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PARAMETRIC AND NONPARAMETRIC MONTE CARLO ESTIMATES OF STANDARD ERRORS OF FORECASTS IN ECONOMETRIC MODELS

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ABSTRACT

In the econometric literature simulation techniques are suggested for estimating standard errors of forecasts, especially in case of nonlinear models, where explicit analytic formulae are not available. For this purpose analytic simulation on coefficients, Monte Carlo on coefficients, Monte Carlo simulation based on parametric estimate of the underlying error distribution have been proposed, and more recently a nonparametric procedure which uses the bootstrap technique is also suggested. Main purpose of this paper is to compare, in empirical applications for real world models, parametric and nonparametric estimates. Furthermore, in case of linear models, the same comparisons are performed with respect to the results obtained via analytic formulae. Additional results are obtained from an *error-in-variables* approach.

CONTENTS

1. INTRODUCTION

Forecasts produced by econometric models are subject to many sources of uncertainty: error terms, coefficient estimates, measurement errors in the variables in the sample estimation period, exogenous variables forecasts and possible misspecification of the model.

Several methods have been proposed for estimating the contribution to forecast errors of some or all of these error sources. In particular there is a wide literature[1] dealing with the first two components: fully analytic methods have been developed for linear models, simulation techniques are usually applied in the nonlinear case. The other sources of uncertainty have been less extensively investigated: in fact, forecasts are generally given supposing no errors in the variables and conditional on forecast period exogenous variables and model's structure.

On the one hand this paper will still follow the main stream of the literature; as a sequel to Bianchi and Calzolari (1982, 1983), it will be mainly concerned with the comparison of methods for estimating the contribution to forecast errors of the first two sources of uncertainty (i.e. error terms and errors in the estimated coefficients). These methods are of different types.

1) Full analytical methods: they were originally designed for linear systems (e.g. Goldberger et al. 1961, or Schmidt, 1974), but even in case of models containing nonlinearities, these methods can be applied to solve a good deal of the problem (Calzolari, 1981).
2) Mixed methods, partially analytical and partially based on numerical

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[1] Reference can be made, for example, to Nagar (1969), Schink (1971), Haitovsky and Wallace (1972), Sowey (1973), Cooper and Fischer (1974), Bianchi and Calzolari (1980), Fair (1980), Calzolari (1981), Mariano and Brown (1983), Freedman and Peters(1984) and Brown and Mariano (1984).

simulation procedures (analytic simulation): conceptually equivalent to the full analytical methods, they allow for a considerable reduction of computational complexity and are suitable for application even to medium-large size models (Bianchi and Calzolari, 1980).

3) Parametric Monte Carlo methods: estimates of the variances are computed from sample variances of replicated simulation experiments, after additive multinormal pseudo-random errors have been inserted into the structural equations of the model (Schink, 1971), or even into model's coefficients (Fair, 1980).

4) Nonparametric Monte Carlo methods as the procedure recently suggested by Freedman and Peters (1984), where estimates of the variances are computed as in the previous case from sample variances of replicated experiments but using the so called bootstrap technique. When using bootstrap, the theoretical distribution of an unobservable disturbance term is approximated by the empirical distribution of an observable set of residuals.

In the two recent papers Bianchi and Calzolari (1982, 1983) have performed some experiments on a set of small, medium and large size real world models, both linear and nonlinear, comparing the results and performances of three different methods (analytic simulation, stochastic simulation and re-estimation, Monte Carlo on coefficients) proposed in the literature for the computation of variances of forecasts. Comparisons were first confined to the case of forecasts one period ahead (1982), then to the case of multiperiod forecasts produced with dynamic simulation (1983).

Main purpose of this paper is to compare the nonparametric bootstrap technique with parametric Monte Carlo and analytical methods, using empirical applications on real world models.

Moreover, another source of forecast uncertainty will be investigated: measurement errors in the variables. For this purpose a parametric measurement error process, proposed by Weihs (1986), will be introduced, which facilitates the consideration of a-priori information on the maximal size of measurement errors. A Monte Carlo procedure will be used to study this process empirically. In the performed experiments, errors in the variables will influence the forecasts both through the estimated coefficients and through the errors in the exogenous variables in the forecast period.

The plan of the paper is as follows. Section 2 summarizes the main assumptions and notations used in the paper. In Section 3 four methods for analyzing the component due to errors in the estimated coefficients are discussed. Section 4 deals with the component due to the error terms. In section 5 some empirical results for real world models are presented and in section 6 the measurement error process is introduced and experiments with this process are commented on.

2. ASSUMPTIONS AND NOTATIONS

A structural econometric model can be represented as

$$f(y_t, y_{t-1}, x_t, a) = u_t; \qquad t=1,2,\dots,T \tag{2.1}$$

where $f = (f_1, f_2, \dots, f_m)'$ is a vector of functional operators, continuously differentiable with respect to the elements of current and lagged y, x and a; $y_t = (y_{1t}, y_{2t}, \dots, y_{mt})'$, $x_t = (x_{1t}, x_{2t}, \dots, x_{nt})'$ and y_{t-1} are the vectors of current endogenous, exogenous and lagged endogenous variables, respectively; $a = (a_1, a_2, \dots, a_s)'$ is the vector of the structural coefficients to be estimated (all the other known coefficients of the model are excluded from this vector and included in the functional operators); $u_t = (u_{1t}, u_{2t}, \dots, u_{mt})'$ is the vector of structural stochastic disturbances (or error terms) at time t, having zero mean and being independently and identically distributed over time, with finite contemporaneous covariance matrix, and independent of all the predetermined variables. In the experiments based on parametric Monte Carlo the contemporaneous distribution of the error terms will be assumed multivariate normal: $u_t \sim N(0,\Sigma)$.

