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Abstract

A special characteristic of the patent system is that it features multiple patent-policy levers that can be employed by policymakers. In this note, we develop an R&D-based growth model to analyze the optimal mix of patent instruments. Specifically, we consider (a) patent breadth and (b) the division of profit in research joint ventures. Our results can be summarized as follows. First, we analytically derive the optimal mix of patent breadth and the profit-division rule. Then, we calibrate the model to quantitatively evaluate the welfare gain from optimizing both patent instruments as compared to optimizing only one patent instrument. In summary, we find that the welfare gain can be quantitatively significant.

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1 Introduction

A special characteristic of the patent system is that it is a multi-dimensional policy system in the sense that it features multiple patent-policy levers, such as patent length and patent breadth, that can be employed by policymakers. Given this notable feature of the patent system, we develop an R&D-based growth model to analyze the optimal mix of patent-policy levers. In this note, we consider two patent instruments, namely, patent breadth and a profit-division rule in research joint ventures (RJVs). Our results can be summarized as follows. In the theoretical analysis, we analytically derive the optimal mix of patent breadth and the profit-division rule. Then, in the quantitative analysis, we calibrate the model to numerically evaluate the welfare gain from choosing both patent instruments optimally as compared to choosing only patent breadth optimally given any profit-division rule. We find that the welfare gain can be as large as 5% of consumption per year.

In this study, we analyze the optimal mix of patent breadth and the profit-division rule in RJVs for the following reasons. First, patent breadth is widely perceived to be an important patent-policy lever in both the microeconomic patent-design literature and the macroeconomic patent-and-growth literature.[1] Second, the division of profit in RJVs is an important focus of a recent patent-policy reform that we will discuss below.

Here we discuss first the importance of RJVs that have also been the focus of some patent-policy reforms in the past. The importance of RJVs was firstly emphasized by Penrose (1959), who argues that forming RJVs is a useful way for firms to gain access to external complementary technological resources for R&D. For example, in order to foster a more cooperative research environment in the US, policymakers enacted the National Cooperative Research Act of 1984 "to promote research and development, encourage innovation, stimulate trade, and make necessary and appropriate modifications in the operation of the antitrust laws."[2] Another example is the Third Amendment to the Chinese Patent Law that was approved in December 2008 and came into effect in October 2009.[3] An important purpose of this patent reform in China is to encourage the exploitation of jointly owned patents. For

[1] See Scotchmer (2004) and O'Donoghue and Zweimuller (2004) for a discussion.

[2] This act was subsequently expanded into the National Cooperative Research and Production Act of 1993.

[3] See, for example, Yang and Yen (2010) for a review of the patent-policy changes in this amendment.

example, "Article 15 provides that the exploitation of patent rights between co-owners should be determined by an agreement. Where an agreement is not available, any co-owner may exploit the patent alone or grant general licenses (i.e., non-exclusive licenses) to others to exploit the patent, and that any licensing fee received shall be shared between the co-owners."[4] In our theoretical analysis, we model this patent-policy lever using a profit-division rule in RJVs. In addition to encouraging the exploitation of jointly owned patents, this patent reform also involves other policy changes, such as increasing statutory damages and administrative fines, and heightening patentability requirement. In other words, this policy amendment involves the reform of multiple patent-policy levers instead of a single patent-policy lever. One purpose of the present study is to develop a quantitative dynamic general-equilibrium (DGE) framework to demonstrate the welfare difference between optimizing multiple patent instruments and optimizing a single patent instrument.

This study relates to the patent-design literature. In his seminal study, Nordhaus (1969) characterizes optimal patent length and shows that it balances between the social benefit of innovation and the social cost of monopolistic distortion. However, Nordhaus only considers patent length as the single patent-policy lever. Subsequent studies by Tandon (1982), Gilbert and Shapiro (1990), Klemperer (1990) and Denicolo (1996) analyze the optimal mix of patent instruments, such as patent length, patent breadth and compulsory licensing.[5] The present study complements these interesting partial-equilibrium analyses by revisiting the optimal mix of patent instruments in a quantitative DGE framework, which allows for an explicit consideration of economic growth and social welfare.

In the literature on patent policy and economic growth, the seminal DGE analysis on optimal patent length is Judd (1985), who shows that the optimal patent length can be infinite in a specific environment. In contrast, Futagami and Iwaisako (2007) show that the optimal patent length is usually finite in the Romer model. While these studies focus on patent length, other studies analyze the growth and welfare effects of other patent instruments in R&D-based growth models. See, for example, Cozzi (2001) on intellectual appropriability, Li (2001) on lagging patent breadth, O'Donoghue and Zweimuller (2004) on leading patent breadth and patentability requirement,

[4]See Yang and Yen (2010, p. 8).

[5]See Scotchmer (2004) for a comprehensive review of this patent-design literature.

Kwan and Lai (2003), Horii and Iwaisako (2007) and Furukawa (2007) on patent protection against imitation, and Chu (2009) on blocking patents. The present paper complements these studies by analyzing the optimal mix of multiple patent instruments, which is often neglected in this literature.[6] Finally, a recent study by Chu (2011) provides a quantitative analysis on the welfare cost of one-size-fits-all patent protection.

The rest of this note is organized as follows. Section 2 describes the model. Section 3 defines the equilibrium and characterizes the equilibrium allocation. Section 4 derives optimal patent policies and calibrates the model to provide a quantitative analysis. Section 5 considers a number of extensions. The final section concludes.

2 The model

To consider the optimal mix of patent-policy levers, we modify the Grossman-Helpman (1991) quality-ladder model by incorporating into the model (a) patent breadth that determines the markup and (b) competitive RJVs in which the division of profit is subject to a profit-division rule. In their seminal study, Kamien *et al.* (1992) define a competitive RJV as an interfirm arrangement in which each firm decides its own R&D investment taking the other firm's R&D investment as given and the firms share their innovation.[7] We adopt this setup to reformulate the R&D sector of the Grossman-Helpman model.[8] Given that the quality-ladder model has been well-studied, we briefly describe the familiar features to conserve space and discuss the new features in more details.

