Coherent optimal prediction with large nonlinear systems: an example based on a French model

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1 September 1986

COHERENT OPTIMAL PREDICTION WITH LARGE NONLINEAR SYSTEMS: AN EXAMPLE BASED ON A FRENCH MODEL[1]

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The drawbacks of predictors obtained with the usual deterministic solution methods in nonlinear systems of stochastic equations have been widely investigated in the literature. Most of the proposed therapies are based on some estimation of the conditional mean of the endogenous variables in the forecast period. This however provides a solution to the problem which does not respect the internal coherency of the model, and in particular does not satisfy nonlinear identities. At the same time, for analogy with univariate skewed distributions, the conditional mean may be expected to lie on the *wrong* side of the deterministic solution, meaning that it moves towards values of the variables which are less likely to occur, rather than towards the most probable values. Estimation of the most likely joint value of all endogenous variables is proposed as an alternative optimal predictor. Experimentation is performed on a large scale macroeconomic model of the French economy, and some considerations are drawn from the results.

[1] The research underlying this paper was carried out in partial fulfillment of a joint research project between INSEE and the IBM Scientific Center of Pisa. Discussions with S.G.Hall and F.P.Sterbenz are gratefully acknowledged; responsibility, however, remains with the authors. Suggestions and comments are welcome.

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1. INTRODUCTION

The drawbacks of predictors obtained with the usual deterministic solution methods in nonlinear systems of stochastic equations have been widely investigated in the literature and are now well known.

Several therapies have been proposed and there seems to be a kind of general consensus on the idea of replacing deterministic solution predictors with an estimate of the conditional means of endogenous variables in the prediction period.

Howrey and Kelejian (1971) point out that, being in general the deterministic solution different from the conditional mean of the endogenous variables, model's validation should not be based on the comparison between deterministic solution and historical values of the variables.

Mariano and Brown (1983) observe that deterministic solution of a nonlinear model with estimated parameters produces asymptotically biased and inefficient estimates of the conditional means. They recommend to use the sample mean of replicated parametric stochastic simulations (using the algorithms proposed by Nagar, 1969, or by McCarthy, 1972, if the random errors are supposed multivariate normal), or (Brown and Mariano, 1984, 1985) to use the sample mean of nonparametric residual-based simulations. Both procedures attain, under certain conditions, asymptotic unbiasedness (and efficiency to some extent).

Wallis (1982) gives examples where the use of deterministic simulation predictors from a nonlinear econometric model may be less efficient than an *extrapolative time-series* forecast, in contrast with a well known result on the relative efficiency in the case of linear systems.

Empirical studies have been performed on a variety of real-world macroeconomic models used in practice by model builders for forecasting purposes (e.g. Bianchi et al, 1976, 1980, 1984, Calzolari, 1979, Fair, 1980, Hall, 1984, 1985, Fisher and Salmon, 1986). They aimed at evidencing whether considering the conditional mean of the endogenous variables in place of the deterministic solution gives or not significant improvements to the forecaster. The conclusion is clearly model-dependent and not univocal; cases with significant differences have been evidenced.

Predictors obtained as conditional means of the endogenous variables are not, however, optimal from all points of view. They are obviously optimal if the forecaster has in mind some kind of quadratic loss function for his forecasts. These predictors have, in fact, two highly undesirable properties, the first of which is that

the means do not necessarily satisfy the equations of the model; if the model is nonlinear and contains identities, given the values of the predetermined variables and of the model's parameters, it is generally impossible to find values of the random error terms which correspond to the mean values of the endogenous variables.

This is a problem in particular for those variables which appear in nonlinear identities; if the means are computed with replicated stochastic simulations (parametric or nonparametric), although all identities hold replication by replication, they do not hold anymore (at least the nonlinear ones) in terms of the means. In other words, the conditional mean does not provide a coherent forecast, and this can lead to some serious misinterpretations of the results.