It is usually assumed that a simultaneous equation system like (2.1) implicitly defines a single inverse relationship (reduced form) for relevant values of the coefficients, the predetermined variables, and any values of the disturbance terms:

$$(2.2) \qquad y_t = g(y_{t-1}, x_t, a, u_t).$$

Of course, the vector of functions g implicitly defined is usually unknown, but can be assumed continuously differentiable, like f.

Using a suitable estimation method, an estimated vector of coefficients, $\hat{a}$, an estimated covariance matrix of the structural disturbance process, $\hat{\Sigma}$, and an estimate of the coefficients' covariance matrix (which will be indicated as $\hat{\Psi}/T$) can be obtained.

In case of linear dynamic or nonlinear static models, few assumptions in addition to those listed above are usually sufficient to ensure consistency and asymptotic normality of $\hat{a}$, produced by suitable estimation methods. In these cases, asymptotically as $T \to \infty$,

$$(2.3) \qquad T^{\frac{1}{2}} (\hat{a} - a) \sim N(0,\Psi).$$

Unfortunately, to the best of our knowledge, there are no general theoretical tools to prove that (2.3) holds when the nonlinear model includes lagged endogenous variables among the predetermined variables. It can, however, be assumed that (2.3) holds under heuristic considerations, as in Gallant (1977, pp.73-74). If (2.3) holds, then

several results which will be derived are asymptotically exact; if (2.3) does not hold exactly, the results which will be derived are not asymptotically exact, but simply "reasonable" approximations.

If $\hat{\Psi}$ is a consistent estimate of Ψ, an estimate of the covariance matrix of a multinormal distribution which approximates the small sample distribution of $\hat{a}$ is obtained as $\hat{\Psi}/T$, that is dividing $\hat{\Psi}$ by the actual length of the sample period (see Schmidt, 1976, p.254). $\hat{\Psi}/T$ is, together with $\hat{a}$, a standard outcome of system estimation methods. When limited information estimation methods are applied, as in our case, this matrix must be built block by block, after the estimated a has been obtained, and the resulting matrix may be singular, in case of undersized samples. In the models used in the following experiments, the blocks of the matrix $\hat{\Psi}/T$ have been computed as in Brundy and Jorgenson (1971, p.215).

Indicating with h the first time period not belonging to the sample estimation period $1,2,\ldots,T$, (i.e. $h=T+1$), making use of the reduced form notation, at time h the actual values for the endogenous variables can be written as:

$$(2.4) \qquad y_h = g(y_{h-1}, x_h, a, u_h).$$

In the same period, supposing to know x_h and y_{h-1} with certainty, the forecast is usually obtained as:

$$(2.5) \qquad \hat{y}_h = g(y_{h-1}, x_h, \hat{a}, 0)$$

where $\hat{a}$ is the vector of the estimated coefficients and the random error terms u_h are set to their expected value (zero).

Introducing the following auxiliary vector:

$$(2.6) \qquad \bar{y}_h = g(y_{h-1}, x_h, a, 0)$$

(where $\bar{y}_h$ could be defined as the vector of forecasts that would be produced by the model if there were no errors of any kind), the vector of forecast errors (e_h) can be written as:

$$(2.7) \qquad e_h = \hat{y}_h - y_h = (\hat{y}_h - \bar{y}_h) + (\bar{y}_h - y_h).$$

This expression explicitly decomposes the forecast error (conditional on the model structure) into two terms: the first takes into account errors in the coefficient estimates and the second the presence of the error terms. These two components, conditional on the exact knowledge of the predetermined variables, are independent when forecasting outside the sample period, so that their statistical properties can be studied separately.

3. FOUR METHODS FOR ANALYSING THE COMPONENT OF FORECAST ERRORS DUE TO ERRORS IN ESTIMATED COEFFICIENTS

Four different methods to analyze the component $(\hat{y}_h - \bar{y}_h)$ are briefly described in this section. For exposition purpose they will be referred to as:

- Parametric stochastic simulation and re-estimation
- Bootstrap simulation and re-estimation
- Monte Carlo on coefficients
- Analytic simulation on coefficients

3.1. Parametric stochastic simulation and re-estimation

This method can be summarized as follows (see Schink, 1971 for more details). Let $\hat{\Sigma}$ be the available estimate of the covariance matrix of the structural disturbances.

1) T vectors of pseudo-random numbers, $\tilde{u}_t$, $t=1,2,\ldots,T$ (each of which having multinormal distribution, zero means and covariance matrix equal to the available $\hat{\Sigma}$), are generated. The method by Nagar (1969) can be applied if $\hat{\Sigma}$ is positive definite; if $\hat{\Sigma}$ is not of full rank, the method by McCarthy (1972,a) can be used.
2) The vectors $\tilde{u}_t$ are inserted into the model, where the structural coefficients are maintained fixed at their originally estimated values, and the model is solved over all the sample period, obtaining for the endogenous variables the vectors $\tilde{y}_t$, $t=1,2,\ldots,T$.
3) The vectors $\tilde{y}_t$ are treated as a new set of observations of the endogenous variables and are used to re-estimate the model, thus obtaining a new vector, $\tilde{a}$, of pseudo-estimated coefficients.
4) The coefficients $\tilde{a}$ are inserted into the model to produce, via deterministic solution, a vector of pseudo-forecasts at time h, $\tilde{y}_h$.

The process is repeated from step 1 to 4 and the desired results follow from the computation of the sample variances of the elements of all the $\tilde{y}_h$ computed in the various replications.

Some complications arise from the treatment of lagged endogenous variables in the simulation phase (in other words simulation can be static or dynamic) and in the re-estimation phase (they can be maintained "static", i.e. fixed at some given *historical* value, or their simulation value can be chosen). This problem is discussed in Schink (1971, pp.101-108). In the experiments here performed different combinations have been adopted.

This method is frequently used in the literature to derive small sample distributions of estimators for simultaneous equation systems, when analytical methods are not available. The main theoretical limitation is in the possible nonexistence of finite moments in the small sample distribution of the structural form or reduced form coefficients (these last directly related to forecasts); this topic is discussed, for example, in Dhrymes (1970, p.182), McCarthy (1972,b), Sargan (1976) and Mariano (1982).

