A tale of two growth engines: The interactive effects of monetary policy and intellectual property rights

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The Interactive Effects of Monetary Policy and Intellectual Property Rights

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Abstract

How do intellectual property rights that determine the market power of firms influence the effects of monetary policy on economic growth and social welfare? To analyze this question, we develop a monetary R&D-based growth model with elastic labor supply. We find that monetary expansion reduces growth and welfare through a decrease in labor supply that reduces R&D. Furthermore, a larger market power of firms strengthens these effects of monetary policy in the R&D model. In contrast, increasing the market power of firms dampens the growth and welfare effects of monetary policy in the AK model. Therefore, the market power of firms has drastically different implications on the welfare cost of inflation under the two growth engines (i.e., innovation versus capital accumulation). We also calibrate the two models using data in the US and the Euro Area to quantitatively evaluate and compare the welfare cost of inflation in the two economies. Finally, we simulate transition dynamics of the R&D model in order to compute the complete welfare changes from reducing inflation.

Keywords**:** economic growth, inflation, monetary policy, patent policy, R&D

JEL classification**:** O30, O40, E41

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1. Introduction

Since the seminal study by Tobin (1965), the relationship between inflation and economic growth has been a fundamental issue in monetary economics, and there is now an established literature on monetary policy and economic growth.[1] This study relates to this literature by analyzing an unexplored implication that the strength of patent protection has an influence on the effects of monetary policy on economic growth and social welfare. To analyze how intellectual property rights that determine the market power of firms influence the growth and welfare effects of monetary policy, we develop a monetary R&D-based growth model with elastic labor supply. We find that monetary expansion raises the cost of consumption relative to leisure consequently reducing labor supply, which in turn reduces growth and welfare by decreasing a factor input for R&D. Interestingly, the magnitude of the growth and welfare effects of monetary policy depends on the strength of patent protection. Specifically, a larger market power of firms strengthens the growth and welfare effects of monetary policy. However, the implication is drastically different in the AK growth model. In an AK model with monopolistic competition,[2] a larger market power of firms has the opposite implication of dampening the growth and welfare effects of monetary policy. In other words, the strength of patent protection has an important effect on the welfare cost of inflation. Whether it amplifies or mitigates the welfare cost of inflation depends on the underlying growth engine (i.e., innovation versus capital accumulation).

The above theoretical finding has an important implication on a recent policy reform. As a result of the World Trade Organization's Agreement on Trade-Related Aspects of Intellectual

[1] See for example Gillman and Kejak (2005) for a recent survey on this literature.

[2] In an AK model with perfect competition, firms do not have market power; therefore, markup has no implication on the effects of monetary policy. This finding is also different from the R&D-based growth model.

Property Rights (TRIPS),[3] many countries have strengthened their protection for intellectual property rights. For example, according to the Ginarte-Park index of patent rights in Park (2008), 107 countries have experienced an increase in the strength of patent rights from 1995 to 2005.[4] In these 107 countries, the average increase in the Ginarte-Park index is 0.82.[5] Our theoretical result implies that the welfare cost of inflation would have increased in some of these countries. Given that innovation is likely to be the main engine of economic growth in developed countries, these countries would experience a larger welfare cost of inflation as a result of stronger patent protection. In contrast, for a developing country in which the main engine of growth is capital accumulation, our result implies that it should experience a smaller welfare cost of inflation as a result of stronger patent protection.

The reason why the strength of patent protection has different implications on the growth and welfare effects of monetary policy under the two growth engines is as follows. For a given supply of labor, increasing the market power of firms raises the incentives for innovation and the share of labor devoted to R&D. This increase in the R&D share of labor tends to *magnify* the growth and welfare effects of the decrease in labor supply driven by monetary expansion. In contrast, in an AK model with monopolistic competition, a larger markup raises the profit share of income and decreases the capital share. This reduction in the capital share of income in turn decreases the incentives for capital accumulation. Because economic growth is determined by the supply of labor and the capital-investment rate in the AK model, a larger markup that decreases capital investment tends to *mitigate* the growth and welfare effects of the reduction in

[3] The WTO's TRIPS Agreement, which was initiated in the 1986-94 Uruguay Round, establishes a minimum level of intellectual property protection that must be provided by all member countries by 2006.

[4] There are a total of 122 countries in the Ginarte-Park index. Of these 122 countries, 119 countries have available measure of patent rights from 1995 to 2005, and only one country, Iraq, has experienced a reduction in the strength of patent rights during this period.

[5] The index is a scale of 0 to 5, and a larger number indicates stronger patent rights. See Park (2008) for details.

labor supply driven by monetary expansion. Therefore, the market power of firms has drastically different implications on the growth and welfare effects of monetary policy when the growth engine is innovation as opposed to capital accumulation. In other words, the effects of monetary policy are influenced by an interaction between the growth engine and the market power of firms. To our knowledge, this interaction has never been explored in the literature.

In the quantitative analysis, we calibrate the models using data in the United States (US) and the Euro Area (EA) to quantitatively evaluate and compare the welfare cost of inflation in the two economies. We consider currency and M1 as alternative measures of money. In both economies, we find that the welfare cost of inflation is much higher under the M1 specification than under the currency specification as in Dotsey and Ireland (1996). We also find a significant difference in the welfare cost of inflation between the EA and the US when we use M1 as the measure of money but a negligible difference between the two economies when we consider currency as the measure of money. As for comparing between the R&D and AK models, the welfare cost of inflation is much higher in the R&D model than in the AK model when we focus on steady-state welfare. However, this is not a proper comparison because the R&D-based growth model exhibits non-trivial transition dynamics while the AK growth model always jumps to the balanced-growth path. This different dynamic property of the R&D-based growth model is due to the endogenous number of varieties as an additional state variable. Once we take into account transition dynamics, the welfare difference between the two models becomes smaller but remains non-negligible for both economies and for the two alternative measures of money.

1.1. Literature review

Tobin (1965) argues that higher monetary growth stimulates the accumulation of physical capital via the substitution with money holding. In contrast to Tobin (1965), when money is required for purchasing capital goods (Stockman, 1981), higher anticipated inflation reduces real balances, capital investment and the level of output (i.e., the reversed Tobin effect). This theoretical result is also consistent with many subsequent studies in the literature that consider variants of the AK model with cash-in-advance constraints and analyze the growth and welfare effects of inflation through elastic labor supply. For example, Gomme (1993) and Mino (1997) introduce money into the two-sector Lucas (1988) model via cash-in-advance constraints and emphasize how the money growth rate affects the consumption-leisure decision.[6] Our finding of a negative effect of inflation on economic growth is consistent with these studies. Another branch of studies, such as Zhang (1996) and Jha *et al.* (2002), highlights the role of money in facilitating monetary transactions for which a change in the rate of monetary growth affects the consumption-leisure decision through transaction costs.[7] These studies in general support the negative relationship between inflation and economic growth regardless of whether the model is based on a cash-in-advance constraint or transaction costs.[8] In this study, we explore a related growth-inflation relationship but consider a different growth engine that is R&D-driven innovation.[9] Specifically, we incorporate a cash-in-advance constraint on consumption goods into the seminal Romer (1990) R&D-based growth model and allow for elastic labor supply.

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[6] In a recent study, Itaya and Mino (2007) use an endogenous-growth model with a cash-in-advance constraint to show an interesting result that the growth effect of money supply depends on the preference structure and production technology. Specifically, if the production technology exhibits strong non-convexity or if the utility function has a high elasticity of intertemporal substitution, then there may be multiple balanced-growth paths that feature different growth effects of inflation.

[7] As for monetary growth models with money in utility, see for example Wang and Yip (1992) and Ho *et al.* (2007).

[8] In contrast, Itaya and Mino (2003) show that the Tobin effect (i.e., a positive growth effect of inflation) may emerge in an endogenous-growth model with transaction costs when labor externalities are sufficiently large.

[9] See also Marquis and Reffett (1994), Funk and Kromen (2006, 2010) and Chu and Lai (2009). Later on, we discuss how the present paper relates to and differs from these other studies in the literature.

In contrast to the well-established literature on monetary policy in the AK model, a small but growing number of studies, such as Marquis and Reffett (1994), Funk and Kromen (2006, 2010) and Chu and Lai (2009), has analyzed the effects of monetary policy on economic growth in the R&D-based growth model. The seminal study by Marquis and Reffett (1994) incorporates a transaction-service sector along with a cash-in-advance constraint into the Romer model. They show that higher inflation reduces growth through a reallocation of factor inputs from R&D and production to transaction services. Our model features a different mechanism from the Marquis-Reffett model by having a negative effect of inflation on economic growth through a reduction in labor supply. Chu and Lai (2009) incorporate money demand into a quality-ladder model similar to Grossman and Helpman (1991) with a money-in-utility specification. They consider how the elasticity of substitution between consumption and the real money balance affects the growth and welfare effects of inflation. Funk and Kromen (2006, 2010) incorporate nominal price rigidity into a quality-ladder model to quantitatively evaluate the effects of inflation on economic growth,[10] and they analyze an interesting channel through which nominal price rigidity transmits the effects of inflation from the short run to the long run. The present paper differs from the abovementioned studies by (a) showing different implications of firms' market power on the effects of monetary policy on R&D and capital investment,[11] (b) comparing the welfare cost of inflation between the US and the EA, and (c) simulating the transition dynamics of the monetary R&D-based growth model to compute the complete welfare changes from reducing inflation.

In an early study, Mansfield (1980) points out that higher inflation may reduce R&D by decreasing investment in the plant and equipment that are necessary for R&D and by increasing

[10] Vaona (2011) incorporates nominal rigidity into an AK-style model with learning by doing to analyze the growth effects of inflation, and he provides empirical evidence that shows a negative effect of inflation on economic growth.
[11] It is useful to note that the quality-ladder models in Chu and Lai (2009) and Funk and Kromen (2006, 2010) do not feature physical capital.

uncertainty on relative prices. Goel and Ram (2001) provide empirical evidence to confirm the latter effect by showing that inflation uncertainty has a negative effect on R&D. A recent study by Chu and Lai (2009) provides further empirical evidence that supports a negative relationship between R&D and the level of inflation using cross-country regressions. In addition to empirical studies, policy-oriented research also suggests that high inflation could potentially reduce R&D investment. For example, in Economic Development Indicators (chapter 8, 2005), "… high and volatile inflation also discourages investment, including human capital and R&D investment."

