When Opposites Attract: Is the Assortative Matching Always Positive?

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WHEN OPPOSITES ATTRACT:
IS THE ASSORTATIVE MATCHING ALWAYS POSITIVE?

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ABSTRACT

This paper shows that the positive assortative matching of Ghatak (1999) and Van Tassel (1999) is not a general result and always depends on the distribution of safe and risky types. Some new implications are: (*i*) borrowers may be better off by forming mixed groups. (*ii*) a mixed pooling equilibrium is possible when homogeneous pooling equilibria do not exist, and even when the reservation income of borrowers is equal to zero.

Keywords: joint liability lending, assortative matching, screening

JEL Classification numbers: D81, G20, O12

1 INTRODUCTION

This paper shows that the positive assortative matching as derived in Ghatak (1999) and Van Tassel (1999) is not a general result and always depends on the distribution of safe and risky types.

In their seminal papers, Ghatak (1999) and Van Tassel (1999) show that joint liability lending always leads to a positive assortative matching in the formation of groups. Namely, in a population of two types of borrowers, safe and risky, safe borrowers will always choose safe partners and risky borrowers will consequently pair with risky partners. Risky types cannot induce safe types, through a side payment, to form mixed groups. The reason is that safe types value safe partners more than risky types as they have a lower probability of failure, and so of repaying for the other.

In the present paper, to simplify the exposition, I follow and refer to Ghatak (2000) that studies the effect of the positive assortative matching on both the underinvestment setup of Stiglitz-Weiss (1981) and the overinvestment of de Meza-Webb (1987).[1] Ghatak (2000) compares the expected utilities of borrowers in case they form either a homogeneous or a mixed lending group. In this comparison, he uses the *same* generic contract (individual and joint liability components offered by

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[1] The theoretical results could be easily extended to all other similar contexts that consider the formation of groups through a self-selection mechanism.

a benevolent or perfectly competitive bank), to derive the payoff of borrowers under homogeneous and mixed groups.

In the paper, I argue that in Ghatak (2000) the payoff of mixed pairs should actually be based on a different contract. Specifically, the bank must offer a menu of two different generic contracts, one designed for homogeneous groups and one for mixed groups. In each of these contracts, the bank must break even, and so let borrowers choose the partners which maximize their expected utilities. This means that borrowers will compare the payoff they receive under homogeneous groups (based on the separating or pooling contract derived in Ghatak, 2000) with the payoff they receive under mixed groups (based on a contract opportunely designed for such groups).

Some new implications are that:

(*i*) borrowers may be better off by forming mixed groups. This occurs, both in the underinvestment and the overinvestment settings of Ghatak (2000), when borrowers cannot be separated and when the proportion of safe types is not high enough. The positive assortative matching, instead, still holds when borrowers receive a separating contract or when the number of safe types is sufficiently high under a pooling contract. The reason is that, when borrowers receive a separating contract or a pooling contract with a high proportion of safe types, safe firms reach their highest possible payoff, so it is difficult for risky firms to persuade them to choose a mixed group after a side payment. If, instead, borrowers receive a pooling contract and the proportion of safe types is not high, safe firms receive a payoff close to their reservation income. In this case, they could be induced to form mixed groups.

(*ii*) a mixed pooling equilibrium is possible when homogeneous pooling equilibria do not exist, and even when the reservation income of borrowers is very low or equal to zero. We know that, in the underinvestment case of Ghatak (2000), safe types cannot receive a pooling individual liability contract since their expected net payoff is not enough to cover their outside option. However, their revenue in case of success is large enough to satisfy the joint liability constraint, so they can obtain a loan in a group-lending scheme. If, on the contrary, we assumed in Ghatak (2000) a reservation income low or equal to zero, we could have at equilibrium both underinvestment and no possibilities for group lending. The present paper shows that, in this case, mixed group can nevertheless exist and solve the rationing problem.

It is important to note that the *negative* assortative matching described here, may arise in Ghatak (2000) as an implicit feature of his paper. Besides, this result does not depend on any additional assumptions. Indeed, it is well established in the literature that, in different contexts, negative assortative matching may occur in group lending. Guttman (2008), for example, shows that the positive matching does not necessarily hold in Ghatak (2000) if dynamic incentives (such as the threat of not being refinanced if the group defaults) are intoduced.

Section 2 considers the case of underinvestment (Stiglitz-Weiss, 1981) where projects are classified in terms of a second-order stochastic dominance. Section 3 considers the case of overinvestment (de Meza-Webb, 1987) where projects are, instead, ranked in terms of a second-order dominance.

2 THE UNDER-INVESTMENT CASE

2.1 The Ghatak (2000) Model

This subsection will briefly review the basic underinvestment setup analyzed in Ghatak (2000).

There are two types of risk-neutral potential entrepreneurs/firms endowed with two different investment projects, safe and risky. Both projects require a unit of investment. Firms have no initial wealth and therefore need an outside loan.

The safe project yields R_s with probability p_s. The risky project yields R_r with probability p_r. Both types yield nothing in case of failure. As in Siglitz-Weiss (1981), assume that $p_s > p_r$ and $p_s R_s = p_r R_r = \overline{R}$. There is a risk-neutral benevolent bank[2] with an opportunity cost of capital equal to ρ. As regards the informational structure, firms know each other's quality. The bank instead only know that, statistically, the probability of financing a risky type (the quality of the environment) is $\theta \in (0,1)$. Assume that the final output produced is imperfectly observable, in the sense that the bank can only verify whether the project was successful or not, but cannot observe the exact amount produced. So, the final output cannot be related with certainty ex post to the borrower's type (for example, a borrower could in theory conceal or invest elsewhere some of the final product). In this case, the optimal form of financing is the debt contract[3].