As pointed out in McCarthy (1972,b, p.761), "...it should be noted that the non-existence of moments has some implications for those engaged Monte Carlo studies. Outliers can be expected. Computation of mean squared forecast errors and the mean squared errors of the restricted reduced form coefficient estimates will not converge as the number of Monte Carlo runs increases. These computations really will not yield meaningful information. Throwing out the outliers in making these calculations is also of questionable value. What is accomplished by throwing them out?...".

3.2. Bootstrap simulation and re-estimation

This method described in Efron (1979), has been recently used by Freedman and Peters (1984) for the purposes in which we are interested.

Like stochastic simulation, bootstrap is a procedure for estimating standard errors by resampling the data using a Monte Carlo approach. The process is exactly the same as the one described in the previous subsection, with the only difference that in the first step, rather than sampling from some assumed parametric distribution (i.e. from a multinormal with zero means and covariance matrix equal to the available $\hat{\Sigma}$), the T vectors $\tilde{u}_t$ are the results of T draws, made at random with replacement, from the T vectors $\hat{u}_t$ $(t=1,2,\ldots,T)$ of the calculated structural residuals corresponding to $\hat{a}$. Steps 2, 3, 4 and the way in which the sample variances of $\tilde{y}_h$ are computed, are exactly the same as described in the previous case.

Using this procedure, the only distributional assumption concerning the disturbances in the system is that the disturbances should be independent and identically distributed over time. Of course, in order to preserve the stochastic relationships of the estimated equations, the pattern of the disturbances across equations does not change in the experiment.

3.3. Monte Carlo on coefficients

This method, described in Fair (1980), can be summarized as follows. Let $\hat{\Psi}/T$ be the available estimate of the covariance matrix of the structural coefficients $\hat{a}$.

1) A vector $\tilde{a}$ of pseudo random numbers, with mean $\hat{a}$ and covariance matrix equal to the available $\hat{\Psi}/T$, is generated.
2) The pseudo-random coefficients vector $\tilde{a}$ replaces the original estimates $\hat{a}$ and the model is solved deterministically in the forecast period h, obtaining the vector of pseudo-forecasts $\tilde{y}_h$.

The process is repeated from step 1 to 2 and the desired results follow from the computation of the sample variances of the elements of all the $\tilde{y}_h$ computed in the various replications.

A difficulty may arise in the generation of the pseudo-random vectors $\tilde{a}$. The usual generation methods are, in fact, based on Choleski triangularization of the matrix $\hat{\Psi}/T$ (see Cooper and Fischer, 1974, or Nagar, 1969, for example) and most of the available algorithms perform the triangular decomposition only if such a matrix is positive definite (see for example Nagar, 1969). Unfortunately, this is not always the case. For example when, in a large scale model, the length of the time series does not allow the application of system estimation methods, the matrix $\hat{\Psi}/T$ must be built block by block (see, for example, Brundy and

Jorgenson, 1971, p.215, for instrumental variables LIVE estimation) and it is not necessarily of full rank. In this case the triangular decomposition must carefully take into account the possible singularity of the matrix. Alternatively the generation of the pseudo-random coefficients vectors $\tilde{a}$ might pass through the generation of shorter vectors with full rank covariance matrix (see, for example, Rao, 1965, pp.498-501) with some additional computational difficulties.

This problem clearly does not arise if only the diagonal blocks of the $\hat{\Psi}/T$ matrix are taken into account, as in the work of Cooper and Fischer (1974), Haitovsky and Wallace (1972) and Fair (1980). In the experiments that will be described in the following sections, the complete matrix $\hat{\Psi}/T$ will be taken into account whenever possible, otherwise only its diagonal blocks will be used. It must, however, be pointed out that all the experiments performed with the complete $\hat{\Psi}/T$ matrix have also been repeated with the diagonal blocks only, obtaining in most cases rather similar results (a similar conclusion is in Bianchi, Calzolari and Corsi, 1981).

With respect to parametric stochastic simulation and re-estimation or bootstrap simulation and re-estimation this method seems to be more sensitive to outliers; a kind of "instability" in the convergence of the Monte Carlo process is encountered more often.

3.4. Analytic simulation on coefficients

This method, described in Bianchi and Calzolari (1980), is an extension, to nonlinear models, of the fully analytical methods developed, for one-period forecast with linear models, in Goldberger et al. (1961) and Calzolari (1981); the case of dynamic (multiperiod) forecast in linear models is treated by Schmidt (1974).

The method relies on the property, well known in large sample theory (see, for example, Rao, 1965, p.322), that asymptotic normality of sample statistics can be maintained through transformations, even nonlinear, provided they are continuous and differentiable.

If we assume that, as T increases to infinity, asymptotically

$$(3.4.1) \qquad T^{\frac{1}{2}}(\hat{a} - a) \sim N(0,\Psi)$$

(and $\hat{\Psi}$ is a consistent estimate of Ψ) then, asymptotically,

$$(3.4.2) \qquad T^{\frac{1}{2}}(\hat{y}_h-\bar{y}_h) = T^{\frac{1}{2}}[g(y_{h-1},x_h,\hat{a},0)-g(y_{h-1},x_h,a,0)] \sim N(0,G_h\Psi G_h')$$

where G_h is the $(m\times s)$ matrix of first order derivatives of the elements of g with respect to the elements of a, computed in the point $(y_{h-1},x_h,a,0)$, provided that y_{h-1} and x_h are given.

If the computation is performed at the point $(y_{h-1},x_h,\hat{a},0)$ and $\hat{\Psi}$ is used in equation (3.4.2), then $\hat{G}_h\hat{\Psi}\hat{G}_h'$ is a consistent estimate of $G_h\Psi G_h'$; the division by the sample period length, T, leads to the result we are looking for, the estimate of the covariance matrix of a multinormal distribution which approximates the small sample distribution of the random vector $(\hat{y}_h-\bar{y}_h)$.