Suppose for example that a model is designed to predict the exchange

rate in francs per dollar (*ER1*). Then, in order to compute the exchange rate in dollars per franc, the model simply takes the reciprocal, that is the identity $ER2 = 1 / ER1$ will be included among the equations. If we use the conditional means as predictors, the mean of *ER2* is not the reciprocal of the mean of *ER1*. The naive analyst who uses the results supplied by the model might believe that arbitrage opportunities exist in this market, while of course they do not.

Considering the problem of incoherency of predictors obtained from conditional means, Hall (1985) observes that also the *modes* of the marginal distributions of the endogenous variables would not escape the problem: a nonlinear identity does not generally hold for the mode values of the variables. Since coherency is an important property, Hall (1985) reconsiders the properties of forecasts obtained from deterministic solution of the nonlinear model. He shows that, in particular conditions and with an appropriate extension of the concept of median to the multivariate case, the deterministic solution of the nonlinear system can be regarded as the multivariate median of the joint distribution of the endogenous variables; and, being a solution of the system, it provides a coherent set of forecasts. It has also some desirable properties in terms of the loss function of the forecaster since, while the mean minimizes the mean squared error of predictors, the median minimizes the mean absolute error.

Considering the mean of each endogenous variable as opposed to median and mode, a second undesirable property is also usually expected from the analysis of simple univariate distributions:

the mean is usually expected to be less probable than the deterministic predictor. In other words, with respect to the deterministic solution *value, the mean of an endogenous variable is shifted towards the side of more unlikely values, rather than towards the side where the probability density grows.*

In fact in skewed univariate distributions which are unimodal and of moderate asymmetry, an empirical relationship holds between mean, median and mode (see Kendall and Stuart, 1969, section 2.11), and it is that the three quantities occur on the distribution in the same order (or in the reverse order) as in the dictionary. This should imply that, if we associate the median of each endogenous variable to the deterministic solution of the model, the most likely values of the variable are systematically expected to lie on the opposite side of the mean.

What we propose in this paper is to reconsider the *mode of the joint distribution* of the endogenous variables as an optimal predictor which preserves identities. Although it does not coincide with the mode of the univariate distributions of the endogenous variables considered separately (i.e. their marginal distributions), it has the quite appealing property of being an estimate of *the most likely joint value* of all the endogenous variables simultaneously. At the same time, since it is obtained as solution of the system corresponding to a particular set of values of the random error terms, it implies coherency.

Strong technical difficulties are encountered when searching for the maximum of the joint density function of all the endogenous variables in a large scale macroeconometric model. We had to overcome these difficulties when performing experiments on the large scale nonlinear model of the French economy considered in this paper. The empirical results related to ex-post optimal forecasts one period ahead are presented in this paper and compared with estimates of the conditional means of the endogenous

variables obtained with parametric and nonparametric stochastic simulation procedures.

Some interesting conclusions are obtained from the experimental results. In particular, since we are considering the mode of the joint distribution of all the variables and not of each variable separately, empirical results show that for most variables the mean is shifted, with respect to the deterministic predictor, towards the side of increasing probability, rather than the opposite side.

2. THE CONDITIONAL MEAN AS PREDICTOR

Let the simultaneous equation model be represented as

$$(1) \qquad f\,(y_t, x_t, a) = u_t \qquad\qquad t=1,2,\ldots,T$$

where y_t is the $M{\times}1$ vector of endogenous variables at time t, x_t is the vector of exogenous variables at time t and a is the vector of all unknown structural coefficients in the model. The model is supposed to contain $m{\leq}M$ stochastic equations and $M{-}m$ identities. The $M{\times}1$ vector of random error terms at time t, $u_t=(u_{1t},u_{2t},\ldots,u_{mt},0,\ldots,0)'$, is assumed to be independently and identically distributed as

$$(2) \qquad N\left\{0,\begin{bmatrix}\Sigma & 0\\ 0 & 0\end{bmatrix}\right\}$$

with the $m{\times}m$ covariance matrix Σ completely unknown, apart from being symmetric and positive definite.