The rest of this paper is organized as follows. Section 2 presents the monetary R&D-based growth model. Section 3 analyzes the effects of monetary policy on economic growth and social welfare. Section 4 considers an AK growth model for comparison. Section 5 calibrates the models to numerically evaluate the growth and welfare effects of monetary policy at the steady state and on the transition path. The final section concludes.

2. A monetary R&D-based growth model

To analyze the interactive effects of monetary policy and patent policy, we modify the seminal R&D-based growth model in Romer (1990) by introducing (a) a cash-in-advance constraint on consumption goods to model money demand, (b) elastic labor supply, and (c) variable patent breadth as in Goh and Olivier (2002). Given that the Romer model has been well-studied, the standard features of the model will be briefly described below to conserve space.

2.1. Households

There is a unit continuum of identical households, who have a life-time utility function given by

(1) $$U=\int_0^{\infty} e^{-\rho t}u_t dt \,,$$

where $u_t = \ln c_t - \psi l_t$.[12] Instantaneous utility u_t is increasing in consumption c_t and decreasing in the supply of labor l_t. As for the exogenous parameters, $\rho > 0$ is the discount rate, and $\psi > 0$ determines the disutility of labor supply. Households maximize utility subject to the following asset-accumulation equation.

(2) $$\dot{a}_t + \dot{m}_t = r_t a_t + w_t l_t + \tau_t - c_t - \pi_t m_t .$$

a_t is the real value of assets owned by households, and these assets consist of tangible and intangible capital. r_t is the real interest rate. Households supply labor to earn a real wage w_t. τ_t is a real lump-sum transfer from the government. π_t is the inflation rate that determines the cost of holding money. m_t is the real money balance held by households to facilitate purchases of consumption goods that are subject to a cash-in-advance constraint given by $\xi c_t \le m_t$, where $0 < \xi \le 1$. The usual cash-in-advance constraint is captured by the special case of $\xi = 1$. Here we follow Dotsey and Ireland (1996) to consider a more general setup in which only a fraction of consumption expenditure is subject to a cash-in-advance constraint.[13] This generalization allows us to perform a more realistic quantitative investigation on the welfare cost of inflation.

Using standard dynamic optimization, the optimality condition for consumption is

(3) $$1/c_t = \lambda_t (1 + \xi i_t),$$

where λ_t is the Hamiltonian co-state variable on (2), and $i_t = r_t + \pi_t$ is the nominal interest rate that captures the opportunity cost of holding money as opposed to accumulating tangible or intangible capital. The optimality condition for labor supply is

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[12] The utility function that we consider is a special case of the more general specification $u_t = \ln c_t - \psi l_t^{1+\varepsilon}/(1+\varepsilon)$. We focus on the special case of $\varepsilon = 0$ in order to obtain closed-form solutions for the steady-state allocations and for the linearized transition path. To ensure the robustness of our results, we have also considered other values of $\varepsilon > 0$ in computing the steady-state welfare cost of inflation. Results are available upon request from the authors.

[13] See also Wu and Zhang (1998) who consider a generalized cash-in-advance constraint.

(4) $$w_t = \psi(1+\xi i_t)c_t .$$

The familiar intertemporal optimality condition is

(5) $$r_t = \rho - \dot{\lambda}_t / \lambda_t .$$

2.2. Final goods

Final goods y_t are produced by a standard CES aggregator using labor $l_{y,t}$ and a continuum of differentiated intermediates goods $x_t(j)$ for $j \in [0, n_t]$ given by

(6) $$y_t = l_{y,t}^{1-\alpha}\left(\int_0^{n_t} x_t^{\alpha}(j)dj\right).$$

This sector is perfectly competitive, and the producers take the output and input prices as given. The conditional demand functions for production labor and intermediate goods are respectively

(7) $$w_t = (1-\alpha)y_t / l_{y,t} ,$$

(8) $$p_t(j) = \alpha(l_{y,t} / x_t(j))^{1-\alpha} ,$$

where $p_t(j)$ is the price of $x_t(j)$ relative to final goods.

2.3. Intermediate goods

There is a continuum of industries producing intermediate goods $x_t(j)$ for $j \in [0, n_t]$. Each industry is occupied by a monopolist who rents capital to produce intermediate goods in an one-to-one fashion; i.e., $x_t(j) = k_t(j)$. The monopolistic profit is

(9) $$\omega_{x,t}(j) = p_t(j)x_t(j) - r_t k_t(j) ,$$

where the rental price of capital is given by r_t due to the simplifying assumption of zero capital depreciation as in Romer (1990).

The unconstrained optimization yields a profit-maximizing markup of $1/\alpha$. Here we follow Goh and Olivier (2002) to introduce patent breadth denoted by η as a policy variable by assuming that the unit cost of producing imitative products is increasing in patent breadth.[14] Thus, without sufficient strength of patent protection, the presence of monopolistic profits attracts imitation. Therefore, stronger patent protection allows monopolistic producers to charge a larger markup without the threat of imitation; see Li (2001) and Chu (2011) for a similar formulation of patent breadth in the quality-ladder model. This formulation is also consistent with Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price". In summary, the maximum markup is determined by η.[15] For the rest of this study, we assume $\eta < 1/\alpha$,[16] so that

(10) $$p_t(j) = \eta r_t$$

for $j \in [0, n_t]$. This formulation also serves to provide a simple way to separate the capital share α and the markup η. The amount of profit is symmetric across industries and given by

(11) $$\omega_{x,t}(j) = \left(\frac{\eta - 1}{\eta}\right) p_t(j) x_t(j) = (\eta - 1)\left(\frac{r_t k_t}{n_t}\right),$$

where the second equality of (11) uses the market-clearing condition for capital goods $x_t n_t = k_t$. Equation (11) shows that a larger markup η increases the amount of monopolistic profits that in turn improves incentives for R&D investment.

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[14] In this study, we focus on patent breadth and make a standard assumption in the literature that the patent length is infinite for simplicity. See for example Iwaisako and Futagami (2003) and Futagami and Iwaisako (2007) for an interesting analysis on finite optimal patent length in the Romer model. See also Palokangas (2011) for an interesting analysis on optimal patent length and breadth in an R&D-based growth model. Chu (2010) shows that at the current patent length of 20 years, extending the patent length would have negligible effects on R&D and social welfare. Therefore, we focus on the analysis of patent breadth in this study.

[15] Alternatively, one can also view the limited markup as price regulation. For example, Evans *et al.* (2003) analyze price regulation in the Romer model without money demand.

[16] Given a capital share of about one-third, the unconstrained markup would be 200% (i.e., $1/\alpha - 1$) that is unrealistically large. Therefore, imposing an upper bound on the markup also helps to separate between the effects of markup and capital share. See for example Jones and Williams (2000) for a discussion on this issue.

2.4. R&D

Denote the value of an invented variety as v_t. The familiar no-arbitrage condition for v_t is

(12) $$r_t v_t = \omega_{x,t} + \dot{v}_t .$$

Intuitively, (12) equates the interest rate to the asset return per unit of asset, where the asset return is the sum of monopolistic profit $\omega_{x,t}$ and potential capital gain $\dot{v}_t$. In the R&D sector, there is a unit continuum of entrepreneurs who hire workers for R&D. The profit of R&D is

(13) $$\omega_{r,t} = v_t \dot{n}_t - w_t l_{r,t} ,$$

where $\dot{n}_t = n_t \varphi l_{r,t}$ is the mass of inventions created by the entrepreneur.[17] The parameter φ determines R&D productivity. The zero-profit condition in the R&D sector is

(14) $$v_t n_t \varphi = w_t .$$

This condition determines the allocation of labor between production and R&D.

2.5. Monetary authority

The growth rate of money supply M_t is denoted by $\mu_t = \dot{M}_t / M_t$ that is exogenously set by the monetary authority. Given the definition of the real money balance $m_t \equiv M_t / P_t$ (where P_t is the price of final goods), the inflation rate π_t is endogenously determined by

(15) $$\pi_t = \mu_t - \dot{m}_t / m_t .$$

Any change in money supply is redistributed to the households as a lump-sum transfer that has a real value of $\tau_t = \dot{M}_t / P_t = \mu_t m_t = \dot{m}_t + \pi_t m_t$, where the last equality follows from (15).

[17] Although we consider a deterministic R&D process as in the original Romer model, it is useful to note an interesting result by Li (1998) who shows that this deterministic R&D process can be derived from an underlying stochastic R&D process.

2.6. Decentralized equilibrium

The equilibrium is a time path of allocations $\{c_t, m_t, l_t, y_t, l_{y,t}, x_t(j), k_t, l_{r,t}\}_{t=0}^{\infty}$, a time path of prices $\{w_t, r_t, v_t, p_t(j), P_t\}_{t=0}^{\infty}$, and a time path of monetary policy $\{\mu_t\}_{t=0}^{\infty}$. Also, at each instant of time,

a. households choose $\{c_t, m_t, l_t\}$ to maximize (1) subject to (2) taking $\{w_t, r_t, \pi_t, \tau_t\}$ as given;

b. competitive final-goods firm produce $\{y_t\}$ to maximize profit taking $\{w_t, p_t(j)\}$ as given;

c. the monopolist in industry $j \in [0,1]$ produces $\{x_t(j)\}$ and chooses $\{p_t(j)\}$ subject to the level of patent breadth η to maximize profit taking $\{r_t\}$ as given;

d. R&D entrepreneurs maximize profit taking $\{w_t, v_t\}$ as given;

e. the market for final goods clears such that $y_t = c_t + \dot{k}_t$;

f. the market for capital goods clears such that $k_t = x_t n_t$;

g. the labor market clears such that $l_t = l_{y,t} + l_{r,t}$;

h. the value of households' assets equals the total value of tangible and intangible capital in the economy such that $a_t = k_t + n_t v_t$;

i. the monetary authority balances its budget such that $\tau_t = \mu_t m_t$.

