Cultural preference on fertility and the long-run growth effects of intellectual property rights

Chu, Angus C. and Cozzi, Guido

Shanghai University of Finance and Economics, Durham University

February 2011

Cultural Preference on Fertility and the Long-Run Growth Effects of Intellectual Property Rights

Angus C. Chu, Shanghai University of Finance and Economics
Guido Cozzi, Durham Business School, Durham University

February 2011

Abstract

How does patent policy affect long-run economic growth through the population growth rate? To analyze this question, we develop an R&D-based growth model with endogenous fertility. In recent vintages of R&D-based growth models in which scale effects are absent, the long-run growth rate depends on the population growth rate that is *assumed* to be exogenous. In this study, we develop a semi-endogenous-growth version of the quality-ladder model with endogenous fertility and human-capital accumulation to analyze an unexplored interaction between intellectual property rights, endogenous fertility and economic growth. We find that strengthening patent protection has a surprisingly negative effect on technological progress in the long run through endogenous fertility. Furthermore, a stronger cultural preference on fertility tends to magnify this negative effect of patent policy on long-run growth.

JEL classification: O31, O34, O40
Keywords: economic growth, endogenous fertility, patent policy

Chu: anguscccc@gmail.com. School of Economics, Shanghai University of Finance and Economics, China. Cozzi: guido.cozzi@durham.ac.uk. Durham Business School, Durham University, UK. The authors would like to thank Silvia Galli for her helpful comments and suggestions. The usual disclaimer applies.

1 Introduction

How does patent policy affect long-run economic growth through the population growth rate? To analyze this question, we develop a scale-invariant R&D-based growth model with endogenous fertility. In the literature on R&D-driven economic growth, there has been a very important debate about the presence of counterfactual scale effects (i.e., a positive relationship between population size and long-run growth) in the first-generation models, such as Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992). In response to this critique, subsequent generations of R&D-based growth models have been developed to remove these scale effects.[1] In these scale-invariant models, the long-run growth rate is either solely or partly determined by the population growth rate that is *assumed* to be exogenous. However, in a more realistic framework, the fertility rate should be treated as an endogenous variable chosen by optimizing households. In this study, we develop two versions of the scale-invariant quality-ladder model with endogenous fertility and apply this growth-theoretic framework to analyze the effects of intellectual property rights on fertility and long-run economic growth. To our knowledge, this interaction between patent policy, endogenous fertility and economic growth has never been explored in the literature. Furthermore, in recent vintages of R&D-based growth models, the long-run growth rate is usually increasing in the population growth rate (i.e., a *weak* scale effect); however, even this weak scale effect is not supported empirically.[2] Therefore, we follow Strulik (2005) to allow for human-capital accumulation in order to further remove this weak scale effect.

In the model, optimizing households choose the fertility rate by trading off the marginal utility of higher fertility against its marginal costs arising from (a) foregone wages, (b) the dilution of financial assets per capita, and (c) the dilution of human capital per capita. In the semi-endogenous-growth version of the quality-ladder model,[3] we find that strengthening patent protection that increases the market power of firms weakens the *foregone-wage* effect and the *human-capital-diluting* effect but strengthens the *asset-diluting* effect of fertility. On the one hand, weakening the foregone-wage effect and the

[1] See Jones (1999) for an excellent review on these subsequent generations of R&D-based growth models.

[2] See for example Strulik (2005) for a discussion.

[3] Early studies on the R&D-based semi-endogenous growth model include Jones (1995), Kortum (1997) and Segerstrom (1998).

human-capital-diluting effect leads to a higher fertility rate. On the other hand, strengthening the asset-diluting effect leads to a lower fertility rate. We find that the effects of patent policy on the dilution of financial assets and the dilution of human capital cancel each other. As a result, strengthening patent protection unambiguously increases fertility through weakening the foregone-wage effect, and this higher rate of fertility in turn reduces the growth rate of human capital and technological progress in the long run.

Furthermore, we calibrate the model to aggregate data of the US economy to provide a quantitative analysis. We find that the magnitude of the negative effect of patent policy on long-run economic growth crucially depends on a preference parameter on fertility and is increasing in its parameter value. Given a reasonable parameter value for the US economy, the long-run growth effect of strengthening patent protection under endogenous fertility is similar to under exogenous fertility. Therefore, applying an R&D-based growth model with exogenous fertility to analyze intellectual property protection in the US may serve as a useful approximation to a more realistic model with endogenous fertility. However, for a culture in which there is a strong preference on fertility, the negative growth effect of patent policy through endogenous fertility could be quantitatively significant. For example, Fernandez and Fogli (2009) provide empirical evidence to show that preference on fertility varies across culture and has a significant effect on fertility outcomes.[4]

In addition to the semi-endogenous-growth version of the quality-ladder model, we also consider an alternative specification that removes scale effects through diluting R&D inputs by the scale of the economy following Laincz and Peretto (2006). Under this specification, the equilibrium growth rate depends on both the population growth rate and the share of labor allocated to R&D resembling a second-generation model.[5] In this case, strengthening patent protection has a positive effect on the R&D share of labor, which increases the growth rate. Interestingly, it has no effect on the population growth rate. Therefore, the different equilibrium effects of patent policy on long-run growth may serve as an indirect test for the two alternative solutions to the scale-effect problem.[6]

[4] Fernandez and Fogli (2009) use the past total fertility rate in the country of ancestry as a proxy for cultural preference on fertility and find that second-generation Americans whose ancestry is from countries with higher fertility rates tend to have more children.

[5] Early studies on the second-generation R&D-based *endogenous* growth model include Young (1998), Dinopoulos and Thompson (1998), and Peretto (1998).

[6] For example, Laincz and Peretto (2006) and Ha and Howitt (2007) provide some inter-

Our study relates to the literature on endogenous fertility and R&D-driven growth for which Growiec (2006) provides an excellent review.[7] Jones (2001) develops an R&D-based growth model with endogenous fertility to analyze the emergence of rapid growth and demographic transitions. To simplify their analysis, both Jones (2001) and Growiec (2006) consider a model in which the allocation of inputs between R&D and production is exogenously determined. The present study differs from Jones (2001) and Growiec (2006) by developing a quality-ladder model in which both fertility and the allocation of factor inputs are endogenously determined through the market equilibrium. Therefore, our model follows more closely the footsteps of Connolly and Peretto (2003), who develop an R&D-based growth model with vertical and horizontal innovations to analyze demographic shocks and industrial policies that affect the costs of R&D and/or entry. However, our model differs from Connolly and Peretto (2003) by featuring human-capital accumulation as well as creative destruction that gives rise to the importance of patent breadth that protects an innovation against previous innovations. Therefore, the present study complements their interesting analysis by analyzing another important set of industrial policy that is the effects of intellectual property rights on fertility and economic growth.

