# Absolute vs. Relative Notion of Wealth Changes

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## Abstract

This paper discusses solutions derived from lottery experiments using two alternative assumptions: that people perceive wealth changes as absolute amounts of money; and that people consider wealth changes as a proportion of some reference value dependant on the context of the problem under consideration. The former assumption leads to the design of Prospect Theory, the latter - to a solution closely resembling the utility function hypothesized by Markowitz (1952). This paper presents several crucial arguments for the latter approach and provides strong arguments for rejecting the Prospect Theory paradigm.

**JEL Classification:** C91, D03, D81, D87.

**Keywords:** Prospect / Cumulative Prospect Theory, Probability Weighting Function, Markowitz Hypothesis, Weber's Law.

### 1. Introduction

Prospect Theory (Kahneman, Tversky, 1979), and its Cumulative version (Tversky, Kahneman, 1992) assert that people are more concerned with changes in wealth than its overall value when making decisions involving small sums of money. Anticipated gains and losses, expressed as monetary amounts, are then used to evaluate the prospects under consideration. An analysis of lottery experiments assuming the absolute notion of wealth changes requires, however, the concept of probability weighting to be incorporated into the descriptive model.

The present paper questions the assumption that people treat gains and losses as absolute values when making decisions under conditions of risk. On the contrary, it asserts that gains and losses are perceived in relative terms, in a context which depends on how attention is focused on it. The result is that gains and losses are perceived in relation to a reference value which, most frequently, is the maximum prospect outcome. Analyzing gains and losses in

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relative terms eliminates the need for the probability distortion concept and leads to a solution closely resembling the utility function hypothesized by Markowitz (1952).

This paper presents several important arguments in support of the latter approach, the major one being that people regard changes in wealth in relative terms is founded on Weber's Law - one of the fundamental laws of psychophysics. This law contradicts the absolute notion of wealth changes. The observation is not new and has been confirmed by modern researchers including Kahneman, Tversky (1984) and Thaler (1985, 1999) since the introduction of Prospect Theory.

## 2. Absolute Notion of Wealth Changes

**2.1.** Consider the following set of experiments[3]:

Experiment 1: *Would you prefer to enter a lottery with a 50% chance of winning either $100 or $0, or to receive a payment of $50?* Most respondents would prefer to receive the certain payment.

Experiment 2: *Would you prefer to enter a lottery with a 50% chance of winning either $200 or $0, or to receive a payment of $100?* Most respondents would prefer the certain payment.

Experiment 3: *Would you prefer to enter a lottery with a 50% chance of winning either $400 or $0, or to receive a payment of $200?* Most respondents would prefer the certain payment.

Analogous experiments can be repeated for other monetary amounts in order to "scan" a hypothetical utility function over a broad range of outcomes. Because the responses invariably indicate a preference for certain payments, researchers conclude that people are averse to risk. This is why their utility curve is assumed to be concave whatever the outcome under consideration.

**2.2.** Consider the following further set of experiments:

Experiment 4: *Would you prefer to enter a lottery with a 10% chance of winning $500 and $0 otherwise, or to receive a payment of $50?* Most respondents would prefer the lottery.

Experiment 5: *Would you prefer to enter a lottery with a 10% chance of winning $1000 and $0 otherwise, or to receive a payment of $100?* Most respondents would prefer the lottery.

[3] These are mental experiments only. However similar experiments have been conducted by numerous authors including Kahneman, Tversky (1979, 1992), and Gonzales, Wu (1999). CPT's parameters have been estimated basing on the results of two-outcome lotteries. Gonzales and Wu proceeded in a similar way.

Experiment 6: *Would you prefer to enter a lottery with a 10% chance of winning $2000 and $0 otherwise, or to receive a payment of $200?* Most respondents would prefer the lottery.

The respondents in these cases exhibit risk seeking behavior and, interestingly, for the same certain payment amounts ($50, $100, and $200) as in Experiments 1 - 3. Researchers conclude that people are generally risk averse (as demonstrated in Experiments 1 - 3) in these situations as well, but that there has to exist an additional effect related to the perception of probabilities in order to explain the risk-seeking behavior observed in Experiments 4 - 6. This effect would have to rely on a non-linear perception of probabilities and especially on an overweighting of low ones. This reasoning thus leads to a theory in which there exists a non-linear probability weighting function in addition to a utility function. This is how Prospect Theory (and other theories using a similar approach) developed.