Continuity and differentiability of the elements of the (unknown) vector of reduced form functional operators g is ensured by the implicit function theorem, which also provides the way of computing the derivatives

$$(3.4.3) \qquad (\partial g/\partial a') = -(\partial f/\partial g')^{-1}(\partial f/\partial a')$$

where the derivatives of the structural form operators vector f (known) can be also analytically computed, once the deterministic solution of the model at time h has been computed.

For medium or large scale models it can be simpler to perform the computation of the above derivatives with numerical methods (finite differences), rather than analytically; this criterion has been followed for all the models used here, with the exception of the Klein-1 model, where both analytical and numerical differentiation has been performed (of course, with coincident results).

3.5. Some remarks on the four methods

It must be pointed out that not only are there technical differences in the computational algorithms, but there are some basic conceptual differences among the methods.

Parametric stochastic simulation and re-estimation as well as bootstrap simulation and re-estimation try to deal with the "small sample" distribution of $(\hat{y}_h-\bar{y}_h)$ directly. Although no formal proof is undertaken in this paper, it is reasonable to expect greater efficiency from parametric stochastic simulation if the distribution of the error terms is correctly specified (since in this case we would use a-priori information). On the contrary we should expect a greater robustness from bootstrap simulation and re-estimation against misspecification, since it does not make explicit assumptions on the error process.

Monte Carlo on coefficients starts from the estimate of the asymptotic covariance matrix of the structural coefficients, treats this matrix as an approximation of the small sample covariance matrix of the coefficients and derives the consequences of this assumption on $(\hat{y}_h-\bar{y}_h)$.

Also analytic simulation on coefficients starts from the estimated asymptotic covariance matrix of the structural coefficients and derives the asymptotic covariance matrix of $(\hat{y}_h-\bar{y}_h)$; only after this computation is performed, the resulting matrix is interpreted as an approximation of the small sample covariance matrix of $(\hat{y}_h-\bar{y}_h)$. In some sense, with respect to

Monte Carlo on coefficients, the approximation is performed at a later stage.

From a purely empirical point of view, however, all methods lead to the same information, i.e. an estimated covariance matrix of the given component of forecast errors.

4. THE COMPONENT DUE TO THE STRUCTURAL DISTURBANCES

When dealing with linear models, the statistical properties of the term $(\bar{y}_h-y_h)$, which is a function of the random disturbances u_h, are well known in the econometric literature.

For nonlinear models, the properties of such a term are generally unknown, as far as the distribution and its parameters are concerned; for example, the conditional expectation of $(\bar{y}_h-y_h)$ is generally different from zero, i.e.: $E[(\bar{y}_h-y_h)|y_{h-1},x_h,a]\neq 0$.

As suggested by Howrey and Kelejian (1971), approximate values for the conditional expectation and the covariance matrix of $(\bar{y}_h-y_h)$ can be obtained using stochastic simulation.

As pointed out in the previous section, stochastic simulation can be performed with Monte Carlo draws from some assumed parametric distribution or drawing directly from the empirical distribution of the calculated residuals using the bootstrap procedure.[2] In both cases stochastic simulation would supply approximate estimates of moments of the distribution of $(\bar{y}_h-y_h)$ and, of course, the accuracy of the approximation

[2] The residual-based procedure recently proposed by Brown and Mariano (1984) for the one-period forecasts in static models, utilizes complete enumeration of the residuals over the sample period. In some way, it could be considered a particular case of bootstrap. Applying the same procedure in case of linear models gives the same results as computing the reduced form variances through the calculation of the mean squared errors over the sample period.

improves as the number of replications increases.

Some improvement in the accuracy of the approximation can be obtained by applying some variance reduction technique (more precisely, some technique to reduce the experimental variance of the sample variance). The control variates technique in Calzolari and Sterbenz (1983) allows for considerable reduction of computation time with respect to straightforward stochastic simulation simply based on independent random drawing.

5. EMPIRICAL RESULTS FROM REAL WORLD MODELS

This section is concerned with numerical results obtained from experiments performed on some real world models which will be briefly described below.

Some general considerations hold for all the experiments and must be taken into account for a clear understanding of the tables of results:

1) Parameter estimates in each model have been obtained by means of limited information instrumental variables efficient method (LIVE). The estimation method is exactly the method of Brundy and Jorgenson (1971), since it make use of the deterministic solution of the models to build the instruments. Therefore the name LIVE has been maintained, although this kind of instruments is generally not fully efficient when applied to nonlinear models (see Amemiya, 1977).
2) If not otherwise specified, for all the models the resampling has been performed using static simulation; forecasts are related to the first period outside the sample estimation period and the standard errors of forecasts generally include both the component due to the errors on the estimated coefficients and the component due to the additive random error terms.
3) In all the tables, the first numerical column displays the deterministic

forecast values of some of the most important endogenous variables of the model; the other columns display the results (standard errors of forecasts) we are interested in. For easier comparison, at the top of each column the corresponding method is referred to in the following way:

ANAL	=	analytic simulation
PSS	=	parametric stochastic simulation and re-estimation
BOOT	=	bootstrap simulation and re-estimation
MCC	=	Monte Carlo on coefficients

4) The variances due to random error terms have been computed using the procedures described in section 4: local linearization of the model in the neighborhood of the solution point has been used for ANAL; for PSS and MCC the parametric stochastic simulation procedure suggested by McCarthy (1972,a) has been used. It must be recalled that, in case of linear models, ANAL furnishes results that are exactly equal to the ones obtained via analytic formulae.
5) The number of replications in the Monte Carlo experiments will be specified when discussing the empirical results; in general this number has been chosen looking at the sample variability of the variances in the Monte Carlo process.
6) In general the results reported in the following tables show the empirical equivalence of the parametric methods ANAL and especially PSS with the nonparametric method BOOT. This conclusion seems to be quite different from the one obtained in Freedman and Peters (1984) where the empirical illustration shows that the parametric analytic method by Goldberger, Nagar and Odeh (1961) (for linear models exactly equivalent to ANAL) systematically underestimates the standard errors as computed by the nonparametric BOOT. One explanation of the divergences in the empirical results can be found in the way in which Freedman and Peters (1984) apply the BOOT method. In fact,

when forecasting, bootstrap is extended to the forecast period in the sense that the forecast period is included in the *pseudo-samples* used in the Monte-Carlo process for estimating the structural coefficients. Therefore their results are not completely comparable with the ones reported in the present paper.