Our study also relates to the literature on patent policy and economic growth. The seminal study in the literature on optimal patent design is Nordhaus (1969).[8] While studies in this patent-design literature mostly analyze patent policy in partial-equilibrium models, the present study follows more closely a related macroeconomic literature by analyzing the effects of patent policy in a quantitative dynamic general-equilibrium model. The seminal dynamic general-equilibrium analysis on optimal patent length is Judd (1985), who finds that the optimal patent length can be infinite. Subsequent studies by Iwaisako and Futagami (2003) and Futagami and Iwaisako (2007) show that the optimal patent length is usually finite in the Romer model due to an additional distortionary effect on intermediate goods that is absent in Judd (1985).[9] While this branch of studies focuses on characterizing the optimal patent length, another branch of studies in the literature analyzes the

esting empirical investigations on the two branches of scale-invariant R&D-based growth models.

[7] See also Barro and Becker (1989) for a seminal study on endogenous fertility in an overlapping-generation model with exogenous growth.

[8] See Scotchmer (2004) for a comprehensive review of this patent-design literature.

[9] See also Horowitz and Lai (1996).

effects of other patent-policy levers on innovation and growth. See, for example, Li (2001) on patent breadth,[10] O'Donoghue and Zweimuller (2004) on forward patent protection and patentability requirement, Cozzi (2001) and Cozzi and Spinesi (2006) on intellectual appropriability, Furukawa (2007, 2010) and Horri and Iwaisako (2007) on patent protection against imitation, and Chu (2009) on blocking patents. Some of these studies find that strengthening patent protection may generate a negative effect on innovation and technological progress, and this finding is consistent with the detailed case studies analyzed in Jaffe and Lerner (2004), Bessen and Meurer (2008) and Boldrin and Levine (2008). The present study contributes to this literature by analyzing a novel channel (i.e., endogenous fertility) through which patent policy may generate a negative effect on long-run economic growth while having a positive effect on R&D.

Finally, this study relates to a growing literature on culture and economic growth. A recent empirical study by Tabellini (2010) provides evidence that cultural traits, such as trust, respect for others, confidence in individual self-determination, and emphasis on children's obedience, have significant causal effects on regional per capita income in Europe. Another interesting empirical study by Alesina and Giuliano (2010) analyzes the effects of family ties on economic outcomes, such as home production and labor force participation of women and youth. In terms of theoretical work, a seminal study by Galor and Moav (2002) shows that individual preferences on offspring quality affect the speed of transition to sustained economic growth. A subsequent study by Ashraf and Galor (2007) analyzes the relative advantage of two interesting cultural characteristics, namely, cultural assimilation and cultural diversity, at different stages of economic development. Another recent study by Chu (2007) argues that cultural variation in entrepreneurial overconfidence can play a role in causing different rates of economic growth across countries. The present study relates to this literature by showing that cultural preference on fertility not only has a direct effect on long-run economic growth but it may also have an indirect effect on growth through intellectual property rights.

The rest of this study is organized as follows. Section 2 describes the semi-endogenous-growth version of the model. Section 3 defines the equilibrium and derives the balanced-growth path. Section 4 analyzes the effects of patent policy on fertility and economic growth. Section 5 considers an extension of

[10] See also Chu (2011) for a quantitative analysis on uniform versus sector-specific optimal patent breadth in a two-sector quality-ladder growth model.

the model with fully endogenous growth. The final section concludes.

2 A quality-ladder model with endogenous fertility and human-capital accumulation

In this section, we consider the semi-endogenous-growth formulation of the Grossman-Helpman (1991) quality-ladder model based on Segerstrom (1998). The key changes in our model are as follows. First, we consider endogenous fertility instead of exogenous fertility following the setup in Razin and Ben-Zion (1975) and Yip and Zhang (1997). Second, we allow for variable patent breadth as in Li (2001) in order to analyze the effects of patent policy. Finally, we introduce human-capital accumulation as in Strulik (2005) to further remove the weak scale effect. Given that the quality-ladder model has been well-studied, we will describe the familiar features briefly to conserve space and discuss the new features in details.

2.1 Households

There is a unit continuum of identical households. As is standard in the literature on endogenous fertility, households derive utility from fertility. Here we consider a continuous-time setup similar to Yip and Zhang (1997). However, considering a discrete-time setup with overlapping generations of households as in Razin and Ben-Zion (1975) would not change our results. The intergenerational utility of households is the discounted sum of per capita utility across time.[11] Specifically, the utility function of a household is given by

$$U = \int_0^\infty e^{-\rho t} u(c_t, n_t) dt, \tag{1}$$

where $u(c_t, n_t) = \ln c_t + \alpha \ln n_t$. c_t is consumption per capita at time t, and n_t is the fertility rate. α is a fertility-preference parameter, and $\rho > 0$ is the discount rate. In this simple model with zero mortality, n_t is also the population growth rate.

[11] See Growiec (2006) for an interesting discussion on alternative ways of modelling endogenous fertility in the growth literature.

Each household maximizes (1) subject to the following asset-accumulation equation.

$$\dot{a}_t = (r_t - n_t)a_t + w_t l_t - c_t. \tag{2}$$

a_t is the amount of financial assets per capita, and r_t is the rate of return on assets. An increase in n_t reduces the amount of assets per capita, and we refer to this effect as the *asset-diluting* effect of fertility. w_t is the wage rate, and l_t is human-capital embodied labor supply. Each person has one unit of time to allocate between fertility, work and education. The time spent on fertility is given by $n_t/\theta < 1$, where θ is a parameter that is negatively related to the time cost of fertility. At time t, the stock of human capital per capita is h_t. Each person combines her remaining time endowment $1 - n_t/\theta$ with her human capital h_t for work l_t and education e_t subject to

$$h_t(1 - n_t/\theta) = l_t + e_t. \tag{3}$$

Increasing n_t reduces the amount of time available for work and education, and this setup captures the *foregone-wage* effect of fertility. The law of motion for human capital per capita is

$$\dot{h}_t = \xi e_t - (\delta + n_t)h_t, \tag{4}$$

where ξ is a productivity parameter for human-capital accumulation. We impose $\xi > \rho$ in order for e_t to be non-negative (to be shown later). $n_t h_t$ captures the *human-capital-diluting* effect of fertility as in Strulik (2005). The parameter δ is the depreciation rate of human capital. Given that this parameter does not affect our results, we set δ to zero for simplicity. Finally, the law of motion for the population size is $\dot{N}_t = n_t N_t$.