**2.3.** There is no reason why these experiments could not have been conducted in the reverse order to arrive at the very opposite conclusions. Researchers could well have concluded that people are generally risk-seeking on the strength of Experiments 4 - 6 and argued that their utility function should be convex over the entire range of outcomes under consideration. The risk-aversion observed in Experiments 1 – 3 would again find its explanation in a non-linear probability weighting, only this time the low probabilities would be perceived accurately whereas probabilities around 0.5 would be heavily underweighted.

**2.4.** Both explanations presented utilize the probability weighting function additionally to the utility function. Both explanations would have the same ability to describe the experimental results, albeit with completely different modi operandi. The fact that changing the sequence of reasoning steps changes the resulting explanation might be a signal that this approach is unsound.

## 3. Relative Notion of Wealth Changes

**3.1.** The foregoing reasoning assumed the absolute notion of wealth changes. This means that gains and losses were represented and analyzed as monetary amounts (such as $50, $100, or $200). This is one of the basic assumptions of Prospect Theory and is best expressed by a value function which supposedly determines the value (utility) of specific amounts of money to people (according to Cumulative Prospect Theory the value function is defined as $v(x) = \lambda|x|^{\alpha}$, where $x$ is the gain or loss expressed as an absolute monetary amount).

People, however, typically consider wealth changes in relative terms. This means that gains and losses are usually perceived as a proportion of a reference value, which depends on the context of the problem. This observation is not new and was also noticed by Kahneman and Tversky in 1984, i.e. 5 years after the introduction of Prospect Theory: *The topical organization of mental accounts leads people to evaluate gains and losses in relative rather than in absolute terms* (emphasis added)[4]. Despite this, the absolute notion of gains and losses remained the underlying assumption of Cumulative Prospect Theory, which appeared in 1992.

**3.2.** The explanation that people regard changes of wealth in relative terms is founded on basic psychophysical laws. In the first half of the nineteenth century, German researcher Ernst Weber conducted experiments on determining the Just Noticeable Difference in weight between objects and concluded that this difference is twice as great with a 2 kg object than with a 1 kg object. The law Weber formulated in 1834 states that this difference is a constant proportion of the initial stimulus magnitude. This is now one of the fundamental laws of psychophysics (Encyclopædia Britannica, 2009):

$$JND/\ S = k$$

where *JND* denotes the Just Noticeable Difference, *S* denotes the initial stimulus magnitude and *k* is a constant. This law holds with reasonable accuracy for most stimuli within a broad range. For example, the value of *k* is 2% for weight, 4.8% for loudness and 7.9% for brightness. It follows from the Weber law that the same change in stimulus (for instance 0.2 kg) can be strongly felt, slightly noticed or not perceived at all depending on the magnitude of the initial stimulus. It further follows that an unambiguous and absolute perception level of a specific stimulus change cannot be determined, as this depends on the situational context.

**3.3.** How the Weber Law works for financial stimuli will be presented with the following example:

Problem 1a: *What is the smallest significant amount of money to a person shopping for goods worth about $100?*

This will possibly be close to 1 dollar (but not as small as 1 cent). Such a person may, for example, consider choosing a rival product that is $0.5 cheaper.

Problem 1b: *What is the smallest amount of money considered by the same person purchasing a house for $500,000?*

This will probably be at least $1,000. A purchase offer of $479,538 is hard to imagine; $480,000 seems far more likely. It follows that ten dollars, a significant amount in the former

[4] Kahneman and Tversky consider a minimal, topical, and comprehensive account in their “Choices, Values, and Frames” paper. They state: *A topical account relates the consequences of possible choices to a reference level that is determined by the context within which the decision arise* and conclude: *People will spontaneously frame decisions in terms of topical accounts.* The idea of mental accounts actually originated with Thaler (1985, 1999).

case, is completely insignificant in the latter. Even \$100, the sum total of a person's expenditure in a shop, is of no significance in a house purchase.

