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Abstract

This paper evaluates the dynamic out of sample nominal exchange rate forecasting performance of the canonical New Keynesian model of a small open economy. A novel Bayesian procedure for jointly estimating the hyperparameters and trend components of a state space representation of an approximate linear panel unobserved components representation of this New Keynesian model, conditional on prior information concerning the values of hyperparameters and trend components, is developed and applied for this purpose. In agreement with the existing empirical literature, we find that nominal exchange rate movements are difficult to forecast, with a random walk generally dominating the canonical New Keynesian model of a small open economy in terms of predictive accuracy at all horizons. Nevertheless, we find empirical support for the common practice in the theoretical open economy macroeconomics literature of imposing deterministic equality restrictions on deep structural parameters across economies, both in sample and out of sample.

JEL Classification: C11; C13; C33; F41; F47

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1. Introduction

There exists an extensive empirical literature concerning the predictability of nominal exchange rates using structural macroeconomic models over the recent flexible exchange rate period. The general conclusion of this literature is that exchange rate movements are difficult to forecast at short horizons, while there exists some evidence of long horizon predictability. The most influential negative empirical evidence was documented by Meese and Rogoff (1983), who evaluated the out of sample forecasting performance of a variety of structural models of nominal exchange rate determination. Their primary result was that all structural macroeconomic models

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were generally dominated by a driftless random walk in terms of predictive accuracy at short horizons, despite generating exchange rate forecasts conditional on out of sample realizations of other macroeconomic variables. The empirical literature concerning the predictability of nominal exchange rates using structural macroeconomic models was recently updated by Cheung, Chinn and Pascual (2005), who found that exchange rate movements remain difficult to forecast, with a random walk generally dominating a variety of structural models of nominal exchange rate determination in terms of predictive accuracy conditional on out of sample realizations of other macroeconomic variables at all horizons. These results suggest that exchange rate movements are difficult to rationalize on the basis of movements in other macroeconomic variables, even retrospectively. This empirical disconnect between nominal exchange rates and other macroeconomic variables out of sample, labeled the exchange rate forecasting puzzle by Obstfeld and Rogoff (2000), has never been decisively resolved in spite of numerous attempts to do so, and a random walk has become the standard benchmark for evaluating the exchange rate forecasting performance of structural macroeconomic models.

The exchange rate forecasting puzzle is an empirical property of a set of structural macroeconomic models which predominantly excludes those arising from revolutionary developments in the theoretical open economy macroeconomics literature during the last decade. Building on the seminal contribution of Obstfeld and Rogoff (1995), a dominant theoretical paradigm for the conduct of open economy macroeconomic analysis has recently emerged based on rigorous microeconomic foundations and short run nominal rigidities. The set of structural macroeconomic models associated with this theoretical paradigm was enriched by Galí and Monacelli (2005), who extended the canonical New Keynesian model of a closed economy exemplified by Woodford (2003) to a small open economy setting by introducing international trade and financial linkages. Variants of the resulting structural macroeconomic model, which we refer to as the canonical New Keynesian model of a small open economy, have since been extensively applied to the analysis of the monetary transmission mechanism and the optimal conduct of monetary policy.

This paper evaluates the dynamic out of sample nominal exchange rate forecasting performance of the canonical New Keynesian model of a small open economy. A novel Bayesian procedure for jointly estimating the hyperparameters and trend components of a state space representation of an approximate linear panel unobserved components representation of this New Keynesian model, conditional on prior information concerning the values of hyperparameters and trend components, is developed and applied for this purpose. In agreement with the existing empirical literature, we find that nominal exchange rate movements are difficult to forecast, with a random walk generally dominating the canonical New Keynesian model of a small open economy in terms of predictive accuracy at all horizons. Nevertheless, we find empirical support for the common practice in the theoretical open economy macroeconomics literature of imposing deterministic equality restrictions on deep structural parameters across economies, both in sample and out of sample.

The organization of this paper is as follows. The next section develops the canonical New Keynesian model of a small open economy. In section three, a panel representation of an approximate linear unobserved components representation of it is described. The development and application of a Bayesian procedure for jointly estimating the hyperparameters and trend components of this approximate linear panel unobserved components representation of the New Keynesian model are the subjects of section four. An evaluation of its dynamic out of sample nominal exchange rate forecasting performance is conducted in section five. Finally, section six offers conclusions and recommendations for further research.

2. Model Development

Consider two open economies which are asymmetric in size, but are otherwise identical. The domestic economy is of negligible size relative to the foreign economy.

2.1. The Utility Maximization Problem of the Representative Household

The representative infinitely lived household has preferences defined over consumption $C_{i,s}$ and labour supply $L_{i,s}$ represented by intertemporal utility function

$$U_{i,t} = \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u(C_{i,s}, L_{i,s}), \tag{1}$$

where subjective discount factor β satisfies $0 < \beta < 1$. The intratemporal utility function is additively separable:

$$u(C_{i,s}, L_{i,s}) = \frac{(C_{i,s})^{1-1/\sigma}}{1-1/\sigma} - \chi \frac{(L_{i,s})^{1+1/\eta}}{1+1/\eta}. \tag{2}$$

This intratemporal utility function is strictly decreasing with respect to labour supply if and only if $\chi > 0$. Given this parameter restriction, this intratemporal utility function is strictly concave if $\sigma > 0$ and $\eta > 0$.

The representative household enters period s in possession of a previously purchased diversified portfolio of internationally traded domestic currency denominated bonds $B_{i,s}$ which completely spans all relevant uncertainty. It also holds a diversified portfolio of shares $\{x_{i,j,s}\}_{j=0}^{1}$ in domestic intermediate good firms which pay dividends $\{\Pi_{j,s}\}_{j=0}^{1}$. The representative household supplies final labour service $L_{i,s}$, earning labour income at nominal wage W_s. These sources of wealth are summed in household dynamic budget constraint:

$$\mathrm{E}_s\, Q_{s,s+1} B_{i,s+1} + \int_{j=0}^{1} V_{j,s} x_{i,j,s+1} dj = B_{i,s} + \int_{j=0}^{1} (\Pi_{j,s} + V_{j,s}) x_{i,j,s} dj + W_s L_{i,s} - P_s^C C_{i,s}. \tag{3}$$

According to this dynamic budget constraint, at the end of period s, the representative household purchases a diversified portfolio of state contingent bonds $B_{i,s+1}$, where $Q_{s,s+1}$ denotes the price of a bond which pays one unit of the domestic currency in a particular state in the following period, divided by the conditional probability of occurrence of that state. It also purchases a diversified portfolio of shares $\{x_{i,j,s+1}\}_{j=0}^{1}$ at prices $\{V_{j,s}\}_{j=0}^{1}$. Finally, the representative household purchases final consumption good $C_{i,s}$ at price P_s^C.

In period t, the representative household chooses state contingent sequences for consumption $\{C_{i,s}\}_{s=t}^{\infty}$, labour supply $\{L_{i,s}\}_{s=t}^{\infty}$, bond holdings $\{B_{i,s+1}\}_{s=t}^{\infty}$, and share holdings $\{\{x_{i,j,s+1}\}_{j=0}^{1}\}_{s=t}^{\infty}$ to maximize intertemporal utility function (1) subject to dynamic budget constraint (3) and terminal nonnegativity constraints $B_{i,T+1} \geq 0$ and $x_{i,j,T+1} \geq 0$ for $T \to \infty$. In equilibrium, selected necessary first order conditions associated with this utility maximization problem may be stated as

$$u_C(C_t, L_t) = P_t^C \lambda_t, \tag{4}$$

$$-u_L(C_t, L_t) = W_t \lambda_t, \tag{5}$$

$$Q_{t,t+1} \lambda_t = \beta \lambda_{t+1}, \tag{6}$$

$$V_{j,t} \lambda_t = \beta \mathrm{E}_t (\Pi_{j,t+1} + V_{j,t+1}) \lambda_{t+1}, \tag{7}$$

where $\lambda_{i,s}$ denotes the Lagrange multiplier associated with the period s household dynamic budget constraint. In equilibrium, necessary complementary slackness conditions associated with the terminal nonnegativity constraints may be stated as:

$$\lim_{T \to \infty} \frac{\beta^T \lambda_{t+T}}{\lambda_t} Q_{t+T,t+T+1} B_{t+T+1} = 0, \tag{8}$$

$$\lim_{T\to\infty} \frac{\beta^T \lambda_{t+T}}{\lambda_t} V_{j,t+T} x_{j,t+T+1} = 0. \tag{9}$$

Provided that the intertemporal utility function is bounded and strictly concave, together with all necessary first order conditions, these transversality conditions are sufficient for the unique utility maximizing state contingent intertemporal household allocation.

The absence of arbitrage opportunities requires that short term nominal interest rate i_t satisfy $\frac{1}{1+i_t} = \mathrm{E}_t\, Q_{t,t+1}$. Combination of this equilibrium asset pricing relationship with necessary first order conditions (4) and (6) yields intertemporal optimality condition

$$u_C(C_t, L_t) = \beta \mathrm{E}_t (1+i_t) \frac{P_t^C}{P_{t+1}^C} u_C(C_{t+1}, L_{t+1}), \tag{10}$$

which ensures that at a utility maximum, the representative household cannot benefit from feasible intertemporal consumption reallocations. Finally, combination of necessary first order conditions (4) and (5) yields intratemporal optimality condition

$$-\frac{u_L(C_t, L_t)}{u_C(C_t, L_t)} = \frac{W_t}{P_t^C}, \tag{11}$$

which equates the marginal rate of substitution between leisure and consumption to the real wage.

