Matching Models of Equilibrium Unemployment: An Overview

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Abstract

This book aims to provide an overview of the labour market's benchmark macroeconomic models. The matching models of equilibrium unemployment are, in fact, the primary and most popular theoretical tools used by economists to evaluate various labour market policies and to study one of the key macroeconomic variables: the unemployment rate. It has been recognised that unemployment has also a structural nature which persists over the business cycle. The matching models, i.e. the models à la Mortensen-Pissarides, explain the co-existence in equilibrium of unemployment and vacancies through frictions in matching workers and firms. Furthermore, these models generate predictions that have the right direction: unemployment goes up in recession and down in boom, while job vacancies shift in the opposite direction. The central role of these models in imperfect labour markets has recently been confirmed by the 2010 Nobel Prize for Economy awarded to the founders of this approach: Peter Diamond, Dale Mortensen and Christopher Pissarides.

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1. INTRODUCTION

The matching models of equilibrium unemployment – i.e. the models *à la* Mortensen & Pissarides – are the primary and most popular theoretical tools used by academic and government economists to evaluate various economic policies and to study the problem of unemployment (Hagedorn and Manovskii, 2008). These models rate the benchmark macroeconomic models of the labour market (Garibaldi, 2006). Nowadays, the leading theory of equilibrium unemployment and vacancies is in fact the Mortensen-Pissarides model, which explains the co-existence of unemployment and vacancies through frictions in matching workers and firms. Furthermore, from an empirical point of view, these models appear to satisfactorily explain what occurs in reality: *« […] in calibrations, matching models are usually compared with Hansen's calibrated model and are shown to perform at least as well.»* (Pissarides, 2000, p. 36). The central role of these models in imperfect labour markets has recently been confirmed by the 2010 Nobel Prize for economy awarded to the founders of this approach: Peter Diamond, Dale Mortensen and Christopher Pissarides.

The awareness of the fact that modern labour markets are characterised by large flows, both of workers in and out of employment and of work positions created and destroyed by firms, has led to this new theoretical approach whose main scope is to derive an empirically realistic equilibrium unemployment theory, in which unemployment persists in equilibrium.

The flow of workers between employment, unemployment and inactivity, and the rich dynamics behind them, is a characteristic common to both the American (Blanchard and Diamond, 1990a) and European (Burda and Wyplosz, 1994) labour market. Although these flows are in theory compatible with labour turnover over a fixed number of jobs, the reallocation of workers is actually associated with substantial annual flows in job creation and destruction at the single firm level (Davis and Haltiwanger, 1992).

Even in the absence of net changes in employment, the simultaneous creation and destruction of jobs is intense (Bagliano and Bertola, 1999; Andolfatto, 2008). Figure 1 is a clear example of this: a small net change in employment, amounting to 15.000 individuals, is consistent with approximately one million individuals transiting *in* and *out* of employment.

========== *Figure 1 about here (now at the end)* ==========

The acknowledged importance of these flows in the persistence of unemployment, even at equilibrium, substantiates the economic mechanism underlying matching models: the matching process between workers and firms. More precisely, employment dynamics are the result of vacancies being created and filled by firms, and the activity of job-seekers, particularly the unemployed.[1] The matching between a firm and worker results in a filled, and thus active, job that therefore produces income and is able to pay wages (Bagliano and Bertola, 1999).

However, matching takes time to finalise since the process is characterised by a decentralised, uncoordinated and costly (in terms of both time and money) search conducted by job-seekers and firms [2] (Bagliano and Bertola, 1999). Worker-firm matching is not instantaneous due to the existence of frictions (i.e. search externalities, heterogeneity of individuals and jobs, incomplete information etc.). Search externalities, also known as congestion externalities are particularly relevant in matching models (see Pissarides, 2000). In fact, every firm that creates new jobs produces externalities that are positive for job-seekers (since the probability of finding a job increases) and negative for other firms (since the probability of filling existing vacancies is reduced); *vice versa*, an increase in job-seekers produces positive externalities for firms and negative externalities for other job-seekers, for precisely the opposite reasons.[3]

2. FROM SEARCH FRICTIONS TO MATCHING FUNCTION

It should be specified that the idea that labour market frictions exist and are significant is not unique to matching models and was already present in Hutt (1939) and Hicks (1963). The latter, in particular, claimed that the short-term disequilibrium in the labour market was due to the fact that wages were slow to adjust in the wake of economic shocks, and that this was attributable to existing frictions. This view has essentially been confirmed by more recent studies (cf. Petrongolo and Pissarides, 2001). Keynes (1936), on the other hand, basically

[1] Furthermore, in these models the definition of unemployment is consistent with that typically used in national job-force surveys: individuals are considered unemployed when they do not have a job but are actively searching for one (Andolfatto, 2008).
[2] This differs from the traditional neoclassic model in which the matching process is centralised and coordinated, and work demand and offer are instantly balanced by variations in wages.
[3] In the matching framework, firms and workers have completely rational expectations, i.e. they are fully aware of the matching process. Nonetheless, they act independently, without attempting to coordinate their actions (Pissarides, 2000).

coined the term "frictional unemployment", in other words unemployment that is compatible with full employment, and believed that this type of unemployment was not particularly significant and as a consequence disagreed that frictions played a major role in the slow adjustment of wages.

The work carried out in the '60s and '70s (e.g. Alchian, 1969; Phelps, 1968, 1970, 1972; Mortensen, 1970) successively emphasised the key role played by search frictions and led to today's *search theory*, i.e. an unemployment theory based on the assumption that labour-market search is an economically costly activity. Basically, in models where the individual must choose how to optimally divide his time between work and leisure, a third option is introduced: the option of searching for a new and/or better job. The *search equilibrium* has two key properties: 1) search frictions that introduce monopoly revenue, subdivided between firm and worker through wage determination once a match has been made; 2) indifference to the so called congestion externalities in individual optimisation problems. In essence, individuals ignore the effects their actions have on the aggregate probability of finding a job and filling a vacancy.

Starting from the late '70s – early '80s, more analytically sophisticated models were constructed, now commonly known as *search and matching* models. Amongst these, a distinction can be made between those that focus on the entire economy, in particular on the presence of multiple equilibria (Diamond, 1982a, 1982b, 1984), and those whose main focus is on the labour market (Pissarides, 1979, 1984, 1985a, 1985b, 1986, 2000; Mortensen, 1987; Mortensen and Pissarides, 1994, 1998 and 1999; and Pissarides, 2000). The first models in which the matching function is not only present but is also the main economic mechanism underlying unemployment, basically replacing the reservation wage,[4] are those of Hall (1979), Pissarides (1979), Diamond and Maskin (1979), Bowden (1980).

The matching function is conceptually equivalent to the production function: the result of the "productive process" is the creation of jobs and the "productive

[4] The reservation wage is the wage that leaves an individual indifferent to working or not. It is deduced by equalling the benefit of being employed and the opportunity cost of being employed. Economies with a lower reservation wage have a higher level of employment, however this does not necessarily imply a greater social wellbeing. There is, in fact, no *a priori* reason for believing that higher levels of employment necessarily correspond to higher levels of social wellbeing (Andolfatto, p. 84, 2008).

factors" are unemployment [5] and vacancies (Bagliano and Bertola, 1999). As a consequence, the use of an aggregate (macroeconomic) type function is justified by its empirical relevance and ability to capture the main characteristics of the matching process (Pissarides, 2000). In this sense, the matching function is a useful modelling tool, as it can describe the job formation process without having to clarify the reasons that make this process challenging and costly. Moreover, the matching function is able to grasp (as will become apparent in paragraph 4.1) variations in both the optimal behaviour of firms and workers [6] and the degree of mismatch present in the labour market.[7]

From an empirical point of view, it is common in the literature to resort to the constant returns to scale hypothesis and utilise a *Cobb-Douglas* type function to describe the matching process. Both of these assumptions are empirically supported (Blanchard and Diamond, 1989, 1990b; Pissarides, 2000; Petrongolo and Pissarides, 2001; Stevens, 2007). However, although the choice of a *Cobb-Douglas* type function is common in the literature, its application lacks a convincing theoretical explanation. It is, in fact, employed mainly due to empirical evidence and not because of consensus at the theoretical (microeconomic) level.[8]

An alternative to the *Cobb-Douglas* matching function, which has received important and recent consensus, is the *stock-flow matching* model (Coles and Smith, 1998; Coles and Muthoo, 1998; Lagos, 2000; Gregg and Petrongolo, 2005; Shimer, 2007; Ebrahimy and Shimer, 2010). The idea behind this approach is the following: when a job-seeker enters the market searching for a job, s/he considers all the available vacancies and applies for the position s/he deems most adequate. If the response is positive, i.e. s/he is hired, s/he becomes employed and stops searching, whereas in the case of a negative response s/he remains in the market awaiting new vacancies, having already discarded the old ones. As a consequence, job-seekers are

[5] In the case where on-the-job search (employed individuals searching for a job) is not possible, the only job-seekers are the unemployed.
[6] Consider a variation in the search intensity of workers and/or a higher or lower publicising of vacancies by firms.
[7] The degree of mismatch is, in fact, an empirical concept. Its increase (decrease) indicates that the matching process, under the same conditions of vacancies and unemployment, has become more difficult (easier).
[8] Despite its importance, few attempts have been made at microfounding the matching function and, above all, no microfoundation is better than another (Pissarides, 2000). The aggregate-type matching function is, in fact, usually described as a *"black-box"* (cf. Petrongolo and Pissarides, 2001).

initially flows and vacancies are stock, while successively job-seekers are stock and vacancies are flows.[9]

3. THE NEGATIVE RELATIONSHIP BETWEEN VACANCIES AND UNEMPLOYMENT: THE "BEVERIDGE CURVE"

Long before the appearance of the matching function in the literature, there was another important analytical tool: the *Beveridge Curve*. The *Beveridge Curve* is still used today for analysing unemployment and it describes the negative relationship between unemployment and job vacancies. This relationship is empirically proven (an example is shown in figure 2) and fully intuitive, since an increase in vacancies corresponds to a decrease in unemployment, and *vice versa*.

========== *Figure 2 about here (now at the end)* ==========

The *Beveridge Curve* was discovered by, and is named after, the British social economist William Beveridge (1944).

Pioneeristic work on the *Beveridge Curve*, independent of the existence of a matching function, was carried out by Dow and Dicks-Mireaux (1958), Holt and David (1966), Hansen (1970).

The first studies analysed the interactions between vacancies and unemployment in order to derive a more solid equilibrium unemployment theory, with renewed interest in the *Phillips Curve* and the natural rate of unemployment theory (Phelps, 1967; Friedman, 1968). Successive studies focused instead on two primary goals: a) understanding the employment dynamics of the modern labour market; b) building new macroeconomic models with frictions able to adequately explain these dynamics (cf. Pissarides, 2000).

A phenomenon related to the *Beveridge Curve*, that is sufficiently widespread to have earned the status of "basic fact" of the economic cycle, is the following: during periods of growth and recession, vacancies and unemployment follow anticlockwise trajectories around the *Beveridge Curve* (cf. figure 3).

========== *Figure 3 about here (now at the end)* ==========

As shown in figure 3, the effects produced by the economic cycle are completely intuitive. In fact, under economic growth (recession) the new equilibrium will be characterised by more (fewer) vacancies and lower (more)

[9] Considering the significant and recent interest generated, the last paragraph of this work will be dedicated to the *stock-flow matching* model.

unemployment. This phenomenon, described ever since the first empirical studies carried out with the *Beveridge Curve* (Dow and Dicks-Mireaux, 1958; Holt and David, 1966), is captured by the basic matching model. More precisely, if during the economic cycle a straight line with slope equal to the vacancy – unemployment ratio is traced from the origin (cf. figure 3), vacancies and unemployment follow anticlockwise trajectories around the *Beveridge Curve,* subsequent to productivity shocks (cf. Pissarides, p. 32, 2000). Intuitively, after an increase (reduction) in productivity, the straight line out of the origin will be displaced towards the top (bottom), since an increase (decrease) in productivity increases (decreases) the advantage for firms to create new vacancies. The microeconomic reasoning behind this is the following: when firms anticipate a decrease in unemployment, they aim to keep fewer vacancies open in the future since they will be more difficult to fill; however, in order to have fewer vacancies in the future, more vacancies need to be opened in the present. These are the dynamics described by the anticlockwise trajectories that vacancies and unemployment trace in periods of economic growth.[10] Essentially, this causes a larger variation in vacancies at the beginning of the adjustment period than once equilibrium has been reached. According to Phelps (1968), Hansen (1970) and Bowden (1980), the insight underlying this phenomenon is that job demand is more flexible than job offer.

Finally, as far as the empirical estimate provided by the *Beveridge Curve* is concerned, there is wide consensus over its outward shift for the majority of European countries, corresponding to the increase in unemployment registered over the last thirty years. The explanations accounting for this change, however, differ: increase of long-term unemployment (Budd *et al.*, 1988), generosity of the employment protection mechanisms and unemployment benefits (Jackman *et al.*, 1989), and lack of suitable active labour market policies (Jackman *et al.*, 1990).

4. THE BASIC MATCHING FRAMEWORK: THE MORTENSEN–PISSARIDES MODEL

4.1 The decentralised equilibrium

This paragraph will introduce the matching model commonly used in theoretical analyses.

[10] The opposite reasoning can be applied to the case in which firms foresee an increase in unemployment.

It is common practice with matching models to consider a match between job and worker as a firm, in other words to assume that each firm only employs one worker (*one-job-firm*). The following approach essentially focuses on analysing the match rather than the firm.[11]

As previously mentioned, the main element underlying these models is the matching function, which expresses the number of jobs created in any given moment in time ($M = m \cdot L$) as a function of the total number of unemployed workers ($U = u \cdot L$) and of vacancies ($V = v \cdot L$):

$$M = m(U, V) \Rightarrow m \cdot L = m(u \cdot L, v \cdot L)$$

where m , u and v are, respectively, the rate of matching, unemployment and vacancy, whereas L is the labour force (generally normalised to 1 and assumed to be constant in time). The matching function basically describes the efficiency of the matching process, highlighting the importance of the two inputs (vacancies and job-seekers) in the creation of jobs (Petrongolo and Pissarides, 2001).

If there were no frictions in the matching process, in other words if a vacancy were immediately filled, the number of jobs created would be defined by the minimum between the number of unemployed workers and the number of vacancies, i.e. $M = min(u \cdot L, v \cdot L)$. However, the presence of frictions determines a lower number of jobs given the same number of vacancies, i.e. $min(u \cdot L, v \cdot L) \geq m(u \cdot L, v \cdot L)$.

Assuming, as is common in the literature, that the matching function is increasing and concave in both arguments and degree 1 homogeneous (i.e. characterised by constant returns to scale), the rates can be simplified and the expression rewritten as:

$$m \cdot L = L \cdot m(u, v) \Rightarrow m = m(u, v)$$

Resorting to the commonly used *Cobb-Douglas* functional form, the matching function becomes:

$$m = u^{\alpha} \cdot v^{1-\alpha}$$

where $0 < \alpha < 1$ is the elasticity of the matching function with respect to the unemployment rate:

$$\varepsilon_{m,u} = \frac{\partial m}{\partial u} \cdot \frac{u}{m} \Rightarrow \alpha \cdot u^{\alpha - 1} \cdot v^{1-\alpha} \cdot \frac{u}{u^{\alpha} \cdot v^{1-\alpha}} = \alpha$$

[11] Matching models that disregard the commonly accepted *one-job-firm* hypothesis are those of Bertola and Caballerro (1994) and Garibaldi (2006).