[6] Some notable exceptions are Iwaisako and Futagami (2003) and Palokangas (2011) on optimal patent length and breadth. However, their studies are qualitative in nature while the present study also provides a quantitative analysis.

[7] See also Greenlee (2005) for an interesting analysis on competitive RJVs.

[8] See also Cozzi (1999) and Cozzi and Tarola (2006) for an interesting analysis on cooperative RJVs in the R&D-based growth model.

2.1 Households

There is a unit continuum of identical households. Their lifetime utility is given by

$$U = \int_0^\infty e^{-\rho t} \ln C_t dt, \tag{1}$$

where $\rho > 0$ is discount rate, and C_t is the consumption of final goods at time t. Households maximize utility subject to asset accumulation given by

$$\dot{A}_t = R_t A_t + W_t - P_t C_t. \tag{2}$$

P_t denotes the price of final goods at time t. Each household supplies one unit of labor (chosen as the numeraire) to earn the wage W_t (normalized to unity). A_t is the value of assets owned by households, and R_t is the nominal rate of return. The familiar Euler equation is

$$\dot{E}_t / E_t = R_t - \rho, \tag{3}$$

where $E_t \equiv P_t C_t$ is the nominal expenditure on consumption.

2.2 Final goods

Final goods are produced by a standard Cobb-Douglas aggregator over a unit continuum of differentiated intermediate goods $X_t(i)$ indexed by $i \in [0, 1]$.

$$Y_t = \exp\left(\int_0^1 \ln X_t(i) di \right). \tag{4}$$

This sector is perfectly competitive, and final-goods firms take both the output and input prices as given. From standard cost minimization, the price index of final goods can be expressed as

$$P_t = \exp\left(\int_0^1 \ln P_t(i) di \right), \tag{5}$$

where $P_t(i)$ is the price of $X_t(i)$. The conditional demand curve for $X_t(i)$ is

$$X_t(i) = P_t Y_t / P_t(i). \tag{6}$$

2.3 Intermediate goods

There is a unit continuum of differentiated intermediate goods indexed by $i \in [0, 1]$. Each intermediate goods i is produced by a monopolistic leader, who holds a patent on the latest innovation. This industry leader dominates the market temporarily until the arrival of the next innovation (i.e., the Arrow replacement effect).[9] The production function for the leader of intermediate goods i is

$$X_t(i) = z^{q_t(i)} L_t(i). \tag{7}$$

The parameter $z > 1$ is the step size of a productivity improvement, and $q_t(i)$ is the number of productivity improvements that have occurred in industry i as of time t. $L_t(i)$ is the number of production workers employed in industry i. Given $z^{q_t(i)}$, the industry leader's marginal cost of production is

$$MC_t(i) = W_t / z^{q_t(i)}. \tag{8}$$

We follow the standard approach in the literature to consider Bertrand competition. Under Bertrand competition, the profit-maximizing price for the current leader is a markup over the marginal cost.

$$P_t(i) = \mu_t MC_t(i), \tag{9}$$

where $\mu_t = z^{b_t}$ and b_t is the level of patent breadth at time t.[10] Grossman and Helpman (1991) assume complete patent protection against imitation (i.e., $b_t = 1$). Li (2001) generalizes the patent regime to allow for incomplete patent protection (i.e., $b_t \in (0, 1]$). Because of incomplete protection, the current leader's innovation enables the former leader to increase her productivity by a factor of z^{1-b_t} without infringing the current leader's patent. Therefore, the limit-pricing markup for the current leader is z^{b_t}. O'Donoghue and Zweimuller (2004) refer to $b_t \in (0, 1]$ as *lagging* patent breadth (i.e., backward protection against imitation), and they also consider *leading* patent breadth $b_t \in \{2, 3, ...\}$, which captures forward protection against subsequent innovations. We follow the formulation in O'Donoghue and Zweimuller (2004) here.

[9] See Cozzi (2007) for a discussion on the Arrow effect in the quality-ladder model.

[10] Li (2001) generalizes (4) to a CES aggregator, in which case, the markup is given by $\min\{z^b, \eta/(1-\eta)\}$, where $\eta \in (1, \infty)$ is the elasticity of substitution between intermediate goods. In the case of a Cobb-Douglas aggregator (i.e., $\eta \to 1$), $z^b < \eta/(1-\eta)$ always holds. As for the case of a CES aggregator, so long as $z^b < \eta/(1-\eta)$, the effect of patent breadth on the markup is the same as in the case of a Cobb-Douglas aggregator.

In the presence of leading breadth, a profit-sharing arrangement between generations of patentholders is needed, and we consider the optimal *front-loading* profit-sharing arrangement that is to allow the most recent innovator to obtain all the profits.[11] Combining lagging breadth and leading breadth with the frontloading profit-sharing arrangement, $b_t \in (0, \infty)$ simply becomes a continuous variable. Let's consider an example of $b_t = 1.5$ for illustration. In this case, the integer 1 refers to the degree of leading breadth, and the decimal 0.5 refers to the degree of lagging breadth. A leading breadth of degree one implies that the most recent innovator infringes the patent of the second-most recent innovator, and they consolidate their market power giving rise to a markup of z^2 if lagging breadth were complete. However, an incomplete lagging breadth of 0.5 implies that the third-most recent innovator is able to imitate half of the innovation owned by the second-most recent innovator, and the resulting Bertrand competition between the third-most recent innovator and the coalition (formed by the most recent innovator and the second-most recent innovator) limits the markup to $z^{1.5}$.[12]

In summary, a larger patent breadth enables the current leader to charge a higher markup, and the resulting increase in profit improves incentives for R&D. For the rest of this study, we use μ_t to denote patent breadth for convenience and consider changes in μ_t coming from changes in b_t only. Finally, the amount of monopolistic profit is

$$\Pi_t(i) = \left(\frac{\mu_t - 1}{\mu_t}\right) P_t(i) X_t(i) = \left(\frac{\mu_t - 1}{\mu_t}\right) P_t Y_t \tag{10}$$

for $i \in [0, 1]$, and the second equality of (10) follows from (6).

