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TESTING FOR NON-CAUSALITY BY USING THE AUTOREGRESSIVE METRIC

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Abstract

A new non-causality test based on the notion of distance between ARMA models is proposed in this paper. The advantage of this test is that it can be used in possible integrated and cointegrated systems, without pre-testing for unit roots and cointegration. The Monte Carlo experiments indicate that the proposed method performs reasonably well in finite samples. The empirical relevance of the test is illustrated via two applications.

Keywords: AR metric, Bootstrap test, Granger non-causality, VAR

1 Introduction[1]

Since the seminal paper of Granger (1969), Granger non-causality test between economic time series have become ubiquitous in applied econometric research. This concept is defined in terms of predictability of variable x from its own past and the past of another variable y. In particularly, we say that y Granger-causes x if the past values of y can be used to predict x more accurately rather than simply using the past values of x alone. Thus Granger causality may have more to do with precedence, or prediction, than with causation in the usual sense. However, apart from these theoretical considerations, there are a number of methodological issues arising from the various applications of Granger causality tests. It was shown that the use of non-stationary data in causality tests can yield spurious causality results (Park and Phillips (1989), Stock and Watson (1989) and Sims *et al.* (1990)). Thus before testing for Granger causality, it is important to establish the properties of the time series involved. The common practice is the following: when both series are I(0), a vector autoregressive (VAR) model in levels is used; when one of the series is found I(0) and the other one I(1), VAR is specified in the level for the I(0) variable and in terms of first difference for the I(1) variable; when both series are determined I(1) but not cointegrated, the proper model is VAR in terms of the first differences. Finally, when the series are cointegrated, we can use a vector error correction (VECM) model or a VAR model in levels. Of course, the weakness of this strategy is that incorrect conclusions drawn from preliminary analysis might be carried over onto the causality tests. An alternative method is the lag-augmented Wald test (see Toda and Yamamoto (1995) and Dolado and Lütkepohl (1996)). This method does not rely so heavily on pre-testing. However, the knowledge of the maximum order of integration is

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still required. Further, the lag-augmented Wald test may suffer from size distortion and low power especially for small samples (Giles (1997) and Mavrotas and Kelly (2001)).

In this paper we propose a new Granger non-causality test based on the notion of the distance between ARMA models, the AR metric introduced by Piccolo (1990). The advantage of this test is that it can be carried out irrespective of whether the variables involved are stationary or not and regardless of the existence of a cointegrating relationship among them. Consequently no pre-testing for unit roots and cointegration is required. Further, our test appears to be well-sized and has reasonably good power properties.

The remainder of the paper is organized as follows. Section 2 introduces the notion the distance between ARMA models and specifies the relationship between AR metric and Granger causality. Section 3 presents the new Granger non-causality test. Section 4 provides some Monte Carlo evidence about the finite sample behavior of our testing procedure in comparison with the lag-augmented Wald test. Section 5 contains two empirical illustrations of testing causality. Section 6 gives some concluding remarks.

2 Granger causality and AR metric

Let z_t a zero mean invertible ARMA model defined as

$$\phi(L)z_t = \theta(L)\epsilon_t$$

where $\phi(L)$ and $\theta(L)$ are polynomials in the lag operator L, with no common factors, and ϵ_t is a white noise process with constant variance σ^2. It is well known that this process admit the representation:

$$\pi(L)z_t = \epsilon_t$$

where the AR(∞) operator is defined by

$$\pi(L) = \phi(L)\theta(L)^{-1} = 1 - \sum_{i=1}^{\infty} \pi_i L^i$$

with $\sum_{i=1}^{\infty} |\pi_i| < \infty$.

Let ℓ the class of ARMA invertible models. If $x_t \in \ell$ and $y_t \in \ell$, following Piccolo (1990), we define the AR metric as the Euclidean distance between the corresponding π-weights sequence, $\{\pi_j\}$,

$$d = \left[\sum_{i=1}^{\infty} (\pi_{xi} - \pi_{yi})^2\right]^{\frac{1}{2}}. \tag{1}$$

The AR metric d is a well defined measure because of the absolute convergence of the π-weights sequences. The asymptotic distribution of the maximum likelihood estimator $\hat{d}^2$ has been studied in Corduas (1996, 2000), D'Elia (2000) and Corduas and Piccolo (2008).