It is usually assumed that a simultaneous equations system like (1) uniquely defines the values of the elements of y_t once values for the coefficients, the predetermined variables, and the disturbance terms are given. This means that the structural form equations (1) implicitly define a system of reduced form equations

$$(3) \qquad y_t = y(x_t, a, u_t).$$

If the vector of functional operators y is representable in closed form, and the analytical computation of the conditional mean is feasible

$$(4) \qquad \bar{y}(x_t, a, \Sigma) = E(y_t | x_t, a, \Sigma)$$

then if $\hat{a}$ and $\hat{\Sigma}$ are the available estimates of the structural form parameters, the estimated conditional mean in the forecast period h, $\bar{y}(x_h, \hat{a}, \hat{\Sigma})$ may be used as predictor. Nonlinearity usually implies this predictor to be different from the usually employed deterministic predictor $y(x_h, \hat{a}, 0)$.

Since, however, the vector of functional operators y is generally not representable in closed form in the case of nonlinear models, and the analytical computation of the mean is usually infeasible, some approximation techniques are usually employed.

2.1. Parametric stochastic simulation

It is the stochastic simulation procedure that is most widely used in the literature (e.g. Bianchi et al., 1976, Fisher and Salmon, 1986, Hall, 1985). The procedure is as follows.

1) A vector of pseudo-random numbers $\tilde{u}_h$, with multivariate normal distribution, zero mean and the available covariance matrix $\hat{\Sigma}$ is generated. The method of Nagar (1969) can be applied if $\hat{\Sigma}$ is positive definite; if $\hat{\Sigma}$ is not of full rank, the method of McCarthy (1972) can be used.
2) The vectors $\tilde{u}_h$ are inserted into the model, where the structural coefficients are maintained fixed at their originally estimated values, and the model is solved in the forecast period, h, obtaining for the endogenous variables the vector $\tilde{y}_h$.

Stages 1 and 2 are repeated and sample means of the elements of $\tilde{y}_h$ are computed.

If finite moments exist, a very large number of replications would lead, in practice, to the exact values of the means, if the parameters of the model (the vector a and the covariance matrix of the structural disturbances) were known with certainty. As, however, we assume only estimates of these parameters, stochastic simulation will lead to an estimate of the conditional means of the endogenous variables in the prediction period.

The experimental variance of the sample mean decreases in inverse proportion with the number of replications. This is often insufficient to allow appreciating significant differences between the mean and the deterministic solution value even with a rather large number of replications (Bianchi et al., 1976, 1980, Fisher and Salmon, 1986). Huge gains in the computational efficiency are often obtained in macroeconometric models by the use of antithetic variates in the stochastic simulation procedure (Calzolari, 1979, Brown and Mariano, 1985).

Replications are performed pairwise, once with pseudo-random error terms $\tilde{u}_h$ generated as discussed above, and once with the same vector of error terms with opposite sign, $-\tilde{u}_h$. The results of the two replications (which will presumably be negatively correlated) are averaged, and the means of the endogenous variables are computed as sample means of the pairwise means.

Whether or not antithetic variates are used, the results do not change, except that the same accuracy may be obtained (and has been obtained for the model we are considering in this paper) with a smaller number of replications. The results we present have been obtained with 40000 couples of antithetic replications, which guaranteed for all the variables displayed in the tables an estimate of the bias (deterministic solution minus conditional mean) at least 50 times larger than its experimental standard deviation: a similar computational accuracy without antithetic variates would be obtained only at the cost of several millions of independent replications.

2.2. *Residual-based procedures*

A residual-based procedure has been proposed by Brown and Mariano (1984) for estimating the conditional means of endogenous variables in the prediction period. The procedure utilizes complete enumeration of the residuals over the sample period. It consists of replicating the solution of the model in the forecast period, h, exactly T times, using the T vectors of estimated residuals $\hat{u}_1$, $\hat{u}_2$, ..., $\hat{u}_T$, and then computing sample variances of the elements of the T vectors of solutions. The computational steps are quite similar to those of the parametric method described above. In the first step, however, we use one vector of estimated residuals rather than using a generator of pseudo-random numbers; moreover, steps 1 and 2 are repeated *exactly* T times (rather than an arbitrary number of times), using each time a different vector of residuals.