[11] In the present study, it is appropriate to consider the frontloading profit-sharing arrangement because our focus is on optimal patent policies. The frontloading profit-sharing arrangement is optimal because it maximizes the incentives for R&D for a given level of patent breadth. See O'Donoghue and Zweimuller (2004) and Chu (2009) for a more detailed discussion.

[12] In a policy environment in which the consolidation of market power through collusion is restricted by antitrust policies, the markup μ would have an upper bound of z (i.e., any degree of patent breadth above 1 cannot yield a larger markup due to the violation of antitrust policies). In this case, patent breadth would not be an effective policy instrument in helping policymakers to achieve the socially optimal allocation if the optimal markup happens to be greater than z. Therefore, in Section 5, we consider R&D subsidies as an alternative policy instrument that can be coordinated with the profit-division rule to achieve the socially optimal allocation.

2.4 R&D joint ventures

Denote $V_t(i)$ as the value of the latest innovation in industry i. Because $\Pi_t(i) = \Pi_t$ for $i \in [0,1]$ from (10), $V_t(i) = V_t$ in a symmetric equilibrium that features an equal arrival rate of innovation across industries.[13] The familiar no-arbitrage condition for V_t is

$$R_t V_t = \Pi_t + \dot{V}_t - \lambda_t V_t, \tag{11}$$

which equates the interest rate to the asset return per unit of asset. The asset return is the sum of (a) the profit Π_t received by the patentholder, (b) the potential capital gain $\dot{V}_t$, and (c) the expected capital loss due to creative destruction $\lambda_t V_t$, where λ_t is the *industry-level* Poisson arrival rate of innovation.

Greenlee (2005) provides a survey of empirical evidence to show that firms in RJVs tend to behave competitively rather than cooperatively. Therefore, we consider competitive RJVs and assume that a successful innovation results from two types of entrepreneurial activities, which we label as type-1 R&D and type-2 R&D.[14] Type-1 R&D is performed by type-1 firms, and type-2 R&D is performed by type-2 firms. This formulation is consistent with the empirical evidence summarized in Greenlee (2005) that "firms perceive gaining access to complementary knowledge as the single most important objective in research consortia." In the economy, there is a unit continuum of each type of firms. For simplicity, we consider a Cobb-Douglas functional form for the arrival rate $\widetilde{\lambda}_t$ of innovation in each RJV.[15,16]

$$\widetilde{\lambda}_t = \varphi (H_{1,t})^{\alpha} (H_{2,t})^{1-\alpha}, \tag{12}$$

where $\alpha \in (0,1)$ is the relative factor share of the two types of R&D activities. $H_{1,t}$ and $H_{2,t}$ denote R&D labors employed by type-1 and type-2 firms respectively.

In the case of a successful innovation, the two firms sell the patent to a manufacturer, and they share the value of the patent according to a profit-division rule $s_t \in (0,1)$. A type-1 firm receives $s_t V_t$ while a type-2 firm

[13] We follow the standard approach in the literature to focus on the symmetric equilibrium. See Cozzi *et al.* (2007) for a theoretical justification for the symmetric equilibrium in the quality-ladder model.

[14] For example, type-1 R&D may be basic R&D, and type-2 R&D may be applied R&D.

[15] In Section 5, we consider optimal patent policies under a more general CES form.

[16] In equilibrium, $\widetilde{\lambda}_t = \lambda_t$.

receives $(1-s_t)V_t$. This division of profit can be viewed as a bargaining outcome, and the bargaining power of each side can be influenced by patent policy. Therefore, it is reasonable to treat s_t as a policy variable. For example, if we treat $H_{1,t}$ and $H_{2,t}$ as basic R&D and applied R&D respectively, then a change in s_t captures a change in the relative bargaining power between basic and applied researchers, which has been witnessed in the US.[17]

The expected return to R&D for a type-1 firm is

$$\pi_{1,t} = s_t V_t \widetilde{\lambda}_t - W_t H_{1,t}, \tag{13}$$

and the expected return to R&D for a type-2 firm is

$$\pi_{2,t} = (1-s_t) V_t \widetilde{\lambda}_t - W_t H_{2,t}. \tag{14}$$

In equilibrium, the first-order conditions for $H_{1,t}$ and $H_{2,t}$ become

$$\alpha s_t V_t \lambda_t = W_t H_{1,t}, \tag{15}$$

$$(1-\alpha)(1-s_t) V_t \lambda_t = W_t H_{2,t}. \tag{16}$$

Equations (15) and (16) imply that the R&D sector generates positive profits, which are transferred back to the households.[18] Combining (15) and (16) yields the equilibrium ratio of $H_{1,t}$ to $H_{2,t}$ given by

$$\frac{H_{1,t}}{H_{2,t}} = \frac{s_t}{1-s_t}\left(\frac{\alpha}{1-\alpha}\right). \tag{17}$$

3 Decentralized equilibrium

The equilibrium is a time path $\{C_t, Y_t, X_t(i), L_t, H_{1,t}, H_{2,t}, W_t, R_t, V_t, P_t, P_t(i)\}$, $t \geq 0$. Also, at each instant of time,

- households maximize utility taking $\{R_t, P_t, W_t\}$ as given;
- competitive final-goods firms produce $\{Y_t\}$ to maximize profit taking $\{P_t, P_t(i)\}$ as given;

[17] See Cozzi and Galli (2009) for a discussion on the strengthening of intellectual property protection for basic research relative to the protection for applied research.

