The canonical econophysics approach to the flash crash of May 6, 2010

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Abstract

We carry out a statistical physics analysis of the flash crash of May 6, 2010 using data from the Dow Jones Industrial Average index sampled at a one-minute frequency from September 1, 2009 to May 31, 2010. We evaluate the hypothesis of a non-Gaussian Levy-stable distribution to model the data and pay particular attention to the distribution-tail behavior. We conclude that there is non-Gaussian scaling and thus that the flash crash cannot be considered an anomaly. From the study of tails, we find that the flash crash followed a power-law pattern outside the Levy regime, which was not the inverse cubic law. Finally, we show that the time-dependent variance of the DJIA-index returns, not tracked by the Levy, can be modeled in a straightforward manner by a GARCH (1, 1) process.

Mathematics Subject Classification: 60G51, 60G18, 91G70

Keywords: flash crash, econophysics, stable distribution, extreme events

1 Introduction

The "flash crash" is the term used to describe the stock-market crash on May 6, 2010 involving US corporate stocks. The Dow Jones Industrial Average (DJIA) suffered its largest intraday decline, 998.5 points. Most of the losses occurred between 2.40 pm and 3.00 pm, with a peak at 2:45 pm (Figure 1). The stocks of Accenture, for example, briefly traded for one cent. The crash was followed by an almost immediate rebound.

The trigger of the crash remains unknown, but some observers point to possible causes, including computer-automated trades and error by human traders. An initial rumor that a trader had typed a sell order for 16 billion shares of Procter & Gamble, instead of 16 million, was later dismissed by regulators. On October 1st, 2010 the Securities and Exchange Commission issued a report blaming a sloppily executed sell order of one mutual-fund group (Waddel & Reed), which started to sell $4.1 billion of "E-Mini" futures contracts through robot trading, taking account only of volume, not time or price. Some analysts blame an intermarket sweep order, anxiety over Greece's bailout package, the British election's outcome, and simply two previous days of declines. Even if not the main cause, robot trading through electronic-trading platforms (such as Direct Edge and BATS), which execute trades in milliseconds, certainly played a role in magnifying the crash. The trigger is uncertain, but a confluence of factors (with no need for a particular trigger) is a more probable explanation for the crash. What is certain, however, is that regulators did not act preemptively; they only reacted in the aftermath. The Securities and Exchange Commission later suggested a market-wide system of "circuit breakers," which would require all exchanges to stop or slow down trading for a few minutes if the market experiences a certain rate of decline. Whether this strategy works well in the future remains to be seen.

If stock markets are viewed as complex systems, there is no need for a trigger to explain a crash [2]. Extreme events need not have a cause when a confluence of factors is involved. Here, we share this viewpoint. As a result, we are interested in assessing the hypothesis of a leptokurtic, fat-tailed non-Gaussian shape of the distribution of index returns. To accomplish this, we collected data from the DJIA index sampled at a one-minute frequency from September 1, 2009 to May 31, 2010, totaling 65,534 observations. We then carried out a canonical statistical physics analysis of the index returns [6, 8, 4] for evaluating the hypothesis of a Levy-stable distribution to model the data. Particular attention was paid to both the tail of the distribution and the time-dependent variance.

The next section presents our analysis, and the conclusions are summarized in Section 3.

2 Methods

Analysis was carried out through eight steps. In the first step, we plotted the probability density function (pdf) of returns, Z, defined as

$$Z_{\Delta t}(t) \equiv Y(t+\Delta t) - Y(t) \tag{1}$$

where Y is an observation of the DJIA index at time t. We first plotted the pdf $P(Z)$ for the returns $Z_{\Delta t}(t)$ using $\Delta t = 1\,\text{min}$ (Figure 2). For better visualization, we plotted the log of $P(Z)$ on the *y*-axis, and we divided the returns Z by the variance calculated from the data, which equals $\sigma = 5.243$, for plotting the values on the *x*-axis. A Gaussian distribution (dotted line) was also plotted for comparison using the same variance value. The pdf of data (circles) is almost symmetric, highly leptokurtic, and non-Gaussian for small index changes. The excess kurtosis is well above 3 ($\kappa = 208.5$), and thus Gaussianity is not detected in any of the standard tests presented in Table 1.

How does the shape of $P(Z)$ change with time? We assessed this (in the second step) by considering increasing values of Δt in the definition of returns (equation 1). Usually, pdfs change in both shape and scale as time changes. However, the Gaussian pdf is stable, that is, it does not change in the functional form but only in the scale (it becomes broader as the time interval increases). The Gaussian pdf is a member of the family of Levy-stable pdfs. There are other pdfs, for example, the Cauchy (Lorentzian) pdf, which is also stable and shows non-Gaussian scaling. Because the data in Figure 2 are compatible with a possible non-Gaussian stable pdf, we evaluated the time behavior of $P(Z)$ by considering several subsets of $Z_{\Delta t}(t)$ for growing values of Δt. The number of records in each set decreased from 65,534 ($\Delta t = 1\,\text{min}$) to 64,535 ($\Delta t = 1{,}000$). Figure 3a shows the pdfs for $\Delta t = 1, 2, \ldots, 10$ (Figure 3b is the corresponding semilog plot). Two patterns emerge: (1) the pdfs spread as Δt increases, as in a random process (Figure 3b); (2) the peaks of the pdfs ($Z = 0$) decrease as Δt increases (Figure 3a), following visible intervals that are suggestive of non-Gaussian scaling.

