Dynamic Market for Lemons with Endogenous Quality Choice by the Seller

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Abstract

We analyze a dynamic market for lemons in which the quality of the good is endogenously determined by the seller. Potential buyers sequentially submit offers to one seller. The seller can make an investment that determines the quality of the item at the beginning of the game, which is unobservable to buyers. At the interim stage of the game, the information and payoff structures are the same as in the market for lemons. Our main result is that the possibility of trade does not create any efficiency gain if (i) the common discounting is low, and (ii) the static incentive constraints preclude the mutually agreeable ex-ante contract under which the trade happens with probability one. Our result does not depend on whether the offers by buyers are private or public.

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1 Introduction

In this paper, we analyze a situation in which one player (seller) invests in an asset that she may sell to another player (from among potential buyers) who cannot observe the investment decision by the seller. Should trade occur, it occurs so before the value of the asset is fully known to the potential buyers. This trading opportunity, therefore, creates a moral hazard problem on the seller's investment decision. In other words, there is a trade-off between (i) the ex-ante efficiency, i.e., the incentive to invest, and (ii) the ex-post efficiency, i.e., the higher probability of trade when there is an efficiency gain from such trade.

Economic situations that exhibit the feature described above abound. For example, the current subprime crisis has proven that the securitization process reduced the incentives of financial intermediaries to carefully screen borrowers. In this setting, the seller is the originator of the loan, and the buyers are the potential investors. Evaluation and screening of the quality of the loan applicant usually involve collecting both "hard" information, such as the credit score, and "soft" information such as the impression held by the loan officer of the borrower's honesty, creditworthiness, and likelihood to defaulting. The fact that investors purchase securitized loans based on hard information reduces the incentive to collect soft information. In other words, the securitization has an adverse effect on the ex-ante screening effort of loan originators. This lax screening results in a large increase in low-quality securitized loans. In fact, Keys, Mukherjee, Seru, and Vig (2010) have found empirically that conditional on being securitized, the portfolio which is easier to securitize defaults by around 10 to 25 percent more often than a similar risk profile group which is harder to securitize.

As another example, consider the founder of a start-up company. Naturally, she is better informed about the company's fundamental than is the market. On any given day, she always has the option of selling the company, even though there may be no reason that actually forces her to do so on that day. With every passing day that she retains ownership, she gains one day's profit.

One question that naturally arises here is whether the possibility of trade contributes to the social welfare. While trade creates the efficiency gain, it also creates ex-ante inefficiency due to the moral hazard problem. By analyzing a dynamic market for lemons with endogenous quality choice by the seller, we show that the possibility of trade does not create any efficiency gain if (i) the common discounting is low, and (ii) the static incentive constraints preclude the mutually agreeable ex-ante contract under which trade happens with probability one.

At the interim stage of the game, the information and payoff structures are as in the market for lemons in Akerlof (1970). One seller is better informed than the potential buyers about the value of the single unit for sale. It is common knowledge that trade is mutually beneficial. More precisely, the item creates a flow payoff every period. The flow payoff is higher when it is in the hands of one of buyers than in the seller's hands. The potential buyers identically value the unit. The seller bargains sequentially with one of the potential buyers until an agreement is reached, if ever, and delay is costly. When it is his turn, a buyer (an uninformed party) makes a take-it-or-leave-it offer to the seller.

The novel feature of our model is that the seller's "type," i.e., the value of the item is an endogenous variable. More precisely, at the beginning of the game, the seller can make a costly investment that increases the value of the item. The potential buyers also benefit from the investment if the good is in their hands. However, the unobservability and the unverifiability of the seller's action, as well as the resulting quality of the item, create a moral hazard problem. In fact, if trade (with a high price) is sure to happen, then the seller does not have any incentive to make a costly investment at all. On the other hand, if trade always fails to happen, then the seller will invest for her own benefit. Therefore, without any pre-agreed contract, the only incentive that induces the seller to invest is the possibility of no trade, i.e., ex-post inefficiency. In other words, the game considered in this paper entails trade-off between (i) creating the ex-ante incentive for the investment, which increases the potential efficiency gain, and (ii) achieving (ex-post) efficient trade.

Despite the moral hazard problem, the seller makes the investment with a positive probability in any equilibrium. Nevertheless, the social welfare without any trade is idential to the social welfare with trade (under any equilibrium) if (i) the common discounting is low, and (ii) the static incentive constraints preclude the mutually agreeable ex-ante contract under which trade occurs with probability one. More precisely, no matter whether the previous offers are public or private, (i) the seller's ex-ante payoff is equal to her payoff from investing and retaining the good to herself, and (ii) all buyers' ex-ante payoffs are zero. Our result is not an asymptotic property.