**3.4.** The foregoing example demonstrates that the human mental system adapts itself to financial quantities, just as its sensory system does to physical ones. The result is that the Just Noticeable Difference remains an approximately constant proportion of different financial amounts. This means that when considering financial prospects (projects, investments, lotteries etc.), the size of the prospect becomes a reference value in the entire mental process, rendering an absolute amount of money (say \$10) relevant or irrelevant depending on the context. This conclusion constitutes a fundamental deviation from Prospect Theory, which regards gains and losses in absolute terms, and attempts to derive a value function in terms of absolute monetary amounts.

## 4. Solution Resulting From Relative Notion of Wealth Changes

**4.1.** In point 2, we demonstrated how the consideration of gains and losses in absolute terms inevitably leads to the concept of probability weighting. We will now show that considering gains and losses as relative values allows the experimental results to be explained without this concept.

**4.2.** The results of the Experiments 1-6 can be explained differently by assuming that people perceive outcomes as proportions of the main payment. No "scanning" of the utility function is performed for the various outcomes in Experiments 1 - 3 because the amount of the certain payment is always half that of the main payment. All three questions therefore deal with the value of the utility function for the relative outcome of $r = 0.5$. The conclusion from these three experiments is that people are risk-averse for this particular *relative* outcome, so the utility function is concave at this point. But this conclusion only concerns this one point. Repeating similar experiments with $p = 0.5$ for different outcome values would still only inform us about this single point of the curve expressed in terms of *relative* outcomes.

Similarly, Experiments 4 - 6 show that people exhibit a risk-seeking attitude by their preference for the lottery over a relative outcome of $r = 0.1$. The utility function is therefore convex at this point. Once again, this conclusion is only valid for this single point. This line of reasoning leads to the plotting of a utility function expressed for *relative* outcome values, which is partially convex and partially concave. This would certainly be a completely different solution from that proposed by Prospect Theory, all the more so since it would not utilize any probability weighting function.

**4.3.** The foregoing reasoning may be repeated for losses as well. Consider the following set of 3 experiments (the outcomes in parentheses correspond with the respective experiment numbers):

Experiment 7 (8, 9): *Would you prefer to enter a lottery with a 90% chance of losing $100 ($200, $400) and $0 otherwise, or to pay $90 ($180, $360) to avoid the game?* Most respondents would prefer the lottery.

Consider now the following further set of 3 experiments:

Experiment 10 (11, 12): *Would you prefer to enter a lottery with a 10% chance of losing $900 ($1800, $3600) and $0 otherwise, or to pay $90 ($180, $360) to avoid the game?* Most respondents would prefer to pay the certain amount.

The respondents in Experiments 7 – 9 exhibit risk seeking behavior, whereas the respondents in Experiments 10 - 12 exhibit risk aversion behavior for exactly the same certain payment amounts ($90, $180, and $360). As in case of gains, considering losses in absolute terms requires a probability weighting function to be incorporated into the descriptive model. Similarly, the order of the reasoning steps influences the conclusion regarding the general attitude to risk. Beginning with Experiments 7 - 9 leads to the conclusion that people are generally risk *seeking* and that a probability weighting function is required to explain the risk aversion observed in Experiments 10 - 12. Conversely, beginning with Experiments 10 - 12 leads to the conclusion that people are generally risk *averse* and that a probability weighting function is required to explain the risk seeking attitude observed in Experiments 7 - 9.

Once again, it follows that completely different modi operandi result from both approaches with the value and probability weighting functions assuming different shapes in each case.

**4.4.** Analyzing the experimental results using the relative notion of losses leads to a completely different and unambiguous result. Experiments 7 - 9 lead to the conclusion that people are risk seeking for the *relative* loss of $r = 0.9$, whereas Experiments 10 - 12 lead to the conclusion that people are risk averse for the *relative* loss of $r = 0.1$. No probability weighting function is required to explain the results of these experiments.

## 5. Relative Utility Function

**5.1.** The shape of the utility function, expressed in terms of *relative* outcomes, can now be hypothesized using the results presented in Point 4. The solution is presented in Fig.1.