Scaling means that the stable distributions are self-similar [8, Ch.4]. The scaling variables for a Levy-stable process of index α are

$$\tilde{Z} \equiv \frac{Z}{(\Delta t)^{1/\alpha}} \tag{2}$$

and

$$\tilde{P}(\tilde{Z}) \equiv P(Z) \cdot (\Delta t)^{1/\alpha} \ . \tag{3}$$

The symmetric Levy-stable distribution $P_L(Z)$—with a zero mean, index α, and a positive scale factor γ—is given by the expression [8, Ch.4]:

$$P_L(Z) \equiv \frac{1}{\pi}\int_0^{\infty} e^{-\gamma|q|^{\alpha}} \cos(qZ)dq \tag{4}$$

where $e^{-\gamma|q|^{\alpha}}$ is the characteristic function of the symmetrical stable process. Analytical forms for $P_L(Z)$ are available only for $\alpha = 1$ (Cauchy) and $\alpha = 2$ (Gaussian) in the symmetrical case, and for $\alpha = \frac{1}{2}$ (Levy-Smirnov) in the asymmetrical case.

In the third step, we then estimated the parameters of the Levy distribution, if any, present in the data. Here, we departed from the analyses presented in the reports by Mantegna and Stanley [6, 8] and calculated α and γ by the maximum-likelihood method, following Nolan's approach [9] and using his software Stable.exe (available at http://academic2.american.edu/~jpnolan/).

The log-likelihood function is given by

$$\sum_Z \ln P(Z_{\Delta t}) = \sum_Z \ln L(Z_{\Delta t}) + \sum_Z \ln f(Z_{\Delta t}) + \text{constant}\,. \tag{5}$$

The maximum-likelihood estimation is carried out by minimizing (5) as a function of the distribution parameters. Equation (5) is made up of three terms. For $\Delta t = 1$, the first term depends only on α and γ, whereas the second term depends on the other parameters. Thus, the estimation process can be carried out separately for $\Delta t = 1$. Estimation of α and γ by maximum likelihood yields the same result as when the stochastic process was generated by a Levy-stable pdf. We considered the software's S0 default parameterization, which is (according to Nolan) "better suited to numerical calculations and modeling data than the standard representations. Unless you have a specific reason to use a different parameterization, I suggest that you use the S0 one." We obtained values of $\alpha = 1.493 \pm 0.05$ and $\gamma = 2.0544 \pm 0.05$. Because $0 < \alpha < 2$, the result is suggestive of a Levy-stable pdf, generating the DJIA index returns.

Using these parameter values, in the fourth step, we derived the theoretical Levy pdf (Figure 4). A nice fit was observed for the central region of the pdf, but data in the extremes were located below the theoretical Levy, thus indicating breaking down of scaling. This is not unexpected because the theoretical Levy presents infinite variance, and the second moment in the data is finite. The fit with the empirical pdf is good for the set of observations

$$|Z| < 2.56\sigma\,. \tag{6}$$

Data of the empirical pdf thus are located below the theoretical Levy pdf for $|Z| \geq 2.56\sigma$. The tails decay precisely because variance is finite in the empirical pdf. We then partially concluded that (1) data can be modeled by a Levy-stable pdf for the central region of the pdf, and (2) the tails deserve further scrutiny.

In the fifth step, we then sought scaling considering all the other regions of the empirical pdf and not only in regions where $Z = 0$. We then considered equations (2) and (3) along with $\alpha = 1.493$. Figures 5a and 5b show that all the empirical pdfs (for all values of Δt) collapse to the $\Delta t = 1$ pdf. Again, the fit is good for the central region of the pdfs, thus suggesting that scaling breaks down for long time intervals. We thus conclude that (1) there is non-Gaussian scaling for the DJIA index returns; and (2) this scaling breaks down for long time intervals in the definition of returns.

In the sixth step, we checked for the robustness of the calculated parameters α and γ. We considered nine subsets (months) of the original database and for each of them, repeated the same analysis previously carried out on the entire database (steps 1 to 5 above).

Figure 6 shows that α is located always inside the non-Gaussian Levy regime ($1.4 < \alpha < 1.8$), thus confirming that the hypothesis of a Levy-stable distribution cannot be dismissed from the data. Figure 7 shows that γ experiences strong fluctuations ($1.43 < \gamma < 4.31$). It can be shown [8, Ch.9] that the parameter γ increases as σ increases. Thus, parameter γ is unstable and the Levy pdf fails to track the volatility present in the data.

To put these results in the context of previous works, we note that the value of our index $\alpha = 1.49$ for the DJIA index matches that found in the pioneering work of the S&P-500 index [6], which reported a value of $\alpha = 1.40$. It also matches that of the Bovespa index, namely, $\alpha = 1.66$, calculated previously by one of us [4]. However, our value of $\gamma = 2.0544$ for the DJIA index is greater than that of the S&P 500 ($\gamma = 0.00375$) and Bovespa ($\gamma = 0.00093$) indices. We speculate that our large value for γ may be related to our sample selection, which was aimed exactly at tracking a shorter period on the eve of the flash crash.

Because the Levy pdf of the index $\alpha = 1.49$ cannot explain the rare occurrences of large positive or negative returns of the DJIA index, which occur whenever $|Z| \geq 2.56\sigma$, in the seventh step, we focused on the properties of the empirical $\Delta t = 1$ pdf tails. Here, we estimated tail decay using the two major existing methods.

The first simple (although robust) technique of estimating tail exponents [3] is to run an ordinary least-squares regression for the sizes of the extreme returns ranked from top to bottom ($Z_{(1)} \geq \ldots \geq Z_{(t)}$), that is,

$$\log\left(t - \tfrac{1}{2}\right) = c - \alpha_T \log Z(t) \tag{7}$$

where α_T is an estimate of the decay exponent, whose asymptotical standard error is given by $\sqrt{\frac{2}{t}}\alpha_T$.

Table 2 shows the results for α_T using equation (7) for the tail sizes of 1, 5, and 10 percent of the series of ranked returns. Figure 8 shows the log of absolute returns versus $\log(\text{rank} - \frac{1}{2})$, that is, locally weighted scatterplot smoothing of the DJIA index. The slope corresponds to the estimate of the coefficient α_T in the regression represented by equation (7). The straight line shows evidence of power-law decay well outside the Levy regime of $0 < \alpha \leq 2$. The decay exponent ($\alpha_T \approx 2.5$) is lower than 3, and thus, an inverse cubic power law for extreme events [5] is absent from the data.

