Time Compression

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Abstract

Economists have generally ignored the notion that perceived time may differ from clock time. Borrowing from the behavioral psychology literature, we investigate the case of *time compression* whereby perceived time passes more quickly than actual time. A framework is presented to embed time compression in economic models. We then apply the principle to a standard lifecycle permanent income model with endogenous labor. Time compression provides an alternative explanation of why older individuals, even those without declining labor productivity, may choose to reduce their work effort.

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1 Introduction

In this paper, we endogenize time. Economists have traditionally treated 'time' as a deterministic process that is outside the control of agents. We deviate from this tradition by allowing agents to make decisions based on the psychological, rather than the actual, passage of time. An agent's inner clock and the actual clock are often out of sync (Flaherty and Meer (1994)). An hour in a dentist's chair can feel like an eternity while a busy day at work can seemingly pass by in an instant. These perceptions of time can have an important influence on economic behavior. Economists have long recognized that beliefs about the future are central in determining economic outcomes (e.g., Keynes (1936); Friedman (1957)). We extend this principle by allowing beliefs about the passage of time to alter behavior. In the process, we endogenize time by creating a formal relationship between perceived and clock time, embed this relationship in a standard economic model and explore the associated behavioral implications.

Our primary interest is in a phenomenon we call *time compression*. Time compression refers to the idea that a person's inner clock may run faster than the actual clock, so for example, one calendar year may seem to pass in just a few months. A commonly expressed feature of time compression is that it tends to worsen with age; life speeds up as you get older. An implication of this phenomenon is that behavioral plans may no longer be time consistent. The lifecycle plans of a 25 year-old may no longer be optimal when the person reaches age 50 and time seems to be speeding by. This leads to a number of interesting questions. For instance, how should someone adjust their allocation of labor and leisure as time begins to speed up and death appears to be approaching at a faster rate? Does this encourage one to work fewer hours and spend more time in leisure activities or does it encourage one to work longer hours and build up a nest egg so as to enjoy future consumption and leisure when time will be speeding by even faster? Are there any consumption or leisure choices that an agent can make to reverse or mitigate time compression? These are fascinating questions that we hope to make progress toward answering.

Toward that end, we embed time compression into the standard lifecycle permanent-income (LCPI) model with endogenous leisure. The standard LCPI individual with constant productivity and a synchronized internal clock will choose a constant level of work intensity throughout the lifecycle. Our primary finding is that when time compression worsens with age, individuals increase work intensity when young and decrease work intensity when old. This effect is much stronger for individuals that anticipate time compression and build it into current plans, but it is also present to a lesser degree for myopic individuals who expect the current degree of time compression to continue throughout the lifecycle. The results provide a new and provocative explanation of why older workers choose to retire or scale back work effort.

2 Brief Review of the Research on Time Perception

Time compression is more than just folk theory. There is an impressive amount of scientific and experimental evidence that subjective and objective time are often out of sync. Here we review the time-perception literature from the fields of psychology and economics.

2.1 Psychology Literature

Hoagland (1933) was one of the first to provide evidence of an internal clock, which could be manipulated by varying body temperature.[1] Rammsayer (1997) shows that dopamine activity, which naturally decreases with age (Volkow *et al.* (1998)), is positively correlated with the perceived duration of short spans of time. Therefore, as we age and the dopamine activity in our brains decreases, we may sense that time is seeming to speed up. In an experimental test of this theory, Mangan and Bolinskey (1997) developed a counting exercise to test whether the perceived passage of time varies with age. They found that time does indeed appear to pass at a higher rate as people grow older. In particular, they found that people in their 20's tended to have synchronized clocks while those in their 60's systematically underestimated intervals of time. In a similar spirit, Crawley and Pring (2000) report that time appears to pass more rapidly in middle-aged individuals through an experiment where individuals of various ages estimate the amount of time that had elapsed since major past public events. Flaherty and Meer (1994) also reports that middle-aged individuals tend to perceive that time passes more rapidly. One possible explanation for these results is given by Chaston and Kingstone (2004), who report that people tend to underestimate the amount of elapsed time when their attention is engaged. To the extent that individuals accumulate responsibilities and develop busier schedules as they reach middle age, this may explain the sense that time is getting compressed.

Philosophers have been studying the nature of time for centuries. In the 4th century A.D., Saint Augustine wrote in his *Confessions*:

[1] Hoagland's discovery apparently occurred during a trip to the local drugstore for his feverishly ill wife. After his return from his 20 minute trip, his wife was certain that he had been gone for well over an hour.

> ...O Lord, we perceive intervals of times, and we compare them with themselves, and we say some are longer, others shorter. We even measure by how much shorter or longer this time may be than that...When, therefore, time is passing, it can be perceived and measured; but when it has passed, it cannot, since it is not.

In more recent times, William James, the prominent American psychologist, wrote on the notion of time compression stating that "the same space of time seems shorter as we grow older" (James (1892)). James attributes this phenomenon to the fact that in old age one is so accustomed to the events of life that they no longer leave individual memories.

Psychologists have established several formal theories to explain how we perceive a duration of time. Ornstein (1969) develops a storage-size model where the perceived duration of an interval is negatively related to the amount of information available during the interval and the process involved to encode the information. Thomas and Weaver (1975) develop a processing-time model to explain the perceived duration of an interval with information content I. Mathematically, the model assumes that perceived duration of the interval (τ) depends on a timer (f) and a stimulus processor (g) according to

$$\tau = af(t, I) + (1 - a)g(I) \tag{1}$$

where t is the actual duration and a is a weighting parameter. Thomas and Weaver apply their model to microseconds, but Poynter (1989) states that the "basic concepts seem well suited to more general applications." When an interval has limited stimulus information, a is close to one and the internal time processor $f(t, I)$ receives the greater weight. For example, when sitting idly at a stop light, the agent will rely almost exclusively on her inner timer to judge the duration of the experience. On the other hand, an interval with a great deal of stimulus will correspond to an a close to zero and most weight placed on the stimulus processor. Michon (1985) describes this latter situation:

> ...an exciting holiday or romantic evening have a content I so absorbing that little or no attention remains for temporal cues (t) that such events might in principle provide. The resulting immediate judgement therefore tends to severely underestimate the actual duration.

Finally, the memory-change model states that duration judgements depend on the number of perceived events during the interval and how memorable they are. James (1892) commented on this over 100 years ago, stating "in general a time filled with varied and interesting experiences seems short in passing ... [while] a tract of time empty of experiences seems long in passing." Several experimental psychology studies have found that filled intervals are perceived to be of shorter duration than empty ones (e.g., Hicks *et al.* (1976); Zakay *et al.* (1983)).

2.2 Economics Literature

The economics literature has largely ignored the differences between perceived and actual time. When economic agents look into the future to solve dynamic optimization problems they are assumed to accurately perceive how much time will pass between now and a known future event. However, the evidence cited above suggests that agents often underestimate the amount of time that elapses between dates, and these misperceptions tend to worsen with age. We are unaware of a single study that has rigorously examined the behavioral implications of age-dependent time misperceptions in a modern economic framework.