The first-best outcome of the game is that the seller makes the investment with probability one, and trade occurs with the first buyer. One might expect that the result will be that the first-best is asymptotically attainable, i.e., that the result is like the conjecture in Coase (1972). [1)]

We shall see, however that the seller invests with a positive probability but not with one, and that trade fails to happen with a positive probability (even at the limit where the discounting is taken to be zero) in any equilibrium. The intuition behind this result is as follows: First, recall that some degree of ex-post inefficiency is needed to induce the seller to invest. Therefore, a positive level of investment necessarily entails a positive probability of failure in trade. This also implies that the seller does not invest with probability one in the equilibrium. Because if she does, then trade happens with probability one with the first buyer, which contradicts the previous claim. Second, if it is known that the seller never invests, then potential buyers never offer a high price. Then, the seller deviates to invest at the beginning of the game and retains the good to herself. Lastly, since the seller does not have any bargaining power, the seller's payoff is driven down to her reservation value, i.e., the payoff she earns by investing and keeping the item.

[1)] In dynamic hold-up literature, the first-best result is asymptotically attainable when (i) investment pays off only if the trade occurs and (ii) uninformed player (single player) repeatedly makes offers (See Gul (2001) and Lau (2008).) Neither assumption is satisfied in our model.

Suppose that there is only one buyer instead of an infinite number of potential buyers. Then the buyer has all the bargaining power in our framework. In this case, the seller who did not make any investment accepts a low price offer if it exceeds her reservation value. Since the buyer's reservation value always exceeds that of the seller, the buyer can earn a positive level of payoff by trading with the seller who did not invest. This also determines the social welfare under the second-best outcome when the buyer has all the bargaining power.

When there are future buyers, however, the seller who did not invest does not have any incentive to accept an offer that exceeds her reservation value but is not high because she may receive a high offer in the future. This competition among buyers is what drives the buyers' ex-ante payoff to zero.

The interim stage of our game is a dynamic adverse selection problem. There are a number of related papers in the literature. Among others, Hörner and Vieille (2009) is closely related to ours.[2)] It demonstrates that if the probability that the seller has a high-quality good is strictly below a certain threshold, then there is an equilibrium in which the bargaining ends up in an impasse when the offers are public, but no such an impasse occurs when the offers are private. When the offers are public, the seller can send a signal that she has invested by rejecting a high offer. In fact, the seller will reject even an offer that exceeds the offer expected in the future (on the equilibrium path) because her rejecting such an offer sends a signal that the good she owns is of high quality.

As a result, an impasse is likely to result when the offers are public.[3)] In contrast, when the probability that the seller has a high-quality good is weakly above the threshold, then trade happens with the first buyer with probability one whether the offer is public or private.

As we shall show, however, when the seller's type is endogenous, the probability

[2)] Other related works include Janssen and Roy (2002), Taylor (1999), Hendel and Lizzeri (1999), Hendel, Lizzeri, and Siniscalchi (2005).

[3)] Note that although public offers might transmit finer information than private offers do, they do not do this in any equilibrium.

that the seller owns a high-quality good (i.e., the probability that the seller makes the investment) will be exactly at the threshold. The intuition is quite simple. If the buyers believe that the seller invests with a very high probability, then the high price is offered at the first period. The seller, therefore, loses the incentive to invest. On the other hand, if the potential buyers initially believe that the seller does not invest with a high probability, then the payoff of the seller who does not invest is strictly below her payoff by investing (and not trading). However, if this really is the case, then the seller prefers to invest.

Moreover, we show that the buyers' beliefs stay at this level all the time, i.e., there is no screening on the equilibrium path no matter whether the offers are public or private. This is why our result does not depend on whether the offers are public or private.

Hörner and Vieille (2009) assume that potential buyers do not receive any signal about the quality of the item. A effects of public news in a dynamic market for lemons with private offers is analyzed in Daley and Green (2010). They show that the equilibrium involves periods of no trade, which ends either when enough good news arrives resulting in immediate trade; or when bad news arrives resulting in partial sell-off of low-value assets.

The dynamic adverse selection problem in all the aforementioned papers is exogenous, i.e., the seller's type is exogenous. There are, however, many situations in which the quality of the good is endogenously determined by the seller as we have discussed above. This paper takes a first step into endogenizing the quality of the good in the market for lemons.

Since the setting of this paper can be interpreted as bargaining that takes place in a market, this paper is also related to the literature on bargaining with interdependent values. In fact, our result is reminiscent of the result in the literature on bargaining with interdependent values.[4)] Deneckere and Liang (2006) characterize the stationary equilibrium of the game between one seller and one buyer with equal discount factors in

[4)]See also Evans (1989) and Vincent (1989).

which the uninformed party (i.e., the buyer) makes all the offers. The limiting bargaining outcome involves agreement but delay, and fails to be the second-best result. Our paper differs from theirs in that the seller's "type" is endogenous; and, instead of one buyer who keeps making offers until an agreement is reached, there is an infinite number of potential buyers and each buyer can make an offer only once.

Another paper related to ours, which represents a third dimension of the relevant literature, is Hermalin (2010). In it, Hermalin considers the optimal ex-ante contract – which is not renegotiation-proof – between one seller and one buyer in which the seller endogenously chooses the quality of the good (a static version of our game), and shows that the possible efficiency gains depends on who has the bargaining power. In fact, if the seller has the bargaining power, she can signal the quality of the good via her offer. As a result the efficiency gain can be larger when the seller has all the bargaining power than when the buyer has all the bargaining power. But there is a continuum of (belief-driven) equilibria in the game where the seller has all the bargaining power. One equilibrium among those - the worst equilibrium - shares the key properties of the equilibrium of our model, i.e., the ex-ante probability of trade, the probability of investment, and the efficiency gain.

