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Abstract

Estimation results obtained with production smoothing models are often imprecise and sensitive to normalisation rules. It is argued here that production costs in these models can be better specified in terms of production factor costs. The resulting (factor demand) model with inventories leads to more efficient estimation results and normalisation rules become even redundant. Furthermore, the (smoothing) role of inventories with respect to production factors is discussed. © 1997 Elsevier Science S.A.

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1. Introduction

In production smoothing models an entrepreneur chooses the inventory stock that minimizes costs (or maximizes profits) by which the 'optimal' production results. Sales (or shipments) are often assumed to be exogenous. Items concerning this class of models, such as the measurement of production, sales and the inventory stock, non-stationarities and estimation techniques to be used, have often been discussed in the literature.

As is argued here and to the best of my knowledge never received attention before, the specification of production costs in production smoothing models is difficult to interpret from an economic point of view. An improvement is found by specifying production factor costs. In contrast to production costs, production factor costs are specified in more detail and observed. The (factor demand) model therefore has the advantages that more efficient estimation results are obtained, and, a normalisation rule is no longer necessary. It is argued further that, instead of the often discussed production smoothing role of inventories, attention should rather be paid to the smoothing role of inventories in relation to production factors.

The outline is as follows. In Section 2 the production smoothing model is specified. In Section 3 its

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production costs specification is discussed in more detail and compared with cost specifications used in factor demand models. In Section 4 the production smoothing and factor demand model are heuristically compared. Section 5 discusses whether entrepreneurs keep inventories to smooth production or production factors. Section 6 concludes.

2. The production smoothing model

Production smoothing models, often used in American studies, are specified as

$$E\left\{\sum_{h=0}^{\infty} \beta^h[\omega_1 Y_{t+h} + 0.5\omega_2 Y_{t+h}^2 + 0.5\gamma(\Delta Y_{t+h})^2 + 0.5\alpha_1(H_{t+h} - \alpha_2 S_{t+h})^2]|\Omega_t\right\}, \tag{1}$$

where Δ is the first difference operator, Y_t represents the production, S_t the sales and H_t the inventory stock at time t, E is the rational expectations operator and Ω_t represents the information set. β represents the constant (real) discount factor. The parameters ω_1, ω_2, γ, α_1, α_2 are to be estimated. The entrepreneur's production in Eq. (1) equals sales in addition to the change in inventory stock, i.e. the identity

$$Y_t = S_t + \Delta H_t \tag{2}$$

holds. This identity certifies that the stock of inventories increases if production exceeds sales and decreases if production falls short of sales.

A representative entrepreneur is assumed to aim at minimizing production and inventory costs over an infinite horizon, see Eq. (1), subject to identity Eq. (2). Production costs in Eq. (1) consist of costs associated with the level of production, $\omega_1 Y_t + 0.5\omega_2 Y_t^2$, and costs associated with the changes in the production level, $\gamma(\Delta Y_t)^2$. Inventory costs are specified as costs incurred when inventory stock diverges from the level of sales, $\alpha_1(H_t - \alpha_2 S_t)^2$. This latter specification is introduced by Holt et al. (1960) and has omnipresently been used. On the role of α_1 and α_2, see for example West (1993).

Production and inventory stock are assumed to be decision variables for the producer whereas sales are assumed not to be within influential reach. Hence, the minimization of Eq. (1) subject to Eq. (2) results in a first order condition,

$$E\{\omega_1(1-\beta) + \omega_2(Y_t - \beta Y_{t+1}) + \gamma(\Delta Y_t - 2\beta\,\Delta Y_{t+1} + \beta^2\Delta Y_{t+2}) + \alpha_1(H_t - \alpha_2 S_t)|\Omega_t\} \tag{3}$$

that is obtained by substitution of Eq. (2) in Eq. (1) and differentiating with respect to H_t. This equation can be estimated directly by the General Method of Moments, for instance, or, by a Full Information estimation method when a 'closed form solution' can be derived. In this case a suitable stochastic process for S_t is to be specified.