3. Growth and welfare effects of monetary policy

In this section, we focus on the balanced-growth path along which the equilibrium allocation of labor is stationary. Applying $x_t(j) = x_t$ and $k_t = x_t n_t$ on (6) yields

(16) $$y_t = k_t^{\alpha}(n_t l_{y,t})^{1-\alpha}.$$

On the balanced-growth path, the growth rates of output and capital are equal, and hence, the balanced growth rate is

(17) $$g \equiv \frac{\dot{y}_t}{y_t} = \frac{\dot{n}_t}{n_t} = \varphi l_r .$$

To derive the steady-state allocation of R&D labor, we first make use of (a) the R&D zero-profit condition $v_t n_t \varphi = w_t$, (b) the market-clearing condition for labor $l_t = l_{y,t} + l_{r,t}$, (c) the share of income that goes to production labor $w_t l_{y,t} = y_t(1-\alpha)$, (d) the share of income that goes to monopolistic profit $n_t \omega_{x,t} = y_t \alpha(\eta-1)/\eta$, (e) the value of an invented variety $v_t = \omega_{x,t}/r$, and (f) the Euler equation $r = \rho + g$ to obtain

(18) $$l_r = \frac{1}{\eta - \alpha}\left(\alpha(\eta-1)l - \eta(1-\alpha)\frac{\rho}{\varphi}\right),$$

where l is still an endogenous variable. For a given labor supply l, R&D labor l_r is increasing in the markup η. Intuitively, a larger markup increases monopolistic profit improving incentives for R&D. We will show that l is also increasing in η in the R&D model. Intuitively, a larger amount of monopolistic profit increases the value of inventions driving up the wage rate through the R&D zero-profit condition (14). The higher wage rate increases labor supply given a constant share of production-labor income (i.e., $1-\alpha = w_t l_{y,t} / y_t$). Thus, increasing patent breadth has a strictly positive effect on R&D and economic growth, and this finding is consistent with earlier studies, such as in Li (2001) and Chu (2011).

Equation (18) also shows another important insight that any effect of monetary policy on economic growth operates through elastic labor supply l. In other words,

(19) $$\frac{\partial l_r}{\partial \mu} = \left(\frac{\alpha(\eta-1)}{\eta-\alpha}\right)\frac{\partial l}{\partial \mu},$$

where the coefficient in front of $\partial l / \partial \mu$ is increasing in the markup η. Therefore, holding the effect of η on $\partial l / \partial \mu$ constant, increasing the market power of firms tends to magnify the effect of monetary policy on economic growth by increasing the *share* of labor devoted to R&D. In Proposition 1, we show that $\partial l / \partial \mu < 0$ (i.e., monetary expansion reduces labor supply) and that this detrimental effect of monetary expansion on economic growth becomes more severe as the market power of firms increases. The reason for $\partial l / \partial \mu < 0$ is standard in the literature that monetary expansion increases the nominal interest rate that in turn raises the cost of consumption (which requires money holding) relative to leisure (which does not require money holding).

We relegate the derivation of l to Appendix A. The equilibrium labor supply is

$$l = \frac{1}{\psi[1+\xi(\rho+\mu)]} - \left(\frac{\eta}{\eta-1}\right)\frac{\rho}{\varphi}, \tag{20}$$

which is increasing in patent breadth η and decreasing in money growth μ. Differentiating (20) with respect to μ yields

$$\frac{\partial l}{\partial \mu} = -\frac{\xi}{\psi[1+\xi(\rho+\mu)]^2} < 0. \tag{21}$$

Substituting (21) into (19) and using (17) yield

$$\frac{\partial g}{\partial \mu} = -\left(\frac{\alpha(\eta-1)}{\eta-\alpha}\right)\frac{\varphi\xi}{\psi[1+\xi(\rho+\mu)]^2} < 0. \tag{22}$$

Equation (22) shows our first result that the absolute value of $\partial g / \partial \mu$ is increasing in the markup η, and this result is summarized in Proposition 1.

Proposition 1: *Economic growth decreases in response to an increase in the monetary target* μ*. In addition, larger patent breadth strengthens the effect of monetary policy on economic growth in the R&D-based growth model.*

Proof: See Appendix A.□

As for social welfare, we can substitute $u_t = \ln c_t - \psi l_t$ into (1) and then impose balanced growth to derive

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g}{\rho} - \psi l\right). \tag{23}$$

Here we define f as the steady-state fraction of capital consumed by the households. It can be shown that

$$f \equiv \frac{c_t}{k_t} = \frac{y_t}{k_t} - \frac{\dot{k}_t}{k_t} = \frac{\eta}{\alpha} r - g = \frac{\eta}{\alpha}\rho + \left(\frac{\eta - \alpha}{\alpha}\right) g . \tag{24}$$

Therefore, $\partial f / \partial \mu$ and $\partial g / \partial \mu$ must have the same sign. Substituting $c_0 = k_0 f$ into (23) and then normalizing the initial k_0 to unity yield

$$U = \frac{1}{\rho}\left(\ln f + \frac{g}{\rho} - \psi l\right), \tag{25}$$

where $g = \varphi l_r$ from (17). Differentiating (25) with respect to μ yields

$$\rho \frac{\partial U}{\partial \mu} = \frac{1}{f} \underbrace{\frac{\partial f}{\partial \mu}}_{-} + \frac{1}{\rho} \underbrace{\frac{\partial g}{\partial \mu}}_{-} - \psi \underbrace{\frac{\partial l}{\partial \mu}}_{-} . \tag{26}$$

Intuitively, a monetary expansion that reduces labor supply has a partially positive effect on welfare by increasing leisure. However, the increase in leisure reduces the supply of labor for R&D and is harmful to welfare by decreasing economic growth and the fraction of capital for consumption. Despite these counteracting forces, we show in Proposition 2 that social welfare is unambiguously decreasing in μ. Due to positive R&D externalities in the Romer model and the cash-in-advance constraint that drives a wedge on the consumption-leisure choice, R&D labor l_r and labor supply l are suboptimally low in the decentralized equilibrium. Therefore, an increase in μ that further decreases these two variables leads to lower social welfare. Furthermore, the nominal interest rate i is increasing in μ as shown in the proof of Proposition 2. Therefore, social welfare increases as the nominal interest rate decreases. Because welfare is monotonically decreasing in the nominal interest rate that gives rise to the distortions explained above, social welfare is maximized as the nominal interest rate approaches the zero lower bound (i.e., the seminal Friedman rule holds in our model).[18] Proposition 2 shows that a larger market power of firms magnifies the effect of monetary policy on welfare as in the case of economic growth.

Proposition 2: *Social welfare decreases in response to an increase in the monetary target μ, and the Friedman rule holds in the R&D-based growth model. In addition, larger patent breadth strengthens the effect of monetary policy on social welfare.*

Proof: See Appendix A.□

4. A monetary AK growth model

In this section, we develop a monetary AK growth model and compare the effects of monetary policy in this model with the effects in the R&D model. Specifically, we want to know how the market power of firms influences the growth and welfare effects of monetary policy in the AK model. We make two changes to the R&D growth model by eliminating the R&D sector and modifying the production function in (6) to

$$y_t = l_t^{1-\alpha}\left(\int_0^1 x_t^{\alpha}(j)dj\right)\bar{k}_t^{1-\alpha}. \tag{27}$$

There is now a unit continuum of competitive firms producing final goods. Each firm takes as given the aggregate capital stock $\bar{k}_t$ that has a positive externality on productivity capturing the

[18] See for example Mulligan and Sala-I-Martin (1997) for a comprehensive discussion on the Friedman rule.

learning-by-doing effect as in Romer (1986). Using the equilibrium condition $\bar{k}_t = k_t = x_t$, we simplify (27) to the following aggregate production function that is standard in the AK model.

$$y_t = l_t^{1-\alpha} k_t . \tag{28}$$

As in the R&D-based growth model, the share of income that goes to capital rent is

$$r_t k_t = \frac{\alpha}{\eta} y_t . \tag{29}$$

Therefore, a larger markup decreases the capital share of income and increases the profit share. As a result, a larger markup decreases the incentives for capital accumulation and results into a lower growth rate. This implication stands in stark contrast to the R&D-based growth model in which a larger markup increases the incentives for R&D and economic growth. Furthermore, a larger markup raises labor supply in the R&D-based growth model as explained before whereas it decreases labor supply in the AK model as shown in Appendix A.

To see how these different implications influence the growth effect of monetary policy in the AK model, we derive the growth rate of capital given by

$$g_{k,t} \equiv \frac{\dot{k}_t}{k_t} = \left(1 - \frac{c_t}{y_t}\right) \frac{y_t}{k_t} = s_t l_t^{1-\alpha} , \tag{30}$$

where $s_t \equiv \dot{k}_t / y_t$ is the share of final goods devoted to capital investment. The growth rate in the AK model is determined by $s_t l_t^{1-\alpha}$. On the balanced-growth path, (30) further simplifies to

$$g_k = r - \rho = \frac{\alpha}{\eta} l^{1-\alpha} - \rho . \tag{31}$$

In Proposition 3, we analytically show that (a) increasing the monetary target μ reduces growth and (b) a larger markup mitigates this growth effect of monetary policy. Intuitively, monetary policy affects economic growth through elastic labor supply l while the markup η mitigates the growth effect of monetary policy by reducing the capital share of income given by $r_t k_t / y_t = \alpha / \eta$.

