Simulation based approach for measuring concentration risk

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Abstract

Asymptotic Single Risk Factor (ASRF) model is used to derive the regulatory capital formula of Internal Ratings-Based approach in the new Basel accord (Basel II). One of the important assumptions in ASRF model for credit risk is that the given portfolio is well diversified so that one can easily calculate the required capital level by focusing only on systematic risk. In real world, however, idiosyncratic risk of a portfolio cannot be fully diversified away, causing the so called concentration risk problem. In this paper we suggest simulation based approach for measuring concentration risk using bank capital dynamic model. This approach is especially suitable for a portfolio with relatively small to medium number of obligors and relatively large sized loans.

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1. Introduction

In recent years, many important advances have been made in modeling credit risk of a portfolio. One of them is Asymptotic Single Risk Factor (ASRF) model, which is used to derive the regulatory capital formula of Internal Ratings-Based approach in the new Basel accord (Basel II).

Under the ASRF framework there are only two sources of risk, systematic risk and idiosyncratic risk. As the number of obligors in a portfolio increases, idiosyncratic risk is diversified away, so its contribution to portfolio risk disappears. Thus, one can easily calculate required capital level by focusing only on systematic risk under the ASRF assumptions. In real world, however, a bank's portfolio is often not sufficiently diversified. The fact that there are some large exposures in the portfolio implies that there is a residual of undiversified idiosyncratic risk in the portfolio. Under these circumstances, IRB formula in the Basel accord underestimates the required regulatory capital. Some historical examples such as insolvency of Enron, Worldcom and Parmalat show the dangers of misunderstanding concentration risk.[1]

The approaches for measuring concentration risk suggested in recent studies can be categorized into two different types. The first approach is to adapt indices of concentration such as Gini coefficient or Herfindahl-Hirschman Index (HHI). This approach is simple and easy to perform. While these indices could be good measures for concentration itself, they do not seem to serve well for concentration risk because they do not take distribution of different quality obligors into consideration. The second approach is granularity adjustment suggested by Gordy (2003). Its difficulties in implementation and huge data requirement make it hard to be performed in practice.

Usually practitioners use both approaches to measure the concentration risk of their portfolio. While the concentration measurement index such as HHI could not measure the actual risk accurately, granularity adjustment sometimes overestimates the actual concentration risk of a portfolio.

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[1] Bundesbank(2006): Concentration risk in credit portfolios, monthly report, June

In this paper, we introduce a simulation based approach to measure concentration risk. We show that HHI could not provide enough information to measure the actual concentration risk. With the proposed approach, we are able to calculate the amount of required capital for concentration risk directly. We believe that the approach is especially suitable for banks with portfolios of relatively small number of obligors with relatively large size of loans.

This paper is organized as follows. In section 2, we present detailed descriptions of concentration risk and Herfindahl-Hirshman Index, respectively. Section 3 explains the frameworks of our simulation based approach to measure concentration risk. Section 4 provides some numerical results based on the actual example and explains the implication of those numbers. Section 5 concludes the paper.

2. Concentration risk under Basel II framework

2.1 The IRB model and concentration risk

In this section, we provide a brief summary on the key assumptions of the Asymptotic Single-Risk Factor (ASRF) model that is used to calculate the regulatory capital requirement by Basel II. In the risk factor model frameworks that underpin the Internal Ratings-Based (IRB) risk weights of Basel II, credit risk of a portfolio is caused by two main sources, systematic and idiosyncratic risks.[2]

Systematic risk represents the effect of unexpected changes in macroeconomic and financial market conditions on the performance of borrowers. Borrowers may differ in their degree of sensitivity to systematic risk, but few firms are completely indifferent to the wider economic conditions in which they operate. Therefore, the systematic component of portfolio risk is unavoidable and only partly diversifiable. Meanwhile idiosyncratic risk represents the effects of risks that are particular to individual borrowers. As a portfolio becomes more fine-grained, in the sense that the largest individual exposures account for a smaller share of total portfolio exposure, idiosyncratic risk is diversified away at the portfolio level. This risk is totally eliminated in an infinitely granular portfolio (one with a very large number of exposures) as unsystematic risk vanishes in Capital Asset Pricing Model.