7) As far as the parametric MCC method is concerned, when there is convergence in the Monte-Carlo process, the results are similar to those produced by the other methods. Nevertheless, as pointed out in Bianchi and Calzolari (1982, 1983), this method is very sensitive to the presence of outliers: the sample variances may not converge as the number of replications increases so that, as reported in table 1-b, there is an abnormal fluctuation in the experimental results.

5.1. Klein-I model: LIVE estimates 1921-1941

The model, proposed in Klein (1950), consists of three stochastic plus three definitional equations; there are 12 estimated coefficients, 4 for each equation. Estimation has been performed with the LIVE on the sample period 1921-1941. Forecasts are related to 1948 using, for the predetermined variables, the values in Goldberger et al. (1961). The results are based on 1000 replications.

MCC1 and MCC2 are related to results obtained with the MCC method but using a different set of *pseudo-random numbers* for the Monte-Carlo process. After 1000 replications the differences in the sample standard errors could anticipate the non convergence of the MCC method for the LIVE estimates of Klein-I model. Furthermore the possible instability of the MCC method can be stressed by LIVE over the sample period 1921-1939. In fact, as reported in table 1-b, the experimental results after 200, 400, 600, 800 and 1000 replications (respectively MCC1, MCC2, MCC3, MCC4 and MCC5) show abnormal fluctuations.

Table 1-a

Klein-I model: LIVE estimates 1921-1941

One-period forecasts at 1948

	Det.sol.	ANAL	PSS	BOOT	MCC1	MCC2
C	78.07	2.51	2.56	2.56	2.61	2.82
I	9.221	1.68	1.68	1.67	1.88	1.96
W1	59.80	2.08	2.11	2.10	2.14	2.31
Y	95.49	3.98	3.99	3.99	4.28	4.57
P	26.98	2.35	2.32	2.37	2.55	2.70
K	206.9	1.68	1.68	1.67	1.88	1.96

Glossary

C	Consumption
I	Net investment
W1	Private wage bill
Y	National income
P	Profits
K	End-of-year capital stock

Table 1-b

Klein-I model: LIVE estimates 1921-1939

One-period forecasts at 1940

	Det.sol.	ANAL	PSS	BOOT	MCC1	MCC2	MCC3	MCC4	MCC5
C	65.56	2.26	2.40	2.47	4.57	8.14	6.82	6.08	6.59
I	2.970	1.62	1.71	1.69	3.20	7.20	5.98	5.31	5.85
W1	45.94	1.92	2.08	2.06	3.33	5.52	4.68	4.22	4.43
Y	74.33	3.79	3.98	4.01	7.68	15.27	12.7	11.3	12.2
P	20.39	2.20	2.23	2.29	4.59	9.90	8.23	7.28	7.90
K	204.2	1.62	1.71	1.69	3.20	7.20	5.98	5.31	5.85

5.2. The Wharton Mini Growth Model (MGM) of the U.S. Economy

The nonlinear model analyzed in this subsection is an annual model of the U.S. economy developed at Wharton Econometric Forecasting Associates (WEFA). The current structure, an updated version of the model described in Sheinin (1982), consists of 16 stochastic equations, 82 economic behavior and national income identities and 31 exogenous variables: it "is designed to analyze the short and long run effect of alternative policies and random shocks on aggregate economic indicators".

It must be recalled that, in this model, supply, demand and prices are simultaneously determined, producing short run equilibrium and long run steady state properties; the particular iterative estimation technique used to estimate the corresponding block of equations is described in Sheinin (1982, pp. 21-22).

In order to respect the above mentioned characteristics, iterative LIVE has been used to re-estimate the model on the sample period 1960-1980; the convergence, generally, has been reached after 12 iterations. In table 2 the displayed results are based on 500 replications and are relative to the standard errors of forecasts due to errors on the estimated coefficients only.

Since the estimated covariance matrix of the structural coefficients is not positive definite, in the MCC case (referred to in table 2 as MCCBD), this matrix has been used in its block-diagonal part only (i.e. only the covariances among coefficients of the same equation have been included), and for comparison purposes, standard errors of analytic simulation have been computed not only with the full covariance matrix (column ANAL), but also with a block-diagonal one (column ANALBD). Of course, the above mentioned computation is impossible for the PSS and BOOT methods.

Table 2

MGM model: LIVE estimates 1960-1980, forecast at 1981

Standard errors of forecasts due to the coefficients only

	Det.sol.	ANAL	PSS	BOOT	ANALBD	MCCBD
CE	904.7	8.55	9.00	9.07	7.82	7.54
ENER.DCE	17.87	.278	.283	.276	.269	.269
IBFR	61.99	4.14	4.11	4.35	4.04	4.12
IBFNE	92.94	3.87	4.24	4.20	3.54	3.54
IBFNS	49.06	.899	.980	1.05	.791	.817
IBIT	.6338	3.27	3.77	3.63	2.53	2.57
ENER.DPR	52.87	.739	.716	.692	.623	.628
ENER.TD	70.74	.876	.854	.834	.718	.718
TMB-E	98.02	2.21	2.30	2.41	2.04	1.93
TMBE	6.940	.395	.385	.376	.324	.324
TEB	174.8	3.35	3.58	3.84	3.39	3.32
GVR	976.9	17.1	17.6	17.4	13.0	12.9
GVE	1021.	8.97	8.81	9.12	8.14	8.13
YPD	985.5	9.47	9.89	9.53	8.46	8.34
GNP	1473.	16.6	17.5	17.4	13.6	13.4
NEHT	101.8	1.01	1.02	.981	.878	.875
CPRGNP	.9029	.008	.008	.008	.007	.007
NLC	108.7	.523	.522	.493	.490	.494
NRUT	6.363	.555	.558	.543	.472	.476
PDGNP	202.4	1.99	1.96	2.05	1.87	1.88
WRC	343.3	3.75	3.73	3.42	3.13	3.11
PENER	900.0	4.72	4.60	4.51	3.87	3.87
FRMLCDS	14.52	.110	.723	.646	.085	.083
PKBFN	211.6	2.48	4.70	4.31	2.37	2.39