The bank can offer one of two alternative forms of contract: individual liability contract or joint liability contract. The individual liability contracts is standard debt contract with a fixed repayment sum (principal plus interest), r. In a joint liability contract, the bank asks the borrowers to form groups of two. Under this arrangement, a successful borrower, in addition to the repayment sum, pays also a joint liability component, c, if the other borrower does not obtain a positive outcome.

Ghatak (2000) assumes that

[2] The theoretical results would remain unchanged if we considered perfectly competitive lenders.

[3] As pointed out by de Meza-Webb (1987), if the final returns were perfectly observable, the bank could reach the optimum also by offering a share finance contract.

$$\overline{R} > \rho + \overline{u}, \qquad (A1)$$

$$\overline{R} < \frac{p_s}{\overline{p}}\rho + \overline{u}, \qquad (A2)$$

where $\overline{p} = (1-\theta)p_s + \theta p_r$, and $\overline{u}$ is the reservation income of both borrowers.

The assumption ($A1$) guarantees that both projects are socially productive. However, under the assumption ($A2$), pooling individual liability contracts do not exist. So, the bank is restricted to offer joint liability contracts, when this is feasible.

Under a joint contract (r,c), the expected payoff of a type *i*, when his group mate is *j*, is

$$U_{ij}(r,c) = p_i p_j (R_i - r) + p_i(1-p_j)(R_i - r - c) = p_i(R_i - r) - p_i(1-p_j)c. \qquad (1)$$

Ghatak (2000) shows that, with joint liability, borrowers always form homogeneous groups. This is the so-called positive assortative matching result.

His argument is as follows. Given the expected loss of a safe type from pairing with a risky mate, $U_{ss}(r,c) - U_{sr}(r,c) = p_s(p_s - p_r)c$, and the expected gain of a risky type from pairing with a safe, $U_{rs}(r,c) - U_{rr}(r,c) = p_r(p_s - p_r)c$, the difference of expected utilities can be written as

$$[U_{ss}(r,c) - U_{sr}(r,c)] - [U_{rs}(r,c) - U_{rr}(r,c)]. \qquad (2)$$

The positive assortative matching of Ghatak (2000) stems from the fact that (2) is always positive. That is, risky types cannot induce safe types to form a heterogeneous group even after a monetary compensation.

The generic contract (r,c) that Ghatak (2000) uses to compare the expected utilities in (2) can be separating or pooling. The separating contract derives from the following bank's zero-profit conditions on safe and risky groups,

$$p_s r_s + p_s(1-p_s)c_s - \rho = 0, \text{ and} \qquad (0\pi C_{ss})$$

$$p_r r_r + p_r(1-p_r)c_r - \rho = 0. \qquad (0\pi C_{rr})$$

The pooling contract derives from the following bank's zero-profit condition on the average group,

$$(1-\theta)[p_s r + p_s(1-p_s)c] + \theta[p_r r + p_r(1-p_r)c] - \rho = 0 . \qquad (0\pi C_{POOL})$$

Note that in ($0\pi C_{ss}$), ($0\pi C_{rr}$) and ($0\pi C_{POOL}$), the probability for the bank of receiving the joint liability payment is always based on $p_i(1-p_i)$, for $i = s, r$. This probability does not take into account the fact that, in theory, borrowers may prefer mixed groups.

All equilibrium contracts must satisfy a limited liability constraint. Namely, the outcome of a successful borrower must be large enough to pay both the individual and the joint liability obligations. That is,

$$R_s \geq r + c , \qquad (LLC)$$

where we only have to consider the (lower) return of the safe borrower.

Ghatak (2000) also considers two participation constraints for both types, that is,

$$U_{ss}(r,c) \geq \bar{u} \text{ , and} \qquad (PC_{ss})$$

$$U_{rr}(r,c) \geq \bar{u} . \qquad (PC_{rr})$$

The aim of the benevolent bank is to choose a contract that maximizes a weighted sum of the expected payoffs of the representative borrowers, that is,

$$\max_{r,c} (1-\lambda)U_{ss}(r,c) + \lambda U_{rr}(r,c) ,$$

where λ is the social weight associated to risky types.

Ghatak (2000) shows that, depending on R_s (and so on the *LLC*), there exist either separating or pooling equilibria. If $R_s \geq \hat{r} + \hat{c}$, where $\hat{r} = \rho(p_s + p_r - 1)/(p_s p_r)$ and $\hat{c} = \rho/(p_s p_r)$, that is, if the assumption

$$\overline{R} > \rho\left(1 + \frac{p_s}{p_r}\right) \qquad (A3)$$

holds, separating equilibria exist[4]. If ($A3$) does not hold, pooling equilibria may exist under the less restrictive assumption

$$\overline{R} > \rho \frac{p_s}{\overline{p}} + \overline{u}\,\frac{(1-\theta)p_s^2 + \theta p_r^2}{p_s \overline{p}}. \qquad (A4)$$

2.2 A Possibility for Mixed Groups

This subsection shows that in the model of Ghatak (2000), borrowers may in some cases pair in heterogeneous groups.