Furthermore, the constant returns to scale hypothesis allows attention to be focalised on a single variable, θ, which expresses the relationship between vacancies and unemployment, i.e. $\theta \equiv v/u$.[12]

The matching function can be used to calculate both the rate with which an unemployed worker finds a job:

$$\frac{u^{\alpha} \cdot v^{1-\alpha}}{u} \Rightarrow \left(\frac{v}{u}\right)^{1-\alpha} \equiv \theta^{1-\alpha}$$

and the rate with which a vacant position is filled:

$$\frac{u^{\alpha} \cdot v^{1-\alpha}}{v} \Rightarrow \left(\frac{v}{u}\right)^{-\alpha} \equiv \theta^{-\alpha}$$

$\theta^{1-\alpha}$ and $\theta^{-\alpha}$ are the two rates that characterise the matching process and express, respectively, the instantaneous probability of finding a job and of filling a vacancy. It immediately follows that the instantaneous probability of finding a job is positive-concave with regards to the vacancies-unemployment ratio, whereas the probability of filling a vacancy is negative-convex. Furthermore, these instantaneous probabilities can (theoretically) tend to infinity in an infinitesimal time interval, *dt*. In particular:

$$\lim_{\theta \to 0} \theta^{1-\alpha} = \lim_{\theta \to \infty} \theta^{-\alpha} = 0 \qquad \lim_{\theta \to \infty} \theta^{1-\alpha} = \lim_{\theta \to 0} \theta^{-\alpha} = \infty$$

It must be pointed out that these properties hold true independently of whether a *Cobb-Douglas* functional form is used.[13]

Employment (n), evolves over time in accordance to inflows (filled vacancies, unemployed workers finding a job) and outflows (existing jobs destroyed with exogenous rate δ).[14] Consequently, the change in employment over time can be expressed as both a function of the firm's transition rate, $\theta^{-\alpha}$,

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[12] In empirical calibrations, it is common practice to introduce a multiplicative factor to the matching function, i.e. $m = \mu\, u^{\alpha}\, v^{(1-\alpha)}$, in order to account for the degree of *mismatch* which, under the same conditions of vacancies and unemployment, makes the matching process more or less difficult. It follows that the larger μ is, the more efficient the matching process and therefore the smaller the degree of *mismatch*. Moreover, as the search intensity and the posting of vacancies may be seen as parameters of technological change in the matching function (see Pissarides, p. 124, 2000), in accordance with the hypothesis of constant returns to scale, the parameter μ may represent the intensity both of job search by individuals and of vacancy posting by firms, as long as the two parameters are assumed equal, i.e. $m = (\mu u)^{\alpha}\, (\mu v)^{(1-\alpha)}$, from which $m = \mu\, u^{\alpha}\, v^{(1-\alpha)}$ is derived.

[13] In order to simplify the explanations and for greater clarity the *Cobb-Douglas* functional form will be used throughout this work.

[14] Jobs are destroyed following shocks specific to the firm, such as technological-organisational changes.

$$\dot{n} \equiv \frac{dn}{dt} = v \cdot \theta^{-\alpha} - n \cdot \delta$$

and as a function of the worker's transition rate, $\theta^{1-\alpha}$,

$$\dot{n} \equiv \frac{dn}{dt} = u \cdot \theta^{1-\alpha} - n \cdot \delta$$

as a result, it must be true that:

$$v \cdot \theta^{-a} - n \cdot \delta = u \cdot \theta^{1-\alpha} - n \cdot \delta \Rightarrow \theta = \frac{v}{u}$$

The relationship between the vacancy rate and the unemployment rate represents a measure of labour market tightness, and as already seen, the probability of finding a job and of filling a vacancy depends on this. The chosen reference point is of utmost importance in understanding how this variable describes labour market frictions: indeed, for the firm, an increase in θ makes filling a vacancy more difficult due to the so called congestion externalities; *vice versa* the situation is improved for the worker since it becomes easier to find a job (the so called positive externalities derived from a "denser" market). In matching models it is common practice to take the firm's point of view as reference, in other words an increase in labour market tensions (or tightness) is associated with an increase in θ.

As previously mentioned, another fundamental labour market analysis tool, often associated with the matching function, is the *Beveridge Curve*, i.e. the inverse relationship between unemployment and vacancy rate. This relationship can be easily obtained from the following expression, which describes how the unemployment rate changes over time:

$$\dot{u} = (1-u) \cdot \delta - u \cdot \theta^{1-\alpha}$$

$(1-u) \cdot \delta$ represents unemployment inflows, i.e. existing jobs destroyed at rate δ, ($1 = n + u$ is, in fact, the normalised labour force), whereas $u \cdot \theta^{1-\alpha}$ describes the unemployment outflows, i.e. unemployed workers that find a job. In steady state equilibrium, where unemployment is constant over time ($\dot{u} = 0$), it follows that:

$$u = \frac{\delta}{\delta + \theta^{1-\alpha}}$$

this equation expresses the reverse relationship between unemployment, *u*, and the measure of labour market frictions θ (and, therefore, between u and v), and is known as the *Beveridge Curve* (BC).[15] The convergence to the equilibrium value of u, given any initial level u_0, is guaranteed due to the negative sign of $\partial \dot{u} / \partial u$,

$$\partial \dot{u} / \partial u = -\left(\delta + \theta^{1-\alpha}\right)$$

where $\left(\delta + \theta^{1-\alpha}\right)$ indicates the rate of convergence.

In order to calculate the equilibrium value of θ, it is necessary to introduce the so called *Bellman equations*, named after the mathematician Richard Bellman who originally presented them in the '50s. The *Bellman equations* describe the expected marginal values (from which the interest rate r has been deducted) associated with the differing conditions of labour market participants, basically comparing them to financial securities.[16] Formally, and very generally, the *Bellman equations* associated with the employment value (W), with the unemployment value (U), with the vacancy value (V) and the filled job value (J), are the following:

$$r \cdot W = w + \delta \cdot (U - W) + \dot{W}$$

$$r \cdot U = z + \theta^{1-\alpha} \cdot (W - U) + \dot{U}$$

$$r \cdot V = -c + \theta^{-\alpha} \cdot (J - V) + \dot{V}$$

$$r \cdot J = y - w + \delta \cdot (V - J) + \dot{J}$$

the terms on the right hand side of the expressions are, respectively, the "dividends" associated with the different conditions (w = wage rate, z = employment opportunity cost, c = cost of opening a vacancy and y = productivity) and the "capital gains or losses", in other words the transition from one condition to the other, influenced by the probability of finding a job, of filling a vacancy and by the job destruction rate.[17] Finally, $\dot{X} \equiv dX / dt$ (where $X = W, U, V, J$) indicates the change over time of the presently considered deducted value. The equilibrium usually characterised by these

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[15] The Beveridge Curve is not only decreasing but is also convex. In fact:

$$\frac{\partial u}{\partial \theta} = -\frac{\delta \cdot (1-\alpha) \cdot \theta^{-\alpha}}{\left(\delta + \theta^{1-\alpha}\right)^2} < 0$$

$$\frac{\partial^2 u}{\partial \theta^2} = \frac{\alpha \cdot \delta \cdot (1-\alpha) \cdot \theta^{-\alpha-1} \cdot \left(\delta + \theta^{1-\alpha}\right)^2 + \delta \cdot (1-\alpha) \cdot \theta^{-\alpha} \cdot 2 \cdot \left(\delta + \theta^{1-\alpha}\right) \cdot (1-\alpha) \cdot \theta^{-\alpha}}{\left(\delta + \theta^{1-\alpha}\right)^4} > 0 \cdot$$

[16] It is common practice in the literature to make use of linear utility functions. Assuming that individuals are risk neutral not only simplifies the analysis, but also allows to focus on the consequences of the search and matching process rather than on the deficiencies of the insurance markets.

[17] Intuitively, the transition from unemployed (vacancy) to employed (filled vacancy) is profitable for the worker (firm). In fact, necessary conditions for non trivial equilibria are $W \geq U$ and $J \geq V$.

models is the "ideal" stationary state, in which the values of the variables are not subject to further changes over time. It therefore follows that $\dot{X}=0\ \forall X$.

The condition which allows the equilibrium value θ to be determined is known as the *zero-profit* or *free-entry* condition: a firm will continue to open new vacancies until the value of a further vacancy becomes equal to zero. In equilibrium, in fact, all the profit opportunities derived from opening new vacancies have been exploited, therefore the value of an additional vacancy is equal to zero.[18] Setting $V=0$ in the *Bellman equations* relative to the value of a filled position and of a vacancy, the following is obtained:

$$\begin{cases}(r+\delta)\cdot J = y-w \\ \dfrac{c}{\theta^{-\alpha}} = J\end{cases} \Rightarrow c\cdot\theta^{\alpha} = \frac{y-w}{r+\delta} \Rightarrow \theta = \left[\frac{y-w}{c\cdot(r+\delta)}\right]^{\frac{1}{\alpha}}$$

The former expression, which shows an inverse relationship between θ and w, is known as the *Job Creation Condition* (JCC).[19] Essentially, the net gain deducted by the firm must cover the expected costs associated with opening a vacancy (the reciprocal probability of filling a vacancy $1/\theta^{-\alpha}\equiv\theta^{\alpha}$ is, in fact, the average length of time for which a vacancy is filled).[20]

With regards to w, wages can be determined in several ways,[21] however it is common practice in the literature to use the generalised *Nash bargaining rule*.[22] Based on this rule, the wage is determined by dividing, between firm and worker, the surplus generated by their matching. The optimisation problem which must be resolved is the following:

$$w = \arg\max_{x}(J-V)^{1-\beta}\cdot(W-U)^{\beta}$$

[18] To be more precise, *"at any given instant, in both stationary equilibrium and adjustment, firms take advantage of all profit opportunities that arise due to the opening of a vacancy:* $V(t)=0,\ \forall t$*. Therefore, even out of stationary equilibrium,* $\dot{V}(t)=0,\ \forall t$ " (Bagliano and Bertola, p.274, 1999). The application of the *zero-profit* condition, which ensures a closed-form solution of the model, was discussed for the first time by Pissarides (1979).

[19] JCC can be seen as a "special" job demand curve. Indeed, if the cost of opening a vacancy were zero, JCC would become a standard work demand, i.e. *y* = *w*.

[20] Similarly, the reciprocal probability of finding a job, is the average duration of unemployment.

[21] See Mortensen and Pissarides (1999) for an overview.

[22] The *Nash rule* is appropriate in this context, since it is assumed that both sides of the labour market implement costly search activities and that, therefore, a successful match is in their best interest.

where $\beta \in (0,1)$ is a measure of the workers' bargaining power, namely the surplus quota owed to the job factor. The relative first-order condition for optimal surplus subdivision is given by:

$$(W-U) = \frac{\beta}{1-\beta} \cdot (J-V)$$

from which the following final expression is obtained (see ***Appendix A***), the so called *Wage Setting* (WS):

$$w = (1-\beta) \cdot b + \beta \cdot y + \beta \cdot c \cdot \theta$$

with $\partial w / \partial \theta > 0$, since an increase in θ increases the probability that an unemployed worker finds a job, thereby improving his/her external opportunities and hence bargaining power.

We now have the three key equations (*Beveridge Curve, Job Creation Condition* and *Wage Setting*) for representing the stationary state equilibrium reached in a labour market with frictions, characterised by four endogenous variables (θ, w, u and v):

$$JCC \Rightarrow c \cdot \theta^{\alpha} = \frac{y-w}{r+\delta}$$

$$WS \Rightarrow w = (1-\beta) \cdot b + \beta \cdot y + \beta \cdot c \cdot \theta$$

$$BC \Rightarrow u = \frac{\delta}{\delta + \theta^{1-\alpha}}$$

========== *Figure 4 and 5 about here (now at the end)* ==========

The equilibrium value of θ and w is determined by the intersection of the *Job Creation Condition* with the *Wage Setting* (cf. figure 4). Finally, the intersection of the *Beveridge Curve* with the origin-based segment of slope θ , allows the equilibrium values of u and v to be determined (cf. figure 5).

4.2 Endogenous job destruction rate

It is often not completely realistic to assume that the job destruction rate is exogenous. In some cases, in fact, the job destruction rate is more sensitive to economic shocks than the job creation rate (Pissarides, 2000).[23]

[23] It must be pointed out that this is, however, mainly empirical evidence relative to the US and not European economy (Boeri, 1996). It is probable that this depends on the restrictions present in the European context that make job closing difficult (Garibaldi, 1998). However, it is unanimously believed that job creation and destruction flows are asymmetric and that there is a negative correlation between the respective rates.

When a shock affects job productivity, the firm can decide whether to continue using the labour factor at the new productivity or whether to destroy it.[24] The choice is made by the firm in accordance with the so called "reserve productivity", R: if the shock that affects the labour factor reduces productivity below this threshold, the firm will destroy the job, *vice versa* it will keep it open. In order to derive the reserve productivity, the overall productivity of the labour factor is indicated by $y \cdot x$, where y is a general productivity parameter, whereas x is the idiosyncratic (or specific) component that describes the change in productivity following the shock. Moreover, it is hypothesised that x is drawn from a known continuous distribution function $G(x)$ and that its value is between 0 and 1.[25] As a consequence, $J(x)$ now represents the value of a filled vacancy with idiosyncratic productivity x, with R satisfying the condition $J(R)=0$. Following a shock, the firm's best choice is to continue producing if and only if $J(x) \geq J(R)$.[26]

In this case, the *Beveridge Curve* of the model will have to account for the fact that not all negative shocks destroy jobs:

$$u = \frac{\delta \cdot G(R)}{\delta \cdot G(R) + \theta^{1-\alpha}}$$

$G(R) = 1 - \int_R^1 x \, dG(x)$ is the probability that a shock lowers productivity below R and destroys the job. Moreover, the threshold value of R must also satisfy the condition $W \geq U$. The rule for determining wages (i.e. the subdivision of surplus) basically excludes voluntary unilateral separations, therefore, in order for the job to be destroyed, it is necessary that firms prefer to do without the labour, i.e. $J(x) < J(R)$, but also that workers prefer to be unemployed, i.e. $W < U$.

The value of a filled vacancy, with idiosyncratic productivity x, and of a vacancy are essentially similar to those described previously:[27]

$$r \cdot J(x) = y \cdot x - w(x) - \delta \cdot J(x) + \delta \cdot \int_R^1 J(s) \, dG(s)$$

---

[24] In the presence of exogenous job destruction, the immediate destruction of the job was hypothesised following a negative shock.

[25] This hypothesis can be generalised by indicating a positive value x_{max} as a maximum value of idiosyncratic (or specific) component x.

[26] A realistic variant of the hypothesis formulated by Pissarides (2000) could foresee job closure even when $J(x) = J(R) = 0$.

[27] It is assumed that all newly created jobs are characterised by maximum productivity, namely $x = 1$.

$$r \cdot V = -c + \theta^{-\alpha} \cdot [J(1) - V]$$

when the shock hits, the firm must discard the value $J(x)$ for another value, $J(s)$, as long as $J(s) \geq J(R)$.

The two former equations allow the "new" JCC and *Job Destruction Curve* (JD) to be obtained, and their intersection will determine the equilibrium values of θ and R (see figure 6 and ***Appendix B***):

========== *Figure 6 about here (now at the end)* ==========

$$JCC \Rightarrow \frac{c}{\theta^{-\alpha}} = \frac{(1-\beta) \cdot y}{(r+\delta)} \cdot (1-R)$$

$$JD \Rightarrow 0 = R - \frac{b}{y} - \frac{\beta \cdot c \cdot \theta}{(1-\beta) \cdot y} + \frac{\delta}{(r+\delta)} \cdot \int_R^1 (s-R)\, dG(s)$$

The results are completely intuitive. JCC has a negative slope even in the (θ, R) interval: in fact, if R increases, the average duration of a job is reduced, and it is for this reason that the firm opens fewer vacancies, thereby decreasing θ. By inverse reasoning, JD increases in R and therefore has a positive slope in the (θ, R) interval.