2.2. The Value Maximization Problem of the Representative Firm

There exists a continuum of intermediate good firms indexed by $j \in [0,1]$. Intermediate good firms supply differentiated intermediate output goods, but are otherwise identical. Entry into and exit from the monopolistically competitive intermediate output good sector is prohibited.

2.2.1. Employment Behaviour

The representative intermediate good firm sells shares $\{x_{i,j,t+1}\}_{i=0}^1$ to domestic households at price $V_{j,t}$. Recursive forward substitution for $V_{j,t+s}$ with $s > 0$ in necessary first order condition (7) applying the law of iterated expectations reveals that the post-dividend stock market value of the representative intermediate good firm equals the expected present discounted value of future dividend payments:

$$V_{j,t} = \mathrm{E}_t \sum_{s=t+1}^{\infty} \frac{\beta^{s-t} \lambda_s}{\lambda_t} \Pi_{j,s}. \tag{12}$$

Acting in the interests of its shareholders, the representative intermediate good firm maximizes its pre-dividend stock market value, equal to the expected present discounted value of current and future dividend payments:

$$\Pi_{j,t} + V_{j,t} = \mathrm{E}_t \sum_{s=t}^{\infty} \frac{\beta^{s-t} \lambda_s}{\lambda_t} \Pi_{j,s}. \tag{13}$$

The derivation of result (12) imposes transversality condition (9), which rules out self-fulfilling speculative asset price bubbles.

Shares entitle households to dividend payments equal to profits $\Pi_{j,s}$, defined as revenues derived from sales of differentiated intermediate output good $Y_{j,s}$ at price $P_{j,s}^{Y}$ less expenditures on final labour service $L_{j,s}$:

$$\Pi_{j,s} = P_{j,s}^{Y} Y_{j,s} - W_s L_{j,s}. \tag{14}$$

The representative intermediate good firm rents final labour service $L_{j,s}$ given labour augmenting productivity coefficient A_s to produce differentiated intermediate output good $Y_{j,s}$ according to production function

$$Y_{j,s} = A_s L_{j,s}, \tag{15}$$

where $A_s > 0$. This production function abstracts from capital accumulation and exhibits constant returns to scale.

In period t, the representative intermediate good firm chooses a state contingent sequence for employment $\{L_{i,s}\}_{s=t}^{\infty}$ to maximize pre-dividend stock market value (13) subject to production function (15). In equilibrium, demand for the final labour service satisfies necessary first order condition

$$\Phi_t = \frac{W_t}{P_t^{Y} A_t}, \tag{16}$$

where $P_s^{Y} \Phi_{j,s}$ denotes the Lagrange multiplier associated with the period s production technology constraint. This necessary first order condition equates real marginal cost Φ_t to the ratio of the real wage to the marginal product of labour.

2.2.2. Output Supply and Price Setting Behaviour

There exist a large number of perfectly competitive firms which combine differentiated intermediate output goods $Y_{j,t}$ supplied by intermediate good firms in a monopolistically competitive output market to produce final output good Y_t according to constant elasticity of substitution production function

$$Y_t = \left[\int_{j=0}^{1} (Y_{j,t})^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \tag{17}$$

where $\theta > 1$. The representative final output good firm maximizes profits derived from production of the final output good

$$\Pi_t^Y = P_t^Y Y_t - \int_{j=0}^{1} P_{j,t}^Y Y_{j,t} dj, \tag{18}$$

with respect to inputs of intermediate output goods, subject to production function (17). The necessary first order conditions associated with this profit maximization problem yield intermediate output good demand functions:

$$Y_{j,t} = \left(\frac{P_{j,t}^Y}{P_t^Y} \right)^{-\theta} Y_t. \tag{19}$$

Since the production function exhibits constant returns to scale, in competitive equilibrium the representative final output good firm earns zero profit, implying aggregate output price index:

$$P_t^Y = \left[\int_{j=0}^{1} (P_{j,t}^Y)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}. \tag{20}$$

As the price elasticity of demand for intermediate output goods θ increases, they become closer substitutes, and individual intermediate good firms have less market power.

In an adaptation of the model of nominal output price rigidity proposed by Calvo (1983), each period a randomly selected fraction $1-\omega$ of intermediate good firms adjust their price optimally. The remaining fraction ω of intermediate good firms adjust their price to account for past steady state output price inflation according to indexation rule:

$$P_{j,t}^{Y} = \frac{\bar{P}_{t-1}^{Y}}{\bar{P}_{t-2}^{Y}} P_{j,t-1}^{Y}. \tag{21}$$

Under this specification, optimal price adjustment opportunities arrive randomly, and the interval between optimal price adjustments is a random variable.

If the representative intermediate good firm can adjust its price optimally in period t, then it does so to maximize to maximize pre-dividend stock market value (13) subject to production function (15), intermediate output good demand function (19), and the assumed form of nominal output price rigidity. Since all intermediate good firms that adjust their price optimally in period t solve an identical value maximization problem, in equilibrium they all choose a common price $P_t^{Y,*}$ given by necessary first order condition:

$$\frac{P_t^{Y,*}}{P_t^{Y}} = \left(\frac{\theta}{\theta-1}\right) \frac{\mathrm{E}_t \sum_{s=t}^{\infty} \omega^{s-t} \frac{\beta^{s-t}\lambda_s}{\lambda_t} \Phi_s \left(\frac{\bar{P}_{t-1}^{Y}}{\bar{P}_{s-1}^{Y}} \frac{P_s^{Y}}{P_t^{Y}}\right)^{\theta} P_s^{Y} Y_s}{\mathrm{E}_t \sum_{s=t}^{\infty} \omega^{s-t} \frac{\beta^{s-t}\lambda_s}{\lambda_t} \left(\frac{\bar{P}_{t-1}^{Y}}{\bar{P}_{s-1}^{Y}} \frac{P_s^{Y}}{P_t^{Y}}\right)^{\theta-1} P_s^{Y} Y_s}. \tag{22}$$

This necessary first order condition equates the expected present discounted value of the revenue benefit generated by an additional unit of output supply to the expected present discounted value of its production cost. Aggregate output price index (20) equals an average of the price set by the fraction $1-\omega$ of intermediate good firms that adjust their price optimally in period t, and the average of the prices set by the remaining fraction ω of intermediate good firms that adjust their price according to indexation rule (21):

$$P_t^{Y} = \left[(1-\omega)(P_t^{Y,*})^{1-\theta} + \omega\left(\frac{\bar{P}_{t-1}^{Y}}{\bar{P}_{t-2}^{Y}} P_{t-1}^{Y}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{23}$$

Since those intermediate good firms able to adjust their price optimally in period t are selected randomly from among all intermediate good firms, the average price set by the remaining intermediate good firms equals the value of the aggregate output price index that prevailed during period $t-1$, rescaled to account for past output price inflation.

2.3. International Trade and Financial Linkages

In an open economy, exchange rate adjustment contributes to both intratemporal and intertemporal equilibration, while business cycles are generated by interactions among a variety of nominal and real shocks originating both domestically and abroad.

2.3.1. International Trade Linkages

The law of one price asserts that arbitrage transactions equalize the domestic currency prices of domestic imports and foreign exports. Let $\mathcal{E}_s$ denote the nominal exchange rate, which measures the price of foreign currency in terms of domestic currency, and define the real exchange rate,

$$\mathcal{Q}_s = \frac{\mathcal{E}_s P_s^{Y,f}}{P_s^Y}, \tag{24}$$

which measures the price of foreign output in terms of domestic output. Under the law of one price, the real exchange rate coincides with the terms of trade, which measures the price of imports in terms of exports.

There exist a large number of perfectly competitive firms which combine a domestic intermediate consumption good $C_{h,t}$ and a foreign intermediate consumption good $C_{f,t}$ to produce final consumption good C_t according to constant elasticity of substitution production function

$$C_t = \left[\phi^{\frac{1}{\psi}} (C_{h,t})^{\frac{\psi-1}{\psi}} + (1-\phi)^{\frac{1}{\psi}} (C_{f,t})^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}, \tag{25}$$

where $0 < \phi < 1$ and $\psi > 1$. The representative final consumption good firm maximizes profits derived from production of the final consumption good

$$\Pi_t^C = P_t^C C_t - P_t^Y C_{h,t} - \mathcal{E}_t P_t^{Y,f} C_{f,t}, \tag{26}$$

with respect to inputs of domestic and foreign intermediate consumption goods, subject to production function (25). The necessary first order conditions associated with this profit maximization problem imply intermediate consumption good demand functions:

$$C_{h,t} = \phi\left(\frac{P_t^Y}{P_t^C}\right)^{-\psi} C_t, \tag{27}$$

$$C_{f,t} = (1-\phi)\left(\frac{\mathcal{E}_t P_t^{Y,f}}{P_t^C}\right)^{-\psi} C_t. \tag{28}$$

Since the production function exhibits constant returns to scale, in competitive equilibrium the representative final consumption good firm earns zero profit, implying aggregate consumption price index:

$$P_t^C = \left[\phi(P_t^Y)^{1-\psi} + (1-\phi)(\mathcal{E}_t P_t^{Y,f})^{1-\psi}\right]^{\frac{1}{1-\psi}}. \tag{29}$$

Combination of this aggregate consumption price index with intermediate consumption good demand functions (27) and (28) yields:

$$C_{h,t} = \phi\left[\phi + (1-\phi)(Q_t)^{1-\psi}\right]^{\frac{\psi}{1-\psi}} C_t, \tag{30}$$

$$C_{f,t} = (1-\phi)\left[(1-\phi) + \phi(Q_t)^{\psi-1}\right]^{\frac{\psi}{1-\psi}} C_t. \tag{31}$$

These demand functions for domestic and foreign intermediate consumption goods are directly proportional to final consumption good demand, with a proportionality coefficient that varies with the real exchange rate.

