	SEN	69	07-Feb-94	01-May-00	0.13	72.38	64.92	41.23	52.86	21.73	280.37
11141	7.000	6.16	SEN	39	14-Mar-94	16-Nov-00	0.25	70.70	65.65	27.96	20.20	18.68	131.75
29100	7.400	28.12	SEN	73	06-Sep-95	01-Dec-00	0.10	104.61	90.55	32.05	22.90	67.09	243.93
37881	7.100	8.59	SEN	46	15-Mar-96	16-Aug-00	0.14	81.58	68.93	44.51	34.18	31.90	244.10
38580	7.700	17.91	SEN	64	10-Apr-96	05-Dec-00	0.11	111.40	108.48	37.33	24.73	56.37	243.36
72797	6.250	6.27	SEN	35	22-Apr-98	30-Oct-00	0.11	75.59	68.06	32.96	36.22	23.57	153.49
72798	6.375	9.13	SEN	40	22-Apr-98	18-Dec-00	0.10	100.19	96.80	35.84	18.58	43.27	184.52
72800	6.750	29.00	SEN	53	06-May-98	19-Dec-00	0.07	133.03	125.65	36.49	34.33	45.47	241.63
Georgia Pacific Corp													
11321	9.500	14.85	SEN	42	18-May-95	30-Nov-99	0.16	107.37	100.83	33.29	20.34	73.46	203.46
26145	7.700	18.53	SEN	47	05-Jun-95	22-Dec-99	0.14	124.23	121.16	26.22	17.13	82.07	202.67
32843	7.375	28.41	SEN	31	05-Dec-95	03-Feb-00	0.20	132.07	128.63	25.19	15.53	94.19	186.20
Columbia Hca Healthcare Corp													
4718	6.500	2.92	SEN	30	16-Mar-94	27-Feb-98	0.19	63.67	50.14	49.49	25.18	23.94	244.47
4719	7.150	7.40	SEN	45	17-Mar-94	11-Feb-00	0.19	144.31	84.94	102.16	41.20	39.64	375.97
39572	7.250	10.56	SEN	44	15-May-96	01-Nov-00	0.15	133.33	62.09	109.14	45.85	39.04	389.20
40885	6.875	4.06	SEN	16	02-Jul-96	28-Sep-98	0.20	114.40	60.31	95.94	37.36	33.10	293.55
IBM Corp													
13191	7.250	5.72	SEN	97	04-Mar-94	18-Jul-00	0.10	44.02	41.46	22.41	15.19	6.19	104.87
13192	6.375	3.67	SEN	107	13-Jan-94	10-Jun-99	0.07	43.30	39.62	21.93	17.61	2.11	119.62
13193	7.500	16.29	SEN	81	07-Jan-94	01-Nov-00	0.12	74.99	71.47	19.84	19.00	35.45	137.18

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Bond Characteristics				Sample Characteristics				Credit Spread Descriptive Statistics (basis points)					
FISD ID	Coupon (%)	Mean $(T-t)$	Rank	n	First Trade	Last Trade	Mean Δt (yrs)	Mean	Median	SD	SD d(1)	Min	Max
International Paper Co													
13295	7.625	8.95	SEN	44	17-May-94	14-Sep-00	0.21	94.50	98.18	41.56	22.08	31.84	168.71
13298	7.500	7.50	SEN	40	19-May-94	20-Mar-00	0.21	72.04	59.99	33.46	30.23	21.78	160.86
13299	6.125	5.90	SEN	42	28-Feb-94	13-Dec-00	0.24	84.56	65.06	45.04	20.87	28.71	178.68
13302	7.625	7.50	SEN	38	04-Aug-94	20-Nov-00	0.24	79.91	66.66	45.26	36.21	18.99	177.72
40082	7.000	3.82	SEN	45	31-May-96	05-Apr-00	0.12	62.48	43.81	43.68	28.51	11.43	181.51
Lehman Brothers Holdings Inc													
14210	8.750	7.64	SEN	45	10-Mar-95	22-Nov-99	0.15	131.21	105.76	62.62	31.61	63.41	301.67
20228	8.875	5.21	SEN	43	10-Jun-94	03-Mar-00	0.19	106.03	98.53	47.61	41.29	40.69	264.88
20230	8.750	5.64	SEN	41	14-Jul-94	30-Jul-99	0.18	114.67	96.02	68.29	57.22	28.06	416.68
25524	8.500	9.63	SEN	48	02-May-95	11-Oct-00	0.17	135.22	103.54	71.38	64.11	49.85	423.05
44745	7.250	5.76	SEN	38	09-Oct-96	09-Nov-00	0.16	96.96	78.32	59.01	45.59	36.95	342.87
54022	7.375	5.66	SEN	42	08-May-97	04-Apr-00	0.10	130.97	113.98	79.29	46.31	39.69	414.48
59238	7.200	10.87	SEN	35	14-Aug-97	08-Oct-99	0.09	151.37	149.10	75.16	44.45	75.57	393.93
61527	6.500	4.02	SEN	35	01-Oct-97	09-Aug-00	0.12	134.27	114.27	87.53	52.05	44.67	437.92
72049	6.250	4.37	SEN	37	02-Apr-98	02-Aug-00	0.09	149.06	161.26	82.68	52.64	56.30	439.81
87559	6.625	6.44	SEN	41	29-Jan-99	06-Dec-00	0.07	166.02	161.27	42.56	35.48	107.60	295.24

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Bond Characteristics				Sample Characteristics				Credit Spread Descriptive Statistics (basis points)					
FISD ID	Coupon (%)	Mean $(T-t)$	Rank	n	First Trade	Last Trade	Mean Δt (yrs)	Mean	Median	SD	SD d(1)	Min	Max
						Merrill Lynch & Co							
15207	7.000	3.36	SEN	40	01-Jul-94	29-Oct-98	0.16	57.59	49.59	30.79	27.58	6.44	201.28
15212	6.250	4.97	SEN	40	08-Jun-94	01-Aug-00	0.22	58.87	46.80	32.73	21.75	15.41	195.00
29589	6.640	5.96	SEN	40	13-Apr-94	15-Aug-00	0.23	77.11	75.75	30.53	29.83	21.00	171.28
39416	7.375	10.95	SEN	49	23-Jun-94	03-Jul-00	0.18	89.03	76.98	39.35	33.20	36.35	199.01
58248	6.550	3.32	SEN	31	27-Jul-95	21-Jul-99	0.19	65.68	53.14	35.73	21.59	27.48	175.34
68436	6.000	3.71	SEN	37	22-Feb-96	05-Jan-00	0.15	53.99	42.00	35.46	27.51	12.43	159.60
77781	6.500	8.48	SEN	45	10-May-96	22-Dec-00	0.15	73.86	61.39	34.56	20.54	31.04	152.72
83254	6.375	6.07	SEN	43	29-Jul-97	06-Dec-00	0.11	64.96	55.55	28.39	15.35	28.36	141.03
84897	6.875	4.07	SEN	86	04-Feb-98	12-Dec-00	0.05	91.13	88.06	41.22	24.18	25.71	201.56
88024	6.000	9.34	SEN	102	09-Feb-99	29-Dec-00	0.03	115.55	110.64	31.22	21.04	10.80	216.58
						Motorola Inc							
15835	7.600	8.55	SEN	42	02-Aug-95	06-Dec-00	0.19	69.10	64.65	41.55	27.16	22.40	201.92
15836	6.500	10.62	SEN	34	22-Mar-95	04-Aug-99	0.19	42.94	33.33	23.15	12.09	12.18	107.69
25589	7.500	27.07	SEN	33	24-May-95	04-Dec-00	0.24	98.81	92.15	47.73	21.42	49.37	224.35
82677	5.800	9.36	SEN	16	15-Oct-98	01-Nov-00	0.19	95.06	99.00	23.63	22.85	56.80	143.69
						Nabisco Group Hldgs Corp							
26811	6.700	4.99	SEN	76	22-Jun-95	05-Dec-00	0.10	83.42	76.38	37.90	21.26	19.94	187.80
26812	6.850	7.92	SEN	67	22-Jun-95	17-Aug-00	0.11	98.76	81.26	48.65	50.91	53.74	366.64
26813	7.550	17.69	SEN	81	22-Jun-95	01-Dec-00	0.10	145.30	118.99	59.76	37.15	82.52	335.43
26975	8.000	3.18	SEN	35	01-May-95	24-Mar-98	0.12	73.99	55.11	63.59	61.37	9.50	305.33
26977	8.300	2.55	SEN	41	16-Feb-95	08-Apr-98	0.11	70.15	54.78	59.45	47.02	16.09	297.13
27158	7.050	9.84	SEN	81	11-Jul-95	29-Sep-00	0.09	108.25	87.12	54.75	30.34	9.05	320.08

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Bond Characteristics				Sample Characteristics				Credit Spread Descriptive Statistics (basis points)					
FISD ID	Coupon (%)	Mean $(T-t)$	Rank	n	First Trade	Last Trade	Mean Δt (yrs)	Mean	Median	SD	SD d(1)	Min	Max
Niagara Mohawk Pwr Corp													
16471	7.375	6.55	SS	51	09-Feb-94	10-May-00	0.18	141.37	119.87	62.69	32.51	61.12	293.52
16475	5.875	5.51	SS	55	02-Mar-94	19-Dec-00	0.18	142.24	112.39	63.89	32.14	72.20	287.91
16476	6.875	4.00	SS	53	25-Feb-94	03-Feb-00	0.16	135.29	121.02	52.84	35.14	67.93	300.55
25729	7.750	8.28	SS	47	08-Aug-95	05-Oct-00	0.16	163.11	142.33	67.86	32.09	72.82	309.86
Northrop Grumman Corp													
16731	8.625	6.42	SEN	40	28-Feb-96	19-Dec-00	0.18	101.95	85.78	45.48	36.24	35.39	189.16
69804	7.000	8.24	SEN	72	27-Feb-96	22-Dec-00	0.10	96.25	77.74	44.10	24.64	18.46	198.68
69808	7.750	17.99	SEN	43	27-Feb-96	20-Sep-00	0.16	143.92	108.02	55.90	28.41	83.06	245.81
69822	7.875	27.91	SEN	59	01-Mar-96	21-Dec-00	0.12	143.50	118.15	54.31	29.71	79.44	272.94
Paine Webber Group Inc													
17692	7.750	5.69	SENS	40	01-Mar-94	02-Nov-00	0.24	111.06	107.99	52.63	51.73	20.77	243.06
17694	7.000	3.51	SEN	35	25-Feb-94	12-Jan-00	0.25	97.11	78.11	55.21	53.95	30.19	287.27
17696	6.500	8.58	SEN	60	24-Feb-94	27-Oct-00	0.16	112.59	97.02	40.25	28.10	56.15	211.50
17697	7.625	16.46	SEN	60	08-Feb-94	03-Nov-00	0.16	145.52	129.58	53.13	41.28	25.23	314.00
17698	8.875	8.20	SEN	33	20-Mar-95	20-Aug-99	0.20	124.50	127.83	40.56	28.13	64.92	211.92
72702	6.550	8.97	SEN	25	20-Apr-98	03-Nov-00	0.15	119.25	123.99	45.34	61.02	32.54	222.47
85254	6.450	4.25	SEN	33	25-Nov-98	28-Aug-00	0.08	139.67	134.94	31.80	32.65	52.83	206.58
94324	6.375	4.46	SEN	20	13-May-99	30-Oct-00	0.11	121.22	122.53	34.24	46.40	3.78	161.08
102067	7.625	9.69	SEN	22	24-Nov-99	31-Oct-00	0.06	155.64	148.76	22.08	20.98	129.51	233.25

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Bond Characteristics				Sample Characteristics				Credit Spread Descriptive Statistics (basis points)					
FISD ID	Coupon (%)	Mean $(T-t)$	Rank	n	First Trade	Last Trade	Mean Δt (yrs)	Mean	Median	SD	SD d(1)	Min	Max
Penney J C Co Inc													
17851	5.375	2.91	SEN	27	20-Jan-94	28-Oct-97	0.20	39.47	40.14	9.94	13.14	11.69	57.84
17852	6.125	6.04	SEN	59	02-Sep-94	04-Dec-00	0.15	181.75	64.04	337.62	174.37	15.56	1948.50
17853	7.125	26.31	SEN	41	15-Feb-94	04-Oct-00	0.24	161.18	106.42	139.21	52.52	48.04	643.71
17854	7.375	7.05	SEN	49	07-Jun-94	17-Nov-00	0.19	138.07	60.09	240.59	169.65	21.89	1617.50
42350	7.375	10.43	SEN	48	14-Aug-96	14-Dec-00	0.13	163.98	69.92	225.22	86.47	38.87	1261.10
42351	7.650	18.19	SEN	39	14-Aug-96	15-Dec-00	0.16	169.32	159.01	163.09	102.76	71.62	1055.40
Philip Morris Companies Inc													
18221	8.250	6.40	SEN	73	11-Jan-94	26-Oct-00	0.14	97.31	81.71	43.96	50.59	10.77	264.90
18222	7.500	4.81	SEN	70	17-Feb-94	20-Dec-00	0.14	94.41	72.30	58.62	43.06	46.72	354.23
18226	7.625	4.71	SEN	43	04-May-94	28-Dec-00	0.23	92.98	77.39	46.23	28.40	36.89	217.93
18228	7.125	6.91	SEN	48	23-Mar-94	15-Aug-00	0.19	106.48	87.02	48.81	28.89	56.86	234.74
18231	7.250	5.80	SEN	58	10-Jan-94	19-Oct-00	0.17	89.29	73.73	40.05	28.77	42.80	213.40
40699	7.650	10.03	SEN	56	26-Jun-96	12-Dec-00	0.12	118.38	97.33	60.41	46.72	46.54	299.49
44048	7.250	3.89	SEN	35	19-Sep-96	27-Jan-00	0.14	81.03	63.42	42.37	34.29	36.10	223.21
47411	6.800	5.29	SEN	53	10-Dec-96	05-Dec-00	0.11	105.19	82.96	53.04	46.20	33.17	227.29
49572	7.200	8.61	SEN	45	07-Feb-97	19-Jul-00	0.11	114.20	98.18	54.47	32.24	52.67	315.37
57203	7.000	6.43	SEN	48	10-Jul-97	07-Dec-00	0.10	126.02	107.74	50.33	32.93	67.86	252.41
Seagram Co Ltd													
20000	8.350	9.21	SEN	60	18-May-94	01-Dec-00	0.16	95.31	77.19	54.84	30.72	16.74	240.59
20002	8.350	24.88	SEN	44	20-Jan-94	01-Dec-00	0.23	120.64	101.79	50.20	29.13	78.18	272.22
20003	6.500	5.61	SEN	46	12-Jan-94	08-Dec-00	0.22	74.38	59.31	39.23	28.15	33.72	191.74
20004	6.875	25.31	SEN	38	12-Oct-94	01-Dec-00	0.24	136.45	119.24	51.31	31.86	75.53	264.99