The ASRF model framework underlying the IRB approach is based on two key assumptions. The first one is that bank portfolios are perfectly fine-grained and the

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[2] BCBS(2006): Studies on credit risk concentration, working paper, Basel

second one is that there is only one source of systematic risk. When these two assumptions hold, one can easily calculate required capital level depending on only one systematic risk. In case of well diversified portfolio, the capital required for a loan does not depend on the portfolio it is added to. This simplicity makes the new IRB framework applicable to a wider range of countries and institutions. However, if any of two assumptions is violated, there is no guarantee that the IRB approach and ASRF model will be accurate. The violation of the assumption of the fine-grained portfolio leads to concentration risk problem. Concentration of exposures in credit portfolios arises from imperfect diversification of idiosyncratic risk in the portfolio. The small to medium size of credit portfolio or some large exposures to specific individual obligors can lead to concentration risk.

2.2 Herfindahl-Hirshman Index

The Herfindahl-Hirshman index[3] (HHI), better known as the Herfindahl index, is a statistical measure of concentration. The HHI accounts for the number of firms in a market, as well as concentration, by incorporating the relative size of all firms in a market. It is calculated by squaring the market shares of all firms in a market and then summing the squares, as follows:

$$HHI = \sum_{i=1}^{n} (MS_i)^2 \, , \tag{1}$$

where MS_i is market share of i th firm and n is the number of firms.

Well-diversified portfolios with a very large number of very small firms have an HHI value close to zero whereas heavily concentrated portfolios can have a considerably higher HHI value. In the extreme case of a monopoly, the HHI takes the value of one.

In the context of the measurement of concentration risk, the HHI formula is included as a main component of a number of approaches. But HHI itself has some drawbacks to be used for measuring concentration risk. At first, it does not consider distribution of exposures across credit ratings, so portfolios with the same HHI values can have different sizes of concentration risks. Secondly, it does not allow concentration risk to be expressed directly as economic capital, so it needs additional functions to

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[3] Board of Governors of the Federal Reserve System (1993): The Herfindahl-Hirshman Index, Federal Reserve Bulletin, March, pp 188-189.

calculate economic capital for concentration risk.

3. Framework for simulations

In this section, We introduce the framework for simulations presented in Peura and Jokivuolle(2003) and how to use this bank capital dynamics model to calculate concentration risk.

3.1 Bank capital dynamics based on rating transitions

To model bank capital dynamics and required capital buffers and to avoid confusion of notations, we use three different types of bank capital, the actual capital, the regulatory capital and the economic capital. The actual capital is bank's actual capital and denoted by A_t. The regulatory capital is the minimum regulatory capital charge of Basel II and denoted by R_t. And the economic capital is minimum capital level calculated by bank without considering regulatory capital. Now let there be a bank with assets consisting of illiquid corporate loans. Under Basel II framework, the actual bank capital must satisfy equation (2).

$$A_t \geq R_t \tag{2}$$

Equation (2) gives us intuition how to determine initial actual capital of a bank. By calculating required initial actual capital subject to equation (2), we can have the required capital amount for credit risk of a portfolio.

Now, to model bank's actual capital dynamics, we assume that the bank's profit occurs before credit losses during period t. The bank's credit loss during period t is denoted by L_t and the dividends paid out of the bank capital at time t by V_t, the issues of new equity at time t by S_t. Now, the bank's capital dynamics can be determined by

$$A_{t+1} = A_t + I_{t+1} - L_{t+1} - V_{t+1} + S_{t+1}. \tag{3}$$

The bank determines the actual capital level preparing for severe macroeconomic downturns. In those conditions when capital is insufficient, it is natural to assume that there are no dividends. And also in macroeconomic downturns, it is hard to issue new

equity. So, with little loss of generality, we can assume that both the V_{t+1} and the S_{t+1} terms in equation (3) equal zero in all scenarios. Now we can express the capital dynamics that we simulate as

$$A_{t+1} = A_t + I_{t+1} - L_{t+1} \qquad (4)$$

By rolling the difference equation (4) forward, we can get the capital at time t from