To sum up, (1) tail decay follows a power law of exponent roughly equal to 2.5, suggesting a pattern for the flash crash; (2) the tail exponent is located outside the Levy regime, thus confirming that the Levy pdf cannot account for the extreme events; and (3) the flash crash cannot be accommodated by an inverse cubic law.

To confirm our analysis of the tails, we considered a second method [5]. We again studied the returns Z for $\Delta t = 1\,\text{min}$ and divided the values by the variance calculated from the data, that is, $\sigma = 5.243$, as shown in equation (8):

$$g(t) = \frac{Z(t)}{\sigma}. \tag{8}$$

We then formed a time-series by ranking the normalized returns from top to bottom, and plotted a cumulative probability distribution (cdf), that is, the probability for a data point being larger than or equal to a threshold g, $P(g) \equiv P\big(g(t) \geq g\big)$, as a function of g. The decay exponent α_T was obtained by calculating the slope of the plot of $\log P(g)$ against $\log g$. In fact, the data are well fit by the following power law:

$$P(g) \approx g^{-\alpha_T} \tag{9}$$

which, in logs, is represented as

$$\log P(g) = -\alpha_T \log g\,. \tag{10}$$

Table 3 shows that the decay exponents are only slightly smaller than the ones calculated using the previous method. Figures 9–11 show the power laws for the tail sizes of 1, 5, and 10 percent of the series of ranked returns.

In the final step of our analysis, we tackled the time-dependence of σ. As described, because the parameter γ is unstable, the Levy pdf fails to track the volatility in data. Here, the natural candidate to capture this time-dependent volatility is a GARCH(1, 1) model [8, Ch.10; 7, 10]

$$\sigma_t^2 = a_0 + a_1 Z_{t-1}^2 + b_1 \sigma_{t-1}^2 \tag{11}$$

where a_1, a_2, and b_1 are the control parameters. In the GARCH(1, 1) model, although σ is locally nonstationary, it is also asymptotically stationary. The asymptotic, unconditional variance is given by

$$\sigma^2 = \frac{a_0}{1 - a_1 - b_1} \tag{12}$$

and the kurtosis is expressed as

$$\kappa = 3 + \frac{6a_1^2}{1 - 3a_1^2 - 2a_1 b_1 - b_1^2}\,. \tag{13}$$

Using equations (12) and (13), considering the values $\sigma = 5.243$ and $\kappa = 208.5$ measured from the empirical data of $\Delta t = 1\,\text{min}$, along with b_1 = 0.9, we get a_0 = 0.2348 and a_1 = 0.0915. The value of $b_1 = 0.9$ is chosen from the finance literature [1]. The conditional variance is then described by

$$\sigma_t^2 = 0.2348 + 0.0915 Z_{t-1}^2 + 0.9\sigma_{t-1}^2\,. \tag{14}$$

Figure 12 shows a comparison of the empirical $\Delta t = 1$ min pdf from the DJIA index data with the unconditional pdf of a GARCH(1, 1) process characterized by a_0 = 0.2348, a_1 = 0.0915, and b_1 = 0.9. The agreement between the two pdfs is good.

Because the GARCH(1, 1) process has finite variance, the central limit theorem for the sum of random variables applies. This means that the GARCH(1, 1) process asymptotically converges to the Gaussian basin of attraction [8, Ch.10]. Because this is an unstable process, it shows no scaling. Thus, the GARCH(1, 1) process is able to describe the pdf of the DJIA index at $\Delta t = 1$ min time horizon; however, it fails to describe properly the scaling properties of pdfs at different time horizons present in the data.

3 Conclusions

Our study of the unique period preceding the flash crash of the DJIA index sampled at a one-minute frequency from September 1, 2009 to May 31, 2010 allows us to conclude that

1. The probability density function of the DJIA index returns is almost symmetric, highly leptokurtic, and non-Gaussian for small index changes (there is heavy excess kurtosis, $\kappa = 208.5$).
2. The pdfs spread as the time interval in the definition of returns increases, as in a random process. In addition, the peaks of the pdfs decrease as the time intervals increase, following a pattern that is indicative of non-Gaussian scaling.
3. An index parameter of $\alpha = 1.49$ can be calculated from the data. Because $0 < \alpha < 2$, this result is suggestive of a Levy-stable pdf generating the DJIA index returns. The fit is good for the central region of the empirical pdf but not for the tails, thereby demanding a closer look at tail behavior.
4. This also means that scaling should break down for long time intervals in the definition of returns.
5. A parameter $\gamma = 2.0544$ is also established, which is greater than those typically found for other stock-market indices. Nevertheless, because γ is related to volatility, the sample selection targeting the crash episode may explain the high value for the parameter.
6. The index α remains inside the Levy-stable regime as we consider subsamples of data. However, the parameter γ experiences strong fluctuations across the subsamples. Because parameter γ is unstable, the Levy pdf fails to track the volatility in the data, thus demanding further analysis of σ.
7. Because the Levy pdf of index $\alpha = 1.49$ cannot also explain the large positive and negative returns of the DJIA index, we estimate the tail decay of the empirical pdf through two major methods. Both find that tail decay follows a power law of exponent greater than two but less than three, thus suggesting a power-law pattern for the flash crash, which is not the inverse cubic law.
8. Because the Levy pdf cannot explain the fact that σ is time-dependent, the natural candidate to complement the analysis is to couple it with a GARCH(1, 1) process. As a result, we find a formula from the GARCH(1, 1) model for the time-evolution of the volatility of the DJIA index data. But, because unstable distributions show no scaling, the GARCH(1, 1) model fails to detect the non-Gaussian scaling present in the data.