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Bond Characteristics				Sample Characteristics				Credit Spread Descriptive Statistics (basis points)					
FISD ID	Coupon (%)	Mean $(T-t)$	Rank	n	First Trade	Last Trade	Mean Δt (yrs)	Mean	Median	SD	SD d(1)	Min	Max
					Sears Roebuck Accep Corp								
26308	6.500	2.77	SEN	31	05-Sep-95	10-Mar-99	0.17	66.89	52.35	38.68	38.43	31.10	205.95
29172	6.750	8.05	SEN	47	11-Sep-95	13-Dec-00	0.16	80.33	57.08	50.61	29.55	30.39	212.58
34174	6.125	8.49	SEN	28	18-Jan-96	13-Jul-00	0.23	71.60	59.32	42.96	15.62	15.89	181.52
41876	6.900	5.27	SEN	29	02-Aug-96	05-Sep-00	0.21	76.22	49.71	50.37	45.63	27.07	224.77
46444	6.700	8.66	SEN	46	19-Nov-96	10-Nov-00	0.13	80.62	63.61	48.40	21.07	28.67	217.56
56492	7.000	8.77	SEN	100	25-Jun-97	27-Dec-00	0.05	93.98	80.55	48.03	23.94	27.29	258.33
60903	6.700	9.23	SEN	23	18-Sep-97	18-Jan-00	0.15	80.07	67.57	36.73	24.62	37.02	159.54
70992	6.000	4.04	SEN	56	13-Mar-98	21-Dec-00	0.07	95.05	97.60	34.90	26.74	33.40	180.70
93083	6.250	9.30	SEN	37	29-Apr-99	20-Dec-00	0.06	154.61	146.52	43.48	22.78	74.61	261.40
					Service Corp Intl								
20172	8.375	7.32	SEN	22	13-Jun-95	13-Mar-00	0.32	257.15	61.96	464.25	315.51	13.48	2033.80
31194	6.875	9.90	SEN	41	05-Oct-95	26-Sep-00	0.18	157.42	69.54	281.42	135.14	43.57	1356.80
31195	6.375	3.22	SEN	33	05-Oct-95	08-Sep-99	0.17	77.48	54.80	59.69	36.67	30.81	250.88
39942	6.750	2.83	SEN	28	23-May-96	12-Apr-00	0.20	700.38	99.57	1102.70	452.91	21.74	3348.60
39943	7.200	8.24	SEN	40	23-May-96	17-Oct-00	0.16	380.06	64.11	557.91	140.78	28.85	1644.30
52596	7.375	5.43	SEN	25	15-Apr-97	20-Sep-00	0.20	508.32	133.99	715.28	287.87	43.01	2125.30
52597	7.700	10.47	SEN	32	14-Apr-97	21-Sep-00	0.16	386.30	126.24	495.74	152.86	40.96	1382.80
70887	6.500	9.19	SEN	29	11-Mar-98	14-Dec-00	0.14	298.13	119.46	384.40	92.86	72.16	1352.70

(continued on next page)

(continued from previous page)

Bond Characteristics				Sample Characteristics				Credit Spread Descriptive Statistics (basis points)					
FISD ID	Coupon (%)	Mean $(T-t)$	Rank	n	First Trade	Last Trade	Mean Δt (yrs)	Mean	Median	SD	SD d(1)	Min	Max
Union Pacific Corp													
25528	7.375	3.36	SEN	33	10-Jan-96	26-Jan-00	0.18	93.82	72.20	49.93	38.26	37.99	202.39
34274	6.400	7.77	SEN	46	23-Jan-96	27-Dec-00	0.16	94.59	80.80	43.06	25.55	31.89	201.01
34275	7.000	17.83	SEN	58	23-Jan-96	19-Dec-00	0.12	123.92	118.07	44.05	26.58	72.27	228.87
45340	7.250	10.34	SEN	53	22-Oct-96	21-Dec-00	0.11	110.90	98.31	49.69	24.61	49.23	199.45
47092	6.700	8.61	SEN	41	03-Dec-96	29-Nov-00	0.14	87.67	79.60	37.59	37.48	36.81	184.88
68031	6.625	8.84	SEN	43	29-Jan-98	19-Dec-00	0.10	127.71	106.41	35.89	23.51	77.37	198.50
68032	7.125	28.66	SEN	30	03-Feb-98	12-Dec-00	0.14	167.79	150.78	45.64	42.64	108.25	247.69
82805	5.780	2.18	SEN	17	16-Oct-98	05-Oct-00	0.17	132.72	132.36	32.34	26.32	93.51	209.17
Viacom Inc													
25799	7.750	6.87	SEN	176	01-Dec-95	27-Dec-00	0.04	135.25	132.75	29.87	17.74	74.98	273.13
33080	6.750	4.22	SEN	46	12-Dec-95	28-Dec-00	0.16	120.54	115.72	24.56	23.23	81.20	174.65
33081	7.625	17.77	SEN	37	12-Dec-95	16-Aug-00	0.18	158.40	160.28	30.81	30.89	118.29	252.36
Wal-Mart Stores Inc													
23421	9.100	3.83	SEN	50	26-Apr-94	09-Jul-99	0.15	41.58	34.48	21.26	14.12	17.15	107.11
23422	8.625	4.14	SEN	115	09-Mar-94	20-Mar-00	0.08	42.64	38.90	18.76	15.09	9.13	106.93
23425	6.125	3.25	SEN	37	15-Feb-94	13-Aug-98	0.18	33.36	31.25	11.17	14.05	12.92	59.10
23427	6.500	5.78	SEN	67	10-Mar-94	29-Dec-00	0.15	43.72	38.94	19.54	15.90	5.63	80.92
23429	7.250	15.38	SEN	45	28-Mar-95	27-Dec-00	0.19	65.41	59.49	23.06	13.14	33.06	135.09
23430	5.875	8.83	SEN	108	02-Mar-94	01-Dec-00	0.09	45.05	40.01	19.18	17.24	2.83	96.63
23432	7.500	7.22	SEN	84	18-May-94	06-Dec-00	0.11	48.83	42.17	24.73	20.05	5.26	163.56
23433	8.000	9.01	SEN	56	20-Sep-94	12-Dec-00	0.16	55.62	45.89	29.49	19.77	3.06	128.57
25644	6.750	5.14	SEN	31	10-May-95	08-Aug-00	0.25	37.38	36.80	15.39	13.59	4.93	71.10

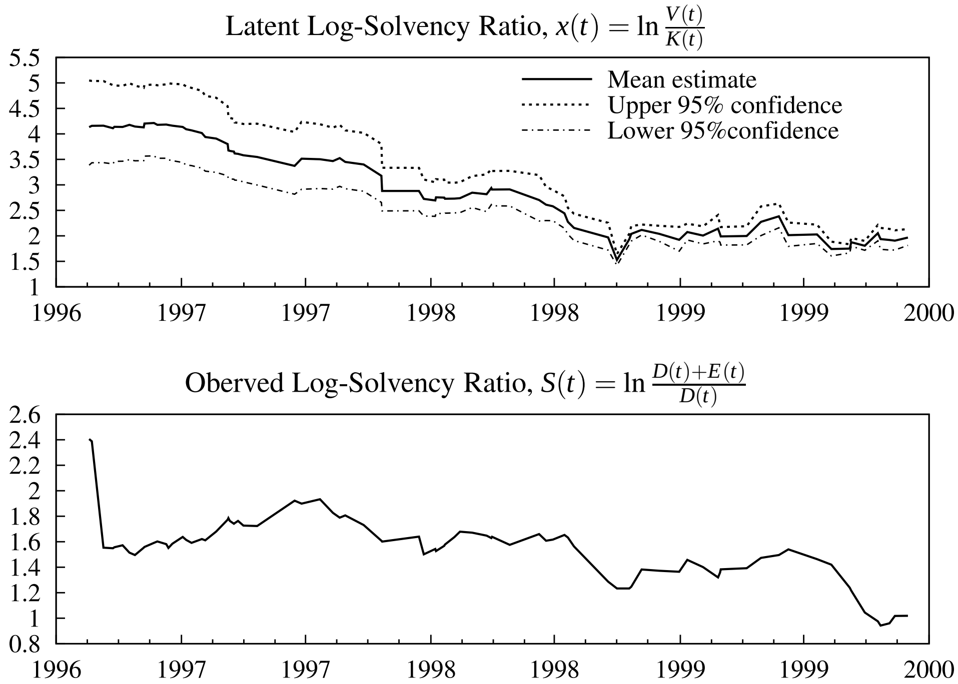
Table C.2: This table details the Refcorp strip data sourced from Bloomberg. Shown are the summary statistics of daily yields and available remaining maturities.

		Maturity (yrs)			Bid yields (%)			
Bloomberg ID	No. of Obs	Mean	Min	Max	Mean	Min	Max	Std. Dev.
76116EAR Govt	695	2.4	1.0	3.8	6.18	4.70	7.88	0.73
76116EAT Govt	804	3.2	1.0	4.8	6.30	4.90	7.94	0.71
76116EAV Govt	838	4.0	1.1	5.8	6.35	5.06	7.88	0.71
76116EAX Govt	871	5.0	1.0	6.8	6.47	5.12	7.90	0.67
76116EAZ Govt	857	6.0	1.8	7.8	6.56	5.02	8.00	0.66
76116EBB Govt	830	7.1	2.9	8.8	6.68	5.42	8.08	0.66
76116EBD Govt	828	8.1	3.9	9.8	6.81	5.60	8.20	0.64
76116EBF Govt	1753	7.3	3.8	10.8	6.46	4.41	8.43	0.79
76116EBH Govt	1753	8.3	4.8	11.8	6.52	4.48	8.46	0.78
76116EBK Govt	1753	9.3	5.8	12.8	6.57	4.51	8.48	0.78
76116EBM Govt	1753	10.3	6.8	13.8	6.61	4.56	8.51	0.78
76116EBP Govt	1753	11.3	7.8	14.8	6.66	4.60	8.55	0.77
76116EBR Govt	1753	12.3	8.8	15.8	6.70	4.71	8.58	0.76
76116EBT Govt	1753	13.3	9.8	16.8	6.75	4.83	8.62	0.75
76116EBV Govt	1753	14.3	10.8	17.8	6.80	4.94	8.66	0.75
76116EBX Govt	1753	15.3	11.8	18.8	6.84	5.06	8.71	0.74
76116EBZ Govt	1753	16.3	12.8	19.8	6.88	5.18	8.76	0.74
76116ECB Govt	1753	16.3	12.8	19.8	6.91	5.23	8.81	0.75
76116ECD Govt	1753	18.3	14.8	21.8	6.92	5.28	8.77	0.74
76116ECF Govt	1753	19.3	15.8	22.8	6.92	5.34	8.73	0.73
76116ECH Govt	1753	20.3	16.8	23.8	6.92	5.39	8.70	0.72
76116ECK Govt	1753	21.3	17.8	24.8	6.92	5.45	8.66	0.70
76116ECM Govt	1753	22.3	18.8	25.8	6.91	5.41	8.63	0.70
76116EGU Govt	1753	23.3	19.8	26.8	6.91	5.38	8.59	0.70
76116EGW Govt	1753	24.3	20.8	27.8	6.89	5.35	8.56	0.69
76116EGY Govt	1753	25.3	21.8	28.8	6.87	5.32	8.52	0.69
76116EHA Govt	1753	26.3	22.8	29.8	6.85	5.28	8.49	0.68
76116EHC Govt	1556	26.9	23.8	30.0	6.70	5.24	8.45	0.63
76116EHE Govt	1307	27.4	24.8	30.0	6.50	5.19	7.60	0.50
76116EHG Govt	1055	27.9	25.8	30.0	6.34	5.14	7.60	0.47
76116EHJ Govt	805	28.4	26.8	30.0	6.14	5.09	7.11	0.40
76116EHL Govt	554	28.9	27.8	30.0	6.24	5.32	7.11	0.42
76116EHN Govt	303	29.4	28.8	30.0	6.42	5.75	7.12	0.30
76116FAD Govt	53	29.9	29.8	30.0	6.19	5.91	6.45	0.17
Total	46416	16.6	1.0	30.0	6.71	4.41	8.81	0.74

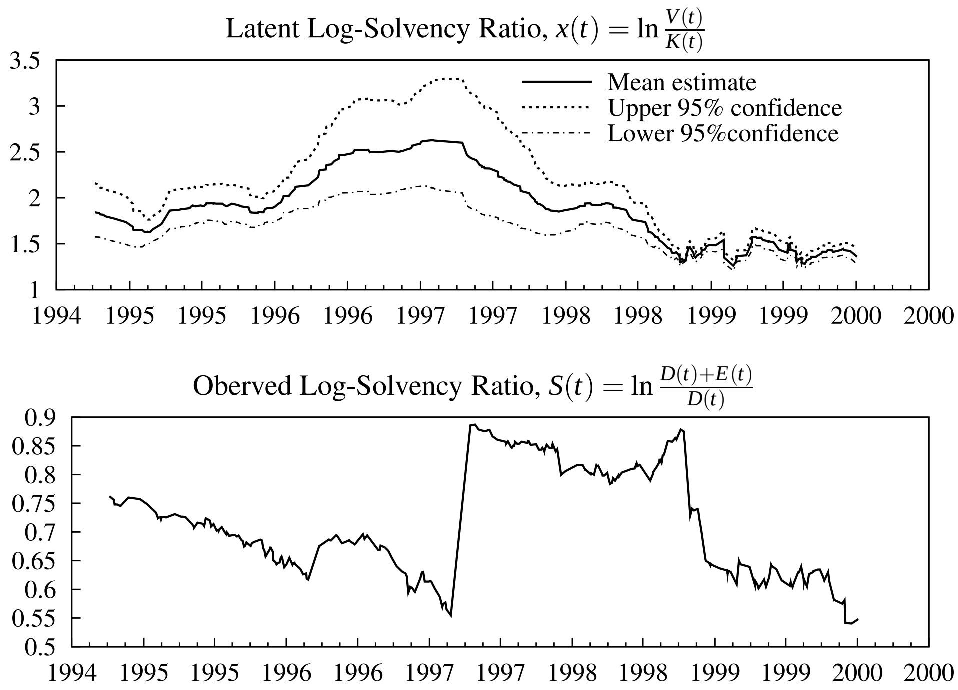
Appendix D

Example of Implied Firm Solvency Paths

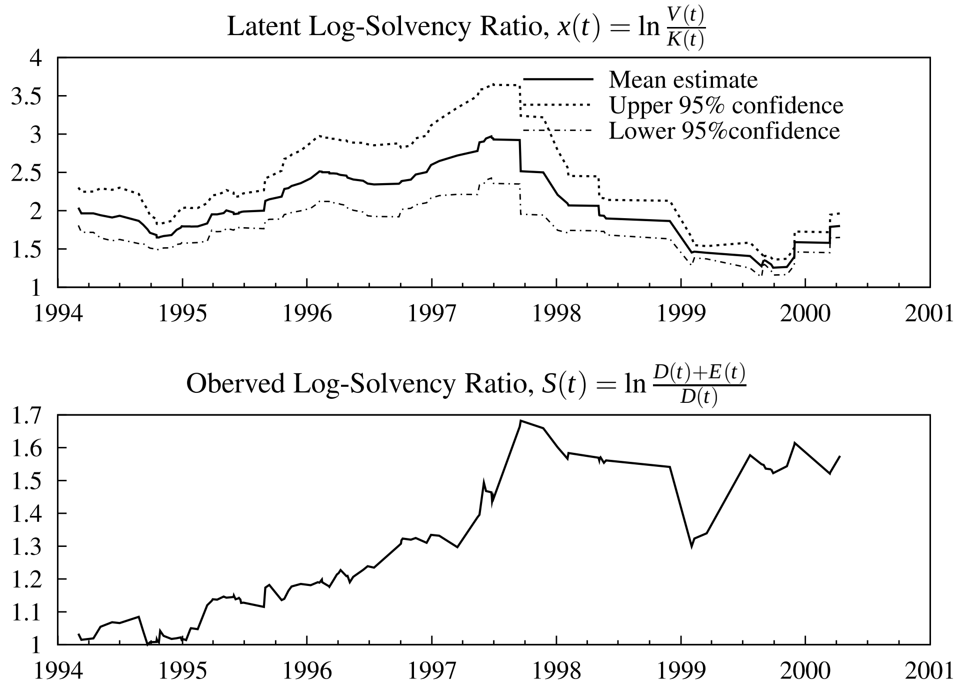
The following plots show the implied time-series path for each firm's log-solvency ratio and the corresponding observed log-solvency ratio. The implied path of solvency is from the obtained from the smoothed estimate of the state process from fitting the extended Merton (1974) (EM) model with time-varying liquidity.