[18] Therefore, households' assets include patents and the ownership of R&D enterprises.

- monopolistic intermediate-goods firms produce $\{X_t(i)\}$ and choose $\{P_t(i)\}$ to maximize profit taking $\{W_t\}$ as given;
- R&D firms choose $\{H_{1,t}, H_{2,t}\}$ to maximize expected profit taking $\{W_t, V_t\}$ as given;
- the labor market clears such that $L_t + H_{1,t} + H_{2,t} = 1$; and
- the goods market clears such that $Y_t = C_t$.

3.1 Equilibrium allocation

Proposition 1 shows that given a stationary path of patent breadth μ and profit-division rule s, the economy is on a stable and unique balanced-growth path, along which the equilibrium allocation of labor inputs is stationary.

Proposition 1 *Given constant μ and s, the economy always jumps to a unique and stable balanced-growth path.*

Proof. See Appendix A. ■

Imposing balanced growth on (11) yields $V_t = \Pi_t/(\rho + \lambda)$. Equation (10) implies that the wage income of production labor is $W_t L_t = P_t Y_t/\mu$. Substituting these two conditions along with (10) and (12) into (15) yields

$$\rho + \varphi(H_1)^\alpha (H_2)^{1-\alpha} = \alpha s(\mu - 1)\varphi \left(\frac{H_2}{H_1}\right)^{1-\alpha} L. \tag{18}$$

Combining (17), (18) and the labor-market-clearing condition yields the equilibrium allocation of labor inputs given by

$$L = \frac{\rho/[\varphi s^\alpha (1-s)^{1-\alpha} \alpha^\alpha (1-\alpha)^{1-\alpha}] + 1/[s\alpha + (1-s)(1-\alpha)]}{(\mu - 1) + 1/[s\alpha + (1-s)(1-\alpha)]}, \tag{19}$$

$$H_1 = \left(\frac{s\alpha}{s\alpha + (1-s)(1-\alpha)}\right)(1 - L), \tag{20}$$

$$H_2 = \left(\frac{(1-s)(1-\alpha)}{s\alpha + (1-s)(1-\alpha)} \right) (1-L). \tag{21}$$

To ensure the non-negativity of R&D labors, we impose a lower bound on R&D productivity φ, which we label as *Condition R*.

$$\varphi > \frac{\rho/(\mu - 1)}{s^{\alpha}(1-s)^{1-\alpha}\alpha^{\alpha}(1-\alpha)^{1-\alpha}}. \tag{R}$$

It is useful to note that *Condition R* implies $L < 1$.

Substituting (7) into (4) yields $Y_t = Z_t L_t$, where the aggregate level of technology is defined as

$$Z_t \equiv \exp\left(\int_0^1 q_t(i) di \ln z \right) = \exp\left(\int_0^t \lambda_\tau d\tau \ln z \right), \tag{22}$$

where the second equality can be obtained by appealing to the law of large numbers. Finally, differentiating the log of (22) with respect to t yields the growth rate of technology given by $g_t \equiv \dot{Z}_t / Z_t = \lambda_t \ln z$.

4 Optimal mix of patent instruments

Before we derive the optimal mix of patent breadth and the profit-division rule, we firstly derive the first-best allocation of labor inputs. Given the balanced-growth behavior of the economy, the lifetime utility of households in (1) can be re-expressed as

$$U = \frac{1}{\rho} \left(\ln C_0 + \frac{g}{\rho} \right), \tag{23}$$

where $g = \lambda \ln z$ and $C_0 = Z_0 L$. Maximizing (23) subject to $L + H_1 + H_2 = 1$ yields the first-best allocations $\{L^*, H_1^*, H_2^*\}$.

$$L^* = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \left(\frac{\rho}{\varphi \ln z} \right), \tag{24}$$

$$H_1^* = \alpha(1 - L^*), \tag{25}$$

$$H_2^* = (1-\alpha)(1 - L^*). \tag{26}$$

To achieve this first-best allocation of labor inputs in the decentralized economy, one can consider the use of patent levers as policy instruments. In our model, patent breadth serves the purpose of optimizing the relative allocation of production labor L and R&D labors $\{H_1, H_2\}$. It is useful to note that a larger patent breadth increases H_1 and H_2 but decreases L. As for the profit-division rule, it serves the purpose of optimizing the relative allocation of R&D labors. An increase in s changes the incentives of type-1 and type-2 R&D firms giving rise to an increase in H_1 relative to H_2. Therefore, we need both patent instruments to achieve the first-best allocation.

When both patent instruments are chosen optimally, the optimal profit-division rule is

$$s^* = 0.5 \tag{27}$$

regardless of the value for α. Equations (25) and (26) show that $H_1^*/H_2^* = \alpha/(1-\alpha)$, which can be satisfied in (17) if and only if $s = 0.5$. Intuitively, the optimal H_1^*/H_2^* is solely determined by the relative input share $\alpha/(1-\alpha)$ whereas the market-equilibrium allocation is determined by the relative division of profit $s/(1-s)$ in addition to the relative input share. Therefore, to achieve the optimal allocation, the effect of $s/(1-s)$ in the market equilibrium should be eliminated by setting $s/(1-s) = 1$. As for optimal patent breadth, equating (19) and (24) and setting $s^* = 0.5$ yields

$$\mu^* = 2\left(\alpha^\alpha (1-\alpha)^{1-\alpha} \frac{\varphi}{\rho} + 1\right) \ln z - 1. \tag{28}$$

Proposition 2 *When both patent instruments are chosen optimally, the optimal profit-division rule is* $s^* = 0.5$ *and optimal patent breadth is given by* μ^* *in (28). Also,* μ^* *increases in* φ *and* z *but decreases in* ρ.