Lowenstein and Prelec (2003) briefly mention the possibility that inner and actual "'clocks' [may] run at different speeds." Samuelson (1976) discusses the various possible rules that relate perceived and actual time (e.g., logarithmic or power rules) and whether the 'speeding up of time with age' is a backward-looking or forward-looking phenomenon. In the backward-looking case, Samuelson postulates that time tends to speed up because of the accumulation of life experiences and, similar to Eisler (1975), suggests a power function of the form

$$\tau = t^{\alpha}/\alpha \tag{2}$$

where $0 \neq \alpha \leq 1$. In the forward-looking case, life tends to speed up as one grows older because the percentage of life remaining diminishes as we grow older. To capture this notion, Samuelson proposes that perceived time may follow

$$\tau = \int_0^t e^0(u)du \tag{3}$$

where $e^0(t)$ represents the average number of days a person of age t expects to live. As a final contribution, Samuelson also presents several axioms that in principle lead to testable hypotheses of the proposed relationships between perceived and actual time.

Ahlbrecht and Weber (1995) use time compression to argue that in many instances hyperbolic discounting can be used in prescriptive theory. They show that a concave time-perception function with standard exponential discounting will lead to a preference structure observationally equivalent to hyperbolic discounting. Ahlbrecht and Weber therefore claim that hyperbolic discount functions are not "arbitrary but are founded on sound principles of rationality." In a similar spirit, several recent studies (i.e., Takahashi (2005); Takahashi (2009); Zauberman *et al.* (2009); Kim and Zauberman (2009)) conclude that nonlinear time compression can explain hyperbolic discounting behavior.

2.3 Limitations of the Previous Literature

One limitation of the literature above, particularly that from psychology, is that it often focuses on short durations in retrospect. For our purposes, we are more interested in forward-looking durations of much longer intervals. A second limitation of the literature on time perception is that it provides only a few concrete relationships between perceived and clock time that can be directly applied to analytical models. Compiling information from over 100 experimental studies, Eisler (1975) suggests a power function $\tau = at^b$ for the relationship between actual time (t) and perceived time (τ), where $0.5 \leq b \leq 1$. Kim and Zauberman (2009) also use a power function, calling a the time-contraction parameter and b the sensitivity-to-time parameter. Michon (1985) states that "longer intervals (>20 s) may be represented proportional to the square root of physical time" (i.e., $b = 0.5$). But he goes on to state that "at the upper end, beyond 20 s most adults will by and large, but not always, linearize their time scales with the help of clocks and calendars and with a great deal of experience in linearizing time." Macar (1985) and Takahashi (2005) rely on Weber's Law (i.e., the just noticeable difference in a stimulus is a constant proportion of the original stimulus) to suggest a logarithmic relationship $\tau = a \ln(t)$. Cohen (1967) develops an experiment where subjects mark various points on a time line representing their lifecycle. He finds evidence for a logarithmic scale (back to 12 months) and linear thereafter. Cohen also mentions that future perceived time intervals may be represented on a logarithmic scale. In a similar experiment, Zauberman *et al.* (2009) also find evidence of logarithmic time compression. Campbell (1990) summarizes the experimental studies on the perceived duration of elapsed time. Nearly all studies find that time is compressed with a typical subjective hour lasting between 1.08 and 1.47 objective hours, or flipping this around, one objective hour is perceived to last somewhere between 40 to 55 minutes.

In sum, there exists both scientific and anecdotal evidence that perceived time not only differs from clock time, but it seems to speed up as we get older. The implications for economic theory are both intriguing and relatively untested.

The rest of the paper is organized as follows. Section 2 presents a formal characterization of time compression, embeds it into an agent's dynamic optimization problem, and distinguishes it from standard notions of discounting. Sections 3 applies time compression to the life-cycle consumption/leisure problem. Section 4 summarizes our findings and discusses possible future research.

3 Economic Framework for Perceived and Clock Time

In this section, we formally define *time compression* and present an analytical framework that captures the relationship between perceived and actual time. Start by considering an agent who is standing at any time period $t_0 \in [0, T]$. The agent chooses a planned path for the control variable $x(t)$ over the interval $t \in [t_0, T]$ in order to maximize

$$\int_{t=t_0}^{T} F(\tau\Delta) \cdot u\left(x(t)\right) dt \tag{4}$$

where $F(\cdot)$ is the discount function; $\Delta = t - t_0$ is the planning horizon; τ is the time-perception parameter; and $u\left(x(t)\right)$ is the utility function with derivatives $u' \geq 0$ and $u'' \leq 0$. The optimal plan involves maximizing (4) by choosing $x(t)$ for all $t \in [t_0, T]$ subject to standard dynamic constraints, initial conditions, and endpoint conditions.

The parameter $\tau > 0$ acts as a scale parameter for durations of time.[2] If $\tau = 1$, the agent's inner and actual clocks are synchronized and the perceived distance of any future date equals the actual distance. If $\tau \neq 1$, then perceived and clock time are asynchronized. This leads to our definition of time compression (and time expansion):

Definition 1 *An agent experiences* ***time compression (expansion)*** *if* $\tau < (>) 1$.

Our primary focus is on time compression but the framework is sufficiently general to allow for time expansion. The definition above is moot about the change in time compression through the lifecycle. To capture the notion that 'time speeds up as you get older', we allow the time-perception parameter τ to be age-dependent: $\tau = \tau(t_0)$.

[2] The studies reviewed above generally treat τ as the perceived duration of the interval $\Delta = t - t_0$, where $t_0 = 0$. We instead treat τ as a scale parameter such that $\tau\Delta$ is the perceived length of Δ.

Definition 2 *An agent experiences* ***age-increasing time compression*** *if* $\tau'(t_0) < 0$ *for any* $t_0 \in [0, T]$.

Although time compression will alter behavior, if time compression stays constant through the lifecycle (i.e., $\tau'(t_0) = 0$), there will be no incentive to deviate from the original plan; the original plan will be time consistent. Age-dependent time compression, however, will produce time inconsistent plans because future events will appear to be approaching at differing rates across the lifecycle and will alter relative intertemporal tradeoffs.

Next, we turn to a discussion of the relationship between time compression and discounting.

3.1 Time Compression and Discounting

Time compression causes future dates to be perceived as closer than they actually are. As a result, the future will not be discounted as heavily, and all else equal, time compression will encourage long-term planning and a greater concern for the future. To better understand these effects, we consider how time compression interacts with standard notions of discounting. The choice of time perception can produce exponential, hyperbolic or age-dependent discounting as special cases.