From standard dynamic optimization, the Euler equation is

$$\frac{\dot{c}_t}{c_t} = r_t - n_t - \rho, \tag{5}$$

and the consumption-fertility optimality condition is

$$\frac{\alpha}{n_t} = \frac{1}{c_t}\left[a_t + \left(\frac{1}{\theta} + \frac{1}{\xi}\right) w_t h_t\right]. \tag{6}$$

This condition equates the marginal utility of fertility given by α/n_t to the marginal utility of consumption (in response to a change in fertility) given by $[a_t + w_t h_t (1/\theta + 1/\xi)] / c_t$. The first term a_t/c_t captures the asset-diluting effect of fertility, and this effect is positively related to the value of assets per capita. The second term $\theta^{-1} w_t h_t / c_t$ captures the foregone-wage effect of fertility, and the third term $\xi^{-1} w_t h_t / c_t$ captures the human-capital-diluting effect of fertility. Both of these effects are positively related to the wage rate. From dynamic optimization, we can also derive an equilibrium condition for the growth rate of w_t given by

$$\frac{\dot{w}_t}{w_t} = r_t - \xi(1 - n_t/\theta). \tag{7}$$

We will show that this condition determines the equilibrium growth rate of human capital.

2.2 Final goods

Final goods are produced by competitive firms that aggregate intermediate goods using a standard Cobb-Douglas aggregator given by

$$Y_t = \exp\left(\int_0^1 \ln X_t(i) di\right). \tag{8}$$

$X_t(i)$ denotes intermediate goods $i \in [0, 1]$. From profit maximization, the conditional demand function for $X_t(i)$ is

$$X_t(i) = Y_t / p_t(i), \tag{9}$$

where $p_t(i)$ is the price of $X_t(i)$.

2.3 Intermediate goods

There is a unit continuum of industries producing differentiated intermediate goods. Each industry is temporarily dominated by an industry leader until the arrival of the next innovation, and the owner of the new innovation becomes the next industry leader.[12] The production function for the leader in industry i is

$$X_t(i) = z^{q_t(i)} L_{x,t}(i). \tag{10}$$

[12] This is known as the Arrow replacement effect in the literature. See Cozzi (2007) for a discussion on the Arrow effect.

The parameter $z > 1$ is the step size of productivity improvement, and $q_t(i)$ is the number of productivity improvements that have occurred in industry i as of time t. $L_{x,t}(i)$ is production labor in industry i. Given $z^{q_t(i)}$, the marginal cost of production for the industry leader in industry i is $mc_t(i) = w_t/z^{q_t(i)}$. It is useful to note that we here adopt a cost-reducing view of vertical innovation as in Peretto (1998, 1999).

Standard Bertrand price competition leads to a profit-maximizing price given by

$$p_t(i) = \mu(z, b)mc_t(i), \tag{11}$$

where $\mu = z^b > 1$ and $b \in (0, 1)$ denotes patent breadth. In the original Grossman-Helpman (1991) model, the patentholder is assumed to have complete protection against imitation such that $b = 1$. Li (2001) considers a more general policy environment with incomplete patent protection against imitation such that $b \in (0, 1)$. Here we follow the formulation in Li (2001). From (9), the amount of monopolistic profit is

$$\pi_t(i) = \left(\frac{\mu - 1}{\mu}\right) p_t(i)X_t(i) = \left(\frac{\mu - 1}{\mu}\right) Y_t. \tag{12}$$

Therefore, a larger patent breadth b increases the markup μ and the amount of monopolistic profit improving the incentives for R&D. For the rest of this study, we use μ to measure the strength of patent protection. Finally, production-labor income is

$$w_t L_{x,t}(i) = \frac{p_t(i)X_t(i)}{\mu} = \frac{Y_t}{\mu}. \tag{13}$$

Equations (12) and (13) show that strengthening patent protection increases the share of profit income (i.e., π_t/Y_t) and decreases the share of wage income (i.e., $w_t L_{x,t}/Y_t$). Through these effects, patent policy affects the equilibrium rate of fertility.

2.4 R&D

Denote $v_t(i)$ as the share value of the monopolistic firm in industry i. Because $\pi_t(i) = \pi_t$ for $i \in [0, 1]$ from (12), $v_t(i) = v_t$ in a symmetric equilibrium that features an equal arrival rate of innovation across industries.[13] In this case, the familiar no-arbitrage condition for v_t is

$$r_t v_t = \pi_t + \dot{v}_t - \lambda_t v_t. \tag{14}$$

This condition equates the interest rate to the asset return per unit of asset. The asset return is the sum of (a) monopolistic profit π_t, (b) potential capital gain $\dot{v}_t$ and (c) expected capital loss $\lambda_t v_t$ from creative destruction for which λ_t is the arrival rate of the next innovation.

There is a unit continuum of R&D firms indexed by $j \in [0,1]$. They hire R&D labor $L_{r,t}(j)$ for innovation. The zero-expected-profit condition of firm j is

$$v_t \lambda_t(j) = w_t L_{r,t}(j), \tag{15}$$

where the firm-level arrival rate of innovation is

$$\lambda_t(j) = \varphi L_{r,t}(j)/Z_t, \tag{16}$$

where Z_t denotes aggregate technology. Here we follow Segerstrom (1998) to remove the strong scale effects (i.e., a counterfactual positive relationship between population *size* and long-run economic growth) by assuming that R&D productivity φ/Z_t falls as technology accumulates. Because $L_{r,t} = \int_0^1 L_{r,t}(j)dj$, the aggregate arrival rate of innovation is $\lambda_t = \varphi L_{r,t}/Z_t$.