Now, consider the following VAR model of order p, for a $n \times 1$ vector time series $\{w_t;\ t \in \mathbb{Z}\}$:

$$A(L)w_t = \epsilon_t \tag{2}$$

where $A(L) = I_n - A_1L - A_2L^2 - \cdots - A_pL^p$ is a $n \times n$ matrix polynomial in the lag operator operator L, and ϵ_t is vector white noise process with positive definite covariance matrix Σ.

Consider the partition $w_t = (x_t, y_t')'$ where x_t is a scalar time series and y_t is a $(n-1) \times 1$ vector of time series. Model (2) accordingly to the partition of w_t:

$$A(L)\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} \epsilon_{x_t} \\ \epsilon_{y_t} \end{bmatrix} \tag{3}$$

$$A(L) = \begin{bmatrix} 1 - A_{11}(L) & A_{12}(L) \\ A_{21}(L) & I - A_{22}(L) \end{bmatrix},\quad E\left(\begin{bmatrix} \epsilon_{x_t} \\ \epsilon_{y_t} \end{bmatrix}\begin{bmatrix} \epsilon_{x_s} & \epsilon_{y_s} \end{bmatrix}\right) = \begin{cases} \Sigma & t = s \\ \mathbf{0} & t \neq s \end{cases}$$

where $A_{ij}(L) = \sum_{h=1}^{p} A_{ij}^{(h)} L^h$ $i, j = 1, 2$ are matrix polynomials in the lag operator L of order p and Σ is a non-singular matrix. We further assume that $\det(A(z)) \neq 0$ for $|z| < 1$. This condition allows nonstationarity fr the series, in the sense that the characteristic polynomial of the VAR model described by equation $\det(A(z)) = 0$ may have roots on the unit circle. Condition $\det(A(z)) \neq 0$ for $|z| < 1$, however, excludes explicitly explosive processes from our consideration.

In this framework it is well known that y_t does not cause x_t (denoted as $y_t \not\Rightarrow x_t$) if and only if $A_{12}(L) = 0$. We note that, if y_t does not cause x_t, then

$$[1 - A_{11}(L)]\, x_t = \epsilon_{x_t} \tag{4}$$

The aim of this paper is to investigate the condition of non causality, $A_{12}(L) = 0$, by using the notion the distance between ARMA models measured by (1). In particular, we will consider the distance between the AR(p) model (4) and the ARMA model for the subprocess $\{x_t;\ t \in \mathbb{Z}\}$ implied by the VAR(p) model (2).

The implied ARMA model can be obtained as follows. Premultiplying both sides of (2) by the adjoint $\mathrm{Adj}(A(L))$ of $A(L)$, we obtain

$$\det(A(L))\, w_t = \mathrm{Adj}(A(L))\, \epsilon_t \tag{5}$$

We note that each component of $\mathrm{Adj}(A(L))\, \epsilon_t$ is a sum of finite order MA processes, thus it is a finite order MA process (see Lutkepohl, 2005, Proposition 11.1). Hence, the subprocess $\{x_t;\ t \in \mathbb{Z}\}$ follows an ARMA model given by:

$$\det(A(L))\, x_t = \delta(L) u_t$$

where u_t is univariate white noise and $\delta(L)$ is an invertible operator. It is possible that $\det(A(L))$ and $\delta(L)$ will have certain factors in common that must be canceled from these operators. Thus we obtain that, in general, the process $\{x_t;\ t \in \mathbb{Z}\}$ has the following ARMA representation: $\phi(L)x_t = \theta(L)u_t$ where $\phi(L) = \det(A(L))$ and $\theta(L) = \delta(L)$ if $\det(A(L))$ and $\delta(L)$ have no common factors.