In the class of production smoothing models according Eq. (1)– Eq. (2), inventories are said to have a smoothing role if they are kept by entrepreneurs to 'smooth' production. This smoothing can occur in light of sales fluctuations, see for example Blinder (1982), and/or cost shocks, see for example Eichenbaum (1989). Assuming sales to be exogenous, inventories will most commonly serve as production smoother if adjusting production is costly (γ is high) in comparison with the costs of keeping inventories (that depend on α_1, α_2). In this case production must be less volatile than sales. This issue will be addressed in Section 5.

Many studies take the model Eq. (1)– Eq. (2) as a vantage model to elaborate on the smoothing role of inventories. For example West (1986) assumes $\omega_1 = 0$ and derives a variance bounds test for Eq. (1). By this test the smoothing role of inventories is rejected. Ramey (1991) adds an additional term, $\omega_3 Y_t^3$, to the production cost to allow for non-convexities in the production cost structure. In her model an entrepreneur bunches production if it is advantageous. Eichenbaum (1989) replaces ω_1 by a stochastic (autoregressive) process, to be interpreted as a cost shock. Among others, Durlauf and Maccini (1993) elaborate on this. They assume these cost shocks to be the price shocks of different production factors. Other studies, as for example West and Wilcox (1993), concentrate on stochastic process specifications for sales, together with alternative methods to estimate Eq. (3).

In Error Correction Models non-stationarities of the time series used in Eq. (1) are investigated by, for example, Rossana (1993). Problems associated with the measurement of inventories, like seasonally adjustments, order backlogs or determination of the volume of inventories, have also often been discussed.

Here the attention will not be on data, on econometric or other technical details, but rather on a better understanding of the production costs part specified in Eq. (1) from an economic point of view.

3. The specification of production costs

The first three terms in Eq. (1) refer to costs directly associated with the amount of the commodity that is produced. The third term, $\gamma(\Delta Y_t)^2$, is supposed to capture the costs that represent, for example, hiring and firing costs (see West (1986), page 379) or adjusting the labour force and reassigning tasks (see Ramey (1991), page 312). As will be argued in the following, production costs can be specified in more detail.

A common assumption in the literature is that the level of production is determined by a number of production factors. Supposing that these factors are physical capital stock (K_t), labour (N_t) and materials and energy (here comprised by M_t), it holds that

$$Y_t = f(K_t, N_t, M_t), \tag{4}$$

where f determines the transformation process of factor inputs into commodities.

Production level costs can be associated directly with each production factor. Similarly, costs associated with adjustments in the level of production can be associated directly with factor specific adjustment costs. A specification for these factor costs is found in factor demand studies, see Pindyck and Rotemberg (1983),

$$C_t I_t + W_t N_t + P_{mt} M_t + 0.5\gamma_1(\Delta K_t)^2 + 0.5\gamma_2(\Delta N_t)^2, \tag{5}$$

where I_t, C_t, W_t, P_{mt} represent the gross investment in physical capital stock, the (real) investment price, the (real) wage rate and the (real) price of materials respectively. If $\gamma_1 \neq 0$ and $\gamma_2 \neq 0$ it is assumed that both capital and labour incur quasi-fixed costs referred to as, 'installation' and 'scrappage' costs, and, 'hiring' and 'firing' costs. Adjustment costs for materials or additional costs, like lump sum costs or interrelated adjustment costs, could of course also be specified but do not change essentially the derivations that follow here.

A factor demand model is then specified by

$$E\left\{\sum_{h=0}^{\infty} \beta^h[C_{t+h}I_{t+h} + W_{t+h}N_{t+h} + P_{m,t+h}M_{t+h} + 0.5\gamma_1(\Delta K_{t+h})^2 + 0.5\gamma_2(\Delta N_{t+h})^2 + 0.5\alpha_1(H_{t+h} - \alpha_2 S_{t+h})^2]|\Omega_t\right\}. \quad (6$$

Necessary conditions for this model are obtained by minimizing Eq. (6) with respect to K_t, N_t, M_t, H subject to Eq. (2) and Eq. (4). S_t is still assumed to be exogenous. Substitution of Eq. (4) in Eq. (2 and rearrangement of terms makes the variable production factor M_t as a function of the othe variables. Substitution of M_t in Eq. (6) then gives an expression in terms of the endogenous variable K_t, N_t, H_t and exogenous variables C_t, W_t, P_{mt}, S_t.[1]

A system of three Euler equations associated with K_t, N_t, H_t and an equation resulting from th Shephard's lemma concerning M_t then result. The first order solution of Eq. (6) subject to Eq. (2) an Eq. (4) consists thus of four conditions that can be estimated jointly.