3 Decentralized equilibrium

The equilibrium is a time path of allocations $\{c_t, n_t, h_t, l_t, N_t, Y_t, X_t(i), L_{x,t}(i), L_{r,t}(j)\}$ and a time path of prices $\{p_t(i), w_t, r_t, v_t\}$. Also, at each instance of time,

- households maximize utility taking $\{r_t, w_t\}$ as given;
- competitive final-goods firms produce $\{Y_t\}$ to maximize profit taking $\{p_t(i)\}$ as given;

[13] We follow the standard approach in the literature to focus on the symmetric equilibrium. See Cozzi *et al.* (2007) for a theoretical justification for the symmetric equilibrium to be the unique rational-expectation equilibrium in the quality-ladder growth model.

- monopolistic intermediate-goods firms produce $\{X_t(i)\}$ and choose $\{L_{x,t}(i), p_t(i)\}$ to maximize profit taking $\{w_t\}$ as given;

- R&D firms choose $\{L_{r,t}(j)\}$ to maximize expected profit taking $\{w_t, v_t\}$ as given;

- the market-clearing condition for human-capital embodied labor supply holds such that $l_t N_t = L_{x,t} + L_{r,t}$;

- the market-clearing condition for final goods holds such that $Y_t = c_t N_t$; and

- the value of household assets adds up to the share value of monopolistic firms such that $v_t = a_t N_t$.

The aggregate production function is given by

$$Y_t = Z_t L_{x,t}, \tag{17}$$

where aggregate technology Z_t is defined as

$$Z_t = \exp\left(\int_0^1 q_t(i) di \ln z\right) = \exp\left(\int_0^t \lambda_\tau d\tau \ln z\right). \tag{18}$$

The second equality of (18) applies the law of large numbers. Differentiating the log of (18) with respect to t yields the growth rate of aggregate technology given by

$$g_{z,t} \equiv \frac{\dot{Z}_t}{Z_t} = \lambda_t \ln z = (\varphi \ln z)\frac{L_{r,t}}{Z_t}. \tag{19}$$

3.1 Balanced growth path

In this section, we focus on the balanced growth path, along which each variable grows at a constant (possibly zero) rate. On the balanced growth path, the arrival rate of innovation is constant so that $L_{r,t}$ and Z_t must grow at the same rate implying that

$$g_z = g_h + n, \tag{20}$$

where g_h is the steady-state growth rate of human capital per capita. In a standard semi-endogenous growth model, g_h equals zero and n is exogenous. However, in our model with endogenous fertility, the long-run growth rate of technology becomes endogenous. Furthermore, with endogenous human-capital accumulation, g_h is decreasing in n, and hence, g_z could also be decreasing in n.

Combining (13) and (17) yields $w_t = Z_t/\mu$, which implies

$$\frac{\dot{Z}_t}{Z_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{c}_t}{c_t} + n_t + \rho - \xi(1 - n_t/\theta), \tag{21}$$

where the second equality of (21) is derived by substituting (5) into (7). The steady-state growth rate of consumption per capita is

$$g_c = g_y - n = g_z + g_h, \tag{22}$$

where g_y is the steady-state growth rate of Y_t. Substituting (22) into (21) yields

$$g_h = \xi(1 - n/\theta) - n - \rho. \tag{23}$$

Therefore, the growth rate of human capital per capita is decreasing in n. The first negative effect (i.e., $-\xi n/\theta$) arises from the crowding out of fertility on the time endowment. The second negative effect (i.e., $-n$) is the human-capital-diluting effect. Substituting (23) into (20) yields

$$g_z = \xi(1 - n/\theta) - \rho. \tag{24}$$

Therefore, the growth rate of aggregate technology is also decreasing in n because the negative effects of n on g_h dominate the (weak) scale effect of n on g_z. Finally, the growth rate of consumption c_t is

$$g_c = 2(\xi - \rho) - (1 + 2\xi/\theta)n, \tag{25}$$

which is also decreasing in n. In other words, by introducing human-capital accumulation into the semi-endogenous growth model, we are able to generate a negative relationship between fertility and long-run growth as in Strulik (2005). Furthermore, in our model, endogenous fertility generates an additional negative effect on human-capital accumulation through the crowding out of time endowment that is absent in the Strulik exogenous-fertility model.

Here we define two useful notations $s_{r,t} \equiv L_{r,t}/(h_t N_t)$ and $s_{x,t} \equiv L_{x,t}/(h_t N_t)$. Using (13) and (15), we derive the first equation for solving the steady-state equilibrium n^* as follows.

$$\frac{v_t \lambda_t}{L_{r,t}} = w_t = \frac{Y_t}{\mu L_{x,t}} \Leftrightarrow \frac{s_r}{s_x} = (\mu - 1)\frac{\lambda}{\rho + \lambda}, \tag{26}$$

where the steady-state equilibrium innovation-arrival rate λ is given by

$$\lambda(\underset{-}{n}) = \frac{g_z}{\ln z} = \frac{\xi(1 - n/\theta) - \rho}{\ln z}. \tag{27}$$

The second equation for solving the model can be obtained by combining the time-endowment constraint and the labor-market clearing condition.

$$1 - \frac{n}{\theta} = \frac{l_t}{h_t} + \frac{e_t}{h_t} = s_r + s_x + \frac{e_t}{h_t}. \tag{28}$$

From (4), the steady-state growth rate of h_t is

$$g_h \equiv \frac{\dot{h}_t}{h_t} = \xi \frac{e_t}{h_t} - n. \tag{29}$$

Equating (29) and (23) yields[14]

$$\frac{e_t}{h_t} = 1 - \frac{n}{\theta} - \frac{\rho}{\xi}. \tag{30}$$

In order for $e_t \geq 0$, it requires that $n \leq \overline{n} \equiv \theta(1 - \rho/\xi)$, which also ensures $\lambda \geq 0$ from (27). Using (30), we can simplify (28) to

$$\frac{\rho}{\xi} = s_r + s_x. \tag{31}$$

Finally, the last equation for solving the model is the consumption-fertility optimality condition in (6).[15]

$$\begin{aligned} \frac{\alpha}{n} &= \frac{a_t}{c_t} + \left(\frac{1}{\theta} + \frac{1}{\xi}\right)\frac{w_t h_t}{c_t} \\ &= \left(\frac{\mu - 1}{\mu}\right)\frac{1}{\rho + \lambda} + \left(\frac{1}{\theta} + \frac{1}{\xi}\right)\frac{1}{\mu s_x}. \end{aligned} \tag{32}$$

[14] If we allow for a positive depreciation rate δ of human capital, each of (23) and (29) would feature δ, so that they cancel each other giving rise to the same condition as (30).