4.3 Labour turnover

If the destruction of jobs by firms is the only determinant of unemployment inflows, i.e. the only reason for destroying a match, the rates of worker turnover and of job reallocation are equal. However, this assumption is not empirically realistic. The rate of worker turnover is, in fact, much higher than the rate of job reallocation (Pissarides, 2000). Negative shocks that affect firm productivity are not the only causes behind variations in unemployment; indeed, the main causes considered in the literature are: i) retirement; ii) quitting into unemployment;[28] iii) labour force growth rate.

Further flows from the job offer side, modelled through simple *Poisson processes*, will be introduced in this paragraph: the labour force inflow rate, b, (for births); the labour force outflow rate (retirement rate), d, (for deaths); the rate of voluntary resignation in order to find another job, q. The retirement rate is unique,

[28] On-the-job search (job-to-job quitting without intervening unemployment) will be discussed in more detail in paragraph 6.3.

regardless of whether the worker is employed or unemployed, whereas all the "new" workers initially enter the unemployed pool.

The rate of labour force growth can be expressed as the difference between labour force inflows and outflows, for any initial level, $L(0)$. In fact, given the labour force at time *t*:

$$L(t) = L(0) \cdot e^{(b-d)\cdot t}$$

from the natural logarithms and deriving with respect to time, we obtain:

$$\frac{\dot{L}}{L} = b - d$$

The model presented in this paragraph has three differences with respect to the model developed in Pissarides (cf. chapter 4, 2000):

i) The job destruction rate is, for simplicity, considered to be exogenous and constant;
ii) In case of job destruction, i.e. worker layoff, the firm must pay a fixed cost, *F*;
iii) In case of retirement, the worker receives a constant (expected) income discounted of benefits (i.e. his pension).

The (partially) modified *Bellman equations* are, therefore:

$$rJ = y - w - \delta \cdot F + (\delta + q + d) \cdot [V - J]$$

$$rW = w + (\delta + q) \cdot [U - W] + d \cdot [P - W]$$

$$rU = z + \theta^{1-\alpha} \cdot [W - U] + d \cdot [P - U]$$

It immediately follows that the value of θ is now lower than the value calculated in absence of retirement, worker resignation and inflow of new job-seekers:

$$\frac{c}{\theta^{-\alpha}} = \frac{y - w - \delta \cdot F}{(r + \delta + q + d)}$$

in fact, the rates of retirement and resignation increase the discount rate of the marginal value of a filled vacancy and the fixed cost, *F,* reduces the "dividends".

The variation over time in unemployment ($u \cdot L$) is now given by:

$$\frac{d}{dt}(u \cdot L) = (\delta + q) \cdot (1 - u) \cdot L + b \cdot L - d \cdot u \cdot L - \theta^{1-\alpha} \cdot u \cdot L$$

since $\frac{d}{dt}(u \cdot L) = \dot{u} \cdot L + u \cdot \dot{L}$, it follows that:

$$\dot{u} = (\delta + q) \cdot (1 - u) + b - d \cdot u - \theta^{1-\alpha} \cdot u - u \cdot (b - d)$$

in stationary state, therefore: [29]

$$\underbrace{\theta^{1-\alpha} \cdot u}_{\text{outflow}} = \underbrace{(\delta + q + b) \cdot (1-u)}_{\text{inflow}}$$

from this we obtain the model's new *Beveridge Curve*:

$$u = \frac{(\delta + q + b)}{\delta + q + b + \theta^{1-\alpha}}$$

ceteris paribus, the countries with the highest inflow, b, and/or voluntary resignation, q, rates should also have the highest equilibrium unemployment rates.

The labour market inflow rate influences the *Beveridge Curve*, but the *Job Creation Condition* does not. This means that if b increases, unemployment and vacancies also increase proportionally. Intuitively, firms react to the increase in job-seekers by increasing the number of vacancies (graphically, the *Beveridge Curve* shifts to the right).

Only the voluntary resignation rate, q, influences both the *Beveridge* Curve and the *Job Creation Condition*. The overall effect of an increase in q is an increase in unemployment and a decrease in labour market tensions (graphically, the *Beveridge Curve* shifts to the right and the *Job Creation Condition* shifts lower).

On the other hand, an increase in the rate of labour force growth, $(b-d)$, increases vacancies and labour market tensions (graphically, the *Beveridge Curve* shifts to the right and the *Job Creation Condition* shifts higher). However, if the effect produced by the increase in b is greater than the effect produced by the decrease in d, even unemployment increases, making the final variation in θ ambiguous.

4.4 Out-of-steady-state dynamics

This paragraph focuses on the behaviour of the unemployment rate and of *out-of-steady-state* labour market tensions, during the adjustment period that leads to equilibrium.

One of the two main differential equations needed to study the dynamic of the model was introduced in paragraph 4.1, i.e.:

$$\dot{u}(t) = (1 - u(t)) \cdot \delta - u(t) \cdot \theta(t)^{1-\alpha}$$

[29] The rate of retirement does not influence the unemployment rate since a single labour market outflow rate is assumed (equal for employed and unemployed).

from the dynamic equation that leads to the *Beveridge Curve,* it immediately follows that the "reaction" (i.e. the variation over time) of $\dot{u}$ with respect to u is negative, $-\left(\delta+\theta^{1-\alpha}\right)$: an increase in u, in fact, reduces the inflows and increases the outflows. This implies (cf. figure 7) that for the points to the left and right of the curve $\dot{u}=0$, the value of u tends to get increasingly closer to its steady state equilibrium value, i.e. for any initial value of u_0, unemployment always converges to its equilibrium value. Due to the properties of the function $\theta^{1-\alpha}$, the relationship of $\dot{u}$ with respect to θ is also negative and equal to $-u\cdot(1-\alpha)\cdot\theta^{-\alpha}$. Intuitively, if the probability of finding a job increases, unemployment decreases.

========== *Figure 7 about here (now at the end)* ==========

On the other hand, it can be formally proven (cf. ***Appendix C***) that the variation of θ over time does not depend (in an independent manner) on the rate of unemployment, but only on the level of θ and on the model's parameters, i.e.:

$$\dot{\theta}(t)=\frac{(r+\delta)}{\alpha}\cdot\theta(t)-\frac{\theta(t)^{1-\alpha}\cdot(1-\beta)\cdot(y-b)}{c\cdot\alpha}+\frac{\beta\cdot\theta(t)^{2-\alpha}}{\alpha}$$

with: $\dfrac{d\dot{\theta}(t)}{d\theta(t)}=\dfrac{r+\delta}{\alpha}-\dfrac{(1-\alpha)\cdot(1-\beta)\cdot(y-b)}{c\cdot\alpha}\cdot\theta(t)^{-\alpha}+(2-\alpha)\cdot\dfrac{\beta}{\alpha}\cdot\theta(t)^{1-\alpha}>0$.

This implies that for the points lying above and below the curve $\dot{\theta}=0$, the value of θ tends to shift increasingly further from its steady state value (cf. figure 7).

The apparently unstable behaviour of θ is due to the fact that firms base their decision to create vacancies on the future expected value of θ, and immediately create more vacancies if they foresee a future increase in vacant jobs in order to avoid creating new ones when their opening cost will be higher. In fact, the higher θ, the lower the probability of filling a vacancy, whereas the average duration of a filled vacancy increases.

This "forward looking" attitude of firms, with regards to vacancies, makes v and θ take on the characteristics of "jump" variables, i.e. they respond immediately to changes in parameters or expectations. For this reason, labour market tension immediately becomes long term and remains present throughout the entire adjustment period.

The presence of a "backward looking" variable, i.e. a predetermined variable (the unemployment rate),[30] and of a "forward looking" variable (the vacancy rate), implies a very simple adjustment dynamic that in turn implies the existence of a unique dynamic path (*saddlepath*) converging at steady state (*saddlepoint*), shown by point *E* in figure 8.

========== *Figure 8 about here (now at the end)* ==========

It is possible to formally verify the nature of an equilibrium *saddlepoint* by linearising the dynamic equations surrounding a generic steady state equilibrium point $(\bar{u},\ \bar{\theta})$:

$$\begin{pmatrix} \dot{u} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} - & - \\ 0 & + \end{pmatrix} \cdot \begin{pmatrix} u-\bar{u} \\ \theta-\bar{\theta} \end{pmatrix}$$

The negative sign of the determinant of the coefficient matrix confirms the nature of the steady state equilibrium *saddlepoint*.[31]

5. THE PROBLEM OF SOCIAL EFFICIENCY IN THE DECENTRALISED EQUILIBRIUM

The existence of externalities, and the fact that they are not taken into account by individual optimisation problems, immediately questions the social efficiency of the decentralised equilibrium.

As shown in Pissarides (chapter 8, 2000) and Bagliano-Bertola (paragraph 5.4, 1999), the decentralised market equilibrium achieved in the matching models coincides with the socially efficient equilibrium solution, in other words, it is efficient when the surplus quota owed to the labour factor is equal to the elasticity (with respect to θ) of the average duration of a vacancy (specifically when $\beta=\alpha$).[32]

Formally, the condition $\beta=\alpha$ can be derived by comparing the decentralised solution, put in place by a representative firm, and the socially efficient solution, put

[30] The variations in u are mediated by the matching process. In fact, as v (and therefore θ) varies, unemployment also varies due the change in the probability of finding a job.

[31] In order to have equilibrium stability, the matrix trace must be negative. In fact, *"The equilibrium is a node that can be stable or unstable depending on whether the matrix trace is, respectively, smaller than or larger than zero"* (cf. Bagliano and Bertola, p.259, 1999).

[32] The average duration of a vacancy is the reciprocal of the probability of filling a vacancy, i.e. θ^{a}. The hypothesis of constant returns to scale implies that the elasticity with respect to θ of the average duration of a vacancy is equal to the elasticity of the matching function with respect to the unemployment rate. According to *Cobb-Douglas*, this elasticity is equal to α.

in place by a social planner. The solutions of the respective optimisation problems are the following (cf. ***Appendix D***):

Decentralised solution	Socially efficient solution
$\frac{y-w}{r+\delta}=\frac{c}{\theta^{-\alpha}}$	$\frac{y-b}{\left(r+\delta+\alpha\cdot\theta^{1-\alpha}\right)}=\frac{c}{(1-\alpha)\cdot\theta^{-\alpha}}$

where b is the utility flow due to unemployed workers (i.e. the unemployment benefit). By comparing the two *Job Creation Conditions* it is deduced that:

a) The "social" discount rate is larger than the "individual" rate $\left(r+\delta+\alpha\cdot\theta^{1-\alpha}\right)>(r+\delta)$. In fact, in the socially efficient solution, congestion externalities created by an increase in vacancies, and therefore θ, are taken into account. Therefore, in the socially efficient solution, the marginal value of a filled vacancy is discounted at a higher rate.

b) The decentralised solution attributes a lower net productivity to a filled job than the socially efficient solution, since $w \geq b$.[33]

c) The expected cost of a filled vacancy evaluated by the socially efficient solution is larger than the estimated provided by the decentralised solution: $c\cdot\theta^{\alpha}/(1-\alpha)>c\cdot\theta^{\alpha}$. This means that, with respect to the decentralised solution, the social planner will open a smaller number of vacancies so as not to further increase the average duration, and therefore the expected cost, of a vacancy.

Basically, the two solutions differ due to interest in congestion externalities in the centralised solution and the presence of wages in the decentralised solution. For this reason, the decentralised equilibrium will most probably be inefficient, since the rule for determining wages by subdividing the surplus between matched workers and firms neglects those (vacancies and unemployed) that are still engaged in search activities.

The decentralised market equilibrium coincides with the socially efficient solution and, consequently, the wage determined by the *Nash rule* "internalises" the research externalities, when the following is true:

$$\frac{c}{(1-\alpha)\cdot\theta^{-\alpha}}=J+W-U$$

[33] The socially efficient solution disregards wages (since it simply constitutes a transfer of income between firms and workers) and considers the utility flows due to unemployed workers.

the efficiency condition requires that the expected cost of a filled vacancy, evaluated by the socially efficient solution, be equal to the surplus created by a match (in equilibrium $V = 0$). Combining the former expression with the optimisation condition $W - U = \frac{\beta}{1-\beta} \cdot J$, it follows that:

$$\frac{c}{(1-\alpha)\cdot\theta^{-\alpha}} = \frac{1}{1-\beta}\cdot J$$

$$\Rightarrow \frac{c}{(1-\alpha)\cdot\theta^{-\alpha}} = \frac{1}{1-\beta}\cdot\frac{c}{\theta^{-\alpha}}$$

where J is the expected cost of a filled vacancy, obtained from the optimisation condition in the decentralised equilibrium. The efficiency condition is therefore: [34]

$$1-\beta = 1-\alpha \Rightarrow \alpha = \beta$$

It should be stressed that social efficiency is most influenced by the allocation of resources, and whether or not an efficient decentralised equilibrium is reached. Unemployment is, in fact, probably the most significant result of the chosen mechanism for resource allocation, but it is not the cause of non efficient allocation. When $\beta \neq \alpha$ the allocation of resources is not efficient since:

i. if $\beta > \alpha$, firms create fewer jobs and workers search with less intensity since the reserve wage is excessively high (result: high unemployment);
ii. if $\beta < \alpha$, the reserve wage is too low and, as a consequence, workers accept a job too easily (result: underemployment).

Therefore, very generally, equilibrium unemployment is greater than the socially efficient rate if $\beta > \alpha$, whereas the reverse is true for $\beta < \alpha$.

6. THE MAIN EXTENSIONS OF THE BASIC MATCHING FRAMEWORK

6.1 Model with career choice

Since the deliberate focus of these models is on the labour market, the matching literature wouldn't be complete without the formalisation of an individual's fundamental economic choice: the decision between entering the market as an entrepreneur or as a worker. However, the formalisation of this choice within a matching framework, is relatively recent (cf. Fonseca *et al.*, 2001; Pissarides, 2002; Uren, 2007).

[34] It must be pointed out that $\beta = \alpha$ is the efficiency condition only when the matching function displays constant returns to scale. For a broader discussion on this subject see Pissarides (2000).

In matching models, the economic decision of an individual to become entrepreneur or worker is based on the comparison of the two values expected from labour market entry, i.e. the unemployment value and the vacancy value.

Indeed, in Uren (2007), the equality condition:

$$rV(\theta) = rU(\theta)$$

$$z - c + \theta^{-\alpha} \cdot [J - V] = z + \theta^{1-\alpha} \cdot [W - U]$$

allows the equilibrium value of labour market tensions to be determined, using the already discussed *Bellman equations*:[35]

$$\theta^{1-\alpha} \cdot \beta \cdot (y - 2z + 2c) - \theta^{-\alpha} \cdot (1-\beta) \cdot (y - 2z) + c \cdot (r + \delta) = 0$$

The existence and the uniqueness of the value of θ that satisfies this former expression is guaranteed by the condition $y - 2 \cdot z > 0$ (see ***Appendix E***).[36]

Unlike the case of the basic model used in paragraph 4.1, the *free-entry* condition ($V = 0$) is no longer used to determine the equilibrium value of θ. Intuitively, in a model in which there is a fixed total number of firms, there is no need to apply the *zero-profit* condition when creating vacancies. In brief, if the number of firms is constant, the unrealistic possibility of infinite vacancy openings can never be true due to the fact that each firm only has one job/worker (*one-job firm*)

In the models that offer a *career choice*, the total population (not the labour force) is normalised to one:

$$1 = (1 - l) + l = n + v + n + u$$

where $(1 - l) = n + v$ and $l = n + u$ represent, respectively, the overall quota of entrepreneurs and of workers in the total population. Since these are *one-job firm* models, the filled jobs, *n*, represent both the incumbent entrepreneurs and the employed workers (the vacancies, therefore, represent the entrant entrepreneurs).

