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Brams, Steven J. and Kaplan, Todd R and Kilgour, D. Marc

New York University, Universities of Exeter and Haifa, Wilfrid Laurier University

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Steven J. Brams[*] Todd R. Kaplan[†] D. Marc Kilgour[‡]

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Abstract

We describe a simple 2-stage mechanism that induces two bargainers to be truthful in reporting their reservation prices in a 1st stage. If these prices criss-cross, the referee reports that they overlap, and the bargainers proceed to make offers in a 2nd stage. The average of the 2nd-stage offers becomes the settlement if both offers fall into the overlap interval; if only one offer falls into this interval, it is the settlement, but is implemented with probability $\frac{1}{2}$; if neither offer falls into the interval, there is no settlement. Thus, if the bargainers reach the 2nd stage, they know their reservation prices overlap even if they fail to reach a settlement, possibly motivating them to try again.

1 Introduction

How to induce players to go to their "bottom lines" in bargaining is an age-old problem in the design of a bargaining mechanism. A solution for sealed-bid auctions is a second-price, or Vickrey, auction [7], whereby the

[*]Department of Politics, New York University, Email: steven.brams@nyu.edu

[†]School of Business and Economics, University of Exeter, Exeter, UK and Department of Economics, University of Haifa, Haifa, Israel; email: Dr@ToddKaplan.com

[‡]Department of Mathematics, Wilfrid Laurier University, E-mail: mkilgour@wlu.ca

high bidder pays the second-highest bid, rendering what the winner pays independent of what he or she bid. The extension of this idea to Vickrey-Clarke-Groves (VCG) mechanisms [6] likewise induces honesty, because the settlement does not depend directly on what either player offers. Similarly, Brams and Kilgour [2] show that that when players bid for rooms in a house in which they share the rent, the "gap procedure" creates a kind of partial independence, motivating the housemates to make truthful bids that sum to the total rent of the house.

When only two bargainers haggle over the price of some good or service, then splitting the difference between their offers by averaging them does not induce honesty, because the bargainers each have an incentive to exaggerate in opposite directions. Indeed, Chatterjee and Samuelson [3] find a simple symmetric equilibrium in which exaggeration is piecewise linear in the bargainers' *reservation prices*—the settlement prices that would make each indifferent between an agreement or none. In that game, the final price is an average of the offers if they *overlap* (i.e., if the buyer's offer does not fall below the seller's); otherwise, there is no agreement, and the players get nothing. Myerson and Satterthwaite [5] prove that this simple mechanism is more efficient than any other, and Leininger, Linhart, and Radner [4] show that there is an infinity of asymmetric equilibria different from the linear symmetric equilibrium of Chatterjee and Samuelson. For background information on mechanism design, see [6].

In this paper, we give a two-stage mechanism that induces two bargainers truthfully to reveal their reservation prices to a referee in stage 1. If the 1st-stage offers overlap, there is the *potential* for an agreement, which is realized—at the mean of the 2^{nd}-stage offers—if *both* bargainers' offers fall in the overlap interval; if *only one* bargainer's offer falls in this interval, this offer becomes the agreement with probability $\frac{1}{2}$, but otherwise not; if *neither* bargainer's offer falls in the overlap interval, there is no agreement.

As we will show, this procedure, like another probabilistic mechanism [1] that we discuss in section 5, is not as efficient as the one given by Chatterjee and Samuelson, in part because of the random draw when exactly one offer falls in the overlap interval. But even if no agreement is *realized*, our procedure does reveal—if it goes on to stage 2—that the reservation prices of the bargainers *allow* for a mutually profitable agreement. Another benefit of our mechanism is that there is a positive probability of settlement even for extreme reservation values; in contrast, bargainers with extreme values have no incentive to bargain under the Chatterjee-Samuelson procedure, because there is no possibility of any agreement.

2 The Mechanism

We consider the possible sale of an object by a Seller to a Buyer. If they can agree on a price p, the object will be transferred from Seller to Buyer, and the Seller will receive p as compensation. Of course, Seller prefers a higher p, whereas Buyer prefers a lower p. If they cannot agree on a p, there is no sale.

For definiteness, we phrase our discussion in terms of a possible sale, but we note that our mechanism has many applications, such as to the settlement of a claim by an insured party against an insurer. In that case, the insured party, which prefers a higher settlement, plays the role of Seller, and the insurer corresponds to the Buyer.