Recall, the discount rate is defined by

$$r(\Delta) = \frac{-F'(\Delta)}{F(\Delta)}. \tag{5}$$

The two most common discount functions are exponential and hyperbolic. The exponential discount function

$$F(\Delta) = \exp(-\rho\Delta) \tag{6}$$

produces a time-invariant discount rate $r(\Delta) = \rho$ and time-consistent optimal plans (Strotz (1956)). To better match experimental data, researchers have recently begun to use a hyperbolic discount function (Ainslie and Haslam (2003)):

$$F(\Delta) = (1 + \alpha\Delta)^{-\rho/\alpha}. \tag{7}$$

When α tends to zero, $F(\Delta)$ approaches the exponential function so that exponential discounting can be thought of as a limiting case of hyperbolic discounting. An important feature of hyperbolic discounting is that the discount rate $r(\Delta) = \rho/(1 + \alpha\Delta)$ is decreasing in Δ, producing time inconsistent optimal plans and preference reversals (Lowenstein and Prelec (1992)). Figure 1 shows the standard discount functions with and without time compression. Hyperbolic discount curves have a present-period bias relative to exponential discounting; they are relatively steeper in the near future and flatter in the distant future. The discount functions with constant time compression are everywhere higher (i.e., less discounting) because future time periods are perceived to be closer.

As Ahlbrecht and Weber (1995) point out, hyperbolic discounting is observationally equivalent to exponential discounting with time compression of the following form:[3]

$$\tau(\Delta) = \frac{\ln(1+\alpha\Delta)}{\alpha\Delta}. \tag{8}$$

In this case, an individual discounts the future exponentially but perceives future dates to be increasingly less distant than they are in actual clock time. Therefore, time compression is capable of producing the same notions such as regret, procrastination, and self control that accompany time-inconsistent preferences. However, our specification of age-dependent time compression, $\tau(t_0)$, is also capable of matching evidence that impatience and discount rates tend to decline with age (Green *et al.* (1999)).

We now turn to specific functional forms for the time compression parameter τ.

3.2 Specifications for Time Compression

We consider two different types of time compression: exogenous, age-dependent time compression ($\tau = \tau_x$) and endogenous, choice-based time compression ($\tau = \tau_y$). Tien and Burnes (2002) classify the distinction as nature (due to the changing levels of dopamine in the brain) versus nurture (due to the set of experiences we have accumulated over our lifetime). Exogenous, age-dependent time compression is modeled as

$$\tau_x = \left(1 - \kappa\frac{t_0}{T}\right)^{\beta} \tag{9}$$

where $\kappa \in [0,1]$ controls the speed at which time compression occurs through the lifecycle and $\beta \in [0,\infty)$ captures the curvature of the relationship. Time compression is treated as an inherent part of an agent's preference structure, similar to diminishing marginal utility, impatience, or aversion to risk. When $\beta = 0$, perceived time and clock time are perfectly synchronized and $\tau_x = 1$ for any t_0. A positive value for β indicates that as the agent ages and t_0 increases, dates of

[3] This can be verified by substituting (8) into $F(\tau\Delta) = \exp(-\rho\tau\Delta)$.

fixed distance in the future are perceived to be getting closer and closer – time is getting increasingly compressed. When β is between zero and one, time compression is a concave function of the aging process; when $\beta = 1$ it is a linear function; and when β is greater than one, time compression is a convex function of the aging process. Figure 2 shows how age-dependent time compression impacts the perceived distance of a fixed 10-year horizon.

One interpretation of t_0/T in (9) is that it represents the inventory of experiences the agent has accumulated over the lifecycle (Cottle (1976); Tien and Burnes (2002)). The term t_0/T measures the fraction of the agent's lifecycle that is complete. When t_0/T is near zero and the agent is young, each experience is relatively new and is perceived to have a longer duration. As the agent becomes older, t_0/T increases and the inventory of experiences grows. Each new experience then contributes less to the inventory, is less memorable and appears to pass by more rapidly. Equation (9) could loosely be interpreted as Weber's law applied to time perception – the perceived length of an interval of time is proportional to number of intervals experienced.

Under the exogenous specification, time compression is an unavoidable part of the aging process. However, recent research in the field of social psychology suggests that agents can make conscious decisions to adjust their internal clocks (Flaherty (2003)). Following Flaherty (2003), we link perceptions regarding the passage of time to the intensity of an individual's actions, denoted as $I(t) \in [0, 1]$. When an individual is completely engaged in an activity, $I(t)$ is near one. Examples include participation in competitive sports, late night cramming for a college exam, or Samuelson's (1972) description of Galois' feverish attempt to transcribe his final scientific contributions the night before a hopeless duel. Conversely, sitting idle at a stop light, an afternoon in a dentist's chair, or waiting in a long line at a department store stretch out perceived time. To capture these features, we specify endogenous time compression as follows:

$$\tau_y(t) = \exp[-\gamma I(t)]. \tag{10}$$

Increased intensity therefore causes a reduction in the time-perception parameter and an acceleration of perceived time:

$$\frac{\partial \tau_y(t)}{\partial I(t)} = -\gamma \exp[-\gamma I(t)] = -\gamma \tau_y(t) < 0, \tag{11}$$

where $\gamma > 0$.

An intriguing possibility is that an individual experiences both exogenous and endogenous time compression. For example, time compression might be a geometric average of the two types:

$$\tau = \tau_x^{\alpha} \tau_y^{1-\alpha}, \tag{12}$$

where $0 \leq \alpha \leq 1$ is a weighting parameter that measures the degree to which time compression is exogenous or endogenous. When $\alpha = 1$, time compression is entirely exogenous and only dependent on the agent's position in the lifecycle. When $\alpha = 0$, time compression is entirely endogenous and only dependent on the agent's chosen intensity level. When α is between zero and one, we get the case where the aging process naturally increases the degree of time compression, but the agent can make conscious decisions to decrease her intensity level to offset the effect.

3.3 Anticipating Time Compression

An important issue is whether agents anticipate time compression. We consider two cases for the anticipation of time compression, following the literature on time-inconsistent preferences (e.g., Pollack (1968)). In the first, agents expect the current degree of time compression to continue for the remainder of the lifecycle. This is the case of *naïveté*, in which

$$E[\tau(t)] = \tau(t_0) \tag{13}$$

for all $t \in [t_0, T]$. Time compression is thought to continue at its current rate for the remainder of the lifecycle.

In the second, agents are *sophisticated*, correctly anticipating the entire path of time compression and the implied self-control problems (O'Donoghue and Rabin (1999)). Expectation of the exogenous component of time compression is known with certainty and always correctly predicted. When making future plans, agents use

$$E[\tau_x(t)] = \left(1 - \kappa \frac{t}{T}\right)^{\beta} \tag{14}$$

for all $t \in [t_0, T]$. For the endogenous component, agents similarly use

$$E[\tau_y(t)] = \exp[-\gamma I(t)] \tag{15}$$

for all $t \in [t_0, T]$. Although the degree of future time compression is known, the individual will still face a time-consistency problem. Even well-intentioned plans made today will not be consistent with the incentives faced by future selves. We apply this framework to the classic life-cycle consumption problem with leisure to formally investigate the behavioral impacts of time compression.