Finally, we observe that $\{x_t;\ t \in \mathbb{Z}\}$ has also the following autoregressive representation of infinite order

$$\varphi(L)x_t = u_t \tag{6}$$

where

$$\varphi(L) = \frac{\phi(L)}{\theta(L)} = 1 + \varphi_1 L + \ldots$$

2.1 Propositions

We consider the distance according to (1) between the models (6) and (4), as follows:

$$d = \left[\sum_{i=1}^{\infty} \left(\varphi_i - A_{11}^{(i)}\right)^2\right]^{\frac{1}{2}}.$$

where $A_{11}^{(i)} = 0$ for $i = p + 1, \ldots$. The following proposition provides a necessary condition for non-causality in terms of the distance d.

Proposition 1. If y_t does not cause x_t, then $d = 0$.

Proof. If y_t does not cause x_t, then $A_{12}(L) = 0$. It follows that

$$\det(A(L)) = \det(1 - A_{11}(L)) \det(I - A_{22}(L))$$

and

$$\delta(L) = \det(I - A_{22}(L)).$$

Thus $\phi(L) = 1 - A_{11}(L)$ and $\theta(L) = 1$. This implies that $\varphi(L) = 1 - A_{11}(L)$ and hence $d = 0$.

□

Under the condition $A_{21}(L) \neq 0$ we obtain the following characterization of non-causality.

Proposition 2. If x_t causes y_t, then y_t does not cause x_t if and only if $d = 0$.

Proof. ($\Rightarrow$) If y_t does not cause x_t, by Proposition 1, it follows that $d = 0$.

($\Leftarrow$) If $d = 0$, then $\varphi_i = A_{11}^{(i)}$ for $i = 1, ..., p$ and $\varphi_i = 0$ for $i = p + 1, ...$ Thus $\varphi(L) = 1 - A_{11}(L)$. On the other hand, we have

$$\varphi(L) = \frac{\phi(L)}{\theta(L)} = \frac{\det(A(L))}{\delta(L)}$$

and hence

$$1 - A_{11}(L) = \frac{\det(1 - A_{11}(L)) \det\left(I - A_{22}(L) - A_{21}(L)(1 - A_{11}(L))^{-1} A_{12}(L)\right)}{\delta(L)}.$$

Thus we have

$$\det\left(I - A_{22}(L) - A_{21}(L)(1 - A_{11}(L))^{-1} A_{12}(L)\right) = \delta(L).$$

Since by hypothesis $A_{21}(L) \neq 0$, it follows that $A_{12}(L) = 0$, that is y_t does not cause x_t.

□

Propositions 1 and 2 allow us to test for non-causality considering the null hypothesis

$$H_0 : \; d = 0 \tag{7}$$

Since the condition $d = 0$ is necessary for non-causality from y_t to x_t, if we reject the null hypothesis (7) we can reject the hypothesis of non-causality. However, it is important to note that if we accept the hypothesis (7) we cannot accept the hypothesis of non-causality.

Proposition 2 establishes that under condition $A_{21}(L) \neq 0$ the non-causality from y_t to $x_t$is equivalent to the condition $d = 0$. Thus in the situations were we know that there is a causal link from x_t to y_t, we can test for non-causality from y_t to x_t considering the null hypothesis $d = 0$.

3 The bootstrap test procedure

As mentioned above, the asymptotic distribution of the maximum likelihood estimator $\hat{d}^2$ has been studied, among others, in Corduas and Piccolo (2008). Now, it is important to note that this distribution has been derived under the hypothesis that the considered ARMA processes are independent. In our case the two processes are equal and so they cannot be considered independent. Thus in order to test for non-causality considering the null hypothesis (7) we use the following bootstrap test procedure.