Glossary

CE	Personal Consumption Expenditures
ENER.DCE	Residential Energy Consumption
IBFR	Residential Investment
IBFNE	Nonresidential Investment in Equipment
IBFNS	Nonresidential Investment in Structures
IBIT	Changes in Business Inventory
ENER.DPR	Demand for Energy
ENER.TD	Demand for Energy Total
TMB-E	Total Import less Energy
TMBE	Import of Energy (Bill. 1972 US dollars)
TEB	Export, Total
GVR	Total Government Receipts
GVE	Total Government Expenditures
YDP	Real Disposable Income
GNP	Gross National Product
NEHT	Demand for Labor
CPRGNP	Capacity Utilization Rate
NLC	Labor Supply
NRUT	Unemployment Rate
PDGNP	Aggregate Price Deflator
WRC	Wage Rate
PENER	Average Energy Price
FRMLCDS	Interest Rate on Large Time Deposits
PKBFN	User Cost of Capital

5.3. A Nonlinear Model of the Italian Economy

The model used for the experiments described in this section is an annual model of the Italian economy. It is an updated version, estimated in the sample period 1953-1982, of one of the first Italian econometric models (Sylos Labini, 1967); for a detailed description see Del Monte (1981).

The model is mildly nonlinear in the endogenous variables (in fact, nonlinearities arise only from some identities); in this version it includes 23 endogenous and 16 exogenous variables (several of which are dummies): 14 are the stochastic equations and 9 the identities, which mainly connect variables expressed in different ways, like levels, first differences, annual percentage differences. The results are based on 500 replications.

The meaning of the endogenous variables displayed in table 3 is

indicated in the following glossary; a final "H" in the name of a variable means that the related variable is expressed in annual percentage differences.

Table 3

Italian Model: LIVE estimates 1953-1982, forecasts at 1983

	Det.sol.	ANAL	PSS	BOOT	MCC
PREAGRH	7.371	4.35	4.42	4.18	4.49
PREINDH	7.745	2.03	2.08	2.07	2.10
PREMINH	10.52	2.11	2.17	2.02	2.29
COSTOCCH	13.05	2.76	2.81	2.85	3.03
RELADIH	13.95	2.27	2.35	2.33	2.44
CORTOTCH	13.51	2.66	2.73	2.37	2.96
OCCSTREH	-1.766	1.12	1.14	1.12	1.13
INVINDKH	-6.135	3.65	3.52	3.63	3.58
VAINDKH	1.881	2.17	2.13	2.23	2.19
IMPFOBCH	5.961	4.80	4.85	5.01	4.64
PREINGH	7.709	2.09	2.23	1.95	2.22
COSVITH	10.68	2.10	2.10	1.93	2.23
RELADINH	10.85	3.48	3.53	3.69	3.67
CONSALCH	10.97	2.85	2.99	2.79	3.07
VAINDK	591.3	12.6	12.3	12.9	12.7
OCCSTRE	145.7	1.67	1.69	1.67	1.67
RELADIN	5818.	182.	185.	194.	192.
CORTOTK	388.6	5.82	6.43	5.82	5.94
CORTOTC	4172.	97.7	100.	87.2	109.
PREIND	693.9	13.1	13.4	13.3	13.6
COSTPROH	8.805	3.63	3.61	3.66	3.92
RELADI	6672.	133.	138.	137.	143.
PREMIN	1073.	20.4	21.0	19.6	22.2

Glossary

PREAGR	Wholesale Price of Agricultural Products
PREIND	Wholesale Price of Industrial Products
PREMIN	Retail Prices
COSTOCC	Gross Wage per Dependent Worker
RELADI	Contractual Earnings
CORTOTC	Domestic Private Consumption (Current Prices)
OCCSTRE	Dependent Workers in Industry
INVINDK	Fixed Industrial Investments (Constant Prices)
VAINDK	Added Value in Industrial Sector (Constant Prices)
IMPFOBC	Import at FOB Prices
PREING	Wholesale Prices
COSVIT	Cost of Living
RELADIN	Contractual Earnings in Industrial Sector
CONSALC	Domestic Food Consumption (Current Prices)
CORTOTK	Domestic Private Consumption (Constant Prices)
COSTPRO	Unit Product Labor Cost

6. MEASUREMENT ERRORS IN THE VARIABLES: A PARAMETRIC ERROR PROCESS AND NUMERICAL ILLUSTRATION

6.1. A parametric process for the generation of measurement errors

Up to now it was supposed that all endogenous and exogenous variables can be measured without error in some relevant time-period. In this section these variables are assumed to be latent only and some assumptions on the structure of the measurement errors are proposed (see Weihs, 1986):

1) The (unlagged and lagged) endogenous variables y_{it} and the exogenous variables x_{jt} are observable in a sample-period, $t=1,\dots,T$, only indirectly by means of measurement variables y^m_{it}, x^m_{jt}, where:

$$y^m_{it} = y_{it} + d_{yit},\ i = 1,\dots,m \quad \text{and}$$
$$x^m_{jt} = x_{jt} + d_{xjt},\ j = 1,\dots,n.$$

2) The measurement errors d_{yit}, d_{xjt} are identically distributed for all $t=1,\dots,T$, and $E(d_{yit}) = E(d_{xjt}) = 0$, $i=1,\dots,m$, $j=1,\dots,n$.