Proposition 3: *Economic growth decreases in response to an increase in the monetary target μ. In contrast to the R&D-based growth model, a larger market power of firms dampens the effect of monetary policy on economic growth in the AK model.*

Proof: See Appendix A.□

As in the R&D-based growth model, we define $f \equiv c_t / k_t$ as the steady-state fraction of capital consumed by households. It can be shown that in the AK model,

(32) $$f = \frac{y_t}{k_t} - \frac{\dot{k}_t}{k_t} = l^{1-\alpha} - g_k = \rho + \frac{\eta - \alpha}{\eta} l^{1-\alpha}.$$

As for the balanced-growth level of social welfare in the AK model, it is given by

(33) $$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g_k}{\rho} - \psi\, l\right) = \frac{1}{\rho}\left(\ln f + \frac{g_k}{\rho} - \psi\, l\right).$$

where $c_0 = k_0 f$ and the initial k_0 is normalized to unity. Differentiating (33) with respect to μ yields

(34) $$\rho \frac{\partial U}{\partial \mu} = \frac{1}{f} \underbrace{\frac{\partial f}{\partial \mu}}_{-} + \frac{1}{\rho} \underbrace{\frac{\partial g_k}{\partial \mu}}_{-} - \psi \underbrace{\frac{\partial l}{\partial \mu}}_{-}.$$

As in the R&D model, monetary expansion has a partial positive effect on welfare by increasing leisure, but the reduction in labor supply is also harmful to welfare by decreasing economic growth and the fraction of capital for consumption. Despite these counteracting forces, we show in Proposition 4 that social welfare decreases in μ. Intuitively, labor supply l in the equilibrium is suboptimally low, so that any increase in μ that further reduces l is harmful to social welfare. Thus, welfare is monotonically decreasing in μ, and hence, the Friedman rule also holds in the AK model. In our numerical analysis, we find that a larger market power of firms dampens the effect of monetary policy on social welfare in the AK model as in the case of economic growth.

Proposition 4: *Social welfare decreases in response to an increase in the monetary target μ, and the Friedman rule also holds in the AK model.*

Proof: See Appendix A.□

5. Quantitative analysis

In this section, we calibrate the two models to provide a numerical analysis on the growth and welfare effects of monetary policy and to quantitatively examine how the markup affects them. We consider two monetary aggregates, currency and M1, as alternative measures of money held by households for the purpose of facilitating transactions. On the one hand, currency holding by households is subject to the cost of inflation. On the other hand, M1 includes interest-bearing assets, such as demand deposits, which are partly immune to the depreciation effect of inflation. Therefore, we report the welfare cost of inflation computed based on currency as a lower bound and the welfare cost computed based on M1 as an upper bound.

We begin by considering the steady-state effects. To calibrate each structural parameter, we either set it to a conventional value or choose a value to match an empirical moment. The capital-share parameter α is set to 0.3, and the discount rate ρ is set to 0.04. For the EA, we consider an initial money growth rate μ of 4.3%, so that the inflation rate is 2.2% (i.e., the average inflation rate in the EA from 1999 to 2008) when the economy experiences an average output growth rate of 2.1% (i.e., the average growth rate in the EA from 1999 to 2008). We set the cash-in-advance parameter ξ to 0.1 when we match the average ratio of currency to households' final consumption expenditure in the EA from 1999 to 2008. We set ξ to 0.63 when we match the ratio of M1 to households' final consumption expenditure in the EA. In addition, we choose a value for the markup η such that R&D as a percentage of GDP is 1.9% as in the EA, and this value of η is 1.22, which is within the range of values considered in Jones and William (2000). Finally, when money is measured by currency, we set the leisure parameter ψ to 5.56, so that the long-run growth rate in the AK model is 2.1% under the benchmark markup of 1.22. As for the R&D model, we also set ψ to 5.56 and then set the R&D-productivity parameter φ to 5.78, so that the long-run growth rate in the R&D model is also 2.1% under the markup of 1.22.[19] When money is measured by M1, we set ψ to 5.33 and φ to 5.78, so that the long-run growth rate in both models is 2.1% under the markup of 1.22. In both models, we lower μ from 0.043 to -0.04 such that the nominal interest rate decreases and approaches zero to achieve the Friedman rule in both money specifications. Table 1a reports the results.

[Insert Table 1a about here]

Table 1a shows that under both currency and M1 specifications, reducing money growth and inflation increases economic growth and social welfare in both the R&D and AK models.[20] However, the growth and welfare effects respond differently to the markup in the two models. In the R&D model, a larger markup *increases* the magnitude of the changes in growth and welfare in response to the lower inflation. In contrast, in the AK model, a larger markup *decreases* the

[19] In this calibration exercise, the implied values of l differ slightly across the models (e.g., l = 0.1365 under the AK model and l = 0.1401 under the R&D model). We have considered an alternative calibration exercise, in which we choose a different value of ψ for the R&D model in order for it to have the same calibrated value of l as in the AK model. In this case, we also choose a different value of φ to match the growth rate of 2.1% in the R&D model. We find similar results under this alternative calibration exercise. Results are available upon request from the authors.

[20] The welfare changes are expressed in terms of equivalent variation in annual consumption.

magnitude of the changes in growth and welfare in response to the lower inflation. Therefore, whether monetary policy generates larger growth and welfare effects in the R&D model or in the AK model depends crucially on the market power of firms. At a larger markup, the growth and welfare effects of monetary policy tend to be larger in the R&D model than in the AK model. Under our benchmark calibration of $\eta = 1.22$, the welfare cost of inflation is almost twice as large in the R&D model as in the AK model. Furthermore, we find that the welfare cost is much larger under the M1 specification than under the currency specification in both models. For example, in the R&D model, the welfare cost of inflation under the M1 specification is 10.30% whereas the welfare cost under the currency specification is 1.55%. As for the AK model, the respective values are 5.48% and 0.84%.

In this study, we also consider the empirical moments in the US to compare the welfare costs of inflation between the EA and the US. As before, we consider $\alpha = 0.3$ and $\rho = 0.04$. By analogous inference, we set an initial money growth rate μ of 5.4%, currency-consumption ratio of 0.08, M1-consumption ratio of 0.15, and a markup η of 1.28 to match the corresponding empirical moments for the US economy. The average inflation rate, output growth rate and R&D share of GDP are 2.8%, 2.6% and 2.6% respectively in the US from 1999 to 2008. We set the leisure parameter ψ to 4.68 and the R&D-productivity parameter φ to 4.3 when money is measured by currency. We set ψ to 4.65 and φ to 4.3 when money is measured by M1. In both the R&D and AK models, we lower μ from 0.054 to -0.04 to achieve the Friedman rule. Table 1b reports the results. Table 1b shows that for either specification, the welfare cost of inflation is almost twice as large in the R&D model than in the AK model under our benchmark calibration of $\eta = 1.28$. It is useful to note that our numerical results for the AK model are in line with the results in Dotsey and Ireland (1996) and Wu and Zhang (1998).

[Insert Table 1b about here]

Under the benchmark markup, the welfare cost of inflation in the US is 2.81% in the R&D model and is only one-quarter of the welfare cost in the EA when we consider M1 as the measure of money due to the substantially larger M1-consumption ratio in the EA than in the US. However, when we consider currency as the measure of money, the welfare cost in the US is 1.49% in the R&D model and is similar to the welfare cost of 1.55% in the EA. As for the AK model, we also obtain a similar finding that the welfare cost of inflation in the EA is over three times as large as the welfare cost in the US under the M1 specification but the welfare costs are similar in the two economies under the currency specification.

5.1. Transition dynamics

To analyze the transition dynamics and evaluate the complete welfare changes, we simulate the transition paths using the technique of linearization by Taylor expansion.[21] In Appendix B, we present the system of dynamic equations and analytically derive the transition paths of the variables that fully characterize the dynamics of the R&D model. As for the AK model, it does not exhibit transition dynamics, so that the welfare gains reported in Tables 1a and 1b are valid.[22] To simulate the transition dynamics of the R&D model, we consider the same set of benchmark parameter values and the same numerical experiments by decreasing μ from 0.043 to -0.04 for the EA and decreasing μ from 0.054 to -0.04 for the US.

[Insert Figures 1a and 1b about here]

[21] We utilize a Taylor expansion to linearize the dynamic system around the new steady-state equilibrium in which the money growth rate is set to -0.04.

[22] In Appendix C (not for publication), we show that the dynamic system of the AK model is characterized by saddle-point stability such that the economy always jumps to a unique and stable balanced-growth path.

Figures 1a and 1b present the transition paths of the variety growth rate for the EA (under its benchmark $\eta = 1.22$) and for the US (under its benchmark $\eta = 1.28$) respectively. When the monetary authority reduces the monetary target, the variety growth rate jumps up as a result of the increase in R&D labor and the magnitude of the jump in the EA is larger than that in the US under the M1 specification. Because the adjustment process takes almost three decades for the economy to reach the new balanced-growth path, we expect the welfare gain adjusted for transition dynamics to be smaller than the steady-state values reported in Table 1. Using the simulated paths of consumption and labor supply, we can compute the lifetime utility of the representative household and compare it to the lifetime utility on the initial balanced-growth path. The results are reported in Tables 2a and 2b. We find that for the EA, the transitional welfare gains using currency and M1 as measures of money are 1.16% and 7.69% respectively,[23] which are smaller than the steady-state welfare gains of 1.55% and 10.30%. Nonetheless, Table 2a also shows that the welfare gain in the R&D model continues to be larger than that of the AK model even after adjusting for transition dynamics. As for the US, we also find that taking into account the transition dynamics, the R&D model exhibits a larger welfare gain than the AK model as shown in Table 2b.