The number of entrepreneurs $(1 - l)$ and of workers (l) is obtained from the equations describing how vacancies and unemployment evolve over time:

$$\dot{v} = \delta \cdot [(1 - l) - v] - \theta^{-\alpha} \cdot v$$

[35] Uren (2007) uses the *z* notation to identify the free-time value. An entrepreneur that places a vacancy deducts the cost of opening a vacancy from the free-time value. Therefore, in the surplus calculation shown in *Appendix A*, the dividend associated to the vacancy value in Uren (2007) is $z - c$. As for the rest, the *Bellman equations* are analogous to those already seen.

[36] This condition arises since a job match generates *p* units of output but requires the input of a worker and an entrepreneur. Each individual may receive a flow utility of *z* when unemployed. For gain from production to exist, $p > 2 \cdot z$ is necessary.

$$\dot{u} = \delta \cdot (l - u) - \theta^{1-\alpha} \cdot u$$

where $(1-l)-v$, the difference between the total number of firms and of vacancies, are the filled jobs, whereas $(l-u)$ represents employed workers, i.e. the difference between the labour force and unemployed workers. It is interesting to note that, unlike the basic model analysed previously (in which, given u and θ, the equilibrium level of vacancies is determined by the relationship $v = u \cdot \theta$), this model also uses a dynamic equation for the vacancies. This is due to the fact that the new expression also makes explicit reference to the quota of entrepreneurs/firms in the total population.

Finally, by applying the definition of labour market tensions, the values of u and v, obtained through use of the steady state condition $\dot{v} = \dot{u} = 0$, are used to find the equilibrium value of l, which completes the model (see ***Appendix E***):

$$\theta = \frac{v}{u} \Rightarrow l = \frac{\delta + \theta^{1-\alpha}}{\delta \cdot (1+\theta) + 2 \cdot \theta^{1-\alpha}}$$

From an economic point of view, a clearer distinction between entrepreneurs and workers can be found in Fonseca *et al.* (2001). The authors, in fact, introduce entrepreneurial ability, ϑ, which follows a known distribution function, $F(\vartheta)$, in the population. This ability is comprised between a positive minimum value, $\vartheta_{\min} > 0$, and a finite maximum value, $\vartheta_{\max}$.

The model's solution is similar to that proposed by Uren, since the threshold value of entrepreneurial ability (S) is obtained from the following inequality:

$$\vartheta \cdot rV(\theta) - K \geq rU(\theta)$$

where K is a fixed cost (*start-up* cost). Since $V(\theta)$ and $U(\theta)$ are both assumed to be independent of ϑ,[37] the inequality satisfies the so called "reservation of entrepreneurial ability property": i.e. a reservation entrepreneurial ability, S, exists, such that an individual becomes entrepreneur if $\vartheta \geq S$; *vice versa*, for $\vartheta < S$, s/he enters the market as a worker. Consequently, $F(S) = u + n = 1 - \int_S^{\vartheta_{\max}} \vartheta \, dF(\vartheta)$ is the quota of individuals that become workers, while $1 - F(S) = v + n = \int_S^{\vartheta_{\max}} \vartheta \, dF(\vartheta)$ is the quota of entrepreneurs. Formally, the threshold value is given by:

[37] Entrepreneurial ability is, in fact, a simple multiplicative parameter. Matching models in which entrepreneurial ability influences firm productivity are those of Lisi and Pugno (2010a, 2010b). These studies will be discussed in the following paragraph since they extend the matching framework of the underground economy.

$$S = \frac{rU(\theta)+K}{rV(\theta)}$$

with $S'(\theta)>0$, since $V'(\theta)<0$ and $U'(\theta)>0$.[38] These properties can be very simply illustrated through the use of the *Bellman equations* introduced earlier (see ***Appendix F***).

Intuitively, the *Job Creation Condition* (JCC) is decreasing in *S*, since if the threshold value is higher, then fewer individuals become entrepreneurs and, as a consequence, fewer vacancies are opened (see ***Appendix F***).

As illustrated graphically (cf. figure 9), the function $S(\theta)$ assumes a small but positive value ($S=\vartheta_{min}$) for $\theta=0$, and tends to infinity for sufficiently large values of θ where $V(\theta)=0$. *Vice versa*, JCC tends to zero for $S=\vartheta_{max}$ (the whole population chooses to become workers), whereas for $S=\vartheta_{min}$ it tends to its maximum value $(\overline{\theta}<\infty)$.[39]

========== *Figure 9 about here (now at the end)* ==========

The $\overline{\theta}$ shown in figure 9 is the value of θ that satisfies the condition $V(\overline{\theta})=0$, i.e. the equilibrium value of θ obtained from the standard matching model in the absence of entrepreneur-worker choice. Essentially, values of θ higher than $\overline{\theta}$ are excluded since if $\theta>\overline{\theta}$ then $V(\theta)<0$.

As in the Uren (2007) model, the number of entrepreneurs in the total population is fixed; therefore the key role of the *zero-profits* condition in creating vacancies is lost. More precisely, in the Fonseca *et al*. (2001) model, the *cut-off* condition (from which the threshold value of entrepreneurial ability is derived) determines – along with JCC – the total number of entrepreneurs (incumbent and entrant) and of workers (employed and unemployed).

Finally, Pissarides (2002) basically enhances the former model. Indeed, the choice is now more detailed since the potential new entrepreneur also decides the number of job vacancies to be created and managed (γ), based on the following maximisation:

$$\max_{\gamma}\{\gamma\cdot rV(\theta)-\vartheta\cdot g(\gamma)\}\geq rU(\theta)$$

[38] Intuitively, this is straightforward to understand since the greater *θ,* the smaller the probability of a firm filling a vacancy, and the greater *θ,* the higher the probability of the worker finding a job.

[39] Fonseca *et al*. (2001) exclude the value $\theta=\infty$ since in this case a vacancy is never filled.

where $g(\gamma)$ is the cost of managing a job. It follows that,

$$\Rightarrow \vartheta \cdot g'(\gamma) = rV$$

i.e. the marginal cost of managing a job, $g'(\gamma) > 0$, is equal to the marginal revenue from the posting of one more job vacancy. The maximisation condition is also used to obtain the threshold value that determines the entrepreneur-worker decision:

$$S = \frac{\gamma \cdot rV(\theta) - rU(\theta)}{g(\gamma)}$$

However, unlike the previous model, individuals now become entrepreneurs when $\vartheta \leq S$, since the increase in entrepreneurial ability decreases the management costs. Basically, the most able entrepreneurs have a lower ϑ, and therefore a lower management cost $g(\gamma)$. Indeed at the limit, when $\vartheta = 0$, the management costs are null, $g(\gamma) = 0$.

6.2 Model with underground sector

The use of matching models can be easily extended in order to analyse other important phenomena, both labour market related and non.[40] In particular, the persistence of the underground economy even in OECD countries – the so-called "*shadow puzzle*" (Boeri and Garibaldi, 2006) – is a very significant problem, strongly connected to unemployment.

As claimed by Bouev (2005), the substantial weakness in the underground economy theory is the lack of proper attention towards the labour market, thus ignoring the fact that the decision to "go underground" is essentially the outcome of a worker-firm match. However, by using matching-type models it is possible to overcome this shortcoming.[41]

The theoretical literature, which has just put together the underground economy theory with the Mortensen-Pissarides model, is growing and relatively new: see, e.g. Albrecht and Vroman (2002), Bouev (2002, 2005), Boeri and Garibaldi (2002, 2006), Kolm and Larsen (2003, 2008), Fugazza and Jacques (2004), Bosch and Esteban-Pretel (2009), Albrecht *et al.* (2009), Lisi and Pugno (2010a, 2010b).

[40] Wasmer and Weil (2004), in fact, show that this framework can also be used to describe matching difficulties between financial backers (banks) and firms.
[41] The explicit differentiation between worker and firm, in fact, exempts the need for using fictitious producer-consumer integration, allowing a more complete understanding of the role played by participants on both sides of the labour market (Laing, Palivos and Wang, 1995).

These models will not be described in detail since an in depth analysis of their results is beyond the scope of this work. Here it is more important to focus on the different hypotheses used for the matching process and on the results regarding the tight relationship between underground economy and unemployment.

As far as the matching process is concerned, the search for a job can be *directed* or *random*. In the case of a *directed search* (see, e.g. Boeri and Garibaldi, 2006), the unemployed workers select the sector in which to search for a job. Hence, there are two different matching functions, one for every sector, i.e. $m_i = v_i^{1-\alpha} u_i^{\alpha}$, where the subscript $i = F, I$ denotes the sector (with F = *formal* and I = *irregular*). Hence, θ_F and θ_I represent the labour market tightness in the two sectors (obtained according to their respective *free-entry* conditions).[42]

Search is *random* or *undirected* (see, e.g. Albrecht and Vroman, 2002; Kolm and Larsen, 2003; Bouev, 2002, 2005) when workers search for any employment and accept the first available job. In the presence of *undirected search*, both formal and informal vacancies have the same probability of being matched to workers. In this case, the total number of vacancies is entered into the matching function. Formally, if the matching function is again given by $m = v^{1-\alpha} u^{\alpha}$, then $v = v_F + v_I$ is the total number of vacancies supplied by firms and $\theta = v / u$ is the "overall" labour market tightness. Hence, the worker's transition rates into the two sectors can be expressed as $\kappa \cdot \theta^{1-\alpha}$ and $(1-\kappa) \cdot \theta^{1-\alpha}$, where $\kappa = v_F / v$ is the fraction of vacancies supplied in the formal sector. In short, with *random* or *undirected search*, the transition rate facing firms is equal across sectors and given by $\theta^{-\alpha}$, whereas $\theta^{1-\alpha}$ can be interpreted as the probability of a worker getting any job offer. The *random search* assumption allows only one value of searching for a job:

$$rU = z + \kappa \cdot \theta^{1-\alpha} \cdot [W_F - U] + (1-\kappa) \cdot \theta^{1-\alpha} \cdot [W_I - U]$$

where W_F and W_I are the values of being employment in the two sectors.

Finally, Lisi and Pugno (2010a, 2010b) use a "modified" *directed search*. In short, the unemployed cannot search for a job in both sectors at the same time (i.e. there is a *directed search*), but irrespective of the sector, if an unemployed person

[42] The elasticity of the matching function with respect to the unemployment rate in the two sectors may be different, but evidence is lacking in this regard.

fails to find a job, s/he falls back into the same pool of unemployment. Formally, they use the following matching function $m_i = v_i^{1-\alpha} u^{\alpha}$.

It is interesting to note how the *Bellman equations* can be easily manipulated in order to allow them to be extended to the underground sector. For example, in Boeri and Garibaldi (2002), the value of a filled job in the underground sector is given by the following expression (recall that the subscript $i = I$ denotes the irregular sector):

$$rJ_I = y_I - w_I - \rho\phi\tau + (\delta + \rho)\cdot[V_I - J_I]$$

by definition, underground activities are subject to checks by the revenue authorities and the government (checks that, specifically, are given by the probability of discovering the illegality, ρ, that therefore represents an increased discount rate), and if discovered the job is destroyed and a fine $\phi\tau$, equal to a multiple $\phi > 1$ of the unpaid tax τ is paid.[43]

As regards the close relationships between underground employment and unemployment, the results are ambiguous. According to Bouev's (2002, 2005) matching model, scaling down the underground sector may lead to a decrease in unemployment, whereas, according to Boeri and Garibaldi's (2002, 2006) matching model, attempts to reduce shadow employment will result in higher open unemployment.

Bosch and Esteban-Pretel (2009) focus on the role of the job destruction rate. According to their matching model, policies that reduce the cost of formality (or those that increase the cost of informality) produce an increase in the share of formal employment while also reducing unemployment because the reallocation between formal and informal jobs has non-neutral effects on the unemployment rate, since informal jobs report much higher separation rates, given that $(\delta + \rho) > \delta$.

In Lisi and Pugno (2010a, 2010b), the role of the monitoring parameter is strengthened, since any policy intended to reduce the irregular sector may also reduce the unemployment rate if ρ is sufficiently high. In fact, in the usual case

[43] The previously seen optimum rule for firms is valid even for the underground sector: in equilibrium, the net discounted value of a filled job must be equal to its expected cost. A very realistic and intuitive assumption foresees that entering the regular sector has higher costs than entering the underground sector. Indeed, this hypothesis is often used as one of the key criteria with which to distinguish the underground sector from the regular sector (cf. Gërxhani, 2004). As a consequence, the cost of opening a vacancy in the underground sector should be lower than that sustained in the regular sector.

where the official sector is higher than the underground one, a reduction of the underground sector increases unemployment if the monitoring parameter ρ is sufficiently low or even zero, and it decreases unemployment if ρ is sufficiently high. Formally, by using the steady state equilibrium conditions in the supply side of the labour market,

$$\textit{Irregular sector}: \quad \underbrace{g\left(\frac{v_I}{u}\right)^{1-\alpha} \cdot u}_{\text{inflow}} = \underbrace{(\delta+\rho)\cdot n_I}_{\text{outflow}}$$

$$\textit{Formal sector}: \quad g\left(\frac{v_F}{u}\right)^{1-\alpha} \cdot u = \delta \cdot n_F$$

and given the unemployment identity $u = L - n_F - n_I$, where L is the labour force and n_F and n_I are the steady state employment rates in the two sectors, it is straightforward to get the unemployment rate:

$$u = \frac{L\cdot\delta\cdot(\delta+\rho)}{\delta\cdot(\delta+\rho)+(\delta+\rho)\cdot g\left(\frac{v_F}{u}\right)^{1-\alpha}+\delta\cdot g\left(\frac{v_I}{u}\right)^{1-\alpha}}$$

which depends on both v_F and v_I. Hence, it is possible to find a threshold value of monitoring ($\rho = \rho^*$) such that $\frac{\partial u}{\partial v_I}\begin{cases} >0 & \text{if } \rho > \rho^* \\ <0 & \text{if } \rho < \rho^* \end{cases}$, under the realistic condition that $v_F > v_I$. This threshold value is equal to:[44]

$$\rho^* = \delta\left(\left(\frac{v_F}{v_I}\right)^{\alpha} - 1\right)$$

For mathematical details see ***Appendix G***.

Some of the previously cited articles introduce the hypothesis of heterogeneity of individuals (Fugazza and Jacques, 2004; Boeri and Garibaldi, 2006; Albrecht *et al*., 2009; Lisi and Pugno, 2010a, 2010b). Fugazza and Jacques (2004), Boeri and Garibaldi (2006) and Albrecht *et al*. (2009) take the heterogeneity present on the job offer side into consideration, whereas Lisi and Pugno (2010a, 2010b) consider the heterogeneity of the demand side. Specifically, the heterogeneity present in Fugazza and Jacques (2004) concerns the moral considerations brought into play by workers at the moment they decide in which sector to work; the heterogeneity introduced by Boeri and Garibaldi (2006) and Albrecht *et al*. (2009)

[44] In fact, if $v_F < v_I$, then the monitoring rate would be negative.

refers to the differing productivity of workers; finally, Lisi and Pugno (2010a, 2010b) concentrate on the heterogeneous ability of entrepreneurs. The main results obtained through the introduction of the heterogeneity hypothesis into a matching framework with an underground sector are the following:

i. Workers with low moral principles are willing to work in the underground sector (Fugazza and Jacques, 2004);
ii. The most productive workers enter the regular sector (Boeri and Garibaldi, 2006; Albrecht *et al.*, 2009);
iii. The most able entrepreneurs open a firm in the regular sector (Lisi and Pugno, 2010a, 2010b).