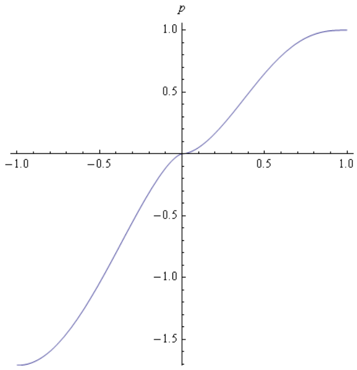


Fig. 1. The relative utility function.

The curve should be concave for high relative gains, reflecting the risk aversion observed in Experiments 1-3, and convex for low relative gains, reflecting the risk seeking attitude observed in Experiments 4-6.

The picture is reversed for losses. The function should be convex for high relative losses, reflecting the risk seeking attitude observed in Experiments 7 - 9, and concave for low relative losses, reflecting the risk aversion observed in Experiments 10 – 12.

The double-S shape of this hypothetical relative utility function offers a simple and concise explanation of risk-seeking and risk-aversion attitudes when making decisions under conditions of risk. This replaces the "fourfold pattern" of risk attitude formulated by Cumulative Prospect Theory. This solution is unambiguous and does not require a probability weighting function to describe the pattern. The curve should be of greater magnitude for losses than for gains as people are generally averse to loss.

**5.2.** Quite surprisingly, the obtained curve strongly resembles the utility function hypothesized by Markowitz (1952) and shown in Figure 2.

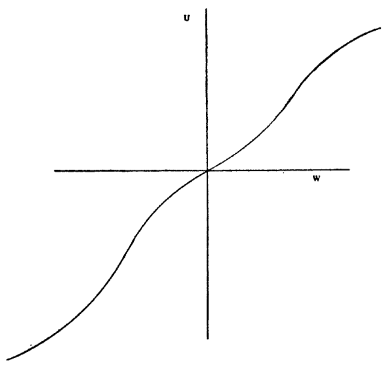


Fig. 2. The shape of the utility function according to the Markowitz hypothesis of 1952.

The only significant difference with the Markowitz curve is that the relative utility function is defined for relative values of outcomes rather than for absolute ones.

## 6. Summary.

This paper discusses solutions derived from lottery experiments using two alternative assumptions: the absolute and relative notion of wealth changes. Several important advantages of the latter approach have been presented in the paper. Most importantly, people do consider changes of wealth in relative terms. This was even confirmed by Kahneman and Tversky – the authors of Prospect Theory.

However, introducing this assumption into an analysis of the experimental data leads to the rejection of Prospect Theory itself. The concept of probability weighting – one of the key planks of Prospect Theory – becomes unnecessary when gains and losses are considered in relative terms.

The relative notion of wealth changes leads to a completely different solution, one which strongly resembles the utility function hypothesized by Markowitz (1952).

This paper strictly demarcates the two approaches as the relative and absolute notions are mutually exclusive. As a result Prospect Theory (Kahneman & Tversky, 1979, 1992) cannot be held to be an accurate explanation of people's behavior while gains and losses are simultaneously held to be perceived in relative terms (Kahneman & Tversky, 1984).

**References**

Gonzales, R., Wu, G., (1999). *On the Shape of the Probability Weighting Function.* Cognitive Psychology, 38, pp 129-166.

Kahneman, D., Tversky, A., (1979). *Prospect theory: An analysis of decisions under risk.* Econometrica, 47, pp 313-327.

Kahneman, D., Tversky, A., (1984). *Choices, Values, and Frames.* American Psychologist, Vol. 39, No. 4, pp 341-350.

Markowitz, H., (1952). *The Utility of Wealth.* Journal of Political Economy, Vol. 60, pp 151-158.

Thaler, R. H., (1985). *Mental accounting and consumer choice.* Marketing Science, 4, pp 199-214.

Thaler, R. H., (1999). *Mental Accounting Matters.* Journal of Behavioral Decision Making, Vol.12, pp 183-206.

Tversky, A., Kahneman, D., (1992). *Advances in Prospect Theory: Cumulative Representation of Uncertainty.* Journal of Risk and Uncertainty, Springer, vol. 5(4), pp 297-323.

Weber's law**.** (2009). In *Encyclopædia Britannica.* Retrieved August 11, 2009, from Encyclopædia Britannica Online: http://www.britannica.com/EBchecked/topic/638610/Webers-law