The Seller's reservation price for the object, S, is the value of a random variable with cumulative distribution function F_S. The Buyer's reservation price, B, is the value of a random variable with cumulative distribution function F_B. Both F_S and F_B have support $[C, D]$. Both players' reservation prices are private information, and their utilities are quasi-linear, so that if a sale takes place at price p, Buyer will receive $B - p$ and Seller will receive $p - S$. If there is no sale, both players receive 0. The players are risk-neutral.

The mechanism is a two-stage procedure:

Stage 1. The players submit *reserves* to the referee: Seller submits $\widehat{S}$ and Buyer submits $\widehat{B}$. The reserves may or may not equal the corresponding reservation prices (i.e., the 1st-stage submissions are not necessarily truthful). If $\widehat{S} \leq \widehat{B}$, the overlap interval is $[\widehat{S}, \widehat{B}]$, and the procedure moves to stage 2. If $\widehat{S} > \widehat{B}$, the reserves do not overlap, there is no settlement, and the procedure ends.

Stage 2. The players submit *offers* to the referee: Seller submits s, and Buyer submits b. If both s and b fall in the overlap interval defined in stage 1, there is a sale at price $p = \frac{s+b}{2}$. If only one of s and b falls in the overlap interval, the name of one player is selected at random; if the selected player's offer is the one in the overlap interval, then it is sale price; if not, there is no sale. If neither offer is in the overlap interval, there is no sale.

This mechanism determines (i) whether there is a sale and (ii) if there is a sale, at what price. As usual, we model each player as privately learning its own (true) reservation price (S or B) prior to stage 1, and using this information to choose its strategy: $((\widehat{S}, s)$ for Seller; $(\widehat{B}, b)$ for Buyer). Thus, a strategy for Seller is a pair of functions $\widehat{S}(S)$ and $s(S)$ that give the values of its strategic variables as a function of its reservation price. Similarly, Buyer's strategy can be thought of as two functions, $\widehat{B}(B)$ and $b(B)$. We assume that all four of these strategy functions are differentiable and increasing in the players' reservation prices.

One strategy for a player *weakly dominates* another strategy for that player if the first yields an expected utility that is at least as great as the second, no matter what strategy is chosen by its opponent. A (Bayesian-Nash) *equilibrium* is a profile of strategies with the property that, for each player, the equilibrium strategy maximizes the player's expected utility, given that the opponent plays according to its equilibrium strategy.

To represent our mechanism, we use two functions,

$$t : \mathbb{R}^4 \longrightarrow [0,1] \text{ and } p : \mathbb{R}^4 \longrightarrow \mathbb{R}$$

with the interpretation that $t(\widehat{S}, s, \widehat{B}, b)$ is the probability that an agreement is reached if the 1st-stage reserves are $\widehat{S}$ and $\widehat{B}$, and the 2nd-stage offers are s and b; similarly, $p(\widehat{S}, s, \widehat{B}, b)$ is the price. Note that both $\widehat{S}$ and s are functions of Seller's true reservation price S; we have written $\widehat{S}$ instead of $\widehat{S}(S)$, and s instead of $s(S)$, for notational simplicity. Observe that, if $t = 0$, the value of p is irrelevant. Using the functions t and p, we can describe our mechanism formally, as follows:

$$(t,p) = \begin{cases} (1, \frac{s+b}{2}) & \text{if } \widehat{S} \leq s, b \leq \widehat{B}, \\ (\frac{1}{2}, b) & \text{if } \widehat{S} \leq b \leq \widehat{B} < s, \\ (\frac{1}{2}, s) & \text{if } b < \widehat{S} \leq s \leq \widehat{B}, \\ (0,0) & \text{otherwise.} \end{cases} \tag{1}$$

We will assume below that players always choose 2nd-stage strategies that are at least as "aggressive" as their 1st-stage strategies, i.e., that $b \leq \widehat{B}$ and $s \geq \widehat{S}$. This assumption is innocuous because, by (1), the players' payoffs are certain to be 0 if it does not hold, while strategies satisfying $b \leq \widehat{B}$ and $s \geq \widehat{S}$ ensure a payoff of at least 0.