The model is described in section two. In section three, we analyze the benchmark case in which there is only one buyer. We then provide our main result in section four. The discussion and concluding remarks are found in section five, and the proofs are found in the Appendix.

2 Model

We consider a dynamic game between (i) a single (female) seller s with one unit of an indivisible good for sale, and (ii) a countably infinite number of potential (male) buyers, $\{b_t\}_{t=1}^{\infty}$, or *buyers* for short. Time is discrete and indexed by $t = 1, 2, \cdots$. At period $t = 1, 2, \cdots$, the seller is matched with a buyer b_t, and they bargain over the price at which to trade the good according to the procedure described later.

The seller receives a flow payoff $(1-\delta)\, v_0^s = 0$ at each period $t = 1, 2, \cdots$ from the good if she owns it. The seller discounts the future payoff by a discount factor δ. Therefore, the reservation value of the good to the seller is v_0^s. At the beginning of period 1, and only then, the seller can make an irreversible investment to boost the value of the good to her. We denote the investment decision by $I \in \{0, 1\}$, where $I = 0$ and 1 represent the "no investment" and the "investment" decisions respectively. If the seller chooses $I = 1$, the flow payoff to her increases to $(1-\delta)\, v_1^s$, but she incurs the one time private cost $c \in \left(0, v_1^s\right)$ at $t = 1$. The seller's choice of I is her private information. Since the investment increases the reservation value of the good to v_1^s and since $v_1^s - c > v_0^s = 0$, the seller prefers to invest if she knows that she will retain the good in the future.

If the good is in a buyer's hands, he receives $(1-\delta)\, v_I^b$ every period when the seller's investment decision is $I \in \{0, 1\}$. The buyer is also assumed to discount the future payoff by the same discount factor δ. Therefore, the value of the good to a buyer is v_I^b when she knows the seller's investment decision is I. The investment increases the quality of the good, i.e., $v_1^b > v_0^b$.

We assume that both trade and the investment are always mutually beneficial:

1. The buyers always value the good higher than the seller does, i.e., $v_I^b > v_I^s$.

2. It is socially optimal that the seller makes the investment and trade occurs, i.e., $v_1^b - v_1^s - c > \max\left\{v_0^b - v_0^s, v_1^s - c\right\}$.

We only consider the case where there is no mutually agreeable contract under which trade happens with probability one, i.e., $v_1^s - c > v_0^b$. Note that if there exists a contract under which trade happens with probability one, the seller never invests. Therefore, if the above condition is violated (i.e., $v_1^s - c \le v_0^b$), then the seller and the first buyer can agree on a contract under which the first buyer is required to pay some price $p \in \left[v_1^s - c, v_0^b\right]$ upon the delivery of the good. However, if $v_1^s - c > v_0^b$, then no such price exists.

The game proceeds as follows. At the beginning of $t = 1$, the seller decides whether or not she will invest. After the investment decision is made, the seller meets the first buyer b_1. Without knowing the seller's investment decision, b_1 makes a take-it-or-leave-it offer $p_1 \geq 0$ to the seller. If the seller accepts the offer, then the agreement is struck and trade occurs. When the investment that the seller has chosen is I, the payoffs of the seller and the buyer b_1, evaluated at $t = 1$, are $p_1 - Ic$, and $v_I^b - p_1$, respectively. If the seller rejects the price offer, the game proceeds to the next period.

At the beginning of each period $t = 2, 3, \cdots$, the seller meets the t-th buyer, b_t, if trade did not occur by the end of period $t - 1$. Then buyer b_t makes a take-it-or-leave-it offer $p_t \geq 0$. In the private-offer case, b_t cannot observe any of the previous offers. On the other hand, if the offers are public, b_t can observe all the previous offers that have been rejected by the seller. If the seller accepts the offer by b_t, then the agreement is struck and trade occurs at period t. If the seller rejects the price offer at t, the game proceeds to $t + 1$, and the seller negotiates with a new buyer.

Note that under autarky, i.e., when trade is "prohibited" or impossible, the seller makes the investment at $t = 1$ and earns ex-ante payoff of $v_1^s - c > 0$ (and all buyers' ex-ante payoffs are zero). Hence if we define the social welfare as the sum of all players' ex-ante payoffs, then the social welfare under autarky is $v_1^s - c$. When trade is possible, all players are weakly better off. We define the efficiency gain created by the possibility of trade (denoted by E) as the difference between the the social welfare under the equilibrium and the social welfare under autarky. Note that $E \in \left[0, v_1^b - \left(v_1^s - c\right)\right]$. Then the efficiency gain E can be decomposed into (i) the gain from trade, and (ii) the loss from the higher probability of there being a lemon. To see this, let σ be the probability that the seller chooses $I = 1$, and π_i be the probability that trade occurs when the seller chooses $I = i$.