[15] It is useful to recall that $a_t = v_t/N_t$.

Combining equations (26), (31) and (32) solve the three endogenous variables n, s_r and s_x. In the next section, we use these equilibrium conditions to analyze the effects of patent policy.

4 Effects of strengthening patent protection

Combining (26), (31) and (32) and then rearranging terms yield

$$\left(\frac{\alpha\rho^2}{\xi}\right)\frac{1}{n}+\left[\left(\frac{\alpha\rho}{\xi}\right)\frac{1}{n}-\left(\frac{1}{\theta}+\frac{1}{\xi}\right)\right]\lambda(n)=\frac{\rho}{\mu}\left(\frac{1}{\theta}+\frac{1}{\xi}\right)+\frac{\rho}{\xi}\left(\frac{\mu-1}{\mu}\right), \quad (33)$$

where $\lambda(n)$ is a function of n given in (27). To ensure the existence of a unique equilibrium $n^* \in (0,\overline{n})$, where $\overline{n} \equiv \theta(1-\rho/\xi)$, we impose the following upper bound on the fertility-preference parameter α.[16]

$$\text{Condition E (equilibrium uniqueness): } \alpha < \frac{\theta(\xi-\rho)}{\rho}\left(\frac{1}{\theta\mu}+\frac{1}{\xi}\right).$$

The left-hand side (LHS) of (33) is initially decreasing in n while the right-hand side (RHS) of (33) is independent of n. It is easy to see that $\lim_{n\to 0} LHS > RHS \in (\rho/\xi, \rho/\xi+\rho/\theta)$. It can be shown that Condition E implies $LHS|_{n=\overline{n}} < RHS$. Therefore, LHS crosses RHS exactly once giving rise to a unique equilibrium $n^* \in (0,\overline{n})$.

As for the comparative statics of n^* with respect to the strength of patent protection μ, we need to find out whether an increase in μ shifts RHS up or down. If an increase in μ shifts RHS down, then n^* would be increasing in μ. See Figure 1 for an illustration. The first term of RHS given by $\rho/(\theta\mu)$ is decreasing in μ, and this term captures the foregone-wage effect of fertility. A larger patent breadth weakens this foregone-wage effect by decreasing $w_t h_t/c_t$ (i.e., the ratio of wage income to consumption) and leads to a higher rate of fertility. The second term of RHS given by $\rho/(\xi\mu)$ is also decreasing in μ. This term captures the human-capital-diluting effect of fertility, and a larger patent breadth also weakens this effect by decreasing $w_t h_t/c_t$. The

[16] When α is sufficiently large such that Condition E is violated, the model features either no equilibrium or two equilibria, in which the additional equilibrium exhibits a high population growth rate. In the numerical analysis, we show that Condition E is satisfied under reasonably calibrated parameter values.

third term of RHS given by $(\rho/\xi)(\mu - 1)/\mu$ is increasing in μ, and this term captures the asset-diluting effect of fertility. A larger patent breadth strengthens this effect by increasing profit income and a_t/c_t (i.e., the ratio of asset value to consumption) resulting in a lower rate of fertility. Although there are two positive effects versus one negative effect, we nonetheless derive an unambiguously positive effect because the human-capital-diluting effect and the asset-diluting effect of fertility cancel each other. To see this result, RHS of (33) simplifies to

$$RHS(\underset{-}{\mu}) = \frac{\rho}{\mu}\left(\frac{1}{\theta} + \frac{1}{\xi}\right) + \frac{\rho}{\xi}\left(\frac{\mu - 1}{\mu}\right) = \frac{\rho}{\theta\mu} + \frac{\rho}{\xi}. \tag{34}$$

Proposition 1 *An increase in the strength of patent protection μ increases the equilibrium fertility rate n^* and decreases the long-run growth rates of human capital, technology and consumption $\{g_h, g_z, g_c\}$.*

Proof. First, note that an increase in μ shifts down RHS of (33). Then, note (23), (24) and (25). ■

Furthermore, we find that as households value fertility more (i.e., an increase in α), they choose a higher rate of fertility n^*. As a result of higher population growth, the economy exhibits a lower growth rate of human capital and slower technological progress in the long run. Therefore, a stronger cultural preference on fertility reduces long-run growth. In the next section, we also analyze the interactive effect between fertility preference and patent protection, and we find that a stronger preference on fertility strengthens the negative growth effect of patent policy.

Proposition 2 *An increase in the fertility-preference parameter α increases the equilibrium fertility rate n^* and decreases the long-run growth rates of human capital, technology and consumption $\{g_h, g_z, g_c\}$.*

Proof. First, note that an increase in μ shifts up LHS of (33); see Figure 2. Then, note (23), (24) and (25). ■

4.1 Quantitative analysis

In this section, we calibrate the model to examine quantitatively the effects of patent breadth on R&D and economic growth. In the previous section, we show that strengthening patent protection reduces long-run economic growth through a higher population growth rate. Therefore, the semi-endogenous growth model with exogenous population growth may not be an accurate framework for analyzing the long-run growth effects of patent policy. Here we analyze this question quantitatively to see whether the effects of patent policy in a model with endogenous fertility are similar to that of a model with exogenous fertility.

There are six structural parameters $\{\rho, \alpha, \theta, \xi, \mu, z\}$ that are relevant for this numerical exercise. First, we set the discount rate ρ to a standard value of 0.04. Then, we consider a range of values for the fertility-preference parameter $\alpha \in \{0.5, 1, 2, 4\}$. Finally, we use the following four empirical moments to pin down the values of the remaining four parameters. We consider a long-run population growth rate of 1% for the US economy, and the equilibrium condition for n is given by (33). We consider an innovation-arrival rate of 0.2, which takes on an intermediate value within the range considered by Acemoglu and Akcigit (2009). The equilibrium condition for λ is given by (27). We consider an R&D share of GDP of 0.025 for the US economy, and this share is given by $S_r \equiv wL_r/Y$ in the model.

$$S_r = \left(\frac{\mu - 1}{\mu}\right) \frac{\lambda}{\rho + \lambda}. \tag{35}$$

We consider a long-run growth rate of total factor productivity of 2%, and the equilibrium condition is $g_z = \lambda \ln z$. Given a chosen value for each of ρ and α, these four empirical moments determine the values of $\{\theta, \xi, \mu, z\}$ respectively. The calibrated parameter values are reported in Table 1. It is useful to note that Condition E (i.e., the parameter condition for equilibrium uniqueness) is satisfied under these sets of parameter values.

