Wassily Leontief and Léon Walras: the Production as a Circular Flow

Akhabbar, Amanar and Lallement, Jérôme

Lausanne University, Centre Walras-Pareto, Centre d'Economie de la Sorbonne

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Amanar AKHABBAR*
Jérôme LALLEMENT**

"The controversial assumption of constant input ratios (fixed "production coefficients") enables us to engage in a factual study of interindustrial relationships and to apply the powerful tools of general equilibrium theory"
Wassily Leontief (1949, 213)

"The leading example of an eminent economist who argues on the basis of the classical theory of prices of production while believing himself to be a Walrasian –as Joan Robinson has observed repeatedly– is W. Leontief"
Bertram Schefold (1989, 333)

Prelude: Models of Leontief as duck-rabbit pictures

If a model was a (logical) picture, Leontief's input-output models would be like the duck-rabbit picture Wittgenstein used to mention (Figure 1). As a matter of perception, facing such a picture one may first recognize a rabbit, or a duck, or even a duck-rabbit. "I see two pictures, with the duck-rabbit surrounded by rabbits in one, by ducks in the other" (Wittgenstein [1953] 2001, 167), and it is always a matter of interpretation: "We can also see the illustration now as one thing or another. —So we interpret it, and *see* it as we *interpret* it" (Wittgenstein [1953] 2001, 165).

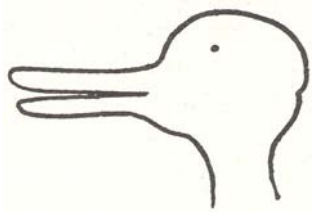

Figure 1 The duck-rabbit picture (Wittgenstein 1953)[1]

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* University of Lausanne – Centre Walras-Pareto, amanar.akhabbar@unil.ch
** University Paris Descartes and Centre d'Economie de la Sorbonne (University Paris 1), jerome.lallement@univ-paris1.fr
[1] Wittgenstein referred to the famous optical illusion of the duck-rabbit picture designed by the American psychologist Joseph Jastrow.

Replace "duck" by "classical" and "rabbit" by "Walrasian" and then one catches the dilemma of the interpretation of the models of Leontief. For instance, Luigi Pasinetti (1981) firmly sees in input-output analysis a simple classical model; Lionel McKenzie (2002) sees a Walrasian general equilibrium model. Wassily Leontief himself played with the ambiguous shape of his models, characterizing them sometimes as classical and sometimes as neoclassical. Is there a way out from this ambiguity? "If asked "What's that?" or "What do you see here?" I should have replied: "A picture-rabbit". If I had further been asked what that was, I should have explained by pointing to all sorts of pictures of rabbits, should perhaps have pointed to real rabbits, talked about their habits, or given an imitation of them" (166). May be that we should simply say that it's a picture. But, if someone tries to convince me that the picture is the one of a rabbit (or a duck) then I may examine his arguments and argue myself.

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When in 1936 Wassily Leontief[2] presented for the first time his input-output table, he specified that his "statistical research […] has been undertaken with the definite aim of supplying an empirical background for the study of the interdependence between the different parts of our national economy on the basis of the theory of general economic equilibrium" (1936, 116). The following year, Leontief exposed the theoretical model of this interdependence and rested explicitly his analysis on a simplified Walrasian model of which he took the equations of production and the coefficients of production. The model permitted to determine simultaneously equilibrium prices and quantities for a *m* goods economy (including households services) in a static framework without capital or land. Obviously, as far as Leontief gave up Walras' equations of supply and demand, the so-called model of Leontief is a sharp simplification of the Walrasian general equilibrium theory. Rejection of the supply and demand theory of prices and of the theory of marginal productivity introduced a distance between his model and usual neoclassical theories –especially concerning the theory of exchange. Indeed, given the state of econometric techniques in the 1930s, Leontief judged supply and demand equations not relevant and too complex for empirical implementation. Nevertheless, the technical coefficients of the input-output model were explicitly defined in the same way as the "coefficients of fabrication" Walras employed to analyze production in the Part 4 of the *Elements of pure economics* (1874). Thereafter, Leontief, like all his contemporaries, associated input-output analysis to an approach framed in the terms of Walrasian general equilibrium theory. For instance, in a report of the meeting of the *Econometric Society*, in Atlantic City in December 1937, Dickson Leavens noticed that "[Leontief's] theoretical scheme was characterized as that of a simplified Walrasian system" (Leavens 1938, 190). In his review in *The American Economic Review*, Hans Neisser –who had worked in Kiel like Leontief– presented *The Structure of American Economy* as "the description of the American economy in 1919 and 1929 in terms of a neoclassical equation system" (Neisser 1941, 608). Neisser underlined the "ingenuity [Leontief showed] in simplifying the Walras system in such a way that it, for the first time, became available to quantitative determination" (610). During the 1950s and 1960s, at a time when linear programming was developed and general equilibrium theory was renewed, the model of Leontief was presented in scientific articles and textbooks as a particular case of general

[2] Wassily Leontief (1905-1999) was born in Munich (Germany) and grew up in Saint Petersburg in a bourgeois family. In 1925 Leontief moved from USSR to Berlin (Weimar Republic). In *Humboldt-Universität zu Berlin* his PhD supervisor was Werner Sombart. At first Sombart's assistant, Leontief also worked at the Kiel *Institut für Weltwirtschaft* in the department for business cycles analysis. His PhD dissertation was published under the title "*Die Wirtschaft als Kreislauf*" (1928) (The economy as a circular flow). In 1931 he moved to the United States.

equilibrium theory (see Dorfman, Solow and Samuelson 1958). With the works of Moore and Schultz on applied general equilibrium, it gave birth to the tradition of Walrasian empirical models. Nowadays, this picture of input-output analysis as an empirical application of general equilibrium theory endures, especially in the United-States. Showen and Whalley underlined that their computable general equilibrium models "extend Wassily Leontief's work on empirical Walrasian models based on fixed input-output coefficients" (1984, 1008) and, according to Dale Jorgenson, "Leontief (1941) is the founder of empirical general equilibrium modeling, beginning with the implementation of the static input-output model" (1998, xiii)[3].

However, from the 1970s onwards, numerous elements have come out to break up the consensus about the nexus between input-output models and general equilibrium theory. Until then faithful to the neoclassical Cambridge, Leontief entered the rhetoric of conflict reactivated by Sraffa's followers. Indeed, Leontief himself underlined the closeness of his approach to classical theory. In his interviews published by Rosier (1986)[4], Leontief admitted that the way he used to refer to general equilibrium theory was a way to make his approach more attractive: "I certainly wanted to legitimate my theory; to give it an aristocratic origin. If there is an influence, it is mainly the one of classical economists. But Walras already had the idea of the [technical] coefficients" (Leontief *in* Rosier 1986, 89). The following year, in his presentation of input-output analysis in the *New Palgrave*, Leontief wrote that input-output analysis was "a practical extension of the classical theory of general interdependence" (Leontief 1987, 860). Even if Leontief gave latter contradictory interpretations of his input-output theory like for example when he said in an interview with Duncan Foley that he "would interpret [input-output analysis] as an outgrowth of neoclassical theory" (Leontief *in* Foley 1998, 129), Leontief's declarations reflected a general change in the theoretical interpretation of his models. Indeed, his hesitations were accompanied by a move to relate input-output analysis to Neo-Ricardian economics notably by Luigi Pasinetti and Giorgio Gilibert. For instance Pasinetti presented "Leontief's input-output analysis and Sraffa's production of commodities scheme" (1981, 17) as similar classical approaches and he dealt with the model of Leontief as a simplified Sraffian model (1977).