We can construct a strategically equivalent mechanism by retaining stage 1 and replacing stage 2 by

Stage 2′. One player, Seller or Buyer, is chosen at random. If Seller is chosen, and if Seller's 2nd-stage offer s satisfies $s \leq \widehat{B}$, then the transaction takes place at price $p = s$; if $s > \widehat{B}$, then there is no sale. Similarly, if the player chosen is Buyer, and if Buyer's 2nd-stage offer b satisfies $\widehat{S} \leq b$, then the transaction takes place at price $p = b$; if $b < \widehat{S}$, there is no transaction.

We assume that the random selection of a player in stage 2′ is independent of the players' reservation prices. The equivalence of the two mechanisms arises because, if stage $2'$ is followed, the players' expected utilities are exactly as in (1). For example, if $\widehat{S} \leq s, b \leq \widehat{B}$, then Seller's expected utility is

$$\frac{1}{2}(s-S)+\frac{1}{2}(b-S)=\frac{s+b}{2}-S=p-S,$$

where p is determined by the first condition of (1). A similar relation holds for Buyer. The verification is immediate if the 2^{nd}-stage offer of only one player, or none, falls in the overlap interval. Below, we will use the stage 2 and stage $2'$ formulations interchangeably.

3 A Simple Truth-Telling Mechanism

We define truth-telling as follows:

Definition 1 *Seller's strategy $(\widehat{S}, s)$ is* ***truth-telling*** *if $\widehat{S}(S) = S$ for all $S \in [C, D]$. Buyer's strategy $(\widehat{B}, b)$ is* ***truth-telling*** *if $\widehat{B}(B) = B$ for all $B \in [C, D]$. A strategy profile $(\widehat{S}, s; \widehat{B}, b)$ is a* ***truth-telling equilibrium*** *if it is an equilibrium and both players' strategies are truth-telling.*

Note that truth-telling refers to the players' reserve strategies (1^{st} stage), not their offer strategies (2^{nd} stage).

A cumulative distribution function for Buyer, $F_B(x)$ has a *monotone hazard rate* iff $\frac{d}{dx}\frac{F_B'(x)}{1-F_B(x)} \geq 0$ for all $x \in [C, D$. Similarly, a cumulative distribution function for Seller, $F_S(x)$ has a *monotone hazard rate* iff $\frac{d}{dx}\frac{F_S'(x)}{F_S(x)} \leq 0$ for all $x \in [C, D]$. If both Buyer's and Seller's distributions satisfy a monotone hazard rate condition, our procedure has a truth-telling equilibrium, as shown next.

Proposition 1 *Any strategy of Seller, $(\widehat{S}, s)$, is weakly dominated by the truth-telling strategy (S, s). Any strategy of Buyer, $(\widehat{B}, b)$, is weakly dominated by the truth-telling strategy (B, b). If $F_S(\cdot)$ and $F_B(\cdot)$ are strictly in-*

creasing and satisfy a monotone hazard rate condition, then there is a truth-telling equilibrium in which the players' 2^{nd}-stage offers, s^* and b^* are the solutions of $1-F_B(s) = (s-S)F_B'(s)$ and $F_S(b) = (B-b)F_S'(b)$, respectively. Moreover, there are no truth-telling equilibria other than $(S, s^*; B, b^*)$.

Proof. First we consider the Seller's expected utility, which we calculate using the procedure of stage $2'$. The Seller knows the value of S and determines strategy $(s, \widehat{S})$ using the functions $s(S)$ and $\widehat{S}(S)$. The expectation must be taken over the Buyer's value B and the random selection of Seller or Buyer. Therefore, Seller's expected utility given S is

$$\frac{1}{2}\int_{b^{-1}(\widehat{S})}^{D} (b(B) - S)\, dF_B(B) + \frac{1}{2}\int_{\widehat{B}^{-1}(s)}^{D} (s - S)\, dF_B(B), \qquad (2)$$

where the first integral is associated with the random selection of Buyer (so the price is b) and the second with the selection of Seller (so the price is s). The first integral must be restricted to values of B such that $b(B) \geq \widehat{S}$, which is equivalent to $B \geq b^{-1}(\widehat{S})$, as indicated in the lower limit. Similarly, the second integral is restricted to those values of B for which $s \leq \widehat{B}(B)$, as reflected in the lower limit.