Table 1: Calibration

α	θ	ξ	μ	z
0.5	0.070	0.070	1.031	1.105
1.0	0.030	0.090	1.031	1.105
2.0	0.019	0.130	1.031	1.105
4.0	0.014	0.211	1.031	1.105

Given these calibrated parameter values, we consider a counterfactual policy experiment by increasing patent breadth such that μ increases from 1.03 to 1.10 (i.e., patent breadth $b = \ln \mu / \ln z$ increases from about 0.3 to 1.0). The numerical results are reported in Table 2. We see that S_r (i.e., the R&D share of GDP) roughly triples regardless of α, and this effect would be similar to a semi-endogenous growth model with exogenous population growth.[17] However, the magnitude of the changes in $\{n, \lambda, g_z\}$ is increasing in α, but these variables would be independent of patent breadth in a semi-endogenous growth model with exogenous population growth. In the second column of Table 2, we see that as α increases, strengthening patent protection has a larger positive effect on n, which in turn leads to a more dramatic reduction in the steady-state values of λ and g_z.[18] However, when α is sufficiently small (e.g., $\alpha \leq 1$), the decreases in λ and g_z are negligible.

Table 2: Policy experiment ($\mu = 1.10$)

α	n	λ	S_r	g_z
0.5	1.005%	0.199	0.076	1.995%
1.0	1.008%	0.198	0.076	1.976%
2.0	1.010%	0.193	0.076	1.933%
4.0	1.011%	0.184	0.075	1.836%
$\mu = 1.03$	1.000%	0.200	0.025	2.000%

From this quantitative analysis, we conclude that whether analyzing the effects of patent policy in a model with exogenous fertility provides a good approximation to a more realistic model with endogenous fertility largely depends on the empirical value of the fertility-preference parameter α. Here we consider the calibrated values of n/θ (i.e., the fraction of time spent on fertility) to narrow down the empirical range of α. Using the calibrated values of θ in Table 1, one can show that $\alpha \in \{0.5, 1, 2, 4\}$ implies the following calibrated values of $n/\theta \in \{0.14, 0.33, 0.54, 0.72\}$. According to the American Time Use Survey from 2005 to 2009, an average person in households with youngest child under 6 spends less than 3 hours per day for child caring as a

[17] To see this result, the steady-state value of λ is independent of μ in a semi-endogenous growth model with exogenous population growth, so that the change in S_r is solely determined by the change in the markup ratio $(\mu - 1)/\mu$, which roughly triples when μ increases from 1.03 to 1.10.

[18] We will also consider an alternative numerical exercise, which is to increase α while holding all the other parameters constant.

primary activity.[19] Assuming an average of 16 hours of non-sleeping time per day, the fraction of time spent on child caring in the data is close to the lower bound of the calibrated values of n/θ implying that the empirical value of α should be reasonably small in the US. Therefore, a semi-endogenous growth model with exogenous fertility should serve as a good approximation (for the purpose of analyzing the long-run growth effects of patent policy in the US) to a semi-endogenous growth model with endogenous fertility.

Finally, we consider an alternative numerical exercise, which is to increase α while holding all the other parameters constant. We use the set of parameter values that corresponds to $\alpha = 0.5$ in Table 1 as our benchmark,[20] and we vary $\alpha \in \{0.2, 0.5, 0.8, 1.1, 1.4\}$.[21] Table 3 shows the same result that the magnitude of the negative growth effect of patent policy is increasing in α. Therefore, for a culture that has a stronger preference on fertility, the negative effect of patent policy on long-run growth could be much larger.

Table 3: Effects of fertility preference

α	$n(\mu = 1.03)$	$n(\mu = 1.10)$	$g_z(\mu = 1.03)$	$g_z(\mu = 1.10)$	Δg_z
0.2	0.400%	0.401%	2.603%	2.602%	−0.001%
0.5	1.000%	1.005%	2.000%	1.995%	−0.005%
0.8	1.601%	1.613%	1.396%	1.384%	−0.012%
1.1	2.206%	2.230%	0.788%	0.764%	−0.024%
1.4	2.823%	2.902%	0.168%	0.088%	−0.080%

5 Extension: Dilution with human capital

In this section, we consider an extension of our model by assuming that R&D inputs are diluted by the scale of the economy following Laincz and Peretto (2006). Under this specification, our model resembles a second-generation R&D-based growth model, in which the long-run growth rate depends on *both* the R&D share s_r of factor inputs and the population growth rate n.

The key modification is to replace aggregate technology Z_t in (16) by the stock of human capital $h_t N_t$ (i.e., the scale of the economy) such that

$$\lambda_t(j) = \varphi L_{r,t}(j)/(h_t N_t). \tag{36}$$

[19] Persons in households with older children spend even less time for child caring.

[20] Our result is robust to considering the other sets of parameter values.

[21] Under this set of parameter values, Condition E is violated at larger values of α.

Under this specification, the growth rate of technology becomes

$$g_{z,t} = \lambda_t \ln z = (\varphi \ln z) s_{r,t}, \tag{37}$$

where $s_{r,t} = L_{r,t}/(h_t N_t)$. The growth rate of (per capita) human capital continues to be given by (23). Therefore, substituting (23) and (37) into (22) yields the growth rate of (per capita) consumption given by

$$g_c = g_z + g_h = (\varphi \ln z) s_r - (1 + \xi/\theta) n + \xi - \rho. \tag{38}$$

It is interesting to note that g_c is now increasing in s_r and continues to be decreasing in n.