As said, in Ghatak (2000), the mixed terms $U_{sr}(r,c)$ and $U_{rs}(r,c)$ used for the comparison in (2) derive either from ($0\pi C_{ss}$) and ($0\pi C_{rr}$) in a separating equilibrium, or from ($0\pi C_{POOL}$) in a pooling equilibrium. In other words, these expected utilities derive from a contract that is based on the presumption that groups are always homogeneous. This also means that in Ghatak (2000) the reaction function of the bank to the possibility of mixed groups is not fully specified.

In the present paper, I argue that the *mixed* terms $U_{sr}(r,c)$ and $U_{rs}(r,c)$ should actually derive from the following bank's *mixed* zero-profit conditions

$$p_s r + p_s(1-p_r)c - \rho = 0, \text{ and} \qquad (0\pi C_{sr})$$

$$p_r r + p_r(1-p_s)c - \rho = 0. \qquad (0\pi C_{rs})$$

In ($0\pi C_{sr}$) and ($0\pi C_{rs}$), the bank takes into account that, whenever borrowers formed mixed groups, the joint liability component is actually $p_i(1-p_j)c$ and not $p_i(1-p_i)c$. That is, the bank must always consider a different contract, (r_{MIX}, c_{MIX}), if the group financed is not homogeneous[5].

As a result, in the case of a *negative* assortative matching, the bank's maximization problem is

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[4] This paper does not consider the extension of the note by Gangopadhyay et al. (2005). There, the authors point out the fact that in Ghatak (2000) the amount of joint liability, c, exceeds the amount of individual liability, r. This could raise the problem that when one member fails, the group may announce that both members had success and pay just the interest-rate obligations.

[5] In other terms, the problem should satisfy the complete formulation of a sub-game perfect Nash equilibrium. This is true if the bank is either benevolent or in a perfectly competitive market.

$$\max_{r,c} U_{sr}(r,c) + U_{rs}(r,c),$$

where, clearly, we do not have to consider a social weight for a specific group.
With mixed groups, the possible financial contract is easily derived. The equilibrium is pooling and unique. Solving ($0\pi C_{sr}$) and ($0\pi C_{rs}$), we obtain

$$r_{MIX} = c_{MIX} = \frac{\rho}{p_s + p_r - p_s p_r}.$$

Under mixed groups, we have neither separating, nor other pooling equilibria in addition to (r_{MIX}, c_{MIX}). This also means that the mixed contractual terms do not depend on the distribution of types.
Two necessary conditions for the existence of mixed groups are: first, a participation constraint for each firm that takes into account the transfer/compensation that one type makes to the other[6]. The second condition is the limited liability constraint for mixed groups, that is,

$$R_s \geq r_{MIX} + c_{MIX} = \frac{2\rho}{p_s + p_r - p_s p_r}. \qquad (LLC_{MIX})$$

The two mixed terms $U_{sr}(r,c)$ and $U_{rs}(r,c)$ become

$$U_{sr}(r_{MIX}, c_{MIX}) = p_s(R_s - r_{MIX}) - p_s(1-p_r)c_{MIX} = p_s\left(R_s - \frac{(2-p_r)\rho}{p_s + p_r - p_s p_r}\right), \text{ and} \qquad (3a)$$

$$U_{rs}(r_{MIX}, c_{MIX}) = p_s(R_r - r_{MIX}) - p_s(1-p_r)c_{MIX} = p_r\left(R_r - \frac{(2-p_s)\rho}{p_s + p_r - p_s p_r}\right). \qquad (3b)$$

The payoff in (3a) can be positive or negative. In the latter case, it would be more difficult to convince safe types to accept mixed groups as they would require a larger compensation.
The difference (2) becomes

[6] It is not necessary to specify the heterogeneous participation constraints because they will be automatically satisfied if borrowers choose mixed groups and if the (PC_{ss}) and (PC_{rr}) hold.

$$[U_{ss}(r,c)-U_{sr}(r_{MIX},c_{MIX})]-[U_{rs}(r_{MIX},c_{MIX})-U_{rr}(r,c)], \qquad (AM)$$

that is, the assortative-matching expression that borrowers use to compare their payoff under homogeneous and mixed groups.

The following proposition shows that the sign of the assortative matching in Ghatak (2000) may depend on the distribution of borrowers' types.

Proposition 1. In the underinvestment case, the assortative matching in Ghatak (2000),
a) if (A3) holds, is always positive.
b) if (A3) does not hold and (A4) holds, is positive if $\theta \le 1/2$*, and negative otherwise.*

Proof.
a) In this case, the equilibrium is separating. The expression (*AM*) evaluated at the contract $(\hat{r},\hat{c})$ is equal to 0, so borrowers prefer homogeneous partners[7]. This is also true for all the other possible separating contracts because safe and risky types reach the same payoff.
b) Pooling equilibria exist when the *LLC* is below the contract $(\hat{r},\hat{c})$. Write (*AM*) as $2r-(p_s+p_r)r-[p_s(1-p_s)+p_r(1-p_r)]c=\gamma(r,c)$.

Write also the $0\pi C_{POOL}$ as $c=\dfrac{\rho-[(1-\theta)(p_s-p_r)+p_r]r}{p_r-p_r^2+(1-\theta)[p_s-p_s^2-(1-p_r)p_r]}$.