The procedure produces an asymptotically unbiased predictor, as the parametric procedure. The parametric stochastic simulation predictor is more efficient if the number of replications is large (it can be made arbitrarily close to the closed form predictor), but the residual-based predictor is more efficient when the number of replications in the parametric procedure is not greater than the sample period size.

Brown and Mariano (1985) propose an antithetic version of the residual-based procedure. Although asymptotic efficiency gains are not guaranteed, a small sample experiment showed that the antithetic residual-based predictor was efficient relatively to all other asymptotically unbiased procedures.

Since the use of the antithetics produces results numerically different, both residual-based procedures have been applied to the model.

Numerical differences in the three computations of conditional means as predictors can be observed in the tables; in many cases the information is *"qualitatively"* the same, since the sign an the magnitude of the difference with respect to the deterministic predictor is the same, but not always. Since Brown and Mariano (1984) show that the parametric procedure is more efficient than the residual-based method (provided that the number of replications is sufficiently large, as it certainly is in our case) we shall confine comments to the former group of results; in any case also the others will be displayed for the sake of completeness.

3. THE MOST LIKELY JOINT VALUE AS PREDICTOR

The log-likelihood of the *t-th* observation can be expressed as

$$L_t = -1/2 \log|\Sigma| + \log|\partial f_t/\partial y_t'| - 1/2\, f_t'\Sigma^{-1}f_t \qquad (5)$$

where the Jacobian determinant $|\partial f_t/\partial y_t'|$ is taken in absolute value.

Since the system includes identities, the following considerations must be kept in mind when computing the log-likelihood (see Oberhofer, 1971, who extends to the nonlinear case the treatment of identities in maximum likelihood estimation given by Rothenberg and Leenders, 1964, pp.71-72; see also Anderson, 1958, appendix 1.3).

1) Σ is computed from the residuals of the m stochastic equations of the model, excluding identities.
2) $\partial f_h/\partial y_h'$ is the $m \times m$ Jacobian matrix of first derivatives of the structural form functions with respect to endogenous variables, *after all identities have been substituted* into the stochastic equations. If we partition the $M \times M$ Jacobian corresponding to all equations

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \qquad (6)$$

where the *1,1* block corresponds to stochastic equations, then the Jacobian determinant that must be used in (5) is the absolute value of $|J|/|J_{22}|$, that is the ratio between the determinant of the $M \times M$ Jacobian matrix of the complete system, and the determinant of the $(M-m)\times(M-m)$ submatrix corresponding to identities.

Conditional on the parameters of the structural form (coefficients and covariance matrix of the random error process, which are set at their estimated values $\hat{a}$ and $\hat{\Sigma}$), the estimate of the most likely joint value of the endogenous variables in the forecast period h is obtained from the vector u_h which maximizes the sum of the second and third term of (5) (the first term is constant).

The maximization of the log-likelihood function has been performed using the well known updating formula due to Broyden, Fletcher, Goldfarb and Shanno (BFGS, see for example Dennis and More', 1977). The algorithm is based on an iterative updating of an initial $(m \times m)$ positive definite matrix. Since the computational efficiency is greatly improved if the initial matrix approximates the Hessian matrix of the function to be maximized $\partial^2 L_h/\partial u_h \partial u_h'$, it was rather obvious in this case to use the available estimate of Σ^{-1}.

The algorithm also requires the evaluation at each step of the gradient of the function. This revealed to be a rather serious problem and at this first stage it has been solved by the numerical computation of first derivatives. This approach however has two main drawbacks. First of all it requires a long computation time, but nevertheless it came out to be computationally more efficient than rival maximizing algorithms which require only the computation of the function value. To quantify this aspect we can mention that in the case of the French model here considered (see below for description) the BFGS algorithm reached the optimum, with a nine significant digits on the value of the log-likelihood, in six iterations and the computation globally took about five minutes on an IBM 3083 computer.

A second main drawback lies in a possible lack of accuracy in the computation of the derivatives. In order to assess the robustness of the results versus the way in which derivatives are computed several formulas (i.e. two, three and five points formulas with different sizes of the increment) were experimented with. The results proved to be very robust versus both the choice of the formula and the choice of the numerical increment, provided that a centered formula is used.