Proof. See (27) and (28). At s^* and μ^*, the equilibrium labor allocations in (19) to (21) coincide with the first-best labor allocations in (24) to (26). ■

Intuitively, an increase in φ or z strengthens the positive effect of R&D on economic growth, so that μ^* is increasing in φ and z. In contrast, an increase in ρ reduces the benefit of a higher growth rate on social welfare, so that μ^* is decreasing in ρ. Substituting s^* and μ^* into $g = (\varphi \ln z)(H_1)^\alpha (H_2)^{1-\alpha}$ yields the first-best growth rate given by

$$g^* = (\varphi \ln z)\alpha^\alpha (1-\alpha)^{1-\alpha} - \rho. \tag{29}$$

4.1 Optimal patent breadth

In this section, we derive optimal patent breadth for any given s. Using (20) and (21), we can rewrite (23) as

$$U = \frac{1}{\rho}\left[\ln L + \frac{\varphi \ln z}{\rho}\left(\frac{s^\alpha (1-s)^{1-\alpha}\alpha^\alpha (1-\alpha)^{1-\alpha}}{s\alpha + (1-s)(1-\alpha)}\right)(1-L)\right], \tag{30}$$

where Z_0 is normalized to unity, and L is given by (19). Differentiating (30) with respect to μ yields

$$\frac{\partial U}{\partial \mu} = \frac{1}{\rho}\left[\frac{1}{L}\frac{\partial L}{\partial \mu} - \frac{\varphi \ln z}{\rho}\left(\frac{s^\alpha (1-s)^{1-\alpha}\alpha^\alpha (1-\alpha)^{1-\alpha}}{s\alpha + (1-s)(1-\alpha)}\right)\frac{\partial L}{\partial \mu}\right], \tag{31}$$

where

$$\frac{\partial L}{\partial \mu} = -\frac{L}{(\mu - 1) + 1/[s\alpha + (1-s)(1-\alpha)]}. \tag{32}$$

Substituting (32) into (31) and setting $\partial U/\partial \mu = 0$ yield μ^{**}, which denotes the optimal patent breadth for any given s.

$$L^{**} \equiv L|_{\mu=\mu^{**}} = \frac{s\alpha + (1-s)(1-\alpha)}{s^\alpha (1-s)^{1-\alpha}\alpha^\alpha (1-\alpha)^{1-\alpha}}\left(\frac{\rho}{\varphi \ln z}\right). \tag{33}$$

Substituting (20), (21) and (33) into $g = (\varphi \ln z)(H_1)^\alpha (H_2)^{1-\alpha}$ yields the second-best growth rate given by

$$g^{**} \equiv g|_{\mu=\mu^{**}} = (\varphi \ln z)\alpha^\alpha (1-\alpha)^{1-\alpha}\Phi - \rho, \tag{34}$$

where $\Phi \equiv s^\alpha (1-s)^{1-\alpha}/[s\alpha + (1-s)(1-\alpha)] \leq 1$ is a composite parameter. Comparing (29) and (34) shows that $g^{**} \leq g^*$. Intuitively, without the optimal profit-division rule, the economy allocates too much labor to production (i.e., $L^{**} \geq L^*$) and fails to achieve the optimal allocation of R&D labors. As a result of the suboptimal allocation of R&D labors, the economy exhibits a lower growth rate than under the first-best allocation.

Proposition 3 *Suppose that only the patent breadth is chosen optimally. Then, the equilibrium growth rate would be lower than the case in which both patent instruments are chosen optimally.*

Proof. Comparing (29) and (34) shows that $g^{**} \leq g^*$ because $\Phi \leq 1$, which becomes a strict inequality unless $s = s^* = 0.5$ or $\alpha \to \{0, 1\}$. ■

As for social welfare, we can derive the welfare difference $\Delta U \equiv U^* - U^{**}$, where $U^{**} \equiv U|_{\mu=\mu^{**}}$.

$$\Delta U = \frac{1}{\rho}\left(\ln \Phi + \frac{(\varphi \ln z)\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\rho}(1-\Phi)\right) \geq 0, \tag{35}$$

where $\Phi = s^{\alpha}(1-s)^{1-\alpha}/[s\alpha + (1-s)(1-\alpha)] \leq 1$ as defined before; therefore, $\ln \Phi \leq 0$ and $1 - \Phi \geq 0$. It can be shown that Φ is an inverted U-shape function in s and reaches its maximum of one at $s = 0.5$. As for the relationship between ΔU and Φ, differentiating (35) with respect to Φ shows that

$$\frac{\partial \Delta U}{\partial \Phi} \leq 0 \Leftrightarrow \Phi \geq \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\left(\frac{\rho}{\varphi \ln z}\right), \tag{36}$$

where the second inequality can be re-expressed as

$$1 \geq \frac{s\alpha + (1-s)(1-\alpha)}{s^{\alpha}(1-s)^{1-\alpha}\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\left(\frac{\rho}{\varphi \ln z}\right) = L^{**}, \tag{37}$$

which is guaranteed to hold by *Condition R*. Therefore, ΔU is an U-shape function in s and reaches a minimum of zero at $s = s^* = 0.5$.

4.2 Quantitative analysis

In this section, we calibrate the model to demonstrate quantitatively the welfare gain from choosing both patent instruments optimally as compared to choosing only patent breadth optimally. Substituting (29) into (35) yields

$$\Delta U = \frac{1}{\rho}\left[\ln \Phi + \left(1 + \frac{g^*}{\rho}\right)(1-\Phi)\right], \tag{38}$$

where $\Phi(s, \alpha) = s^{\alpha}(1-s)^{1-\alpha}/[s\alpha + (1-s)(1-\alpha)]$. Therefore, we need to assign value to three parameters $\{\rho, s, \alpha\}$ and the optimal growth rate g^* in order to evaluate ΔU. For the discount rate ρ, we consider a conventional value of 0.04. For the profit-division rule s, we consider a range of values $s \in [0.20, 0.80]$ around the optimal rule $s^* = 0.5$.[19] For the relative factor share α of R&D activities, we report our results for $\alpha \in [0.05, 0.95]$, but we consider $\alpha = 0.5$ as our benchmark because empirical evidence suggests that RJVs tend to be formed among symmetric firms of similar sizes; see for example Roller *et al.* (2007). Finally, we choose the empirical long-run growth rate of 1.5% as a conservative lower bound for the optimal growth rate g^*. Given the empirical finding of R&D underinvestment, see for example Jones and Williams (1998, 2000), the socially optimal level of R&D investment is higher than the market equilibrium level implying that the optimal growth rate should be higher than the equilibrium growth rate observed in the data.