This value of c, into $\gamma(r,c)$, gives $\dfrac{[2(1-\theta)-1](p_s-p_r)[(p_s+p_r-1)\rho-p_sp_rr]}{(1-\theta)(p_s-p_r)(p_s+p_r-1)+(1-p_r)p_r}=\gamma(r)$. Starting from the contract $(\hat{r},\hat{c})$, where the difference in expected utilities is 0, we have that (*AM*) is increasing in r, along the $0\pi C_{POOL}$, if

$$\left.\frac{d\gamma(r)}{dr}\right|_{0\pi C_{POOL}}=\frac{[2(1-\theta)-1](p_s-p_r)p_sp_r}{(1-\theta)(p_s-p_r)(p_s+p_r-1)+(1-p_r)p_r}\ge 0, \qquad (4)$$

that is always true if $\theta \le 1/2$ (and if, as assumed in Ghatak, 2000, $p_s+p_r>1$ to rule out negative interest-rate repayments). ■

[Fig. 1a and 1b HERE]

[7] The implicit assumption is that, in case of indifference, safe types prefer homogeneous groups.

The intuition behind proposition 1 is straightforward. Under (*A*3), the equilibrium is separating and safe firms reach their highest possible payoff, $p_s R_s - \rho$, so it is difficult for risky firms to compensate their heterogeneous peers if they agree to form a mixed group. If instead (*A*3) does not hold and (*A*4) holds, the equilibrium will be pooling. For example, fig. 1a and 1b depict the possible pooling equilibria in Ghatak (2000), respectively for a low and high level of θ. All contracts between points *A* and *B* in fig. 1a and between *C* and *D* in fig. 1b are optimal pooling contracts. If $\theta \le 1/2$, safe types obtain a payoff close to their first-best level (fig. 1a). So, again, it is difficult for safe types to be induced to choose mixed pairs. On the other hand, if $\theta > 1/2$, safe types obtain a payoff close or equal to their reservation income (fig. 1b). In this case, safe firms need a relatively low transfer from their risky partners to accept mixed groups[8]. The actual equilibrium contract will depend on the social weight, λ, of risky types. For example, if $\lambda = 1$, the (lowest possible) pooling equilibrium is at the intersection between the bank's pooled break-even line, $0\pi C_{POOL}$, and the binding participation constraint of safe types, $PC_{ss} = 0$ (point *B* in fig. 1a or *D* in fig. 1b). For such a contract, borrowers prefer homogeneous groups if (*AM*) is positive, that is, if $\frac{[2(1-\theta)-1](p_s R_s - \bar{u} - \rho)}{\theta} \ge 0$, that is true if $\theta \le 1/2$ (since $p_s R_s - \bar{u} - \rho > 0$).

2.3 Low Reservation Income

In Ghatak (2000), if the reservation income is low or equal to 0, we can observe a situation where there are both underinvestment and no possibilities for homogeneous group lending. This is equivalent to say that the *LLC* is below the point where the bank's pooled break-even line and the binding participation constraint of safe types intersect (as point *B'* in fig. 2a). This subsection shows that, in this case, heterogeneous groups can solve the rationing problem.

Assume first that $\bar{u} = 0$. In such a case, only risky types receive credit and the equilibrium contract is $(\rho/p_r, 0)$.

The difference (*AM*) can be written as

$$[0 - U_{sr}(r_{MIX}, c_{MIX})] - [U_{rs}(r_{MIX}, c_{MIX}) - U_{rr}(r, c)] = \rho - p_s R_s < 0, \qquad (AM_{\bar{u}=0})$$

where the equilibrium payoff of safe firms in homogeneous groups is 0.

[8] An interesting feature not considered in the present model (as well as in Ghatak, 2000) is the fact that, if $\theta \ne 1/2$, the number of mixed pairs is limited by the proportion of one of the two borrowers' types. In such cases, when all mixed groups already received credit, the remaining borrowers (either safe or risky) are forced to form homogeneous groups.

As ($AM_{\bar{u}=0}$) is negative, borrowers will always choose to form mixed groups (they cannot form homogeneous groups). Besides, as risky types receive the same contract, $(\rho/p_r, 0)$, when the *LLC* is below the intersection of the bank's pooled break-even line and the binding participation constraint of safe types, ($AM_{\bar{u}=0}$) is negative for all possible values of $\bar{u}$ such that the *LLC* is below point *B'*. As a result, a pooling mixed equilibrium may exist (as point E_{MIX} in fig. 2b) when pooling homogeneous equilibria are not possible.

Clearly, the condition for the existence of a mixed pooling equilibrium is that the (LLC_{MIX}) must be satisfied. We know that homogeneous pooling equilibria do not exist when (*A*4) does not hold. So, we only need to prove that (LLC_{MIX}) is less restrictive than (*A*4). If $\bar{u}=0$, by comparing (*A*4) with (LLC_{MIX}), it is $\frac{2\rho}{p_s + p_r - p_s p_r} < \rho\frac{1}{\bar{p}}$ when

$$(1-\theta) \geq \frac{p_s - p_r - p_s p_r}{2(p_s - p_r)}\,. \tag{5}$$

For $\bar{u}=0$, (5) is satisfied for a range of values of θ (this range is increasing in $\bar{u}$). For example, if $p_s = 0.8$ and $p_r = 0.4$, it is satisfied when $\theta \leq 0.1$.