To sum up, the hypothesis that the DJIA index returns are generated by a Levy-stable distribution cannot be dismissed. Scaling is present, and it is clearly non-Gaussian. This means that the flash crash cannot be considered as an outlier: it was an unpleasant fact rather than an anomaly. A closer look at the extreme events in the distribution tails shows the presence of power-law decay outside the Levy regime, but not the inverse cubic law. This means that the flash crash follows a power-law pattern outside the Levy regime, which is not also the inverse cubic law. Levy distributions are characterized by infinite variance, and thus, by definition, they cannot model finite data on variance. The time-dependent variance of the DJIA index has therefore been modeled using a GARCH (1, 1) process. Nevertheless, because the latter is an unstable process, there is no scaling. Non-Gaussian scaling in data thus suggest that a specific type of Levy-stable distribution (rather than any unstable distribution, such as the GARCH(1, 1) pdf) is the primary source generating the DJIA-index returns.

Table 1 Normality tests for the $\Delta t = 1$ empirical pdf of the DJIA index

Lilliefors	Cramer-von Mises	Anderson-Darling	Kurtosis
0.1215 (0.0000)	454.8362 (0.0000)	2553.2260 (0.0000)	208.50

Note: p-values shown in brackets. All the p-values are close to zero; therefore, the null of Gaussianity is rejected for any standard (0.01, 0.05, and 0.10) significance level

Table 2 Estimates of the decay exponents of the extreme returns of the DJIA index: method of Gabaix and Ibragimov [3]

Tail size	α_T (positive tail)	α_T (negative tail)
1%	2.30 (0.1778)	2.20 (0.1738)
5%	2.53 (0.0875)	2.41 (0.0851)
10%	2.54 (0.0620)	2.38 (0.0595)

Note: standard error in brackets

Table 3 Estimates of the decay exponents of the extreme returns of the DJIA index: method of Gopikrishnan *et al.* [5]

Tail size	α_T (positive tail)	α_T (negative tail)
1%	2.23 (0.0622)	2.12 (0.0553)
5%	2.39 (0.0652)	2.26 (0.0534)
10%	2.33 (0.0681)	2.21 (0.0593)

Note: standard error in brackets

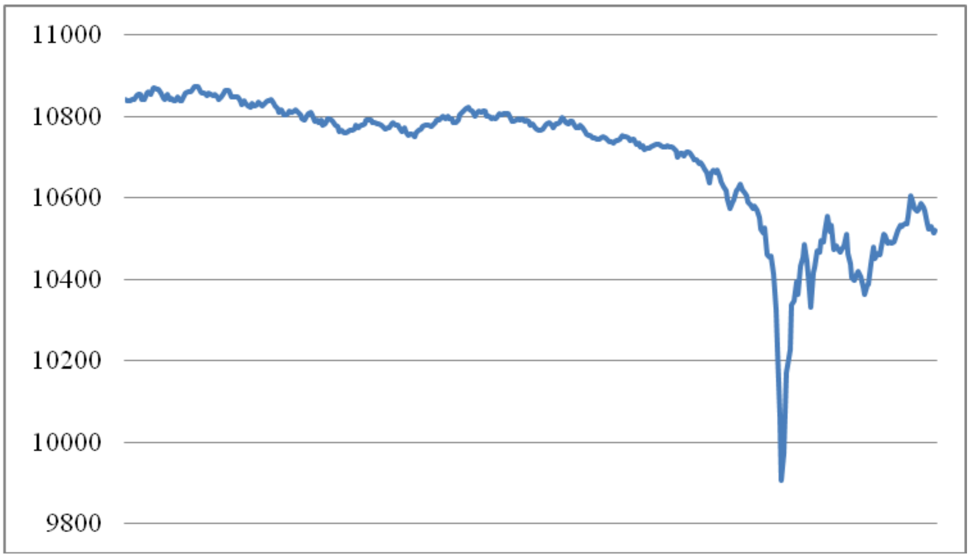

Figure 1 Daily chart of the Dow Jones Industrial Average index during the May 6, 2010 flash crash

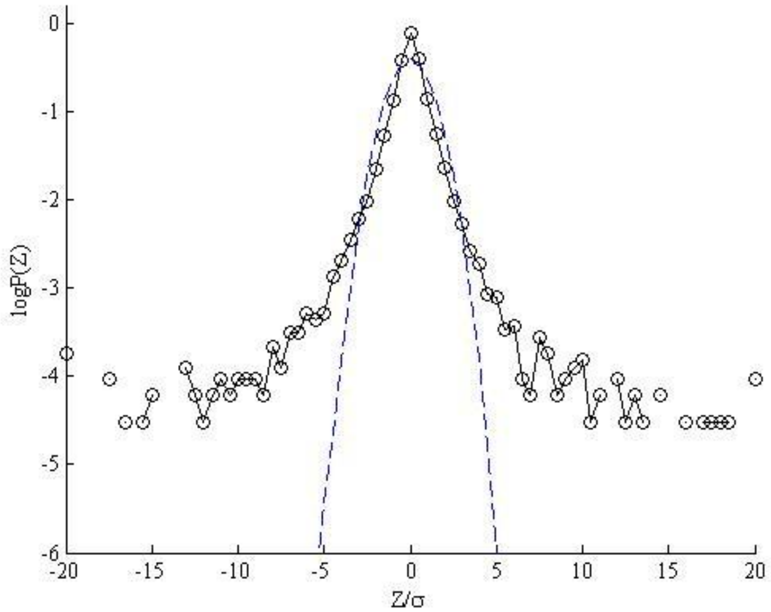

Figure 2 The $\Delta t = 1$ min pdf for the DJIA index from September 1, 2009 to May 31, 2010