Therefore, all workers converge on the (rather reasonable) hypothesis that the underground sector is a backward part of the economy. In particular, these theoretical conclusions account for La Porta and Shleifer's (2008) empirical finding that growth requires the most productive firms, which hence cannot be informal.[45]

6.3 Model with "on-the-job search"

The assumption that an already employed individual may participate in the search process is realistic, considering the fact that the search does not end with employment but aims to find the best work "partner" possible.

Having reviewed the basic matching model and its extension to the underground economy, the simplest (from an analytical point of view) and most intuitive manner to introduce the hypothesis of on-the-job search is to refer to Boeri and Garibaldi's (2002) model with underground sector.[46] The model makes three fundamental hypotheses:

(1) Regular sector jobs are considered *good jobs* whereas irregular jobs are *bad jobs*, because of the differences in productivity and salary, i.e. irregular jobs are considered to be low productivity (assumption supported by empirical evidence). However, at exogenous rate λ *good jobs* become *bad jobs*.
(2) All jobs start out as regular, i.e. the vacancies are all *good*. As a consequence, there will be a unique θ that only expresses the tensions in the regular sector.

[45] Indeed, Lisi and Pugno (2010b) use a matching framework to study the effects produced by the underground economy not only on unemployment but also on growth.
[46] The model presented in this paragraph is a slightly modified version of the original.

(3) Considering the first hypothesis (1), the model's job-seekers are not only the unemployed but also the irregularly employed. Since the unemployed and irregularly employed search for a regular job with the same intensity, i.e. they are equally good at searching for a job, the probability of finding work is the same, i.e. $\theta^{1-\alpha}$. Formally, this is expressed in the following *Bellman equations*:

$$rU = b + \theta^{1-\alpha} \cdot \left(W_{good} - U\right)$$

$$rW_{bad} = w_{bad} - \kappa + (\delta + \rho) \cdot (U - W_{bad}) + \theta^{1-\alpha} \cdot \left(W_{good} - W_{bad}\right)$$

where ρ is the probability of the irregularity being discovered, $\phi\tau$, with $\phi > 1$, is a multiple of the unpaid tax τ, i.e. the fine (penalty), and $\kappa > 0$ is the on-the-job search cost for irregular workers. Even though this cost is not present in Boeri and Garibaldi (2002), Pissarides (2000) considers it to be sufficiently small to make on-the-job search optimal in some conditions.[47] Furthermore, given the model's hypotheses, the tensions in the labour market are given by:

$$m = v^{1-\alpha} \cdot (u + n_{bad})^{\alpha} \Rightarrow \theta = \frac{v}{u + n_{bad}}$$

where the sum of unemployed (u) and underground employed (n_{bad}) identifies the overall quota of job-seekers.[48]

From the view point of an irregular firm, the probability of finding a job $\theta^{1-\alpha}$ represents an additional discount rate since it reduces the average duration of irregular unemployment (since irregular vacancies do not exist, the job is immediately destroyed if the worker leaves):

$$J_{bad} = \frac{y_{bad} - w_{bad} - \rho\phi\tau}{r + \delta + \rho + \theta^{1-\alpha}}$$

The *Bellman equations* that characterise the regular sector are the following (the exogenous rate λ represents an additional discount rate since it reduces the average duration of regular unemployment):

$$V_{good} = 0 \Rightarrow J_{good} = \frac{c}{\theta^{-\alpha}}$$

$$J_{good} = \frac{y_{good} - w_{good} - \tau + \lambda \cdot J_{bad}}{r + \delta + \lambda}$$

[47] The on-the-job search model described by Pissarides (chapter 4, 2000) foresees a productivity threshold value of jobs below which it is always optimal to look for a higher productivity job.

[48] If the hypothesis of on-the-job search is extended to the entire labour force and the labour force is normalised to one, there will be equality between vacancies and labour market tightness ($\theta = v$).

$$W_{good} = \frac{w_{good} + \delta \cdot U + \lambda \cdot W_{bad}}{r + \delta + \lambda}$$

From the above equations, and using the now well known surplus subdivision rule, i.e. $W_i - U = \frac{\beta}{1-\beta} \cdot J_i$ (where i = *good, bad*), the following is obtained:[49]

$$\left(w_{good} + \lambda \cdot W_{bad} - (r+\lambda) \cdot U\right) \cdot (1-\beta) = \beta \cdot \left(y_{good} - w_{good} - \tau + \lambda \cdot J_{bad}\right)$$

$$\left(w_{bad} - \kappa - \left(r + \theta^{1-\alpha}\right) \cdot U + \theta^{1-\alpha} \cdot W_{good}\right) \cdot (1-\beta) = \beta \cdot \left(y_{bad} - w_{bad} - \rho\phi\tau\right)$$

from which it is possible to obtain the negotiated wages in the two sectors:

$$w_{good} = (1-\beta) \cdot b + \beta \cdot \left(y_{good} - \tau + c \cdot \theta\right)$$

since $rU = b + \theta^{1-\alpha} \cdot \frac{\beta}{1-\beta} \cdot c \cdot \theta^{\alpha}$, and

$$w_{bad} = (1-\beta) \cdot (b+\kappa) + \beta \cdot \left(y_{bad} - \rho\phi\tau\right)$$

since $-rU = -b + \theta^{1-\alpha} \cdot U - \theta^{1-\alpha} \cdot W_{good}$.

As shown in Pissarides (2000) and confirmed by Boeri and Garibaldi (2002), the salary of individuals that try to modify their occupational state (their starting condition) does not depend on labour market tensions. Consistent with the hypothesis that both sides of the market have good knowledge of the matching and separation process, irregular firms know that a worker will search for a regular job once employed, thus reducing the marginal value of a filled irregular job. However, the *Nash rule* divides the costs and benefits of on-the-job search: the worker receives a part of the sustained cost $(1-\beta) \cdot \kappa$, but compensates the irregular firm for the procured cost by giving up part of the salary $\beta \cdot c \cdot \theta$. Intuitively, the wages of job-seekers must be lower than that of those who are not searching on-the-job. In fact, the irregular worker will search for regular employment if and only if:

$$w_{good} - w_{bad} > 0$$

$$\beta \cdot \left[y_{good} - y_{bad} - \tau \cdot (1-\rho\phi) + c \cdot \theta\right] - (1-\beta) \cdot \kappa > 0$$

$$0 < \kappa < \frac{\beta}{(1-\beta)} \cdot \left[y_{good} - y_{bad} - \tau \cdot (1-\rho\phi) + c \cdot \theta\right]$$

In essence, the worker must benefit from the activity of on-the-job search, i.e. the search cost must not be too high.

[49] In Boeri and Garibaldi (2002), the bargaining power of workers is identical in both sectors.

In this last part, the effects of on-the-job search on the unemployment theory and on the efficiency problem are derived.

Normalising the labour force to 1 and considering three possible working states (unemployed, irregularly employed and regularly employed), the identity of unemployment is the following:

$$1 = u + n_{good} + n_{bad}$$

In steady state equilibrium, given the *in* and *out* flows characterising the three possible worker states, it follows that:

$$n_{good} = \frac{\theta^{1-\alpha} \cdot (u + n_{bad})}{\delta + \lambda}$$

$$u = \frac{\delta \cdot n_{good} + (\delta + \rho) \cdot n_{bad}}{\theta^{1-\alpha}}$$

$$n_{bad} = \frac{\lambda \cdot n_{good}}{\delta + \rho + \theta^{1-\alpha}}$$

The previous equations form a rank-deficient system in u, n_{good} and n_{bad}, which, together with the unemployment identity yield the following unemployment rate:[50]

$$u = \frac{\rho \cdot (\delta + \lambda) + \delta \cdot \left[\delta + \lambda + \theta^{1-\alpha}\right]}{\left[\delta + \lambda + \theta^{1-\alpha}\right] \cdot \left[\rho + \theta^{1-\alpha} + \delta\right]}$$

The existence of on-the-job search influences unemployment only as far as the equilibrium value of the labour market tensions is concerned. In fact, $\lim_{\theta \to 0} u = 1$; $\lim_{\theta \to \infty} u = 0$, by the *l'Hôpital rule.* In short, the unemployment theory obtained through the on-the-job search hypothesis, therefore, is not significantly different from that obtained without this hypothesis (Pissarides, 2000).

Finally, an important consideration regarding the efficiency problem discussed in paragraph 5 must be made. Following Boeri and Garibaldi (2002), the social planner problem is reformulated considering the fact that the inflow of (regular) employment no longer regards only the unemployed but also a part of the workers (the irregular ones). The optimisation problem is now the following:

$$\max_{\theta} \int_0^{\infty} \left[y_{good} \cdot n_{good} + y_{bad} \cdot n_{bad} + b \cdot u - c \cdot \theta \cdot u\right]$$

subject to the constraint given by the evolution over time of (regular) employment:

[50] Making use of $1 = u + n_{good} + n_{bad}$ to eliminate n_{good} from the other equilibrium conditions (i.e. $n_{good} = 1 - u - n_{bad}$), one obtains a solvable system of two equations in two unknowns u and n_{bad}.

$$\dot{n}_{good} = \theta^{1-\alpha} \cdot (u + n_{bad}) - \delta \cdot n_{good}$$

The relative first order condition for θ is the following:

$$\frac{c \cdot u}{(1-\alpha) \cdot \theta^{-\alpha}} = \Lambda \cdot (u + n_{bad})$$

where Λ is the *Lagrange multiplier* associated with the marginal value of a *"good"* filled job. Now, the decentralised equilibrium does not coincide with the centralised equilibrium even when the efficiency condition is satisfied (i.e. even when $\beta = \alpha$). This result is typical of models with on-the-job search and does not depend on the existence of an underground sector (cf. Boeri and Garibaldi, 2002). In fact, if there were no on-the-job search, i.e. if only u were present in the dynamic constraint, the same socially optimal condition found previously would hold true (cf. *Appendix D*).

7. STOCK-FLOW MATCHING: AN INTRODUCTION

7.1 The basic idea

The previously described Mortensen and Pissarides model is the main theoretical model used in the literature to describe how job-seekers and vacancies match and create a functioning job. The standard matching model is also known as the *random matching* model since the process of matching is casual, i.e. some matches form functioning jobs whereas others do not.

Recently however, a (partially) alternative approach has been proposed in the literature. The underlying idea is that most matches occur between the inflow on one side of the labour market and the existing stock on the other (Coles and Smith, 1998; Coles and Muthoo, 1998; Lagos, 2000; Gregg and Petrongolo, 2005; Shimer, 2007; Ebrahimy and Shimer, 2010). For this reason, this approach is known as *stock-flow matching*.

If a "new" job-seeker is unable to find a suitable position among the available jobs, s/he becomes part of the existing stock of unemployed and will have to wait for new vacancy inflows in order to become employed. In the same manner, if a "new" vacancy is not filled through the existing stock of unemployed, it will increase the existing vacancies stock and won't be filled until there is new inflow of unemployed individuals.

With this approach, a match is attained through the "marketplace" (i.e. through the internet, newspapers, employment agencies, etc.) where all parties are involved in search activities for only a short period of time. Following the search activity, the parties will simply wait for new inflow from the other side of the market (Andrews *et al*., 2009). As a consequence, in the *stock-flow matching* model, unemployed individuals and vacancies persist in equilibrium since it is possible that an adequate partner with which to start a work relationship is not found during the brief period of time in which each party is busy searching. Not all parties are, in fact, constantly and actively involved in their search.

Frictions consistent with the *stock-flow matching* approach and that prevent firms from filling their vacancies derive from the collapse of specific market sectors (professional and/or regional) or from skill shortages (scarcity of qualified labour) rather than from search externalities (Andrews *et al*., 2009). In any case, the frictions analysed by the two approaches should be seen as complementary and not alternative, and a more general model should take them both into consideration (Ebrahimy and Shimer, 2010).

Such a model was recently calibrated and empirically tested (cf. Coles and Petrongolo, 2008; Ebrahimy and Shimer, 2010; Andrews *et al*., 2009). Coles and Petrongolo (2008) basically confirm the interesting point of view described by Lagos (2000), according to which at micro level the match occurs between flows and stock, whereas at the aggregate level (in steady state), the matching process appears to be consistent with the standard matching model. In fact, the matching function describes the aggregate data quite well and does not reject the constant returns hypothesis (Coles and Petrongolo, 2008). However, out of steady state, the *random matching* model is inconsistent with the observed in and out flow dynamics of vacancies and unemployment (Coles and Petrongolo, 2008). Using micro level data, the significant positive effect of new vacancies on the probability of finding a job is higher than that caused by the total vacancy stock. Similarly, the impact of new job-seekers on the probability of filling a new vacancy is higher than that of the total stock of job-seekers. This appears to illustrate the greater empirical validity of the *stock-flow matching* model (Andrews *et al*., 2009).

7.2 The stock-flow matching model

Following on from this reasoning, it should be clear that the standard matching model (i.e. the *random matching* model) can be considered a particular case of *stock-flow matching*, in which matches occur between the global stock of vacancies and unemployed individuals.[51]

Instead, in the *stock-flow matching* model, even *flow-flow*, *flow-stock* and *stock-stock* matches are possible. As a consequence, this approach is characterised by eight rather than two transition probabilities, *h,* (*hazard rates*). In fact, given the overall number of vacancies (V) and unemployed (U) and the new unemployment ($\tilde{u}$) and vacancies ($\tilde{v}$) inflows, the following is obtained:

$$h_w(\tilde{u},\tilde{v}), \qquad h_w(\tilde{u},\tilde{V}), \qquad h_w(\tilde{U},\tilde{v}), \qquad h_w(\tilde{U},\tilde{V})$$

$$h_f(\tilde{v},\tilde{u}), \qquad h_f(\tilde{v},\tilde{U}), \qquad h_f(\tilde{V},\tilde{u}), \qquad h_f(\tilde{V},\tilde{U})$$

Where $\tilde{V} \equiv V - \tilde{v}$ and $\tilde{U} \equiv U - \tilde{u}$ are the pre-existing stock of vacancies and job-seekers (i.e. prior to the new inflows). The subscripts *w* and *f* identify, respectively, the transition probabilities of the unemployed (*worker hazard rates*) and of firms (*firm hazard rates*). These probabilities depend on the type of match considered. For example, the *worker hazard rate* $h_w(\tilde{U},\tilde{v})$ refers to the probability that a *job-seeker* belonging to the pre-existing stock of unemployed matches with the new vacancy inflow.

Intuitively, since flows are quantitatively smaller than stocks, the probability of finding a job and, similarly, of filling a vacancy, should be higher for the new flows of job-seekers ($\tilde{u}$) and vacancies ($\tilde{v}$):

$$h_w(\tilde{u},\tilde{V}) > h_w(\tilde{U},\tilde{v}); \quad h_f(\tilde{v},\tilde{U}) > h_f(\tilde{V},\tilde{u})$$

Furthermore, under the *stock-flow matching* model, the job-seekers (the vacancies) belonging to the pre-existing stock of unemployed individuals (of vacancies) match almost exclusively with the new vacancy (new job-seeker) flows:

$$h_w(\tilde{U},\tilde{v}) > h_w(\tilde{U},\tilde{V}) \approx 0; \qquad h_f(\tilde{V},\tilde{u}) > h_f(\tilde{V},\tilde{U}) \approx 0$$

in fact, in the *stock-flow matching* model, *stock-stock* matches are excluded, since if they were profitable for both sides they would have occurred previously. However, a *stock-stock* match can occur when the searching parties modify their behaviour (by modifying their preferences) or, more simply, when the overall stock of vacancies and job-seekers is sufficiently large.

[51] The model analysed in this subparagraph is tightly related to the model described by Andrews *et al*. (2009).