3) The measurement errors in the lagged endogenous variables are equal to the errors in the corresponding unlagged endogenous variables in the corresponding pre-period.

4) The structure of the measurement errors d_{yit}, d_{xjt} can be interpreted as the result of some a-priori-information. On the one hand it is assumed that measurement errors of different variables are uncorrelated and that all measurement errors are not correlated to any of the endogenous and exogenous latent variables. On the other hand it is assumed to be known that more than 99% (say) of the absolute values of the measurement errors of a variable are smaller than p% of the latent variable itself, where p is a-priori information and may vary for different variables.

The following *measurement generating process (MGP)* has the desired properties:

Let $D := diag\,(q_1^2,\dots,q_m^2,q_{m+1}^2,\dots,q_{m+n}^2)$ be a diagonal matrix with $q_k := p_k/300$, $p_k \in [0,100]$ fixed a-priori. Let $(\tilde{d}_{yt}'\ ;\ \tilde{d}_{xt}')'$ be independently identically $N(0,D)$-distributed random variables, $t=1,\dots,T$, which are independent of y_s, x_s, $s=1,\dots,T$. Then define the measurement variables y^m_t, x^m_t as

$$y^m_{it} := y_{it}\,(1+\tilde{d}_{yit}) =: y_{it} + d_{yit},\ i = 1,\dots,m$$
$$x^m_{jt} := x_{jt}\,(1+\tilde{d}_{xjt}) =: x_{jt} + d_{xjt},\ j = 1,\dots,n,\ t=1,\dots T.$$

Relative error sizes like the p_k may be taken from the official data producers. Otherwise rough guesses should be sufficient.

Unfortunately this measurement generating process may cause measurement variables which do not have the property to follow the same definitional equation system as the latent variables. But some statistical offices force their measurement variables to have just that property. In this case a somewhat different version of the error process has to be used. To ensure the existence of measurement variables of that kind, one has to assume that another implicit function exists besides the reduced form.

Suppose that the structural econometric model (2.1) is formalized in the following way

$$(6.1) \qquad y_t = y(y_t, y_{t-1}, x_t, a) + u_t, \qquad t=1,2,\ldots,T$$

and that it consists of m_B behavioral equations and $m_D = m - m_B$ definitional equations (equilibrium conditions etc.). Then besides the reduced form

$$(6.2) \qquad y_t = g(y_{t-1}, x_t, a, u_t),$$

there has to exist a continuously differentiable function g_D so that

$$(6.3) \qquad y_{Dt} = g_D(y_{Bt}, y_{t-1}, x_t, a, u_t), \qquad t=1,\ldots,T.$$

where y_{Bt}, y_{Dt} represent the endogenous variables on the left hand side of the behavioral and the definitional model equations respectively.

Property (6.3) leads to the following *adapted measurement generating process* (AMGP):

Compute y^m_{it}, $i=1,\ldots,m_B$, and x^m_{jt}, $j=1,\ldots,n$, in the way defined in MGP. Then compute the measurement variables y^m_{it}, $i=m_B+1,\ldots,m$, by using (6.3) so that for the measurement variables the same definitional equations hold as for the latent variables.

To be able to measure the effect on the forecast error caused by measurement errors in the variables, Schink's parametric stochastic simulation and re-estimation method (see 3.1) is extended to deliver not only the estimates for mean and standard deviation of the forecast error for data free of measurement errors, but also the corresponding values for the data-generating-process AMGP in which measurement errors are included. For this experiment Schink's algorithm is adopted using both the static-static and dynamic-dynamic combinations (see 3.1). As an approximation to the latent variables in the sample period the observed values of the model variables are used. In the forecast period the same error sizes are used as in the sample period for convenience. For the latent exogenous variables in the forecast period any probable course may be taken. The endogenous starting values, i.e. the values of the lagged endogenous variables outside the sample period, are never disturbed.

6.2. A nonlinear model for the German economy

For experimenting on measurement errors in the variables the model for the economy of the Federal Republic of Germany described in this subsection was utilized. It is a log-linearized version of a small model built for simulation purposes from a big macroeconomic model for the FRG (see Weihs, 1986).

The model includes 32 equations, 5 of which are behavioral. The model is nonlinear in the variables only. To reduce computer-time consumption for estimation, nonlinearities in the parameters were removed by log-linearizing two of the five behavioral equations of the original model. The behavioral equations include 18 unknown coefficients, for which LIVE-estimations were computed based on the sample period 1962-1982.

The model stands for the development of the private sector of the German economy and its dependence on governmental decisions and on the development of foreign trade and of interest rates. Especially all the public sector is exogenous. The main part of the non-behavioral equations represents national accounting. Behavioral equations are included for private consumption, private non-inventory investment, the private labor coefficient (employees in the private sector per unit of private gross national product), and for the price indices of consumption and investment. According to Friedman's permanent-income-hypothesis, real private consumption is modeled to be dependent on real disposable income of private households and on their expectations of income in the future. The estimation of the other variables is based on the concept of a representative firm, optimizing its decisions.

This model was used to evaluate the effect of measurement errors in the variables using the error process described in 6.1. The percentages p_k of maximal relative error (see 6.1 MGP), used in the experiment, are based on suggestions of the German statistical office (*Statistisches Bundesamt*). But note that these suggestions did not include the percentages themselves but only a rough ordering. Therefore there is not only one set of percentages compatible with a-priori-information. For the experiment from the possible sets of percentages one set was fixed with percentages of reasonable size. These percentages vary between *0%* (e.g. for dummies) and *20%* (for inventory investment only). Most of the percentages were chosen to be equal to *3%* or *5%*. This set will be called *basic* in what follows. To demonstrate the effect of error size, the basic percentages were also used multiplied by *0.5* and by *1.5*.