Then,

$$
\begin{aligned}
E &= \sigma\left(\pi_1 v_1^b + (1-\pi_1)\, v_1^s - c\right) + (1-\sigma)\left(\pi_0 v_0^b + (1-\pi_0)\, v_0^s\right) - \left(v_1^s - c\right) \\
&= \underbrace{\sigma\alpha_1\left(v_1^b - v_1^s\right) + (1-\sigma)\,\pi_0\left(v_0^b - v_0^s\right)}_{\text{gain (i)}} - \underbrace{(1-\sigma)\left(v_1^s - c - v_0^s\right)}_{\text{loss (ii)}}.
\end{aligned} \tag{1}
$$

Let $G \equiv \sigma\alpha_1\left(v_1^b - v_1^s\right) + (1-\sigma)\,\pi_0\left(v_0^b - v_0^s\right)$ and $L \equiv (1-\sigma)\left(v_1^s - c - v_0^s\right)$. As it is clear from (1), the gain from trade G is increasing in σ and in π_i. As we shall see, however, the increase in the probability of trade, i.e., the increase in π_i, creates the moral hazard problem. As a result, the probability of investment σ becomes smaller, and hence, the larger value of L results.

3 Benchmark Case

In this section, we consider a benchmark case in which $\delta = 0$. The situation can also be interpreted as the game with $\delta \in (0,1)$ in which there is only one potential buyer, b_1, and as a result, b_1 has all the bargaining power. The objective is to show that (i) the first-best outcome is unattainable in this game due to the moral hazard problem; (ii) the efficiency gain created by the possibility of trade is nevertheless strictly positive when the buyer has all the bargaining power; and (iii) the existence of trade-off between creating the ex-ante incentive for the investment (ex-ante efficiency) and achieving a higher probability of trade (ex-post efficient trade).

Theorem 1 *When $\delta = 0$, there exists a unique equilibrium. In this equilibrium,*

1. *the seller chooses $I = 1$ with probability σ^*, and $I = 0$ with probability $1-\sigma^*$, where $\sigma^* = v_1^s / v_1^b \in (0,1)$;*
2. *the buyer offers $p = v_1^s$ with probability α^*, and $p = v_0^s = 0$ with probability $1-\alpha^*$, where $\alpha^* = \left(v_1^s - c\right)/v_1^s \in (0,1)$; and*
3. *the efficiency gain created by the possibility of trade E is $\left(1 - v_1^s/v_1^b\right) v_0^b > 0$.*

Proof. In the Appendix. ■

Recall that the first-best outcome of the game is that the seller makes the investment with probability one, and trade happens with probability one at $t = 1$. The theorem above shows that the first-best outcome is not attainable due to the moral hazard problem. Nevertheless, the seller invests with a positive probability, and trade happens with a positive probability. Furthermore, the efficiency gain created by the possibility of trade is positive. In other words, both G and L are positive because $\sigma = \sigma^*$, $\pi_0 = 1$, and $\pi_1 = \alpha^*$, but $G > L$.

Note that $p = 0$ is offered with a positive probability in the equilibrium. This low offer plays two key roles. First, the low offer mitigates the moral hazard problem. If the seller chooses $I = 0$ and faces $p = 0$, then the seller's ex-post payoff is zero. If the seller has chosen $I = 1$ instead, she rejects the offer and can earn $v_1^s - c$ by retaining the good. Therefore, the possibility of this low offer creates an incentive to invest ex-ante, i.e., it induces higher ex-ante efficiency. On the other hand, the seller who has chosen $I = 1$ does not accept the low offer and decides to retains the good. Therefore, even though trade is mutually beneficial at some price $p \in \left[v_1^s, v_1^b\right]$, trade fails to occur. As a result, the low offer which created the incentive to invest also creates the ex-post inefficiency.

As we have seen above, the benchmark case analysis reveals that the game considered in this paper entails trade-off between (i) creating the ex-ante incentive for the investment, which increases the potential efficiency gain, and (ii) achieving (ex-post) efficient trade. The objective of the following section is to examine the effect of competition among buyers on this trade-off, which in turn changes the probability of investment, the probability of trade, and the efficiency gain created by the possibility of trade.

4 Analysis

In this section, we consider the case in which the seller meets with a new buyer if no trade has yet occurred. In such a situation, a buyer "competes" with future potential buyers, and thus a particular buyer's bargaining power will now be weaker. In fact, the seller who has chosen $I = 0$ does not accept $p_1 = 0$ any more. We show that when the common discount factor is close to one, the efficiency gain created by the possibility of trade is zero, i.e., $E = 0$. This result does not depend on whether the offers are private or public.

Assumption 1 *The common discount factor is close to one:* $\delta > \frac{v_0^b}{v_1^s - c}$.