In order to simplify the analysis, it is assumed that all matches become jobs (i.e. they are accepted). Following Andrews *et al*. (2009) it is possible to specify the *hazard rates* by illustrating the relationship between the standard matching model (*random matching* model) and the *stock-flow matching* model:[52]

$$h_w(\tilde{u},\tilde{v}) = \frac{\left(\frac{\tilde{u}\tilde{v}}{UV}\right)\cdot \mu_w(\tilde{u},\tilde{v})\cdot U^{\alpha}\cdot V^{1-\alpha}}{\tilde{u}}$$

$$h_w(\tilde{u},\tilde{V}) = \frac{\left(\frac{\tilde{u}\tilde{V}}{UV}\right)\cdot \mu_w(\tilde{u},\tilde{V})\cdot U^{\alpha}\cdot V^{1-\alpha}}{\tilde{u}}$$

$$h_w(\tilde{U},\tilde{v}) = \frac{\left(\frac{\tilde{U}\,\tilde{v}}{UV}\right)\cdot \mu_w(\tilde{U},\tilde{v})\cdot U^{\alpha}\cdot V^{1-\alpha}}{\tilde{U}}$$

$$h_w(\tilde{U},\tilde{V}) = \frac{\left(\frac{\tilde{U}\tilde{V}}{UV}\right)\cdot \mu_w(\tilde{U},\tilde{V})\cdot U^{\alpha}\cdot V^{1-\alpha}}{\tilde{U}}$$

The numerator on the right hand side of each equation expresses the average number of jobs created per unit time for each match type considered. As in standard matching models, the number of matches is modelled using *Cobb-Douglas*. The difference is that in the *random matching* model μ is unique, regardless of the match type considered, i.e.:

$$\mu_w(\tilde{u},\tilde{v}) = \mu_w(\tilde{u},\tilde{V}) = \mu_w(\tilde{U},\tilde{v}) = \mu_w(\tilde{U},\tilde{V}) = \mu$$

Moreover, in order to obtain the average number of jobs, the expression $\mu_w(\cdot)\cdot U^{\alpha}\cdot V^{1-\alpha}$ must be multiplied by the possible matches, considered as a quota of the total number of matches (the total number of contacts is, obviously, UV). Finally, the relative probability of finding a job is obtained by dividing the entire numerator by $\tilde{u}$ or $\tilde{U}$, according to whether flows or stock are considered.

For example, the probability of finding a job for a job-seeker belonging to the new inflow is:

$$h_w(\tilde{u},\cdot) = h_w(\tilde{u},\tilde{v}) + h_w(\tilde{u},\tilde{V})$$

$$\Rightarrow h_w(\tilde{u},\cdot) = U^{\alpha-1}\cdot V^{-\alpha}\cdot\left[\tilde{v}\cdot\mu_w(\tilde{u},\tilde{v}) + \tilde{V}\cdot\mu_w(\tilde{u},\tilde{V})\right]$$

[52] For simplicity, only the probabilities of finding a job are shown, however the same procedure can obviously be applied to obtain the vacancies' *hazard rate*.

Similarly, the probability of finding a job for a job-seeker belonging to the pre-existing unemployed stock is:

$$h_w\left(\tilde{U},\cdot\right)=h_w\left(\tilde{U},\tilde{v}\right)+h_w\left(\tilde{U},\tilde{V}\right)$$

$$\Rightarrow h_w\left(\tilde{U},\cdot\right)=U^{\alpha-1}\cdot V^{-\alpha}\cdot\left[\tilde{v}\cdot\mu_w\left(\tilde{U},\tilde{v}\right)+\tilde{V}\cdot\mu_w\left(\tilde{U},\tilde{V}\right)\right]$$

Intuitively, and as previously mentioned, in *stock-flow matching* the probability of a job-seeker belonging to the pre-existing unemployed stock finding a job should be (realistically) lower for two reasons: (1) *stock-stock* matching is improbable; (2) it is easier for a flow to match with a stock rather than the opposite.

In general, the probability of finding a job depends on both the flow of new vacancies ($\tilde{v}$) and the pre-existing stock of vacancies ($\tilde{V}$).[53]

7.3 Stock-flow matching vs. random matching

The empirical literature supporting the *random matching* model, and which makes extensive use of a *Cobb-Douglas* functional form, focuses either on the estimate of the matching function (using aggregate data) or on the estimate of the probability of finding employment using data at the individual level (Petrongolo and Pissarides, 2001).

The comparison between the *stock-flow matching* model and the *random matching* model proposed by Andrews *et al*. (2009) is more precisely based on the estimate of the probability of finding employment using data at the microeconomic level. This interesting comparison is made particularly clear by the fact that Andrews *et al*. (2009) derive the standard matching model as a special case of the *stock-flow matching* model. In particular, under the null hypothesis that the true model is *random matching*, i.e.:

$$H_0:\ \mu_w\left(\tilde{u},\tilde{v}\right)=\mu_w\left(\tilde{u},\tilde{V}\right)=\mu_w\left(\tilde{U},\tilde{v}\right)=\mu_w\left(\tilde{U},\tilde{V}\right)=\mu$$

the workers' log-*hazard rate* is simply given by (recall that $\tilde{V}\equiv V-\tilde{v}\Rightarrow V=\tilde{V}+\tilde{v}$):[54]

$$\log h_w=\log(\mu)+(\alpha-1)\cdot\log(U)+(1-\alpha)\cdot\log(V)+\varepsilon_w$$

where the non observable heterogeneity of the job-seekers is only captured by the

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[53] Similar reasoning can obviously be extended to the probability of filling vacancies and depends on both the flow of new unemployed and the pre-existing stock of job-seekers.

[54] The hypothesis that only one parameter, μ, exists implies that the number of possible matches over the total number of contacts is equal to 1, i.e. *UV*/*UV*.

error term ε_w. The probability of finding employment, therefore, depends only on the overall number of vacancies and unemployed individuals.

Under the alternative hypothesis H_1, the *stock-flow matching* model is valid and the log-probabilities of finding employment are given by:

$$\log h_w(\tilde{u},\cdot) = \log\left[\tilde{v}\cdot\mu_w(\tilde{u},\tilde{v}) + (V-\tilde{v})\cdot\mu_w(\tilde{u},\tilde{V})\right] + (\alpha-1)\cdot\log(U) - \alpha\cdot\log(V) + \varepsilon_w$$

$$\log h_w(\tilde{U},\cdot) = \log\left[\tilde{v}\cdot\mu_w(\tilde{U},\tilde{v}) + (V-\tilde{v})\cdot\mu_w(\tilde{U},\tilde{V})\right] + (\alpha-1)\cdot\log(U) - \alpha\cdot\log(V) + \varepsilon_w$$

Andrews *et al.* (2009) propose a very simple and intuitive method for testing the validity of the *random matching* model with respect to the *stock-flow matching* model. Formally, the following is obtained:

$$\frac{\partial\log h_w(\tilde{u},\cdot)}{\partial\tilde{v}} = \frac{\mu_w(\tilde{u},\tilde{v}) - \mu_w(\tilde{u},\tilde{V})}{\left[\tilde{v}\cdot\mu_w(\tilde{u},\tilde{v}) + (V-\tilde{v})\cdot\mu_w(\tilde{u},\tilde{V})\right]}$$

$$\frac{\partial\log h_w(\tilde{U},\cdot)}{\partial\tilde{v}} = \frac{\mu_w(\tilde{U},\tilde{v}) - \mu_w(\tilde{U},\tilde{V})}{\left[\tilde{v}\cdot\mu_w(\tilde{U},\tilde{v}) + (V-\tilde{v})\cdot\mu_w(\tilde{U},\tilde{V})\right]}$$

under H_0 the variation of the (log) probability of finding a job with respect to a variation in the vacancies inflow is null (the numerator is in fact zero), whereas it is positive under the alternative hypothesis H_1, since $\mu_w(\tilde{U},\tilde{v}) > \mu_w(\tilde{U},\tilde{V}) \approx 0$, and $\mu_w(\tilde{u},\tilde{v}) > \mu_w(\tilde{u},\tilde{V})$.[55] In short, in the *random matching* model, the probability of finding a job depends only on the overall number of vacancies and of unemployed, therefore, the variation in the "*flow – stock*" composition has no effect.

Andrews *et al.* (2009) find that the stock of new vacancies has a significant positive impact on the job-seeker hazard, over and above that of the total stock of vacancies. Furthermore, there is an equivalent robust result for vacancy hazards. Thus they find evidence in favour of *stock-flow matching*.

However, an important consideration must be made. The fact that the flow of new vacancies has a positive impact on the probability of finding a job could be attributed to some form of unobservable heterogeneity. For example, a *"good-jobs / bad-jobs"* scenario could be prefigured in which the most attractive vacancies (usually the new ones) are filled more quickly and with greater ease (Coles and Petrongolo, 2008). Furthermore, as remarked by Andrews *et al.* (2009), the data

[55] The larger the flow of new vacancies, the lower the pre-existing stock of vacancies. As a consequence, the probability that the flow of newly unemployed matches with the pre-existing stock of vacancies is lower.

used in their empirical analyses refer to a particular labour market that offers an explicit matching service between job-seekers and vacancies (i.e. “The Lancashire Careers Service data”).

In conclusion, the *stock-flow matching* approach does not infer that search frictions are of scarce importance, it is simply a more realistic description of the behaviour of players active in search activity and in the labour market, such as the long term unemployed and workers that are well qualified for different jobs. These often prefer to wait for better work opportunities on the demand side, if a suitable job is not immediately available (Coles and Petrongolo, 2008).

MATHEMATICAL APPENDIXES

➢ Appendix A

Wages are determined starting from the first order condition for the optimal subdivision of surplus:

$$w = \arg\max (J - V)^{1-\beta} \cdot (W - U)^{\beta} \qquad \Rightarrow (W - U) = \frac{\beta}{1-\beta} \cdot (J - V)$$

by using the *Bellman equations*, it immediately follows that:

$$\Rightarrow \frac{w + \delta \cdot U}{r + \delta} - U = \frac{\beta}{1-\beta} \cdot \frac{y - w + \delta \cdot V}{r + \delta} - V$$

$$\Rightarrow w - rU = \frac{\beta}{1-\beta} \cdot (y - w - rV)$$

$$\Rightarrow w = (1-\beta) \cdot rU + \beta \cdot (y - rV)$$

since the *free-entry* condition ($V = 0 \Rightarrow J = c \cdot \theta^{\alpha}$) is valid, it is possible to deduce $rU = b + \frac{\beta}{1-\beta} \cdot c \cdot \theta$, from which the final expression is easily obtained:

$$w = (1-\beta) \cdot b + \beta \cdot y + \beta \cdot c \cdot \theta \qquad \textbf{[A.1]}$$

The surplus of a job, S, is defined as the sum of the worker's and firm's value of being on the job, net of the respective external options, so that:

$$S = J + W - V - U$$

applying basic algebra and using the *Bellman equations* the following is obtained:

$$rS = y + \delta \cdot (V - J + U - W) - \left[z + \theta^{1-\alpha} \cdot (W - U)\right] - \left[-c + \theta^{-\alpha} \cdot (J - V)\right]$$

finally, knowing that $(W - U) = \beta \cdot S$ and $(J - V) = (1-\beta) \cdot S$, we obtain:[56]

$$S = \frac{y - b + c}{r + \delta + \beta \cdot \theta^{1-\alpha} + (1-\beta) \cdot \theta^{-\alpha}} \qquad \textbf{[A.2]}$$

➢ Appendix B

In order to obtain the "new" *Job Creation Condition* (JCC), the equation for determining wages is substituted into the expression for $r \cdot J(x)$:

$$r \cdot J(x) = y \cdot x - \left[\underbrace{(1-\beta) \cdot b + \beta \cdot y \cdot x + \beta \cdot c \cdot \theta}\right] - \delta \cdot J(x) + \delta \cdot \int_R^1 J(s)\, dG(s)$$

[56] These rates can be obtained very simply from the first order condition for determining wages, i.e. $(W - U) = \beta / (1 - \beta)\ (J - V)$.

$$(r+\delta)\cdot J(x)=(1-\beta)\cdot(y\cdot x-b)-\beta\cdot c\cdot\theta+\delta\cdot\int_R^1 J(s)\,dG(s) \qquad [B.1]$$

The value of the equation [B.1] is found for $x=R$, with $J(R)=0$:

$$0=(1-\beta)\cdot(y\cdot R-b)-\beta\cdot c\cdot\theta+\delta\cdot\int_R^1 J(s)\,dG(s) \qquad [B.2]$$

The value of the equation [B.2] is subtracted from the equation [B.1], obtaining:

$$(r+\delta)\cdot J(x)=(1-\beta)\cdot(y\cdot x-y\cdot R)$$

$$\Rightarrow J(x)=\frac{(1-\beta)\cdot(y\cdot x-y\cdot R)}{(r+\delta)} \qquad [B.3]$$

considering equation [B.3] for $x=1$, (since the firm creates new jobs with maximum productivity), and using the expression for $J(1)$ obtained through the *zero-profit* condition, i.e.:

$$r\cdot V=-c+\theta^{-\alpha}\cdot[J(1)-V]\Rightarrow J(1)=\frac{c}{\theta^{-\alpha}}$$

the "new" *Job Creation Condition* (JCC) is obtained:

$$JCC\ \Rightarrow\ c\cdot\theta^{\alpha}=\frac{(1-\beta)\cdot y}{(r+\delta)}\cdot(1-R) \qquad \textbf{[B.4]}$$

from which it immediately follows that $\frac{d\theta}{dR}<0$.

The *Job Destruction Curve* is determined in the following way. Starting with equation [B.1]:

$$(r+\delta)\cdot J(x)=(1-\beta)\cdot(y\cdot x-b)-\beta\cdot c\cdot\theta+\delta\cdot\int_R^1 J(s)\,dG(s)$$

$J(s)$ is substituted with [B.3], where, obviously, $x=s$

$$(r+\delta)\cdot J(x)=(1-\beta)\cdot(y\cdot x-b)-\beta\cdot c\cdot\theta+\delta\cdot\int_R^1\frac{(1-\beta)\cdot y}{(r+\delta)}\cdot(s-R)\,dG(s) \qquad [B.5]$$

[B.5] is evaluated for $x=R$, which is the threshold productivity value of a job, below which the job itself is destroyed:

$$0=(1-\beta)\cdot y\cdot R-(1-\beta)\cdot b-\beta\cdot c\cdot\theta+\delta\cdot\frac{(1-\beta)\cdot y}{(r+\delta)}\cdot\int_R^1(s-R)\,dG(s) \qquad [B.6]$$

In order to obtain a clearer expression, all the members of [B.6] are divided by $(1-\beta)\cdot y$, obtaining:

$$JD\ \Rightarrow\ 0=R-\frac{b}{y}-\frac{\beta\cdot c\cdot\theta}{(1-\beta)\cdot y}+\frac{\delta}{(r+\delta)}\cdot\int_R^1(s-R)\,dG(s) \qquad \textbf{[B.7]}$$

completely differentiating this equation, we obtain:

$\frac{\beta \cdot c}{(1-\beta)\cdot y} \cdot d\theta = dR \cdot \left\{1-\left(\frac{\delta}{(r+\delta)}\right)\cdot[1-G(R)]\right\}$, with $\frac{d\theta}{dR} > 0$, since the last term between brackets is a product of two numbers, both smaller than one.