The reason why risky types prefer mixed groups, when $\bar{u}$ is very low or equal to 0, is that they would always receive the least favourable homogeneous separating contract.

We can summarize the discussion of this subsection in the following

Proposition 2. A pooling equilibrium in mixed groups can exist even if the reservation income of borrowers is very low or equal to 0.

[Fig. 2a and 2b HERE]

3 THE OVER-INVESTMENT CASE

This section extends the overinvestment analysis of Ghatak (2000) and shows that, even in this case, the positive assortative matching is not a general result.

Consider an environment *a là* de Meza-Webb (1987) with $p_s > p_r$ and $R_s = R_r = R$.

Assume also that risky firms are socially inefficient,

$$p_r R_r < \bar{u} + \rho\,, \qquad (A1')$$

and that they find it profitable to ask for outside financing, that is,

$$p_r\left(R - \frac{\rho}{\bar{p}}\right) > \bar{u}\,. \qquad (A2')$$

In his paper, Ghatak (2000) is not much interested in the overinvestment case. He also chooses not to extend the analysis to pooling equilibria and shows that if the *LLC* is satisfied, that is if

$$R > \rho\left(\frac{1}{p_s} + \frac{1}{p_r}\right), \qquad (A3')$$

the optimal separating contract, $(\hat{r}, \hat{c})$, exists. Under (*A*3'), joint liability lending can solve the overinvestment problem.

[Fig. 3a and 3b HERE]

If R is not so high as to reach the equilibrium $(\hat{r}, \hat{c})$, other optimal separating or pooling equilibria may exist in this overinvestment scenario. We can distinguish two types of contracts.

The first type of contract (separating) is characterized by the fact that the *LLC* is on or above the intersection between the bank's pooled break-even line, $0\pi C_{POOL}$, and the binding risky firm's participation constraint, $PC_{rr} = 0$, as points *X* in fig. 3a and 3b (which highlights a detail of fig.3a with a lower *LLC*). In this case, the equilibrium is unique and separating (risky firms are kept out). Depending on the extent of the *LLC*, the equilibrium contract can be above or below the intersection between the bank's break-even line on good types, $0\pi C_{ss}$, and the $PC_{rr} = 0$ line (point *Y* in fig. 3a and 3b). If the *LLC* is on or above, the separating equilibrium is where the *LLC* and the $0\pi C_{ss}$ lines intersect (as contract *H* in fig. 3a). If the *LLC* is below point *Y*, the separating equilibrium is where the *LLC* and the $PC_{rr} = 0$ lines intersect (as contract *Z* in fig. 3b). Under both equilibria, risky types do not ask for a loan, so the difference (*AM*) can be written as

$$[U_{ss}(r,c) - U_{sr}(r_{MIX}, c_{MIX})] - [U_{rs}(r_{MIX}, c_{MIX}) - \bar{u}]\,, \qquad (AM')$$

where the term $U_{rr}(r,c)$ is replaced by $\bar{u}$.

The second type of contract (pooling) arises when the *LLC* is below the intersection between the $0\pi C_{POOL}$ and the $PC_{rr}=0$ lines (that is point *X'* in fig. 4a or *X''* in fig. 4b). For example, all contracts between point *M* and $(\rho/\bar{p},0)$ along the $0\pi C_{POOL}$ line in fig. 4a and between point *N* and $(\rho/\bar{p},0)$ in fig. 4b are possible pooling contracts. In this case, we need again (*AM*) and not (*AM'*) to compare homogeneous and mixed contracts, because risky types are not kept out of the credit market.

The condition for the existence of a pooling equilibrium is simply

$$R \geq \frac{\rho}{\bar{p}}, \tag{A4'}$$

that is, the same condition that would arise in an individual liability context. The following proposition describes the composition of groups in this kind of environment.

Proposition 3. In the overinvestment case, the assortative matching in Ghatak (2000),

a) if (A3') holds, is always positive.

b) if (A3') does not hold and (A4') holds, is positive if $\theta \leq 1/2$*, and negative otherwise.*

Proof.

a) In this case, (*AM'*) evaluated at the contract $(\hat{r},\hat{c})$ is equal to $\bar{u}+\rho-p_rR_r$ that is always positive since risky project are inefficient.

b) If the *LLC* is below $(\hat{r},\hat{c})$ but on or above the intersection between the lines $0\pi C_{POOL}$ and $PC_{rr}=0$, the contract is still separating. Consider first the contract *X* in fig. 3a or in fig. 3b where $0\pi C_{POOL}$ and $PC_{rr}=0$ intersect. For this contract, (*AM'*) is positive if

$$\frac{(2(1-\theta)-1)(\bar{u}+\rho-p_rR_r)}{(1-\theta)} \geq 0, \tag{6}$$

that is always true if $\theta \leq 1/2$. As (*AM'*) is increasing in $U_{ss}(r,c)$, this is also true if the equilibrium contract is either as depicted in fig. 3a or in fig. 3b, that is, for all the possible contracts between *X* and $(\hat{r},\hat{c})$.