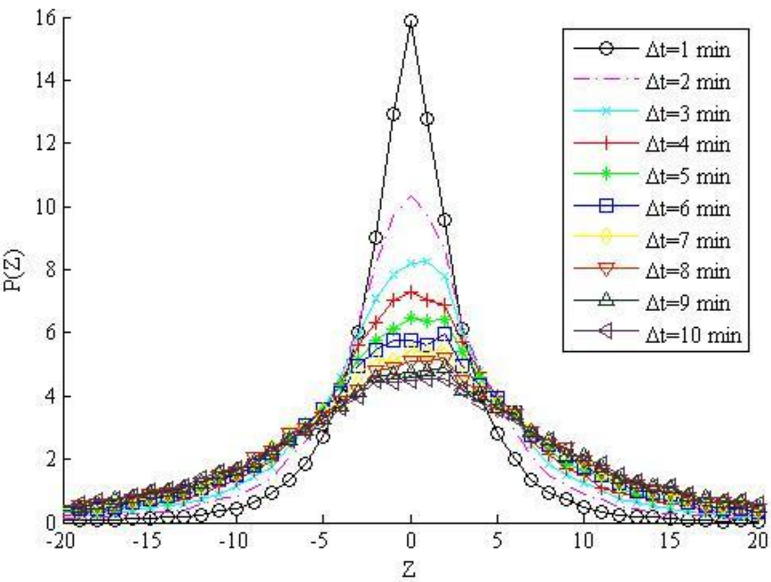

Figure 3a Pdfs of the DJIA index changes measured at different time horizons $\Delta t = 1, 2, \ldots, 10$

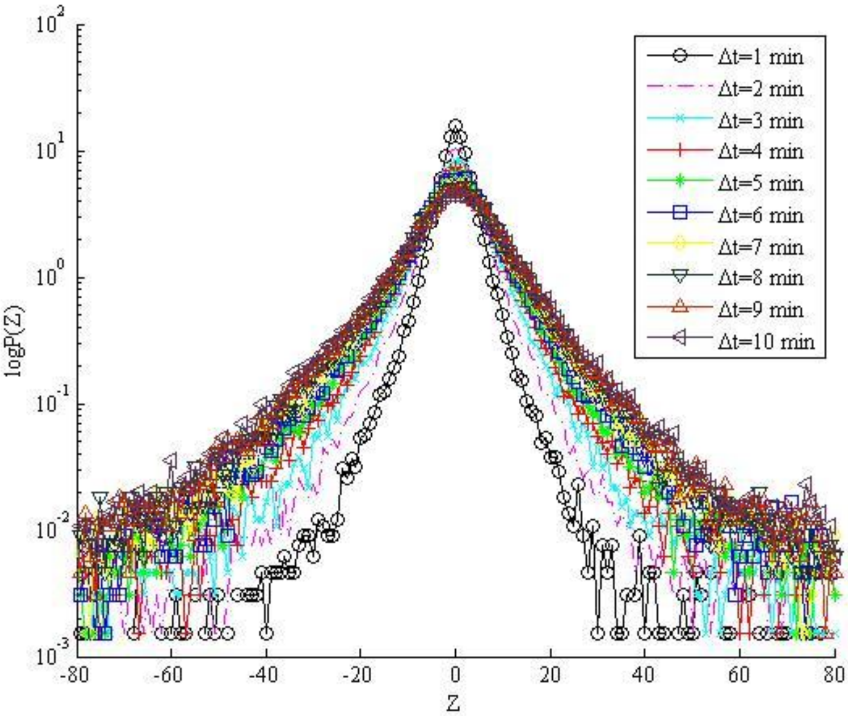

Figure 3b Semilog plot of the pdfs of the DJIA index changes measured at different time horizons $\Delta t = 1, 2, \ldots, 10$

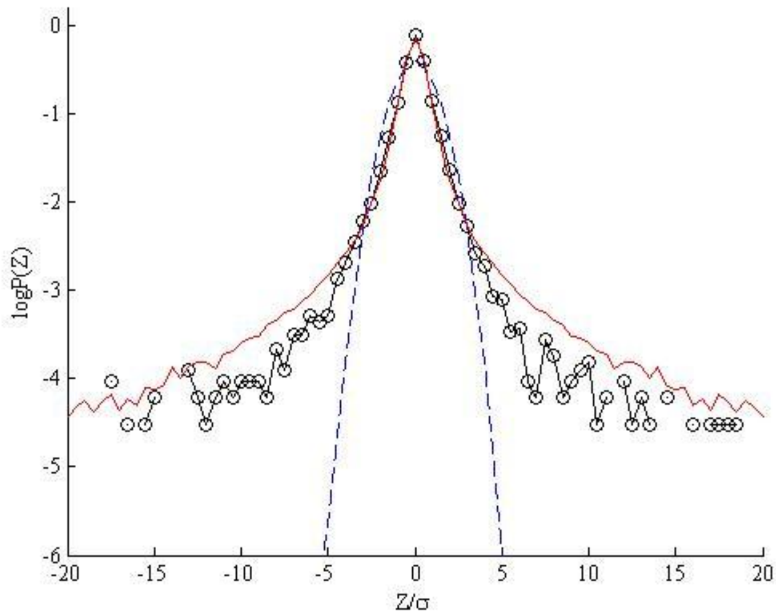

Figure 4 The $\Delta t = 1$ min pdf for the DJIA index from September 1, 2009 to May 31, 2010. The Gaussian pdf (dotted line), using the variance $\sigma = 5.243$ obtained from data, is also plotted for comparison. The Levy pdf (solid line) is further obtained using $\alpha = 1.493$ and $\gamma = 2.0544$