[Insert Tables 2a and 2b about here]

6. Conclusion

In this study, we have revisited a fundamental question in monetary economics originally raised by Tobin (1965) on the relationship between inflation and economic growth. The key departure from the literature is that we focus on innovation (instead of capital accumulation) as the engine

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[23] See also Wen (2010). In a heterogeneous-agent model where money serves as the main form of assets that can be adjusted easily to buffer idiosyncratic shocks, Wen (2010) also finds a substantial welfare cost of inflation in the US.

of economic growth in the long run. In summary, we find that the growth and welfare effects of inflation and monetary policy are largely influenced by an unexplored interaction between the growth engine and the strength of patent protection that determines the market power of firms. We believe that this interaction sheds some light on the importance of the growth engine and an interaction between monetary and patent policies that has been neglected in the growth-inflation literature. Finally, it is well-known that the seminal Romer model exhibits scale effects.[24] In this study, we normalize the size of population to unity, so that population size no longer appears in the equilibrium growth rate. Instead, it is the supply of labor that affects growth; in other words, when R&D scientists and engineers devote more time to research, they generate more innovation. We believe that this implication is more plausible than the original version of scale effects based on population size. Nevertheless, it may be fruitful for future studies to further revisit the growth and welfare effects of monetary policy using other vintages of the R&D-based growth model.

[24] See Jones (1995) and Jones (1999) for an excellent discussion on scale effects in R&D-based growth models.

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Appendix A: Proofs

Proof of Proposition 1: The Euler equation is $r = \rho + g$ and the balanced growth rate is $g = \varphi l_r$. From $\pi = \mu - g$ and $i = r + \pi$, we can derive

(A1) $$i = \mu + \rho .$$

Using (4), (7) and $l = l_y + l_r$, we can rewrite the optimality condition for labor supply in (4) as

(A2) $$(1-\alpha)(y/k) = \psi(1+\xi i)(c/k)(l - l_r) .$$

Substituting (7) into (14) yields $v_t n_t \varphi = (1-\alpha) y_t / l_{y,t}$. Then, we can substitute this condition and $n_t \omega_{x,t} = y_t \alpha(\eta - 1)/\eta$ into (12) to derive

(A3) $$r = \frac{\alpha(\eta-1)}{\eta(1-\alpha)}\varphi(l - l_r) ,$$

where $\dot{v}_t = 0$ on the balanced-growth path. Moreover, the share of income that goes to capital is $r_t k_t = \alpha y_t / \eta$. Applying this equation to (A3) yields

(A4) $$\frac{y}{k} = \left(\frac{\eta-1}{1-\alpha}\right)\varphi(l - l_r) .$$

In addition, using (19), $y = c + \dot{k}$ and $g = \varphi l_r$, we have

(A5) $$\frac{c}{k} = \left(\frac{\eta-\alpha}{\alpha}\right)\varphi l_r + \frac{\eta}{\alpha}\rho .$$

Substituting (A1), (A4) and (A5) into (A2) yields the following condition that determines l_r.

(A6) $$[1+\xi(\rho+\mu)][\eta\rho + (\eta-\alpha)\varphi l_r] = \alpha\varphi(\eta-1)/\psi .$$

Rearranging terms in (A6), we find that equilibrium l_r is

(A7) $$l_r = \frac{\alpha(\eta-1)}{\psi(\eta-\alpha)[1+\xi(\rho+\mu)]} - \frac{\eta\rho}{\varphi(\eta-\alpha)} .$$

To ensure equilibrium l_r is positive, we impose the following lower bound on R&D productivity φ.

Condition R (R&D productivity): $\varphi > \psi\rho\left(\frac{1+\xi(\rho+\mu)}{\alpha}\right)\frac{\eta}{\eta-1}$.

Differentiating (A7) with respect to μ yields

(A8) $$\frac{\partial l_r}{\partial\mu} = -\left(\frac{\alpha(\eta-1)}{\eta-\alpha}\right)\frac{\xi}{\psi[1+\xi(\rho+\mu)]^2} < 0.$$

Using (A7) and (18) yields

(A9) $$l = \frac{1}{\psi[1+\xi(\rho+\mu)]} - \left(\frac{\eta}{\eta-1}\right)\frac{\rho}{\varphi}.$$

Differentiating (A9) with respect to μ yields

(A10) $$\frac{\partial l}{\partial\mu} = -\frac{\xi}{\psi[1+\zeta(\rho+\mu)]^2} < 0.$$

Using $g = \varphi l_r$ and (A8), we obtain

(A11) $$\frac{\partial g}{\partial\mu} = \varphi\frac{\partial l_r}{\partial\mu} < 0.$$

Therefore, an increase in μ reduces long-run economic growth. Moreover, from (A11) the absolute value of $\partial g/\partial\mu$ is $\left|\partial g/\partial\mu\right| = -\varphi\partial l_r/\partial\mu > 0$. Substituting (A8) into this condition and performing a few steps of mathematical manipulation, we have

(A12) $$\frac{\partial}{\partial\eta}\left|\frac{\partial g}{\partial\mu}\right| = \left(\frac{\alpha(1-\alpha)}{(\eta-\alpha)^2}\right)\frac{\varphi\xi}{\psi[1+\xi(\rho+\mu)]^2} > 0.$$

As a result, an increase in η magnifies the growth effect of monetary policy in the R&D-based growth model.□

Proof of Proposition 2: First, we prove that the Friedman rule holds. Based on (19), (24) and $g = \varphi l_r$, equation (26) can be rearranged as

(A13)
$$\rho \frac{\partial U}{\partial \mu} = \frac{\partial g}{\partial \mu}\left[\frac{\eta-\alpha}{\alpha f} + \frac{1}{\rho} - \frac{\psi(\eta-\alpha)}{\alpha\varphi(\eta-1)}\right].$$

Substituting (A2) and (A4) into (A13), we can obtain

(A14)
$$\rho \frac{\partial U}{\partial \mu} = \underbrace{\frac{\partial g}{\partial \mu}}_{-}\left[\frac{\xi i \psi(\eta-\alpha)}{\alpha\varphi(\eta-1)} + \frac{1}{\rho}\right] < 0.$$

From (A1), it is easy to see that

(A15)
$$\frac{\partial i}{\partial \mu} = 1 > 0.$$

(A14) shows that an increase in μ reduces social welfare while (A15) shows that the nominal rate increases in μ. Therefore, social welfare increases (is maximized) as the nominal interest rate decreases (approaches zero), i.e., the Friedman rule holds.

Second, we prove that a larger market power of firms magnifies the effect of monetary policy on social welfare. Differentiating (A14) with respect to η yields

(A16)
$$\frac{\partial}{\partial \eta}\left|\rho \frac{\partial U}{\partial \mu}\right| = \left(\frac{\xi i \psi(\eta-\alpha)}{\alpha\varphi(\eta-1)} + \frac{1}{\rho}\right) \times \frac{\partial}{\partial \eta}\left|\frac{\partial g}{\partial \mu}\right| + \left|\frac{\partial g}{\partial \mu}\right| \times \left(-\frac{\xi i \psi(1-\alpha)}{\alpha\varphi(\eta-1)^2}\right).$$

Substituting (A8), (A11) and (A12) into (A16), we can obtain

(A17)
$$\frac{\partial}{\partial \eta}\left|\rho \frac{\partial U}{\partial \mu}\right| = \frac{1}{\rho} \times \frac{\partial}{\partial \eta}\left|\frac{\partial g}{\partial \mu}\right| > 0.$$

Therefore, a larger η magnifies the effect of monetary policy on social welfare.□

Proof of Proposition 3: The balanced growth rate is $g_k = (\alpha/\eta)l^{1-\alpha} - \rho$. From $\pi = \mu - g_k$ and $i = r + \pi$ we can derive

(A18) $$i = \rho + \mu .$$

Based on (4), (7) and (28), the equilibrium condition for labor market in the AK model can be expressed as

(A19) $$\psi f(1+\xi i) = (1-\alpha)l^{-\alpha} .$$

Substituting (A19) and (32) into (A18), we can derive the equilibrium l that satisfies following condition.

(A20) $$[1+\xi(\rho+\mu)][\eta\rho+(\eta-\alpha)l^{1-\alpha}] = \frac{\eta(1-\alpha)l^{-\alpha}}{\psi} .$$

Differentiating (A20) with respect to μ and η gives rise to

(A21) $$\frac{\partial l}{\partial \mu} = -\frac{\xi[\eta\rho+(\eta-\alpha)l^{1-\alpha}]}{[1+\xi(\rho+\mu)](\eta-\alpha)(1-\alpha)l^{-\alpha}+\alpha\eta\psi^{-1}(1-\alpha)l^{-\alpha-1}} < 0 .$$

(A22) $$\frac{\partial l}{\partial \eta} = -\frac{\alpha\eta^{-1}[1+\xi(\rho+\mu)]l^{1-\alpha}}{[1+\xi(\rho+\mu)](\eta-\alpha)(1-\alpha)l^{-\alpha}+\alpha\eta\psi^{-1}(1-\alpha)l^{-\alpha-1}} < 0 .$$

Based on $g_k = (\alpha/\eta)l^{1-\alpha} - \rho$ and (A21), we can obtain

(A23) $$\frac{\partial g_k}{\partial \mu} = \frac{\alpha(1-\alpha)}{\eta l^{\alpha}}\frac{\partial l}{\partial \mu} < 0 .$$

Differentiating (A23) with respect to η yields

(A24) $$\frac{\partial}{\partial \eta}\left|\frac{\partial g_k}{\partial \mu}\right| = -\frac{\alpha(1-\alpha)}{\eta^2 l^{\alpha}}\left|\frac{\partial l}{\partial \mu}\right| - \frac{\alpha^2(1-\alpha)}{\eta l^{1+\alpha}}\frac{\partial l}{\partial \eta}\left|\frac{\partial l}{\partial \mu}\right| + \frac{\alpha(1-\alpha)}{\eta l^{\alpha}}\left(\frac{\partial}{\partial \eta}\left|\frac{\partial l}{\partial \mu}\right|\right),$$

where $$\frac{\partial}{\partial \eta}\left|\frac{\partial l}{\partial \mu}\right| = -\frac{\xi\alpha\eta^{-1}(1-\alpha)[1+\xi(\rho+\mu)][\eta\rho+(\eta-\alpha)l^{1-\alpha}]l^{-2\alpha}}{\{[1+\xi(\rho+\mu)](\eta-\alpha)(1-\alpha)l^{-\alpha}+\alpha\eta\psi^{-1}(1-\alpha)l^{-\alpha-1}\}^3}$$

$$\times\{[1+\xi(\rho+\mu)](\eta-\alpha)(1-\alpha)+\alpha\eta\psi^{-1}(2-\alpha)l^{-1}\} < 0 .$$