If the *LLC* is below the intersection between the lines $0\pi C_{POOL}$ and $PC_{rr}=0$, all contracts between point *X'* and $(\rho/\overline{p},0)$ in fig. 4a and *X''* and $(\rho/\overline{p},0)$ in fig. 4b are possible pooling contracts. For such contracts, we can use again the second part of proposition 1. For example, for $\lambda=1$, that is for the contract $(\rho/\overline{p},0)$, (*AM'*) equals $\rho\left(2-\frac{p_s+p_r}{(1-\theta)p_s+p_r-\theta p_r}\right)$, that is positive when $\theta\leq 1/2$. ■

[Fig. 4a and 4b HERE]

4 CONCLUSION

The main contribution of this paper is to explain why the assortative matching in group formation of Ghatak (1999, 2000) and Van Tassel (1999) cannot be always positive or negative. The matching depends on the distribution of safe and risky types and, thus, on the contractual terms associated to each possible group formation. If the proportion of safe types is high, the contractual terms for homogeneous pairs will be relatively favourable. So, borrowers will end up choosing partners of the same type. On the other hand, if the proportion of safe types is low, borrowers may prefer mixed groups. This paper argues that the payoff of homogeneous groups under a particular contractual arrangement (separating or pooling) must be compared to the payoff of mixed groups under a completely different contractual arrangement.

Another implication of the paper is that a mixed pooling equilibrium may exist when the reservation income of borrowers is very low or equal to zero. A reservation income sufficiently high is a key feature of the underinvestment section of Ghatak (2000). Indeed, with a high reservation income, it is possible to observe a situation where the participation constraint of safe types cannot be satisfied in a pooling contract and, at the same time, the revenue in case of success is large enough to satisfy the joint limited liability constraint. If, in contrast, the reservation income is low or zero, we can observe both underinvestment for the safe and no possibilities for group lending. This paper shows that, in this case, a mixed joint liability contract can nevertheless exist and solve the underinvestment problem.

REFERENCES

de Meza, D. and Webb, D. C. (1987). 'Too Much Investment: A Problem of Asymmetric Information'. *Quarterly Journal of Economics*, 10, 281-292.

Gangopadhyay, J. E., and Ghatak, M. and Lensink, R. (2005). 'Joint Liability Lending and the Peer Selection Effect', *The Economic Journal*, 115, 1005–1015.

Ghatak, M. (1999), 'Group Lending, Local Information and Peer Selection', *Journal of Development Economics*, 60, 27-50.

Ghatak, M. (2000), 'Screening by the company You Keep: Joint Liability Lending and the Peer Selection Effect', *The Economic Journal*, 110, 601-631.

Guttman, J. M. (2008), 'Assortative Matching, Adverse Selection, and Group Lending', *Journal of Development Economics*, 87, 51-56.

Stiglitz, J. E. and Weiss, A. (1981). 'Credit Rationing in Markets with Imperfect Information', *American Economic Review*, 71, 393–410.

Van Tassel, E. (1999). 'Group Lending Under Asymmetric Information', *Journal of Development Economics*, 60, 3–25.

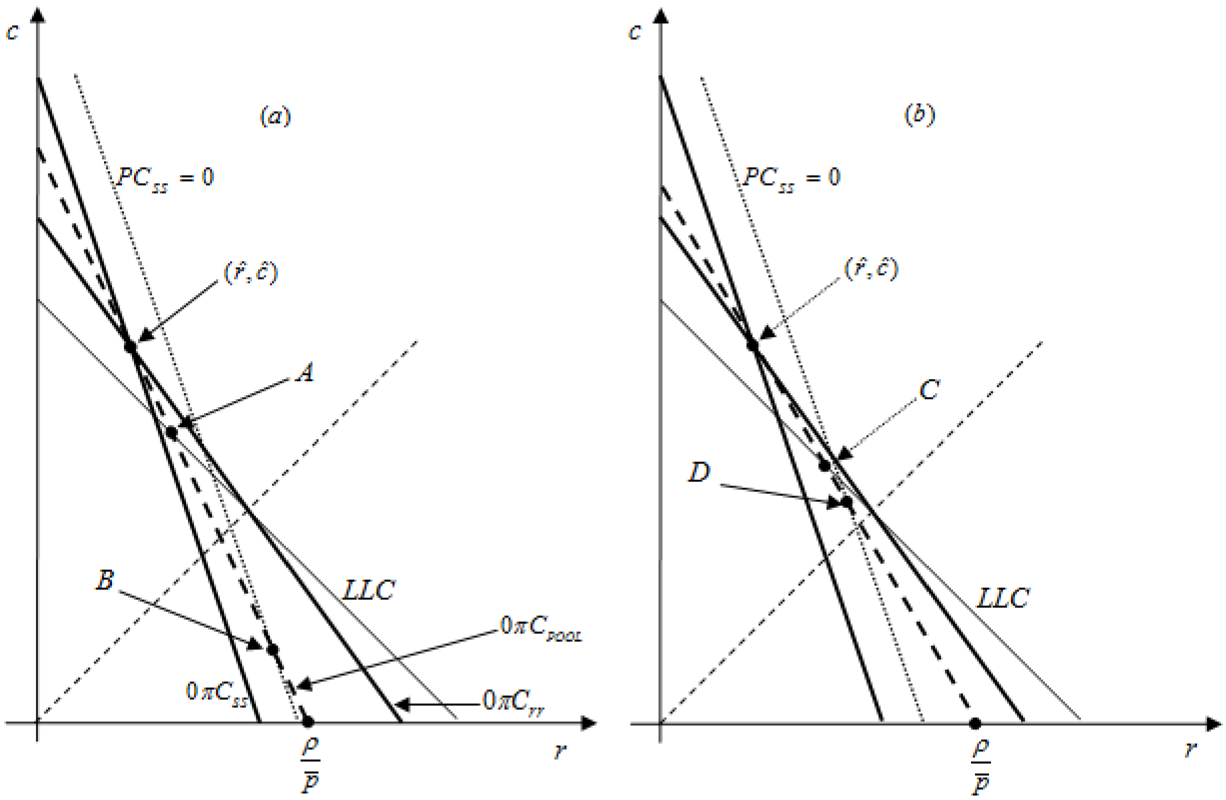

Figure 1 – Underinvestment (pooling equilibria)
a) Low θ: contracts *A* – *B*. *b*) High θ: contracts *C* – *D*.