2.3.2. International Financial Linkages

Under the assumption of complete international financial markets, utility maximization by domestic and foreign households implies intertemporal optimality conditions

$$Q_{t,t+1} = \frac{\beta u_C(C_{t+1}, L_{t+1})}{u_C(C_t, L_t)} \frac{P_t^C}{P_{t+1}^C}, \tag{32}$$

$$Q_{t,t+1} = \frac{\beta u_C(C_{t+1}^f, L_{t+1}^f)}{u_C(C_t^f, L_t^f)} \frac{P_t^{Y,f}}{P_{t+1}^{Y,f}} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}, \tag{33}$$

respectively. Combination of these intertemporal optimality conditions with real exchange rate definition (24) yields international risk sharing condition:

$$\mathcal{Q}_t \propto \frac{u_C(C_t^f, L_t^f)}{u_C(C_t, L_t)} \frac{P_t^C}{P_t^Y}. \tag{34}$$

Under the assumption that the domestic economy is of negligible size relative to the foreign economy, this international risk sharing condition induces stationarity of consumption and the real net foreign asset position.

2.4. *Monetary Policy*

The government consists of a monetary authority which implements monetary policy through control of the nominal interest rate according to monetary policy rule

$$i_t - \bar{i}_t = \xi(\pi_t^C - \bar{\pi}_t^C) + \zeta(\ln Y_t - \ln \bar{Y}_t) + \nu_t, \tag{35}$$

where $\xi > 1$ and $\zeta > 0$. As specified, the deviation of the nominal interest rate from its deterministic steady state equilibrium value is a linear increasing function of the contemporaneous deviation of consumption price inflation from its target value, and the contemporaneous proportional deviation of output from its deterministic steady state equilibrium value. Persistent departures from this monetary policy rule are captured by serially correlated monetary policy shock ν_t.

2.5. *Market Clearing Conditions*

A rational expectations equilibrium in this New Keynesian model of a small open economy consists of state contingent intertemporal allocations for domestic and foreign households and firms which solve their constrained optimization problems given prices and policy, together with state contingent intertemporal allocations for domestic and foreign governments which satisfy their policy rules, with supporting prices such that all markets clear.

Clearing of the final output good market requires that production of the final output good equal the cumulative demands of domestic and foreign households:

$$Y_t = C_{h,t} + C_{f,t}^f. \tag{36}$$

The assumption that the domestic economy is of negligible size relative to the foreign economy is represented by parameter restriction $\phi^f = 1$, under which $P_t^{Y,f} = P_t^{C,f}$ in equilibrium.

3. The Approximate Linear Panel Unobserved Components Model

Estimation and forecasting are based on a state space representation of a panel representation of an approximate linear unobserved components representation of this New Keynesian model of a small open economy. In constructing the approximate linear unobserved components representation, cyclical components are modeled by linearizing equilibrium conditions around a stationary deterministic steady state equilibrium which abstracts from long run balanced growth, while trend components are modeled as random walks while ensuring the existence of a well defined balanced growth path. In constructing the panel representation, this approximate linear unobserved components representation is replicated across a set of structurally isomorphic small open economies. Parameter homogeneity across economies is imposed in deriving the cyclical component specifications associated with the approximate linear unobserved components representation, but is relaxed in constructing the cyclical component specifications associated with its panel representation.

In what follows, $\mathrm{E}_t\, x_{k,t+s}$ denotes the rational expectation of variable $x_{k,t+s}$ associated with small open economy k, conditional on information available at time t. Also, $\hat{x}_{k,t}$ denotes the cyclical component of variable $x_{k,t}$, while $\bar{x}_{k,t}$ denotes the trend component of variable $x_{k,t}$. Cyclical and trend components are additively separable, that is $x_{k,t} = \hat{x}_{k,t} + \bar{x}_{k,t}$.

3.1. Cyclical Components

The cyclical component of output price inflation depends on the expected future cyclical component of output price inflation and the contemporaneous cyclical component of real marginal cost according to output price Phillips curve:

$$\hat{\pi}_{k,t}^{Y} = \beta \mathrm{E}_t \hat{\pi}_{k,t+1}^{Y} + \frac{(1-\omega_k)(1-\omega_k\beta)}{\omega_k}\left\{\left(\frac{1}{\phi_k\sigma_k}+\frac{1}{\eta_k}\right)\ln \hat{Y}_{k,t} - (1-\phi_k)\left[\frac{1}{\phi_k\sigma_k}\ln \hat{Y}_t^{f} - \left(1-\frac{\psi_k(1+\phi_k)}{\phi_k\sigma_k}\right)\ln \hat{Q}_{k,t}\right] - \frac{1+\eta_k}{\eta_k}\ln \hat{A}_{k,t}\right\}. \tag{37}$$

Reflecting the existence of international trade linkages, the cyclical component of real marginal cost depends not only on the contemporaneous cyclical component of domestic output, but also on the contemporaneous cyclical components of foreign output and the real exchange rate.

The cyclical component of consumption price inflation depends on the expected future cyclical component of consumption price inflation and the contemporaneous cyclical component of real marginal cost according to consumption price Phillips curve:

$$\begin{aligned}\hat{\pi}_{k,t}^{C} = \beta \mathrm{E}_t \hat{\pi}_{k,t+1}^{C} + \frac{(1-\omega_k)(1-\omega_k\beta)}{\omega_k}\left\{\left(\frac{1}{\phi_k\sigma_k}+\frac{1}{\eta_k}\right)\ln \hat{Y}_{k,t} - (1-\phi_k)\left[\frac{1}{\phi_k\sigma_k}\ln \hat{Y}_t^f - \left(1-\frac{\psi_k(1+\phi_k)}{\phi_k\sigma_k}\right)\ln \hat{\mathcal{Q}}_{k,t}\right] - \frac{1+\eta_k}{\eta_k}\ln \hat{A}_{k,t}\right\} \\ +(1-\phi_k)\ln \frac{\hat{\mathcal{Q}}_{k,t}}{\hat{\mathcal{Q}}_{k,t-1}} - \beta(1-\phi_k)\mathrm{E}_t \ln \frac{\hat{\mathcal{Q}}_{k,t+1}}{\hat{\mathcal{Q}}_{k,t}}.\end{aligned} \tag{38}$$

Reflecting the entry of the price of imports into the aggregate consumption price index, the cyclical component of consumption price inflation also depends on contemporaneous and expected future proportional changes in the cyclical component of the real exchange rate.

The cyclical component of output depends on the expected future cyclical component of output and the contemporaneous cyclical component of the real interest rate according to approximate linear consumption Euler equation:

$$\ln \hat{Y}_{k,t} = \mathrm{E}_t \ln \hat{Y}_{k,t+1} - \phi_k\sigma_k(\hat{i}_{k,t} - \mathrm{E}_t\hat{\pi}_{k,t+1}^{C}) - (1-\phi_k)\left[\mathrm{E}_t \ln \frac{\hat{Y}_{t+1}^f}{\hat{Y}_t^f} + \psi_k(1+\phi_k)\mathrm{E}_t \ln \frac{\hat{\mathcal{Q}}_{k,t+1}}{\hat{\mathcal{Q}}_{k,t}}\right]. \tag{39}$$

Reflecting the existence of international trade linkages, the cyclical component of output also depends on expected future proportional changes in the cyclical components of foreign output and the real exchange rate.

The cyclical component of the nominal interest rate depends on the contemporaneous cyclical components of consumption price inflation and output according to monetary policy rule:

$$\hat{i}_{k,t} = \xi_k \hat{\pi}_{k,t}^{C} + \zeta_k \ln \hat{Y}_{k,t} + \nu_{k,t}. \tag{40}$$

This monetary policy rule ensures convergence of the level of consumption price inflation to its target value in deterministic steady state equilibrium.

The cyclical component of the real exchange rate depends on the contemporaneous cyclical component of the output differential according to approximate linear international risk sharing condition:

$$\ln \hat{\mathcal{Q}}_{k,t} = \frac{1}{\phi_k^2\sigma_k + \psi_k(1+\phi_k)(1-\phi_k)}(\ln \hat{Y}_{k,t} - \ln \hat{Y}_{k,t}^f). \tag{41}$$

The cyclical component of the real interest rate satisfies $\hat{r}_{k,t} = \hat{i}_{k,t} - \mathrm{E}_t\,\hat{\pi}_{k,t+1}^{C}$, while the cyclical component of the real exchange rate satisfies $\ln \hat{\mathcal{Q}}_{k,t} = \ln \hat{\mathcal{E}}_{k,t} + \ln \hat{P}_t^{Y,f} - \ln \hat{P}_{k,t}^{Y}$.

Variation in cyclical components is driven by two exogenous stochastic processes. The cyclical components of the productivity and monetary policy shocks follow stationary first order autoregressive processes:

$$\ln \hat{A}_{k,t} = \rho_{\hat{A},k} \ln \hat{A}_{k,t-1} + \varepsilon_{k,t}^{\hat{A}},\ \varepsilon_{k,t}^{\hat{A}} \sim \text{iid } \mathcal{N}(0, \sigma_{\hat{A},k}^2), \tag{42}$$

$$\hat{v}_{k,t} = \rho_{v,k} \hat{v}_{k,t-1} + \varepsilon_{k,t}^{v},\ \varepsilon_{k,t}^{v} \sim \text{iid } \mathcal{N}(0, \sigma_{v,k}^2). \tag{43}$$

The innovations driving these exogenous stochastic processes are assumed to be independent, which combined with our distributional assumptions implies multivariate normality.