4.1 Private-Offer Case

First, we consider the private-offer case. Since the offer is private, b_t cannot observe the offers that are rejected at $\tau \le t-1$. A few definitions are now required. Let μ_t be b_t's posterior belief assigned to the seller who has chosen $I = 1$. Let $\tilde{\sigma}$ be b_t's belief at which he is indifferent between offering v_1^s and 0, i.e., $\tilde{\sigma} v_1^b + (1-\tilde{\sigma}) v_0^b - v_1^s = 0$. In other words, $\tilde{\sigma}$ is defined as $\tilde{\sigma} \equiv \frac{v_1^s - v_0^b}{v_1^b - v_0^b}$. We first show that there exists an equilibrium in which the efficiency gain created by the possibility of trade is zero. Consider the following pair of strategies and buyers' beliefs:

- At any t, $\mu_t = \tilde{\sigma}$; and b_t offers either v_1^s with probability $\alpha(\delta) \equiv \frac{(1-\delta)\left(v_1^s - c\right)}{(1-\delta)v_1^s + c\delta}$ or zero with probability $(1-\alpha(\delta))$.
- At any t,
  - the seller who has chosen $I = 1$ accepts p_t with probability one if and only if $p_t \ge v_1^s$, and rejects any $p_t < v_1^s$ with probability one.
  - the seller who has chosen $I = 0$ accepts p_t with probability one if and only if $p_t \ge \delta\left(v_1^s - c\right)$, and rejects any $p_t < \delta\left(v_1^s - c\right)$ with probability one.

To see why this is an equilibrium, note that the ex-ante probability that the seller receives offer v_1^s is $\frac{\alpha(\delta)}{1-(1-\alpha(\delta))\delta} = \frac{v_1^s - c}{v_1^s}$. Therefore, the ex-ante payoff from not investing is $\frac{\alpha(\delta)}{1-(1-\alpha(\delta))\delta} v_1^s = v_1^s - c$, that is, the seller is indifferent between investing and not investing.

Furthermore, at any point, the discounted value of the future offers (in expectation) is $\delta\left(v_1^s - c\right)$. This is the maximum payoff the seller who has chosen $I = 0$ can gain by rejecting the current offer. Therefore, the seller's actions are optimal. As for the buyers, if b_t's offer is accepted only by the seller who has chosen $I = 0$, then such an offer is in $\left[\delta\left(v_1^s - c\right), v_1^s\right)$. However, b_t's reservation value for the item with $I = 0$ is $v_0^b < \delta\left(v_1^s - c\right)$. Hence b_t's actions are optimal.

The equilibrium described above exhibits the following properties: (i) the seller's ex-ante payoff is $v_1^s - c$, and (ii) the buyers' ex-ante payoffs are zero. Therefore, (iii) the efficiency gain created by the possibility of trade is zero, (iv) the ex-ante probability of the investment is $\tilde{\sigma} \in (0,1)$, and (v) the ex-ante probability of trade is $\frac{v_1^s - c}{v_1^s} \in (0,1)$. Furthermore, the probability that trade occurs within any given finite period converges to zero as $\delta \to 1$.

The equilibrium is not unique in general, but we show that every equilibrium exhibits properties (i) to (v).

Theorem 2 *Suppose the buyers' offers are private. Then every equilibrium satisfies the following properties: (i) the efficiency gain created by the possibility of trade is zero, (ii) the ex-ante probability of investment is* $\frac{v_1^s - v_0^b}{v_1^b - v_0^b}$*, and (iii) the ex-ante probability of trade is* $\frac{v_1^s - c}{v_1^s}$*.*

4.2 Public-Offer Case

In this subsection, we consider the case in which $b_t, t \geq 2$ can observe all the previous offers that have been rejected. It is not hard to see that there exists an equilibrium such that on the equilibrium path b_t offers either v_1^s with probability $\alpha(\delta) \equiv \frac{(1-\delta)\left(v_1^s - c\right)}{(1-\delta)v_1^s + c\delta}$ or zero with probability $(1 - \alpha(\delta))$; and trade occurs if and only if v_1^s is offered.

As in the private-offer case, the equilibrium described above exhibits the following properties: (i) The seller's ex-ante payoff is $v_1^s - c$, and (ii) the buyers' ex-ante payoffs are zero. Therefore, (iii) the efficiency gain created by the possibility of trade is zero, (iv) the ex-ante probability of the investment is $\tilde{\sigma} \in (0,1)$, and (v) the ex-ante probability of trade is $\frac{v_1^s - c}{v_1^s}$. Furthermore, the probability that trade occurs within any given finite period converges to zero as $\delta \to 1$.

In this case too, the equilibrium is not unique in general, but we show that every equilibrium exhibits properties (i) to (v).