Consider the information provided by (2) about Seller's choice of strategy functions. The first integral of (2) depends on $\widehat{S}(S)$ but not s, and the second integral of (2) depends on $s(S)$ but not $\widehat{S}$. Therefore, Seller maximizes its expected utility by choosing $\widehat{S}$ to maximize the first integral and s to maximize the second.

The first integral of (2) is

$$I_1(\widehat{S}) = \frac{1}{2}\int_{b^{-1}(\widehat{S})}^{D} (b(B) - S)\, dF_B(B).$$

This integral depends on $\widehat{S}$ only through its lower limit, so it can be differentiated easily to produce

$$\begin{aligned} \frac{d}{d\widehat{S}} I_1(\widehat{S}) &= -\tfrac{1}{2}\left(b(b^{-1}(\widehat{S})) - S\right) F_B{}'(b^{-1}(\widehat{S}))\tfrac{db^{-1}(\widehat{S})}{d\widehat{S}} \\ &= -\tfrac{1}{2}\left(\widehat{S} - S\right) F_B{}'(b^{-1}(\widehat{S}))\tfrac{db^{-1}(\widehat{S})}{d\widehat{S}} \end{aligned}$$

Now ${F_B}'(b^{-1}(\widehat{S}))$ can be assumed positive, as $F_B(\cdot)$ is strictly increasing, and $b^{-1}(\cdot)$ is an increasing function because $b(\cdot)$ is. It follows that $I_1(\widehat{S})$ is a strictly increasing function of $\widehat{S}$ if $\widehat{S} < S$ and a strictly decreasing function of $\widehat{S}$ if $\widehat{S} > S$. Thus, for any 2$^{\text{nd}}$-stage (offer) strategy $s = s(S)$ of Seller, it follows that Seller's expected utility using strategy $(\widehat{S}, s)$, where $\widehat{S} \neq S$, is not greater than Seller's expected utility using strategy (S, s). Therefore, we can conclude that any strategy for Seller is weakly dominated by a truth-telling strategy, and that we can assume that $\widehat{S}(S) = S$ at equilibrium. A similar argument for Buyer leads to the conclusion that $\widehat{B}(B) = B$ can be assumed at equilibrium.

Now we address Seller's choice of $s(S)$ to maximize the second integral of (2). With the substitution $\widehat{B}(B) = B$, this integral becomes

$$I_2(s) = \frac{1}{2}\int_s^D (s - S)\, dF_B(B).$$

We now differentiate with respect to s to produce

$$\begin{aligned} \frac{d}{ds} I_2(s) = \; & \tfrac{1}{2}\left[-(s-S){F_B}'(s) + \textstyle\int_s^D \, dF_B(B)\right] \\ = \; & -\tfrac{1}{2}\left[-(s-S){F_B}'(s) + 1 - F_B(s)\right]. \end{aligned}$$

Setting this derivative equal to zero and solving for $s = s^*$ produces the required condition, using the monotone hazard rate condition for $F_B(\cdot)$. Moreover, the maximizing value s^* is clearly unique. An analogous calculation, utilizing the monotone hazard rate condition for $F_S(\cdot)$, produces the condition on b^*, and again shows that it is the unique maximizer. Thus, $(S, s^*; B, b^*)$ is the unique truth-telling equilibrium. ■

Our proof that $(S, s^*; B, b^*)$ is an equilibrium thus relies on the fact that non-truth-telling strategies are weakly dominated by truth-telling strategies, so there must be a truth-telling equilibrium. Then the offer strategies are obtained by maximizing the players' expected utilities under the assumption of truth-telling. To understand why truth-telling dominates, note that each player benefits from maximizing the width of the overlap interval, up to its reservation price—Seller from below and Buyer from above—in order to ensure, insofar as possible, that the 2^{nd}-stage bids, s and b, fall in the interval, thereby meeting a necessary condition for an agreement.

In fact, truthfully reporting one's reservation price in stage 1 is analogous to bidding one's reservation price in a Vickrey auction: Just as a player cannot win in a Vickrey auction without being the highest bidder, a bargainer cannot reach a settlement unless there is an overlap interval, leading to stage 2. In each case, a player goes to its bottom line for two reasons: (i) failing to do so in stage 1 could preclude a favorable outcome in stage 2 (or cause an unfavorable outcome) and (ii) once in stage 2, the outcome does not depend on what the player reported in stage 1.