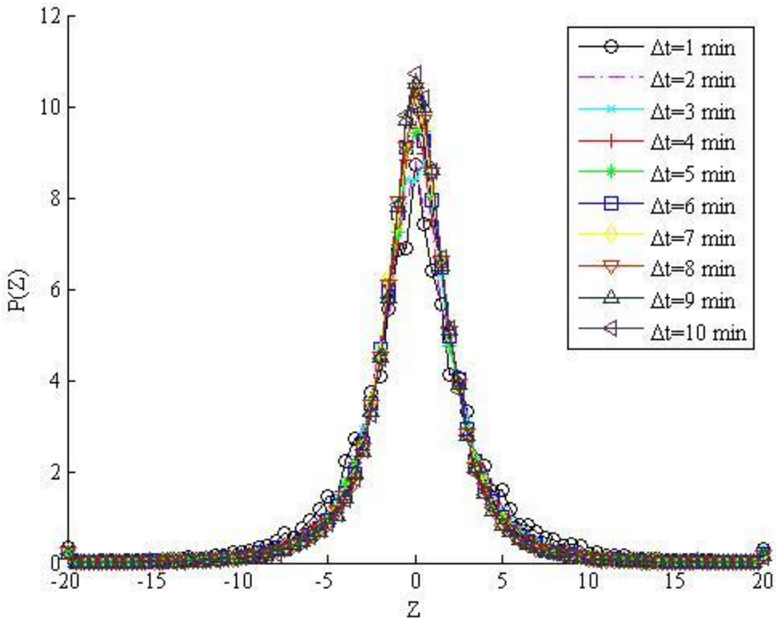

Figure 5a The same pdfs as in Figure 3a plotted in scaled units. Scaling is performed using the value $\alpha = 1.493$

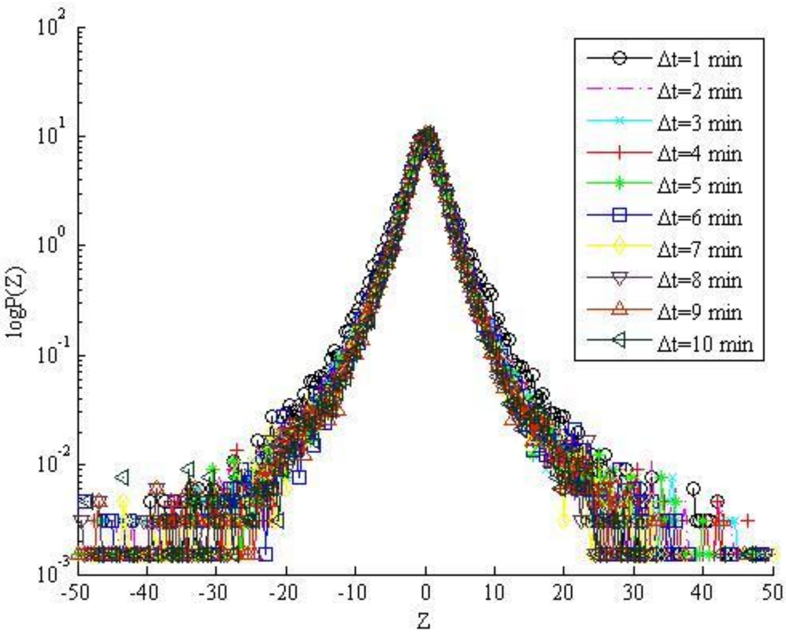

Figure 5b The same pdfs as in Figure 3b plotted in scaled units. Scaling is performed using the value $\alpha = 1.493$

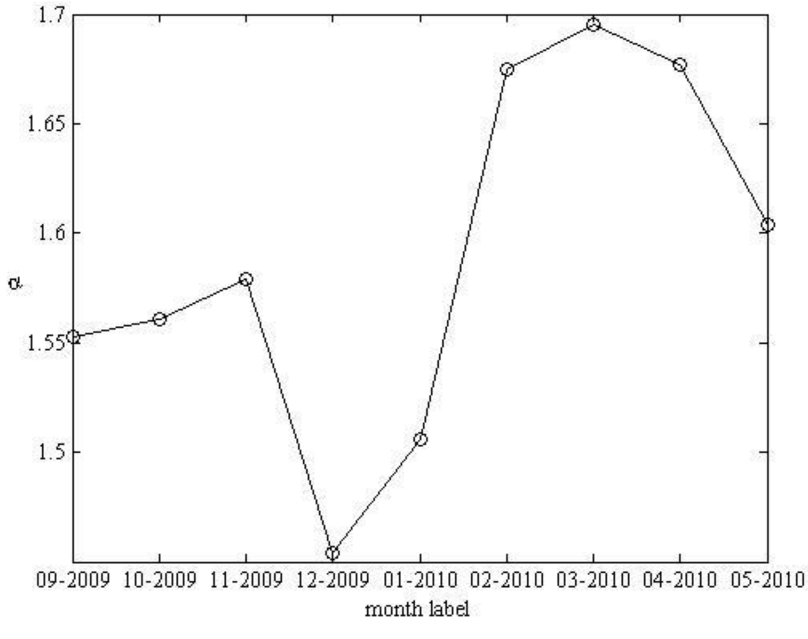

Figure 6 Time-dependence of the index α analyzed on a monthly time scale. Parameter α remains inside ($1.4 < \alpha < 1.8$) the non-Gaussian Levy regime ($0 < \alpha < 2$)

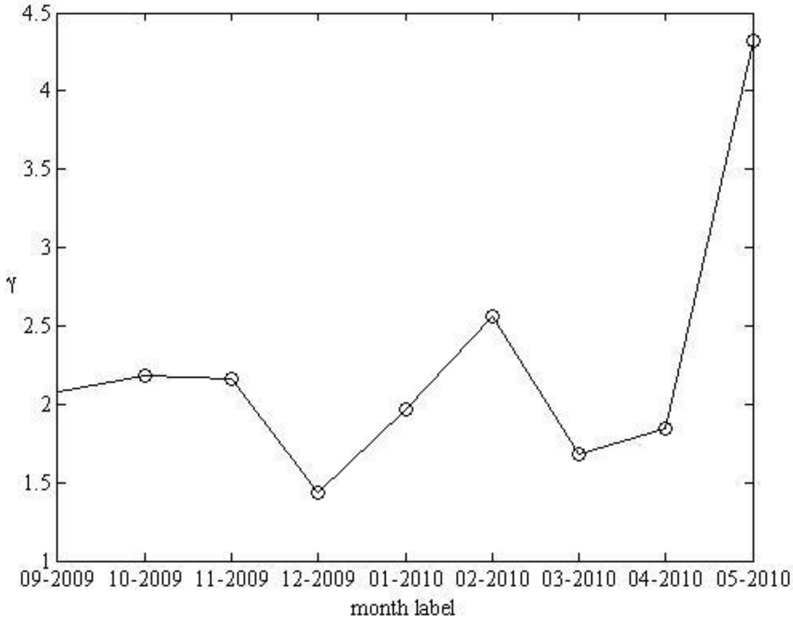

Figure 7 Time-dependence of the parameter γ analyzed on a monthly time scale. Because γ is unstable, the Levy pdf fails to track the volatility in data