Theorem 3 *Suppose the buyers' offers are public. Then every equilibrium satisfies the following properties: (i) the efficiency gain created by the possibility of trade is zero, (ii) the ex-ante probability of investment is* $\frac{v_1^s - v_0^b}{v_1^b - v_0^b}$*, and (iii) the ex-ante probability of trade is* $\frac{v_1^s - c}{v_1^s}$*.*

Remark 1 *It is straightforward to see that in comparison with the benchmark case G is smaller and L is larger. As a result, now* $G = L$ *because of the smaller values of* π_0 *and* σ*. The probability the seller who has chonsen* $I = 0$ *will trade, i.e.,* π_0*, is smaller because such a seller has no incentive to accept a low offer, given that there are future sellers who might make a high offer. The probability of investment is smaller because now buyers are indifferent between making (i) a high offer that is accepted with probability one, i.e.,* v_1^s*, and (ii) an offer that is rejected with probability one, i.e.,* 0*.*

Remark 2 *The possibility that the seller can trade with another buyer drives the bargaining power of a particular buyer to zero. In fact, consider a game in which* $\delta = 0$ *but the seller has all the bargaining power. In other words, the seller makes a take-it-or-leave-it offer only once, and the negotiation breaks down if the seller's offer is not accepted. In such a game, there are multiple equilibria because the seller can signal her own interim type. There is one equilibrium among those in which (i) the efficiency gain created by the possibility of trade is zero, (ii) the seller invests with probability* $\frac{v_1^s - v_0^b}{v_1^b - v_0^b}$ *and offers* v_1^s *with probability one irrespective of her investment decision, and (iii) the buyer accepts the offer* v_1^s *with probability* $\frac{v_1^s - c}{v_1^s}$*, and rejects any offer but* v_1^s *with probability one. It is clear that such an equilibrium has all the properties described in Theorem 2 and Theorem 3.*

Remark 3 *Recall that while the reservation value of the good to the seller who has chosen* $I = 1$ *is* v_1^s*, it is zero for the seller who has chosen* $I = 0$*. Suppose that the reservation value is* *exogenously given, and her reservation value is either v_1^s with probability $\frac{v_1^s - v_0^b}{v_1^b - v_0^b}$, or zero with probability $1 - \frac{v_1^s - v_0^b}{v_1^b - v_0^b}$. Then there is an equilibrium in which b_1 offers v_1^s with probability one, and the game ends at $t = 1$. In such an equilibrium, the payoff of the seller who has chosen $I = 1$ is $v_1^s - c$, which is exactly the same as her reservation value. On the other hand, the seller who has chosen $I = 0$, whose reservation value is zero, receives v_1^s. Since trade occurs without delay, the efficiency gain created by the possibility of trade is positive.*

However, in the game considered here, the seller's reservation value is endogenous. Therefore, the result described above fails to happen. If the seller knows that trade happens with probability one or very close to one, the seller does not have much incentive to invest.

Remark 4 *Suppose that there are only T buyers instead of infinite buyers. It is straightforward to show that irrespective of the value of T, the efficiency gain created by the possibility of trade is positive. This is because the T-th buyer has the bargaining power, and therefore, he gains from trading with the seller who did not invest.*

5 Conclusion

In this paper, we analyzed a dynamic search model with endogenous quality choice and showed that the efficiency gain created by the possibility of trade is zero if (i) the common discounting is low, and (ii) the static incentive constraints preclude the mutually agreeable ex-ante contract under which trade happens with probability one. One implication of our result is that a lower search cost may not necessarily improve the social welfare. To see this, compare the following three scenarios: (i) the case in which the seller cannot meet with any buyer, so the search cost is extremely high; (ii) the case in which the seller can negotiate with only one buyer, so the search cost is only slightly high; and (iii) the case in which the seller can always negotiate with another buyer if the negotiation fails, so the search cost is low. Note that while the social welfare under case (iii) is the same as under case (i), the social welfare under case (ii) is strictly higher than both cases (i) and (iii). In other words, a mildly high search cost induces higher social welfare than either a low or an extremely high search cost. As we have seen, the reasoning behind this result is that a low search cost weakens the incentive for the investment.

Our simple setting can be extended into various directions. Below, we briefly provide several possible alternative specifications, and discuss how our results change. Future directions for research are also discussed.

One difficulty that many search models have to deal with is the familiar Diamond's Paradox (Diamond (1971)). Diamond's paradox is that *inter-temporal* competition among buyers does nothing to increase the surplus of the seller when the time is discrete. One may wonder if our result would be robust to the introduction of *intra-temporal* competition. To see that it is, suppose that there are two buyers every period, and the seller runs a second price auction with the reservation value v_1^s. Recall that no buyer earns a positive payoff in the original equilibrium. Hence, we can construct an equilibrium in which one buyer offers v_1^s with probability $\alpha(\delta)$, and the other buyer offers zero with probability one every period. In this sense, our result is robust to the introduction of intra-temporal competition. [5)]

One may find our assumption that the investment can be made only at the beginning of the game to be quite restrictive. If the seller can invest at the beginning of every period, the moral hazard problem is more severe. Our result – that the efficiency gain created by the possibility of trade is zero – still holds under the more severe moral hazard situation. In fact, the efficiency gain created by the possibility of trade is zero even when there is only one buyer (and the discount factor is large enough) because the seller never accepts a low offer, i.e., $p_t < \delta\left(v_1^s - c\right)$ even if there is no future buyers. Our result shows that even when the moral hazard problem is milder, trade cannot create any efficiency gain.