In summary, we consider $\{\rho, \alpha, g^*\} = \{0.04, 0.5, 0.015\}$ as our benchmark parameter values and quantify the growth and welfare effects as s deviates from $s^* = 0.5$. For sensitivity analysis, we will also discuss how the numerical results change when g^* and ρ vary. For easier interpretation, we express the welfare difference in terms of equivalent variation in consumption flow denoted by $\delta \equiv \exp(\rho\Delta U) - 1$. More formally, δ is defined as $U(C_0^*, g^*) = U(C_0^{**}(1+\delta), g^{**})$.

Table 1: Effects of s on economic growth g^{**}

α/s	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0.05	1.33%	1.43%	1.48%	1.50%	1.48%	1.38%	1.13%
0.20	0.90%	1.24%	1.43%	1.50%	1.42%	1.14%	0.54%
0.35	0.59%	1.11%	1.40%	1.50%	1.39%	1.04%	0.36%
0.50	0.40%	1.04%	1.39%	1.50%	1.39%	1.04%	0.40%
0.65	0.36%	1.04%	1.39%	1.50%	1.40%	1.11%	0.59%
0.80	0.54%	1.14%	1.42%	1.50%	1.43%	1.24%	0.90%
0.95	1.13%	1.38%	1.48%	1.50%	1.48%	1.43%	1.33%

Table 1 shows that the second-best growth rate g^{**} is lower than the first-best growth rate g^* unless the profit-division rule is set to its optimal value $s^* = 0.5$. As s deviates from s^* in either direction, g^{**} decreases and the magnitude of this reduction in growth is the most dramatic at $\alpha = 0.5$ (i.e.,

[19]For some values of s outside the range $s \in [0.20, 0.80]$, $g^{**} = (g^* + \rho)\Phi - \rho$ from (34) and (29) becomes negative, which in turn imply that Condition R is violated.

a symmetric factor share of R&D activities). As α decreases or increases from 0.5, the reduction in growth in response to a change in s becomes less dramatic. In fact, as α approaches zero or one, the reduction in growth approaches zero because the model becomes a conventional quality-ladder model with one type of R&D activity, in which optimal patent breadth alone is sufficient for achieving the first-best optimal growth rate.

Table 2: Effects of s on social welfare δ

α/s	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0.05	1.1%	0.5%	0.1%	0.0%	0.2%	0.8%	2.3%
0.20	3.5%	1.7%	0.4%	0.0%	0.5%	2.2%	4.9%
0.35	4.8%	2.4%	0.7%	0.0%	0.7%	2.8%	5.4%
0.50	5.3%	2.8%	0.7%	0.0%	0.7%	2.8%	5.3%
0.65	5.4%	2.8%	0.7%	0.0%	0.7%	2.4%	4.8%
0.80	4.9%	2.2%	0.5%	0.0%	0.4%	1.7%	3.5%
0.95	2.3%	0.8%	0.2%	0.0%	0.1%	0.5%	1.1%

Table 2 shows that going from optimizing only patent breadth to optimizing both patent instruments can lead to a welfare gain of as large as 5% of consumption per year. The second-best level of social welfare U^{**} is lower than the first-best level U^* unless the profit-division rule is set to its optimal value $s^* = 0.5$. As s deviates from s^* in either direction, the welfare difference δ increases. Although the rise in δ is the most dramatic at our benchmark $\alpha = 0.5$, the welfare differences are also significant at other values of α except for α close to zero or one (in which case, the model behaves like a conventional quality-ladder model with one type of R&D). The welfare differences reported in Table 2 corresponds to $\{g^*, \rho\} = \{0.015, 0.04\}$, which is a conservative set of parameter values. If the optimal growth rate g^* is higher than the empirical growth rate (which is likely to be case due to R&D underinvestment), the welfare differences become even more significant. For example, if g^* increases to 2%, the mean of the welfare differences reported in Table 2 would increase from 1.8% to 2.6% with the upper bound increasing from 5.4% to 8.2%. Similarly, (38) implies that the welfare difference is decreasing in the discount rate ρ. Suppose we consider a lower discount rate of 3% (which is still within the conventional range) and the benchmark optimal growth rate g^* of 1.5%. In this case, the mean of the welfare differences reported in Table 2 would also increase from 1.8% to 2.6% with the upper bound increasing from 5.4% to 8.2% because g^*/ρ increases from 0.375 to 0.5 in both cases. In summary, our numerical results imply a quantitatively significant welfare gain from choosing both patent instruments optimally as compared to choosing only patent breadth optimally given a suboptimal profit-division rule.

5 Extensions

In this section, we discuss the implications of the following extensions (a) a CES innovation function, (b) R&D subsidies, and (c) the removal of scale effects. In what follows, we consider these extensions one at a time. In summary, our main result that the socially optimal allocation can be achieved by appropriately coordinating the policy instruments continues to hold under these extensions.