3.2. Trend Components

The trend components of the prices of output and consumption follow random walks with time varying drift $\pi_{k,t}$, while the trend component of output follows a random walk with time varying drift $g_{k,t}$:

$$\ln \overline{P}_{k,t}^{Y} = \pi_{k,t} + \ln \overline{P}_{k,t-1}^{Y} + \varepsilon_{k,t}^{\overline{P}^Y},\ \varepsilon_{k,t}^{\overline{P}^Y} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{P}^Y,k}^2), \tag{44}$$

$$\ln \overline{P}_{k,t}^{C} = \pi_{k,t} + \ln \overline{P}_{k,t-1}^{C} + \varepsilon_{k,t}^{\overline{P}^C},\ \varepsilon_{k,t}^{\overline{P}^C} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{P}^C,k}^2), \tag{45}$$

$$\ln \overline{Y}_{k,t} = g_{k,t} + \ln \overline{Y}_{k,t-1} + \varepsilon_{k,t}^{\overline{Y}},\ \varepsilon_{k,t}^{\overline{Y}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{Y},k}^2). \tag{46}$$

It follows that the trend component of the relative price of consumption follows a driftless random walk. This implies that along a balanced growth path, the level of this relative price is time independent but state dependent.

The trend components of the nominal interest rate and nominal exchange rate follow driftless random walks:

$$\overline{i}_{k,t} = \overline{i}_{k,t-1} + \varepsilon_{k,t}^{\overline{i}},\ \varepsilon_{k,t}^{\overline{i}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{i},k}^2), \tag{47}$$

$$\ln \overline{\mathcal{E}}_{k,t} = \ln \overline{\mathcal{E}}_{k,t-1} + \varepsilon_{k,t}^{\overline{\mathcal{E}}},\ \varepsilon_{k,t}^{\overline{\mathcal{E}}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{\mathcal{E}},k}^2). \tag{48}$$

It follows that along a balanced growth path, the levels of the nominal interest rate and nominal exchange rate are time independent but state dependent. The trend component of the real interest rate satisfies $\overline{r}_{k,t} = \overline{i}_{k,t} - \mathrm{E}_t \overline{\pi}_{k,t+1}^{C}$, while the trend component of the real exchange rate satisfies $\ln \overline{\mathcal{Q}}_{k,t} = \ln \overline{\mathcal{E}}_{k,t} + \ln \overline{P}_{t}^{Y,f} - \ln \overline{P}_{k,t}^{Y}$.

Long run balanced growth is driven by two common stochastic trends. Trend inflation and growth follow driftless random walks:

$$\pi_{k,t} = \pi_{k,t-1} + \varepsilon_{k,t}^{\pi},\ \varepsilon_{k,t}^{\pi} \sim \text{iid } \mathcal{N}(0, \sigma_{\pi,k}^2), \tag{49}$$

$$g_{k,t} = g_{k,t-1} + \varepsilon_{k,t}^{g},\ \varepsilon_{k,t}^{g} \sim \text{iid } \mathcal{N}(0, \sigma_{g,k}^2). \tag{50}$$

It follows that along a balanced growth path, growth rates are time independent but state dependent. As an identifying restriction, all innovations are assumed to be independent, which combined with our distributional assumptions implies multivariate normality.

4. Estimation

If our approximate linear panel unobserved components representation of the canonical New Keynesian model of a small open economy is correctly specified, then estimating its deep structural parameters conditional on deterministic cross economy equality restrictions may be expected to yield mean squared error optimal exchange rate forecasts at all horizons. However, the empirical adequacy of many of the assumptions underlying this particular version of the New Keynesian model have been called into question, including but not limited to the assumptions of intertemporally additive preferences, perfectly flexible wages, complete international financial markets, and complete exchange rate pass through. Under such extensive and diverse potential forms of model misspecification, it may instead be mean squared error optimal from an exchange rate forecasting perspective to estimate these deep structural parameters conditional on stochastic cross economy equality restrictions of horizon dependent tightness.

This section develops and applies a novel Bayesian procedure for jointly estimating the hyperparameters and trend components of a state space representation of a panel unobserved components representation of a multivariate linear rational expectations model, conditional on prior information concerning the values of hyperparameters and trend components. Prior information concerning the values of hyperparameters is summarized by a hierarchical prior distribution which represents different levels of subjective beliefs. The first tier of this hierarchical prior distribution is informative only for deep structural parameters, identified as those parameters associated with the conditional mean function, and represents the belief that their values are approximately equal across economies. The second tier of this hierarchical prior distribution is diffuse, and represents the belief that the common values to which these deep structural parameters are approximately equal are completely unknown.

4.1. Estimation Procedure

Let $\boldsymbol{x}_t$ denote a vector stochastic process consisting of the levels of N nonpredetermined endogenous variables, of which M are observed. The cyclical components of this vector stochastic process satisfy second order stochastic linear difference equation

$$\boldsymbol{A}_0\hat{\boldsymbol{x}}_t = \boldsymbol{A}_1\hat{\boldsymbol{x}}_{t-1} + \boldsymbol{A}_2\mathrm{E}_t\hat{\boldsymbol{x}}_{t+1} + \boldsymbol{A}_3\hat{\boldsymbol{v}}_t, \tag{51}$$

where vector stochastic process $\hat{\boldsymbol{v}}_t$ consists of the cyclical components of K exogenous variables. This vector stochastic process satisfies stationary first order stochastic linear difference equation

$$\hat{\boldsymbol{v}}_t = \boldsymbol{B}_1\hat{\boldsymbol{v}}_{t-1} + \boldsymbol{\varepsilon}_{1,t}, \tag{52}$$

where $\boldsymbol{\varepsilon}_{1,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_1)$. If there exists a unique stationary solution to this multivariate linear rational expectations model, then it may be expressed as:

$$\hat{\boldsymbol{x}}_t = \boldsymbol{C}_1\hat{\boldsymbol{x}}_{t-1} + \boldsymbol{C}_2\hat{\boldsymbol{v}}_t. \tag{53}$$

This unique stationary solution is calculated with the matrix decomposition based algorithm due to Klein (2000).

The trend components of vector stochastic process $\boldsymbol{x}_t$ satisfy first order stochastic linear difference equation

$$\boldsymbol{D}_0\bar{\boldsymbol{x}}_t = \boldsymbol{D}_1\boldsymbol{u}_t + \boldsymbol{D}_2\bar{\boldsymbol{x}}_{t-1} + \boldsymbol{\varepsilon}_{2,t}, \tag{54}$$

where $\boldsymbol{\varepsilon}_{2,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_2)$. Vector stochastic process $\boldsymbol{u}_t$ consists of the levels of L common stochastic trends, and satisfies nonstationary first order stochastic linear difference equation

$$\boldsymbol{u}_t = \boldsymbol{u}_{t-1} + \boldsymbol{\varepsilon}_{3,t}, \tag{55}$$

where $\boldsymbol{\varepsilon}_{3,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_3)$. Cyclical and trend components are additively separable, that is $\boldsymbol{x}_t = \hat{\boldsymbol{x}}_t + \bar{\boldsymbol{x}}_t$.

Let $\boldsymbol{y}_t$ denote a vector stochastic process consisting of the levels of M observed nonpredetermined endogenous variables. Also, let $\boldsymbol{z}_t$ denote a vector stochastic process consisting of the levels of $N-M$ unobserved nonpredetermined endogenous variables, the cyclical components of N nonpredetermined endogenous variables, the trend components of N nonpredetermined endogenous variables, the cyclical components of K exogenous variables, and the levels of L common stochastic trends. Given unique stationary solution (53), these vector stochastic processes have linear state space representation

$$\boldsymbol{y}_t = \boldsymbol{F}_1 \boldsymbol{z}_t, \tag{56}$$

$$\boldsymbol{z}_t = \boldsymbol{G}_1 \boldsymbol{z}_{t-1} + \boldsymbol{G}_2 \boldsymbol{\varepsilon}_{4,t}, \tag{57}$$

where $\boldsymbol{\varepsilon}_{4,t} \sim \text{iid } \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_4)$ and $\boldsymbol{z}_0 \sim \mathcal{N}(\boldsymbol{z}_{0|0}, \boldsymbol{P}_{0|0})$. Let $\boldsymbol{w}_t$ denote a vector stochastic process consisting of preliminary estimates of the trend components of M observed nonpredetermined endogenous variables. Suppose that this vector stochastic process satisfies

$$\boldsymbol{w}_t = \boldsymbol{H}_1 \boldsymbol{z}_t + \boldsymbol{\varepsilon}_{5,t}, \tag{58}$$

where $\boldsymbol{\varepsilon}_{5,t} \sim \text{iid } \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_5)$. Conditional on known parameter values, this signal equation defines a set of stochastic restrictions on selected unobserved state variables. The signal and state innovation vectors are assumed to be independent, while the initial state vector is assumed to be independent from the signal and state innovation vectors, which combined with our distributional assumptions implies multivariate normality.