A player's utility, if positive, does not depend on the reserves, $\widehat{S}$ and $\widehat{B}$, submitted in stage 1 but, instead, on its bid, s or b, in stage 2. The independence between a player's reserve and its offer implies that it can "afford" to be truthful in stage 1. In fact, a player cannot do worse by reporting its reservation price truthfully in stage 1, and may do better, so we say that under our mechanism each player has an incentive to report its reservation prices truthfully.

The story is different, however, in stage 2: Each player will have an incentive to shade its offer, depending the distribution of the opponent's reservation price. In the next section, we illustrate, for two particular distribution functions, how much shading is optimal.

4 Examples

For our examples, we assume $C = 0$ and $D = 1$, so that $0 \leq S, B \leq 1$.

Example 1 *Uniform distribution:* $F_S(x) = F_L(x) = x$.

The players' optimal offers, $s^*(S) = \frac{1+S}{2}$ and $b^*(B) = \frac{B}{2}$, are shown in Figure 1 below. Notice that each of these strategies halves the distance between the reservation prices, S and B (shown as the line $S = B$) and the endpoints, 1 and 0 respectively, of the bargaining range. In particular, Seller never offers below $\frac{1}{2}$, and Buyer never offers above $\frac{1}{2}$.

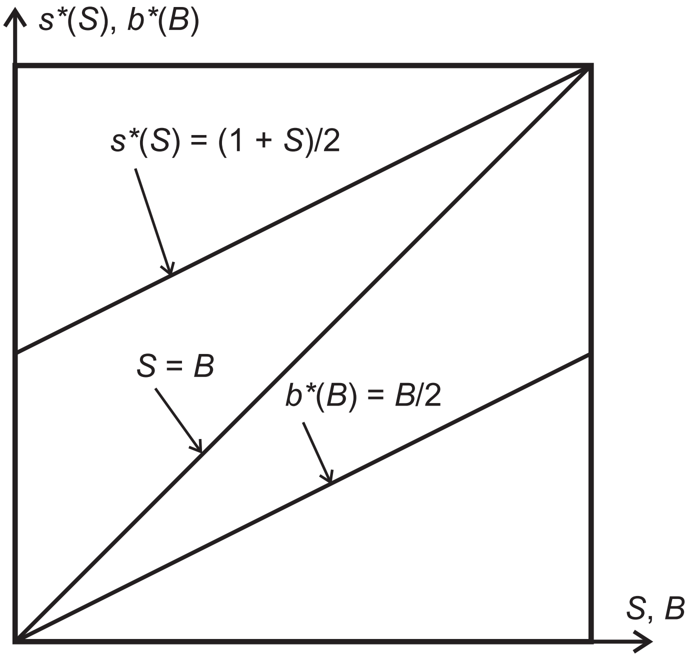

Figure 1: Offer Strategies: Example 1

Notice from Figure 1 that if Seller's (true) reservation price is, for example, $S = \frac{3}{4}$, then Seller's 2^{nd}-stage bid will be $s = s^*(\frac{3}{4}) = \frac{7}{8}$ at equilibrium, and there will be a sale with probability $\frac{1}{2}$ if $B > \frac{7}{8}$; otherwise, there is no possibility of a sale. Hence, in the 1^{st}-stage, Seller will be indifferent between reporting $\frac{3}{4}$ and, say, $\frac{5}{8}$ (provided the 2^{nd}-stage bid remains $s = \frac{7}{8}$). Thus, truthful reporting is weakly but not strictly dominant in the 1^{st} stage.

Figure 2 graphs the results of the equilibrium strategies we have identified for all possible values of S and B. A sale occurs with certainty when $B < 2S$ and $B > \frac{1+S}{2}$; these two conditions define the region with darker shading in Figure 2. Notice that this is the region of small values of S and large values of B; the difference between S and B is so great that the offers s^* and b^* fall in the overlap interval. A transaction occurs with probability $\frac{1}{2}$ when $2S < B < \frac{1+S}{2}$ and when $\frac{1+S}{2} < B < \min\{2S, 1\}$, which are the two regions with lighter shading in Figure 2.