4 Application. Lifecycle Consumption/Leisure Planning

Consider a representative agent faced with the classic control problem introduced by (Modigliani and Brumberg (1954)) modified to include endogenous labor supply, $h(t)$ (Heckman (1976)). The agent enters the workforce at $t = 0$ and passes away at date $t = T$, which is exogenous and certain. In our model, the labor supply decision is not one of choosing hours, but rather choosing the intensity level, $I(t) \in [0, 1]$, of the h hours spent at work. The product of the two, $h(t) = I(t)h$, determines effective labor supply. The chosen intensity level also influences τ, creating a tension between higher labor productivity and time compression. Our focus on labor intensity (or effort) rather than labor hours is similar to the efficiency-wage literature (e.g., Shapiro and Stiglitz (1984); Akerlof and Yellen (1990)), but we abstract from concepts such as shirking and asymmetric information. Labor intensity has also been modeled by Filer (1987), Fairris and Alston (1994), Saez (2002), and Green (2004).[4]

The total time endowment is normalized to one so that hours at work, h, and hours in leisure, l, sum to one. Effective leisure, $l(t) = 1 - h(t)$, is the sum of time away from work, l, and unproductive time at work, $(1 - I(t))h$ (Butler (2001)). The agent receives wage income at rate w, with total income equal to $w \cdot h(t)$. Although we only consider interior solutions and do not explicitly model retirement, individuals may rationally scale back the intensity of their work effort as they age. Because the properties of the basic lifecycle model are well understood, the effects of time compression on the agent's choice will be transparent.

4.1 Optimal Control Solution with Age-Independent Time Compression

Begin with the classic control problem where time compression does not vary with age (i.e., $\tau'(t_0) = 0$). The optimal plan is time consistent and will serve as a benchmark for later analysis with age-

[4] In our framework, the endogenous time compression parameter, τ_y, depends negatively on the chosen intensity level, $I(t)$. This is consistent with the psychological literature on perceived time (e.g., Chaston and Kingstone (2004); Hicks *et al.* (1976); Zakay *et al.* (1983)). However, since there exists a one-to-one mapping between $I(t)$ and either effective labor $h(t)$ or effective leisure $l(t)$, one can equivalently think of time compression as occurring because of either higher levels of effective labor or lower levels of effective leisure.

dependent time compression. The representative agent solves the following control problem:

$$\max_{\{c(t),I(t)\}} \int_{t=t_0}^{T} e^{-\rho\tau(t-t_0)} u\left(c(t), l(t)\right) \ dt \tag{16}$$

for $t_0 \in [0, T]$ subject to (12) and

$$\begin{aligned} \dot{k}(t) &= rk(t) + wh(t) - c(t) \quad (17) \\ k(0) &= k(T) = 0 \quad (18) \end{aligned}$$

where ρ is the discount rate; $c(t)$ is consumption; $k(t)$ is the stock of assets; $h(t) = hI(t)$ is effective labor supply; w is the wage rate; and r is the real rate of return on savings. The utility function, u, is positively related to $c(t)$ and $l(t)$ with the standard curvature properties to ensure a well-behaved, interior solution.

Individuals make optimal consumption $c(t)$ and effective labor $h(t)$ plans. The present-value Hamiltonian is

$$\mathcal{H} = e^{-\rho\tau(t-t_0)} u\left(c(t), l(t)\right) + \lambda_k(t)[rk(t) + wh(t) - c(t)] \tag{19}$$

where $\lambda_k(t)$ is the shadow price of capital. The necessary conditions from the maximum principle are

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial c(t)} &= e^{-\rho\tau(t-t_0)} u_c\left(c(t), l(t)\right) = \lambda_k(t) \quad (20) \\ \frac{\partial \mathcal{H}}{\partial I(t)} &= e^{-\rho\tau(t-t_0)} u_l\left(c(t), l(t)\right) \frac{\partial l(t)}{\partial I(t)} = -\lambda_k(t) w \frac{\partial h(t)}{\partial I(t)} \quad (21) \\ \dot{\lambda}_k(t) &= -\lambda_k(t) r. \quad (22) \end{aligned}$$

We assume a constant elasticity of substitution (CES) utility function:

$$u\left(c(t), l(t)\right) = \frac{\left[c(t)^{\theta} l(t)^{(1-\theta)}\right]^{1-\sigma}}{1-\sigma}, \tag{23}$$

where θ is the weight between consumption and effective leisure and $1/\sigma$ is the intertemporal elasticity of substitution of the consumption-leisure bundle. Combining equations (20), (21) and (22) produces:

$$c(t) = \frac{\theta}{1-\theta} w l(t), \tag{24}$$

the standard optimal relationship between $c(t)$ and $l(t)$ in a neoclassical consumption model with endogenous leisure (Butler (2001)). The planned lifecycle paths for $c(t)$ and $l(t)$ are given by

$$\frac{\dot{c}(t)}{c(t)} = \frac{\dot{l}(t)}{l(t)} = \frac{1}{\sigma}(r - \rho\tau). \tag{25}$$

It is clear from (25) that the solution is equivalent to the classic solution with a scaled discount rate, $\tilde{\rho} = \rho\tau$. However, with the introduction of age-dependent time compression, $\tau(t_0)$, the optimal plan is no longer time consistent. As is standard in the literature on time-inconsistent preferences and self-control problems, we consider two solution criteria. In the first, the individual fails to anticipate future changes in time compression – exogenous or endogenous – and believes that the current rate of time compression will continue through the remainder of the lifecycle. In the second, we assume the individual fully anticipates future changes in the rate of time compression and acknowledges the inherent self-control problems. Following Laibson (1997), the sophisticated agent solves for a subgame perfect Markov equilibrium with her future selves.

4.2 Solution Concepts with Age-Dependent Time Compression

We now consider the case where time compression worsens with age, $\tau'(t_0) < 0$. The degree of time compression varies exogenously with age due to the accumulation of lifetime experiences described by James (1892) and Samuelson (1976). However, time compression also varies endogenously with the intensity level, $I(t)$, of one's activities (Hicks *et al.* (1976)). To integrate endogenous time compression within the standard lifecycle model, we assume that the intensity of an individual's activities at work determine the perceived speed of time.

4.2.1 *Naïve* Solution

The *naïve* individual assumes that $\tau(t) = \tau_x(t_0)^\alpha \tau_y(I(t_0))^{1-\alpha}$ for $t \in [t_0, T]$, which is expressed as τ for simplicity. Time compression makes future dates appear closer so that the effective discount rate is $\rho\tau$. However, as the individual moves through the lifecycle, the degree of time compression continually gets updated. The naïve individual reacts to the new degree of time compression and expects it to continue, but he does not anticipate future changes in the rate of time compression. Actual consumption and effective leisure paths are then given by the envelope of all the initial values from the lifecycle planned paths (Caliendo and Aadland (2007)) as described in the age-independent time compression case.