To solve the model, we use the R&D no-arbitrage condition (26) and the resource constraint (31) as before. Combining these two conditions with $\lambda = \varphi s_r$ yields

$$s_r = \left(\frac{\mu - 1}{\mu}\right) \frac{\rho}{\xi} - \frac{\rho}{\varphi \mu}, \tag{39}$$

$$s_x = \frac{\rho}{\mu} \left(\frac{1}{\xi} + \frac{1}{\varphi}\right). \tag{40}$$

Equation (39) shows that R&D share s_r is increasing in the strength of patent protection μ. Substituting $\lambda = \varphi s_r$, (39) and (40) into the consumption-fertility condition (32) yields

$$n^* = \alpha \left[\left(\frac{\mu - 1}{\mu}\right) \frac{1}{\rho + \lambda} + \left(\frac{1}{\theta} + \frac{1}{\xi}\right) \frac{1}{\mu s_x} \right]^{-1}, \tag{41}$$

where

$$\left(\frac{\mu - 1}{\mu}\right) \frac{1}{\rho + \lambda} = \frac{1}{\rho} \left(\frac{1}{1 + \varphi/\xi}\right).$$

Therefore, although the equilibrium fertility rate n^* continues to be increasing in the fertility-preference parameter α, n^* is now independent of patent strength μ. Because the equilibrium fertility rate is independent of patent strength, the positive effect of μ on R&D share leads to a strictly positive effect on long-run growth. This rather different implication of patent policy may serve as an indirect test for the two alternative solutions to the scale-effect problem.

To understand why n^* is independent of μ under this specification of the model, recall that the consumption-fertility condition from (6) is

$$\frac{\alpha}{n_t} = \frac{a_t}{c_t} + \frac{w_t h_t}{c_t} \left(\frac{1}{\theta} + \frac{1}{\xi}\right).$$

In this case, n^* is independent of μ because a_t/c_t and $w_t h_t/c_t$ are independent of μ. The ratio of asset value to consumption is given by

$$\frac{a_t}{c_t} = \left(\frac{\mu - 1}{\mu}\right) \frac{1}{\rho + \lambda}. \tag{42}$$

Although the markup ratio $(\mu - 1)/\mu$ is increasing in μ, a larger μ also increases λ in such a way that a_t/c_t remains unchanged. Similarly, the ratio of wage income to consumption is given by

$$\frac{w_t h_t}{c_t} = \frac{1}{\mu s_x}, \tag{43}$$

where an increase in μ decreases s_x in such a way that $w_t h_t/c_t$ remains unchanged. These two properties are driven by the constant returns to scale of $\lambda = \varphi s_r$ in s_r.

Proposition 3 *In the model with human-capital dilution, an increase in the strength of patent protection μ increases R&D share s_r as well as the long-run growth rates of technology and consumption $\{g_z, g_c\}$. An increase in the fertility-preference parameter α increases the equilibrium fertility rate n^* and decreases the long-run growth rates of human capital and consumption $\{g_h, g_c\}$.*

Proof. First, note that an increase in μ raises s_r in (39), which in turn increases g_z in (37) and g_c in (38). Then, note that an increase in α raises n^* in (41), which in turn decreases g_h in (23) and g_c in (38). ■

6 Conclusion

In this study, we have considered two versions of the scale-invariant quality-ladder model with endogenous fertility to analyze the effects of patent policy on long-run economic growth. Under the semi-endogenous-growth specification, we find that although strengthening patent protection has a positive effect on R&D, it has a negative effect on technological progress in the long run through endogenous fertility. In the quantitative analysis, we find that the magnitude of this channel depends on the empirical value of a preference parameter on fertility. Calibrating this parameter to a reasonable value for the US economy, we find that the long-run growth effects of patent policy through endogenous fertility are negligible. In this case, our model with endogenous fertility resembles the semi-endogenous growth model with exogenous fertility. However, for a culture that has a stronger preference on fertility, the negative growth effect of patent policy through endogenous fertility could be quantitatively significant. Therefore, an interesting direction for future research would be to empirically examine these effects across countries. Under the fully endogenous-growth specification, strengthening patent protection has a positive effect on R&D and no effect on fertility. Therefore, the different equilibrium effects of patent policy on long-run growth under the two specifications may serve as an indirect test for the two alternative solutions to the scale-effect problem.

References

[1] Acemoglu, D., and Akcigit, U., 2009. State-dependent intellectual property rights policy. manuscript.

[2] Aghion, P., and Howitt, P., 1992. A model of growth through creative destruction. *Econometrica* 60, 323-351.

[3] Alesina, A., and Giuliano, P., 2010. The power of the family. *Journal of Economic Growth* 15, 93-125.

[4] Ashraf, Q., and Galor, O., 2007. Cultural assimilation, cultural diffusion and the origin of the wealth of nations. CEPR Discussion Papers No. 6444.

[5] Barro, R., and Becker, G., 1989. Fertility choice in a model of economic growth. *Econometrica* 57, 481-501.

[6] Bessen, J., and Meurer, M., 2008. Patent Failure: How Judges, Bureaucrats, and Lawyers Put Innovators at Risk. Princeton University Press.

[7] Boldrin, M., and Levine, D., 2008. Against Intellectual Monopoly. Cambridge University Press.

[8] Chu, A., 2007. Confidence-enhanced economic growth. *B.E. Journal of Macroeconomics* (Topics), Vol. 7, Article 13.

[9] Chu, A., 2009. Effects of blocking patents on R&D: A quantitative DGE analysis. *Journal of Economic Growth* 14, 55-78.

[10] Chu, A., 2011. The welfare cost of one-size-fits-all patent protection. *Journal of Economic Dynamics and Control*, forthcoming.

[11] Connolly, M., and Peretto, P., 2003. Industry and the family: Two engines of growth. *Journal of Economic Growth* 8, 115-148.

[12] Cozzi, G., 2001. Inventing or spying? Implications for growth. *Journal of Economic Growth* 6, 55-77.

[13] Cozzi, G., 2007. The Arrow effect under competitive R&D. *The B.E. Journal of Macroeconomics* (Contributions), Vol. 7, Article 2.

[14] Cozzi, G., Giordani, P., and Zamparelli, L., 2007. The refoundation of the symmetric equilibrium in Schumpeterian growth models. *Journal of Economic Theory* 136, 788-797.

[15] Cozzi, G., and Spinesi, L., 2006. Intellectual appropriability, product differentiation, and growth. *Macroeconomic Dynamics* 10, 39-55.

[16] Dinopoulos, E., and Thompson, P., 1998. Schumpeterian growth without scale effects. *Journal of Economic Growth* 3, 313-335.