Why such a turn? The re-appropriation of the models of Leontief by classical authors rests on Sraffa's famous distinction between marginalism and classical economics based on the representation of production: the former would be a one-way avenue process, from factors of production to final goods, and the latter a circular flow where commodities are produced by means of commodities. Obviously Leontief' input-output model would belong to the circular flow view. Nowadays, in Europe mainly, it seems to be admitted that models of Leontief are classical. Such a view may be found in the works of authors like Christian Bidard, Heinz D. Kurz, Bertram Schefold, and Neri Salvadori notably. Neo-Ricardian interpretations referred in particular to Leontief's PhD dissertation (1928), "The economy as a circular flow", and his first interindustrial input-output matrices (1937, 1941). Neo-Ricardians' interpretation of the so-called 'models of Leontief' may be summed up in two points: first, this move insisted on the embedment of Leontief's works in classical thought, both concerning its value side and its production side; second, these authors denied the theoretical relationship between input-output analysis and general equilibrium theory. We will put under scrutiny the Neo-Ricardian economists' arguments who aim to show that the world-view and the theory underlying the models of Leontief are not only classical by essence but that they are not compatible with Walrasian theory. The idea that input-output analysis and Walras' theory are absolutely foreign one to another seems suspicious to us.

[3] See also ten Raa (2006) and Baumol and ten Raa (2009).
[4] Interviews of Leontief published by Rosier (1986) are in French. We translate.

The paper falls into four parts. We first present the Walrasian interpretation and the neo-Ricardian interpretation of the models of Leontief. We show that, ultimately, disagreements rest on the definition of the so-called "circular flow" that appears to be the key concept of the Neo-Ricardian interpretation of input-output models. The second part deals with Leontief's definitions and implementations of the concept of the circular flow. In the third part we continue with a comparative examination of Walras and Leontief's view of production as a circular flow. Section 4 concludes.

I. Postwar interpretations of the models of Leontief: the rhetoric of conflict

Since Leontief's first publications of the input-output table (1936) and of the so-called closed model of Leontief (1937) the stories that have been told about input-output analysis's basis have changed. Two main interpretations struggled. A common point to all them, beginning with Leontief's interpretation, is their claim for a unique truth and their shared rhetoric inherited from neo-positivism: the project of a unified science which gets rid of metaphysical conceptions like "utility" or "labor value" and which is firmly based on observable facts[5]. We first examine the neoclassical claims for input-output theory and then the arguments of neo-Ricardian economists in favor of a Ricardo-Marx view of the models of Leontief[6].

Models of Leontief as particular cases of general equilibrium theory

In neoclassical economics' extensive move, Samuelson tried to show that the model of Leontief was both compatible with the classical theory (according to Samuelson's definitions) and with the neoclassical theory in order to show that classical economics is a particular case of Walrasian general equilibrium theory. Samuelson asked: "Do you think that God created the earth with smooth Wicksteed homogeneous production functions involving a few aggregative factors, Socially Necessary Labor, Efficiency-unit Land, and Catch-all Dollar Capital? To deny such a belief is not to confirm a belief in fixed coefficients" (1957, 907). What mattered to Samuelson was the role of constant technical coefficients, of demand, and of marginal productivity theory in general equilibrium models. Samuelson tried to show that the "constant technical coefficients" case is a particular case of the "factors substitution" case. Of course the following results rest on Samuelson's (1951) and Georgescu-Roegen's (1951) non-substitution theorems.

In a long triptych, Samuelson (1957, 1959a and 1959b) formulated a family of Marxian and Ricardo-like models in the language of linear programming[7]. According to Samuelson, Marxian and Ricardian models are first of all models with one primary factor only (labor),

[5] See Samuelson's revealed preferences and operationist methodology; see also Sraffa's objectivist methodology and his claim for a theory based only on observable data, etc. Schefold wrote: "the difference between classical and neoclassical economics represents the most important divide in the history of economic thought. There is also a different point of view, which allows us to speak of an 'age of ideology' in which the explanation of value through the metaphysical concept of abstract labour or, alternatively, subjective utility became unduly prominent; this period was preceded by an age of more pragmatic 'Ricardian' analysis and was followed by more technocratic approaches with Leontief and Keynes each introducing tools for planning under capitalism" (1989, 336).

[6] In this article we deal only with models of Leontief where quantities of goods are expressed in physical terms (Leontief 1928, 1937, 1941) and not in monetary terms like in the input-output tables. For a comparison of models of Leontief in monetary terms with models in physical terms see for instance Bidard (2004, 26).

[7] According to Samuelson, "David Ricardo propounded a number of what we today should call linear programming problems. Except in the simplest cases he was not able to give complete and correct answers. Yet, despite a number of false conjectures, he did intuitively perceive properties of the equilibrium configuration which he probably could not have rigorously proved. I intend to cast a few of his problems in today's symbolism" (Samuelson 1959a, 1).

like the open model of Leontief[8]. All the other goods may be produced by means of other goods under constant returns to scale technologies. According to Samuelson this model corresponds to a special age: "before good land was scarce or capital goods dreamed of, life was simple and fitted a one-primary-factor theory. Smith, Ricardo, and for that matter the prewar Leontief, would call this single factor "labor"." (1959a, 3) In such a model, the technical relations determine prices and, even if substitutions are possible, they will not occur: hence, technical coefficients are constant[9]. The model of Leontief is considered as a classical model in this sense according to Samuelson: the system is determined by technology, and demand is not relevant to explain the prices and the optimal allocation of factors. But this result rests on the special assumptions on primary factors and joint production. Indeed, in a generalized model of Leontief with more than one primary factor and with joint production, demand is instrumental to determine the optimal allocation of factors and substitutions among inputs occur: "Ricardo and Smith would probably have admitted that the relative prices of joint products –of venison and deer skin, for example– would have to be determined by a demand theory and not from labor or land costs alone. One wonders why they did not worry more about this "jointness", which every student of Walrasian equilibrium knows to be an intrinsic part of the actual pricing relations among diverse factors and goods" (Samuelson 1959a, 18).

Hence, because of the restrictive assumptions made by Leontief, no substitutions take place in the model and demand is not at the first stage. According to Samuelson, the generalized model of Leontief is the proper Walrasian general equilibrium model to analyze economic systems, in contrast with simple models of Leontief (with constant technical coefficients). Samuelson considered that the classical theory was a particular case of general equilibrium theory and thus that the model of Leontief was a particular case of the latter.

To sum up, *(i)* models of Leontief represent the case of a simple Walrasian model with constant technical coefficients, which correspond to Walras' view of production before 1896[10] and also to the Ricardian case ; *(ii)* generalized models of Leontief with joint production and multiple primary factors correspond to the general equilibrium model with substitutions among inputs. Hence, Samuelson's results are important for Walrasian general equilibrium theory as it states in a rigorous way the relationships between the Walrasian models with complementary factors of production (Walras' theory previous to 1896, models of Leontief, Ricardo-like models) and Walrasian models with factor substitutions and marginal productivity theory.

Sraffian economics and the models of Leontief: production as a one-way avenue versus circular flow economics

At first a brilliant intellectual demonstration, Samuelson's analysis took a different significance when Sraffa published his 1960 book, *Production of Commodities by Means of*

[8] According to Samuelson's interpretation of models of Leontief, in the *open* model of Leontief, final demand is exogenous and labor is a primary factor, while in the *closed* model of Leontief demand is an input for households to produce labor. Hence, in the closed model, labor is an output and final demand (the bill of goods) is the input for labor. In other words, in the static closed model without land and capital *n* industries (including households) produce *n* goods (including labor) and, in the open model of Leontief without land and capital, given an amount L of labor and given D the "bill of goods", *n* goods produce *n-1* goods.