1. Estimate on the observed data the VAR(p) and obtain $\hat{A}(L)$, $\hat{\Sigma}$ and the residuals $\hat{\epsilon}_t$;
2. using the estimated parameters from step 1, obtain the univariate ARMA implied by the estimated VAR for the sub-process x_t;
3. evaluate the AR(∞) representation truncated a some suitable lag p_1 of the ARMA model in step 2;
4. estimate for x_t using the observed data, an AR(p) model under null hypothesis of non causality $H_0 : y_t \nRightarrow x_t$;
5. evaluate the distance $\hat{d}$ between the AR(p_1) and the AR(p) model obtained in step 3 and 4;

6. estimate the VAR(p) model under the null hypothesis $H_0 : y_t \nRightarrow x_t$ obtaining the estimates $\tilde{A}(L)$ and $\tilde{\Sigma}$;
7. apply Bootstrap on $\hat{\epsilon}_t$ and obtain the pseudo-residuals ϵ_t^*;
8. generate the pseudo-data $\{(x_t^*, y_t^*)'\}$ obeying to the null of Granger non-causality using $\tilde{A}(L)(x_t^*, y_t^*)' = \epsilon_t^*$ with $\tilde{\Sigma}$;
9. using the pseudo data $\{(x_t^*, y_t^*)'\}$, repeat steps from 1 to 5 obtaining the bootstrap estimate of the distance d^*
10. repeat steps from 7 to 9 for B times
11. evaluate the bootstrap p-value as proportion of the B estimated bootstrap distance d^* that exceed the same statistic evaluated on the observed data $\hat{d}$, that is $pval_B = prop(d^* > \hat{d})$

An essential feature to be taken into account is the dependency across the sub-process expressed by Σ. In order to reproduce it in the pseudo-data, we simply need to apply the resampling algorithm to the entire $T \times n$ matrix of the residuals $\hat{\epsilon}_t$.

4 A Monte Carlo experiment

In this section, we conduct two simulation studies in order to investigate the performances of the proposed Granger non-causality test. For the first simulation study we consider as DGP a bivariate cointegrated VAR(2) model. Then we use a trivariate cointegrated VAR(1) model.

In order to better evaluate the performance of the proposed procedure, we compare the size and power of our test with the size and power obtained with the lag-augmented Wald test suggested by Toda and Yamamoto (1995) and Dolado and Lütkepohl (1996). They use a modified Wald test for restrictions on the parameters of a VAR(p) model. This test has an asymptotic χ^2-distribution with p degrees of freedom when a VAR($p + d_{max}$) is estimated, where d_{max} is the maximal order of integration for the series in the system. However, it is well known that the modified Wald test based on asymptotic critical values may suffer from size distortion and low power especially for small samples (Giles (1997) and Mavrotas and Kelly (2001)). Thus to overcome this problem, we apply the same bootstrap algorithm described above using the Wald test from an augmented VAR(2+d_{max}), with $d_{max} = 1$ and we evaluate the bootstrap p-values.

4.1 Bivariate cointegrated VAR(2) model

Consider the following cointegrated VAR(2) model:

$$\begin{bmatrix} 1 - 1.5L + 0.5L^2 & -\alpha_1 L - \alpha_2 L^2 \\ -0.8L + 0.3L^2 & 1 - L + 0.5L^2 \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} \epsilon_{xt} \\ \epsilon_{yt} \end{bmatrix} \tag{8}$$

with covariance matrix $\Sigma_\epsilon = \begin{bmatrix} 5 & 2 \\ 2 & 3 \end{bmatrix}$.

In our study, the tests of the null hypothesis

$$H_0 : \alpha_1 = \alpha_2 = 0$$

were carried out using nominal significance levels of 1%, 5%, and 10%. To analyze the power of the test we considered the two cases below:

Power 1. $\alpha_1 = -\alpha_2 = 0.3$

Power 2. $\alpha_1 = -\alpha_2 = 0.6$

The results, obtained from 1000 Monte Carlo replications and 1000 Bootstrap redrawings, are collected in Table 1, considering as sample size $T = 50$, a sample size medium in terms of annual data but small for a quarterly frequency, and $T = 100$, that is a time span large in terms of annual data, but pretty common for quarterly data, so to make it relevant for actual empirical applications.

The comparison between the power estimates for our test and the lag-augmented Wald test shows that our test has relatively high power properties in all situations,while the size is very close to the nominal values for both tests.