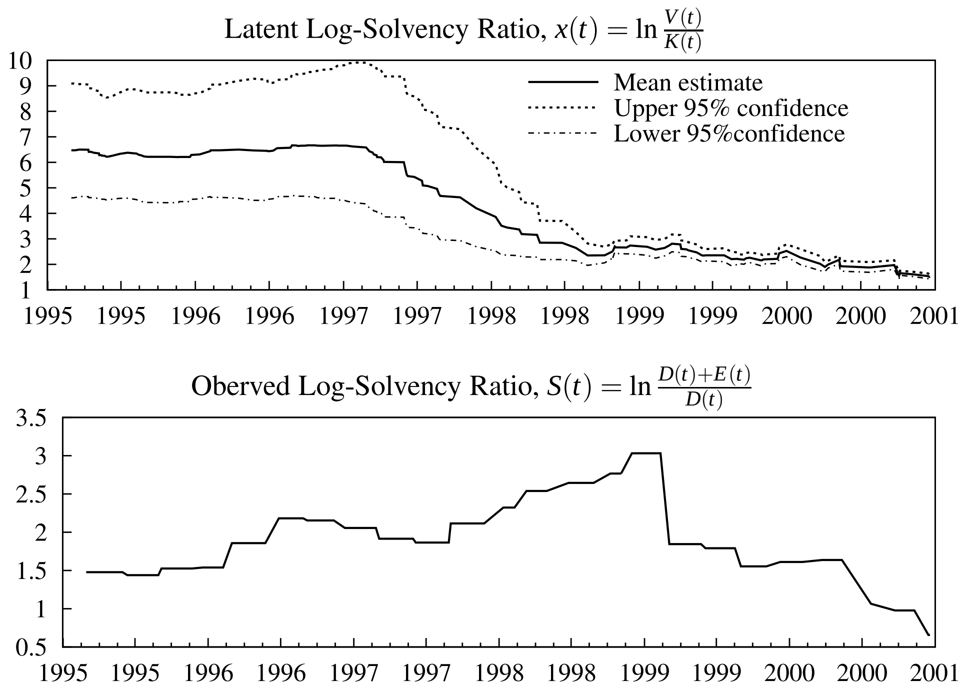
D.1.i Aetna Inc



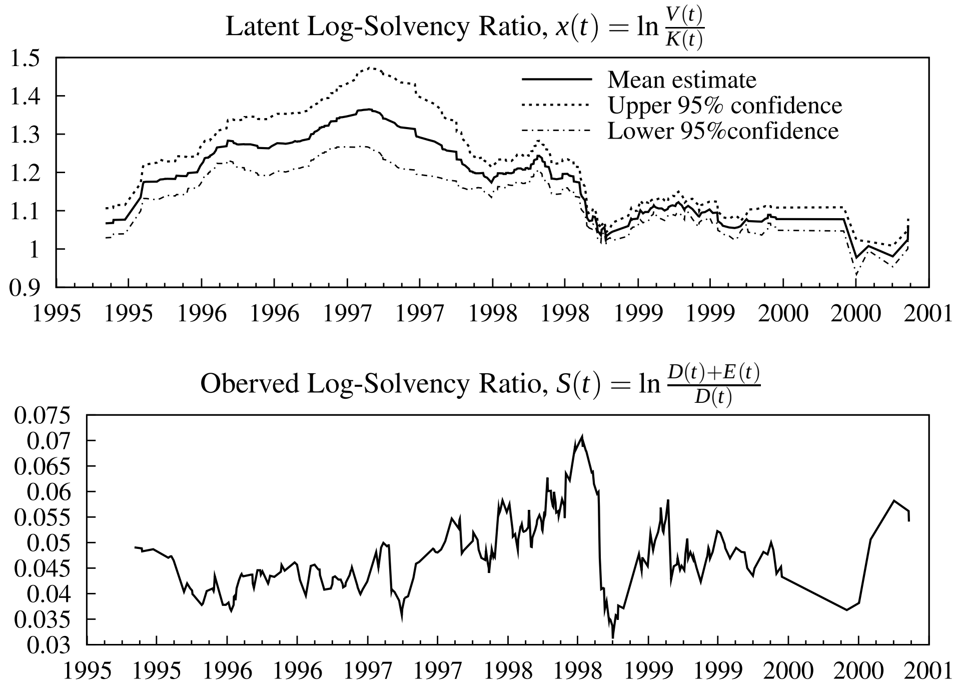
D.1.ii Associates Corp



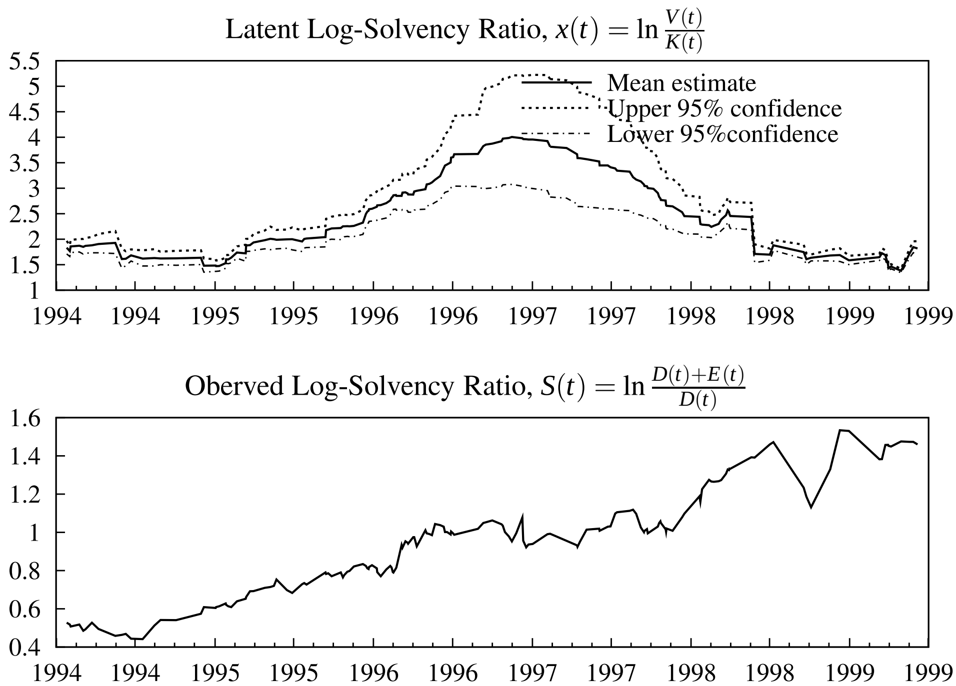
D.1.iii Atlantic Richfield Co



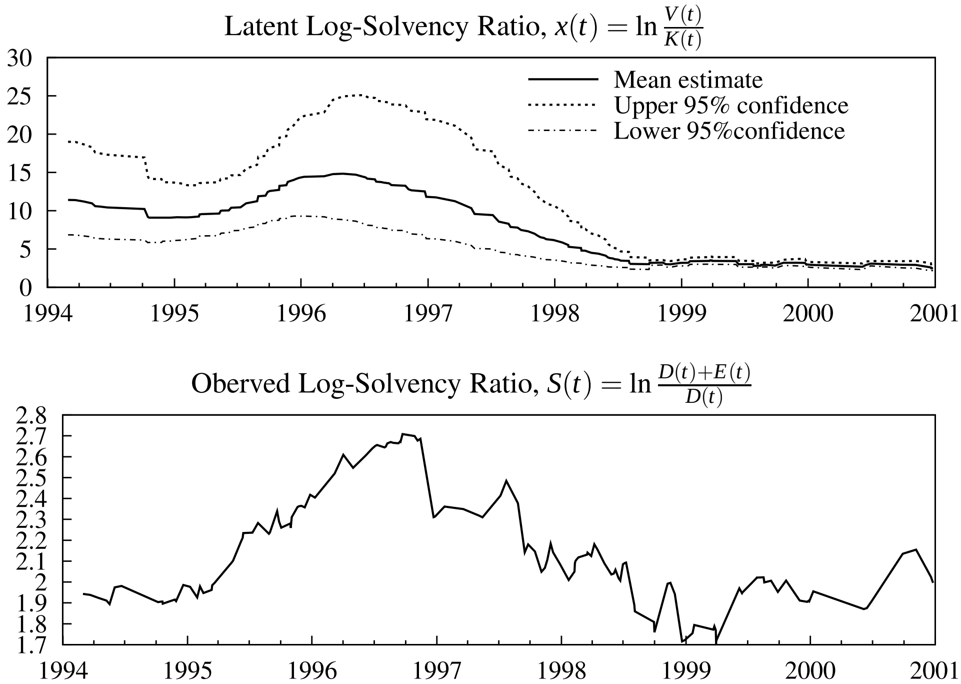
D.1.iv A T & T Corp



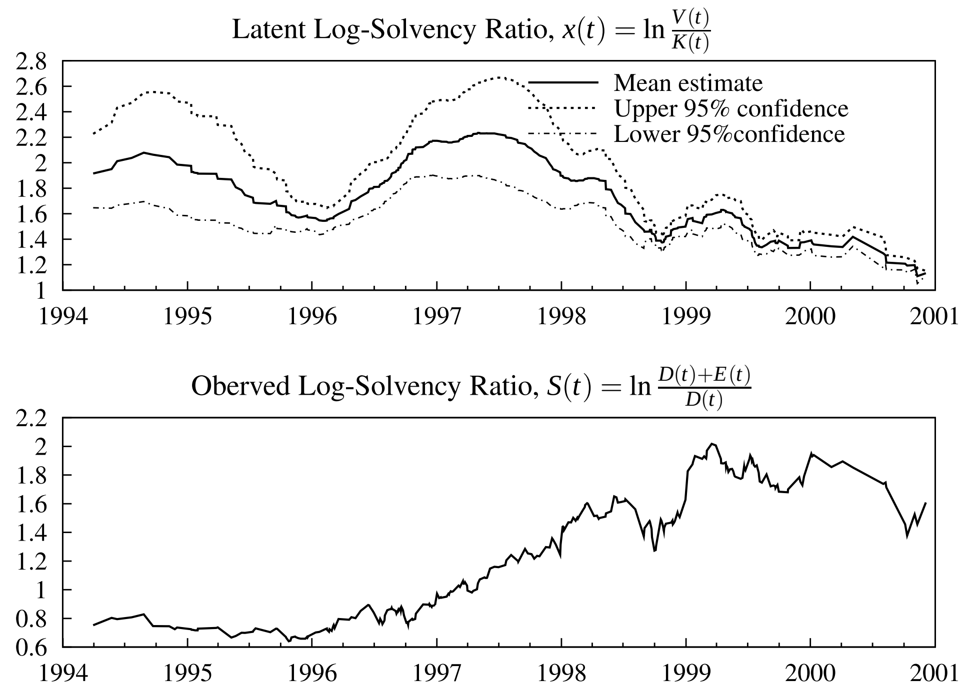
D.1.v Bear Stearns Companies Inc



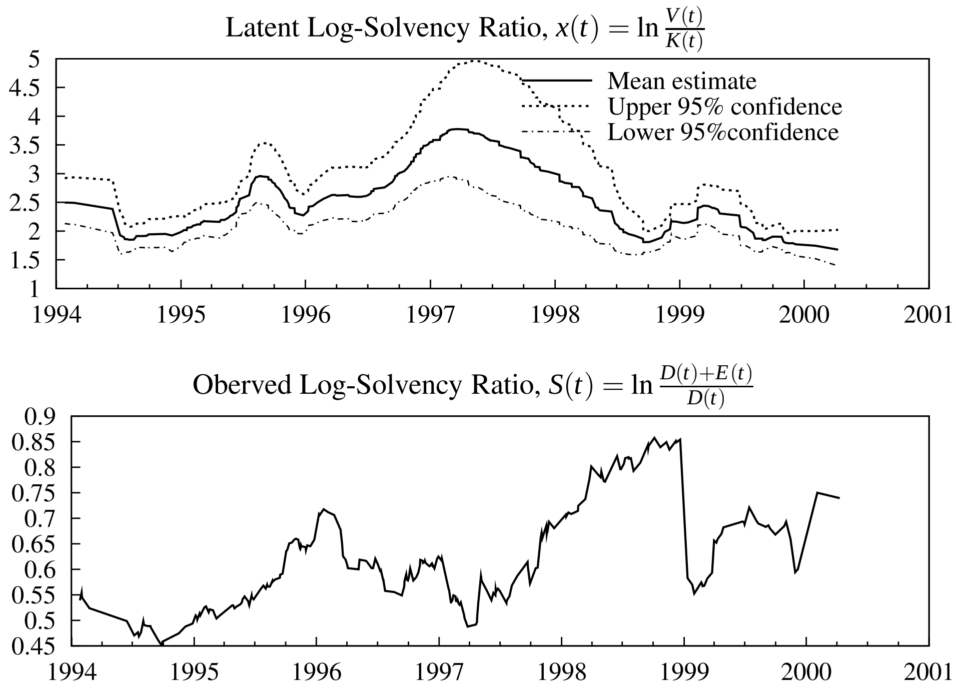
D.1.vi Black & Decker Corp



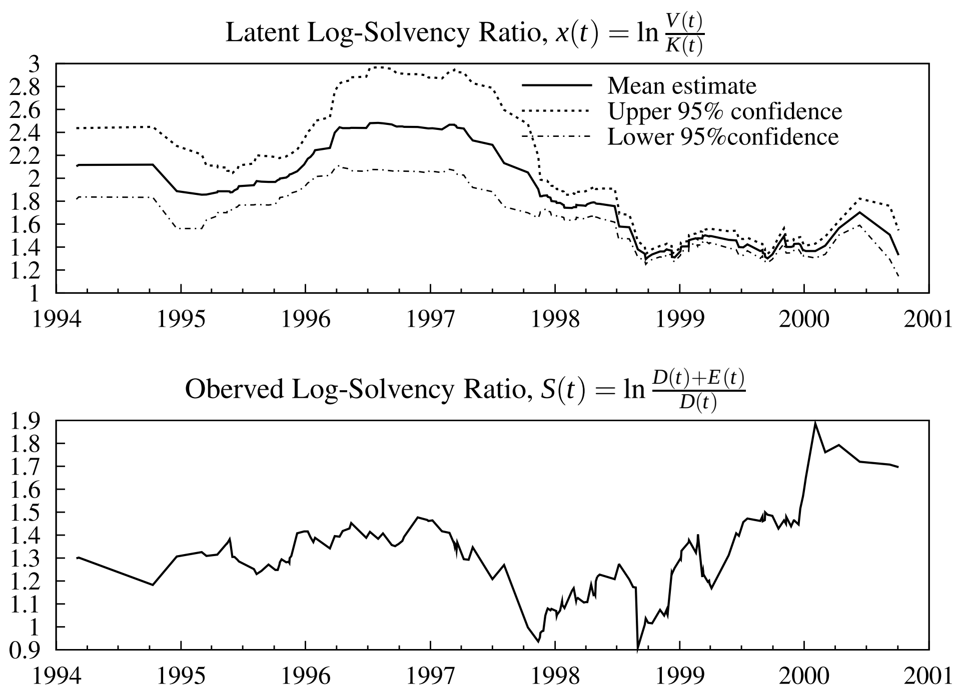
D.1.vii Boeing Co



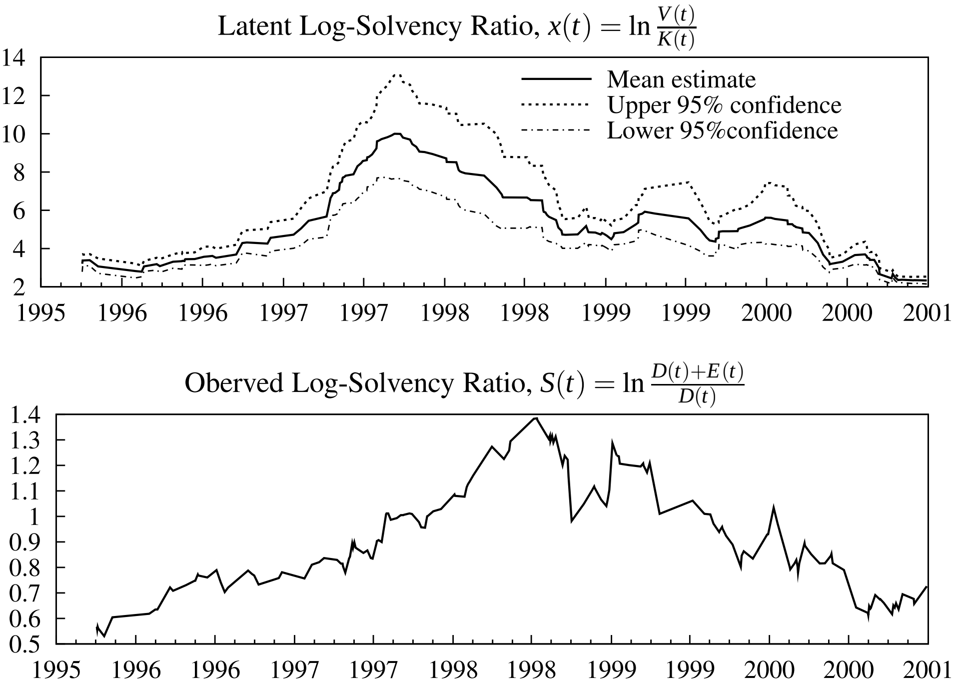
D.1.viii Dayton Hudson Corp



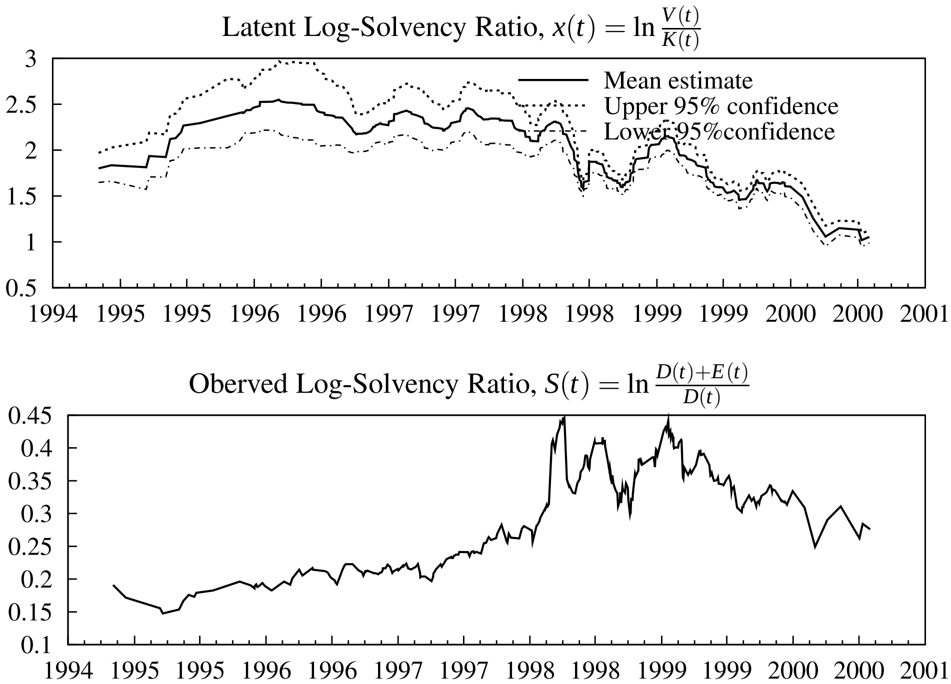
D.1.ix Commonwealth Edison Co



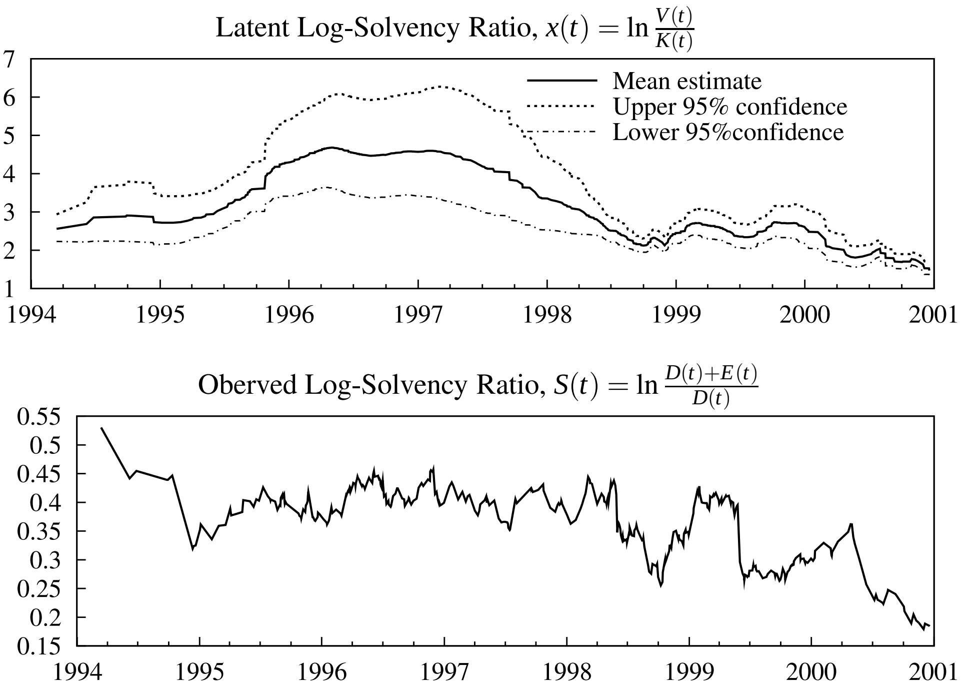
D.1.x Enron Corp