[17] Fernandez, R., and Fogli, A., 2009. Culture: An empirical investigation of beliefs, work, and fertility. *American Economic Journal: Macroeconomics* 1, 146-177.

[18] Furukawa, Y., 2007. The protection of intellectual property rights and endogenous growth: Is stronger always better? *Journal of Economic Dynamics and Control* 31, 3644-3670.

[19] Furukawa, Y., 2010. Intellectual property protection and innovation: An inverted-U relationship. *Economics Letters* 109, 99–101.

[20] Futagami, K., and Iwaisako, T., 2007. Dynamic analysis of patent policy in an endogenous growth model. *Journal of Economic Theory* 132, 306-334.

[21] Galor, O., and Moav, O., 2002. Natural selection and the origin of economic growth. *Quarterly Journal of Economics* 117, 1133-1192.

[22] Grossman, G., and Helpman, E., 1991. Quality ladders in the theory of growth. *Review of Economic Studies* 58, 43-61.

[23] Growiec, J., 2006. Fertility choice and semi-endogenous growth: Where Becker meets Jones. *The B.E. Journals of Macroeconomics* (Topics), Vol. 6, Article 10.

[24] Ha, J., and Howitt, P., 2007. Accounting for trends in productivity and R&D: A Schumpeterian critique of semi-endogenous growth theory. *Journal of Money, Credit and Banking* 39, 733–774.

[25] Horii, R., and Iwaisako, T., 2007. Economic growth with imperfect protection of intellectual property rights. *Journal of Economics* 90, 45-85.

[26] Horowitz, A., and Lai, E., 1996. Patent length and the rate of innovation. *International Economic Review* 37, 785-801.

[27] Iwaisako, T., and Futagami, K., 2003. Patent policy in an endogenous growth model. *Journal of Economics* 78, 239-258.

[28] Jaffe, A., and Lerner, J., 2004. Innovation and Its Discontents: How Our Broken System is Endangering Innovation and Progress, and What to Do About it. Princeton University Press.

[29] Jones, C., 1995. R&D-based models of economic growth. *Journal of Political Economy* 103, 759-784.

[30] Jones, C., 1999. Growth: With or without scale effects. *American Economic Review* 89, 139-144.

[31] Jones, C., 2001. Was an industrial revolution inevitable? Economic growth over the very long run. *The B.E. Journals of Macroeconomics* (Advances), Vol. 1, Article 1.

[32] Judd, K., 1985. On the performance of patents. *Econometrica* 53, 567-586.

[33] Kortum, S., 1997. Research, patenting, and technological change. *Econometrica* 65, 1389-1419.

[34] Laincz, C., and Peretto, P., 2006. Scale effects in endogenous growth theory: An error of aggregation not specification. *Journal of Economic Growth* 11, 263-288.

[35] Li, C.-W., 2001. On the policy implications of endogenous technological progress. *Economic Journal* 111, C164-C179.

[36] Nordhaus, W., 1969. Invention, Growth, and Welfare. The MIT Press.

[37] O'Donoghue, T., and Zweimuller, J., 2004. Patents in a model of endogenous growth. *Journal of Economic Growth* 9, 81-123.

[38] Peretto, P., 1998. Technological change and population growth. *Journal of Economic Growth* 3, 283-311.

[39] Peretto, P., 1999. Cost reduction, entry, and the interdependence of market structure and economic growth. *Journal of Monetary Economics* 43, 173-195.

[40] Razin, A., and Ben-Zion, U., 1975. An intergenerational model of population growth. *American Economic Review* 65, 923-933.

[41] Scotchmer, S., 2004. Innovation and Incentives. The MIT Press.

[42] Segerstrom, P., 1998. Endogenous growth without scale effects. *American Economic Review* 88, 1290-1310.

[43] Strulik, H., 2005. The role of human capital and population growth in R&D-based models of economic growth. *Review of International Economics* 13, 129-145.

[44] Tabellini, G., 2010. Culture and institutions: Economic development in the regions of Europe. *Journal of the European Economic Association* 8, 677-716.

[45] Yip, C., and Zhang, J., 1997. A simple endogenous growth model with endogenous fertility: Indeterminacy and uniqueness. *Journal of Population Economics* 10, 97-110.

[46] Young, A., 1998. Growth without scale effects. *Journal of Political Economy* 106, 41-63.

Figure 1: Effect of patent policy on fertility

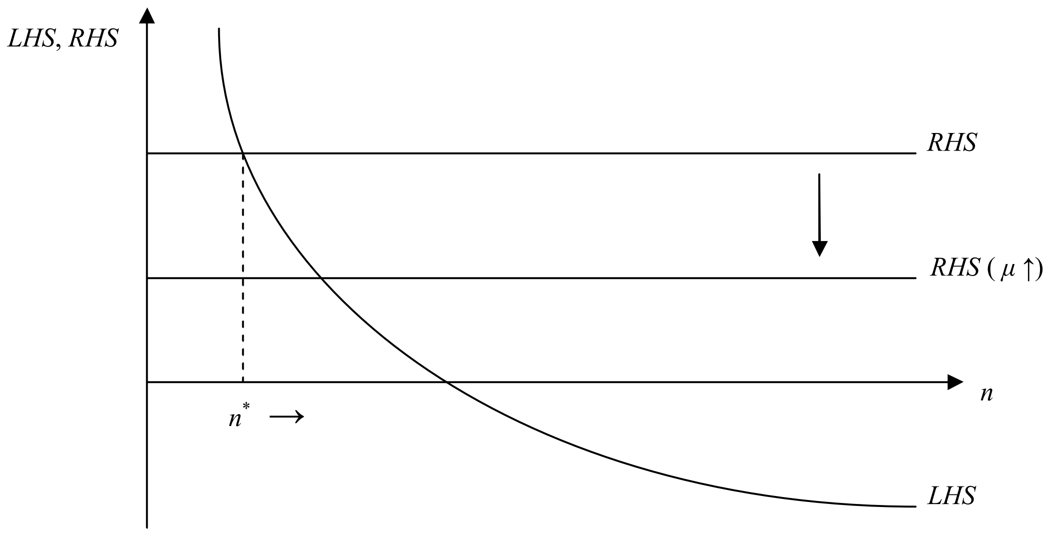

Figure 2: Effect of fertility preference