[9] Georgescu-Roegen noticed that "the equilibrium input-coefficients of the elementary processes are independent of the bill of goods. Therefore, in a closed model, the equilibrium input-coefficients used by each industry are determined only by the technological knowledge and are, consequently, independent of the consumers' choice" (1950, 217).

[10] Walras introduced the possibility of substitutions among factors of production only in the third edition of *The Elements* (1896).

Commodities –which marked a revival of classical thought. The intellectual situation, until then devoted to the making of a unified vision of economics (*e.g.* the neoclassical synthesis), radically changed. A new opposition line appeared that opposed classical analysis and neoclassical analysis: on the one side the mainstream, on the other the heterodoxy. Soon, the question was to know how to distinguish one approach from the other; that is to say to define the exact lines of opposition. According to Joan Robinson, for instance, input-output analysis belongs to classical theory and is a "breakaway from neoclassical orthodoxy. In the input-output system, real wages are treated as an input _a thoroughly classical conception _ and there is a single interconnected set of physical relations of production which leave no room for marginal productivity, substitution, or the principle of economic choice" (1968, 432)[11]. After the publication of Sraffa's book, some similarities brought attention to the classical foundations of input-output analysis. Indeed Sraffa noticed that in economics two different views of the production process were opposed: the one inherited from Quesnay's *Tableau économique* representing production and consumption as a circular flow and, the other representing production as a one-way avenue from 'factors of production' to 'final consumption goods' (the neoclassical view) (Sraffa 1960). The circular framework is the one where *n* commodities are means to produce *n* commodities[12] and, in the (neoclassical) one-way avenue worldview *m* factors of production are means to produce *n* final goods. As noticed Pasinetti "Sraffa's book [1960] brought theoretical attention back to the process of production considered as a circular process" (1981, xii)[13]. In this view, the model of Leontief is of course related to the Quesnay line and is opposed to the neoclassical view.

After the 1960s, numerous Neo-Ricardian economists considered models of Leontief as classical models which had strictly nothing to do with Walrasian models. Gilibert (1981, 1987, 1998) is one of the first commentators who underlined that before developing input-output analysis, Leontief wrote a PhD dissertation (1928) dealing with circular flow in a straight circular flow framework. From the point of view of the history of political economy, introduction of Leontief's dissertation to analyze models of Leontief was a major contribution to the debate. After Pasinetti (1975, 1981) and Gilibert's works, Kurz and Salvadori (1995, 2000, 2006) revived the debate about 'the classical roots' of input-output analysis and they suggested to interpret input-output analysis as the fulfillment of the project Leontief designed in his 1928 PhD dissertation, that is to say the making of a classical model. Kurz and Salvadori (2006) offered the first detailed analysis of Leontief's 1928 dissertation on "the economy as a circular flow". Moreover, Neo-Ricardians scholars like Schefold (1989) and Bidard (1991) seem to admit the idea that the models of Leontief are primarily classical and have little to do with Walras. Neo-Ricardian views may be summed up in three purely theoretical arguments and two historical arguments. Let's begin with the theoretical arguments:

[11] See also Robinson (1971, 600-601): "Professor Leontief confused the issue [of capital analysis] by maintaining that his input-output tables correspond to a picture of Walrasian equilibrium …"

[12] Of course this representation of the circular flow is a simplified one. Indeed, in the classical circular flow, land and natural resources are not produced like other commodities. As noticed Sraffa (1960) land is a non-produced element of the circular flow. Moreover, some commodities are produced but are not used to produce other commodities (Sraffa called them "non fundamental commodities").

[13] According to Arena (1990, 195), Sraffa's use of the idea of "circular process" (*i.e.* of "circular flow"), in a reproduction framework, makes him "the first author of the 20th century who revived the classical tradition by delivering back the most essential part of its speech".

-P1. Proposition 1: Following Sraffa, they underlined that production is a circular flow which means that, essentially, *m* commodities produce *m* commodities[14]. In contrast with Walrasian and Neoclassical economics in general, Classical theory of production represent production as a circular flow. Hence, Leontief's 1928 model and 1937 model are Classical and family likeness with Walrasian models or Cassel's model of general equilibrium is a rough misunderstanding. This argument is shared without exception by Pasinetti, Gilibert, Kurz and Salvadori.

-P2. Proposition 2: In models of Leontief prices are determined independently of utility and demand. Here, Kurz and Salvadori introduced a distinction between the 1928 Leontief essay and his later input-output models. Indeed, while the 1928 model contains a strong a Classical theory of value, the 1937 model of Leontief gave up this strong theory:

(i) In his 1928 model "the exchange ratios of goods reflect not only 'natural', that is to say, essentially, technological factors, but also 'social causes'. For example assuming free competition, as the classical economists did in much of their analysis, the surplus is distributed in terms of uniform rate of return on capital across all industries of the economy. With this specification, the general rate of profit together with the relative prices can be determined in terms of the system of production in use and given real wages. 'But this is the "law of value" of the so-called objective theory' ([Leontief] p. 601 [p.196]), Leontief insisted. These are remarkable propositions. They show that the young Leontief was possessed of a deep understanding of the classical economists' approach to the problem of value and distribution […]" (Kurz and Salvadori 2006, 380). Hence it makes no doubt that, according to Kurz and Salvadori, Leontief's 1928 essay contains a strong Classical theory of value.

(ii) In the closed model of Leontief (1937), value-added are endogenous but "the [value-added] represents the sum total of payments to the owners of productive factors: wages, rents, interest and profits" (Kurz and Salvadori 2006, 387). According to Kurz and Salvadori in this model "two things are lost sight of: the constraint binding changes in the distributive variables, and the dependence of relative prices on income distribution –facts well understood and for perfectly good reasons, it seems, stressed by Leontief in his 1928 paper" (2006, 387). Thus, input-output models subsequent to 1928 do not develop a strong Classical theory of value.

-P3. Proposition 3: Even if Leontief gave up the strong Classical value theory in his closed and in his open input-output models (1937, 1944) it doesn't represent a fundamental change. Indeed, according to Kurz and Salvadori "scrutiny shows […] that in his input-output analysis [Leontief] preserved the classical concept of circular flow and did not, as is maintained by some interpreters, adopt the Walras-Cassel view of production" (2006, 387) which is a representation of production as a one-way avenue.

The last theoretical argument is critical as it shows that the circular flow representation of production is a *necessary and sufficient condition* to characterize Leontief's models as Classical models. Hence Kurz and Salvadori rest their argumentation, concerning input-output analysis, on the first theoretical argument: circular flow of production.

Let's now turn to the historical arguments:

-P4. Proposition 4: "There is of course reason to think that Leontief's perspective on the classical authors was at least partly shaped by the supervisor of his PhD dissertation in Berlin,

[14] As Kurz and Salvadori noticed, in the classical approach "production is essentially a circular flow: commodities are produced by (means of) commodities" (2006, 377).

the eminent theorist and statistician Ladislaus von Bortkiewicz" (2006, 380). Bortkiewicz is considered as the bridge between classical economics and Leontief.