Substituting (A21) and (A22) into (A24), (A24) can be rearranged as

(A25) $$\frac{\partial}{\partial \eta}\left|\frac{\partial g_k}{\partial \mu}\right| = -\frac{\alpha(1-\alpha)}{\eta^2 l^{\alpha}}\left|\frac{\partial l}{\partial \mu}\right| - \frac{\xi\{\alpha\eta^{-1}(1-\alpha)[1+\xi(\rho+\mu)]\}^2[\eta\rho+(\eta-\alpha)l^{1-\alpha}]l^{-2\alpha}}{\{(1-\alpha)(\eta-\alpha)[1+\xi(\rho+\mu)]l^{-\alpha}+\alpha\eta\psi^{-1}(1-\alpha)l^{-\alpha-1}\}^3}$$

$$\times\{2\alpha\eta\rho l^{-1}+(\eta-\alpha)l^{-\alpha}\}<0\,.$$

Therefore, a rise in μ is associated with a reduction in the balanced growth rate. However, in contrast to the R&D-based growth model, a larger η mitigates the effect of monetary policy on economic growth in the AK model.□

Proof of Proposition 4: Based on (30) and (32), equation (34) can be rearranged as

(A26) $$\rho\frac{\partial U}{\partial \mu} = \frac{\partial l}{\partial \mu}\left[\frac{(1-\alpha)(\eta-\alpha)}{\eta f}l^{-\alpha} + \frac{\alpha(1-\alpha)l^{-\alpha}}{\eta\rho} - \psi\right].$$

Substituting (A25) into (A32) and using (32) condition, we can obtain

(A27) $$\rho\frac{\partial U}{\partial \mu} = \underbrace{\frac{\partial l}{\partial \mu}}_{-}\left[\frac{[\rho(\eta-\alpha)+\alpha f]i\psi}{\eta\rho} + \frac{\alpha\varphi(f-\rho)}{\eta\rho}\right]<0\,.$$

From (A18), it is easy to see that

(A28) $$\frac{\partial i}{\partial \mu} = 1 > 0\,.$$

As in the R&D model, equations (A27) and (A28) show that a rise in μ reduces social welfare and the nominal rate is increasing in μ. Thus, social welfare is maximized as the nominal interest rate approaches zero, i.e., the Friedman rule also holds in the AK model.□

Appendix B: Dynamics of the R&D model

This appendix presents the system of equations that determines the dynamics of the R&D model. The endogenous variables are $\{n_t, k_t, v_t, \lambda_t, m_t, c_t, w_t, r_t, \omega_{x,t}, y_t, l_t, l_{r,t}, l_{y,t}, \pi_t\}$.

(B1) $$n_t: \ \dot{n}_t = n_t \varphi l_r ,$$

(B2) $$k_t: \ \dot{k}_t = y_t - c_t ,$$

(B3) $$v_t: \ \dot{v}_t = r_t v_t - \omega_{x,t} ,$$

(B4) $$\lambda_t: \ \dot{\lambda}_t = \lambda_t(\rho - r_t) ,$$

(B5) $$\pi_t: \ \pi_t = \mu_t - \dot{m}_t / m_t ,$$

(B6) $$m_t: \ m_t = \xi c_t ,$$

(B7) $$c_t: \ c_t = \frac{1}{\lambda_t[1+\xi(r_t+\pi_t)]} ,$$

(B8) $$w_t: \ w_t l_{y,t} = (1-\alpha) y_t ,$$

(B9) $$r_t: \ r_t k_t = \alpha\, y_t / \eta ,$$

(B10) $$\omega_{x,t}: \ n_t \omega_{x,t} = \alpha\, y_t (\eta - 1)/\eta ,$$

(B11) $$y_t: \ y_t = k_t^{\alpha}(n_t l_{y,t})^{1-\alpha} ,$$

(B12) $$l_t: \ w_t = \psi[1+\xi(r_t+\pi_t)]c_t ,$$

(B13) $$l_{r,t}: \ v_t n_t \varphi = w_t ,$$

(B14) $$l_{y,t}: \ l_t = l_{y,t} + l_{r,t} ,$$

where μ_t is chosen exogenously by the monetary authority.

Substituting (B7) and (B13) into (B12), we can obtain $\psi = \lambda_t v_t n_t \varphi$. Differentiating this equation with respect to time t yields

(B15) $$0 = \frac{\dot{\lambda}_t}{\lambda_t} + \frac{\dot{v}_t}{v_t} + \frac{\dot{n}_t}{n_t}.$$

In addition, substituting (B8) into (B13) yields $v_t n_t \varphi = (1-\alpha) y_t / l_{y,t}$. Then, we can substitute this condition and (B10) into (B3) to derive

(B16) $$\frac{\dot{v}_t}{v_t} = r_t - \frac{\alpha(\eta-1)}{\eta(1-\alpha)} \varphi (l_t - l_{r,t}).$$

Based on (B1), (B4), (B14) and (B16), (B15) can be rearranged as

(B17) $$\varphi l_t = \frac{\eta - \alpha}{\eta(1-\alpha)} \varphi l_{y,t} - \rho.$$

From (B8), (B11) and (B13), we can obtain $l_{y,t} = [(1-\alpha) n_t^{-\alpha} k_t^{\alpha} / \varphi v_t]^{1/\alpha}$. Differentiating this equation with respect to time t yields

(B18) $$\frac{\dot{l}_{y,t}}{l_{y,t}} = \frac{\dot{k}_t}{k_t} - \frac{\dot{n}_t}{n_t} - \frac{1}{\alpha} \frac{\dot{v}_t}{v_t}.$$

Substituting (B1), (B2), (B14), and (B16) into (B18), (B18) can rearranged as

(B19) $$\frac{\dot{l}_{y,t}}{l_{y,t}} = \frac{\eta - 1}{\eta} \frac{y_t}{k_t} - \frac{c_t}{k_t} - \varphi l_t + \left(1 + \frac{\eta - 1}{\eta(1-\alpha)}\right) \varphi l_{y,t}.$$

Based on $x_t n_t = k_t$, differentiating this equation with respect to time t obtain

(B20) $$\frac{\dot{x}_t}{x_t} = \frac{\dot{k}_t}{k_t} - \frac{\dot{n}_t}{n_t}.$$

Substituting (B1), (B2) and (B14) into (B20), (B20) can rearranged as

(B21) $$\frac{\dot{x}_t}{x_t} = \frac{y_t}{k_t} - \frac{c_t}{k_t} - \varphi (l_t - l_{y,t}).$$

Moreover, from (B8) and (B12) we can rewrite the optimality condition for labor supply as

(B22) $$\frac{c_t}{k_t} = \frac{(1-\alpha) y_t / k_t}{\psi [1 + \xi (r_t + \pi_t)] l_{y,t}}.$$

Based on (B5) and (B6), we can derive $\dot{c}_t / c_t = \mu - \pi_t$. Substituting (B8) and (B22) into this condition, we can obtain

(B23) $$\frac{\dot{c}_t}{c_t} = \mu - \frac{1}{\xi}[\frac{(1-\alpha)y_t / k_t}{\psi(c_t / k_t) l_{y,t}} - 1] + \frac{\alpha y_t}{\eta k_t}.$$

We define $f_t \equiv c_t / k_t$ as the ratio between consumption and capital. In addition, using (B11) and $x_t n_t = k_t$ yield $y_t / k_t = l_{y,t}^{1-\alpha} x_t^{\alpha-1}$. Substituting this condition and (B17) into (B2), (B19), (B21) and (B23), the $l_{y,t}$, f_t and x_t evolve as

(B24) $$\frac{\dot{l}_{y,t}}{l_{y,t}} = \frac{\eta-1}{\eta} l_{y,t}^{1-\alpha} x_t^{\alpha-1} - f_t + \rho - \frac{\eta-\alpha}{\eta(1-\alpha)} \varphi l_{y,t} + \left(1 + \frac{\eta-1}{\eta(1-\alpha)}\right) \varphi l_{y,t},$$

(B25) $$\frac{\dot{f}_t}{f_t} = \frac{\dot{c}_t}{c_t} - \frac{\dot{k}_t}{k_t} = \mu - \frac{1}{\xi}[\frac{(1-\alpha) l_{y,t}^{-\alpha} x_t^{\alpha-1}}{\psi f_t} - 1] + \frac{\alpha}{\eta} l_{y,t}^{1-\alpha} x_t^{\alpha-1} - l_{y,t}^{1-\alpha} x_t^{\alpha-1} + f_t,$$

(B26) $$\frac{\dot{x}_t}{x_t} = l_{y,t}^{1-\alpha} x_t^{\alpha-1} - f_t + \rho - \frac{\eta-\alpha}{\eta(1-\alpha)} \varphi l_{y,t} + \varphi l_{y,t}.$$

Linearizing (B24), (B25) and (B26) around the steady-state equilibrium yields

(B27) $$\begin{pmatrix} \dot{l}_{y,t} \\ \dot{f}_t \\ \dot{x}_t \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} l_{y,t} - l_y \\ f_t - f \\ x_t - x \end{pmatrix} + \begin{pmatrix} a_{14} \\ a_{24} \\ a_{34} \end{pmatrix} d\mu,$$

where $a_{11} = \dfrac{(\eta-1)(1-\alpha)}{\eta} l_y^{1-\alpha} x^{\alpha-1} + \dfrac{\eta-1}{\eta} \varphi l_y$, $a_{12} = -l_y$, $a_{13} = \dfrac{(\eta-1)(\alpha-1)}{\eta} l_y^{2-\alpha} x^{\alpha-2}$,

$a_{14} = 0$, $a_{21} = \dfrac{\alpha(1-\alpha)}{\xi\psi} l_y^{-\alpha-1} x^{\alpha-1} - \dfrac{(1-\alpha)(\eta-\alpha)}{\eta} f l_y^{-\alpha} x^{\alpha-1}$, $a_{22} = \dfrac{(1-\alpha)}{\xi\psi f} l_y^{-\alpha} x^{\alpha-1} + f$,

$a_{23} = \dfrac{(1-\alpha)^2}{\xi\psi} l_y^{-\alpha} x^{\alpha-2} - \dfrac{(\eta-\alpha)(\alpha-1)}{\eta} f l_y^{1-\alpha} x^{\alpha-2}$, $a_{24} = f$,

$a_{31} = (1-\alpha) l_y^{-\alpha} x^{\alpha} - \dfrac{\alpha(\eta-1)}{\eta(1-\alpha)} \varphi x$, $a_{32} = -x$, $a_{33} = (\alpha-1) l_y^{1-\alpha} x^{\alpha-1}$, $a_{34} = 0$.