5.1 CES innovation function

Suppose we consider a CES generalization of (12) given by

$$\widetilde{\lambda}_t = \varphi \left(\alpha H_{1,t}^{(\varepsilon-1)/\varepsilon} + (1-\alpha) H_{2,t}^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}, \tag{39}$$

where $\varepsilon \in (0, \infty)$ is the elasticity of substitution between the two types of R&D activities. Our Cobb-Douglas specification corresponds to the special case of $\varepsilon \to 1$. For this more general CES specification, one can also analytically derive optimal patent policies.[20] We find that the optimal profit-division rule continues to be $s^* = 0.5$ whereas optimal patent breadth becomes

$$\mu^* = 2 \left([\alpha^\varepsilon + (1-\alpha)^\varepsilon]^{1/(\varepsilon-1)} \frac{\varphi}{\rho} + 1 \right) \ln z - 1. \tag{40}$$

In other words, the finding of a symmetric optimal profit-division rule is robust to generalizing the innovation function to a CES form.

[20] The derivations are relegated to an unpublished appendix available upon request from the authors.

5.2 R&D subsidies

In this section, we consider the case in which patent breadth is exogenously given and derive the optimal coordination between fiscal policy (in the form of an R&D subsidy) and patent policy (in the form of a profit-division rule). Given the R&D subsidy rate $\sigma < 1$, (13) and (14) become

$$\pi_{1,t} = s_t V_t \widetilde{\lambda}_t - (1-\sigma) W_t H_{1,t}, \tag{41}$$

$$\pi_{2,t} = (1-s_t) V_t \widetilde{\lambda}_t - (1-\sigma) W_t H_{2,t}. \tag{42}$$

In the case of financing the R&D subsidy with non-distortionary taxes, such as a lump-sum tax and a labor-income tax,[21] the equilibrium allocation of production labor becomes[22]

$$L = \frac{\rho/[\varphi s^{\alpha}(1-s)^{1-\alpha}\alpha^{\alpha}(1-\alpha)^{1-\alpha}] + 1/[s\alpha + (1-s)(1-\alpha)]}{(\mu-1)/(1-\sigma) + 1/[s\alpha + (1-s)(1-\alpha)]}, \tag{43}$$

whereas the equilibrium allocation of R&D labors are (20) and (21) as before.

Comparing these conditions with the optimal allocations in (24) - (26), the optimal profit-division rule s^* continues to be 0.5, and the optimal R&D subsidy is characterized by

$$\frac{\mu-1}{1-\sigma^*} = 2\left(\alpha^{\alpha}(1-\alpha)^{1-\alpha}\frac{\varphi}{\rho} + 1\right)\ln z - 2. \tag{44}$$

As in the case of optimal patent breadth, the optimal R&D subsidy σ^* is increasing in φ and z and decreasing in ρ. Furthermore, σ^* is decreasing in patent breadth μ. In fact, there is a one-to-one mapping between optimal R&D subsidy and optimal patent breadth implying that these two policy instruments are perfectly substitutable in enabling policymakers to achieve the socially optimal allocation. However, this equivalence result relies on the presence of non-distortionary taxes.

[21] In our model, a labor-income tax is non-distortionary due to inelastic labor supply.

[22] The derivations are relegated to an unpublished appendix available upon request from the authors.

5.3 Scale effects

In this section, we discuss the implications of scale effects. Our model belongs to the class of first-generation R&D-based growth models, such as Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), in which scale effects are present.[23] Scale effects refer to the properties that (a) a larger population implies faster technological progress and (b) a growing population implies a rising growth rate rather than a constant growth rate on the balanced growth path. Whether these scale effects are counterfactual is an empirical question. Jones (1995a) shows that scale effects are inconsistent with time-series evidence based on modern data in a number of OECD countries. However, Kremer (1993) considers pre-historic data and shows that scale effects of endogenous growth models are not necessarily inconsistent with empirical evidence when one considers economic growth in the very long run.

Given that our focus is on the effects of intellectual property rights on innovation in the modern era characterized by a balanced growth path, it would be useful for us to explore our results in a scale-invariant version of the model. In our model, we have normalized the supply of labor to unity, so that population size does not appear in the equilibrium growth rate and it is the share of labor in R&D that determines equilibrium growth. This property is consistent with the second-generation R&D-based endogenous growth models.[24] Here we consider a semi-endogenous growth version of the quality-ladder model with population growth as in Segerstrom (1998).[25]

With population growth, the lifetime utility of households becomes

$$U = \int_0^\infty e^{-(\rho-n)t} \ln C_t dt, \tag{45}$$

where $n \in (0, \rho)$ is the exogenous rate of population growth. Population size evolves according to $\dot{N}_t = nN_t$, and the labor-market-clearing condition becomes $L_t + H_{1,t} + H_{2,t} = N_t$. To achieve semi-endogenous growth, we

[23] See Jones (1999) for a discussion on scale effects in R&D-based growth models.

[24] See for example Young (1998) and Peretto (1998) for early contributions in the scale-invariant R&D-based endogenous growth model that combines quality improvement and variety expansion.

[25] See Jones (1995b) for an early contribution in the variety-expanding semi-endogenous growth model.

follow Segerstrom (1998) to consider increasing complexity of innovation. Specifically, we assume that R&D productivity is decreasing in aggregate technology Z_t such that (12) becomes

$$\widetilde{\lambda}_t = \overline{\varphi}_t (H_{1,t})^{\alpha} (H_{2,t})^{1-\alpha} = \frac{\varphi}{Z_t} (H_{1,t})^{\alpha} (H_{2,t})^{1-\alpha}, \tag{46}$$

where $\overline{\varphi}_t = \varphi / Z_t$ decreases as Z_t increases. With these modifications (while keeping the rest of the model as before), the steady-state growth rate of technology becomes exogenously determined by the population growth rate.

$$g = \lambda \ln z = (\varphi \ln z) \left(\frac{H_{1,t}}{Z_t} \right)^{\alpha} \left(\frac{H_{2,t}}{Z_t} \right)^{1-\alpha} = n. \tag{47}$$

In this case, the steady-state equilibrium allocations are[26]

$$L_t = \left[1 + [s\alpha + (1-s)(1-\alpha)] \left(\frac{(\mu - 1)\lambda}{\rho - n + \lambda} \right) \right]^{-1} N_t, \tag{48}$$

$$H_{1,t} = s\alpha \left(\frac{(\mu - 1)\lambda}{\rho - n + \lambda} \right) L_t, \tag{49}$$

$$H_{2,t} = (1-s)(1-\alpha) \left(\frac{(\mu - 1)\lambda}{\rho - n + \lambda} \right) L_t. \tag{50}$$

where the steady-state arrival rate of innovation $\lambda = n / \ln z$ is determined by exogenous parameters $\{n, z\}$.