➢ Appendix C

The *free-entry* condition for equilibrium is valid even out of the stationary state:

$$V(t)=0 \Rightarrow J(t)=\frac{c}{\theta(t)^{-\alpha}} \ \forall \ t$$

The same applies to the rule for subdividing surplus. The wage is, therefore, determined in the same way in both stationary equilibrium and during adjustment:

$$w(t)=(1-\beta)\cdot b+\beta\cdot y+\beta\cdot c\cdot\theta(t)$$

The dynamic of $J(t)$ out of the equilibrium state is, instead, given by:

$$r\cdot J(t)=(y-w(t))+\delta\cdot[V(t)-J(t)]+\dot{J}(t)$$

$$\Rightarrow \dot{J}(t)=(r+\delta)\cdot J(t)-(y-w(t)) \qquad \text{[C.1]}$$

Differentiating $J(t)=c\cdot\theta(t)^{\alpha}$ with respect to time we obtain:

$$\dot{J}(t)=c\cdot\alpha\cdot\theta(t)^{\alpha-1}\cdot\dot{\theta}(t) \qquad \text{[C.2]}$$

Substituting [C.2] into [C.1], we obtain:

$$c\cdot\alpha\cdot\theta(t)^{\alpha-1}\cdot\dot{\theta}(t)=(r+\delta)\cdot J(t)-(y-w(t))$$

$$c\cdot\alpha\cdot\theta(t)^{\alpha-1}\cdot\dot{\theta}(t)=(r+\delta)\cdot c\cdot\theta(t)^{\alpha}-y+w(t)$$

Since $J(t)=c\cdot\theta(t)^{\alpha}$; substituting wage into the previous expression we get:

$$\frac{c\cdot\alpha}{\theta(t)^{1-\alpha}}\cdot\dot{\theta}(t)=(r+\delta)\cdot c\cdot\theta(t)^{\alpha}-y+\underbrace{(1-\beta)\cdot b+\beta\cdot y+\beta\cdot c\cdot\theta(t)}_{w(t)}$$

And finally we obtain the differential equation for θ:

$$\dot{\theta}(t)=\frac{(r+\delta)}{\alpha}\cdot\theta(t)-\frac{\theta(t)^{1-\alpha}\cdot(1-\beta)\cdot(y-b)}{c\cdot\alpha}+\frac{\beta\cdot\theta(t)^{2-\alpha}}{\alpha} \qquad \textbf{[C.3]}$$

with: $\frac{d\dot{\theta}(t)}{d\theta(t)}=\frac{r+\lambda}{\alpha}-\frac{(1-\alpha)\cdot(1-\beta)\cdot(y-b)}{c\cdot\alpha}\cdot\theta(t)^{-\alpha}+(2-\alpha)\cdot\frac{\beta}{\alpha}\cdot\theta(t)^{1-\alpha}>0$.

Note that the $lim_{\theta\to 0}$ of the expression [C.3] tends to $-\infty$, whereas the $lim_{\theta\to\infty}$ of the expression [C.3] tends to $+\infty$.

Appendix D

The representative firm i solves the following optimisation problem:[57]

$$\max_{v_i} \int_0^{\infty} \left[y \cdot n_i - w \cdot n_i - c \cdot v_i \right] \cdot e^{-r \cdot t} \, dt$$

subject to the constraint given by employment's evolution over time:

$$\dot{n}_i = \theta^{-\alpha} \cdot v_i - \delta \cdot n_i$$

The representative firm, that can open more than one vacancy at a time, takes the value of labour market tension as given, ignoring the effects that their own decisions will have on the aggregate conditions of labour market tension. Setting up the *Hamiltonian* we have that:

$$H(t) = \left\{ y \cdot n_i - w \cdot n_i - c \cdot v_i + \Lambda(t) \cdot \left[\theta^{-\alpha} \cdot v_i - \delta \cdot n_i \right] \right\} \cdot e^{-r \cdot t}$$

where v_i is the control variable, n_i is the state variable and $\Lambda(t)$ is the so called "*shadow value*" that specifically expresses the marginal value of a filled job for the firm. The optimisation solutions to the problem are the following:[58]

$$\frac{\partial H(t)}{\partial v_i} = 0 \Rightarrow \left[-c + \Lambda(t) \cdot \theta^{-\alpha} \right] \cdot e^{-r \cdot t} = 0$$

$$\Rightarrow \Lambda(t) = \frac{c}{\theta^{-\alpha}} \qquad \text{[D.1]}$$

$$\frac{\partial H(t)}{\partial n_i} = -\frac{d\left(\Lambda(t) \cdot e^{-r \cdot t}\right)}{dt} \Rightarrow \left[y - w - \Lambda(t) \cdot \delta \right] \cdot e^{-r \cdot t} = -\left[\dot{\Lambda}(t) - r \cdot \Lambda(t) \right] \cdot e^{-r \cdot t}$$

dove $-\frac{d\left(\Lambda(t) \cdot e^{-r \cdot t}\right)}{dt} = -\left[\dot{\Lambda}(t) \cdot e^{-r \cdot t} + \Lambda(t) \cdot (-r) \cdot e^{-r \cdot t} \right]$

$$\Rightarrow (y - w) = (r + \delta) \cdot \Lambda(t) - \dot{\Lambda}(t) \qquad \text{[D.2]}$$

[D.1] is a standard optimality condition: in equilibrium, the marginal value of a filled job must be equal to the expected cost of a vacancy. [D.2] on the other hand expresses the evolution in time of the marginal value of a filled vacancy. In the steady state, with $\dot{\Lambda}(t) = 0$, combining the two solutions, the standard equilibrium condition is obtained for the job demand side, i.e. the *Job Creation Condition*:

$$\frac{y - w}{r + \delta} = \frac{c}{\theta^{-\alpha}} \qquad \textbf{[D.3]}$$

[57] For simplicity, as is common in matching models, it is assumed that the marginal productivity of labour is a linear function of employment.

[58] The optimisation solutions also include the necessary trasversality condition: $\lim_{t \to \infty} \Lambda(t) \cdot e^{-r \cdot t} \cdot n_i = 0$.

which is exactly the same as the JCC obtained in paragraph 4.1.

As regards the socially efficient solution, the maximisation problem is the following:

$$\max_{v} \int_0^\infty [y \cdot n + b \cdot (1-n) - c \cdot v] \cdot e^{-r \cdot t} \, dt$$

$$\dot{n} = v \cdot \left(\frac{v}{1-n} \right)^{-\alpha} - \delta \cdot n$$

In this case, the value of labour market tension is endogenous. Moreover, the socially efficient solution ignores the wage and considers the utility flows obtained from unemployed workers, i.e. $b \cdot (1-n)$, where the labour force is, for simplicity, normalised to 1; hence, $(1-n)$ is the unemployment rate. As before, the optimisation solutions are obtained by formulating the *Hamiltonian*:

$$H(t) = \left[y \cdot n + b \cdot (1-n) - c \cdot v + \Lambda(t) \cdot \left[v \cdot \left(\frac{v}{1-n} \right)^{-\alpha} - \delta \cdot n \right] \right] \cdot e^{-r \cdot t}$$

$$\frac{\partial H(t)}{\partial v} = 0 \Rightarrow \left\{ -c + \Lambda(t) \cdot \left[\left(\frac{v}{1-n} \right)^{-\alpha} + v \cdot (-\alpha) \cdot \left(\frac{v}{1-n} \right)^{-\alpha-1} \cdot \frac{1}{1-n} \right] \right\} \cdot e^{-r \cdot t} = 0$$

$$\Rightarrow \Lambda(t) = \frac{c}{\theta^{-\alpha} - \alpha \cdot \theta^{-\alpha}} = \frac{c}{(1-\alpha) \cdot \theta^{-\alpha}} \qquad \text{[D.4]}$$

$$\frac{\partial H(t)}{\partial n_i} = -\frac{d\left(\Lambda(t) \cdot e^{-r \cdot t}\right)}{dt}$$

$$\left\{ y - b - \Lambda(t) \cdot \delta + \Lambda(t) \cdot \left[v \cdot v^{-\alpha} \cdot \alpha \cdot (1-n)^{\alpha-1} \cdot (-1) \right] \right\} \cdot e^{-r \cdot t} = -\left[\dot{\Lambda}(t) - r \cdot \Lambda(t) \right] \cdot e^{-r \cdot t}$$

$$\Rightarrow (y-b) = \left(r + \delta + \alpha \cdot \theta^{1-\alpha} \right) \cdot \Lambda(t) - \dot{\Lambda}(t) \qquad \text{[D.5]}$$

combining [D.4] with [D.5] the marginal value of a filled job is obtained in the steady state, i.e. the *Job Creation Condition* in the decentralised equilibrium:

$$\frac{y-b}{\left(r + \delta + \alpha \cdot \theta^{1-\alpha} \right)} = \frac{c}{(1-\alpha) \cdot \theta^{-\alpha}} \qquad \textbf{[D.6]}$$

➢ Appendix E

Using the *Bellman equations*,

$$rV(\theta) = rU(\theta)$$

$$\Rightarrow (z-c) + \theta^{-\alpha} \cdot [J - V] = z + \theta^{1-\alpha} \cdot [W - U]$$

$$-c + \theta^{-\alpha} \cdot (1-\beta) \cdot S = \theta^{1-\alpha} \cdot \beta \cdot S$$

with $S = \dfrac{y - 2z + c}{r + \delta + (1-\beta)\cdot\theta^{-\alpha} + \beta\cdot\theta^{1-\alpha}}$, we obtain:

$$0 = \theta^{1-\alpha}\cdot\beta\cdot(y - 2z + 2c) - \theta^{-\alpha}\cdot(1-\beta)\cdot(y - 2z) + c(r+\delta) \qquad \textbf{[E.1]}$$

which is defined in the following way,

$$C(\theta) = \theta^{1-\alpha}\cdot\beta\cdot(y - 2z + 2c) - \theta^{-\alpha}\cdot(1-\beta)\cdot(y - 2z) + c(r+\delta)$$

given the so called *Inada* conditions:

$$\lim_{\theta\to 0}\theta^{1-\alpha} = \lim_{\theta\to\infty}\theta^{-\alpha} = 0,\ \lim_{\theta\to\infty}\theta^{1-\alpha} = \lim_{\theta\to 0}\theta^{-\alpha} = \infty$$

with $y - 2z > 0$, we obtain:

$$C'(\theta) = (1-\alpha)\cdot\theta^{-\alpha}\cdot\beta\cdot(y - 2z + 2c) - (-\alpha)\cdot\theta^{-\alpha-1}\cdot(1-\beta)\cdot(y - 2z) > 0$$

$$\lim_{\theta\to 0} C(\theta) = -\infty$$

$$\lim_{\theta\to\infty} C(\theta) = +\infty$$

as a consequence, the *intermediate value theorem* implies the existence of a solution and the monotonic nature of $C(\theta)$ guarantees uniqueness.

Once the uniqueness of the equilibrium value of the vacancy-unemployment relationship is guaranteed, it is possible to describe the allocation of the individuals between entrepreneurship and labour force (i.e. to know the equilibrium values of l). In steady state we have:

$$v = \frac{\delta\cdot(1-l)}{\delta + \theta^{-\alpha}}$$

$$u = \frac{\delta\cdot l}{\delta + \theta^{1-\alpha}}$$

for the steady level of vacancies and unemployment to be consistent with the equilibrium value of labour market tensions, the $\theta = \dfrac{v(l)}{u(l)}$ relationship must be respected. Solving the former expression for l, it is possible to obtain the equilibrium value of workers and, as a consequence, of entrepreneurs $(1-l)$:

$$\Rightarrow \theta = \frac{\delta\cdot(1-l)}{\delta + \theta^{-\alpha}}\cdot\frac{\delta + \theta^{1-\alpha}}{\delta\cdot l} \Rightarrow \theta = \frac{(1-l)}{l}\cdot\frac{\delta + \theta^{1-\alpha}}{\delta + \theta^{-\alpha}}$$

$$\Rightarrow \theta\cdot l\cdot\left(\delta + \theta^{-\alpha}\right) = (1-l)\cdot\left(\delta + \theta^{1-\alpha}\right)$$

$$\Rightarrow \theta\cdot l\cdot\left(\delta + \theta^{-\alpha}\right) + l\cdot\left(\delta + \theta^{1-\alpha}\right) = \delta + \theta^{1-\alpha}$$

$$l = \frac{\delta + \theta^{1-\alpha}}{\delta\cdot(1+\theta) + 2\cdot\theta^{1-\alpha}} \qquad \textbf{[E.2]}$$

➢ Appendix F

From the *Bellman equations*,

$$r \cdot V = -c + \theta^{-\alpha} \cdot (J - V) \qquad r \cdot J = y - w + \delta \cdot (V - J)$$

$$r \cdot W = w + \delta \cdot (U - W) \qquad r \cdot U = z + \theta^{1-\alpha} \cdot (W - U)$$

very simple algebra gives:

$$[J - V] = \frac{y - w + c}{r + \delta + \theta^{-\alpha}}; \qquad [W - U] = \frac{w - z}{r + \delta + \theta^{1-\alpha}}$$

Hence, it is straightforward to get:

$$rV = \frac{\theta^{-\alpha} \cdot (y - w) - c \cdot (r + \delta)}{r + \delta + \theta^{-\alpha}} \qquad \textbf{[F.1]}$$

$$rU_r = \frac{\theta^{1-\alpha} \cdot w + z \cdot (r + \delta)}{r + \delta + \theta^{1-\alpha}} \qquad \textbf{[F.2]}$$

with $\frac{\partial rV}{\partial \theta} = -(y - w + c) \cdot (r + \delta) < 0$ and $\frac{\partial rU}{\partial \theta} = (w - z) \cdot (r + \delta) > 0$, since it must be true that $w > z$. Furthermore,

$lim_{\theta_r \to 0} rV_r = y - w$, by the l'*Hôpital rule*; $lim_{\theta_i \to 0} rU = z$

$lim_{\theta_i \to \infty} rV = -c$; $lim_{\theta_i \to \infty} rU = w$, by the l'*Hôpital rule*.

The evolution of employment can be expressed in terms of both firm's transition rates ($\theta^{-\alpha}$) and worker's transition rates ($\theta^{1-\alpha}$), i.e.:

$$\dot{n} = \left(\int_S^{\vartheta_{\max}} \vartheta \, dF(\vartheta) - n \right) \cdot \theta^{-\alpha} - \delta \cdot n \qquad \dot{n} = [F(S) - n] \cdot \theta^{1-\alpha} - \delta \cdot n$$

Hence, in steady-state ($\dot{n}_r = 0$), we get:

$$n = \frac{\left(\int_S^{\vartheta_{\max}} \vartheta \, dF(\vartheta) \right) \cdot \theta^{-\alpha}}{\theta^{-\alpha} + \delta} \qquad n = \frac{F(S) \cdot \theta^{1-\alpha}}{\theta^{1-\alpha} + \delta}$$

It follows that for any level of employment *n*,

$$\frac{\left(\int_S^{\vartheta_{\max}} \vartheta \, dF(\vartheta) \right) \cdot \theta^{-\alpha}}{\theta^{-\alpha} + \delta} = \frac{F(S) \cdot \theta^{1-\alpha}}{\theta^{1-\alpha} + \delta}$$

Straightforward algebra gives:

$$\frac{\int_S^{\vartheta_{\max}} \vartheta \, dF(\vartheta)}{F(S)} = \frac{\theta^{1-\alpha} + \delta \cdot \theta}{\theta^{1-\alpha} + \delta} \qquad \textbf{[F.3]}$$

By the properties of the matching function, the right-hand side is increasing in θ; whereas, the left-hand side is decreasing in S. Therefore, $\frac{d\theta}{dS} < 0$.