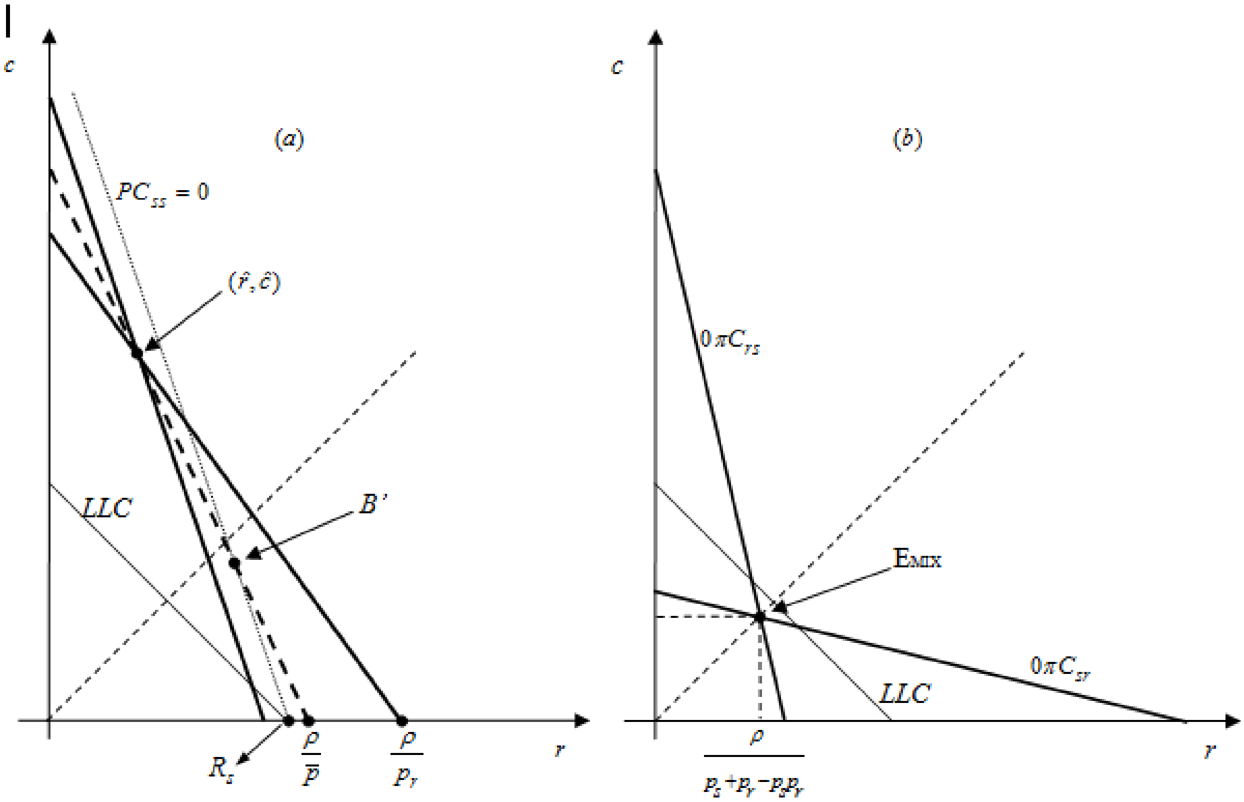

Figure 2 – Underinvestment and $\bar{u} = 0$
a) Separating and pooling joint liability equilibria do not exist.
b) The mixed pooling equilibrium.