$$A_t = A_0 + \sum_{s=1}^{t} I_s - \sum_{s=1}^{t} L_s \qquad (5)$$

The equation (5) implies that the bank's capital dynamics are determined by two stochastic factors, the cumulative net profit and the cumulative change in the minimum capital requirement. Now, we need to model bank income, credit losses and regulatory capital in order to simulate the dynamics of a bank's capital. The bank income and credit losses depend on rating transitions because they depend on default events of obligors. Obligors' defaults can be simulated based on rating transitions model. And also regulatory capital can be simulated by rating transitions because IRB formula of Basel II needs credit ratings of obligors as a component. In Peura and Jokivuolle (2003), they used a one-factor version of the CreditMetrics™ framework (J.P. Morgan, 1997) as rating transitions, extended with an underlying conditioning variable which is interpreted as business cycle state. The Creditmetrics model takes the transition probability matrix of ratings as given, which is determined by the business cycle state in Peura and Jokivuolle (2003). In particular, they assume that the business cycle variable is a two-state, time homogenous, Markov Chain, whose possible states are 'expansion' and 'recession'. In Bangia et al.(2002), they used models of ratings dynamics of this type.

Credit portfolio models are typically implemented as one-period simulations with an annual horizon. However, because banks in most countries report their capital adequacy to their regulators quarterly, multi-period simulations of rating changes should be performed in quarterly time increments. Both the rating transition probabilities and the regime transition probabilities in this simulation are quarterly probabilities estimated based on US data. The conditional transition matrices for the expansion and the recession states are from Bangia et al.(2002), which are based on Standard and Poor's data on US corporate ratings over the period 1981-1998. The regime switching probabilities have been estimated from quarterly data on US business cycles over 1959-

1998. The stationary distribution of the business cycle state implied by this transition matrix is (79%, 21%).

Now we will explain how the evolution of ratings in this model determines bank income, credit losses. For convention of notations, we define an indicator variable $D_{i,t}$ which assigns 1 when i th obligor of the bank's portfolio defaults at time t, otherwise 0.

$$D_{i,t} \begin{cases} 1 & \text{with probability } PD(k_{i,t}) \\ 0 & \text{otherwise} \end{cases}, \tag{6}$$

where unconditional default probability corresponding to rating $k_{i,t}$ by $PD(k_{i,t})$. Regulatory capital is determined by the capital charge function of Basel II and rating transitions. Bank income is determined by interests of loans and usually interests are determined directly proportional to expected loss of loans. Credit losses are determined by default events. Using these properties, we can express the variables R_t, I_t and L_t defined earlier equations in terms of the following sums over obligors in the bank's portfolio:

$$R_t = \sum_{i=1}^{n} f(PD(k_{i,t})) \cdot EAD_i \cdot (1 - D_{i,t}), \tag{7}$$

$$I_t = \sum_{i=1}^{n} \beta \cdot LGD_i \cdot PD(k_{i,t}) \cdot EAD_i \cdot (1 - D_{i,t}), \tag{8}$$

$$L_t = \sum_{i=1}^{n} LGD_i \cdot EAD_i \cdot (D_{i,t} - D_{i,t-1}), \tag{9}$$

where $f(\cdot)$ is the capital charge function of Basel II, which takes the default probability as an argument. $k_{i,t}$ is the credit rating of obligor i at time t. EAD_i is the nominal exposure of obligor i. β is a parameter which indicates the ratio of the nominal loan margin to the expected loss rate (the unconditional default probability times the loss given default percentage) in the portfolio. LGD_i is the loss given default to nominal exposure ratio. n is the number of obligors in the bank's portfolio at time 0. Formula (9) implies that the bank earns income as a fixed multiple of its unconditional expected loss rate.

We assume that the underlying asset value correlations, which together with the transition probabilities determine the rating transition correlations, do not depend on the state of the business cycle. Consistent with the IRB capital charge formula, we use

correlation formula of Basel II.

Now, we have bank's capital dynamics and this result must satisfy equation (2). For convenience of notation, we define capital buffer B_t, which is the difference between the bank's actual capital and the regulatory capital. It can be interpreted as capital buffer to absorb the risk from uncertainty and given by

$$B_t = A_t - R_t \tag{10}$$

Holding capital buffer means an opportunity cost for banks. In this point of view, requiring equation (2) to hold in all possible states of the world is not economical to the bank. Therefore banks use value-at-risk type probabilistic regulatory capital requirement to calculate the size of capital buffer. Value-at-risk is defined as the α th percentile of the distribution and the constraint of VaR can be expressed as

$$P\left[\min_{0 \le t \le T} B_t \ge 0\right] \ge \alpha, \tag{11}$$

where α is a confidence level associated with regulatory capital adequacy, such as 99% or 99.9%. The dynamics of B_t depend on the initial capital buffer B_0 because B_t is increasing in B_0. By substituting equation (5) into equation (10), and applying the inequality (2), we can express the regulatory capital requirement at time t:

$$B_t = B_0 + \sum_{s=1}^{t} I_s - \sum_{s=1}^{t} L_s - R_t + R_0 \ge 0 \tag{12}$$