Table 1: VAR(2) AR-metric and lag-augmented Wald test Size and Power - Bootstrap p-values

			T=50			
		$AR-metric$			$Aug-Wald$	
nom	Size	Power1	Power2	Size	Power1	Power2
0.01	0.02	0.22	0.64	0.01	0.05	0.35
0.05	0.07	0.42	0.82	0.04	0.18	0.62
0.10	0.12	0.56	0.89	0.08	0.27	0.73
			T=100			
0.01	0.01	0.54	0.98	0.01	0.18	0.78
0.05	0.04	0.78	1.00	0.04	0.38	0.92
0.10	0.11	0.85	1.00	0.09	0.50	0.95

4.2 Trivariate cointegrated VAR(1) model

In this experiment, we considered two different trivariate VAR(1) for generating the data, presented in Lach(2010). The first model (Model 1) is

$$\begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} = \begin{bmatrix} 1 & \beta_1 & \beta_2 \\ 0 & 1 & 0 \\ 0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{xt} \\ \epsilon_{yt} \\ \epsilon_{zt} \end{bmatrix}, \quad \Sigma_\epsilon = \begin{bmatrix} 1 & 0 & 0.3 \\ 0 & 1 & 0.9 \\ 0.3 & 0.9 & 1 \end{bmatrix} \tag{9}$$

The second model (Model 2) is

$$\begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} = \begin{bmatrix} 0.25 & \beta_1 & \beta_2 \\ 0 & 1 & 0 \\ -0.75 & 0 & 0.875 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{xt} \\ \epsilon_{yt} \\ \epsilon_{zt} \end{bmatrix}, \quad \Sigma_\epsilon = \begin{bmatrix} 1 & 0 & 0.3 \\ 0 & 1 & 0.9 \\ 0.3 & 0.9 & 1 \end{bmatrix} \tag{10}$$

The tests of the null hypothesis

$$H_0 : \beta_1 = \beta_2 = 0$$

were carried out using the typical significance levels of 1%, 5%, and 10%. In Model 1, the power of the tests has been estimated by calculating the rejection frequencies in 1000 replications using the following values of the β coefficients.

Power 1. $\beta_1 = 0, \ \beta_2 = -0.125$

Power 2. $\beta_1 = 0, \ \beta_2 = -0.375$

For Model 2 the power is evaluated using just $\beta_1 = 0, \ \beta_2 = -0.125$. In all these cases, the models provide specific cointegration properties. Model 1 is characterized by 2 cointegration vectors, while Model 2 is characterized by 1 cointegration vector.

Further, the same the Monte Carlo experiment has been conducted considering a break in variance. The break has been located in $T/2$. In particular, we have posed

$$\Sigma_\epsilon = \begin{bmatrix} 1 & 0 & 0.3 \\ 0 & 1 & 0.9 \\ 0.3 & 0.9 & 1 \end{bmatrix} \text{ for } t = 1, ...T/2 \text{ and } \Sigma_\epsilon = 2 \begin{bmatrix} 1 & 0 & 0.3 \\ 0 & 1 & 0.9 \\ 0.3 & 0.9 & 1 \end{bmatrix} \text{ for } t = T/2+1, ..., T$$

In Table 2 and Table 3 we present the results of our Monte Carlo experiment. First, we note that our test exhibits always higher power. For Model 1 the performance in size is good even when the sample size is small, while the power performance needs a larger sample size to perform in an appreciable way when the alternative hypothesis refers to the "power1" case, that is when the alternative is closer to the null one. When the alternative is far from the null the improvement in power is remarkable especially for $T = 100$. So we can assess that the proposed test is reliable for medium-sized samples. For Model 2 we have had similar simulation results in terms of the power. Both tests have been shown to reject too often, under the null hypothesis, when Model 1 with a break in variance is considered. It is interesting to note that the performace AR-metric test becomes very good in terms of the size and power for Model 2 when a break in variance is present.