-P5. Proposition 5: According to Kurz and Salvadori "in the years 1927-1928 Leontief and Sraffa may be said to have been independently pursuing similar lines of thought […] We are thus confronted with the fact that two major economists of the 20th century developed, independently of each other, similar approaches to the problem of production" (Kurz and Salvadori 2006, 376). "At the centre of the interest" of the problem of production is "that relative prices can be determined exclusively in terms of observable amounts of commodities that are respectively produced within a year" (2006, 374). Hence, the strong classical theory of value is at the centre of the stage. However, proposition 2 (P2) shows that P5 is true only for Leontief's 1928 essay. Once more, P5 stresses the role of the representation of production as a circular flow as the common denominator between models of Leontief (1928, 1937) and also between models of Leontief and Classical thought.

Synthesis. Rabbits hunting ducks…

It is difficult to draw a univocal conclusion from all these contradictory statements, especially those of Leontief (quoted in the introduction). The latter should be enlightened by their context. In the 1930s, Leontief was looking for intellectual and material funding to fulfill his project on the empirical study of general interdependence and he cleverly insisted on the reference to general equilibrium theory. The Harvard University Committee for Economic Research (HUCER), which hired Leontief, was primarily interested in statistical economics and Leontief was known for his econometric works on supply and demand equations. Leontief's archives, at Harvard, show that he had numerous contacts with Henry Schultz and other econometricians, especially at the Cowles Commission. See for instance Leontief's annual reports to HUCER: "In contrast with my previous studies devoted to different problems on price analysis, this project will represent an attempt to abandon partial supply and demand analysis and investigate more directly the connections between the price and the quantity change of more important groups of commodities" (1934)[§]; in 1935 he noticed, that his study of the American economy was a "statistical study of price-quantity changes from the point of view of general equilibrium"[§]. Thus, shifting from the Moore-Schultz econometric program, that is to say partial equilibrium analysis, to general equilibrium econometrics was a rational argument.

The reference to Walras was not only opportunist and persisted, even in his late interview with Duncan Foley (1998). However Leontief didn't deny the ideas exposed in his PhD dissertation and he recognized the classical influence on his work. Leontief's definitions of classical theory were often vague and he hesitated between 'general equilibrium' and 'general interdependence'. These hesitations encouraged the numerous contradictory interpretations of input-output analysis. Since the end of the 1920s, the evolution of economic theories and numerous analytical debates have clearly changed the questions, the stakes and the answers. Whether the late 1940s were characterized by the willingness to build a wide theoretical synthesis and bring together all the different schools, the revival of classical thought in the 1960s changed thoroughly the situation. It is sometimes difficult not to interpret economic theory with a post-1960 apprehension of economics. Such an interpretation is based on a rhetoric structured by, on the one hand, a "mainstream" ("neoclassical") movement absorbing the whole economics field and even other fields in social sciences (from Marxian-Classical thought to sociology of crime for instance), and, on the other hand, a minority claiming for legitimacy and truth. In such a rhetorical game, players stress oppositions rather than common

[§] Leontief's Papers, Harvard University Archives, [HUG.451.5 Correspondence, General, 1932-1946], Box 1.

points: according to Neo-Ricardians, Neoclassical thought is inconsistent; and Neoclassical economists tried "to show that there is no correct Neo-Ricardian proposition which is not contained in the set of propositions which can be generated by orthodoxy. [… Hence] the Neo-Ricardian attack *via* logic is easily beaten off" (Hahn 1982, 353).

Despite the confusing numerous and contradictory arguments, the discussion may be summed up on three points:

i. The first point benefits from a general intersubjective consensus among economists: models of Leontief present clear classical features, from the point of view of price theory and the theory of production as well. This point is admitted by all commentators, from Samuelson to Pasinetti.
ii. The second point is the controversial one: models of Leontief are closely related to the Walrasian model of general equilibrium. This point is admitted by Samuelson who added that the open model of Leontief is a particular case of general equilibrium models. However this claim is rejected by Neo-Ricardian authors. According to the latter, models of Leontief belong exclusively to the Classical thought because they rest on the strong doctrine of the circular flow which is incompatible with Walrasian theory.
iii. The third point concerns only Neo-Ricardian economists. According to Pasinetti, Gilibert, Kurz and Salvadori, interpreting Sraffa (1960), what matters primarily for a theory to be Classical is the representation of production as a circular flow. That is to say that even if Leontief didn't defend a strong Classical theory of value in his input-output analysis he preserved the Classical concept of circular flow and did not adopt the Walras-Cassel view of production.

We admit the first point, which is supported by theoretical arguments (those exposed previously) and historical arguments (Leontief was continuing the work of soviet economists on national accounts and Marx's scheme of reproduction[15]). Hence, the entire question is to know whether the view of the circular flow supported by Leontief was a strong Classical view and whether it is incompatible with the Walrasian representation of the production process. Because of point three above, this paper focuses on the circular flow as a production process.

II. Leontief's view of the circular flow

In this part it is argued that although Leontief's works were rooted in Classical political economy it was compatible with Walras' theory of general equilibrium and this permitted Leontief to use the Walrasian model of production. Our demonstration will only deal with the 1928 essay of Leontief and his 1937 closed model as far as later input-output models were variations on the 1937 model.

One of the stories Leontief used to tell about the making of input-output analysis was that he developed it as an answer to the weakness of Marshallian partial equilibrium analysis. According to Leontief, his econometric work on partial equilibrium supply and demand curves (between 1928 and 1934), convinced him of the superiority of general equilibrium analysis. This statement is obviously wrong. In fact, his view of the economy as a general interdependent system was already in progress (not to say firmly expressed) in his PhD dissertation (1928). Leontief's dissertation was characterized by the rejection of the subjectivist point of view in economics: Leontief refused to found explanation of economic phenomenon on individual behaviors and psychological motivations. In his dissertation, he

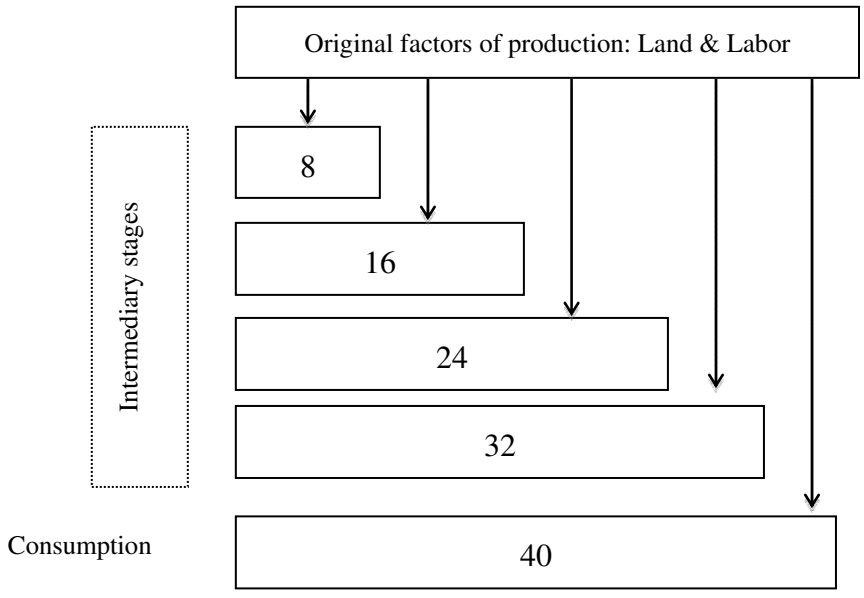
[15] See *Infra*, Section II.

declared that "the circular flow principle seems to deserve a higher logical rating than the extremely complex notion of economic man" ([1928] 1991, 208); moreover, according to Leontief, "psychology has only in a very limited fashion explained its idea of the laws governing the satisfaction of needs" (*Ibid.*, 181). Instead of a theory based on individual behaviors, Leontief built "a model of a system of economic flows" (*Ibid*, 198): he pictured "the economy as a circular flow". This approach is called by Leontief the *material point of view*. We present now Leontief's theory of the circular flow.