Let ϑ_1, ϑ_2 and ϑ_3 be the three characteristic roots of the dynamic system. We then have

(B28) $$\vartheta_1 + \vartheta_2 + \vartheta_3 = \frac{(1-\alpha)}{\xi\psi f} l_y^{-\alpha} x^{\alpha-1} + f > 0,$$

(B29) $$\vartheta_1\vartheta_2\vartheta_3 = -\frac{\varphi(\eta-1)(1-\alpha)(\eta-\alpha)}{\eta^2\xi\psi f} l_y^{2(1-\alpha)} x^{2(\alpha-1)} < 0.$$

As indicated in (B28) and (B29), the dynamic system exists two positive and one negative characteristic roots. This implies that the dynamic system displays the saddle-point stability.

For expository convenience, in what follows let ϑ_1 be the negative root and ϑ_2 and ϑ_3 be the positive roots. From (B24)-(B26) that general solution for $l_{y,t}$, f_t and x_t can be described by

(B30) $$l_{y,t} = l_y + A_1 e^{\vartheta_1 t} + A_2 e^{\vartheta_2 t} + A_3 e^{\vartheta_3 t},$$

(B31) $$f_t = f + d_1 A_1 e^{\vartheta_1 t} + d_2 A_2 e^{\vartheta_2 t} + d_3 A_3 e^{\vartheta_3 t},$$

(B32) $$x_t = x + \frac{\vartheta_1 - a_{11} - a_{12}d_1}{a_{13}} A_1 e^{\vartheta_1 t} + \frac{\vartheta_2 - a_{11} - a_{12}d_2}{a_{13}} A_2 e^{\vartheta_2 t} + \frac{\vartheta_3 - a_{11} - a_{13}d_3}{a_{13}} A_3 e^{\vartheta_3 t},$$

where $d_1 = [(\vartheta_1 - a_{33})(\vartheta_1 - a_{11}) - a_{31}a_{13}]/[a_{32}a_{13} + a_{12}(\vartheta_1 - a_{33})]$,

$d_2 = [(\vartheta_2 - a_{33})(\vartheta_2 - a_{11}) - a_{31}a_{13}]/[a_{32}a_{13} + a_{12}(\vartheta_2 - a_{33})]$,

$d_3 = [(\vartheta_3 - a_{33})(\vartheta_3 - a_{11}) - a_{31}a_{13}]/[a_{32}a_{13} + a_{12}(\vartheta_3 - a_{33})]$.

In addition, A_1, A_2 and A_3 are yet undermined coefficients. We assume that the monetary authorities decrease monetary target μ from μ_0 to μ_1 at $t=0$. Based on (B30)-(B32), we use the following equations to express the feature of such a shock

(B33) $$l_{y,t} = \begin{cases} l_y(\mu_0), & t = 0^-, \\ l_y(\mu_1) + A_1 e^{\vartheta_1 t} + A_2 e^{\vartheta_2 t} + A_3 e^{\vartheta_3 t}, & t \geq 0^+, \end{cases}$$

(B34) $$f_t = \begin{cases} f(\mu_0), & t = 0^-, \\ f(\mu_1) + d_1 A_1 e^{\vartheta_1 t} + d_2 A_2 e^{\vartheta_2 t} + d_3 A_3 e^{\vartheta_3 t}, & t \geq 0^+, \end{cases}$$

(B35)

$$x_t = \begin{cases} x(\mu_0), & t = 0^-, \\ x(\mu_1) + \dfrac{\vartheta_1 - a_{11} - a_{12}d_1}{a_{13}} A_1 e^{\vartheta_1 t} + \dfrac{\vartheta_2 - a_{11} - a_{12}d_2}{a_{13}} A_2 e^{\vartheta_2 t} + \dfrac{\vartheta_3 - a_{11} - a_{12}d_3}{a_{13}} A_3 e^{\vartheta_3 t}, & t \geq 0^+, \end{cases}$$

where 0^- and 0^+ denote the instant before and the instant after the monetary contraction, respectively. To solve appropriate values for A_1, A_2 and A_3. These values are determined by

(B36) $$x_{0^-} = x_{0^+},$$

(B37) $$A_2 = A_3 = 0.$$

Equation (B36) indicates intermediate good remains intact at the time of its policy implementation. Equation (B37) is stable condition which ensures intermediate good to converge the new steady-state equilibrium. Using (B36) yields

(B38) $$A_1 = \frac{a_{13}}{\vartheta_1 - a_{11} - a_{12}d_1}[x(\mu_0) - x(\mu_1)].$$

Substituting (B37) and (B38) into (B33)-(B35), we can obtain

(B39) $$l_{y,t} = \begin{cases} l_y(\mu_0), & t = 0^-, \\ l_y(\mu_1) + \dfrac{a_{13}}{\vartheta_1 - a_{11} - a_{12}d_1}[x(\mu_0) - x(\mu_1)]e^{\vartheta_1 t}, & t \geq 0^+, \end{cases}$$

(B40) $$f_t = \begin{cases} f(\mu_0), & t = 0^-, \\ f(\mu_1) + \dfrac{a_{13}d_1}{\vartheta_1 - a_{11} - a_{12}d_1}[x(\mu_0) - x(\mu_1)]A_1 e^{\vartheta_1 t}, & t \geq 0^+, \end{cases}$$

(B41) $$x_t = \begin{cases} x(\mu_0), & t = 0^-, \\ x(\mu_1) + [x(\mu_0) - x(\mu_1)]e^{\vartheta_1 t}, & t \geq 0^+. \end{cases}$$

Substituting $y_t / k_t = l_{y,t}^{1-\alpha} x_t^{\alpha-1}$ and (B39)-(B41) into (B2), with a given initial capital k_0, we can obtain that the time path of capital can thus be expressed as

(B42) $$k_t = k_0 \exp\left(\int_0^t g_{k,\tau}\, d\tau \right),$$

where $g_{k,\tau} = y_\tau / k_\tau - f_\tau$. In addition, using (B17) yields $l_t = (\eta - \alpha) l_{y,t} / \eta(1-\alpha) - \rho / \varphi$. Substituting this condition, (B40) and (B42) into (1) and normalizing the initial k_0 to unity, social welfare at $t \geq 0^+$ that the transition is taken into account can be expressed as

(B43) $$U = \int_0^\infty e^{-\rho t} \left(\ln f_t + \int_0^t g_{k,\tau}\, d\tau - \psi\, l_t \right) dt .$$

Appendix C (not for publication): Dynamics of the AK model

This appendix shows that the AK model does not exhibit transition dynamics. The optimality condition for labor supply in the AK model is $\psi = \lambda_t(1-\alpha)l_t^{-\alpha}k_t$. Differentiating this equation with respect to time t yields

(C1)
$$\frac{\dot{l}_t}{l_t} = \frac{1}{\alpha}\left(\frac{\dot{\lambda}_t}{\lambda_t} + \frac{\dot{k}_t}{k_t}\right).$$

As in the R&D-based growth model, we define $f_t \equiv c_t / k_t$ as the ratio between consumption and capital. Substituting (5), (29) and $\dot{k}_t/k_t = l_t^{1-\alpha} - f_t$ into (C1), (C1) can be rearranged as

(C2)
$$\frac{\dot{l}_t}{l_t} = \frac{1}{\alpha}\left(\rho + \frac{\eta-\alpha}{\eta}l_t^{1-\alpha} - f_t\right).$$

In addition, from (B8), (B9), (B12) and (28) the equilibrium condition for labor market in the AK model can be expressed as

(C3)
$$f_t = \frac{(1-\alpha)l^{-\alpha}}{\psi[1+\xi(\alpha l^{1-\alpha}/\eta + \pi_t)]}.$$

Based on $\dot{c}_t / c_t = \mu - \pi_t$, we can substitute (C3) into this condition to derive

(C4)
$$\frac{\dot{c}_t}{c_t} = \mu - \frac{1}{\xi}[\frac{(1-\alpha)l_t^{-\alpha}}{\psi f_t} - 1] + \frac{\alpha}{\eta}l_t^{1-\alpha}.$$

Using (C2), (4) and $\dot{k}_t/k_t = l_t^{1-\alpha} - f_t$, the following dynamic system in terms of l_t and f_t can be described by

(C5)
$$\frac{\dot{l}_t}{l_t} = \frac{1}{\alpha}\left(\rho + \frac{\eta-\alpha}{\eta}l_t^{1-\alpha} - f_t\right).$$

(C6)
$$\frac{\dot{f}_t}{f_t} = \mu - \frac{1}{\xi}[\frac{(1-\alpha)l_t^{-\alpha}}{\psi f_t} - 1] + \frac{\alpha}{\eta}l_t^{1-\alpha} - l_t^{1-\alpha} + f_t.$$

Linearizing (C5) and (C6) around the steady-state equilibrium yields

(C7) $$\begin{pmatrix} \dot{l}_t \\ \dot{f}_t \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} l_t - l \\ f_t - f \end{pmatrix},$$

where $b_{11} = \frac{(1-\alpha)(\eta-\alpha)}{\alpha\eta} l^{1-\alpha}$, $b_{12} = -l$, $b_{21} = \frac{\alpha(1-\alpha)}{\xi\psi} l^{-\alpha-1} - \frac{(1-\alpha)(\eta-\alpha)}{\eta} f$,

$b_{22} = \frac{(1-\alpha)}{\xi\psi f} l^{-\alpha} + f$.