In the first of these regions (lower left), $s^* > B$ but $b^* > S$, so there is a sale at $p = b^*$ when Buyer's name is drawn in stage $2'$, and no sale otherwise. Similarly, in the upper right region, $s^* < B$ and $b^* < S$, so there is a sale at $p = s^*$ when Seller's name is drawn in stage $2'$, and no sale otherwise.

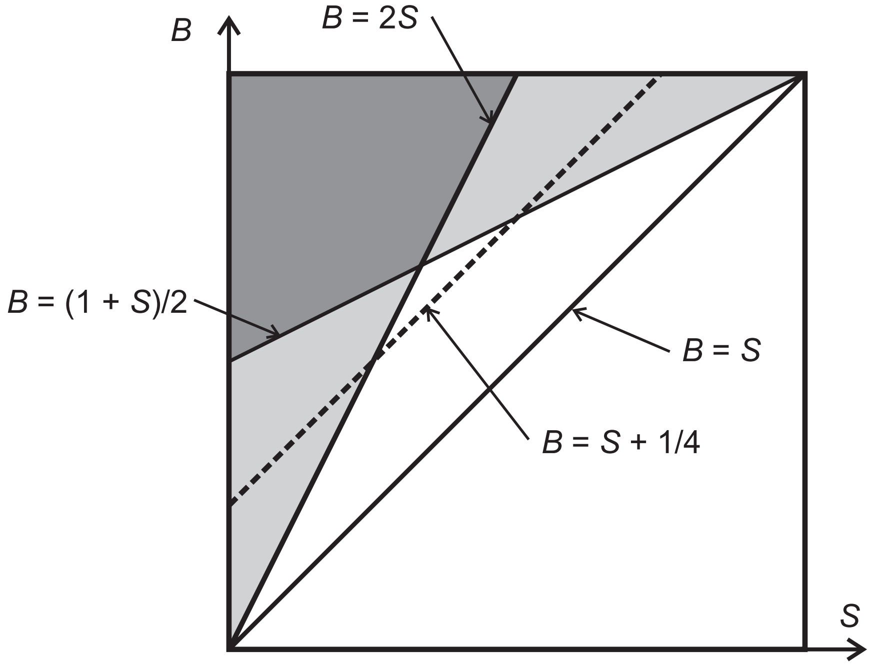

Figure 2: Conditions for a Sale: Example 1

It is instructive to compare our mechanism with the Chatterjee-Samuelson procedure [3]. The Chatterjee-Samuelson procedure produces a transaction, for certain, if and only if $B \geq S + \frac{1}{4}$, which is the area above the dashed line in Figure 2. To compare mechanisms, we use the expected surplus they produce, which because of our assumptions equals the total expected utility of Buyer and Seller after the transaction, if any. For an "ideal" procedure, which produces a settlement whenever the players' reservation prices overlap, the total surplus is

$$\int_0^1 \int_S^1 (B - S)\, dSdB = \frac{1}{6}.$$

Myerson and Satterthwaite [5] demonstrated that no mechanism can produce a larger surplus than the Chatterjee-Samuelson procedure, which gives

$$\int_0^{\frac{3}{4}} \int_{S+\frac{1}{4}}^1 (B - S)\, dBdS = \frac{9}{64}.$$

The surplus from our mechanism is

$$\frac{1}{2}\int_0^1 \int_{\frac{1+S}{2}}^1 (B - S)\, dBdS + \frac{1}{2}\int_0^{\frac{1}{2}} \int_{2S}^1 (B - S)\, dBdS = \frac{1}{8},$$

which is $\frac{8}{9} = 88.9\%$ of the maximally possible surplus.

But there are other ways to compare mechanisms. One positive feature of ours is the potential for trade at all possible values of S and all possible values of B. The Chatterjee-Samuelson procedure does not share this feature. For instance, if $S = 0.8$ and $B \geq 0.9$, a sale occurs with probability 0.5 under our mechanism, but the Chatterjee-Samuelson mechanism rules out any possibility of a transaction.

Example 2 *Power distribution:* $F_S(x) = x^\alpha$, $F_B(x) = 1 - (1-x)^\beta$, *for* $\alpha, \beta > 0$.