Conditional on the parameters associated with these signal and state equations, estimates of unobserved state vector $\boldsymbol{z}_t$ and its mean squared error matrix $\boldsymbol{P}_t$ may be calculated with the filter proposed by Vitek (2007), which adapts the filter due to Kalman (1960) to incorporate prior information. Given initial conditions $\boldsymbol{z}_{0|0}$ and $\boldsymbol{P}_{0|0}$, estimates conditional on information available at time $t-1$ satisfy prediction equations:

$$\boldsymbol{z}_{t|t-1} = \boldsymbol{G}_1 \boldsymbol{z}_{t-1|t-1}, \tag{59}$$

$$\boldsymbol{P}_{t|t-1} = \boldsymbol{G}_1 \boldsymbol{P}_{t-1|t-1} \boldsymbol{G}_1^\top + \boldsymbol{G}_2 \boldsymbol{\Sigma}_4 \boldsymbol{G}_2^\top, \tag{60}$$

$$\boldsymbol{y}_{t|t-1} = \boldsymbol{F}_1 \boldsymbol{z}_{t|t-1}, \tag{61}$$

$$\boldsymbol{Q}_{t|t-1} = \boldsymbol{F}_1 \boldsymbol{P}_{t|t-1} \boldsymbol{F}_1^\top, \tag{62}$$

$$\boldsymbol{w}_{t|t-1} = \boldsymbol{H}_1 \boldsymbol{z}_{t|t-1}, \tag{63}$$

$$\boldsymbol{R}_{t|t-1} = \boldsymbol{H}_1 \boldsymbol{P}_{t|t-1} \boldsymbol{H}_1^\top + \boldsymbol{\Sigma}_5. \tag{64}$$

Given these predictions, under the assumption of multivariate normally distributed signal and state innovation vectors, together with conditionally contemporaneously uncorrelated signal vectors, estimates conditional on information available at time t satisfy updating equations

$$\boldsymbol{z}_{t|t} = \boldsymbol{z}_{t|t-1} + \boldsymbol{K}_{y_t}(\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1}) + \boldsymbol{K}_{w_t}(\boldsymbol{w}_t - \boldsymbol{w}_{t|t-1}), \tag{65}$$

$$\boldsymbol{P}_{t|t} = \boldsymbol{P}_{t|t-1} - \boldsymbol{K}_{y_t}\boldsymbol{F}_1\boldsymbol{P}_{t|t-1} - \boldsymbol{K}_{w_t}\boldsymbol{H}_1\boldsymbol{P}_{t|t-1}, \tag{66}$$

where $\boldsymbol{K}_{y_t} = \boldsymbol{P}_{t|t-1}\boldsymbol{F}_1^{\mathsf{T}}\boldsymbol{Q}_{t|t-1}^{-1}$ and $\boldsymbol{K}_{w_t} = \boldsymbol{P}_{t|t-1}\boldsymbol{H}_1^{\mathsf{T}}\boldsymbol{R}_{t|t-1}^{-1}$. Under our distributional assumptions, these estimators of the unobserved state vector are mean squared error optimal.

Let $\boldsymbol{\theta} \in \boldsymbol{\Theta} \subset \mathbb{R}^J$ denote a J dimensional vector containing the hyperparameters associated with the signal and state equations of this linear state space model. The Bayesian estimator of this hyperparameter vector has posterior density function

$$f(\boldsymbol{\theta} \mid \mathcal{I}_T) \propto f(\mathcal{I}_T \mid \boldsymbol{\theta}) f(\boldsymbol{\theta}), \tag{67}$$

where $\mathcal{I}_t = \{\{\boldsymbol{y}_s\}_{s=1}^t, \{\boldsymbol{w}_s\}_{s=1}^t\}$. Under the assumption of multivariate normally distributed signal and state innovation vectors, together with conditionally contemporaneously uncorrelated signal vectors, conditional density function $f(\mathcal{I}_T \mid \boldsymbol{\theta})$ satisfies:

$$f(\mathcal{I}_T \mid \boldsymbol{\theta}) = \prod_{t=1}^{T} f(\boldsymbol{y}_t \mid \mathcal{I}_{t-1}, \boldsymbol{\theta}) \cdot \prod_{t=1}^{T} f(\boldsymbol{w}_t \mid \mathcal{I}_{t-1}, \boldsymbol{\theta}). \tag{68}$$

Under our distributional assumptions, conditional density functions $f(\boldsymbol{y}_t \mid \mathcal{I}_{t-1}, \boldsymbol{\theta})$ and $f(\boldsymbol{w}_t \mid \mathcal{I}_{t-1}, \boldsymbol{\theta})$ satisfy:

$$f(\boldsymbol{y}_t \mid \mathcal{I}_{t-1}, \boldsymbol{\theta}) = (2\pi)^{-\frac{M}{2}} \, |\boldsymbol{Q}_{t|t-1}|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1})^{\mathsf{T}}\boldsymbol{Q}_{t|t-1}^{-1}(\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1})\right\}, \tag{69}$$

$$f(\boldsymbol{w}_t \mid \mathcal{I}_{t-1}, \boldsymbol{\theta}) = (2\pi)^{-\frac{M}{2}} \, |\boldsymbol{R}_{t|t-1}|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\boldsymbol{w}_t - \boldsymbol{w}_{t|t-1})^{\mathsf{T}}\boldsymbol{R}_{t|t-1}^{-1}(\boldsymbol{w}_t - \boldsymbol{w}_{t|t-1})\right\}. \tag{70}$$

Estimation of the hyperparameters is conditional on both the levels of observed nonpredetermined endogenous variables and preliminary estimates of their trend components.

Prior information concerning hyperparameter vector $\boldsymbol{\theta}$ is summarized by a hierarchical prior distribution

$$f(\boldsymbol{\theta}) = f(\boldsymbol{\theta}_1 \mid \boldsymbol{\theta}_2) f(\boldsymbol{\theta}_2), \tag{71}$$

where $\boldsymbol{\theta} = (\boldsymbol{\theta}_1^{\mathsf{T}}, \boldsymbol{\theta}_2^{\mathsf{T}})^{\mathsf{T}}$. Prior information concerning parameter vector $\boldsymbol{\theta}_1$, which contains those J_1 parameters associated with the signal and state equations of this linear state space model under parameter heterogeneity across economies, is summarized by a conditional multivariate normal prior distribution having mean vector $\boldsymbol{\theta}_{1|2}$ and covariance matrix $\boldsymbol{\Omega}_{1|2}$:

$$f(\boldsymbol{\theta}_1 \mid \boldsymbol{\theta}_2) = (2\pi)^{-\frac{J_1}{2}} \mid \boldsymbol{\Omega}_{1|2} \mid^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_{1|2})^{\mathsf{T}} \boldsymbol{\Omega}_{1|2}^{-1}(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_{1|2})\right\}. \tag{72}$$

Prior information concerning hyperparameter vector $\boldsymbol{\theta}_2$, which contains those $J_2 = J - J_1$ parameters associated with the signal and state equations of this linear state space model under parameter homogeneity across economies, is summarized by an unconditional multivariate normal prior distribution having mean vector $\boldsymbol{\theta}_3$ and covariance matrix $\boldsymbol{\Omega}_3$:

$$f(\boldsymbol{\theta}_2) = (2\pi)^{-\frac{J_2}{2}} \mid \boldsymbol{\Omega}_3 \mid^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\boldsymbol{\theta}_2 - \boldsymbol{\theta}_3)^{\mathsf{T}} \boldsymbol{\Omega}_3^{-1}(\boldsymbol{\theta}_2 - \boldsymbol{\theta}_3)\right\}. \tag{73}$$

Independent priors are represented by diagonal covariance matrices, under which parameter homogeneity across economies is represented by $\boldsymbol{\Omega}_{1|2} = \mathbf{0}$, while parameter heterogeneity is represented by $\boldsymbol{\Omega}_{1|2} \neq \mathbf{0}$.

Inference on the hyperparameters under either parameter homogeneity across economies or parameter heterogeneity is based on an asymptotic normal approximation to the posterior distribution around its mode. Under regularity conditions stated in Geweke (2005), posterior mode $\hat{\boldsymbol{\theta}}_T$ satisfies

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \xrightarrow{d} \mathcal{N}(\mathbf{0}, -\mathcal{H}_0^{-1}), \tag{74}$$

where $\boldsymbol{\theta}_0 \in \boldsymbol{\Theta}$ denotes the pseudotrue hyperparameter vector. Following Engle and Watson (1981), Hessian $\mathcal{H}_0$ may be estimated by

$$\begin{aligned} \hat{\mathcal{H}}_T = {} & \frac{1}{T}\sum_{t=1}^{T} \mathrm{E}_{t-1}\left[\nabla_{\boldsymbol{\theta}}\nabla_{\boldsymbol{\theta}}^{\mathsf{T}} \ln f(\boldsymbol{y}_t \mid \mathcal{I}_{t-1}, \hat{\boldsymbol{\theta}}_T)\right] + \frac{1}{T}\sum_{t=1}^{T} \mathrm{E}_{t-1}\left[\nabla_{\boldsymbol{\theta}}\nabla_{\boldsymbol{\theta}}^{\mathsf{T}} \ln f(\boldsymbol{w}_t \mid \mathcal{I}_{t-1}, \hat{\boldsymbol{\theta}}_T)\right] \\ & + \frac{1}{T}\nabla_{\boldsymbol{\theta}}\nabla_{\boldsymbol{\theta}}^{\mathsf{T}} \ln f(\hat{\boldsymbol{\theta}}_T), \end{aligned} \tag{75}$$

where $\mathrm{E}_{t-1}\left[\nabla_{\boldsymbol{\theta}}\nabla_{\boldsymbol{\theta}}^{\mathsf{T}} \ln f(\boldsymbol{y}_t \mid \mathcal{I}_{t-1}, \hat{\boldsymbol{\theta}}_T)\right] = -\nabla_{\boldsymbol{\theta}} \boldsymbol{y}_{t|t-1}^{\mathsf{T}} \boldsymbol{Q}_{t|t-1}^{-1} \nabla_{\boldsymbol{\theta}} \boldsymbol{y}_{t|t-1} - \frac{1}{2}\nabla_{\boldsymbol{\theta}} \boldsymbol{Q}_{t|t-1}^{\mathsf{T}} (\boldsymbol{Q}_{t|t-1}^{-1} \otimes \boldsymbol{Q}_{t|t-1}^{-1}) \nabla_{\boldsymbol{\theta}} \boldsymbol{Q}_{t|t-1}$ and $\mathrm{E}_{t-1}\left[\nabla_{\boldsymbol{\theta}}\nabla_{\boldsymbol{\theta}}^{\mathsf{T}} \ln f(\boldsymbol{w}_t \mid \mathcal{I}_{t-1}, \hat{\boldsymbol{\theta}}_T)\right] = -\nabla_{\boldsymbol{\theta}} \boldsymbol{w}_{t|t-1}^{\mathsf{T}} \boldsymbol{R}_{t|t-1}^{-1} \nabla_{\boldsymbol{\theta}} \boldsymbol{w}_{t|t-1} - \frac{1}{2}\nabla_{\boldsymbol{\theta}} \boldsymbol{R}_{t|t-1}^{\mathsf{T}} (\boldsymbol{R}_{t|t-1}^{-1} \otimes \boldsymbol{R}_{t|t-1}^{-1}) \nabla_{\boldsymbol{\theta}} \boldsymbol{R}_{t|t-1}$.