Table 2: Model 1: AR-metric and lag-augmented Wald tests Size and Power - Bootstrap p-values

		T=50			T=100		
nom. val.		0.01	0.05	0.10	0.01	0.05	0.10
				$AR-metric$			
Model 1 no break	Size	0.01	0.06	0.10	0.01	0.06	0.10
	Power1	0.03	0.12	0.23	0.33	0.55	0.64
	Power2	0.57	0.77	0.86	1.00	1.00	1.00
				$Aug-Wald$			
	Size	0.01	0.06	0.10	0.01	0.06	0.10
	Power1	0.03	0.09	0.17	0.05	0.17	0.27
	Power2	0.26	0.51	0.64	0.75	0.90	0.94
				$AR-metric$			
Model 1 with break	Size	0.02	0.07	0.15	0.02	0.08	0.15
	Power1	0.17	0.36	0.47	0.36	0.57	0.66
	Power2	0.84	0.94	0.97	0.99	1.00	1.00
				$Aug-Wald$			
	Size	0.02	0.08	0.14	0.01	0.07	0.13
	Power1	0.04	0.12	0.20	0.06	0.18	0.27
	Power2	0.28	0.58	0.63	0.64	0.83	0.89

Table 3: Model 2: AR-metric and lag-augmented Wald tests Size and Power - Bootstrap p-values

		T=50			T=100		
nom. val.		0.01	0.05	0.10	0.01	0.05	0.10
				$AR-metric$			
Model 2 no break	Size	0.01	0.03	0.06	0.01	0.04	0.06
	Power	0.48	0.62	0.71	0.85	0.92	0.94
				$Aug-Wald$			
	Size	0.01	0.05	0.10	0.01	0.06	0.10
	Power	0.02	0.08	0.17	0.05	0.15	0.26
				$AR-metric$			
Model 2 with break	Size	0.01	0.06	0.10	0.01	0.05	0.11
	Power	0.60	0.75	0.81	0.91	0.95	0.97
				$Aug-Wald$			
	Size	0.02	0.07	0.13	0.02	0.07	0.12
	Power	0.04	0.12	0.20	0.06	0.17	0.28

Figure 1 gives a plot of the power for AR-metric and lag-augmented Wald tests for 50 (left panel), and 100 (right panel) observations, in both cases (without and with break in variance), for Model 1. The figure makes clear that the AR-metric test performs satisfactorily in all situations, and that it is superior to the lag-augmented Wald test in small ($T = 50$) and medium ($T = 100$) samples.

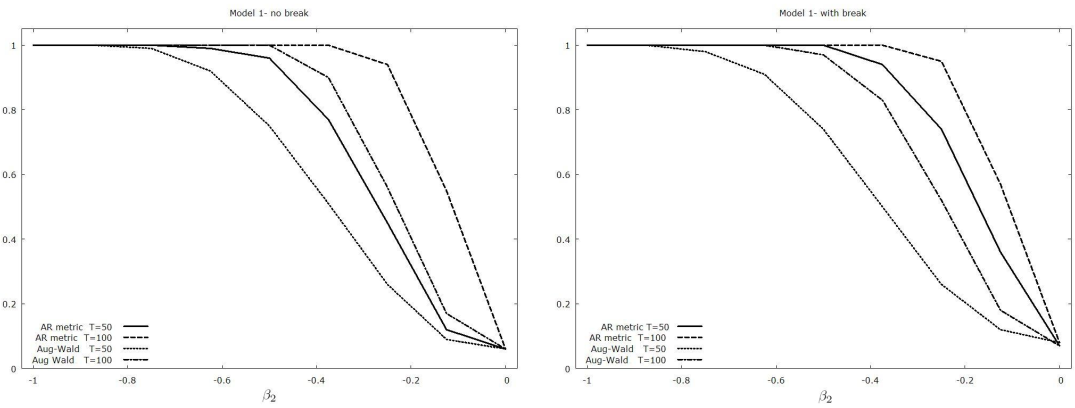

Figure 1: Model 1. Monte Carlo rejection rates (power) of the AR-metric and lag-augmented Wald tests, for different values of β_2. The nominal significance level is 5%, the sample size are 50 and 100

The overall experiment results can be summarized as follows. The proposed test performs well in terms of the size and power for bivariate VAR(2) and trivariate VAR(1) processes. The empirical power of AR-metric test is higher than that of lag-augmented Wald test. If a break in variance at the same point in time is present, our test seems even more preferable to the lag-augmented Wald test.