The first order condition for production smoothing model Eq. (1) is however only one equation, se Eq. (3). As both the production smoothing model and the factor demand model aim at specifying th same objective by entrepreneurs, their specifications should not differ much.

To make a comparison between production smoothing model Eq. (1) and factor demand model Ec (6), production function Eq. (4) will be approximated here. A second order approximation of Eq. (4 can be given as

$$Y_t \doteq f^* + \nabla f^{*\prime}[X_t - X^*] + 0.5[X_t - X^*]'\nabla^2 f^*[X_t - X^*], \quad (7$$

where f^* represents f evaluated in K^*, N^*, M^*, the * denotes the stationary solution $X_t \equiv [K_t\ N_t\ M_t]'$ $X^* \equiv [K^*\ N^*\ M^*]'$, ∇ represents the gradient and ∇^2 represents the hessian. Substitution of thi approximation in Eq. (1) and omitting terms that do not depend on time t, gives the factor deman model

$$E\left\{\sum_{h=0}^{\infty} \beta^h[\omega_1(\nabla f^{*\prime} - X^{*\prime}\nabla^2 f^*)X_{t+h} + \omega_1 X'_{t+h}\nabla^2 f^* X_{t+h} + 0.5\omega_2(f^* + \nabla f^{*\prime}[X_{t+h} - X^*] + 0.5[X_{t+h} - X^*]'\nabla^2 f^*[X_{t+h} - X^*])^2 + 0.5\gamma(\nabla f^{*\prime}\Delta X_{t+h} + 0.5\Delta[X_{t+h} - X^*]'\nabla^2 f^*[X_{t+h} - X^*])^2 + 0.5\alpha_1(H_{t+h} - \alpha_2 S_{t+h})^2]|\Omega_t\right\}. \quad (8$$

A comparison of the factor demand model Eq. (6) and the approximated production smoothing mode Eq. (8) shows three main differences.[2]

Firstly, Eq. (8) incorporates unknown parameters ω_1 and ω_2 in the production cost level par whereas Eq. (6) does not. For a given production function the associated cost function can be derivec or a flexible cost function can be specified. Although this cost function will never be identical to th production smoothing model, the production smoothing model Eq. (1) can be regarded as a roug

[1] It is here assumed that gross investments equal net investments in addition to depreciation, i.e. $I_t = \Delta K_t + \kappa K_{t-1}$ where represents depreciation.

[2] A comparison between Eq. (6) and Eq. (8) could also be made when both the production smoothing model and the factc demand model are formalized in profits instead of costs. This would lead to similar differences.

approximation. It can easily be seen that ω_1 and ω_2 depend on (relative) factor prices and thus should be time varying. The specification Eq. (8) seems thus inferior to specification Eq. (6) on this point.

Secondly, the production (adjustment) costs in Eq. (8) incorporate the marginal productivity $\nabla f(K^*, N^*, M^*)$ of each production factor and $\nabla^2 f(K^*, N^*, M^*)$. In contrast to this, Eq. (6) associates (adjustment) costs directly with the (change) in the amount of capital stock and/or labour.

Thirdly, an average adjustment cost structure (γ) of capital, labour and materials is specified in Eq. (8). In contrast to this, the curvature of adjustment costs of production factors (γ^1, γ^2) in Eq. (6) can be estimated for each quasi-fixed production factor separately. Specification Eq. (8) is thus less specific than specification Eq. (6).