4.2.2 Sophisticated Equilibrium

To make the *sophisticated* case operational, consider a single representative agent that lives for $T+1$ discrete periods: $\{0, 1, 2, ..., T\}$. The subgame perfect time-consistent plan involves choosing a consumption-labor intensity path $\{c(t), I(t)\}_{t=t_0}^{T}$ that maximizes

$$\sum_{t=t_0}^{T} e^{-\rho(t-t_0)\bar{\tau}(t)} u\left(c(t), l(t)\right), \tag{26}$$

for $t_0 \in [0, T]$ while taking into account the law of motion for the asset

$$k(t+1) = (1+r)k(t) + w(1 - l(t)) - c(t), \tag{27}$$

relationship between effective leisure and labor intensity

$$l(t) = 1 - I(t)h, \tag{28}$$

the time-compression parameter

$$\tau(t) = \tau_x(t)^{\alpha} \tau_y(I(t))^{1-\alpha}, \tag{29}$$

the optimal choices of all future selves $c(t) = c(k(t), t)$, $l(t) = l(k(t), t)$ and $k(0) = k(T+1) = 0$. The average time-compression parameter over the interval (t_0, t) is given by

$$\bar{\tau}(t_0, t) = \frac{1}{(t - t_0)} \sum_{s=t_0+1}^{t} \tau(s). \tag{30}$$

Extending Pollack (1968) to include capital accumulation and leisure, the equilibrium conditions are

$$\begin{aligned} u_c(t) &= \sum_{s=t+1}^{T} e^{-\rho\bar{\tau}(t,s)(s-t)} \left[\prod_{j=t+2}^{s} \frac{\partial k(j)}{\partial k(j-1)}\right] \times \\ &\quad \left[u_c(s)\frac{\partial c(k(s), s)}{\partial k(s)} + \left\{u_l(s) - \rho\Lambda(s)\frac{\partial \tau(s)}{\partial I(s)}\frac{\partial I(s)}{\partial l(s)}\right\}\frac{\partial l(k(s), s)}{\partial k(s)}\right], \qquad (31) \\ u_l(t) &= w u_c(t), \qquad (32) \end{aligned}$$

for $t = t_0, ..., T-1$ and

$$c(T) = c(k(T), T) = (1+r)k(T) + w(1-l(T)) \tag{33}$$

$$u_l(T) = wu_c(T), \tag{34}$$

where $\Lambda(s) = \sum_{j=s}^{T} e^{-\rho[\bar{\tau}(t,j)(j-t) - \bar{\tau}(t,s)(s-t)]} u(j)$. The problem can be solved via backward induction starting at period T.

To clarify, consider a simple example similar to the one presented in Pollack (1968) where effective labor is supplied inelastically (i.e., $h(t) = 1$ for all t) and $T = 2$. The optimal choice for period 2 self is

$$c(2) = c(k(2), 2) = (1+r)k(2) + w. \tag{35}$$

Given $k(1)$, period 1 self then chooses $c(1)$ to maximize

$$u\left(c(1)\right) + e^{-\rho\bar{\tau}(1,2)} u\left(c((1+r)k(1) + w - c(1), 2)\right) \tag{36}$$

given how self 2 will respond to the consumption choice of self 1. The first-order condition is

$$u_c\left(1\right) = e^{-\rho\bar{\tau}(1,2)} u_c\left(2\right)\left(1+r\right). \tag{37}$$

Finally, given $k(0) = 0$ and anticipating how self 1 and self 2 will behave (i.e., $c(k(1), 1)$ and $c(k(2), 2)$), self 0 then chooses $c(0)$ to maximize

$$\begin{aligned} & u\left(c(0)\right) + \\ & e^{-\rho\bar{\tau}(0,1)} u\left(c(w - c(0), 1)\right) + \\ & e^{-2\rho\bar{\tau}(0,2)} u\left(c((1+r)(w - c(0)) + w - c(w - c(0), 1), 2)\right). \end{aligned} \tag{38}$$

The first-order condition is

$$u_c\left(0\right) = e^{-\rho\bar{\tau}(0,1)} u_c\left(1\right) \frac{\partial c(1)}{\partial k(1)} + e^{-2\rho\bar{\tau}(0,2)} u_c\left(2\right) \frac{\partial c(2)}{\partial k(2)} \frac{\partial k(2)}{\partial k(1)}. \tag{39}$$

Together, the optimal choices from (35), (37), and (39) produce a time-consistent consumption path that accounts for the intertemporal conflict faced by a sophisticated individual experiencing age-dependent time compression.

4.3 Parameters and Simulation Results

In this section we discuss the chosen parameter values and the simulation results for three cases: (1) no time compression, (2) equally weighted exogenous and endogenous naïve time compression, and (3) equally weighted exogenous and endogenous sophisticated time compression.

4.3.1 Parameter Values

The parameter values are shown in Table 1.

Table 1. Annual Parameter Values

Parameters	$k(0)$	T	ρ, r	κ, h, α	w	θ	σ	β, γ
Value	0	60	0.04	0.5	5	0.33	3	1

Individuals begin at $t = 0$ (age 20) with no capital and live for $T = 60$ years (age 80). The discount and interest rates are set equal ($\rho = r$) so the baseline lifecycle planner with exponential discounting will choose flat consumption and leisure profiles and will not accumulate capital. As a result, any observed capital accumulation will be solely due to time compression. The exogenous component of time compression is linear in age ($\beta = 1$) and progresses at a rate such that $\tau_x(T) = 0.5$. The wage rate per effective hour worked is $w = 5$. The hours worked parameter is $h = 0.5$ so that an individual choosing maximum work intensity (i.e., $I(t) = 1$) is working the equivalent of half the available hours or 12 hours per day. The weight between consumption and effective leisure is $\theta = 0.33$, which implies that synchronized individuals will choose effective labor equal to one third the available hours or 8 hours per day. The curvature parameter in utility is $\sigma = 3$ leading to an intertemporal elasticity of substitution $1/\sigma = 0.\overline{33}$ for the composite leisure-consumption good. The endogenous time compression parameter is $\gamma = 1$. The endogenous component for naïve time compression is normalized so that $\tau_y(0) = 1$ and the individual's internal clock is synchronized at age 20.[5] Finally, with $\alpha = 0.5$, time compression is equally divided between exogenous and endogenous parts.

[5] The normalization is accomplished by modifying the endogenous component of time compression to be

$$\tau_y(t) = \exp\left(-\gamma(I(t) - I(0))\right).$$

The normalization value for naïve individuals is also applied to sophisticated individuals so the two types will experience equal degrees of time compression for the same choice of labor intensity.

4.3.2 Simulation Results and Discussion

The results of the simulations are shown in Figures 3 and 4. For the baseline agent with a synchronized clock ($\tau = 1$), the chosen labor intensity is $I = 1 - \theta = 0.67$ so that approximately two-thirds of the 12 working hours are spent concentrating on job-related tasks. The model does not identify whether the agent (*i*) exerts two-thirds effort throughout the day, (*ii*) exerts full effort for two-thirds of the working hours and no effort for the remainder, or (*iii*) chooses some mix of the two. The important point is that the synchronized agent is effectively putting in 8 hours of work per day.