Marx' reproduction scheme as the theoretical basis of Leontief's view of the circular flow

While Leontief was skeptical about the labor theory of value he supported Marxian economics' worldview: "[Marx] also developed the fundamental scheme describing the interrelations between consumer and capital goods industries. Far from being the *ultima ratio* of this line of analysis, the Marxian scheme still constitutes one of the few propositions concerning which there seems to exist a tolerable agreement among the majority of business cycle theory. It is interesting to note in this connection that even Professor Hayek, as been seen from his recent articles, is busy reconstructing his own triangular investment diagram. One does not need to be a prophet to predict that sooner or later he will present to us a circular arrangement of the orthodox Marxian type" (Leontief 1938, 5). Indeed, the Böhm-Bawerkian representation of production characterized most of the Austrian marginalist tradition, including Hayek who developed this view in *Prices and Production* (1932).

Figure 2 The Austrian "triangle" of the circular flow, Hayek ([1932] 1941)



On Figure 2, representing the Austrian view of the production process, original factors (land and labor) are employed in a first stage to produce intermediate goods (which value is 8). These first intermediate goods are then employed to produce other intermediate goods (which value is 16) and so on, until consumption goods are produced. The triangle shows that the consumption goods are the farthest from original factors. One can characterize a good according to his distance from original factors of production. In Leontief's view, there is nothing like a "first stage", "original factors" or "factors of production". Leontief was attached to the idea of a circular flow, rejecting the very Austrian concept of "original factors of production" (land and labor): "If Böhm-Bawerk did actually set out in search of this

hypothetical first stage, he would find himself now still on the road" (Leontief 1938, 4). Here, Leontief didn't stress the macroeconomic or structural reproduction conditions, but representation of production as a circular flow. What matters is representation of production as a circular flow. Leontief already developed this idea in his 1928 dissertation with a static circular flow model and a dynamic circular flow model. Leontief defined two kinds of situations: first the simple reproduction case "which allow[s] us to return to the initial starting point" ([1928] 1991, 182) and, second, the "changing circular flow system" ([1928] 1991, 190) which deals with economic change like economic development and business cycles. From Leontief's 1928 definitions of the circular flow and his 1938 analysis of the Marxian reproduction schemes as the basic macroeconomic framework, it makes little doubts that the latter was his starting point. This is not surprising as far as Leontief's studies were directly inspired by the Soviet works on the statistical balance of the USSR which was based on Marx's reproduction schemas (Popov 1926) (Leontief 1925). Let's turn to the detailed analysis of Leontief's 1928 model of the circular flow.

Leontief exposed a static circular flow with a four branches model (A, B, C, D). In this model, elements A and B are combined to produce elements C and D and, reversely, C and D produce A and B: $(A \oplus B) \rightarrow (C \oplus D) \rightarrow (A \oplus B) \rightarrow (C \oplus D)$, etc. This is a case of joint production and, as Leontief noticed, "(A+B) and (C+D) exist simultaneously" ([1928] 1991, 186) like in a static general interdependence model. Such a circular flow, in the case of simple reproduction, is represented below in figure 3.

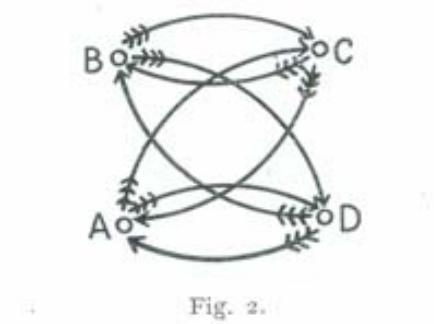


Fig. 2.

Figure 3 Simple reproduction in a four branches circular flow (Leontief 1928, 589)

Combinations between the different elements of the circular flow rest on given constant coefficients. According to Leontief, "individual economic elements appear in the form of specific units which cannot be subjected to division and thus represent the lowest limit of technical coefficients" (186). To define combined elements and the way they are combined, Leontief (1928) referred to his 1927 article on "The theory and statistical description of concentration" where he explicitly linked his coefficients back to Cassel's general equilibrium model: "for every production process there exist some ideal proportions in which all the factors of production involved in that process must be brought together. Cassel calls this relationship the 'technical coefficient'." ([1927] 1985, 259)[16] Hence, in his 1928 model, Leontief used constant technical coefficients, that is to say that he supposed constant returns to scale technologies. The model is expressed in terms of 'coefficients of distribution': a is the proportion of total output A employed to produce C, $(1-a)$ is the portion of A employed to produce D, b the portion of B employed to produce C; $(1-b)$ the portion of B employed to produce D, d the portion of D employed to produce A and c the portion of C employed to

[16] And Cassel took his technical coefficients in Walras' *Elements*.

produce B. Thus, Leontief wrote the combinations of A, B, C, D as follow (Leontief's notations):

$$aA + bB \rightarrow C \qquad dD + cC \rightarrow A$$
$$(1-a)A + (1-b)B \rightarrow D \qquad (1-d)D + (1-c)C \rightarrow B$$

Leontief did not propose a formulation of this model[17] using usual technical coefficients but this one is implicit. Even if he didn't explore the detailed mathematical conditions of the existence of the static equilibrium, he implicitly assumed such mathematical conditions on coefficients that ensure existence of equilibrium.

Leontief also developed a dynamic circular flow model. He didn't offer a mathematical exposition of his 1928 dynamic model and reconstruction of it would show that the model is based on such a relation: $X(t+1) = MX(t)$, where X(t) is the output vector at time *t* and *M* is a coefficient matrix based on constant technical coefficients. Two cases are taken into consideration: the case of simple reproduction (reproduction) and the case of changing circular flow. In the reproduction case, let's suppose that goods A at time *t=0* (A^0) are used to produce goods at time *t=1* (A^1) which in their turn are used to produce goods at time *t=2* (A^2) and so on until goods A^0 are reproduced at time *n* ($A^n \equiv A^0$)[18]. Hence, a circle might be drawn from A^0 to A^0 representing a reproducing circular flow (see Figure 4). This way to represent and note goods according to their distance to an "original point" –here the starting point – shows that Leontief knew well Böhm-Bawerk's work, which is not surprising as far as the latter was widely spread, taught and discussed in Russia[19] and, of course, in Germany.

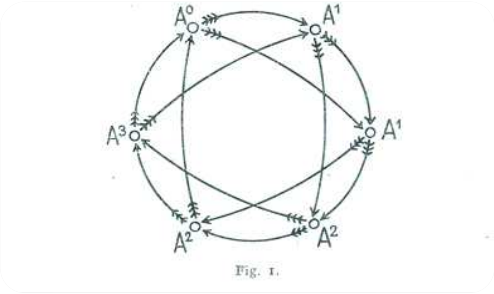


Figure 4 Simple reproduction in the moving circular flow (Leontief 1928, 587)

Economic elements that do not enter in the circular flow process are put aside. For instance, goods that are produced but that do not serve for production will not be represented in the circular image. In a dynamic circular flow, elements like overproduced goods, for instance, stand outside the circular flow until they are consumed; capital goods that are produced at time *t* and consumed at time *t+n* will appear in the circular flow only when they are produced and consumed. This representation of the economy is a "tool [permitting to] identify those causal [input – output] relationships that are specific to the economic sphere" ([1928] 1991, 182). This 'model of economic flows' deals not only with the representation of the production process, but also with the value and prices of commodities produced. Indeed,

[17] Leontief (1928) wrongly wrote "$(1+a)A + (1-b)B \rightarrow D$" (1928, 589). And in the English translation it is written "$(1+a)A + (1+b)B \rightarrow D$" ([1928] 1991, 186) which does not make sense either.