4. A BRIEF NOTE ON MINI-DMS MODEL FOR THE FRENCH ECONOMY

The Mini-DMS model (Brillet, 1981) constitutes a smaller version of the Dynamic Multi Sectorial model of the French economy (Fouquet et al., 1978) built in 1974-1976 at INSEE (National Institute for Statistics and Economic Studies) to be used as a medium term forecasting tool, in particular for national planning studies conducted through the Commissariat General au Plan (General Planning Agency). Largely reduced in size (the present version contains 225 equations, 71 of which are behavioral, as compared to more than 2400 for the larger version), Mini-DMS nevertheless preserves the same economic structure as well as most of the theoretical mechanism of the original model.

In its present state, the Mini-DMS model can be considered as being half way between a forecasting tool and a model for economic policy decisions. Its acceptable forecasting qualities, as well as its rather detailed set of decisional variables, can lead to its use for simple enough macro-economic studies, and for carrying out mathematical economic

experiments.

Estimates of the structural coefficients of the model, on the sample period 1962-1980, were obtained by means of a straightforward extension of Brundy and Jorgenson's (1971) instrumental variables method (limited information) to the case of nonlinear models. The method was applied iteratively, till convergence was reached, so that the final estimates of coefficients are not affected by the choice of the values of the initial coefficients values. In each iteration, the instrumental variables are computed as the deterministic solution values of the system (which is the simplest choice, although not the *best* in the class of nonlinear estimators as is explained in Amemiya, 1983).

5. RESULTS ON MINI-DMS MODEL FOR THE FRENCH ECONOMY

As already mentioned, the model has been estimated by means of instrumental variables (limited information) on the sample period 1962-1980. Static forecasts are related to the first year outside the sample estimation period.

Table 2 presents, for some of the main endogenous variables of the model, the difference between the deterministic and the conditional mean predictors. This difference is displayed in percentage form

$$(7) \qquad \frac{y_{det} - y_{mean}}{y_{det}} \cdot 100$$

where y_{mean} is computed with parametric stochastic simulation and with the residual-based methods (with and without antithetics).

The last column of the table displays, still in percentage form, the difference between the deterministic predictor and the estimate of the most likely joint value (y_{mode})

$$(8) \qquad \frac{y_{det} - y_{mode}}{y_{det}} \cdot 100$$

Table 1

Variables in real terms

Q1 = Added value of the industrial product in millions of 1970 francs.
QQ2 = Added value of the non-industrial product in millions of 1970 francs.
PIBZ = GDP in millions of 1970 francs.
DF1 = Final demand of the industrial product (demand in France not including internal consumption) in millions of 1970 francs.
DF2 = Final demand of the non-industrial product (demand in France not including internal consumption) in millions of 1970 francs.
C1 = Household consumption of industrial product in millions of 1970 francs.
C2 = Household consumption of non-industrial product in millions of 1970 francs.
C = Total household consumption in millions of 1970 francs.
IL2M = Household lodgings investment in millions of 1970 francs.
I1 = Productive investment of firms in industrial product in millions of 1970 francs.
I2 = Productive investment of firms in non-industrial product in millions of 1970 francs.
I = Total productive investment of firms in millions of 1970 francs.
M1 = Imports of industrial product in millions of 1970 francs.
M2 = Imports of non-industrial product in millions of 1970 francs.
M = Total imports in millions of 1970 francs.
X1 = Exports of industrial product in millions of 1970 francs.
UT = Rate of use of productive capacities in the industrial sector.

Employment

N1 = Employment in the industrial sector in thousands.
N2 = Employment in the non-industrial sector in thousands.
N = Total employment in thousands.
PDRE = Unemployment in thousands (Definition of the *Bureau International du Travail*).
OEFM = Offers for jobs at the end of the month presented at the ANPE (National Agency for Labor) in thousands.