4. Disadvantages of the production smoothing model

A major disadvantage of the specification of production costs like in Eq. (1) or Eq. (8) is the necessity of a normalisation rule. After all, as follows from the Euler Eq. (3) of model Eq. (1), the optimal solution is the solution where all parameters equal zero. For the solution of model Eq. (6) this does not hold since variable costs $C_tI_t + W_tN_t + P_{m,t}M_t$ are observed. The solution of the minimization problem will thus in general not be the solution where all parameters are zero. West and Wilcox (1993) pay attention to the high sensitivity of the normalisation rule chosen in production smoothing models, and West (1993) compares the results of Eichenbaum (1989); Ramey (1991) and West (1986). An advantage of the model Eq. (6) is that a normalisation rule becomes redundant.

Another disadvantage of Eq. (8) is that, except for some very specific cases, production costs are mis-specified if they are to be associated with the individual production factors. To simplify, assume that only labour is a production factor and constant returns to scale holds, i.e. $f(N_t) = aN_t$. It then follows that the estimate of γ_2 in Eq. (6) will only equal the estimate of γ in Eq. (8) if $\nabla f(N^*) = a = 1$. If both models are the same, $\gamma_2 = \gamma \nabla f(N^*) \Rightarrow \gamma = \gamma_2 / \nabla f(N^*)$ by which the adjustment cost parameter of production, γ, 'incorporates' the marginal productivity of labour. If more than one quasi-fixed production factor exists, for example capital and labour, and their adjustment costs differ ($\gamma_1 \neq \gamma_2$ in Eq. (6)), not only the marginal productivity but also the interrelation between the production factors gives rise to differences between the two specifications.

As a large part of the costs that are associated with the production process occurs at the input side of the production process, the specification Eq. (6) seems to be preferred to Eq. (8). Assuming Eq. (6) to be correct, the parameter γ in Eq. (8) that is a main indicator of the production smoothing role of inventories, is biased. Marginal productivity, its derivative and the interrelation of the production factors in this context play crucial roles.

5. Production smoothing or factor demand smoothing?

On macroeconomic levels, time series on production are mostly non-stationary. The series ΔY_t, though, is stationary and has a variance given by (see Eq. (2))

$$V\{\Delta Y_t\} = V\{\Delta S_t\} + V\{\Delta^2 H_t\} + 2\,COV\{\Delta S_t, \Delta^2 H_t\}, \tag{9}$$

where V and COV indicate the variance and covariance respectively. By substitution of the approximation Eq. (7) if follows that

$$(\nabla f^{*\prime} - X^{*\prime}\nabla^2 f^*)V\{\Delta X_t\}(\nabla f^* - \nabla^2 f^* X^*) + 0.25V\{\Delta Z_t\} + 0.5(\nabla f^{*\prime} - X^{*\prime}\nabla^2 f^*)\, COV\{\Delta X_t, \Delta Z_t\} \doteq V\{\Delta S_t\} + V\{\Delta^2 H_t\} + 2\, COV\{\Delta S_t, \Delta^2 H_t\}, \tag{10}$$

where $Z_t \equiv X_t' \nabla^2 f^* X_t$.

If the key role of inventories is to smooth production in light of sales, and taking as a measurement of volatility the variance, it should hold that $V\{\Delta Y_t\} < V\{\Delta S_t\}$. In literature this inequality was often found to be violated (see for example Blinder (1982)). Instead of production, we want to concentrate on production factors here. In order to verify necessary conditions concerning production factors, for production to be smoothed, three simple examples will be considered.

As a first example, assume that $Y_t = f(N_t)$ and constant returns to scale hold, i.e. $\nabla^2 f(N^*) = 0$ by which $Z_t = 0$. From Eq. (9)– Eq. (10) then follows that $V\{\Delta Y_t\} \doteq V\{\Delta N_t\}(\nabla f(N^*))^2$. $V\{\Delta Y_t\} < V\{\Delta S_t\}$ can thus be violated, even if $V\{\Delta N_t\}$ is low, since the marginal labour productivity can be sufficiently high. High labour adjustment costs -i.e. γ_2 high- by which labour demand will be smoothed, do not necessarily imply that production is smoothed.