[18] Leontief noticed that "all items whose distance in the production flow from point 'A' is equal to one unit period, i.e. all those where element 'A' is consumed as a cost element, may be labeled as group 'A^1'. Points in the next group are then called 'A^2', and so on […]." ([1928] 1991, 185)

[19] See, for instance, Nikolai Bukharin's 1917 book, *The Theory of Leisure Class* (Bukharin used to be Böhm-Bawerk's student in Vienna). See also the works of Dmitriev and Tugan-Baranovsky.

definition of the circular flow distinguishes two dimensions: first it represents the economy as a "system of economic interrelationships between economic processes" (182) where "certain elements are generated by certain other elements and are then themselves used and consumed in further production" (181); the second dimension examines the conditions of reproduction and exchange in the economy and determines the equilibrium prices. Equilibrium prices do not ensure equality of supplies and demands of commodities but first of all, the continuation of the circular flow process.

Did the same view of the circular flow appear in Leontief's 1937 static input-output model? There is little doubt that Leontief kept the main characteristics of his 1928 model, notably concerning the first dimension, and the major change was formal, that is to say the exposition of his model in matrix algebra. In the 1937 model one finds the constant technical coefficients, and the two-dimension circular flow. Let A be a matrix of coefficients, P the vector of prices and X the vector of outputs: on the one hand, prices equations determine equilibrium prices ($A^{T}P=0$); on the other hand, quantity equations determine output structure ($AX=0$). In both the 1928 models and the 1937 model, one finds representation of production and consumption process as a circular flow, that this to say that m "industries" (single output) produce m goods –and households' consumption produces labor and entrepreneurial services. We can now describe Leontief's main principles of political economy as follow:

i. Development of objectivism and rejection of the concept of utility and of the "economic man";
ii. Rejection of the supply-demand explanation of prices;
iii. Representation of the production process as a circular process instead of a one-way avenue (first dimension of the circular flow) like in a Marxian schema of reproduction;
iv. Explanation of prices by the technological data mainly (second dimension of the circular flow);

As noticed earlier –and admitted by most of Leontief's commentators– these theoretical postulates are clearly Classical in nature and make Leontief's early theoretical preoccupations very close to Sraffa's. As we underlined above, we fully acknowledge the Classical nature of Leontief's framework in his 1928 and his 1937 models. Leontief obviously took support on the schema of reproduction which was already at the basis of Popov's (1926) interindustrial model. Now, the question is to know whether this classical basis should exclude significant references to Walrasian models or not? A first stage to answer this question is the analysis of Leontief's theory of prices in his 1928 and 1937 models.

Value and prices in Leontief's models of the circular flow and Walras' "equations of production"

No doubt that Leontief's theory of prices is largely inspired by Classical thought, especially in his 1928 essay. Indeed, Leontief was attempting to build an objectivist theory of prices and quantities, that is to say a theoretical system independent of utility and demand concepts. Moreover, according to Kurz and Salvadori, in 1928 "Leontief had independently arrived at a view very similar to the one Sraffa elaborated at around the same time. In particular, Leontief had clearly understood that the classical approach provided a coherent explanation of value exclusively in material terms and could entirely do without any reference to labour values" (2006, 380). However, the peculiarity of Leontief's way to deal with prices compared with economists like Dmitriev, Bortkiewicz, or Sraffa is striking. Thereafter, we comment Leontief's theories of prices and value, even if, as noticed by Neo-Ricardians, what

matters ultimately is the representation of the production process as a circular flow as far as Leontief gave up his early theory of value in his 1937 model of the circular flow[20].

Compared with contemporary classical prices theories, Leontief's approach is at least original and, in the end, his 1937 model definitely took distance with Classical thought. Indeed, Leontief's 1928 equilibrium prices are the result of exchanges between ownerships groups to whom parts of the branches of the economy belong. Incomes of owners (i.e. producers) are labor incomes plus so-called "income from ownership". However, in contrast to what Kurz and Salvadori wrote (see *Supra*, **P2**[21]), determination of prices in Leontief's 1928 model did not assume that 'the general rate of profit together with the relative prices can be determined in terms of the system of production in use and given real wages'. Such an analysis looks more like an interpretation of a card in a Rorschach test than a completed demonstration. Indeed, Leontief did specify neither a profit rate nor real wages: they are indistinctly included in the costs items. Owners exchange goods, and equilibrium prices are the one which permits (re)production to take place. Hence, in a two commodity model (A and B, quantities are given; commodity *B* is the *numéraire*), relative prices are determined by the 'law of costs': $a\mathrm{Ap} + b\mathrm{B} - A\mathrm{p} = 0$[22].

According to Kurz and Salvadori, a necessary data for a classical theory is "the ruling real wage rate(s) (or, alternatively, the general rate of profit)" (already quoted, 2000, 154-155), like in Sraffa's 1960 book. Comparison of Leontief's 1928 theory of prices and that of Sraffa is beyond the scope of this paper and we simply underline that in Leontief (1928) prices are determined independently of a ruling real wage rate(s) (or the general rate of profit)[23]. The absence of a rate of profit and of the issue of the sharing rule of the surplus (wages + profits) is quite intriguing for a Classical theory of value[24]. To compare with, Sraffa wrote in 1928 a set of prices equations in the traditional Classical fashion[25]:

[20] Part 3 of this paper is entirely dedicated to the representation of production as a circular flow in the models of Leontief and in the Walras-Cassel models.

[21] According to proposition P2: In models of Leontief, prices are determined independently of utility and demand; Moreover, according to Kurz and Salvadori, in Leontief (1928), "assuming free competition, as the classical economists did in much of their analysis, the surplus is distributed in terms of uniform rate of return on capital across all industries of the economy. With this specification, the general rate of profit together with the relative prices can be determined in terms of the system of production in use and given real wages" (already quoted, 2006, 380).

[22] We can write this equation with usual technical coefficients. As a and b are 'coefficients of distribution" then $a = \frac{x_{AA}}{X_A}$ and $b = \frac{x_{BA}}{X_B}$ with x_{IJ} the quantity of I used to produce J, and X_I the total quantity of I. Hence: $aAp + bB - Ap = O \mapsto a_{AA}p + a_{BA} - p = 0 \mapsto -(1 - a_{AA})p + a_{BA} = 0$.

[23] In a two-commodity (A and B) and two-ownership group model, Leontief distinguished two configurations: (I) the one with no profits (the entire surplus is distributed as labor income or there is no surplus), where prices are trivial; (II) and the other with positive profits but where he didn't give a sharing rule of the surplus. In the latter configuration two possibilities arise: *(i)* if "the individual branches of production are distributed proportionally across the two ownership groups" (Leontief [1928] 1991, 196) then there is no "stimulus toward exchange" that is to say that A and B may be reproduced without exchanges between the two ownerships group; *(ii)* otherwise, to produce A and B the two ownership groups exchange at the rate p deduced from the set of technical coefficients and the rule according to which there "is a tendency to attribute the same value to cost items in a given ownership group and the output items produced from them" (196): p (the price of A, B is the *numéraire*) is given by the 'law of costs' that is to say $a\mathrm{Ap} + b\mathrm{B} - \mathrm{Ap} = 0$.