Prices and wages

W1 = Wage rate per hour in the industrial sector (the dimension is not important).
W2 = Wage rate per hour in the non-industrial sector (the dimension is not important).
SALT = Wage rate per hour for all wage earners.
RDM = Household revenue (all household resources minus income tax) in millions of current francs.
PVA1 = Price index of added value in the industrial sector (=1 in 1970).
PVA2 = Price index of added value in the non-industrial sector (=1 in 1970).
PC = Price index of household consumption (=1 in 1970).
PU1 = Price index of the industrial product used in France (not including VAT); =1 in 1970. This is the ratio between current and real values of the sum = production + imports - exports.
PU2 = Price index of the non-industrial product used in France (not including VAT); =1 in 1970. This is the ratio between current and real values of the sum = production + imports - exports.
PPIB = GDP deflator (=1 in 1970).

Ratios and balances

TEP = Savings ratio of Households.
TPRO1 = Profits ratio of the industrial sector.
TPRO2 = Profits ratio of the non-industrial sector.
AUT1 = Autofinancing of the industrial sector in million of current francs (numerator of the profits ratio + gains on the value of stocks).
AUT2 = Autofinancing of the industrial sector in millions of current francs (numerator of the profits ratio + gains on the value of stocks).
BF1 = Balance of industrial firms in millions of current francs (+ means losses).
BF2 = Balance of non-industrial firms in millions of current francs(+ means losses).
CFF = Balance of Credit Institutions and Insurance Companies in millions of current francs.
CFM = Balance of Households in millions of current francs.

CFX = External balance in millions of current francs.
CFG = Government Balance in millions of current francs.
PIB = Current GDP in millions of current francs.
CM = Household consumption in millions of current francs.

In the four columns of table 2, the following values are displayed.

Pss = Percentage difference between the deterministic solution forecast and the estimate of the conditional mean, computed with parametric stochastic simulation (40000 couples of replications with antithetic variates).
Rb = Percentage difference between the deterministic solution forecast and the estimate of the conditional mean, computed with the residual-based procedure.
Rba = Percentage difference between the deterministic solution forecast and the estimate of the conditional mean, computed with the antithetic residual-based procedure.
Mode = Percentage difference between the deterministic solution forecast and the estimate of the most likely joint value.

Table 2

One-period forecast at 1981. Static simulation

Percentage deviations from deterministic forecasts

	Determin. Forecast	*Pss*	*Rb*	*Rba*	*Mode*
Q1	258326.0	0.100	0.286	0.114	0.056
QQ2	536543.0	0.064	0.073	0.059	-0.008
UT	.8080540	0.087	0.273	0.101	0.043
SALT	66.72890	0.061	-0.580	-0.063	0.106
PU1	2.514670	-0.464	-1.520	-0.713	0.011
PU2	3.079440	-0.054	-0.665	-0.131	-0.009
RDM	2296130.	0.047	-0.705	-0.075	0.014
CM	1981490.	0.039	-0.580	-0.086	-0.028
C1	216234.0	0.309	0.573	0.345	0.142
C2	494296.0	0.100	0.187	0.112	0.007
N1	4581.480	0.057	0.104	0.065	0.024
N2	13025.80	0.035	-0.090	0.034	-0.040
PDRE	1636.860	-0.158	-0.155	-0.173	0.247
OEFM	58.04020	-1.110	4.140	-2.004	12.603
W1	1.900640	0.036	-0.635	-0.089	0.074
W2	1.534870	0.073	-0.567	-0.050	0.118
PVA1	2.468430	0.054	-0.631	-0.052	-0.032
PVA2	2.839010	0.032	-0.784	-0.034	-0.077
PC	2.758950	-0.180	-0.938	-0.301	-0.075
TEP	.1125830	0.490	-0.647	0.281	0.331
IL2M	59275.80	0.039	-0.513	0.038	0.061
BF1	60955.80	-1.196	-11.177	-5.562	0.003
BF2	95328.70	2.853	17.230	8.570	1.124
AUT1	99158.80	1.093	5.278	3.888	0.223
AUT2	220742.0	-2.460	-15.167	-6.697	-0.849
TPRO1	.0298494	3.763	18.860	9.928	0.977
TPRO2	.0487430	-2.163	-11.816	-5.714	-1.178
DF1	336288.0	0.113	0.361	0.134	0.079
DF2	576187.0	0.057	0.032	0.048	-0.020
PIB	3148560.	-0.079	-0.941	-0.224	-0.049
CFG	-83026.60	-1.391	-4.776	-2.018	-0.757
CFM	142544.0	0.725	-1.923	0.108	0.665
CFX	-34661.10	3.629	8.762	2.062	0.665
PIBZ	1132290.	0.062	0.117	0.060	-0.001
PPIB	2.780710	-0.154	-1.087	-0.303	-0.047