Next, consider the naïve agent. At the beginning of the lifecycle (age 20), the naïve agent has a synchronized internal clock and expects no future time compression. The initial choice is therefore the same as the synchronized agent. In the following year (age 21), the individual begins to experience a small amount of time compression, which she expects to persist throughout the rest of the lifecycle. In terms of planning, the effective discount rate for future years is now slightly smaller because perceived time has sped up. The optimal response shown in Figure 3 is to work harder and begin to save for later years, as the rate of return on saving is now higher than the perceived discount rate. The increased work intensity combines with exogenous time compression to speed up perceived time even further. This pattern continues until about age 50 when the naïve agent starts to reduce work intensity relative to the baseline synchronized agent. Between ages 50 and 80, the naïve agent gradually reduces work intensity to slow down time compression, increases effective leisure, and consumes the wealth accumulated from labor effort when young. The observed path for the naïve agent is time inconsistent – future plans are not realized and continually reset. As shown by Caliendo and Aadland (2007), the observed path for the naïve agent is the envelope of all initial values from the continuum of revised plans.

The sophisticated agent is the antithesis of the naïve agent. While the naïve agent continually underestimates the degree of future time compression, the sophisticated agent perfectly anticipates time compression and recognizes the internal conflict with future selves. In response, the sophisticated agent solves for the time-consistent subgame perfect equilibrium path (Laibson (1997)). Figures 3 and 4 show the path for labor intensity and the resulting degree of time compression. Labor intensity begins substantially higher than the naïve or synchronized case, allowing for significant asset accumulation early in the lifecycle but also contributing to time compression. As shown in Figure 4, the sophisticated agent experiences immediate time compression because she chooses a relatively high degree of labor intensity. Labor intensity then falls throughout the lifecycle in order to mitigate naturally occurring time compression. In the final years of the lifecycle, the sophisticated agent is supplying only one third of the effective labor as at age 20, successfully smoothing time compression and financing a significant portion of consumption from early asset accumulation.

One of most robust trends in labor economics over the last century is the steady decline in retirement age (Costa (1998)). Many explanations have been offered: rising incomes, improving health, introduction of social security, etc. We offer a new possible explanation. Older workers may intentionally take more leisure to slow down the passage of perceived time. While 20th century technological advance has certainly elevated incomes and standards of living, it has also led to an accelerated pace of life (Gleick (1999); Wajcman (2008)) and an increasing sense that time is flying by. To offset this effect, those most sharply affected by this phenomenon may choose to reduce labor intensity with the purpose of slowing down their subjective clocks.

5 Conclusion

Building on extensive prior research documenting a widespread discrepancy between clock time and perceived time, we explore the behavioral implications of that discrepancy. We show that, when people base their choices on perceived time, the outcomes mimic some empirically observed patterns. Thus, time compression is not only a psychological phenomenon but also can drive economic behavior.

When time compression increases with age, a formal lifecycle permanent-income model predicts that individuals will choose to work harder when young than when old, particularly if the individual correctly anticipates the future pattern of perception. In comparison with our theoretical predictions, empirical patterns of work-leisure choices might suggest significant individual heterogeneity with respect to the degree and anticipation of time compression. Measuring this heterogeneity and accounting for it in formal lifecycle models of consumption and labor choices remains an open area for further research.

Because time compression alters the rate at which individuals discount the future, many problems of collective choice are likely to be affected by time compression. Policies addressing intertemporal problems such as climate change, depletable resources, pollution, and investment in infrastructure necessarily embody implicit discount rates. The endogeneity of individual discount rates due to time compression thus complicates the challenge of matching social policies to appropriate aggregations of individuals' preferences. A related implication is that demographic trends such as a baby boom will alter the (weighted or unweighted) average discount rate across individuals over time. Hence, the effects of time compression on optimal public policy is another broad and untapped area of future research.