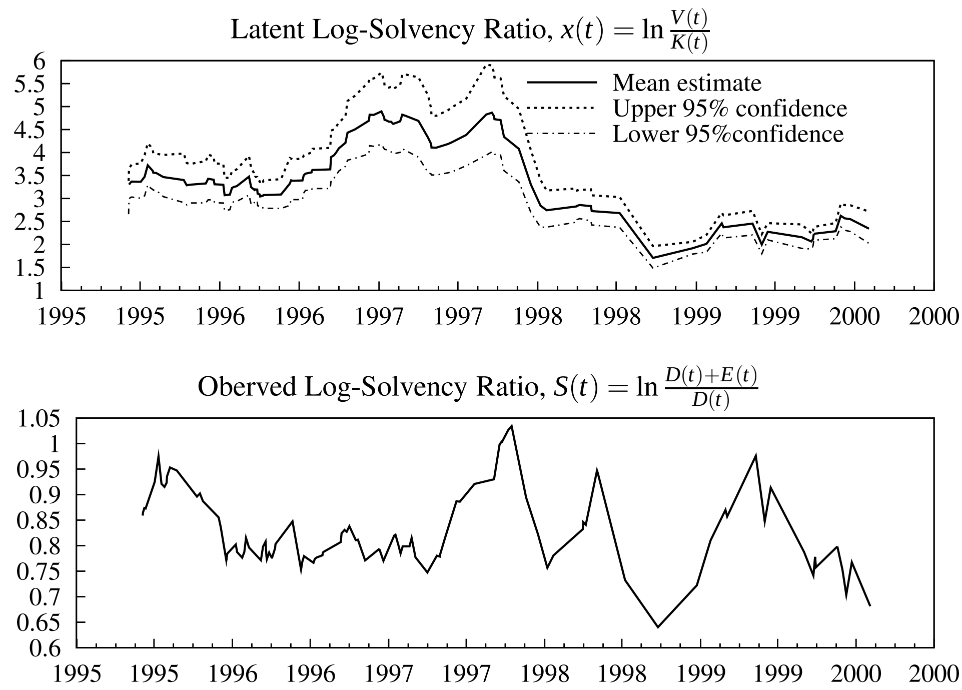
D.1.xi Federated Dept Stores



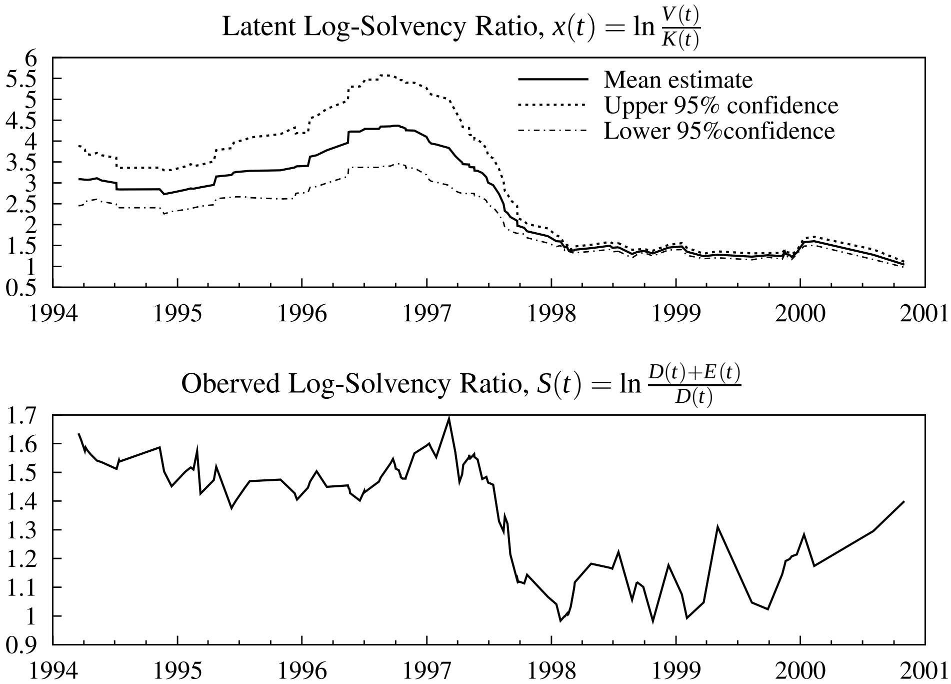
D.1.xii Ford Mtr Co



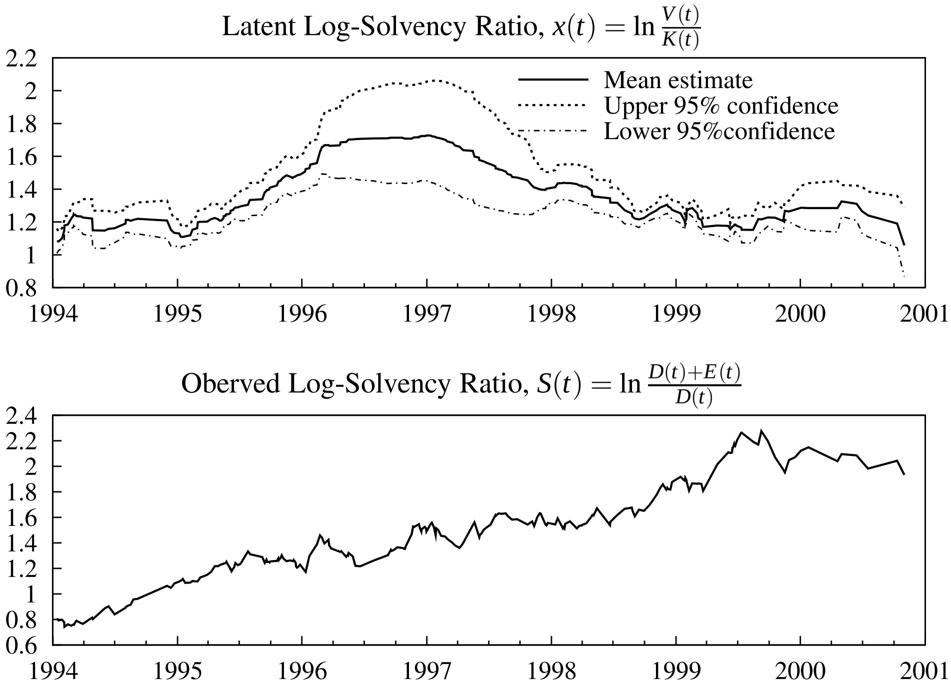
D.1.xiii General Mtrs



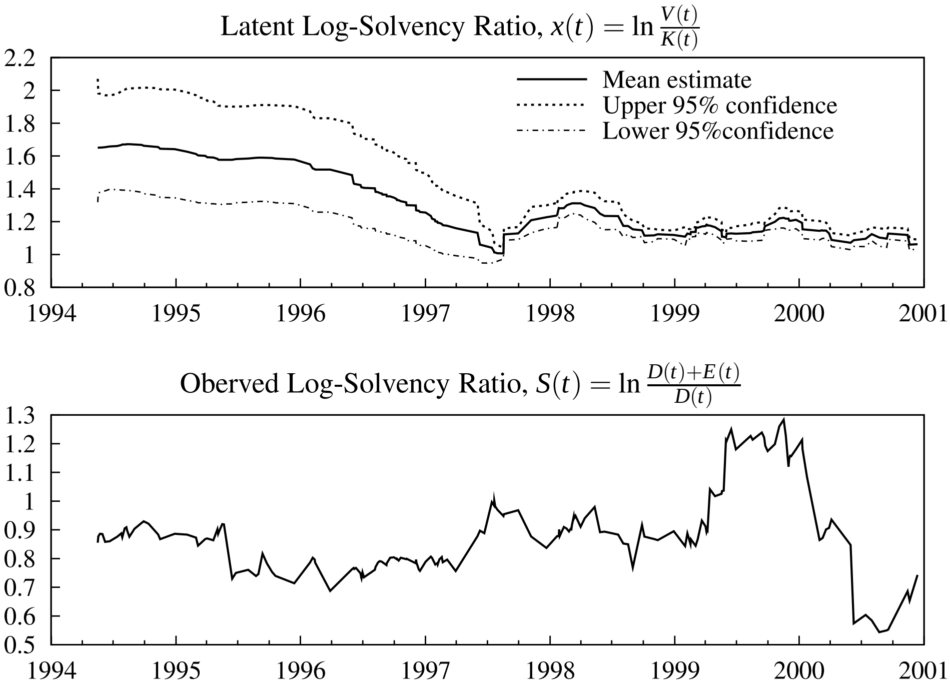
D.1.xiv Georgia Pacific Corp



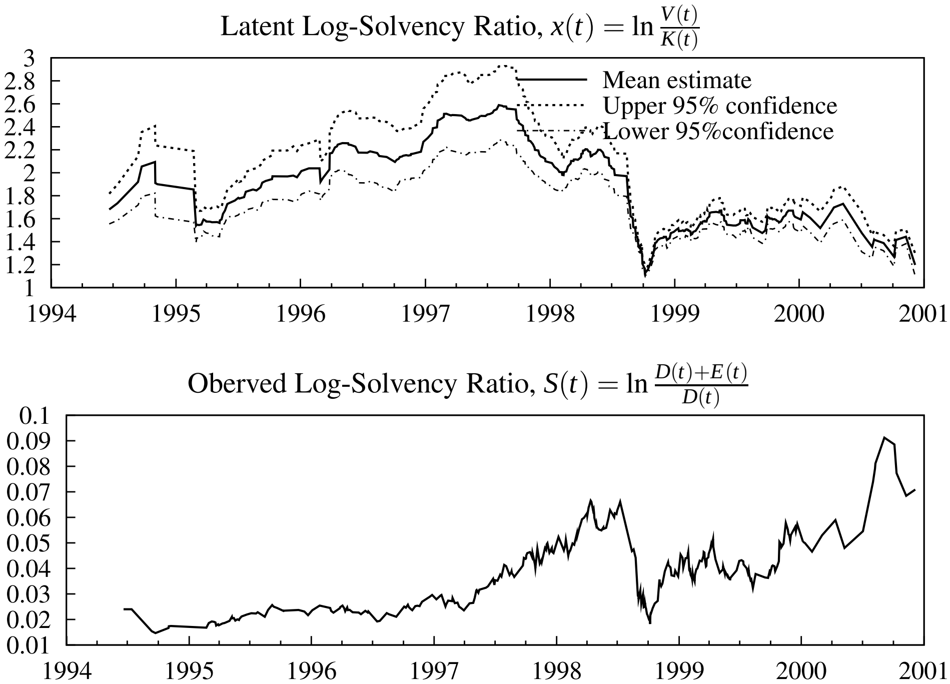
D.1.xv Hca Healthcare Corp



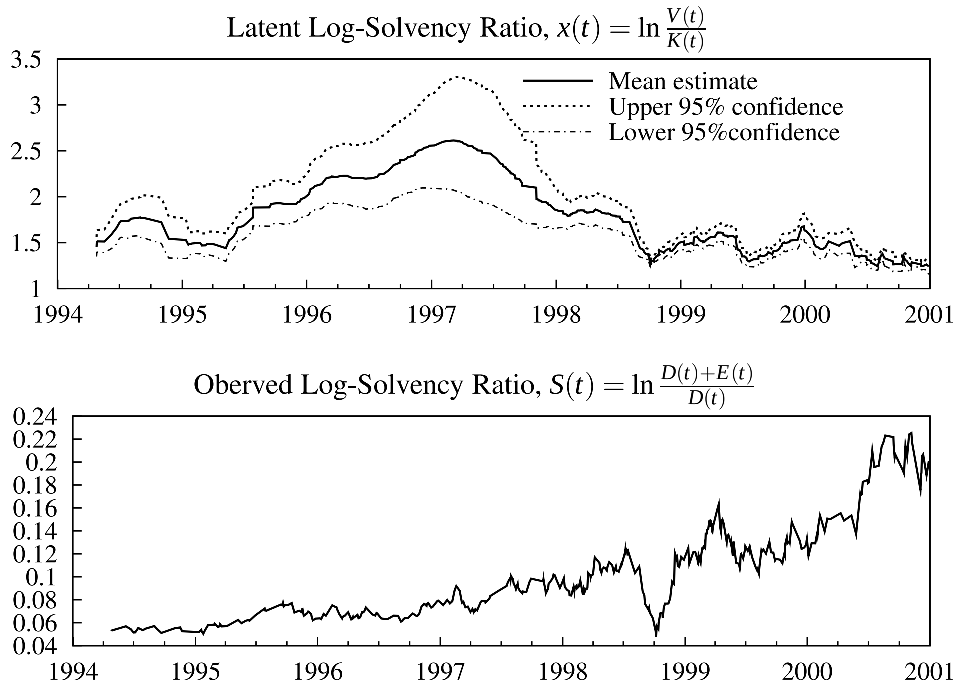
D.1.xvi IBM Corp



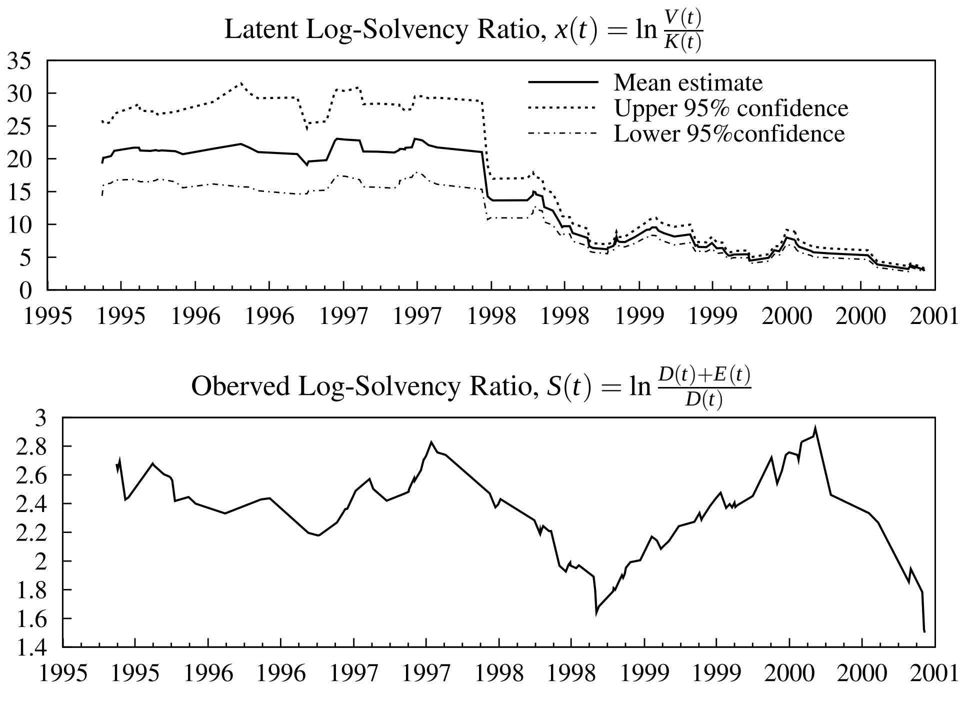
D.1.xvii International Paper Co



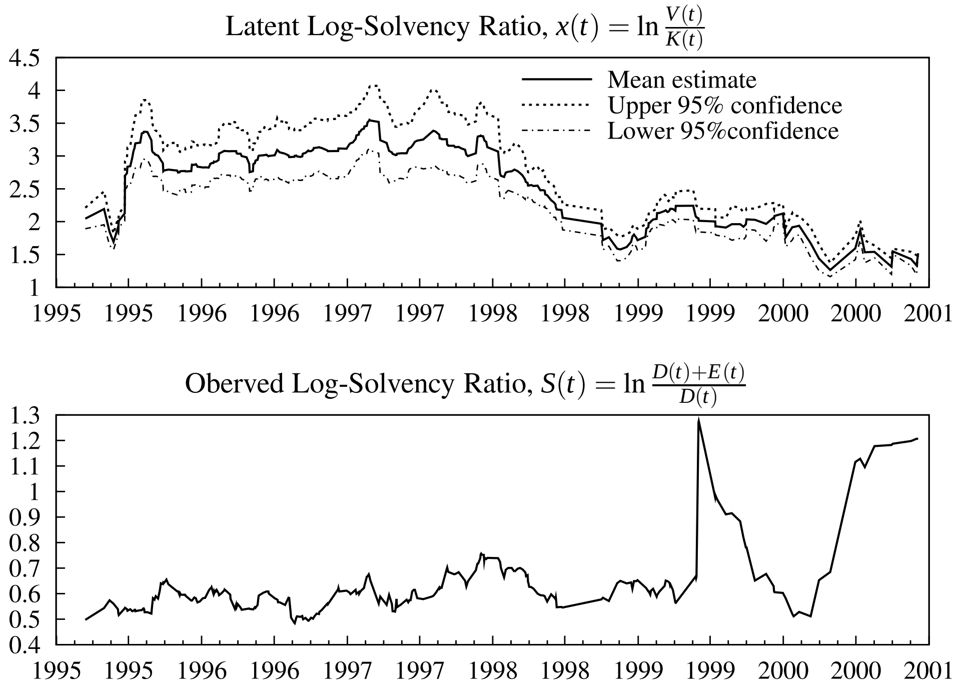
D.1.xviii Lehman Brothers Holdings Inc



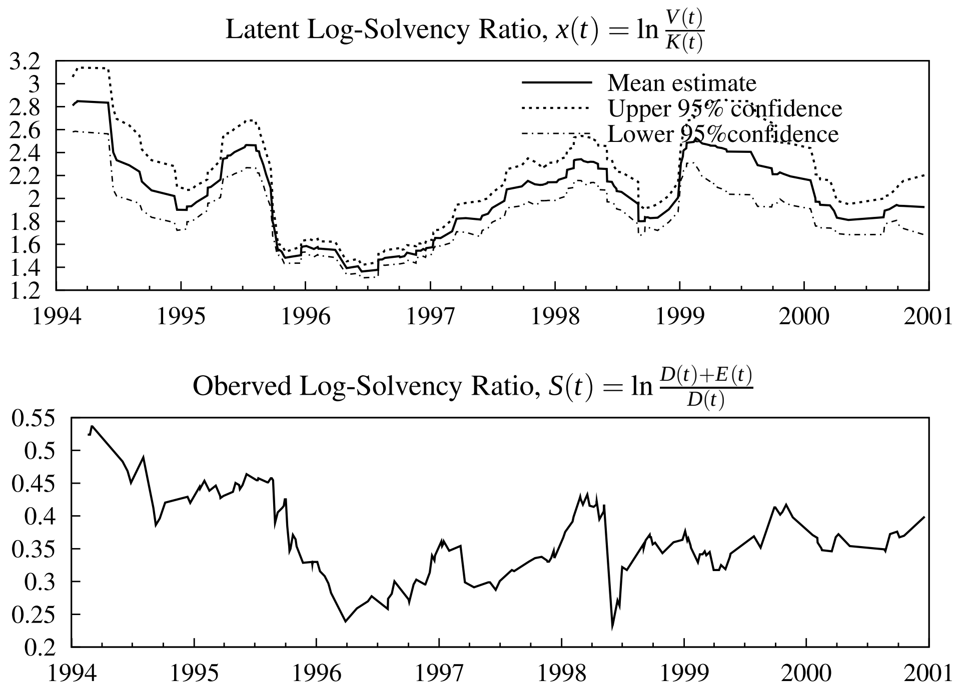
D.1.xix Merrill Lynch & Co



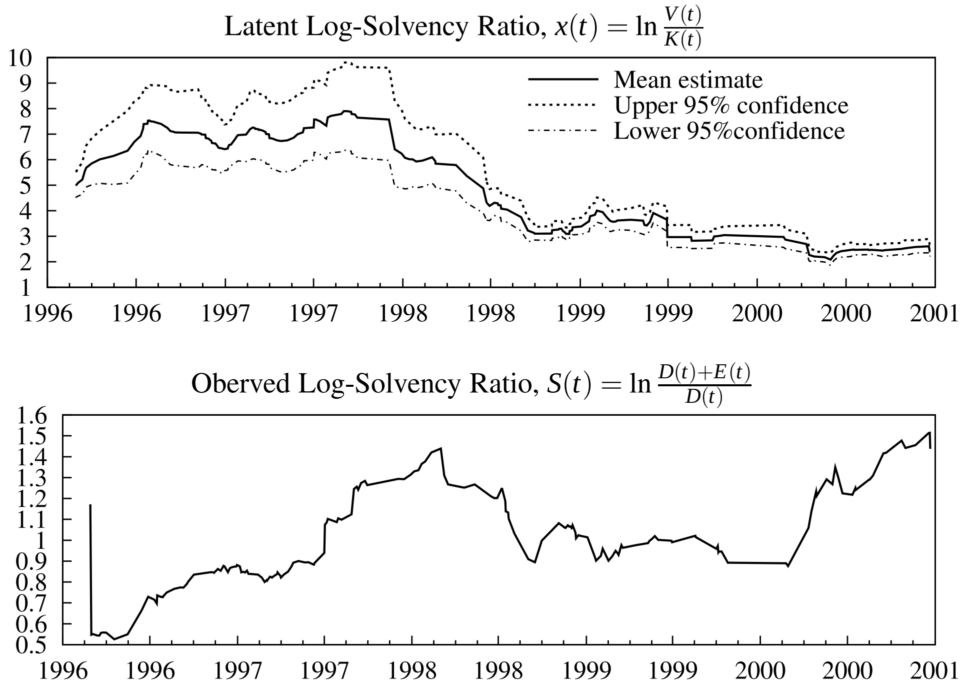