Here the capital buffer at time t is expressed in terms of the initial capital buffer, the inflows and outflows of capital between time 0 and time t, as well as the change in the regulatory capital charge R_t between time t and time 0. In particular, R_t is the capital charge associated with the bank's initial portfolio evaluated based on ratings of the assets in the portfolio at time t. By simulation based on capital dynamics explained above, we can calculate a minimum value for B_0 which satisfies equation (12) and we denote it with $\hat{B}_0$. The required initial capital buffer $\hat{B}_0$ is given by

$$\hat{B}_0 = \inf\left\{ B_0 : P\left[\min_{0 \le t \le T} B_t \ge 0\right] \ge \alpha \right\} \tag{13}$$

Now, by assuming that equation (13) determines the capital buffer, we can calculate initial bank capital as

$$\hat{A}_0 = R_0 + \hat{B}_0,\qquad(14)$$

where $\hat{A}_0$ is required initial capital for credit risk of a loan portfolio.

3.2 Measuring concentration risk

Simulations based on bank capital dynamics model introduced in previous section provide required capital minimum level directly from distribution of bank's initial actual capital. However, in this paper, we need additional simulation and model extensions to calculate concentration risk. Any given portfolio, there exist a benchmark portfolio which have no concentration. Now using simulations, we can calculate required initial capital levels for these two portfolios, real one and benchmark case. The difference between these two values is additional required capital caused by concentration in the real portfolio. In Peura and Jokivuolle(2003), they form portfolios according the given quality distributions that each have 100 equal sized loans to stress test bank capital adequacy. In this framework, there is no concentration in the portfolio. It is not only unrealistic but also unsuitable for our main goal which is to calculate concentration risk. Therefore, we form portfolios which have differently sized loans. In order to perform the simulation, we need business cycle scenarios. In Peura and Jokivuolle(2003), they used various assumptions concerning the initial business cycle state as well as the duration of recessions. But we used randomly selected scenarios because the main purpose of our model is just to calculate the VaR type criterions from the distributions.

3.3 Testing Herfindahl-Hirschman Index

In the context of the measurement of concentration risk, the HHI formula is included as a main component of a number of approaches. But there are two types of shortcomings of HHI to be used for measuring concentration risk. Firstly, HHI doesn't take quality of a portfolio into consideration. It implies that the portfolios differently distributed across the credit ratings can have the same HHI. In the next section, we form two portfolios which have different distributions with the same HHI and show these

portfolios have different sizes of concentration risk. Secondly, HHI doesn't reflect location of concentration in a portfolio. Even though distributions of loans in portfolios are same, the locations of concentration can differ. If the sizes of loans in portfolios are same, then they still have same HHI regardless the locations of concentration. It means the portfolios that have concentrations in different credit ratings can have same HHI. To show this we form two portfolios which have same distribution of loans across the credit ratings but have concentrations in different grades and also show these portfolios have different sizes of concentration risk in the next section.

4. Numerical Results

Our simulations results are calculated by following steps. First, we form two sets of portfolios. Second, we determine business cycle scenarios. Last, we perform Monte-Carlo simulations based on bank capital dynamics described above.

We present our main results subject to the following base case parameters: portfolio maturity T equal to 2.5 years, a bank income equal to the unconditional expected credit loss($\beta = 1$), an loss given default of 45% across all obligors($LGD_i = 0.45$), and a confidence level α of 99%. We use 1,000 scenarios selected randomly. The number of simulations is 1,000 for each scenario. So we have 1,000,000 samples.

We take representative portfolios of banks from a Federal Reserve Board survey as reported by Gordy(2000). The portfolios are reported in Table 1.