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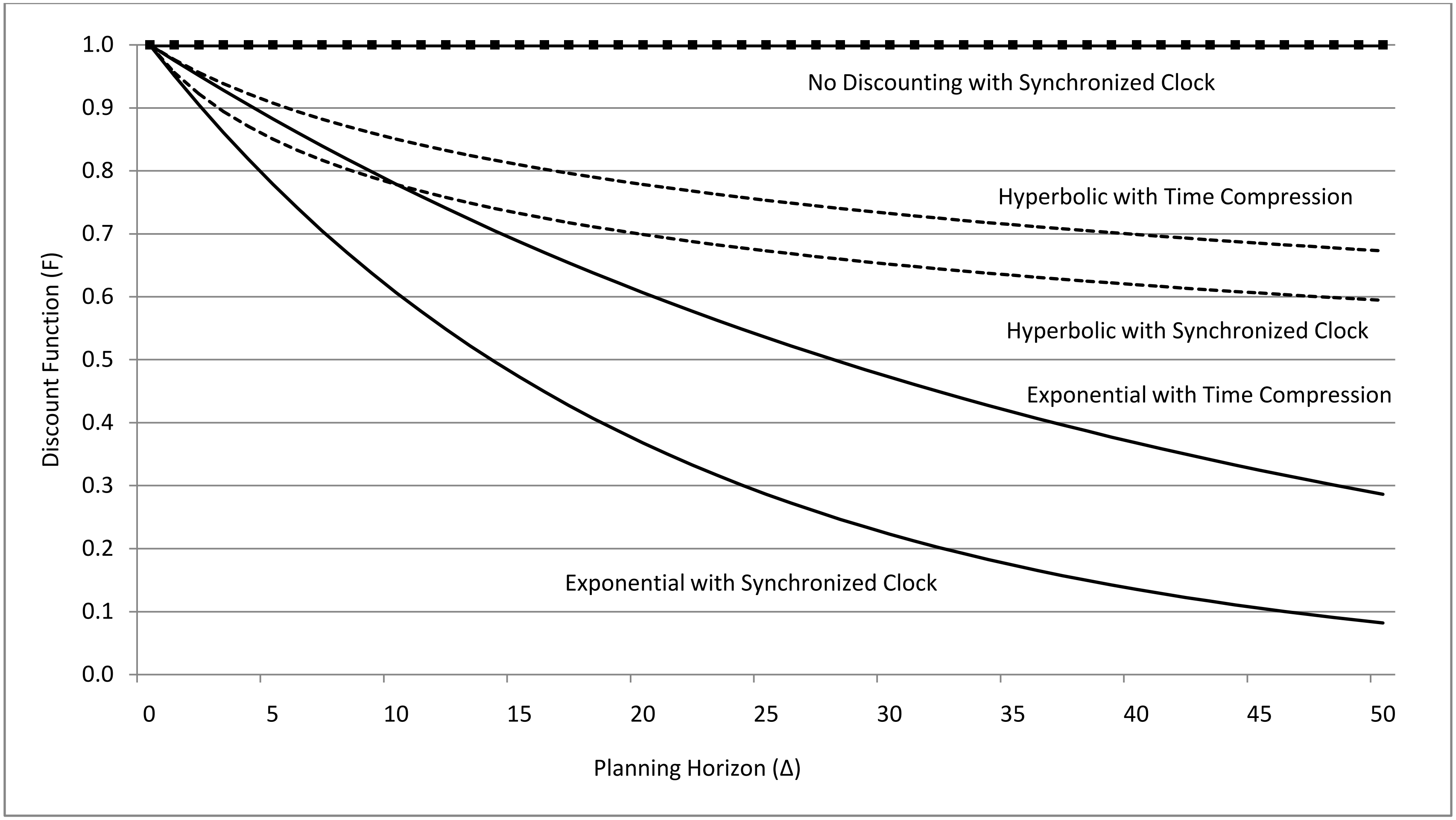
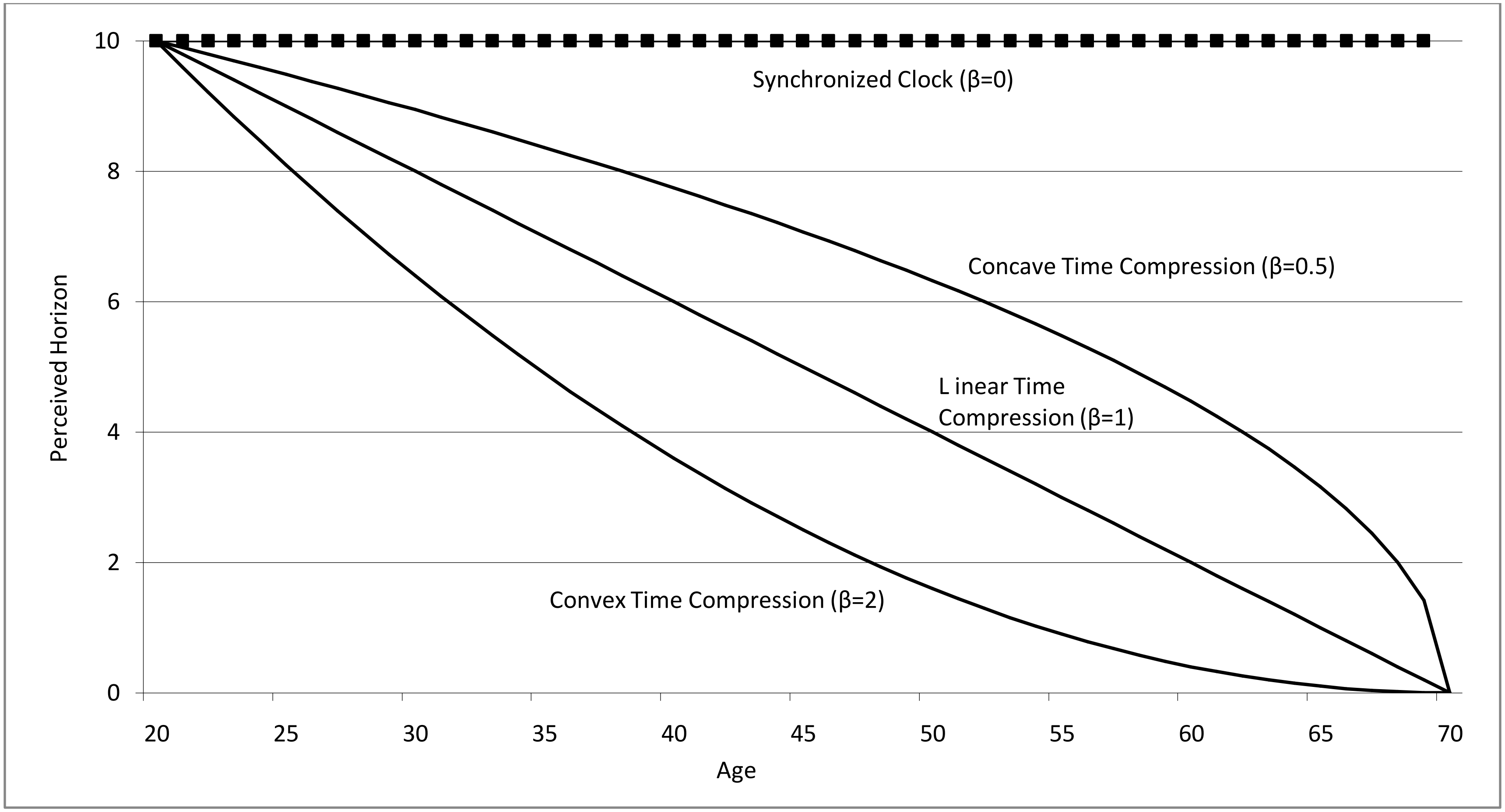
Figure 1. Discount Functions and Age-Independent Time Compression



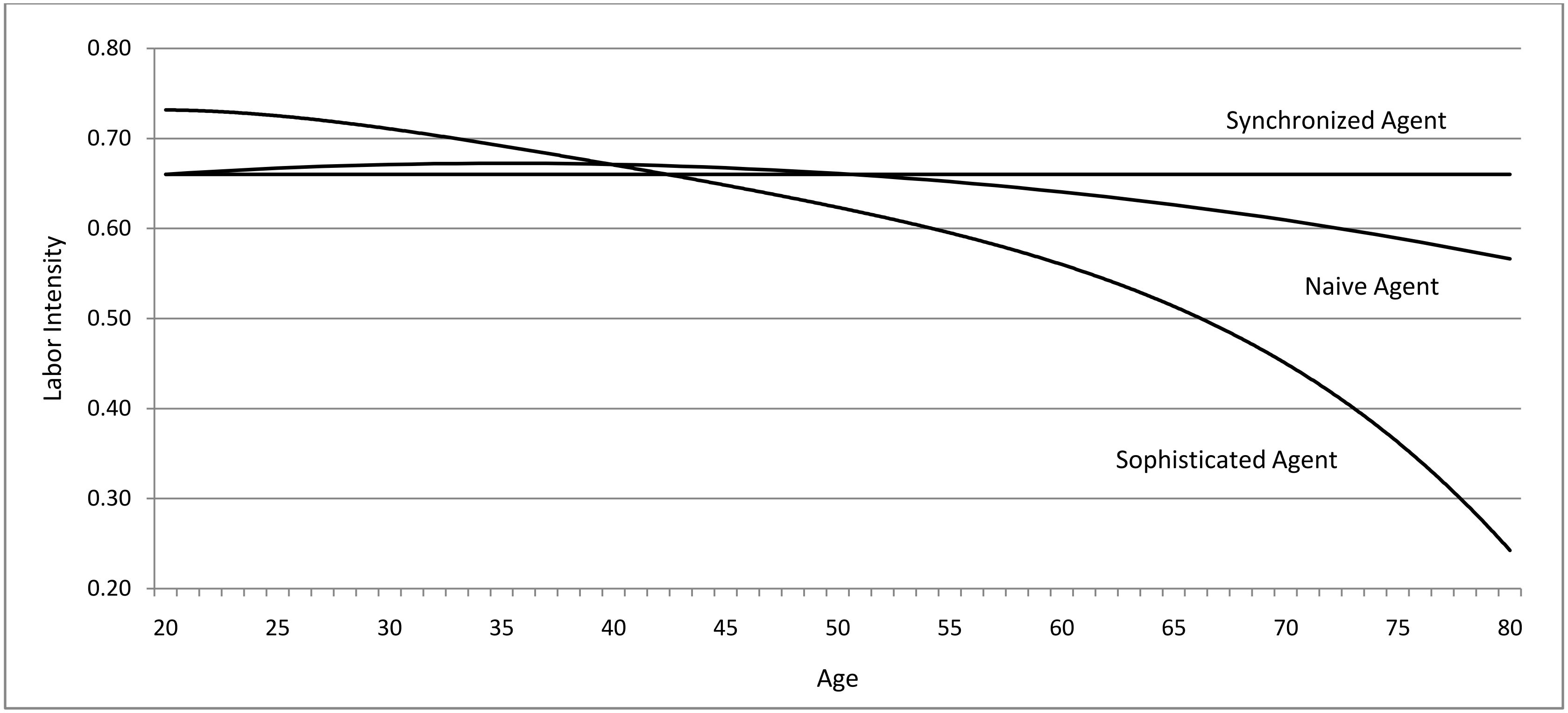
Notes. The exponential discount function is $F = \exp(-\rho\tau\Delta)$ and the hyperbolic discount function is $F = 1/(1 + \alpha\tau\Delta)^{\rho/\alpha}$. The parameters are set at $\rho = 0.05$ and $\alpha = 0.25$. The time compression parameter is set at $\tau = 1$ for no time compression and $\tau = 0.5$ with time compression.