[24] In the 1928 essay, there is no explicit sharing of the surplus between wages and profits, and, if profits are positive, there are rules of surplus sharing between producers only.

[25] Unpublished papers of Sraffa quoted by Kurz and Salvadori (2006, 384); Trinity College Library (Cambridge) reference: (D3/12/2:29).

$$v_a A = (v_a a_1 + v_b b_1 + c_1) r$$
$$v_b B = (v_a a_2 + v_b b_2 + c_2) r$$
$$C = (v_a a_3 + v_b b_3 + c_3) r$$

In this three commodity model (*A*, *B*, *C*), v_i is the price of commodity *i*, the commodity *C* is a standard of value, and *r* is the interest factor. This set of prices equations contrast with Leontief's 1928 and 1937 prices equations. Indeed, the lack of interest in the issue of the sharing of surplus between profits and salaries is a strong common point between Leontief's 1928 theory of value and the one of the closed model (1937). Hence, in contrast with proposition **P2** which introduced a radical break between the 1928 theory of value and the 1937 one, there is some degree of continuity between the two.

Indeed, in the 1937 model, households (industry *n*) "are treated […] exactly as any other "industry" […]. The analogy can be easily extended by identifying the service-output of a household with the production of an enterprise" (1937, 113-114). According to Leontief, "there is an obvious connection between the expenditures of an individual and the amount of his earnings" (114). Like in the 1928 model, income is partly used to produce labor capacity. Rest of the income is the income from ownership. Hence, while *production* coefficients indicate the quantity of each commodity necessary to produce one unit of a commodity, *consumption* coefficients measure the quantity of each commodity necessary to produce one unit of "household services". X_n is the net output of household services and p_n its price. Household services are not only labor but also "capital and entrepreunarial services" (see Leontief 1936, 112-114). Incomes from capital and entrepreunarial services are all the incomes that are not wages and salaries wherever they come from (monopolistic revenue or speculative profits or proper entrepreunarial returns). Hence p_n contains all kinds of incomes but, primarily, income of labor and income of capital. Equilibrium prices, the vector P, are determined by the homogeneous system $M\mathrm{P}=0$, where *M* is a matrix of coefficients (production coefficients, consumption coefficients, productivity coefficients, investment and saving coefficients). As a consequence, there is no need to know the distribution of income, between wages and profits, to determine prices –which is in line with the 1928 theory of prices. The 1937 prices theory is far from the Classical theory of value: as far as prices are independent from the way surplus is shared, Leontief's 1928 and 1937 theories of prices are a breakaway from Classical theories.

Let's note a_{ij} the quantity of commodity *j* employed to produce one unit of commodity *i*, and p_i the price of commodity *i*, Leontief wrote[26]:

$$a_{i1}p_1 + a_{i2}p_2 + a_{i2}p_3 + \ldots - p_i + \ldots + a_{ik}p_k + \ldots + a_{in}p_n = 0$$

These analysis and equations set the stage on which one may compare Leontief's theory and Walras'one. Indeed, one may interpret p_n as the sum of Walras' prices of the "services of capital" (p_k) and of "personal services" (p_p)[27]. In that case, the 1937 prices equations are similar to Walras' "equations of production". The prices equations of Leontief and the

[26] In order to simplify the quite complex and unfamiliar 1937 model of Leontief we took, as Leontief proposed, the productivity coefficient *A* as equal to 1. See Leontief (1937, 116). Moreover, one should note that in contrast with his 1928 model, Leontief employed net outputs: "total minus the amount of its products consumed within the same industry" (1937, 110) –which is only a formal difference which explains why there is no coefficient ($1-a_{ii}$) before p_i.

[27] If $p_n = p_k + p_p$ one doesn't support the marginal productivity theory but that entrepreneurs make neither profit nor loss (profit in the sense of Walras, which is actually a sur-profit).

equations of production of Walras are similar, but the general explanations of prices are different as in the *Elements of pure economics*, demand also explains prices (equations of production are not sufficient to determine prices: prices are determined by the equalities of demands and supplies). However, beyond the formal similarities, shall one find some theoretical connections between Leontief's 'law of costs' and Walras' equations of production?

According to Bortkiewicz, the 'law of costs' was the common result of Walras' general equilibrium theory and Classical theory[28]. Indeed, Bortkiewicz was not only one of the best commentators of Walras' works but he also knew Walras himself; Furthermore, Walras expected Bortkiewicz to become his successor in Lausanne University (see Bridel 2008). Hence, in sharp contrast to Neo-Ricardians' proposition **P4**, Bortkiewicz may be considered not only as the bridge between Marx and Leontief but also, and may be *first of all*[29], the connection between Walras and Leontief. Indeed, Bortkiewicz wrote: "I believe that a good theory of political economy should rest on the equality between selling prices and cost prices [return prices], otherwise this theory would be incomplete; Moreover I believe that the system of Mr. Walras satisfies the required condition" (1890, 83). This means that Bortkiewicz believed in the 'law of costs' and that, according to him, Walras' analysis of exchange and production verified it. Of course, Bortkiewicz added, "in this system, the condition of equality between the selling prices and the cost prices does not appear among the theory of exchange [Parts 2 & 3] as far as this one considers the quantities of products as given. But it appears in the theory of production [Part 4] which considers the quantities of the products as unknowns and which demonstrates that they are determined in order to satisfy the equality between selling prices and cost prices" (*Ibid.*, 83).

This simple remark erases one of Neo-Ricardians' critic about the meaning of the link between Walras' analysis and Leontief's one: "setting aside purely formal similarities, the analyses of Leontief and Walras have little in common" (Kurz and Salvadori, 2000, 174); this claim was based on the comparison between models of Leontief and "Walras' models of pure exchange in parts II and III of the *Eléments*" (Kurz and Salvadori, 2000, 174): there was nothing in common between a "pure exchange economy" and Leontief's production economy. This argument has no significance as one should compare models of Leontief with Walras' complete theory of exchange, production and circulation. As a consequence, it is useless to look for the law of costs in Walras' theory of exchange, but it should be found in the following parts, notably the "theory of production". In contrast to most Russian economists, Bortkiewicz knew perfectly well Walras' theory of production and capital[30]. Hence, he paved the way for the analysis of Walras' theory of production within a Classical framework. Bortkiewicz's view of the relationships between Classical and Neoclassical theory of value is far from the rhetoric of conflict which stresses oppositions rather than intersections. The way Bortkiewicz thought the relationships between general equilibrium theory and Classical theories was not exclusive but complementary. Indeed, "Bortkiewicz believed that both objective and subjective influences on prices should be recognized and that his cost equations [in the classical meaning] could be inserted into the wider setting of general equilibrium analysis" (Meldolesi 1987, 264). According to Bortkiewicz, the Classical system and Walras' system were not contradictory and he believed in a possible synthesis between Ricardian

[28] In Walras' analysis of value, although the law of supply and demand determines prices, the equilibrium is reach only if costs (including income of labor and capital) and prices are the same.

[29] We underline "first of all" as far as the young Leontief couldn't ignore Marx' work and the problem of transformation… but may have not been aware of Walras during his studies in Saint-Petersburg (Leningrad).