Table 1. Average bank portfolios

S&P grade	Default Probability (%)	US average quality (%)	US high quality (%)
AAA	0.00	3	4
AA	0.00	5	6
A	0.04	13	29
BBB	0.24	29	36
BB	1.01	35	21
B	5.45	12	3
CCC	23.69	3	1

US portfolios are from Federal Reserve Board survey, as reported in Gordy(2000). Default probabilities are annual default frequencies from S&P data 1981-1998

These distributions of portfolios do not reflect concentration because they are calculated using exposure based data. In the first case, we form two portfolios using US average quality portfolio and two portfolios using US high quality portfolio. In each case, the benchmark portfolios have the distributions that each obligor has same nominal exposure. Portfolio 1 has concentration in credit rating of BB of average quality portfolio. Portfolio 2 has concentration in the same grade of high quality portfolio. In order to eliminate the effect of location of concentration, we let both portfolios have concentrations in the same grades.
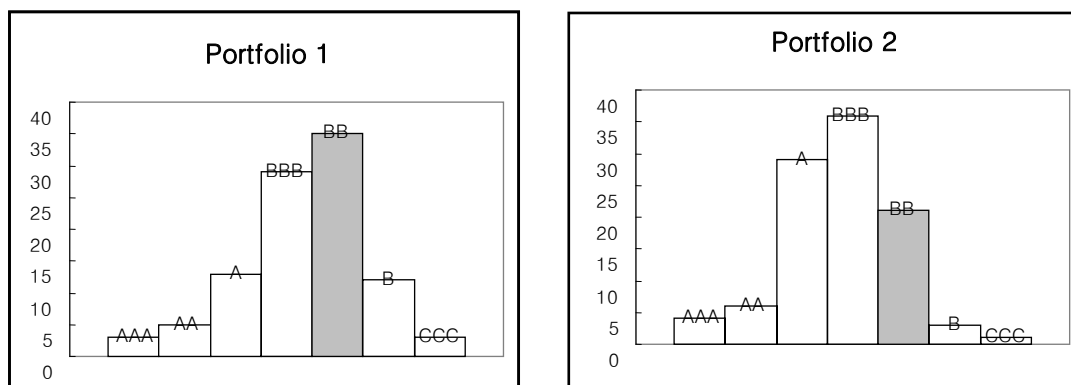

Fig. 1. The distributions of portfolio 1 and 2. Each portfolio has concentration in shaded area.

These two portfolios have two obligors which have exposures of about 10% out of total portfolio exposures. And two portfolios have the same HHI (0.272). Table 2 shows the main result of simulations.

Table 2. Initial actual capitals of portfolios (%)

	Average quality portfolio		High quality portfolio	
	Benchmark	Portfolio 1	Benchmark	Portfolio 2
Initial actual capital	8.7	11.1	5.8	7.9
Additional capital		2.4		2.1

The differences between each portfolio and benchmark portfolio are additional required capitals arise from concentration. It can be interpreted as additional risks from concentrations. In this case, the additional risk of portfolio 1 is 2.4% and the additional

risk of portfolio 2 is 2.1%. The difference 0.3% is large enough to conclude that the concentration risk from differently distributed portfolios with same HHI can be different.

In the second case, we form three portfolios using US average quality portfolio. The benchmark portfolio has the distribution that each obligor has same nominal exposure. Portfolio 1 has concentration in credit rating of BBB. Portfolio 2 has concentration in BB.
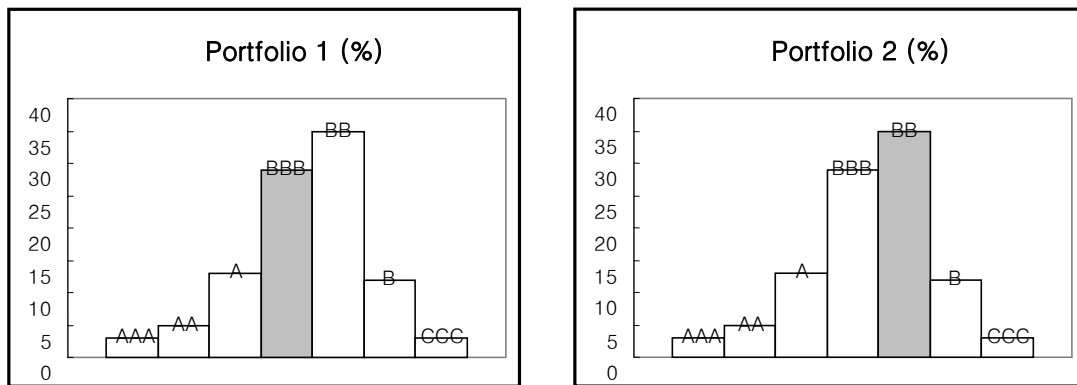

Fig. 2. The distributions of portfolio 1 and 2. Each portfolio has concentration in shaded area.