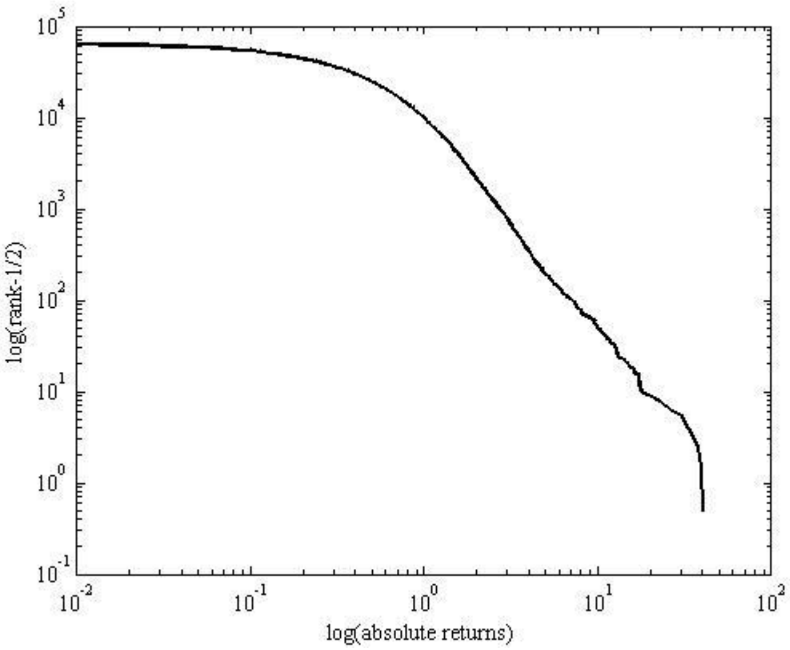

Figure 8 Log of absolute returns versus log(rank $-\frac{1}{2}$) of the DJIA index. The slope corresponds to the estimate of the coefficient α_T in regression (7). Power-law decay is observed with an exponent of roughly 2.5, outside the Levy regime and below the level of 3 (inverse cubic law)

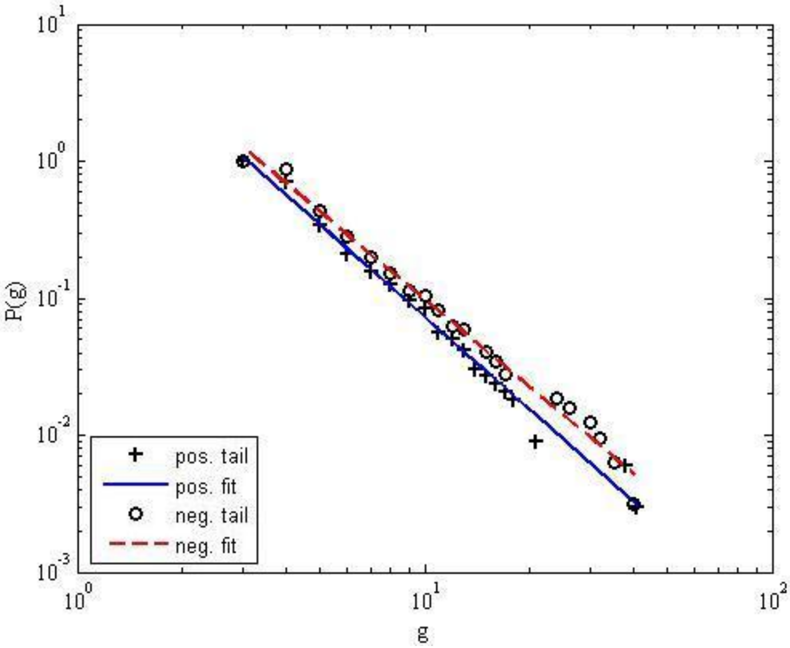

Figure 9 Log-log plot of the cdf of $P(g)$ for the tail size of 1 percent of the ranked, normalized returns $g(t)$: power laws emerge from the data with decay exponents greater than 2 but less than 3

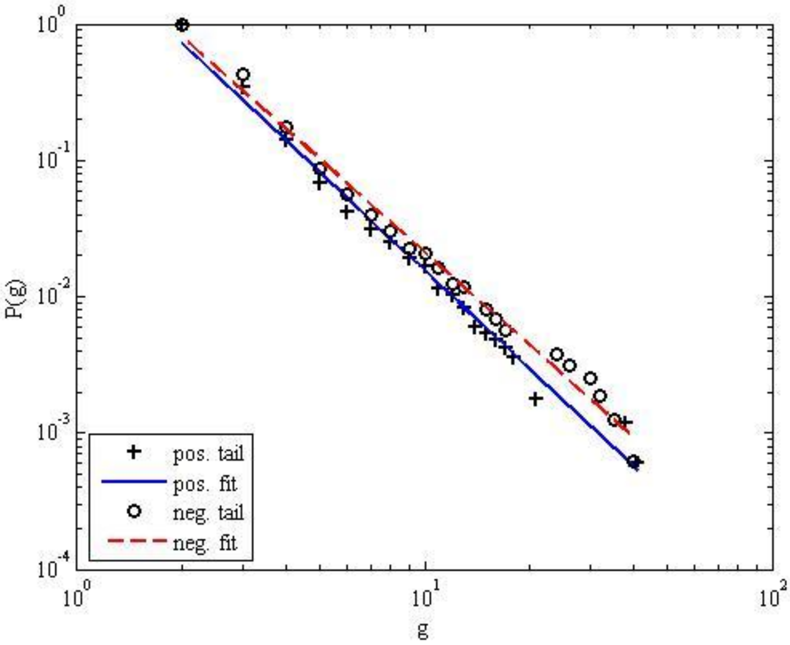

Figure 10 Log-log plot of the cdf of $P(g)$ for the tail size of 5 percent of the ranked, normalized returns $g(t)$: power laws emerge from the data with decay exponents greater than 2 but less than 3