D.1.xx Motorola Inc



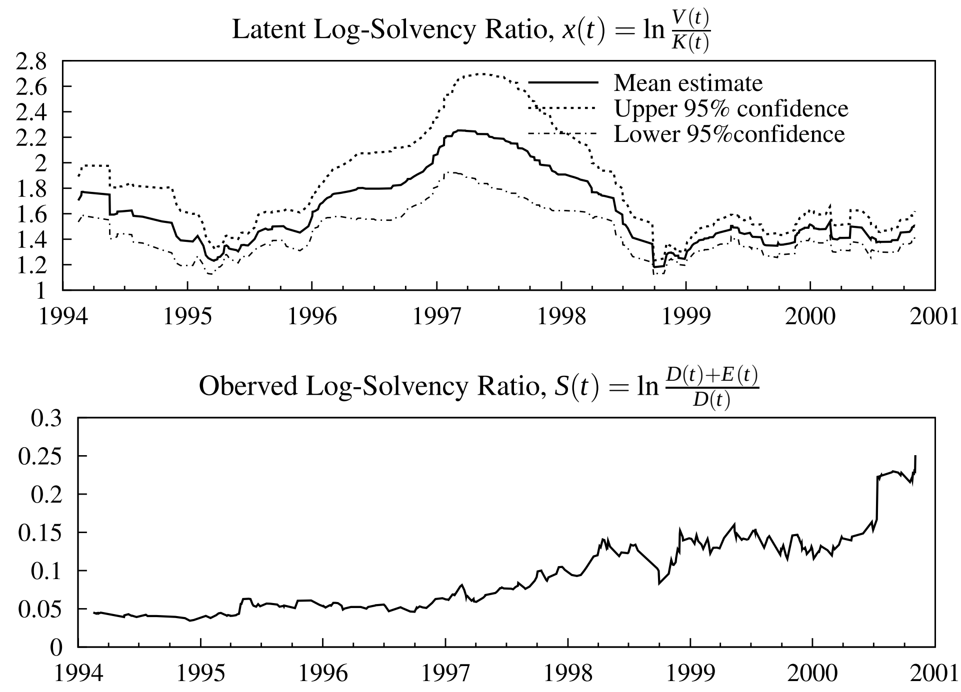
D.1.xxi Nabisco Group Hldgs Corp



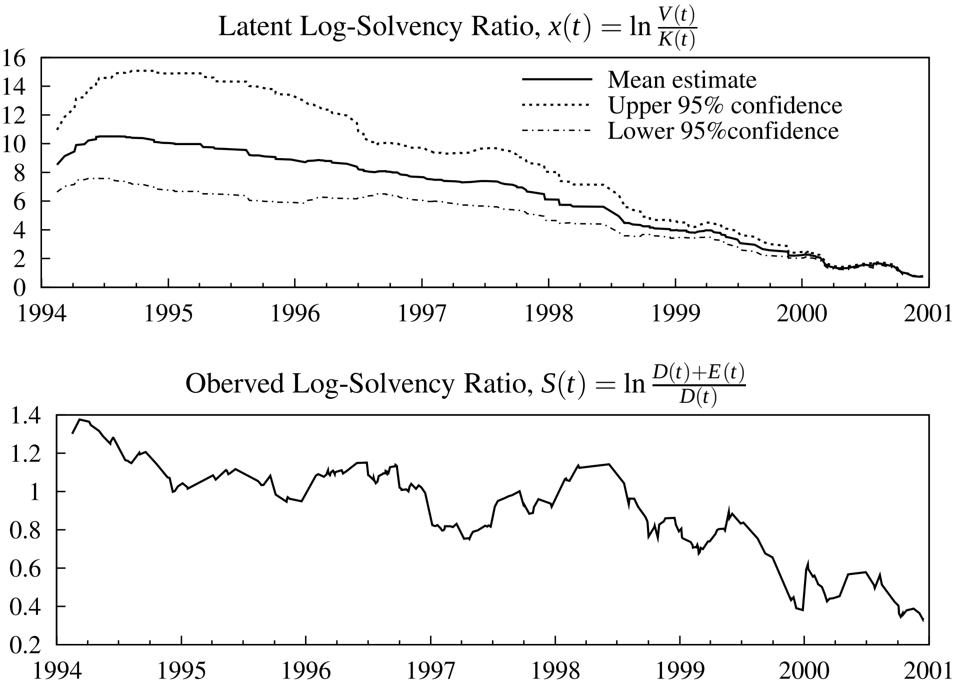
D.1.xxii Niagara Mohawk Pwr Corp



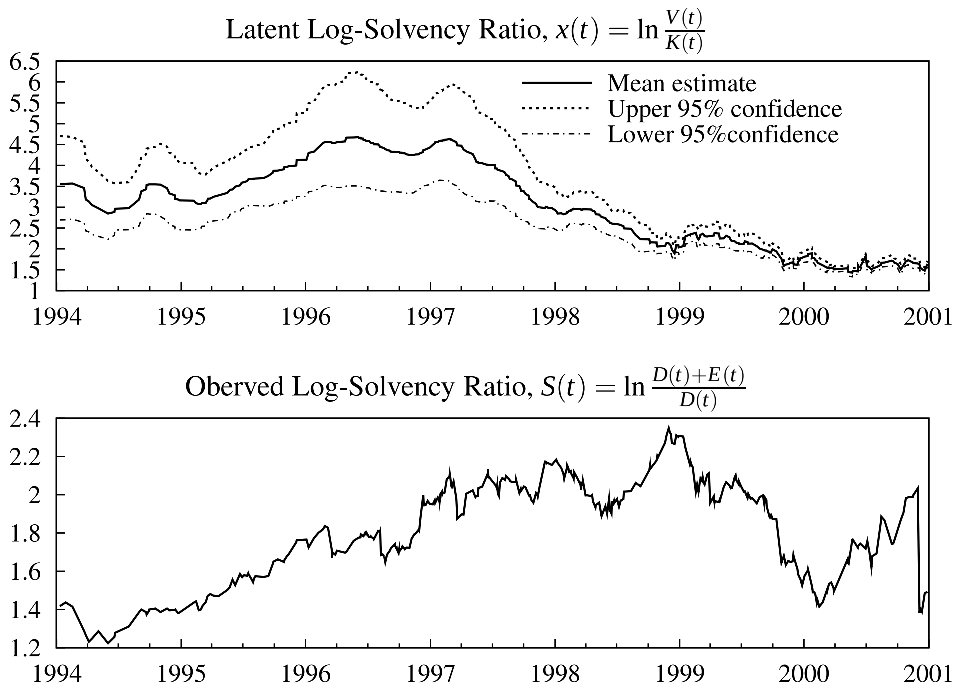
D.1.xxiii Northrop Grumman Corp



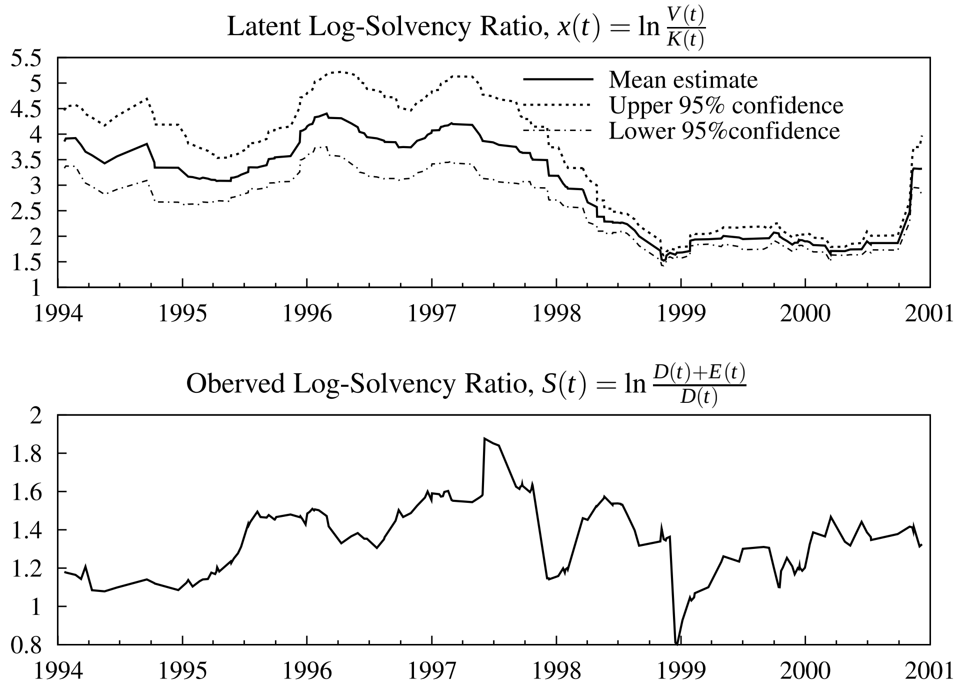
D.1.xxiv Paine Webber Group Inc



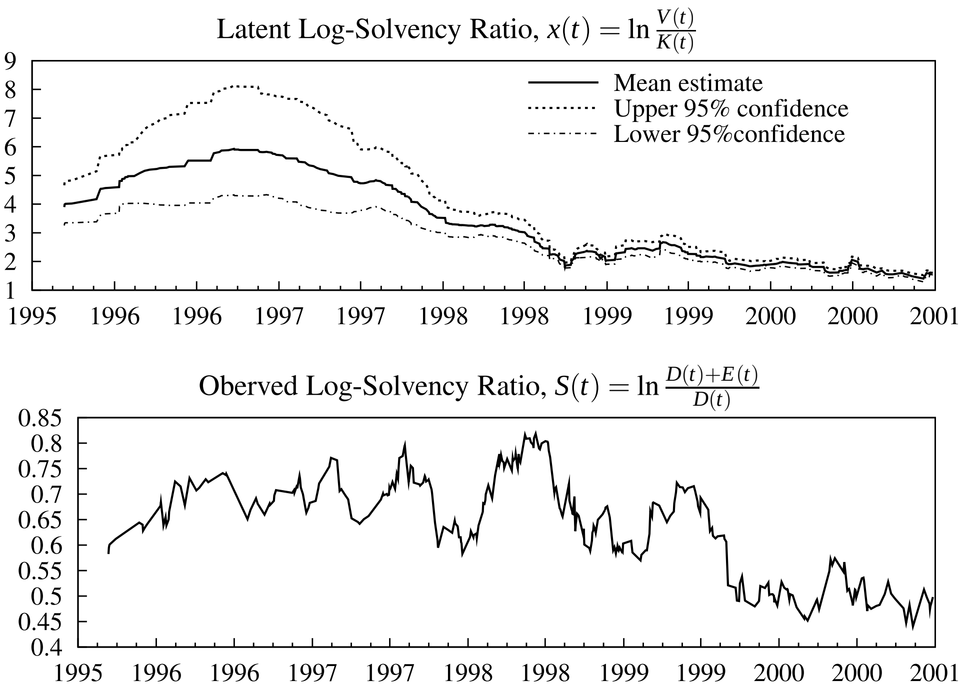
D.1.xxv Penney J C Co Inc



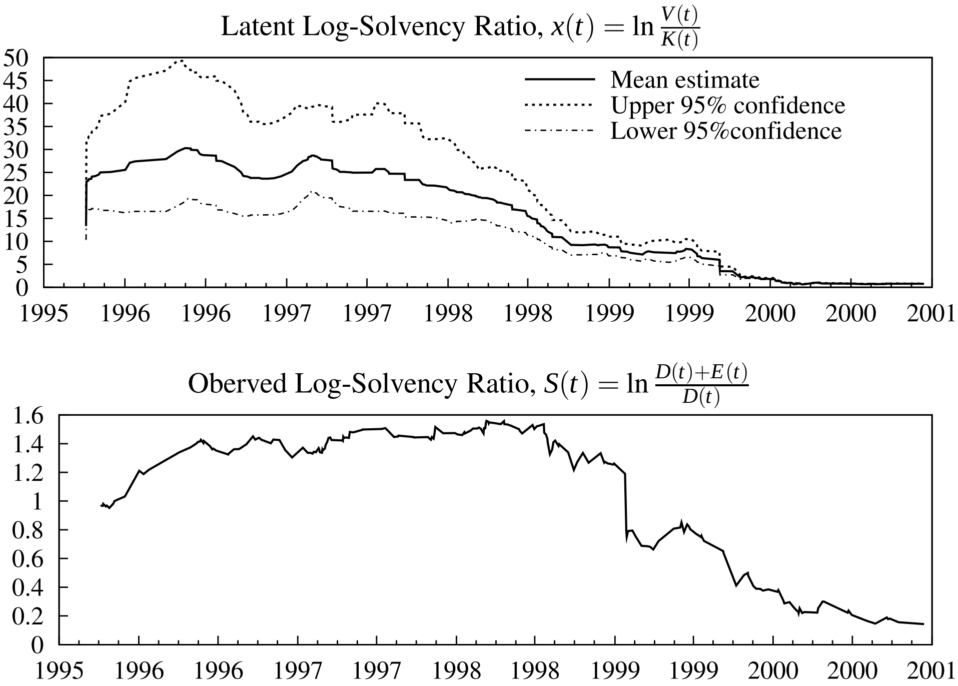
D.1.xxvi Philip Morris Companies Inc



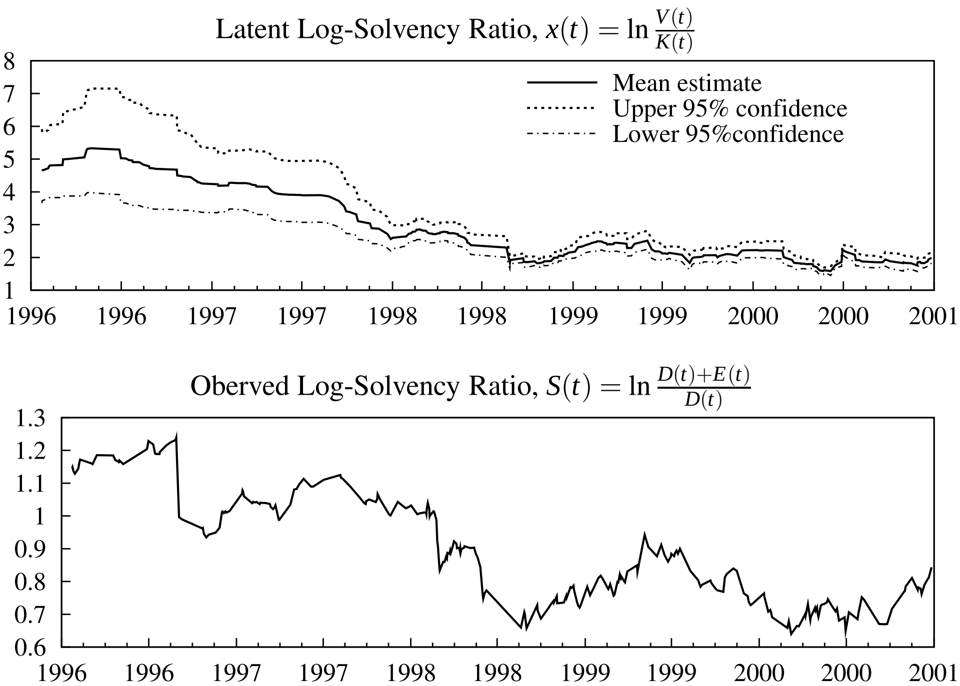
D.1.xxvii Seagram Co Ltd



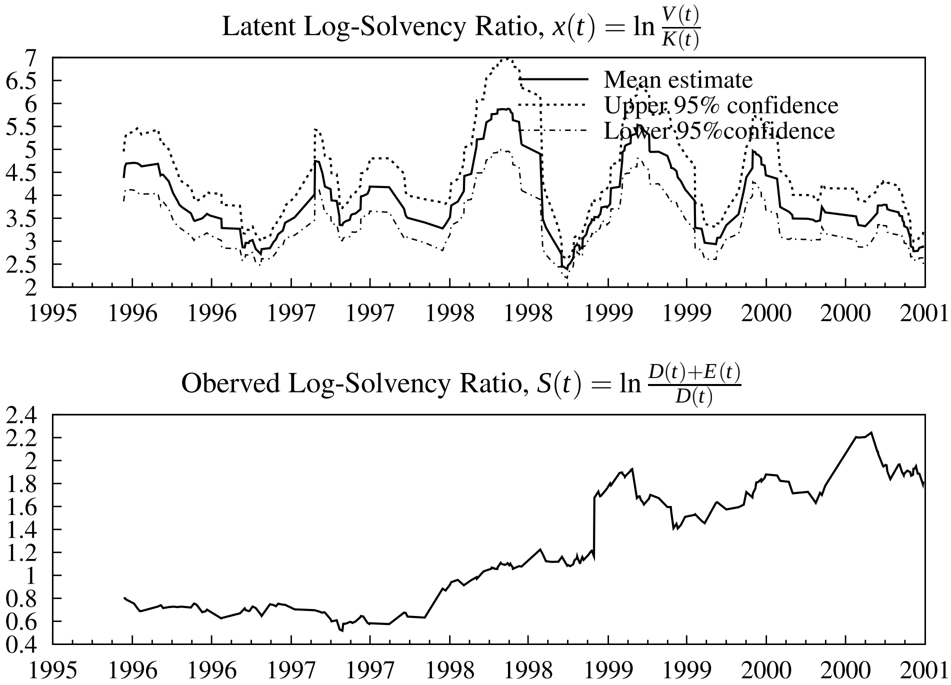
D.1.xxviii Sears Roebuck Accep Corp



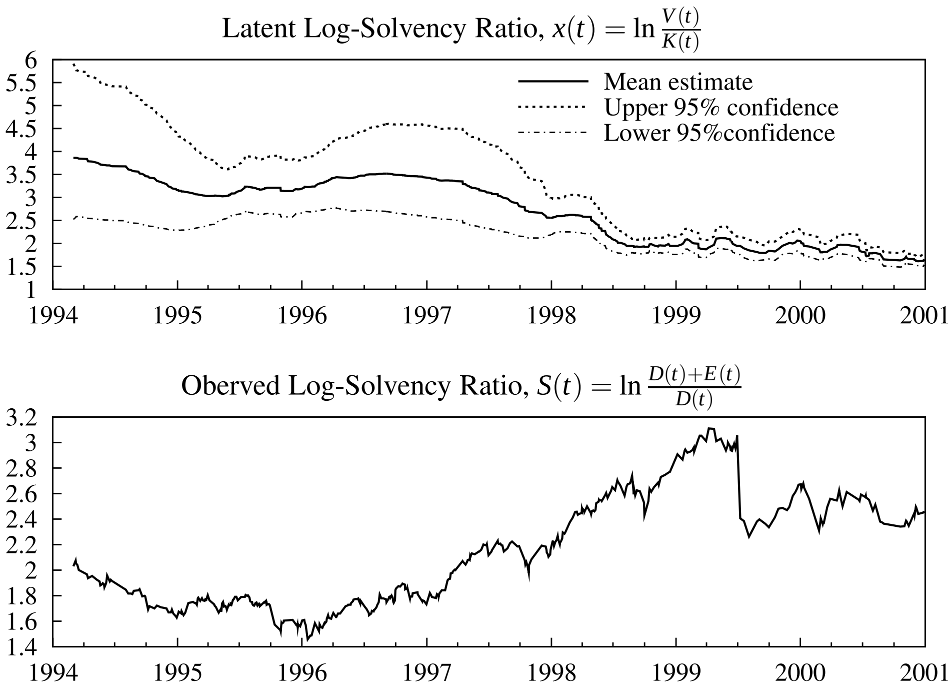
D.1.xxix Service Corp Intl



D.1.xxx Union Pacific Corp



D.1.xxxi Viacom Inc



D.1.xxxii Wal-Mart Stores Inc

Appendix E

Example Ox Code

The following programs are written in OX, a C-style matrix programming language designed for econometric modelling. Further details of Ox are described in Doornick (2002).

In the section, the extended Merton (1974) (EM) model with time-varying liquidity is shown as an example. Other models vary in their state-space specification and bond valuations, but in all other respects, share common code. The main routine is contained in Appendix E.1, and the subroutine function for calculating the credit spread, given parameters passed to it, is contained in Appendix E.2.