One of the crucial assumptions of our model is that buyers do not receive any private or public signals. This is why the future-period buyers do not have any knowledge

[5)]This comes with one caveat, however. There is a continuum of equilibrium outcomes that are belief driven. In fact, there are equilibria in which the efficiency gain is positive.

that is superior to that of current-period buyer in our equilibrium. However, if we consider the examples of the securitized loans or the start-up company, it is conceivable that the market, i.e., potential buyers receive some public information – e.g., profitability, performance, or customer base – as time goes by.[6)] In contrast, if we consider the housing owner example, it is conceivable that each potential buyer receives a private signal.

Some of the equilibrium properties of our result still holds even when the buyers receive some signal. For example, the seller never invests with probability one. The ex-ante probability that trade occurs is positive, but not one.

When the signals are public, buyers gradually learn whether the seller has invested or not. Therefore, it is conceivable that the efficiency gain created by the possibility of trade is positive. But this is not necessarily the case when the signals are private because the following form of information cascade may occur: if the first few buyers receive bad signals and therefore decide not to buy the good, then all the following future buyers may rationally ignore their private signals. Note that such a cascade never occur when the signals are public. Therefore, if the seller invests with the same probability as when the signals are public, it may be the case that the ex-ante probability of trade is lower. That in turn implies that the seller has more incentive to invest at the beginning of the game. Therefore, the welfare comparison of the different types of signals is intricate. Whether or not such a cascade occurs may depend on if the offers are public or private as well. Extending our model in this direction would be a valuable question to explore in future research.

6 Appendix

6.1 Proof of Theorem 1

We solve the game by backward induction. Let σ be the probability that the seller makes the investment. The belief of b_1 on σ is denoted by $\hat{\sigma}$. It is straightforward to see that

[6)]See Daley and Green (2010) for the exogenous-type case.

b_1 offers either 0 or v_1^s. The buyer's optimal offer depends on her belief $\hat{\sigma}$. The payoffs from offering v_1^s and 0 are $\hat{\sigma} v_1^b + (1-\hat{\sigma})\, v_0^b - v_1^s$ and $(1-\hat{\sigma})\, v_0^b$, respectively. Let $\bar{\sigma} = v_1^s / v_1^b$. If $\hat{\sigma} > \bar{\sigma}$, then b_1 offers $p = v_1^s$ with probability one. On the other hand, if $\hat{\sigma} < \bar{\sigma}$, his offer is $p = 0$ with probability one. If $\hat{\sigma} = \bar{\sigma}$, then b_1 is indifferent between offering 0 and v_1^s.

We show that there is no equilibrium with $\hat{\sigma} \neq \bar{\sigma}$. To see this, first suppose $\hat{\sigma} > \bar{\sigma}$. If the seller chooses $I = 0$, the her payoff is v_1^s. On the other hand, if she chooses $I = 1$, her payoff is $v_1^s - c < v_1^s$. Therefore, the seller deviates to $\sigma = 0$, which is a contradiction. Next suppose $\hat{\sigma} < \bar{\sigma}$. If the seller chooses $I = 0$, then her payoffs is 0. On the other hand, the seller can always earn $v_1^s - c > 0$ by choosing $I = 1$. Therefore, the seller deviates to $\sigma = 1$, which is a contradiction.

Therefore, if there is an equilibrium, the seller chooses $I = 1$ with probability $\bar{\sigma}$ and $I = 0$ with probability $1 - \bar{\sigma}$. When the seller takes such a strategy, b_1 is indifferent between offering $p = v_0^s$ and v_1^s. Let α be the probability that the buyer offers $p = v_1^s$. If $\alpha = \left(v_1^s - c\right) / v_1^s$, the ex-ante expected payoffs of choosing $I = 0$ and $I = 1$ are $\alpha v_1^s = v_1^s - c$ and $v_1^s - c$, respectively. Hence, the seller is in fact indifferent between $I = 0$ and 1.

The seller's ex-ante payoff is $v_1^s - c$. Therefore, the buyer's ex-ante payoff corresponds to E, which is $(1-\sigma^*)\, v_0^b$. **Q.E.D.**

6.2 Proof of Theorem 2

Let σ^* be the probability that the seller makes chooses $I = 1$.

Claim 1 *b_t never offers $p_t > v_1^s$.*

Proof. This follows from the facts that b_{t+n} never offers $p_{t+n} > v_1^b$, and therefore the seller who has chosen $I = 1$ accepts any offer $p_t \geq \max\left\{\delta^n v_1^b, v_1^s\right\}$ at t. In addition, v_1^s is accepted by the sellers with $I = 0$ and $I = 1$ with probability one. ■

Claim 2 *If v_1^s will be offered with probability one at t, then $\mu_t = \mu_{t-1}$.*

Proof. If v_1^s is offered with probability one at t, then the seller who has chosen $I = 0$ rejects any offer $p_t \leq v_0^b$ at $t - 1$ because $v_0^b < \delta v_1^s$. Therefore, $\mu_{t-1} = \mu_t$. ■

Claim 3 *In any equilibrium, $\mu_t = \tilde{\sigma}$ for all t.*

Proof. Consider b_t with belief μ_t. If he were to make an offer that will be accepted only by the seller who has chosen $I = 0$, then the highest possible payoff for him would be $(1 - \mu_t)\, v_0^b - 0$. Therefore, b_t offers v_1^s with probability one when $\mu_t v_1^b + (1 - \mu_t)\, v_0^b - v_1^s > (1 - \mu_t)\, v_0^b$, i.e. $\mu_t > m_1 \equiv \frac{v_1^s}{v_1^b}$.