As a second example, assume $Y_t = f(K_t, N_t, M_t)$, $Z_t = 0$ and no interrelations in the production function. In this case $V\{\Delta Y_t\} \doteq V\{\Delta K_t\}(\partial f/\partial K^*)^2 + V\{\Delta N_t\}(\partial f/\partial N^*)^2 + V\{\Delta M_t\}(\partial f/\partial M^*)^2$. Consequently, capital, labour and materials can all be smoothed whereas production is not. Here again, the marginal productivity of each production factor can compensate the volatility of the production factors.

As a third example, assume that interrelations in the production function exist (i.e. $\nabla^2 f^*$ is non-diagonal). In this case production can be more votatile than sales, even when each production factor is smoothed and its marginal productivity is low. This will occur when production factors are strong complements or strong substitutes.

To summarize, these examples show that production smoothing does not imply production factor smoothing and vice versa. If entrepreneurs keep inventories to smooth production factors, instead of production, the literature thus far on the smoothing role of inventories wrongly concentrated on the statistics $V\{\Delta Y_t\}$ and $V\{\Delta S_t\}$.

The derivations above will further be analyzed by the use of data of the period 1976.I–1992.IV for five French industrial sectors. In Table 1 some statistics are given.[3]

The first line shows that production is not smoothed for the intermediate goods and transport equipment since $V\{\Delta Y_t\} > V\{\Delta S_t\}$. For the intermediate goods sector the third, fourth and fifth lines show that the three production factors capital, labour and materials are rather smooth since their variance is low in comparison with production. For the professional equipment sector on the contrary, capital is clearly not smoothed, whereas production is smooth in comparison with sales. Therefore these results re-emphasize the reasonings, see above, that production smoothing does not imply factor demand smoothing or vice versa.

The last six lines in Table 1 give the contributions of each production factor in the calculation of the variance of ΔY_t. They follow from the expression on the left side of the equality sign in Eq. (10),

[3]For a detailed data description see Peeters (1997).

Table 1
Statistics

	Intermediate goods	Professional equipment	Consumer equipment	Transport equipment	Consumer goods
$\frac{V\{\Delta S_t\}}{V\{\Delta Y_t\}}$	0.61	1.71	2.91	0.98	1.00
$\frac{V\{\Delta^2 H_t\}}{V\{\Delta Y_t\}}$	0.24	1.22	3.87	1.10	0.79
$\frac{V\{\Delta^2 K_t\}}{V\{\Delta Y_t\}}$	0.36	**91.36**	**2.03**	**60.69**	0.12
$\frac{V\{\Delta N_t\}}{V\{\Delta Y_t\}}$	0.01	0.03	0.01	0.02	0.02
$\frac{V\{\Delta M_t\}}{V\{\Delta Y_t\}}$	**0.39**	0.40	0.17	0.38	**0.40**
$V\{\Delta K_t\}\left(\frac{\partial f}{\partial K^*}\right)^2 TO$	5	3	1	13	2
$V\{\Delta N_t\}\left(\frac{\partial f}{\partial N^*}\right)^2 TO$	34	**70**	15	0	0
$V\{\Delta M_t\}\left(\frac{\partial f}{\partial M^*}\right)^2 TO$	**44**	7	**74**	**84**	**98**
$2\,COV\{\Delta K_t, \Delta N_t\}\frac{\partial f}{\partial K^*}\frac{\partial f}{\partial N^*} TO$	7	2	3	−1	0
$2\,COV\{\Delta K_t, \Delta M_t\}\frac{\partial f}{\partial K^*}\frac{\partial f}{\partial M^*} TO$	2	0	1	5	0
$2\,COV\{\Delta N_t, \Delta M_t\}\frac{\partial f}{\partial N^*}\frac{\partial f}{\partial M^*} TO$	8	18	6	−1	−1

V, COV are the sample variance and the covariance. $\partial f/\partial X^*$ is the marginal productivity of X^*, obtained by estimating a Translog production function (in first differences) by OLS and evaluating the marginal productivity in the 'stationary' point (assumed to be the observation in 1992.IV). Figures printed bold indicate the highest variance of production factors, or, the highest contribution within the variance of ΔY_t. TO is the inverted sum of all 'contributions' within the variance of ΔY_t (i.e. the left part of Eq. (10) with $Z_t = 0$), multiplied by 100.

where the assumption $Z_t = 0$ is made here. For example, the results for professional equipment sector indicate that the contribution of labour in the total variance of ΔY_t is highest whereas labour seems to be smoothed more than capital and materials (see the lines three until five). For the other four sectors the contribution for materials is highest while this is not always the factor that is smoothed most in these sectors. These results thus reemphasize the important role that can be played by the contribution of a production factor's marginal productivity in the calculation of the production volatility. The contributions of production factors interrelations (see the last three lines) seem to be minor here.