The results of the Monte Carlo experiment for the evaluation of the effect of measurement errors to forecast errors are summarized in table 4. The results are based on *500* replications. The meaning of the endogenous variables displayed in tables 4-a, 4-b and 4-c is indicated in the glossary ahead of the tables. The titles displayed at the top of each column in the tables are explained in section 5, with the exception of ERR.5, ERR1., ERR1.5, which stand for the extension of Schink's method to measurement errors in the variables applied using .5, 1., 1.5 times the error percentages of the basic set.

Glossary

P'C	Price index for consumption (1976=100)
P'I	Price index for investments (1976=100)
I'FP	Real non-inventory private investment, Billions-76DM
C'P	Real private consumption, Billions-76DM
KFN'PN	Capital stock in private sector, Billions-DM
YDP'P	Real private gross national product, Billions-76DM
P	Price index for private sector (1976=100)
Y'WP	Wages in private sector, Billions-DM
Y'PNET	Net-profits in private sector, Billions-DM
Y'DIS	Real disposable income of private households, Billions-76DM

Table 4-a

Model for Germany: standard errors computed at 1983

Static resampling

	Det.sol.	ANAL	BOOT	MCC	PSS	ERR1.
P'C	137.6	1.38	1.36	1.36	1.47	1.46
P'I	136.0	1.36	1.36	1.33	1.36	1.51
I'FP	201.4	5.60	6.17	5.52	5.77	6.65
C'P	694.3	6.22	6.38	6.30	6.58	12.5
KFN'PN	4007.	39.9	40.1	39.1	40.0	44.6
YDP'P	1109.	7.82	8.46	7.69	8.20	15.0
P	132.2	1.10	1.06	1.10	1.15	1.90
Y'WP	738.3	10.5	10.5	10.5	10.0	16.1
Y'PNET	285.5	14.3	13.4	14.2	14.7	20.2
Y'DIS	789.0	5.72	6.14	5.57	6.05	13.9

Table 4-b

Model for Germany: standard errors computed at 1983

Dynamic resampling

	Det.sol.	ANAL	BOOT	MCC	PSS	ERR.5	ERR1.	ERR1.5
P'C	137.6	1.38	1.44	1.34	1.40	1.38	1.44	1.57
P'I	136.0	1.36	1.38	1.42	1.44	1.41	1.59	2.16
I'FP	201.4	5.60	5.62	5.53	5.59	6.41	7.37	8.64
C'P	694.3	6.23	6.12	6.25	6.40	8.45	13.0	18.6
KFN'PN	4007.	40.0	40.7	41.7	42.6	41.5	46.8	63.6
YDP'P	1109.	7.82	7.70	7.65	7.83	10.5	15.5	21.3
P	132.2	1.10	1.15	1.08	1.14	1.29	1.84	2.55
Y'WP	738.3	10.5	10.9	10.4	10.6	11.8	15.5	20.2
Y'PNET	285.5	14.3	14.7	14.6	15.2	15.0	19.1	24.8
Y'DIS	789.0	5.72	5.57	5.61	5.69	8.98	14.9	21.6

Table 4-c

Model for Germany: standard errors computed at 1988

		Dynamic simulation			Dynamic resampling			
	Det.sol.	ANAL	BOOT	MCC	PSS	ERR.5	ERR1.	ERR1.5
P'C	161.0	1.94	1.95	2.00	1.90	2.23	2.58	3.17
P'I	163.1	2.24	2.30	2.25	2.22	2.33	2.80	3.95
I'FP	207.5	13.4	12.6	13.8	12.9	13.7	17.7	23.9
C'P	720.3	11.0	10.8	11.0	11.1	13.6	19.8	27.0
KFN'PN	5202.	115.	111.	112.	113.	120.	151.	201.
YDP'P	1195.	17.1	16.5	17.3	16.9	19.6	27.5	37.1
P	146.4	1.62	1.64	1.67	1.59	2.10	2.88	3.93
Y'WP	928.1	17.7	18.3	18.2	17.5	20.5	28.7	38.5
Y'PNET	283.9	17.4	18.2	18.2	18.0	20.7	26.7	34.4
Y'DIS	817.6	10.3	10.3	10.5	10.6	13.0	19.4	27.1

The results in tables 4-a, 4-b and 4-c can be summarized as follows:

- As for the other models discussed in this paper the four methods not including errors in the variables are generating very similar estimates for the standard error of forecast for one-period forecasts and static resampling. But note that at least for this model this is true also for dynamic resampling and one-period forecasts and even for the estimates in the 6th forecast period (1988) having used dynamic resampling and dynamic simulation.
- Using static or dynamic resampling does not affect the estimates of standard errors very much. This is also true for the extension of Schink's method described above, which was included to demonstrate the effect of measurement errors (see ERR1. in tables 4-a and 4-b).
- Looking at the results for the basic set of percentages (see PSS and ERR1.), the effect of errors in the variables to standard errors of forecast varies from nearly no effect (see P'C in 1983) up to an increase of more than 100% (see Y'DIS in 1983). Fortunately the maximal percentage effects do not grow with time using dynamic resampling and dynamic simulation (see 1983 and 1988). Moreover note that standard errors never exceed 10% of the deterministic solution and most of them came out less than 3%. Considering the percentages, the standard errors in I'FP and Y'PNET are the biggest. Indeed, this result is in accordance with a-priori information, since measurement errors in these variables are suspected to be relatively high.
- Comparing the results for the different sets of percentages (see PSS, ERR.5, ERR1., ERR1.5), one should note that the change in the standard error of forecast is increasing faster than linear in relation to the percentages. This behavior is similar for all the forecast periods.

Reviewing the results of this Monte Carlo experiment, one should be really sure that measurement errors in the variables can be excluded intending to estimate reliable sizes for the standard errors of forecast by analytic simulation etc.

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