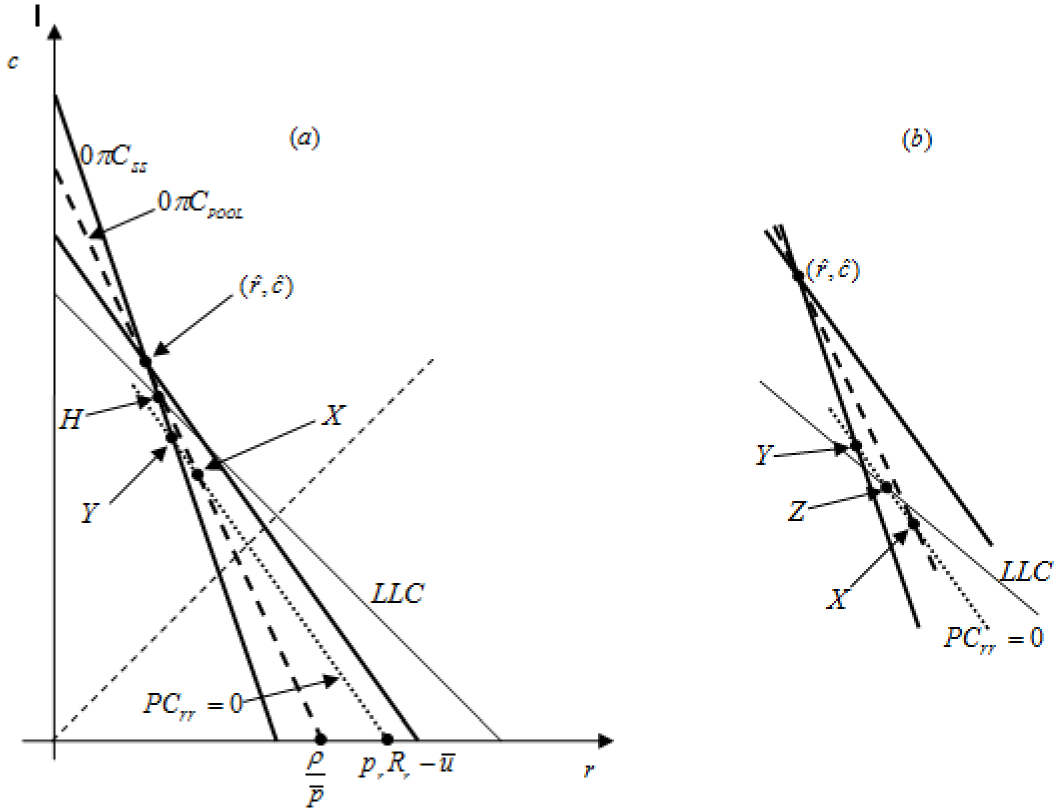

Figure 3 – Overinvestment (separating equilibria)
a) *LLC* below $(\hat{r},\hat{c})$ and above *Y*: equilibrium at *H*.
b) *LLC* below $(\hat{r},\hat{c})$ and below *Y*: equilibrium at *Z*.

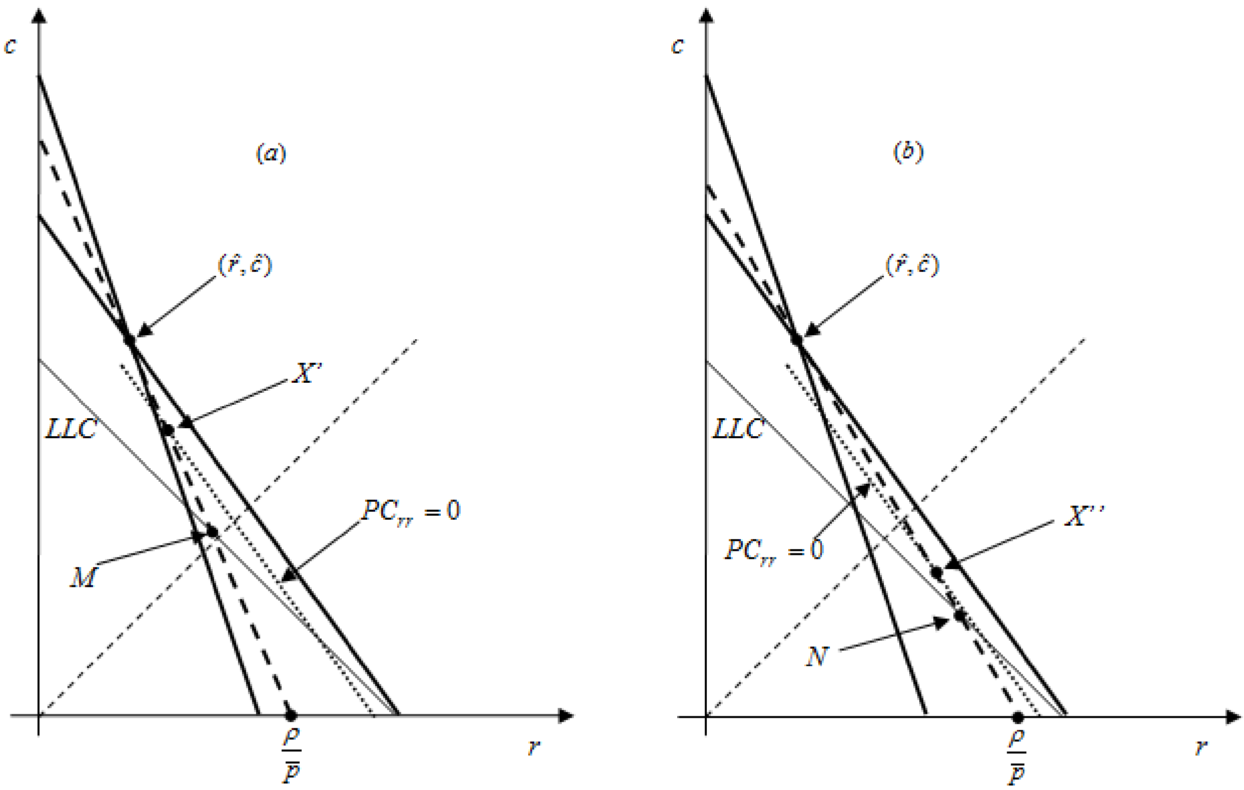

Figure 4 – Overinvestment (pooling equilibria)
a) Low θ: contracts M – $(\rho/\overline{p},0)$. b) High θ: contracts N – $(\rho/\overline{p},0)$.