E.1 Extended Kalman Filter for EM model

```
/*-------------------------------------------------------------------------
EXTENDED KALMAN FILTER - Merton

This procedure returns the quasi-maximum likelihood estimates of
the Extended Merton model

Extended Merton's specification. Coupon bond priced
as the sum of zeros (each valued with Merton zero-coupon model).
Spot rate is maturity matched smoothed estimate obtained from the Vasicek model.

Program returns parameter estimates, asymptotic standard errors
using Hessian method, smoothed estimates of historic latent log-solvency
ratio with upper and lower 95% confidence bounds. Includes constant risk premium
and time-varying liquidity premium.

State-space notation follows Harvey (1989):
Measurement equation: y=Z(t)+a(t)+d(t)+eps(t), Var(eps)=H(t)
Transition equation: a(t)=T(t)a(t-1)+c(t)+R.eta(t), Var(eta)=Q(t)

by Iain Maclachlan
October 2006
-------------------------------------------------------------------------*/
#include <oxstd.h>
#include <oxfloat.h>
#import <database>
#import <maximize>
#include <packages/gnudraw/gnudraw.h>
#include "SpreadFunctionMerton.ox"
```

```
/*-----------------------------------------------------------------------
Declare Global Variables
------------------------------------------------------------------------*/
static decl cm=1; //number of elements in state vector
static decl cg=1; //number of columns in R vector
static decl cN; //number of bonds issued by firm
static decl cT; //final time step indexed from zero
static decl mDt; //matrix of observed steps between trades in years
static decl my; //matrix of observed spreads
static decl mr; //array of smooth estimate short-rates
static decl mP_0; //initial log-solvency variance
static decl ma_0; //initial log-solvency value
static decl mypred; //predicted credit spread
static decl dFunc2; //storage for maximum likelihood
static decl ae; //standardised prediction errors
static decl d; //number of diffuse priors
static decl obs; //number of observations assuming no missing
static decl lag; //shifts row of data input
static decl cmethod; //optimisation method: 1=Simplex, 2=BFGS, 3=Simplex then BFGS
static decl smethod; //optimisation method used
static decl ct; //time step counter
static decl mvk; //filtered leverage ratio
static decl csumNt; //number of obs adjusted for missing values
static decl mindex; //missing observation index: 1 if missing, zero otherwise
static decl mse; //matrix of standardised prediction errors. Missing obs = .NaN
static decl mve; //matrix of prediction errors. Missing obs = .NaN
static decl mtenors; //array of remaining maturities
static decl g_a; //array of state vectors
static decl g_P; //array of state covariances
static decl g_P_prior; //array of prior state estimates (fwd filter)
static decl g_T;  //array of transition matrices
static decl a_T;  //array of prior state estimates (backward smoother recursion)
static decl P_T;  //array of prior state covariance (backward smoother recursion)
static decl mcoupons; //array of coupon rates (p.a.)
static decl gr_sigma; //Vasicek short-rate volatility
static decl gr_theta; //Vasicek long-run short-rate
static decl gr_alpha; //Vasicek short-rate mean reversion rate
static decl gr_lambda; //Vasicek price of interest rate risk
static decl gx_0; //initial observed log-solvency
static decl mRefcorp; //array of 10 CMT Refcorp-Treasury spreads
/*-----------------------------------------------------------------------
The Extended Kalman Filter
Precedes the main function.
Returns the Log-Likelihood function value
------------------------------------------------------------------------*/
EKF(const vLP, const adFunc, const avScore,  const amHessian)
{
g_a=new array[obs]; //array of state vectors
g_P=new array[obs]; //array of state covariances
g_T=new array[obs]; //array of transition matrices
g_P_prior=new array[obs]; //array of prior state estimates
ae=new array[obs]; //array of standardised prediction errors
decl ma=zeros(cm); //state vector = implied log-solvency ratio
decl mP=zeros(cm,cm); //state covariance
decl ma_prior=(cm); //prior estimate of state vector
decl mP_prior=(cm,cm); //prior estimate of state covariance
decl mT=(cm,cm); //transition matrix
decl mc=(cm); //constant liquidity spread premium
decl mQ=(cg,cg); //transition error covariance identity
decl mR=(cm,cg); //transition error covariance scalar
decl mZ; //measurement equation matrix
decl mH; //measurement equation error
decl mF; //prediction error covariance
decl mInvF; //inverse of mF
decl mInvH; //inverse of mH
decl mK; //Kalman gain
decl cL; //log-likelihood
```

```
decl ci; //counter
decl mv; //raw prediction error vector
decl me; //standardised prediction error vector
decl cNt; //no of time varying obs excluding missing obs, cNt<=cN
decl asign; //address of the log of the determinant of mF
decl cit; //measurement row counter excl. missing trades
decl v_delta; //asset payout rate
decl v_sigma; //asset volatility
decl m_sigma; //measurement volatility
decl v_constant; //constant spread premiums
decl refcorp; //Refcorp spread slope coefficient
/*-----------------------------------------------------------------------
Hyperparameters not in the state vector
are passed to global storage for calling
by spread function. Transformed to lie on real line.
-----------------------------------------------------------------------*/
v_sigma=exp(vLP[0]/2);
m_sigma=exp(vLP[1]/2);
v_delta=exp(vLP[2])/(1+exp(vLP[2]));
refcorp=exp(vLP[3]);
v_constant=exp(vLP[4:]);
/*-----------------------------------------------------------------------
 Kalman state matrices are dimensioned before calculation
-----------------------------------------------------------------------*/
mvk=zeros(1,obs); //filtered leverage ratio
csumNt=0; //number of obs adjusted for missing values
mindex=zeros(cN,obs); //vectors of missing observation rows where element=1
mse=zeros(cN,obs); //matrix of standardised prediction errors including missing data = .NaN
mve=zeros(cN,obs);  //matrix of prediction errors including missing data = .NaN
mypred=zeros(cN,obs); //predicted spreads
cL=0; //initialise log-likelihood store
/*-----------------------------------------------------------------------
Recursive filter loop in Kalman Filter: forward step through time
-----------------------------------------------------------------------*/
for (ct=0;ct<=cT;++ct)
{
/*
Index missing obs at each time step
*/ mindex[][ct]=isdotnan(my'[][ct]); //1 if missing, zero otherwise,
cNt=cN-sumc(mindex[][ct]);  //Missing values are =0
csumNt=csumNt+cNt;  //cumulative sum of valid observations
/*
Transition equation
*/  mc=(mr[ct]-v_delta-v_sigma^2/2)*mDt[ct];
mT=(1);
mR=v_sigma*sqrt(mDt[ct]);
mQ=1;
/*
Initialise measurement equation
*/  mH=unit(cNt,cNt)*m_sigma^2;
mZ=zeros(cNt,cm);
mF=zeros(cNt,cNt);
mv=zeros(cNt);
me=zeros(cNt);
/*
Initial prediction step
*/ if (ct==0)
{
ma_prior=mT*gx_0+mc;
mP_prior=mT*mP_0*mT'+mR*mQ*mR';
}
/*-----------------------------------------------------------------------
Iteration loop start

Construct Z matrix with gradients of the state vector
for Extended Kalman filter. Dimensioned to remove missing
data rows. ci=original observation vector row number.
```

```
cit=compressed vector row number excluding missing obs.
-----------------------------------------------------------------------*/
cit=0;
for (ci=0; ci<cN; ++ci) //inner loop across obs at time t
{
/*
Check if data not missing (mindex=0)
Construct Z on reduced dimension
Call predicted spread from sub-routine.
*/
if (mindex[ci][ct]==0)  //mindex=0 for valid obs
{
decl spd=new SpreadFunctionMerton(ma_prior, mtenors[ct][ci],
v_sigma, v_delta, mr[ct], mcoupons[ci], gr_alpha, gr_theta, gr_sigma, gr_lambda);

decl spd2=new SpreadFunctionMerton(ma_prior+0.001, mtenors[ct][ci],
v_sigma, v_delta, mr[ct], mcoupons[ci], gr_alpha, gr_theta, gr_sigma, gr_lambda);

mypred[ci][ct]=spd.GetSpread()+refcorp*mRefcorp[ct]+v_constant[ci];
mZ[cit]=(spd2.GetSpread()-spd.GetSpread())/0.001;
cit=cit+1;
delete spd;
delete spd2;
}//end construction of Z loop
else
{
mypred[ci][ct]=.NaN;
}
} //end inner obs loop at time t
mv=deleter(my'[][ct]-mypred[][ct]); //prediction error
mF=mZ*mP_prior*mZ'+mH;  //variance of prediction error
mInvF=invertgen(mF);    //inverse pred error variance
me=mv./(sqrt(diagonal(mF)')); //standardised prediction error
/*
update state vector
*/ mK=mP_prior*mZ'*mInvF; //Kalman gain
ma=ma_prior+mK*mv;  //update state prediction
mP=(unit(cm)-mK*mZ)*mP_prior; //update covariance vector
/*
prior estimates of state values
*/ ma_prior=mT*ma+mc;   //next period prior estimate of state mean
mP_prior=mT*mP*mT'+mR*mQ*mR'; //next period prior estimate of state covariance
mvk[ct]=exp(ma[0]); //store path of filtered leverage ratio estimate
/*
Sum likelihood excluding diffuse priors
*/ if (ct>=d)
{
cL=cL-1/2*(logdet(mF,&asign)+ mv'* mInvF * mv);
}
/*
Store for backward recursive smoothing and residual analysis
note that dimensions and therefore smoother are unaffected by missing observations
*/ g_a[ct]=ma;
g_P[ct]=mP;
g_T[ct]=mT;
g_P_prior[ct]=mP_prior;
/*
Restore dimension of prediction errors for diagnosis of errors with missing data.
mse=matrix of standardised prediction errors diagnostics on
the matrix require removal of missing .NaN entries
me=matrix of standardised residuals dimensioned to compress missing data
*/ cit=0;
for (ci=0; ci<cN; ++ci)
{
if(mindex[ci][ct]==0)
{
mse[ci][ct]=me[cit];
```

```
mve[ci][ct]=mv[cit];
cit=cit+1;
}
else
{
mse[ci][ct]=.NaN;
mve[ci][ct]=.NaN;
}
}  //end re-dimensioning loop
} //filter loop ends
/*------------------------------------------------------------------------
Return log-likelihood
Note: log-likelihood uses valid number of obs
in numerator and number of time steps (summation) in denominator
------------------------------------------------------------------------*/
adFunc[0]=double((cL-1/2*csumNt*log(2*M_PI))/(obs-d));

return 1; /* Kalman Filter completed */
}
/*---------------------Main Routine--------------------------------------
Performs:
Initial parameter starting values
Data input
Calls Kalman filter from within Ox maximisation routine
------------------------------------------------------------------------*/
main()
{
/*
Local variables declared
*/
decl dbase; //temp input data storage for reading Excel
decl v_sigma_0; //initial asset volatility
decl cmeasure_0; //initial measurement error
decl v_delta_0; //initial asset payout rate
decl m_sigma_0; //initial measurement volatility
decl v_constant_0; //initial constant liquidity premium
decl refcorp_0;    //initial Refcorp slope coefficient
decl ir; //result of Ox maximisation routine. 1=success
decl dFunc; //maximised value of likelihood
decl mhess; //Hessian matrix of the logliklihood with respect to hyperparameters
decl vP; //transformed hyperparameters
decl stateout; //optimised parameter estimates
decl maxit; //maximum number of iterations in the maximisation search
decl ci; //counter per bond
decl cfirm; //counter for the issuer
decl vlabels; //text identifier of bond
decl sname; //text identifier of issuer
decl tvalue;    //estimated t-value of parameter estimates
decl standerrors;  //estimated standard error of parameter estimates
decl mlev; //observed log-solvency S(t)
/*------------------------------------------------------------------------
Identify firm and bonds IDs to load input data files