Then by Claim 2, $\mu_t \leq m_1$ for all t in any equilibrium. Otherwise, $\mu_1 > m_1$, and then $p_1 = v_1^s$ with probability one. If this is the case, however, then the seller never invests, a contradiction.

Now consider b_t with belief $\mu_t \leq m_1$. If he were to make an offer that will be accepted only by the seller who has chosen $I = 0$, then the highest possible payoff for him would be $(m_1 - \mu_t)\, v_0^b - 0$. Therefore, b_t offers v_1^s with probability one if $\mu_t v_1^b + (1 - \mu_t)\, v_0^b - v_1^s > (m_1 - \mu_t)\, v_0^b$, i.e., $\mu_t > m_2$, where m_2 solves $m_2 v_1^b + (1 - m_2)\, v_0^b - v_1^s = (m_1 - m_2)\, v_0^b$. Note that $m_2 < m_1$. Then by Claim 2 again, $\mu_t \leq m_2$ for all t in any equilibrium.

Applying the argument sequentially, we can show that for any t, b_t offers v_1^s with probability one if $\mu_t > m_n$, where m_n solves the following recurrence formula:

$$m_n v_1^b + (1 - m_n)\, v_0^b - v_1^s > (m_{n-1} - m_n)\, v_0^b.$$

Since $m_n > m_{n+1}$, and $\lim_{n\to\infty} m_n = \tilde{\sigma}$, we know that $\mu_t \leq \tilde{\sigma}$ for all t.

We are done if we show that $\mu_1 = \tilde{\sigma}$ since μ_t is weakly increasing. Suppose that $\mu_1 < \tilde{\sigma}$. Then, the maximum payoff of the seller who has chosen $I = 0$ is v_0^b. However, by choosing $I = 1$, she can earn $v_1^s - c$, a contradiction. ∎

Claim 4 *In any equilibrium, the efficiency gain created by the possibility of trade is zero.*

Proof. Note that $\mu_t = \mu_{t+1} = \tilde{\sigma}$ by Claim 3. Therefore b_t's offer is either (i) accepted by both the sellers with $I = 0$ and $I = 1$, i.e., $p_t = v_1^s$ or (ii) rejected by both. In either case, b_t's payoff is zero. The seller's ex-ante payoff is $v_1^s - c$. Therefore, the claim follows. ∎

Claim 5 *In any equilibrium, the ex-ante probability of trade is $\frac{v_1^s - c}{v_1^s}$.*

Proof. The seller is indifferent between investing and not investing. Furthermore, trade occurs if and only if v_1^s is offered. Therefore, the ex-ante probability of trade has to be $\frac{v_1^s - c}{v_1^s}$. ■

Q.E.D.

6.3 Proof of Theorem 3

Claim 6 *If $\mu_t > \tilde{\sigma}$, then b_t offers v_1^s with probability one independent of the history.*

Proof. Recall m_n defined in Claim 3. If $\mu_t > m_1$, then b_t offers v_1^s with probability one. Suppose b_t has belief $\mu_t \leq m_1$. Then b_t's offer cannot lead to a posterior $\mu_{t+1} > m_1$. If it can, then $\mu_{t+1} > \mu_t$ implies that the seller who has chosen $I = 0$ accepts some offer $p_t \leq v_0^b$ with a positive probability. However, if the seller who has chosen $I = 0$ rejects such an offer, she can increase her payoff to δv_1^s, a contradiction. Therefore, the highest possible payoff from an offer that will be accepted only by the seller who has chosen $I = 0$ is $(m_1 - \mu_t)\, v_0^b$. Therefore, if $\mu_t > m_2$, then b_t offers v_1^s with probability one.

Repeating the same argument, we can show that if $\mu_t > m_n$, then b_t offers v_1^s with probability one. Since $\lim_{n\to\infty} m_n = \tilde{\sigma}$, we obtain the result. ■

Claim 7 *In any equilibrium, $\mu_t = \tilde{\sigma}$ for all t.*

Proof. By Claim 6, we know that $\mu_1 \leq \tilde{\sigma}$. But if $\mu_1 < \tilde{\sigma}$, then the highest possible payoff for the seller who has chosen $I = 0$ is v_0^b. Then she prefers to choose $I = 1$, a contradiction. Therefore $\mu_1 = \tilde{\sigma}$. Since it is straightforward to see that there is no history such $\mu_{t+1} > \mu_t = \tilde{\sigma}$ for some t, we are done. ■

The rest of the proof is the same as Theorem 2. **Q.E.D.**

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