6. Conclusions

The specification of production costs in production smoothing models can be improved by using information on production factors. Variable and fixed factor costs can be associated with each production factor involved. Cost shocks are in this way more precisely specified than, for example, in Eichenbaum (1989) or Ramey (1991).

Factor demand models including inventories, are thus preferred to the often used production smoothing models since, costs are more appropriately and more specifically specified from an economic point of view. As a second advantage, from an econometric point of view factor demand models give more efficient estimation results (see Peeters (1997)). As a third advantage, normalisation rules become redundant. Imprecise estimates often obtained with production smoothing models (see West and Wilcox (1993) for an overview) might have their origin in this (mis-)specification of production costs rather than or besides, often investigated data issues, non-stationarity properties or estimation techniques.

In production smoothing models the smoothing role of inventories has always been addressed to production. As was shown here, smoothing production factors is neither a sufficient nor a necessary condition for smoothing production. Besides the volatility of production factors, production volatility also depends on marginal factor productivities, their changes and substitution possibilities among production factors.

The question whether or not inventories 'smooth', could better be addressed to production factors rather than to the production level. As follows from French industrial sectorial data, labour smoothing possibly occurs. Adjusting the labour force can be costly and/or takes time, by which labour smoothing will occur. Hoarding labour or delaying new labour recruitments is in many production processes possible by the existence of inventory stocks, thus inventories smooth labour. Capital smoothing seems only partly probable. Capital stock expansions can take time (due to gestation lags) or can be costly (due to high adjustment costs) but capital can be under-utilized or even scrapped without impediments. Hence, inventories might only be addressed to smooth capital expansions. By similar reasonings, materials smoothing by inventories might or might not occur.

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References

Blinder, A.S., 1982. Inventories and sticky prices: More on the microfoundations of macroeconomics. Am. Econ. Rev. 72, 334–348.

Durlauf, S.N., Maccini, L.J., 1993. Measuring noise in inventory models. National Bureau of Economic Research, Working paper No. 4487, Cambridge, Massachusetts.

Eichenbaum, M., 1989. Some empirical evidence on the production smoothing level and production cost smoothing models of inventory investment. Am. Econ. Rev. 79, 853–864.

Holt, C.C., Modigliani, F., Muth, J.F. Simon, H.A., 1960. Planning Production, Inventories and Work Force. Prentice Hall, Inc., Englewood Cliffs, N.J.

Peeters, H.M.M., 1997. A comparison of a production smoothing model and a dynamic factor demand model with inventories: Applications to French industrial sectors. Annales d'Economie et de Statistique 46, 141–160.

Pindyck, R.S., Rotemberg, J.J., 1983. Dynamic factor demands and the effects of energy price shocks. Am. Econ. Rev. 73, 1066–1079.

Ramey, V., 1991. Nonconvex costs and the behaviour of inventories. J. Political Econ. 99, 306–334.

Rossana, R.J., 1993. The long-run implications of the production smoothing model of inventories: An empirical test. J. Appl. Econometrics 8, 295–306.

West, K.D., 1986. A variance bounds test of the linear quadratic inventory model. J. Political Econ. 94, 374–401.

West, K.D., 1993. Inventory models. National Bureau of Economic Research, Inc. Technical Working Paper No. 143, Cambridge, Massachusetts.

West, K.D., Wilcox, D.W., 1993. Some evidence on finite sample behaviour of an instrumental variables estimator of the linear quadratic inventory model. National Bureau of Economic Research technical paper no. 139, Cambridge, Massachusetts.