5 Empirical applications

In this section we present two empirical examples to illustrate the application of the test suggested in the paper. First we consider the relationship between income and CO_2 emissions. It is well known that the conjecture of the Environmental Kuznets Curve (EKC) hypothesis (Coondoo and Dinda, 2002)is such that, initially as per capita income rises, environmental degradation intensifies, but in later levels of economic growth it tends to subside. Thus, it is presumed that income Granger-causes CO_2 emissions. Hence, we investigate the causal relationship from CO_2 emissions to income by using our test. To establishes if the CO_2 emissions Granger cause or not the GDP may be useful for policy implication.

For example, if for a given country the CO_2 emissions does not Granger-cause the GDP, then any effort to reduce CO_2 emissions does not restrain the development of the economy. If, on the other hand, the causality runs from CO_2 emissions to income, reducing energy consumption (by a carbon tax policy, say) may lead to fall in income.

We use annual data on per capita Real Gross Domestic Product (y) and per capita of Carbon Dioxide Emissions (c) in United States, for the period 1960-2006. All data are from World Development Indicators and are in natural logarithms.

Based on Bayesian Information Criterion, a VAR model of order 1 was selected. The estimated model is given by:

$$y_t = \underset{(0.10)}{0.18} + \underset{(0.01)}{0.99}\, y_{t-1} - \underset{(0.03)}{0.05}\, c_{t-1} + \epsilon_{1_t}$$

$$c_t = \underset{(0.17)}{0.43} - \underset{(0.01)}{0.02}\, y_{t-1} + \underset{(0.06)}{0.88}\, c_{t-1} + \epsilon_{2_t}$$

The estimated distance is $\hat{d} = 0.0073$ and the bootstrap p-value is 0.58. Thus we can conclude that there is no evidence of Granger causality from CO_2 emissions to output.

We now examine the causal relationship between the log of real per capita income (y) and inflation (Δp) in the United States over the period 1953-1992. In particular, we have re-examined the data set used by Ericsson *et al.* (2001). We downloaded the annual time series data from the *Journal of Applied Econometrics* Data Archive. The following bivariate VAR model is estimated.

$$y_t = \underset{(0.21)}{0.03} + \underset{(0.15)}{0.93}\, y_{t-1} + \underset{(0.16)}{0.93}\, y_{t-2} - \underset{(0.24)}{0.82}\, \Delta p_{t-1} + \underset{(0.23)}{0.53}\, \Delta p_{t-1} + \epsilon_{1_t}$$

$$\Delta p_t = -\underset{(0.12)}{0.35} + \underset{(0.09)}{0.34}\, y_{t-1} - \underset{(0.09)}{0.33}\, y_{t-2} + \underset{(0.13)}{1.15}\, \Delta p_{t-1} - \underset{(0.13)}{0.33}\, \Delta p_{t-1} + \epsilon_{1_t}$$

The order of the VAR has been chosen using the Bayesian Information Criterion. The computed $\hat{d}$-statistic is equal to 0.35 with a bootstrap p-value 0. This result indicates the presence of Granger causality from output to inflation. This finding is in accordance with the results of Ericsson *et al.* (2001). The same result is obtained using the lag-augmented Wald test.

6 Conclusions

In this paper we have investigated the relationships between the condition of Granger non-causality in a VAR framework and the notion of distance between ARMA models and we have proposed a new Granger non-causality test. The advantage of this test is that it can be can be carried out irrespective by of whether the variables involved are stationary or not and regardless of the existence of a cointegrating relationship among them. Our method for detecting causality is validated by the Monte Carlo results. The conducted simulation study has shown that our test exhibits a good performance in terms of size and power properties, even in small-samples. Further, it outperforms the lag-augmented Wald test. Finally, we have showed that this test can be usefully applied in practical situations to test causality between economic time series.

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