➢ Appendix G

Let us rewrite the unemployment rate equation as:

$$u = \frac{L \cdot \delta \cdot (\delta + \rho) \cdot u^{1-\alpha}}{\delta \cdot (\delta + \rho) \cdot u^{1-\alpha} + (\delta + \rho) \cdot (v_F)^{1-\alpha} + \delta \cdot (v_I)^{1-\alpha}}$$

$$u^{\alpha} = \frac{L \cdot \delta \cdot (\delta + \rho)}{\delta \cdot (\delta + \rho) \cdot u^{1-\alpha} + (\delta + \rho) \cdot (v_F)^{1-\alpha} + \delta \cdot (v_I)^{1-\alpha}}$$

$$\left(\delta \cdot (\delta + \rho) \cdot u^{1-\alpha} + (\delta + \rho) \cdot (v_F)^{1-\alpha} + \delta \cdot (v_I)^{1-\alpha}\right) \cdot u^{\alpha} = L \cdot \delta \cdot (\delta + \rho)$$

$$\delta \cdot (\delta + \rho) \cdot u + \left((\delta + \rho) \cdot (v_F)^{1-\alpha} + \delta \cdot (v_I)^{1-\alpha}\right) \cdot u^{\alpha} = L \cdot \delta \cdot (\delta + \rho)$$

$$u + \left(\frac{(v_F)^{1-\alpha}}{\delta} + \frac{(v_I)^{1-\alpha}}{\delta + \rho}\right) \cdot u^{\alpha} = L$$

Now, let us calculate $\frac{\partial u}{\partial v_I}$:

$$\frac{\partial}{\partial u}\left(u + \left(\frac{(v_F)^{1-\alpha}}{\delta} + \frac{(v_I)^{1-\alpha}}{\delta + \rho}\right) \cdot u^{\alpha} - L\right) = 1 + \alpha \cdot u^{\alpha-1} \cdot \left(\frac{(v_F)^{1-\alpha}}{\delta} + \frac{(v_I)^{1-\alpha}}{\delta + \rho}\right)$$

$$\frac{\partial}{\partial v_I}\left(u + \left(\frac{(v_F)^{1-\alpha}}{\delta} + \frac{(v_I)^{1-\alpha}}{\delta + \rho}\right) \cdot u^{\alpha} - L\right) = u^{\alpha} \cdot \left(\frac{(1-\alpha)}{(\delta + \rho) \cdot {v_I}^{\alpha}} + \frac{(\alpha - 1)}{\delta \cdot {v_F}^{\alpha}}\right)$$

since $v_F = E - v_I$, where E is the number of entrant entrepreneurs. Hence, we get:

$$\frac{\partial u}{\partial v_I} = -\frac{(1-\alpha) \cdot u^{\alpha} \cdot \left(\frac{1}{(\delta + \rho) \cdot {v_I}^{\alpha}} - \frac{1}{\delta \cdot {v_F}^{\alpha}}\right)}{1 + \alpha \cdot u^{\alpha-1} \cdot \left(\frac{(v_F)^{1-\alpha}}{\delta} + \frac{(v_I)^{1-\alpha}}{\delta + \rho}\right)}$$

Finally, let us find the threshold value for ρ where $\frac{\partial u}{\partial v_I} = 0$:

$$\frac{1}{(\delta + \rho) \cdot {v_I}^{\alpha}} = \frac{1}{\delta \cdot {v_F}^{\alpha}} \Rightarrow \delta \cdot {v_F}^{\alpha} = (\delta + \rho) \cdot {v_I}^{\alpha}$$

$$\rho = \delta \cdot \left(\left(\frac{v_F}{v_I}\right)^{\alpha} - 1\right) \equiv \rho^* \qquad \textbf{[G.1]}$$

FIGURES

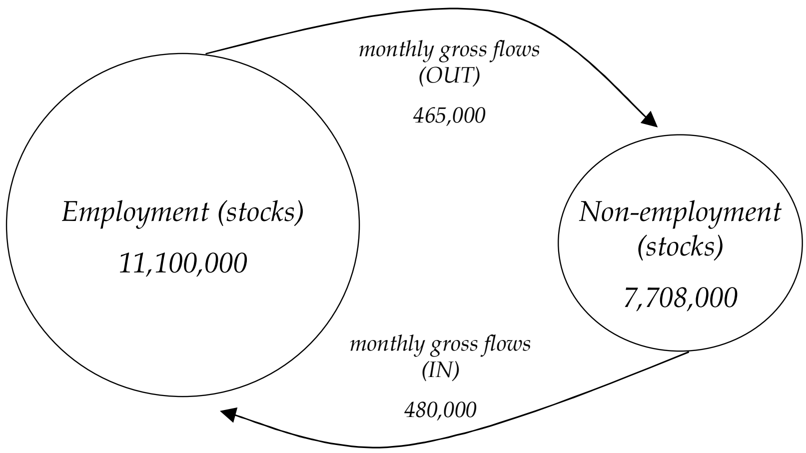

Figure 1. *Average labour market stocks and flows, Canada (1976 – 1991)*
Source: Jones (1993), quoted in: Andolfatto (2008)

Figure 2. *Canadian unemployment and vacancies rates (1966 – 1988).*
Source: Jones (1993), quoted in: Andolfatto (2008)

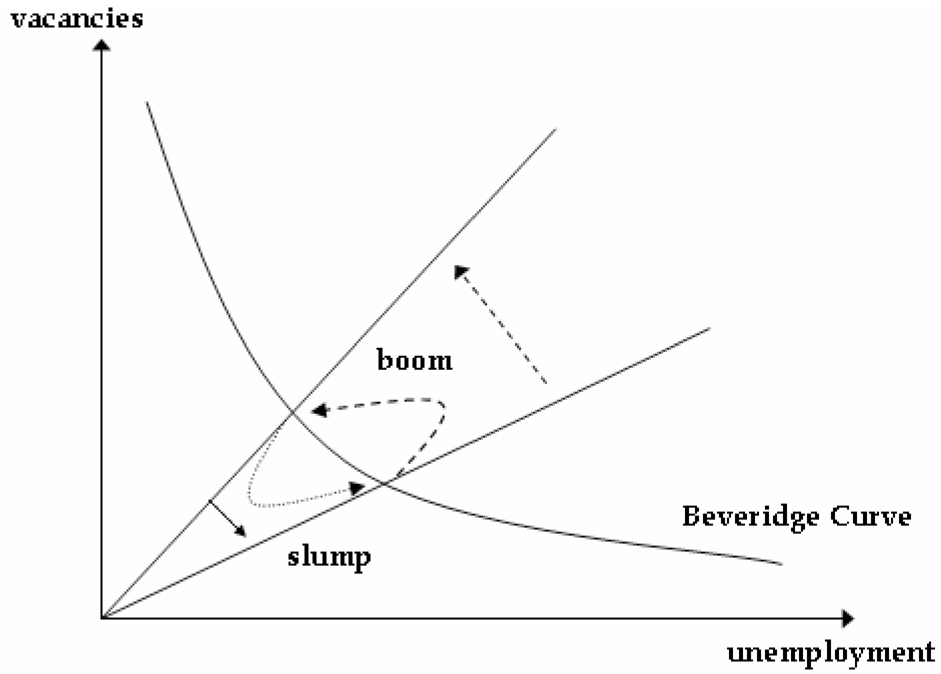

Figure 3. *Beveridge curve and business cycle*

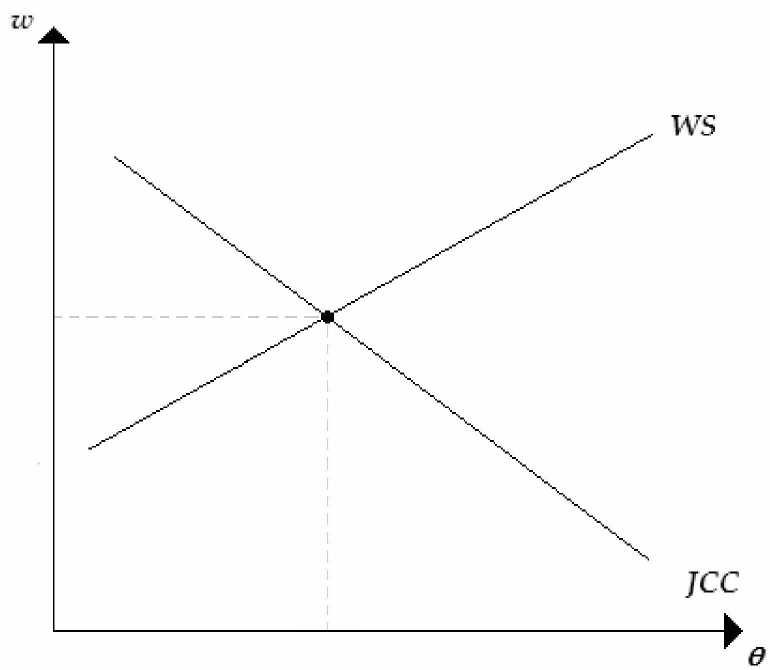

Figure 4. *Equilibrium wage and market tightness*

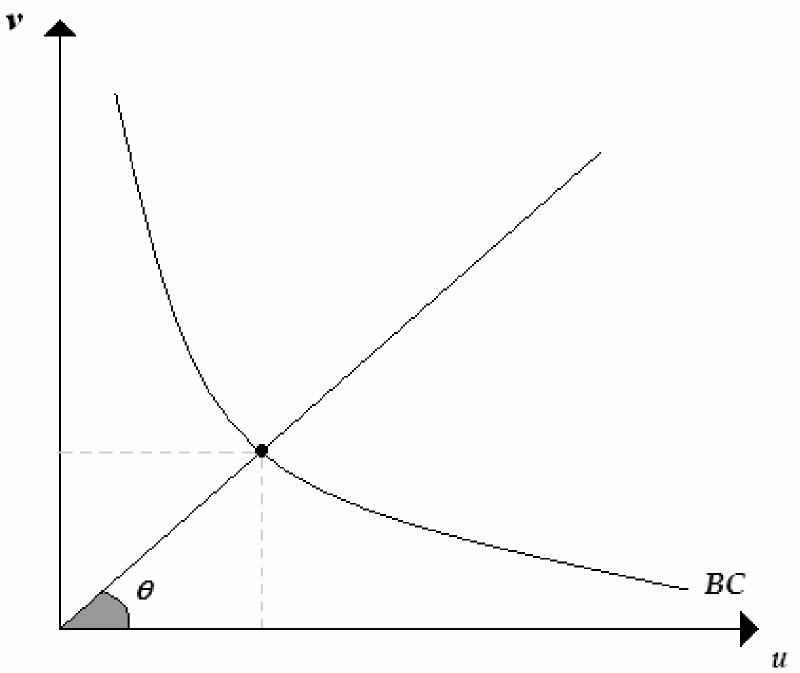

Figure 5. *Equilibrium vacancies and unemployment*

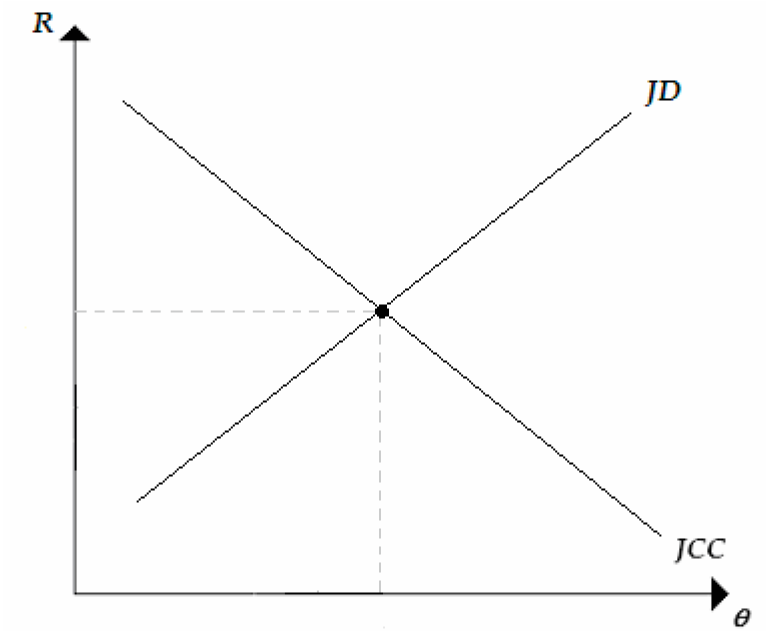

Figure 6. *Equilibrium reservation productivity and market tightness*

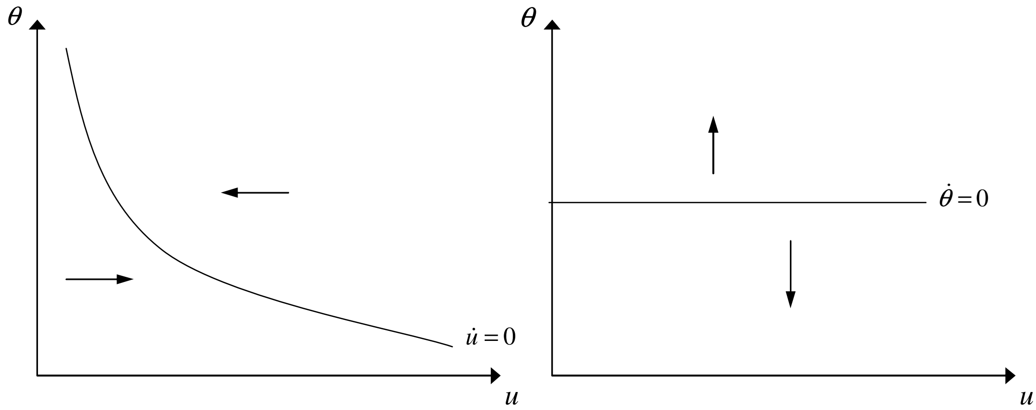

Figure 7*. Out-of-steady-state dynamics of unemployment and market tightness*

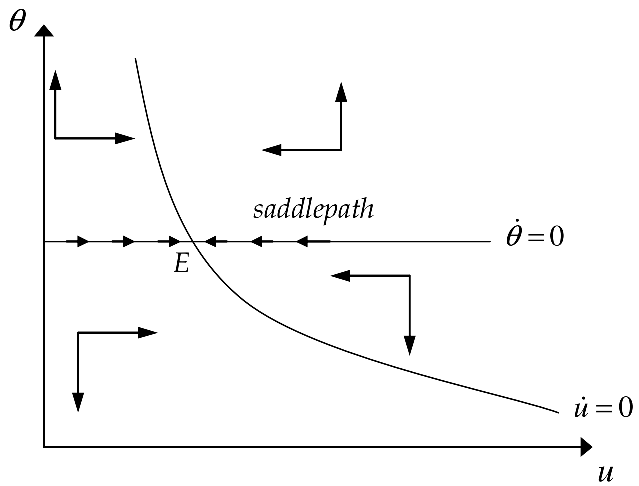

Figure 8*. Adjustment paths in labour-market tightness and unemployment space*

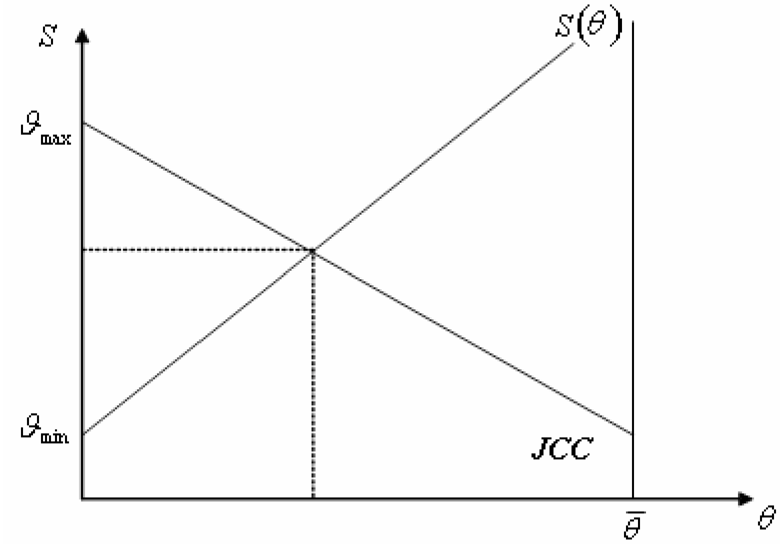

Figure 9*. Equilibrium reservation (entrepreneurial) ability and market tightness*

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