4.2. Estimation Results

The hyperparameters and trend components of our approximate linear panel unobserved components representation of the canonical New Keynesian model of a small open economy are jointly estimated with the Bayesian procedure described above, conditional on prior information concerning the values of hyperparameters and trend components. Estimation is based on the levels of five observed endogenous variables for each of Australia, Canada and the United Kingdom, which are treated as small open economies, and three observed endogenous variables for the United States, which is treated as a closed economy. Descriptions of the variables employed are contained in the appendix.

Prior information concerning the values of hyperparameters is summarized by a hierarchical prior distribution which represents different levels of subjective beliefs. The first tier of this hierarchical prior distribution is informative only for deep structural parameters, and represents the belief that their values are approximately equal across economies. Under the case of parameter homogeneity across economies, corresponding to deterministic cross economy equality restrictions, this conditional prior distribution is degenerate. The second tier of this hierarchical prior distribution is diffuse, and represents the belief that the common values to which these deep structural parameters are approximately equal are completely unknown.

The hyperparameters and trend components of our approximate linear panel unobserved components representation of the canonical New Keynesian model of a small open economy are jointly estimated with the Bayesian procedure described above in two steps. In the first step, parameter homogeneity across economies is imposed, and a set of objective beliefs concerning the common values to which deep structural parameters are exactly equal is generated. In the second step, parameter homogeneity across economies is systematically relaxed, and these deep structural parameters are repeatedly estimated conditional on different sets of subjective beliefs concerning the common values to which they are approximately equal derived from the first step. These subjective beliefs correspond to stochastic restrictions on deep structural parameters having conditional means equal to posterior modes estimated in the first step, and conditional standard errors proportional to corresponding estimates of posterior standard errors. All stochastic restrictions are independent, represented by a diagonal covariance matrix, and are harmonized, represented by a common factor of proportionality. This common factor of proportionality indexes different sets of subjective beliefs, ranging from strong convictions in parameter homogeneity across economies for low values, to weak convictions for high values.

Prior information concerning the values of trend components is generated by fitting third order deterministic polynomial functions to the levels of all observed endogenous variables by ordinary least squares. Stochastic restrictions on the trend components of all observed endogenous variables have conditional means equal to the predicted values associated with these ordinary least squares regressions, and conditional standard errors proportional to corresponding estimates of prediction standard errors assuming known parameters. All stochastic restrictions are independent, represented by a diagonal covariance matrix, and are harmonized, represented by a common factor of proportionality. Reflecting little confidence in these preliminary trend component estimates, this common factor of proportionality is set equal to one.

We jointly estimate the hyperparameters and trend components of our approximate linear panel unobserved components representation of the canonical New Keynesian model of a small open economy over the period 1973Q3 through 2006Q2. Estimation results corresponding to different sets of subjective beliefs concerning parameter homogeneity across economies are reported in Table 1 through Table 4. Initial conditions for the cyclical components of exogenous variables are given by their unconditional means and variances, while the initial values of all other state variables are treated as parameters, and are calibrated to match functions of initial realizations of the levels of observed endogenous variables, or preliminary estimates of their trend components calculated with the linear filter described in Hodrick and Prescott (1997). The posterior mode is calculated as stochastic cross economy equality restrictions are systematically relaxed by numerically maximizing the logarithm of the posterior density kernel with a modified steepest ascent algorithm. The sufficient condition for the existence of a unique stationary rational expectations equilibrium due to Klein (2000) is always satisfied in a neighbourhood around the posterior mode, while our estimator of the Hessian is never nearly singular at the posterior mode, suggesting that our state space representation of our approximate linear panel unobserved components model is locally identified.

Under the case of parameter homogeneity across economies, the posterior modes of the deep structural parameters associated with our approximate linear panel unobserved components representation of the canonical New Keynesian model of a small open economy are all well within the range of estimates reported in the existing literature and are generally precisely estimated, as evidenced by relatively small posterior standard errors. Under the case of parameter heterogeneity across economies, the posterior modes of these deep structural parameters all remain well within the range of estimates reported in the existing literature but are generally less precisely estimated, revealed by larger posterior standard errors. The estimated variances of shocks driving variation in cyclical components are all well within the range of estimates reported in the existing literature, after accounting for data rescaling. The estimated variances of shocks driving variation in trend components are relatively high, indicating that the majority of variation in the levels of observed endogenous variables is accounted for by variation in their trend components.

Table 1. Posterior parameter estimates, Australia

Parameter	$\alpha = 0$		$\alpha = 10^0$		$\alpha = 10^1$		$\alpha = 10^2$		$\alpha = \infty$	
	Mode	SE	Mode	SE	Mode	SE	Mode	SE	Mode	SE
σ	0.957900	0.000007	0.957840	0.000007	0.957860	0.000065	0.958330	0.000655	0.949930	0.142230
η	0.947560	0.000213	0.947060	0.000213	0.947040	0.002128	0.946980	0.021277	0.947020	3.878300
ϕ	0.561370	0.020487	0.540320	0.018237	0.540270	0.057736	0.537110	0.090779	0.540320	0.098650
ψ	1.667200	0.024524	1.715500	0.024322	1.715600	0.203650	1.724400	0.415780	1.715500	0.467760
ω	0.558210	0.003391	0.556750	0.003388	0.556740	0.032816	0.556170	0.135780	0.556750	0.436640
ξ	1.325300	0.009418	1.275900	0.009334	1.275800	0.068703	1.268400	0.111940	1.275900	0.364030
ζ	0.123080	0.000076	0.121000	0.000076	0.120980	0.000756	0.120670	0.007542	0.120970	0.147030
ρ_A	0.516320	0.023025	0.516810	0.021461	0.516810	0.062318	0.516800	0.096858	0.516810	0.101640
ρ_v	0.572680	0.018652	0.618500	0.017321	0.618640	0.049545	0.627920	0.055030	0.618490	0.060834
σ_A^2	0.695410	0.099488	0.697100	0.100080	0.697110	0.191500	0.697510	0.515460	0.697100	0.752860
σ_v^2	0.017443	0.003376	0.017826	0.003287	0.017827	0.005470	0.017896	0.006465	0.017826	0.021152
$\sigma_{\bar{P}^Y}^2$	0.966740	0.118750	0.970860	0.119300	0.970870	0.119390	0.971630	0.119510	0.970860	0.120070
$\sigma_{\bar{P}^C}^2$	0.806240	0.103170	0.805800	0.103000	0.805790	0.103050	0.805720	0.103060	0.805800	0.103150
$\sigma_{\bar{Y}}^2$	0.097556	0.014655	0.101230	0.015985	0.101240	0.016674	0.101800	0.017385	0.101230	0.017268
$\sigma_{\bar{i}}^2$	0.001952	0.000352	0.001965	0.000351	0.001965	0.000353	0.001968	0.000356	0.001965	0.000362
$\sigma_{\bar{\varepsilon}}^2$	1.137400	0.127470	1.223600	0.128560	1.223900	0.129370	1.239500	0.131690	1.223600	0.130570
σ_π^2	0.000219	0.000045	0.000220	0.000045	0.000220	0.000045	0.000220	0.000045	0.000220	0.000045
σ_g^2	0.000024	0.000011	0.000024	0.000011	0.000024	0.000011	0.000024	0.000012	0.000024	0.000012

Note: Prior standard errors under parameter heterogeneity across economies are generated by scaling posterior standard errors under parameter homogeneity by proportionality factor α. Subjective discount factor β is restricted to equal 0.99. All observed endogenous variables are rescaled by a factor of 100.