------------------------------------------------------------------------*/
for (cfirm=0;cfirm<32;++cfirm)
{
if (cfirm==0){
  sname="aetna";
vlabels={"42328","42329","42330"};}
else if (cfirm==1){
sname="associates2";
vlabels={"1555","1563","1568","1569","1575","26127","31674","32803","45820","91982"};}

loop repeats for each firm loading name and bond identifer

else if (cfirm==31){
    sname="walmart2";
vlabels={"23421","23422","23425","23427","23429","23430","23432","23433","25644"};}
```

```
/*----------------------------------------------------------------------
Load data files
 ----------------------------------------------------------------------*/
decl sfilein="C:\\thesis\\DataIn\\"~sname~".xls";
decl sfileparams="C:\\thesis\/media/idedisk3\\firm_assumptions.xls";
/*
Kalman Filter Settings
*/ cmethod=3;
maxit=200;
MaxControlEps(1e-4,5e-3); //Maximisation convergence criteria:(1e-4,5e-3)
/*
State vector settings
*/ d=1; //number of diffuse elements
lag=2; //due to time differencing
/*
Load bond data. Missing observation data represented by .NaN
*/ cN=columns(vlabels);
dbase=loadmat(sfilein);
mDt=dbase[lag:][1];
my=dbase[lag:][2:cN+1];
mtenors=dbase[lag:][cN+2:2*cN+1];
mcoupons=dbase[1][2*cN+2:3*cN+1]./100;
mr=dbase[1:][3*cN+2];
mlev=dbase[lag:][3*cN+5];
mRefcorp=dbase[lag:][4*cN+12];
delete dbase;
/*
load initial firm assumptions
*/ dbase=loadmat(sfileparams);
v_sigma_0=dbase[][1];
v_constant_0=dbase[][3];
m_sigma_0=dbase[][4];
delete dbase;
/*
Find time dimensions of data
*/ obs=rows(my);
cT=obs-1;
/*
Set Vasicek parameters. Exogenously estimated.
*/ gr_alpha=0.0232; //interest-rate reversion
gr_theta=0.1605; //interest-rate long-run
gr_lambda=0.0043; //price of interest rate risk
gr_sigma=0.0147; //interest-rate volatility
/*
Set initial diffuse state vector variance
*/ mP_0=v_sigma_0[cfirm]^2*mDt[0]*1000; //arbitrary increase in variance
v_delta_0=0.0483;  //initial as per mean reported by EHH
gx_0=mlev[0]; //initial observed log-solvency
refcorp_0=1; //arbitrary initial
/*
Transform hyperparameters to meet economic constraints
*/ vP=
log(v_sigma_0[cfirm]^2)|  //log asset variance
log(m_sigma_0[cfirm]^2)|  //log measure variance
log(v_delta_0/(1-v_delta_0))|  //asset payout
log(refcorp_0)| //Refcorp slope
log(ones(cN,1)*v_constant_0[cfirm]); //constant liquidity premiums
/*----------------------------------------------------------------------
Maximise Loglikehood by calling EKF routine
Transformed hyperparameters passed through vP
Returned value for vP is at optimised estimates
Choice of method: 1=simplex, 2=BFGS, 3=simplex then BFGS
----------------------------------------------------------------------*/
if (cmethod==1)  //simplex
{
MaxControl(maxit, 1);
smethod="Simplex";
```

```
ir=MaxSimplex(EKF, &vP, &dFunc2, 0);
}
if (cmethod==2)  //BFGS
{
MaxControl(maxit, 1);
smethod="BFGS";
ir=MaxBFGS(EKF, &vP, &dFunc2, 0, TRUE);
}
if (cmethod>2) //Simplex then BFGS
{
smethod="Simplex then BFGS";
MaxControl(50, 1);        // No. of simplex initial runs
MaxSimplex(EKF, &vP, &dFunc2, 0); //simplex Ox routine
MaxControl(maxit, 1);    //maxit is limit on BFGS runs
ir=MaxBFGS(EKF, &vP, &dFunc2, 0, TRUE); //BFGS Ox routine
}
/*----------------------------------------------------------------------------
Fixed interval filter smoothing using whole data set
Refer Harvey (89) page 154, section 3.6.2
----------------------------------------------------------------------------*/
decl mP_star;  //interim variable in recursion to get P_T. refer Harvey
decl vkvolsmooth; //stand. deviation of state estimate
decl vksmooth;  //smoothed state
decl mysmooth; //smoothed predictions
a_T=new array[obs]; //array of prior state estimates
P_T=new array[obs]; //array of prior state covariance
vkvolsmooth=zeros(1,obs);
vksmooth=zeros(3,obs);
mysmooth=zeros(cN,obs);
/*
initial values at T
*/ a_T[cT]=g_a[cT];
P_T[cT]=g_P[cT];
/*
construct inital values including confidence levels
*/ vkvolsmooth[cT]=sqrt(P_T[cT][0][0]); //leverage estimate deviation
vksmooth[0][cT]=exp(a_T[cT][0][0]); //mean smoothed estimate of x(t)
vksmooth[1][cT]=exp(a_T[cT][0][0]+1.97*vkvolsmooth[cT]); //upper confidence of x(t)
vksmooth[2][cT]=exp(a_T[cT][0][0]-1.97*vkvolsmooth[cT]); //lower confidence of x(t)
/*
Backward recursion
*/ for (ct=cT-1; ct>=0; --ct)
 {
mP_star=g_P[ct]*g_T[ct+1]'*invertgen(g_P_prior[ct]);
a_T[ct]=g_a[ct]+mP_star*(a_T[ct+1]-g_T[ct+1]*g_a[ct]);
P_T[ct]=g_P[ct]+mP_star*(P_T[ct+1]-g_P_prior[ct])*mP_star';
vkvolsmooth[ct]=sqrt(P_T[ct][0][0]);
vksmooth[0][ct]=exp(a_T[ct][0][0]);  //smoothed mean x(t)
vksmooth[1][ct]=exp(a_T[ct][0][0]+1.97*vkvolsmooth[ct]); //upper confidence x(t)
vksmooth[2][ct]=exp(a_T[ct][0][0]-1.97*vkvolsmooth[ct]); //lower confidence x(t)
 }
/*----------------------------------------------------------------------------
Estimate standard errors by computing Hessian
----------------------------------------------------------------------------*/
Num2Derivative(EKF, vP, &mhess);
println("Hessian=",mhess);
tvalue=fabs(vP./sqrt(diagonal(-invertgen(mhess,0)/(obs-d))'));
/*----------------------------------------------------------------------------
Store untransformed parameter estimates
----------------------------------------------------------------------------*/
stateout=
exp(vP[0]/2)| //asset volatility
exp(vP[1]/2)| //measurement error volatility
exp(vP[2])/(1+exp(vP[2]))| //asset payout
exp(vP[3])| //Refcorp slope
exp(vP[4:]); //constant liquidity premium
/*----------------------------------------------------------------------------
```

```
Compute standard errors of estimates
-------------------------------------------------------------------------*/
standerrors=stateout./tvalue;
/*-------------------------------------------------------------------------
Print Results
-------------------------------------------------------------------------*/
print("\nInputs:");
print("\nEKF estimation of Extended Merton, run on ", date());
print("\nFirm: ",sname);
print("\nFile in: ",sfilein);
print("\nStarting values: \n", vP);
print("\nNumber of time periods = ", obs);
print("\nNumber of bonds = ",cN);
print("\nNumber diffuse = ",d);
print("\nInitial state variance = ",mP_0);
print("\nr_alpha = ",gr_alpha);
print("\nr_theta = ",gr_theta);
print("\nr_lambda = ",gr_lambda);
print("\nr_sigma = ",gr_sigma);
print("\nLog-likelihood: ", dFunc2);
print("\nParameter Estimates: ",
"%r",
{"v_sigma",
"m_sigma",
"delta",
"refcorp",
"constant"},
"%c",{"Estimate","Stand. Error","t-Value"},
"%cf",{"%12.4g","    %12.4g","   %12.4g"},
stateout~standerrors~stateout./standerrors);
/*
compute prediction errors per bond expressed in basis points
*/ decl mrmsetable=zeros(3,cN);
for (ci=0;ci<cN;++ci)
{
mrmsetable[0][ci]=(meanc((deleter(vecr(mve[ci][d:]))).^2).^0.5)*10000;
mrmsetable[1][ci]=meanc(fabs(deletec(mve[ci][d:])'))*10000;
mrmsetable[2][ci]=meanc(deletec(mve[ci][d:])')*10000;
}
/*
print average prediction errors
*/ print("\nAverage smoothed V/K =",meanr(vksmooth[0][:]));
print("\nMin smoothed V/K =",min(vksmooth[0][:]));
print("\nMax smoothed V/K =",max(vksmooth[0][:]));
print("\nAverage RMSE =",meanr(mrmsetable[0][:]));
print("\nAverage MAE =",meanr(mrmsetable[1][:]));
/*
print log-solvency smoothed estimates
*/ println("Smoothed V/K");
println("Mean, Lower @95%., Upper @95%");
println(vksmooth');
}//end firm loop
}//end main function
```

E.2 Yield Spread Function for EM model

```
/*-------------------------------------------------------------------------
Values coupon bond under extended Merton model
Arguments in:
x_0 = firm log-solvency=ln(V/K)
T = bond maturity
v_sigma = firm asset volatility
v_delta = firm payout rate
r_0 = risk-free rate
coupon = coupon rate
```

```
r_alpha = mean reversion rate of r
r_theta = long-term level of r
r_sigma = interest rate volatility
r_lambda = market price of interest rate risk
Returns:
Yield to maturity spread
assuming semi-annual compounding
--------------------------------------------------------------------------*/
#include <oxstd.h>
#include "cashflow.ox"
#include "vasicekfun.ox"
#include "ValueFunctionMerton.ox"
#include "ytm.ox"
class SpreadFunctionMerton
{
SpreadFunctionMerton(const x_0, const T, const v_sigma,
const v_delta, const r_0, const coupon, const r_alpha,
const r_theta, const r_sigma, const r_lambda);

decl time,payments,n,num_pmts,i,t,c,r,risky_pv,
riskless_pv,risky_ytm,riskless_ytm,spread,qsi;

GetSpread();
};
SpreadFunctionMerton::SpreadFunctionMerton(const x_0, const T, const v_sigma,
const v_delta, const r_0, const coupon, const r_alpha,
const r_theta, const r_sigma, const r_lambda)
{
payments=cashflow(T,coupon);
num_pmts=rows(payments);
risky_pv=0;
riskless_pv=0;

for (i=0;i<num_pmts;++i)
{
t=payments[i][0];
c=payments[i][1];
if(t==0)
{
r=r_0;
}
else
{
r=-log(vasicekfun(t, r_0, r_alpha, r_theta, r_sigma, r_lambda))/t;
}
riskless_pv=riskless_pv+exp(-r*t)*c;
risky_pv=risky_pv+
ValueFunctionMerton(x_0, v_delta, t, r, v_sigma, c);
}
risky_ytm=ytm(payments, risky_pv, 1);
riskless_ytm=ytm(payments,riskless_pv, 1);
spread=risky_ytm-riskless_ytm;
}
SpreadFunctionMerton::GetSpread()
{
  return (spread);
}


/*-------------------------------------------------------------------------
Returns an array of bond coupons and time to receive
Arguments in:
years = years to maturity
coupon = semi-annual coupon rate
Argumensts out: T x 2 matrix [years to next cashflow][cash]
--------------------------------------------------------------------------*/
#include <oxstd.h>
```

```
#include <oxfloat.h>
cashflow(const maturity, const coupon)
{
decl n; //number of time steps
decl c; //counter
decl mcash; //cash matrix
n=trunc((maturity*2))+1;  //years by 2 then round up to find number of rows
mcash=zeros(n,2);   //initialise
mcash[n-1][0]=maturity;   //last node = maturity in years
mcash[n-1][1]=(coupon/2)+1; //last payment including face value
for (c=n-2;c>=0;--c)    //recursively loop from last payment deducting .5 years
{
mcash[c][0]=mcash[c+1][0]-.5;
mcash[c][1]=(coupon/2);
}
if (mcash[0][0]==0)
{
 mcash[0][1]=0;  //exclude cum coupon
}
return mcash;
}
/*------------------------------------------------------------------------
    Returns continuous ytm using bisection search
    Arguments in:
        payments: array of cash flows
        value: bond value
        upper: maximum boundary of ytm
    Argument out:
        annualised ytm with semi-annual compounding
------------------------------------------------------------------------*/
#include <oxstd.h>
/* Value to minimise to zero */
funval(const payments, const y, const value)
{
decl t,disc,cash,result;
t=payments[][0];    //time of cash in years
cash=payments[][1]; //cash
disc=exp(-y*t);     //vector of discount rates
result=disc'*cash-value; //difference in calculated value from target
return result;
}
/* Bisection search to return ytm */
ytm(const payments, const value, const upper)
{
decl a, b, c, k, fc;
a=0;
b=upper;
decl result;
/* Iterations */
for (k = 0; k < 100; ++k)
   {
    c=(a+b)/2;
    fc=funval(payments, c, value);
        if(fc==0)
        {
            a=c;
            b=c;
        }
        else if(fc<0)
        {
            b=c;
        }
        else
        {
            a=c;
        }
        if(fabs(b-a)<1e-20){break}
```

```
    }
//convert to annualised semi-annual compound
result=2*(exp(c/2)-1);
return result;
}
```

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