CFF	62106.10	-2.343	8.615	3.599	-1.179
I1	35797.10	0.510	2.159	1.091	0.411
I2	83001.00	-0.512	-2.839	-1.369	-0.284
M1	180087.0	0.079	-0.021	0.026	0.011
M2	73167.80	0.109	0.126	0.110	0.090
X1	199824.0	-0.047	0.522	0.060	0.019
C	710529.0	0.164	0.305	0.183	0.048
I	118798.0	-0.204	-1.333	-0.627	-0.075
M	253255.0	0.088	0.021	0.034	0.034
N	17607.30	0.041	-0.039	0.042	-0.023

For the variables which represent ratios and balances it would be more appropriate to give the differences in absolute value and sign, rather than in percentage form, given the nature of the variables themselves. In table 2 they are nevertheless given in percentage form for homogeneity with the other variables, but are in table 3 given in absolute value and sign.

Table 3

One-period forecast at 1981. Static simulation

Deviation from deterministic forecasts

	Determin. Forecast	*Pss*	*Rb*	*Rba*	*Mode*
TEP	.1125830	0.00055	-0.00073	0.00032	0.00037
BF1	60955.80	-729.3180	-6813.20	-3390.37	1.94
BF2	95328.70	2720.02	16424.70	8169.56	1071.46
AUT1	99158.80	1083.59	5234.08	3855.27	221.31
AUT2	220742.0	-5430.88	-33479.50	-14782.30	-1874.52
TPRO1	.0298494	0.00112	0.00563	0.00296	0.00029
TPRO2	.0487430	-0.00105	-0.00576	-0.00279	-0.00057
CFG	-83026.60	1154.85	3964.94	1675.23	628.21
CFM	142544.0	1033.56	-2741.08	153.95	947.30
CFX	-34661.10	-1257.74	-3036.96	-714.84	-230.33
CFF	62106.10	-1455.44	5350.69	2235.16	-732.44

Let us first look at the two groups of statistics independently from each other.

5.1. Deterministic predictor versus conditional mean

As already observed, for the conditional mean predictor we refer to results obtained with parametric stochastic simulation (second column). The difference with respect to the mean is hereunder indicated as *bias*.

The first thing we can observe is the size of the bias, which seems rather small for almost all variables, although we must consider that we are dealing only with one-year-ahead forecasts; for instance a value of .06 for total domestic product (PIBZ), of -.15 on domestic product deflator (PPIB) are small compared to the actual growth rate of these variables; the case is somewhat worse for household consumption of industrial product (C1: 0.3), for the price of the same product (PU1: -0.4) or investment per sector (I1 and I2: 0.5 and -0.5); in these cases the error is not negligible compared with the actual rate of growth.

It is also interesting to remark that the size of these biases is closely related to the forecasting uncertainty of the associated variable, which is of course natural since the two statistics are dependent on the variance of the variables itself.

On the whole it would seem that the size of the bias is sometimes large enough to reduce the trust we should put in the forecasts produced by the model using the deterministic solution, if the conditional mean is supposed to be the most appealing result.