Table 2. Posterior parameter estimates, Canada

Parameter	$\alpha = 0$		$\alpha = 10^0$		$\alpha = 10^1$		$\alpha = 10^2$		$\alpha = \infty$	
	Mode	SE	Mode	SE	Mode	SE	Mode	SE	Mode	SE
σ	0.957900	0.000007	0.957890	0.000007	0.957900	0.000065	0.957950	0.000655	0.956730	0.142730
η	0.947560	0.000213	0.941630	0.000213	0.941320	0.002128	0.940820	0.021276	0.941110	1.980200
ϕ	0.561370	0.020487	0.558190	0.019325	0.558180	0.058882	0.557540	0.069062	0.558190	0.089588
ψ	1.667200	0.024524	1.726600	0.024427	1.726700	0.188600	1.732400	0.309990	1.726600	0.419450
ω	0.558210	0.003391	0.528470	0.003377	0.528430	0.026977	0.525900	0.049500	0.528470	0.206310
ξ	1.325300	0.009418	1.260300	0.009406	1.260100	0.087408	1.252700	0.244280	1.260300	0.374100
ζ	0.123080	0.000076	0.123890	0.000076	0.123900	0.000756	0.123810	0.007551	0.123900	0.178810
ρ_A	0.516320	0.023025	0.521570	0.021768	0.521590	0.065632	0.522490	0.072282	0.521570	0.079076
ρ_v	0.572680	0.018652	0.603190	0.016289	0.603270	0.036155	0.608420	0.043760	0.603190	0.047885
σ_A^2	0.063533	0.012352	0.064379	0.012734	0.064381	0.017198	0.064526	0.023690	0.064379	0.025812
σ_v^2	0.021498	0.004208	0.021805	0.004071	0.021805	0.006961	0.021854	0.013597	0.021805	0.024377
$\sigma_{\bar{P}^Y}^2$	0.781520	0.095662	0.784280	0.096031	0.784290	0.096136	0.784790	0.096169	0.784280	0.096421
$\sigma_{\bar{P}^C}^2$	0.546810	0.067672	0.546430	0.067486	0.546430	0.067514	0.546360	0.067477	0.546430	0.067733
$\sigma_{\bar{Y}}^2$	0.111400	0.015131	0.119510	0.016658	0.119530	0.016955	0.120970	0.017287	0.119510	0.017158
$\sigma_{\bar{i}}^2$	0.001769	0.000344	0.001779	0.000353	0.001779	0.000367	0.001781	0.000373	0.001779	0.000384
$\sigma_{\bar{\varepsilon}}^2$	0.625500	0.076499	0.647000	0.070238	0.647060	0.070402	0.650940	0.071020	0.647000	0.070687
σ_π^2	0.000174	0.000035	0.000175	0.000035	0.000175	0.000035	0.000175	0.000035	0.000175	0.000035
σ_g^2	0.000013	0.000010	0.000013	0.000011	0.000013	0.000011	0.000013	0.000011	0.000013	0.000011

Note: Prior standard errors under parameter heterogeneity across economies are generated by scaling posterior standard errors under parameter homogeneity by proportionality factor α. Subjective discount factor β is restricted to equal 0.99. All observed endogenous variables are rescaled by a factor of 100.

Table 3. Posterior parameter estimates, United Kingdom

Parameter	$\alpha = 0$		$\alpha = 10^0$		$\alpha = 10^1$		$\alpha = 10^2$		$\alpha = \infty$	
	Mode	SE	Mode	SE	Mode	SE	Mode	SE	Mode	SE
σ	0.957900	0.000007	0.957900	0.000007	0.957900	0.000065	0.957990	0.000655	0.956970	0.146560
η	0.947560	0.000213	0.925260	0.000213	0.924160	0.002128	0.923030	0.021276	0.923400	1.969500
ϕ	0.561370	0.020487	0.559560	0.019172	0.559560	0.055069	0.559570	0.063203	0.559560	0.076638
ψ	1.667200	0.024524	1.618900	0.024388	1.619000	0.175130	1.625200	0.264540	1.618900	0.342760
ω	0.558210	0.003391	0.524470	0.003371	0.524400	0.025141	0.520140	0.043093	0.524470	0.209820
ξ	1.325300	0.009418	1.156800	0.009402	1.156700	0.085011	1.150900	0.208910	1.156700	0.319400
ζ	0.123080	0.000076	0.124820	0.000076	0.124830	0.000756	0.124730	0.007551	0.124840	0.192300
ρ_A	0.516320	0.023025	0.522710	0.021312	0.522720	0.056669	0.523670	0.062582	0.522710	0.065314
ρ_v	0.572680	0.018652	0.598610	0.016313	0.598680	0.036038	0.603100	0.041378	0.598610	0.044592
σ_A^2	0.068570	0.012354	0.070055	0.013072	0.070058	0.018119	0.070314	0.023968	0.070054	0.027489
σ_v^2	0.023586	0.004515	0.023906	0.004629	0.023906	0.008393	0.023957	0.015003	0.023906	0.027455
$\sigma_{\bar{P}^Y}^2$	1.347900	0.167050	1.355200	0.166920	1.355200	0.167010	1.356600	0.167100	1.355200	0.167190
$\sigma_{\bar{P}^C}^2$	1.391000	0.174370	1.391600	0.174660	1.391600	0.174750	1.391700	0.174770	1.391600	0.174920
$\sigma_{\bar{Y}}^2$	0.086880	0.012347	0.092830	0.013567	0.092846	0.013786	0.093892	0.014068	0.092830	0.013872
$\sigma_{\bar{i}}^2$	0.000891	0.000189	0.000895	0.000192	0.000895	0.000197	0.000895	0.000200	0.000895	0.000203
$\sigma_{\bar{\xi}}^2$	0.856160	0.096675	0.955150	0.094015	0.955420	0.094705	0.973340	0.097694	0.955140	0.096215
σ_π^2	0.000204	0.000042	0.000205	0.000042	0.000205	0.000042	0.000205	0.000042	0.000205	0.000042
σ_g^2	0.000032	0.000018	0.000032	0.000019	0.000032	0.000019	0.000032	0.000019	0.000032	0.000019

Note: Prior standard errors under parameter heterogeneity across economies are generated by scaling posterior standard errors under parameter homogeneity by proportionality factor α. Subjective discount factor β is restricted to equal 0.99. All observed endogenous variables are rescaled by a factor of 100.

Table 4. Posterior parameter estimates, United States

Parameter	$\alpha = 0$		$\alpha = 10^0$		$\alpha = 10^1$		$\alpha = 10^2$		$\alpha = \infty$	
	Mode	SE	Mode	SE	Mode	SE	Mode	SE	Mode	SE
σ	0.957900	0.000007	0.957750	0.000007	0.957790	0.000065	0.959040	0.000655	1.040500	0.178320
η	0.947560	0.000213	0.890180	0.000213	0.887620	0.002128	0.886780	0.021277	0.885780	7352.100
ω	0.558210	0.003391	0.550690	0.003373	0.550680	0.028778	0.549910	0.125230	0.550690	721.0200
ξ	1.325300	0.009418	1.249600	0.009405	1.249600	0.090148	1.247100	0.622940	1.249600	1179.700
ζ	0.123080	0.000076	0.122130	0.000076	0.122130	0.000756	0.122100	0.007559	0.122120	943.2600
ρ_A	0.516320	0.023025	0.520310	0.019204	0.520330	0.034718	0.521070	0.036746	0.520310	0.038552
ρ_v	0.572680	0.018652	0.577300	0.014668	0.577310	0.031230	0.578060	0.042622	0.577300	0.045933
σ_A^2	0.056605	0.006644	0.056977	0.006759	0.056978	0.010385	0.057036	0.036893	0.056977	9.350300
σ_v^2	0.040991	0.007615	0.041222	0.006849	0.041222	0.010950	0.041264	0.054280	0.041222	145.4200
$\sigma_{\bar{P}^Y}^2$	0.601680	0.068736	0.605330	0.072712	0.605340	0.072831	0.605990	0.073046	0.605330	0.073017
$\sigma_{\bar{Y}}^2$	0.106320	0.013355	0.115140	0.014701	0.115160	0.014805	0.116720	0.015131	0.115140	0.015028
$\sigma_{\bar{i}}^2$	0.001257	0.000310	0.001266	0.000226	0.001266	0.000235	0.001267	0.000245	0.001266	0.000247
σ_π^2	0.000201	0.000055	0.000201	0.000057	0.000201	0.000057	0.000201	0.000057	0.000201	0.000057
σ_g^2	0.000026	0.000013	0.000026	0.000013	0.000026	0.000013	0.000026	0.000013	0.000026	0.000014

Note: Prior standard errors under parameter heterogeneity across economies are generated by scaling posterior standard errors under parameter homogeneity by proportionality factor α. Subjective discount factor β is restricted to equal 0.99. All observed endogenous variables are rescaled by a factor of 100.

The distance between the posterior modes of the deep structural parameters associated with our approximate linear panel unobserved components representation of the canonical New Keynesian model of a small open economy and their prior means is generally increasing in the common factor of proportionality applied in generating prior standard errors, as expected. However, this distance is generally relatively small, both economically and statistically, even under the case of diffuse cross economy equality restrictions, lending empirical support to the common practice in the theoretical open economy macroeconomics literature of imposing deterministic cross economy equality restrictions on deep structural parameters.

5. Forecasting

Our evaluation of the dynamic out of sample nominal exchange rate forecasting performance of the canonical New Keynesian model of a small open economy is multidimensional. First, we examine whether and to what extent the model yields incremental predictive power relative to a driftless random walk across different horizons. This is facilitated by nesting this New Keynesian model within an approximate linear unobserved components framework in which the trend component of the nominal exchange rate follows a driftless random walk. Second, we examine whether and to what extent imposing stochastic cross economy equality restrictions on the deep structural parameters of the model yields incremental predictive power across different horizons, as these parameter restrictions are systematically tightened. This is facilitated by nesting our approximate linear unobserved components representation of this New Keynesian model within a panel framework.

While it is desirable that forecasts be unbiased and efficient, the practical value of any forecasting model depends on its relative predictive accuracy. In the absence of a well defined mapping between forecast errors and their costs, relative predictive accuracy is generally assessed with mean squared prediction error based measures.