To derive the optimal allocation, the social planner chooses a time path of $\{L_t, H_{1,t}, H_{2,t}\}$ to maximize (45) subject to (a) $\dot{Z}_t = (\varphi \ln z)(H_{1,t})^{\alpha}(H_{2,t})^{1-\alpha}$, (b) $C_t = Z_t L_t / N_t$, and (c) $L_t + H_{1,t} + H_{2,t} = N_t$. From standard dynamic optimization, the optimal allocations on the balanced growth path are[27]

$$L_t^* = \left(1 + \frac{n}{\rho} \right)^{-1} N_t, \tag{51}$$

$$H_{1,t}^* = \frac{\alpha n}{\rho} L_t^*, \tag{52}$$

[26] The derivations are relegated to an unpublished appendix available upon request from the authors.

[27] The derivations are relegated to an unpublished appendix available upon request from the authors.

$$H_{2,t}^{*}=\frac{(1-\alpha)n}{\rho}L_{t}^{*}. \tag{53}$$

Comparing (48) - (50) with (51) - (53), we find that the optimal profit-division rule s^* continues to be 0.5 whereas optimal patent breadth is

$$\mu^{*}=2\left(\frac{(\rho-n)\ln z+n}{\rho}\right)+1. \tag{54}$$

In this case, optimal patent breadth μ^* is increasing in z as before. Furthermore, if $\ln z < 1$, then μ^* would be increasing in n and decreasing in ρ; otherwise, μ^* would be decreasing in n/ρ. To understand the intuition behind the comparative statics of μ^* with respect to n/ρ, we combine (48) and (51) to express

$$\frac{L_t}{L_t^{*}}=\frac{1}{s\alpha+(1-s)(1-\alpha)}\left(\frac{(1-n/\rho)\ln z+n/\rho}{\mu-1}\right). \tag{55}$$

An increase in n/ρ has both positive and negative effects on L_t/L_t^*. When the positive effect dominates (which occurs if $\ln z$ is smaller than one), a larger n/ρ makes it more likely for the market economy to allocate too much labor to production (i.e., $L_t > L_t^*$) or equivalently, too little labor to R&D. In this case, optimal patent breadth must increase in order to offset this effect.

6 Conclusion

In this note, we have developed a simple R&D-based growth model to analyze the optimal mix of patent instruments. Even in our simple model, we find that optimizing a single patent instrument is insufficient for the economy to achieve the socially optimal allocation of factor inputs. Therefore, in the more complicated real world, it is unlikely that optimizing a single patent instrument would be sufficient for achieving the social optimum. This finding suggests that future studies on optimal patent protection may want to further explore the multiple dimensionality of the patent system in order for their analysis to be more suitable for policy applications.

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Appendix A

Proof of Proposition 1. Substituting (17) into (15) yields

$$\Gamma V_t = W_t, \tag{A1}$$

where $\Gamma \equiv \varphi s^{\alpha}(1-s)^{1-\alpha}\alpha^{\alpha}(1-\alpha)^{1-\alpha}$ is a composite parameter. Normalizing W_t to unity implies that $\dot{V}_t = 0$ for all t. Consequently, (11) becomes

$$R_t = \frac{\Pi_t}{V_t} - \lambda_t. \tag{A2}$$

Combining (10) and (A1) yields

$$\frac{\Pi_t}{V_t} = \Gamma\left(\frac{\mu-1}{\mu}\right)E_t. \tag{A3}$$

Substituting (17) into (12) yields

$$\lambda_t = \varphi\left(\frac{H_{2,t}}{H_{1,t}}\right)^{1-\alpha} H_{1,t} = \varphi\left(\frac{(1-s)(1-\alpha)}{s\alpha}\right)^{1-\alpha} H_{1,t}. \tag{A4}$$

Substituting (17) into the labor-market-clearing condition yields

$$1 = L_t + H_{1,t} + H_{2,t} = L_t + \left(\frac{s\alpha + (1-s)(1-\alpha)}{s\alpha}\right) H_{1,t}. \tag{A5}$$

Using the production-labor share of output (i.e., $W_t L_t = E_t/\mu$), we have

$$L_t = E_t/\mu. \tag{A6}$$

Substituting (A2) - (A6) into (3) yields

$$\frac{\dot{E}_t}{E_t} = \frac{\Gamma}{\mu}\left(\mu - 1 + \frac{1}{s\alpha + (1-s)(1-\alpha)}\right)E_t - \frac{\Gamma}{\alpha s + (1-\alpha)(1-s)} - \rho. \tag{A7}$$

Equation (A7) implies that the dynamics of E_t is characterized by saddle-point stability such that E_t always jumps to its interior steady-state value given by

$$E = \mu\left(\frac{\rho/\Gamma + 1/[\alpha s + (1-\alpha)(1-s)]}{(\mu-1) + 1/[s\alpha + (1-s)(1-\alpha)]}\right). \tag{A8}$$

Otherwise, $E_t = \mu L_t$ approaching zero violates the utility maximization of households while $E_t = \mu L_t$ approaching μ violates the profit maximization of R&D firms. Equation (A6) implies that the stationarity of E_t ensures the stationarity of L_t, which in turn ensures the stationarity of $H_{1,t}$ and $H_{2,t}$. ■