Now let us consider the coherence of the biases between variables, trying in particular to determine whether, for behavioral equations, this bias comes from the random error term in the associated equation or from the influence of explanatory variables. We can see that the second reason is predominant: most of the quantities and employment show a positive bias; an exception is given by exports (of course influenced negatively by activity) and by investment in the non-industrial sector (but the most important determinant, profits ratio, shows a negative bias too). This evolution of the profits ratio looks due to the positive difference in biases between the wage rate (W2) and the price (PVA2), which is not true for the industrial sector. Indeed, the only major incoherence lies in the price system: while the behavioral variable (price in added value) shows a positive bias, all the other variables show a negative one; this could be linked (but not explained) to the fact that the bias on export price is much more negative than the one on import price.

As to the evolution of the different balances, it is not surprising to find a negative bias on trade balance, a positive one for Government, households, industrial sector, and a negative one for the non-industrial sector.

Finally, we can try to measure, for some variables, the power of the argument pointing out the fact that the conditional mean does not respect the nonlinear identities; for instance if we look at domestic product, we see that the bias on current domestic product (PIB): -.079% is not compatible with a percentage bias of 0.62% on real domestic product (PIBZ) and of -.154% on its deflator (PPIB).

5.2. Deterministic predictor versus multivariate mode

Let us now consider the difference between the deterministic solution and the mode of the joint distribution of the endogenous variables.

The first remark will be that in most cases the size of the value looks rather small, again compared to the actual rate of growth of the associated variables. Again it looks correlated to the variance of the variable itself. As to economic coherence inside the sample, it is again globally verified; but it is only in the industrial sector, this time, that quantities show a positive difference (deterministic solution is greater than the mode); in the other sector, the large difference on imports (considering that it comes from the equation itself, as demand shows an opposite sign) draws final demand, production and employment down (even global employment, considering the relative importance of non-industrial employment).

As to prices, they show this time a negative difference for almost all variables, except for wages, where the growth of purchasing power is due to the rise of job offers, coming mainly from the industrial sector.

The consequent evolution of the trends on the different balances is the same as in the previous case, for the same reasons.

5.3. Mean versus most likely joint value

Now let us come to the most interesting element: the comparison between the two statistics. It has been shown (see (Kendall and Stuart, 1969, section 2.11) that in most univariate cases the difference between mean and mode is about three times the difference between mean and median (which can be assimilated to the deterministic value). This is of course equivalent to saying that the second statistic (median - mode) should be minus two times the first one (median - mean). This is obviously not true in our case.

- First the sign is generally the same: out of the 45 main variables presented in table 2, only 12 show an opposite sign; indeed we have already shown that the statistics showed globally the same sign whether we considered real values or prices; the main differences come from employment in the non-industrial sector, from global activity in that sector, and from Value Added prices, the negative value of which can be associated with the decrease in production costs coming from the decrease in employment (higher than the one on production).
- Second, the absolute size of the second statistic is generally lower (in 36 cases out of 45); out of the nine exceptions, at least six have a single cause: the high relative difference in the second case on OEFM (offers for jobs) which enters by its logarithm in the determination of the wage rate; this explains by itself the high value for wages, and investment in lodgings. Otherwise, the lower (sometimes much lower) value of the second statistic allows to smooth the message given at the beginning: if we consider the mode as the most likely value for the model forecast value, the use of the deterministic solution instead does not look so dangerous. In other terms, in most cases we shall find the mean and deterministic values to be on opposite sides relative to the mode. Using one or the other introduces then an error of different sign, but of a size which is globally of the same order.

5.4. Some univariate modes

For five variables of the model we have collected the results of a parametric stochastic simulation experiment with 200000 replications. The histograms allow to locate, with some rough approximation, the modes of the marginal distributions of the five variables.

OEFM: the deterministic solution exceeds the mode by some 8%; the difference is smaller than for the mode of the joint distribution, but has the same sign. The mean (computed with parametric stochastic simulation, see Table 2) and the univariate mode are on the opposite sides of the deterministic solution.

TPRO1: this time the mode exceeds the deterministic solution (the difference is between 2% and 8%); the sign has changed, with respect to the multivariate case. Again mean and univariate mode are on opposite sides with respect to the deterministic solution.

TPRO2: the mode is not distinguishable from the deterministic solution.

CFG: again the mode and the deterministic solution are not distinguishable.

CFX: the histogram is flat in a wide region around the deterministic solution.

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