Let δ_1 and δ_2 be the two characteristic roots of the dynamic system. We then have

(C8) $$\delta_1 + \delta_2 = \frac{(1-\alpha)(\eta-\alpha)}{\alpha\eta} l^{1-\alpha} + \frac{(1-\alpha)}{\xi\psi f} l^{-\alpha} + f > 0,$$

(C9) $$\delta_1 \delta_2 = \frac{1-\alpha}{\xi\psi} l^{-\alpha} [\frac{(1-\alpha)(\eta-\alpha)}{\alpha\eta f} l^{1-\alpha} + \alpha] > 0.$$

Given that $\delta_1 > 0$ and $\delta_2 > 0$ and l and f are jump variables, we can thus conclude that the balanced growth equilibrium is locally determinate. In other words, the dynamic system of the AK model is characterized by saddle-point stability so that the economy always jumps to the balanced-growth path.

Table 1a: Growth and welfare effects of a lower μ in the EA

	η = 1.19	η = 1.20	η = 1.21	**η = 1.22**	η = 1.23	η = 1.24	η = 1.25
Currency specification							
The R&D model							
g (μ = 4.3%)	1.262%	1.547%	1.827%	**2.100%**	2.367%	2.629%	2.885%
g (μ = - 4%)	1.317%	1.605%	1.886%	**2.161%**	2.431%	2.695%	2.953%
Δg	0.055%	0.057%	0.059%	**0.061%**	0.064%	0.066%	0.068%
ΔU	1.385%	1.441%	1.497%	**1.552%**	1.605%	1.657%	1.709%
The AK model							
g_k (μ = 4.3%)	2.288%	2.224%	2.161%	**2.100%**	2.040%	1.981%	1.923%
g_k (μ = - 4%)	2.329%	2.265%	2.202%	**2.140%**	2.080%	2.021%	1.962%
Δg	0.0416%	0.0412%	0.0408%	**0.0404%**	0.0400%	0.0396%	0.0392%
ΔU	0.864%	0.856%	0.847%	**0.839%**	0.830%	0.822%	0.814%
M1 specification							
The R&D model							
g (μ = 4.3%)	1.262%	1.547%	1.827%	**2.100%**	2.367%	2.629%	2.885%
g (μ = - 4%)	1.608%	1.907%	2.200%	**2.487%**	2.768%	3.042%	3.311%
Δg	0.346%	0.360%	0.374%	**0.387%**	0.400%	0.413%	0.426%
ΔU	9.165%	9.552%	9.931%	**10.304%**	10.670%	11.029%	11.382%
The AK model							
g_k (μ = 4.3%)	2.288%	2.224%	2.161%	**2.100%**	2.040%	1.981%	1.923%
g_k (μ = - 4%)	2.548%	2.482%	2.417%	**2.353%**	2.290%	2.229%	2.169%
Δg	0.2606%	0.2580%	0.2553%	**0.2528%**	0.2503%	0.2478%	0.2454%
ΔU	5.651%	5.593%	5.536%	**5.481%**	5.427%	5.373%	5.321%

Table 1b: Growth and welfare effects of a lower μ in the US

	η = 1.25	η = 1.26	η = 1.27	**η = 1.28**	η = 1.29	η = 1.30	η = 1.31
Currency specification							
The R&D model							
g (μ = 5.4%)	1.944%	2.167%	2.386%	**2.600%**	2.810%	3.016%	3.217%
g (μ = - 4%)	1.998%	2.223%	2.443%	**2.659%**	2.870%	3.078%	3.281%
Δg	0.054%	0.056%	0.057%	**0.059%**	0.060%	0.062%	0.063%
ΔU	1.367%	1.407%	1.446%	**1.485%**	1.522%	1.559%	1.596%
The AK model							
g_k (μ = 5.4%)	2.792%	2.726%	2.663%	**2.600%**	2.539%	2.478%	2.419%
g_k (μ = - 4%)	2.832%	2.766%	2.702%	**2.639%**	2.577%	2.516%	2.457%
Δg	0.0401%	0.0397%	0.0393%	**0.0390%**	0.0386%	0.0382%	0.0379%
ΔU	0.852%	0.844%	0.836%	**0.828%**	0.820%	0.813%	0.805%
M1 specification							
The R&D model							
g (μ = 5.4%)	1.944%	2.167%	2.386%	**2.600%**	2.810%	3.016%	3.217%
g (μ = - 4%)	2.045%	2.272%	2.493%	**2.710%**	2.923%	3.132%	3.336%
Δg	0.102%	0.105%	0.107%	**0.110%**	0.113%	0.116%	0.119%
ΔU	2.583%	2.659%	2.734%	**2.807%**	2.878%	2.948%	3.017%
The AK model							
g_k (μ= 5.4%)	2.792%	2.726%	2.663%	**2.600%**	2.539%	2.478%	2.419%
g_k (μ= - 4%)	2.867%	2.801%	2.736%	**2.673%**	2.611%	2.550%	2.490%
Δg	0.0751%	0.0744%	0.0737%	**0.0730%**	0.0723%	0.0716%	0.0710%
ΔU	1.607%	1.591%	1.576%	**1.561%**	1.547%	1.533%	1.519%

Table 2a: Comparing the steady-state and transitional welfare gains from a lower μ in the EA

	η = 1.19	η = 1.20	η = 1.21	**η = 1.22**	η = 1.23	η = 1.24	η = 1.25
Currency specification							
ΔU (R&D: steady state)	1.385%	1.441%	1.497%	**1.552%**	1.605%	1.657%	1.709%
ΔU (R&D: transition)	0.985%	1.044%	1.100%	**1.156%**	1.211%	1.264%	1.316%
ΔU (AK)	0.864%	0.856%	0.847%	**0.839%**	0.830%	0.822%	0.814%
M1 specification							
ΔU (R&D: steady state)	9.165%	9.552%	9.931%	**10.304%**	10.670%	11.029%	11.382%
ΔU (R&D: transition)	6.552%	6.939%	7.320%	**7.693%**	8.060%	8.420%	8.774%
ΔU (AK)	5.651%	5.593%	5.536%	**5.481%**	5.427%	5.373%	5.321%

Table 2b: Comparing the steady-state and transitional welfare gains from a lower μ in US

	η = 1.25	η = 1.26	η = 1.27	**η = 1.28**	η = 1.29	η = 1.30	η = 1.31
Currency specification							
ΔU (R&D: steady state)	1.367%	1.407%	1.446%	**1.485%**	1.522%	1.559%	1.596%
ΔU (R&D: transition)	1.011%	1.052%	1.092%	**1.132%**	1.170%	1.208%	1.245%
ΔU (AK)	0.852%	0.844%	0.836%	**0.828%**	0.820%	0.813%	0.805%
M1 specification							
ΔU (R&D: steady state)	2.583%	2.659%	2.734%	**2.807%**	2.878%	2.948%	3.017%
ΔU (R&D: transition)	1.912%	1.989%	2.065%	**2.140%**	2.213%	2.285%	2.355%
ΔU (AK)	1.607%	1.591%	1.576%	**1.561%**	1.547%	1.533%	1.519%

Figure 1a: Transition dynamics of the R&D growth model in the EA

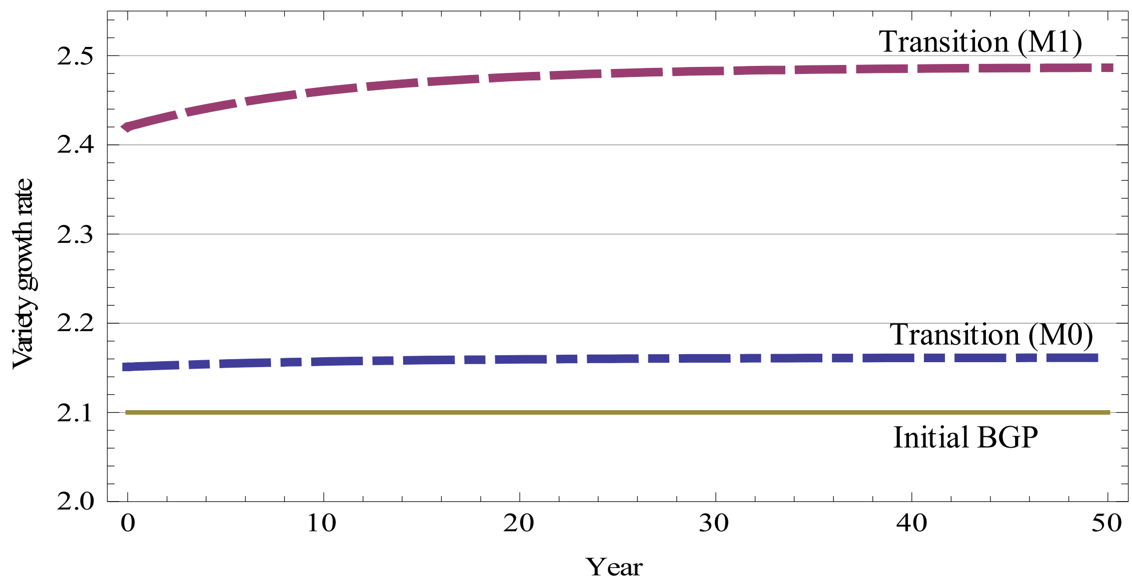

Figure 1b: Transition dynamics of the R&D growth model in the US