(It is easy to verify that these distributions satisfy the monotone hazard rate conditions.) Buyer's optimal offer is $b^* = \frac{\alpha B}{1+\alpha}$ and Seller's is $s^* = \frac{1+\beta S}{1+\beta}$, in agreement with Example 1, which corresponds to $\alpha = \beta = 1$. For example, when $\alpha = \beta = 2$, $b^*(B) = \frac{2}{3}B$ and $s^*(S) = \frac{1}{3} + \frac{2}{3}S$, and when $\alpha = \beta = \frac{1}{2}$, $b^*(B) = \frac{1}{3}B$ and $s^*(S) = \frac{2}{3} + \frac{1}{3}S$.

5 Conclusion

We have demonstrated a simple and elegant 2-stage mechanism that induces two bargainers to be truthful in reporting their reservation prices in the 1^{st}

stage; if these prices criss-cross, the referee reports that there is an overlap interval, and the bargainers make offers in a 2^{nd} stage. The mean of these offers becomes the settlement if they both fall in the overlap interval. If only one offer does, it is implemented as the settlement price with probability $\frac{1}{2}$, whereas if neither offer does, there is no settlement.

Our mechanism is less efficient than the Chatterjee-Samuelson mechanism, but does have several features to recommend it, including the possibility of a transaction even for extreme reservation prices. Also, the truth-telling equilibrium we found is unique.

Part of the inefficiency of our mechanism stems from the random implementation of a 2^{nd}-stage offer, s or b, as the exchange price when only one offer falls in the overlap interval. Randomizing the implementation of a single inside offer is the penalty one pays to render a player's reserve independent of its offer in the expected-payoff calculation, thereby making it optimal for the player to report truthfully its reservation price. This independence would be broken, and it would be suboptimal for a player truthfully to report its reservation price, if single inside offers were implemented with certainty.

Brams and Kilgour [1] analyze other mechanisms that induce two bargainers to be truthful, including a "bonus procedure" in which a third party induces the bargainers to be truthful by paying them a bonus when their bids criss-cross. But it is their "penalty procedure" that is closest to the present mechanism in inducing truth-telling behavior.

Under it, the bargainers make simultaneous offers in a single stage, with the proviso that the probability of implementation of a settlement is a function of the *degree* of overlap, if any, in the bids: the greater the overlap, the higher this probability.[1] It yields a surplus of $\frac{1}{12}$, which is 50% of the maximum possible surplus, so falls short of the 88.9% achieved by the present

[1]The probability of a *certain* settlement in the present mechanism increases as the overlap of the stage 1 reserves increases, because a greater overlap increases the likelihood

mechanism. And, unlike the present mechanism, the players never learn whether their failure to settle was because (i) their reservation prices did not criss-cross (as in stage 1), or (ii) they did criss-cross but probabilistic implementation prevented a settlement (as in stage 2). In principle, however, they could be told whether (i) or (ii) prevented a settlement; if (ii), they might be motivated to try again (as discussed below).

An advantage of the present mechanism is that the players *always* learn if stage 2 is reached and, therefore, that there is an overlap interval and the potential for a mutually profitable settlement. While our mechanism does not reveal the amount of regret—for example, by close the 2^{nd}-stage offers are to the overlap interval—we see no reason why the values of $\widehat{S} = S$, $\widehat{B} = B, s$, and b could not be revealed by the referee, making public the reason why implementation failed in stage 2.

If the optimality of shading one's "bottom line" in stage 2 is the reason that a settlement eluded the players, this outcome might motivate them, or a third party, to try to find a settlement—though, of course, under our model the players must assign probability 0 to this eventuality during the bargaining. But would they in good conscience walk away from the possibility of a mutually profitable settlement that they know exists? While failure is final in the model, perhaps in reality the bargainers would have good reason to jettison the conclusion of the mechanism and try again.[2]

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that *both* players' stage 2 offers will fall into the overlap interval, ensuring a settlement.

[2]One possible solution would be to force a settlement at the mean of the reservation prices if there is an overlap interval but no settlement in stage 2. But this is a different mechanism, and truth-telling would not be optimal for the players in stage 1. Moreover, our demonstration above that the optimal 2^{nd}-stage offers are the form $s = \frac{1+\widehat{S}}{2}$ and $b = \frac{\widehat{B}}{2}$ would no longer apply, so neither the reserves nor the offers could be expected to be related to the players' (truthful) reservation prices.

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