These two portfolios have two obligors which have exposures of about 10% out of total portfolio exposures. And two portfolios have the same HHI (0.272). Table 3 shows the main result of simulations.

Table 3. Initial actual capitals of portfolios (%)

	Benchmark	Portfolio 1	Portfolio 2
Initial actual capital	8.7	10.3	11.1
Additional capital		1.6	2.4

In this case, the additional risk of portfolio 1 is 1.6% and the additional risk of portfolio 2 is 2.4%. The difference 0.8% is large enough to conclude that the concentration risk from portfolios that have concentrations in different grade with same HHI can be different.

In order to show the problems caused by using HHI for concentration risk measure more clearly, we form 1000 randomly selected portfolios(with HHI from 0.012~0.015) of average quality portfolio. Fig. 3 shows the scatter diagram for HHI and concentration

risk. Using simple linear regression, we found R^2 equal to 0.043. It implies that HHI could not provide enough information to measure the actual concentration risk.
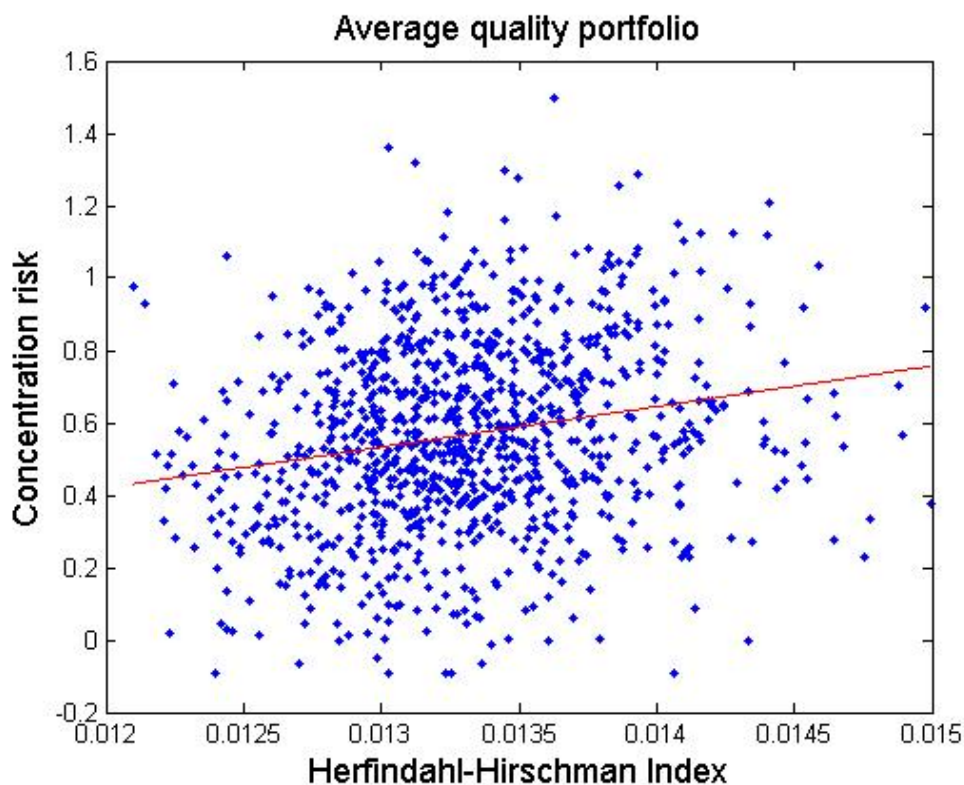

Fig. 3. Scatter diagram for Herfindahl–Hirschman Index and concentration risk of average quality portfolio

5. Conclusion

This paper provides a simulation based approach to measure concentration risk. In addition, it is shown that Herfindahl-Hirshman Index can not be a good measure for concentration risk. Given bank capital dynamic model, simulations directly provide the amount of required capital for concentration risk of a loan portfolio while more simple methods such as HHI or Gini coefficient need an additional function. And also it provides more precise result, compared with approximation methods such as granularity adjustment. It might be more time-consuming than other methods, but it is still the better way especially for banks with portfolios of relatively small number of obligors with relatively large size of loans.

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