We measure the dynamic out of sample nominal exchange rate forecasting performance of the canonical New Keynesian model of a small open economy relative to that of a driftless random walk over a holdout sample of size R at various horizons $h \leq H$ on the basis of the U statistic due to Theil (1966), which equals the ratio of root mean squared prediction errors:

$$U_{k,h} = \frac{\sqrt{\frac{1}{R-H+1}\sum_{s=0}^{R-H}(\ln \mathcal{E}_{k,T-R+h+s} - \ln \mathcal{E}_{k,T-R+h+s|T-R+s})^2}}{\sqrt{\frac{1}{R-H+1}\sum_{s=0}^{R-H}(\ln \mathcal{E}_{k,T-R+h+s} - \ln \mathcal{E}_{k,T-R+s})^2}}. \tag{76}$$

If $U_{k,h} < 1$ then the exchange rate forecasting performance of this New Keynesian model dominated that of a random walk for small open economy k at horizon h over the holdout sample under consideration, and vice versa.

Forecast performance evaluation exercises differ with respect to the manner in which data dependent inputs are updated as the forecast origin rolls forward. Motivated by computational cost considerations, we combine a fixed scheme for updating prior and posterior parameter distributions, which are estimated conditional on information available at the initial forecast origin, with a recursive scheme for updating prior and posterior state variable distributions, which are estimated conditional on information available at the actual forecast origin.

To compare the dynamic out of sample nominal exchange rate forecasting performance of the canonical New Keynesian model of a small open economy with that of a driftless random walk, forty quarters of observations are retained to evaluate forecasts one through twenty quarters ahead. The results of this forecast performance evaluation exercise are reported in Table 5. Exacerbating the exchange rate forecasting puzzle, we find that the New Keynesian model generally yields economically small negative incremental predictive power relative to a random walk at all horizons, measured in terms of root mean squared error. To elaborate, under the case of diffuse cross economy equality restrictions, it yields incremental predictive power of −1.8% for Australia, −3.9% for Canada, and −0.3% for the United Kingdom, averaged across horizons. Nevertheless, we find that imposing and systematically tightening stochastic cross economy equality restrictions on the deep structural parameters of the New Keynesian model generally yields economically small positive incremental predictive power at all horizons, measured in terms of root mean squared error, with predictive power generally maximized under the case of deterministic cross economy equality restrictions. In particular, imposing deterministic cross economy equality restrictions yields incremental predictive power relative to imposing diffuse restrictions of 0.2% for Australia, 1.4% for Canada, and 0.3% for the United Kingdom, averaged across horizons.

Table 5. Forecast performance evaluation of New Keynesian model versus random walk

h	Australia					Canada					United Kingdom				
	$\alpha=0$	$\alpha=10^0$	$\alpha=10^1$	$\alpha=10^2$	$\alpha=\infty$	$\alpha=0$	$\alpha=10^0$	$\alpha=10^1$	$\alpha=10^2$	$\alpha=\infty$	$\alpha=0$	$\alpha=10^0$	$\alpha=10^1$	$\alpha=10^2$	$\alpha=\infty$
1	1.014	1.007	1.007	1.001	1.006	1.007	1.004	1.004	1.006	1.005	1.006	1.006	1.006	1.006	1.007
2	1.021	1.008	1.008	0.989	1.006	1.023	1.007	1.007	1.012	1.008	1.010	1.009	1.009	1.008	1.009
3	1.019	1.012	1.011	0.991	1.010	1.032	1.022	1.022	1.027	1.022	1.020	1.018	1.018	1.021	1.019
4	1.020	1.014	1.014	0.991	1.012	1.045	1.046	1.046	1.052	1.046	1.023	1.026	1.026	1.029	1.026
5	1.018	1.014	1.014	0.992	1.012	1.043	1.047	1.048	1.054	1.047	1.020	1.026	1.026	1.029	1.026
6	1.017	1.014	1.014	0.993	1.012	1.047	1.051	1.051	1.060	1.051	1.011	1.015	1.015	1.016	1.014
7	1.017	1.015	1.015	0.999	1.014	1.048	1.058	1.058	1.066	1.058	1.004	1.007	1.007	1.008	1.007
8	1.018	1.017	1.017	1.002	1.015	1.042	1.056	1.056	1.061	1.056	1.001	1.003	1.003	1.004	1.003
9	1.018	1.019	1.019	1.011	1.018	1.032	1.048	1.048	1.052	1.048	0.998	1.000	1.000	1.000	1.000
10	1.019	1.021	1.021	1.023	1.021	1.024	1.042	1.042	1.047	1.042	0.996	0.998	0.998	0.997	0.997
11	1.018	1.023	1.023	1.034	1.023	1.018	1.039	1.039	1.046	1.039	0.994	0.995	0.995	0.993	0.994
12	1.017	1.025	1.025	1.045	1.025	1.015	1.038	1.038	1.048	1.038	0.991	0.993	0.993	0.989	0.992
13	1.016	1.024	1.024	1.052	1.024	1.014	1.038	1.038	1.049	1.038	0.989	0.991	0.991	0.987	0.990
14	1.014	1.023	1.024	1.051	1.024	1.014	1.038	1.038	1.049	1.038	0.991	0.993	0.993	0.989	0.993
15	1.013	1.022	1.022	1.048	1.022	1.014	1.037	1.037	1.049	1.038	0.991	0.996	0.996	0.991	0.995
16	1.012	1.021	1.021	1.050	1.022	1.015	1.038	1.038	1.050	1.039	0.990	0.996	0.996	0.992	0.995
17	1.010	1.021	1.021	1.050	1.021	1.015	1.039	1.039	1.051	1.040	0.989	0.995	0.995	0.991	0.995
18	1.009	1.021	1.021	1.053	1.021	1.015	1.039	1.039	1.050	1.040	0.989	0.996	0.995	0.991	0.995
19	1.008	1.020	1.020	1.054	1.021	1.015	1.039	1.039	1.051	1.040	0.990	0.997	0.997	0.992	0.997
20	1.007	1.021	1.021	1.059	1.022	1.015	1.039	1.039	1.049	1.039	0.991	0.999	0.999	0.992	0.999
Mean	1.015	1.018	1.018	1.024	1.018	1.025	1.038	1.038	1.046	1.039	1.000	1.003	1.003	1.001	1.003

Note: Table entries are U statistics at horizon h.

6. Conclusion

This paper evaluates the dynamic out of sample nominal exchange rate forecasting performance of the canonical New Keynesian model of a small open economy. In agreement with the existing empirical literature, we find that nominal exchange rate movements are difficult to forecast, with a random walk generally dominating this New Keynesian model in terms of predictive accuracy at all horizons. Nevertheless, we find empirical support for the common practice in the theoretical open economy macroeconomics literature of imposing deterministic cross economy equality restrictions on deep structural parameters, both in sample and out of sample.

The empirical adequacy of many of the assumptions underlying the canonical New Keynesian model of a small open economy have been called into question. An evaluation of whether and to what extent systematically relaxing these assumptions yields incremental predictive power remains an objective for future research.

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Appendix

The data set consists of quarterly observations on several macroeconomic variables for three approximately small open economies and one approximately closed economy over the period 1973Q1 through 2006Q2. The approximately small open economies under consideration are Australia, Canada and the United Kingdom, while the approximately closed economy under consideration is the United States.

The macroeconomic variables under consideration are the price of output, the price of consumption, output, the nominal interest rate, and the nominal exchange rate. The price of output is proxied by the seasonally unadjusted producer price index, while the price of consumption is proxied by the seasonally unadjusted consumer price index. Output is proxied by seasonally adjusted real industrial production. The nominal interest rate is measured by the three month Treasury bill rate expressed as a period average, while the nominal exchange rate is quoted as an end of period value. All data was extracted from the International Financial Statistics database maintained by the International Monetary Fund.

References

Calvo, G. (1983), Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics*, 12, 983-998.

Cheung, Y., M. Chinn and A. Pascual (2005), Empirical exchange rate models of the nineties: Are any fit to survive?, *Journal of International Money and Finance*, 24, 1150-1175.

Engle, R. and M. Watson (1981), A one-factor multivariate time series model of metropolitan wage rates, *Journal of the American Statistical Association*, 76, 774-781.

Galí, J. and T. Monacelli (2005), Monetary policy and exchange rate volatility in a small open economy, *Review of Economic Studies*, 72, 707-734.

Geweke, J. (2005), *Contemporary Bayesian Econometrics and Statistics*, Wiley.

Hodrick, R. and E. Prescott (1997), Post-war U.S. business cycles: A descriptive empirical investigation, *Journal of Money, Credit, and Banking*, 29, 1-16.

Kalman, R. (1960), A new approach to linear filtering and prediction problems, *Transactions ASME Journal of Basic Engineering*, 82, 35-45.

Klein, P. (2000), Using the generalized Schur form to solve a multivariate linear rational expectations model, *Journal of Economic Dynamics and Control*, 24, 1405-1423.

Meese, R. and K. Rogoff (1983), Empirical exchange rate models of the seventies: Do they fit out of sample?, *Journal of International Economics*, 14, 3-24.

Obstfeld, M. and K. Rogoff (1995), Exchange rate dynamics redux, *Journal of Political Economy*, 103, 624-660.

Obstfeld, M. and K. Rogoff (2000), The six major puzzles in international macroeconomics: Is there a common cause?, *NBER Working Paper*, 7777.

Theil, H. (1966), *Applied Economic Forecasting*, North Holland Press.

Vitek, F. (2007), An unobserved components model of the monetary transmission mechanism in a small open economy, *Journal of World Economics Review*, forthcoming.

Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.