Figure 2. Perceived Length of a 10-Year Planning Horizon for Various Types of Age-Dependent Time Compression



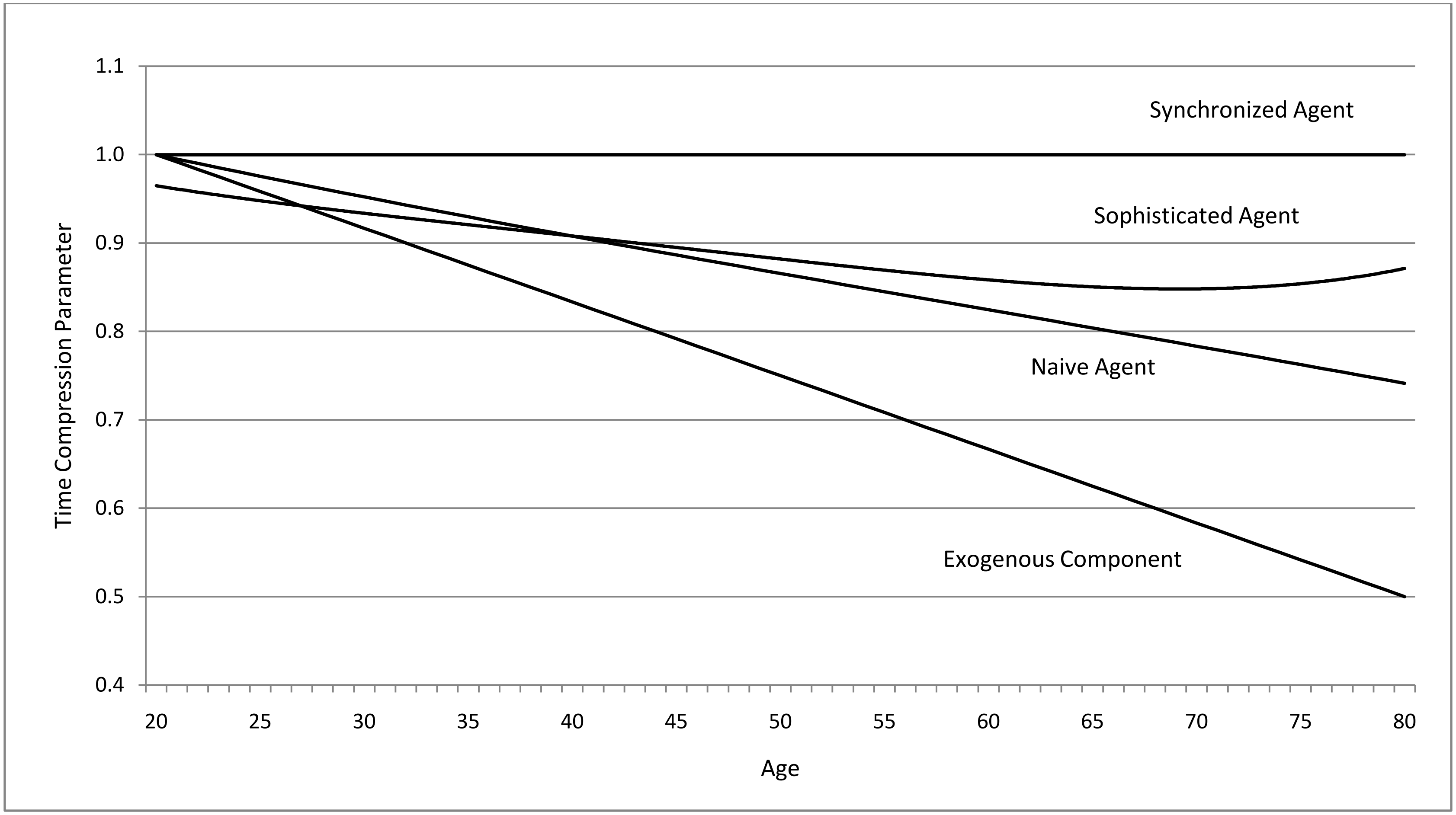
Notes. Age-dependent time compression is given by $\tau(t_0) = (1 - t_0/T)^{\beta}$. The perceived planning horizon is $\tau\Delta$, where $\Delta = 10$.

Figure 3. Chosen Labor Intensity for Synchronized, Naïve and Sophisticated Agents



Notes. Effective labor is given by $h(t) = I(t)h$, where $I(t)$ is labor intensity. The parameter values for the simulation are shown in Table 1. The simulations for the sophisticated agent are linearized approximations to the equilibrium conditions (i.e., equations (31) and (32)) at five-year intervals. The points of approximation are calculated sequentially. In the first round, the points of approximation for $c(t)$ and $k(t)$ are set at the baseline values for the synchronized agent. In the second round, the realized paths for $c(t)$ and $k(t)$ from round one are used as the revised points of approximation.

Figure 4. Time Compression Parameter for Synchronized, Naïve and Sophisticated Agents



Notes. The time compression parameter, τ, for the synchronized, naïve and sophisticated agents includes both exogenous and endogenous components, $\tau = \tau_x^{\alpha}\tau_y^{1-\alpha}$. The exogenous component is given by $\tau_x = 1 - 0.5(t_0/T)$. The simulations for the sophisticated agent are linearized approximations to the equilibrium conditions (i.e., equations (31) and (32)) at five-year intervals. The points of approximation are calculated sequentially. In the first round, the points of approximation for $c(t)$ and $k(t)$ are set at the baseline values for the synchronized agent. In the second round, the realized paths for $c(t)$ and $k(t)$ from round one are used as the revised points of approximation.