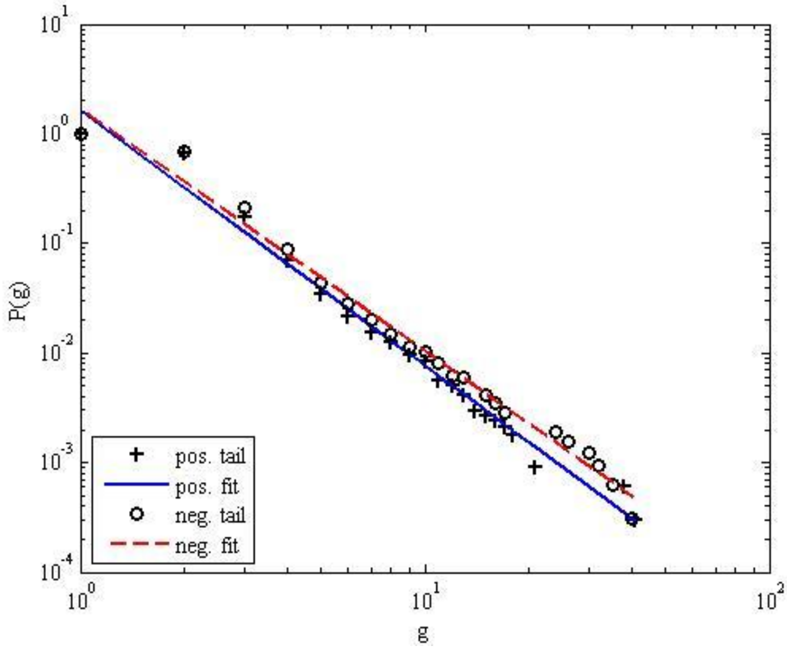

Figure 11 Log-log plot of the cdf of $P(g)$ for the tail size of 10 percent of the ranked, normalized returns $g(t)$: power laws emerge from the data with decay exponents greater than 2 but less than 3

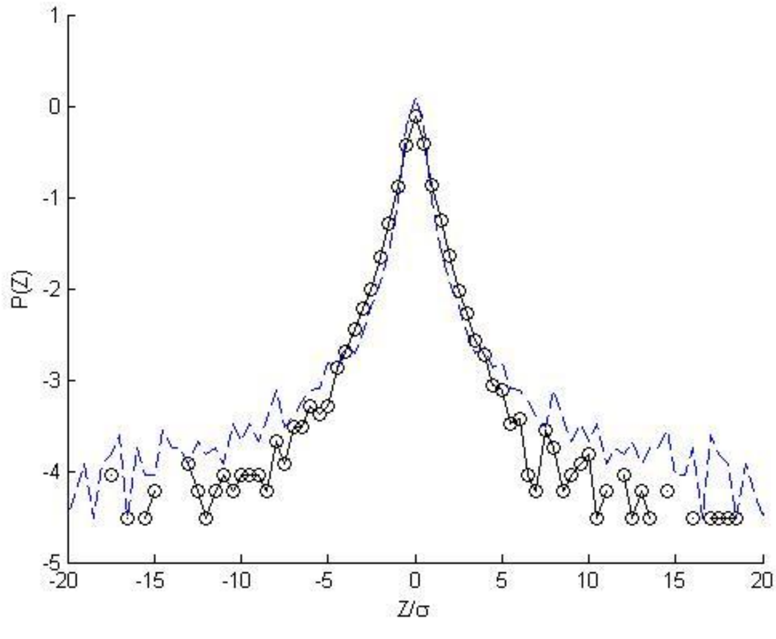

Figure 12 Fit of a GARCH(1, 1) process (dashed line) to the empirical $\Delta t = 1\,\text{min}$ pdf from the DJIA index data (circles). The unconditional pdf of the GARCH(1, 1) process is characterized by $a_0 = 0.2348$, $a_1 = 0.0915$, and $b_1 = 0.9$

References

[1] V. Akgiray, Journal of Business, **62** (1989), 55.

[2] P. Bak and M. Paczuski, Proceedings of the National Academy of Sciences of the USA, **92** (1995), 6689.

[3] X. Gabaix and R. Ibragimov, NBER Working Papers, **342** (2007), 1.

[4] I. Gleria, R. Matsushita and S. Da Silva, Economics Bulletin, **7** (2002), 1.

[5] P. Gopikrishnan, M. Meyer, L.A.N. Amaral and H.E. Stanley, European Journal of Physics B, **3** (1998), 139.

[6] R. Mantegna and H.E. Stanley, Nature, **376** (1995), 46.

[7] R. Mantegna and H.E. Stanley, Physica A, **254** (1998), 77.

[8] R. Mantegna and H.E. Stanley, An Introduction to Econophysics: Correlations and Complexity in Finance, Cambridge University Press, Cambridge, 2000.

[9] J.P. Nolan, in: O.E. Barndorff-Nielsen, T. Mkosch and S. Resnick (Eds.), Lévy Processes, Brikhauser, Boston.

[10] B. Podobnik, P.C. Ivanov, I. Grosse, K. Matia, H.E. Stanley, Physica A, **344** (2004), 216.