[30] As noticed by Allisson, "for our Continental Russians, Bortkiewicz and Winiarski, Walras' theories of exchange and production are logically connected. For the others, there is no theory of production at all" (2009, 28).

economics and subjectivist marginalist analysis (Bortkiewicz 1921). During the 1890-1930 period, some Russian economists shared such a view, trying to put together Neoclassical and Classical theories, like Tugan-Baranowski and Dmitriev. Study of Leontief's theory of exchange and his use of Walras' theory of production indicates that he belonged to the latter move, which explains why he did not employ the rhetoric of conflict Neo-Ricardian chose. Authors like Tugan-Baranowski, Dmitriev, Bortkiewicz, and Leontief clearly shaped a scientific program of synthesis between Classical and Neoclassical theory.

The formal identity between Walras' production equations and Leontief's prices equations is not accidental. Leontief underlined the universal value of the 'law of costs' and purposefully expressed his prices equations in Walrasian terms. It didn't mean that Leontief's theory of prices is ruled by demand and supply principles but that 'law of costs' is true at the equilibrium. If we follow Bortkiewicz' analysis of the law of costs, Leontief's equilibrium prices (1937) are compatible with Walras' equations of production. However this is true only if in Walras' equations, the 'costs' are not costs of the 'production factors' or of the 'original factors'. Indeed, in a circular flow model there are no such things as "production factors" but only commodities producing commodities. Otherwise, similarities are theoretically meaningless, nothing more than misleading family likeness. We then have to turn to the way production is represented in the so-called Walras-Cassel model.

III. Walras' theory: production is not a one-way avenue?

According to Sraffa, the circular flow approach is proper to Classical economists and is opposed to the view of production as a one-way avenue from 'factors of production' to 'consumption goods': "The connection of this work [Sraffa 1960] with the theories of the old classical economists has been alluded to in the preface […]. It is of course in Quesnay's *Tableau Economique* that is found the original picture of the system of production and consumption as a circular process, and it stands in striking contrast to the view presented by modern theory of a one-way avenue that leads from 'Factors of production' to 'Consumption goods'." (Sraffa [1960] 1975, 93).

Since Sraffa's 1960 book, this distinction has been widely spread and strongly anchored in Neo-Ricardian's doctrines: "In contradistinction [to the conceptualization of production as a circular flow], in Austrian and in much of neoclassical analysis, production is conceived of as a linear flow which leads from the services of the primary factors of production to final goods" (Kurz and Salvadori 1995, 379). Schefold gave an enlightening exposition of the circular flow approach as opposed to the "one-way avenue" one, the latter stressing the role of "factors of production" and the former stressing reproduction conditions: "If one is used viewing the economy as a 'one-way street' from factors to consumption goods, the possibility of explaining prices by reproduction will seem strange, but its validity has to be acknowledged, because every economic theory taking its point of departure from these simple assumptions has to be able to explain the economy as a system that reproduces itself with labour (the only original factor) and capital, consisting of raw materials, produces the capital goods necessary for reproduction and the necessary consumption goods […]. Now we find that [prices] already follow from the circular character of production so that the impact of subjective preferences cannot find its expression primarily in prices" (Schefold 1989, 285). Christian Bidard also supported the Sraffian distinction criteria: "The connection suggested by Sraffa between Marginalist theory and "factors of production" may take different meanings: for the neoclassical authors of our pantheon [general equilibrium economists] one can easily dispels suspicion of having use of the unsound concept of aggregated capital; hence the relevant question [in order to find a separation criteria] is the one of a possible distinction

between factors employed for the production of consumption goods and the goods of this latter category: such a distinction would exclude the reproduction of commodities by means of produced commodities. Sraffa's statement is fundamentally correct for the Austrian and Walrasian versions [of neoclassical theory]" (1991, 267)[31]. According to this criteria, Bidard underlined that "nowadays Leontief is the most famous and one of the most faithful follower of the classical economists" (1991, 327). In deference to Hume, we shall call this criterion "Sraffa's Guillotine" as it makes a univocal distinction between Classical and Neoclassical economics.

Following this criterion, at first sight Leontief is a Classical economist. Indeed, the view of production as a circular flow on which Leontief based his PhD dissertation was maintained in his later input-output models (1937, 1941). Leontief wrote about the "factors of production", in a disconcertingly close fashion to Sraffa's statement: "The venerable trinity of 'land, labor, and capital' seems still to dominate the field of theoretical discussion. It is closely linked with the traditional approach to the system of national economy as if the latter were a single tremendously complicated production process absorbing the services of 'land, labor, and capital' on the one side and throwing out the 'national product' on the other"[32] (Leontief 1937, 112)[33]. However, at the same time, Leontief presented his approach as an application of general equilibrium theory. Were the two different assertions contradictory or was the Walras-Cassel view of the economic system compatible with the circular flow view of production? To answer this question we first examine Cassel's 1918 mathematical general equilibrium model and then we examine Walras' *Elements of pure economics*.

Cassel and the circular flow

One of the peculiarity of Cassel's approach of general equilibrium theory was his despise for the strong subjectivism doctrine and the concept of utility. Nevertheless, in Cassel's framework, prices are determined by subjective and objective forces, respectively demand and supply. However, demand is not microfounded on utility concept. We restrict our analysis to Cassel's mathematical model of general equilibrium published in his 1918 *Theoretische Sozialökonomie*[34], which is the most relevant for the understanding of Leontief's use of technical coefficients and general equilibrium prices equations (Leontief quoted Cassel in 1927 and Walras in 1935-1936-1937).

In the Cassel model, *r* factors of production, with "R_1, R_2, … R_r, the quantities of them which are available in a given period" ([1918] 1932, 14) produce *n* goods with a production function characterized by constant technical coefficients: "to produce the unit quantity of commodity 1, the quantity a_{11}, … a_{1r} of the factors of production may be necessary" (142). Writing q_1, … q_r the factor prices and p_i the price of good *i* we reproduce Cassel's equations:

[31] See also Gilibert (1981) who considered that Walras' view of production, from factors of production to final goods is strictly incompatible with Leontief's view of production as a circular flow.

[32] See Marx's famous statement in the *Capital*: "Capital – profit (profit of enterprise plus interest), land – ground-rent, labour – wages, this is the trinity formula which comprises all the secrets of the social production process" (1885 [1960], 193).

[33] One may recall that at the same period John von Neumann (1937) built a model of general interdependence based on the same idea of circularity. Robert Leonard Leonard accepted the Neo-Rocardian interpretation of von Neumann's model because "unlike the Walras-Cassel approach where factor services are transformed into final goods, von Neumann production involves time: it is the creation of commodities by means of commodities. (…) The many features of von Neumann's model may be found, in one form or another, in earlier classical contributions, ranging from William Petty to Ladislaus von Bortkiewicz" (1995, 736). Among other things, one may however stress that Sraffa's view of circular flow doesn't necessarily involve time.

[34] This mathematical model is only one short chapter of Cassel's two-volume book.

$$a_{11}q_1 + a_{12}q_2 + ... + a_{1r}q_r = p_1$$
$$a_{21}q_1 + a_{22}q_2 + ... + a_{2r}q_r = p_2$$
$$...$$
$$a_{n1}q_1 + a_{n2}q_2 + ... + a_{nr}q_r = p_n$$

As a conclusion, Cassel's mathematical model represents production as one-way avenue from *r* primary factors R_i to *n* final goods. Sraffa's Guillotine is perfectly functional concerning Cassel's mathematical theory of general equilibrium: it is not a circular flow analysis. Let's now turn to Walras' theory.

Walras' theory of general equilibrium

Walras developed in a considerable way the theory of production and of general interdependence. In the *Elements of pure economics*, his theory of production was developed into different parts: "The theory of production" (Part 4), "The theory of capitalization and credit" (Part 5), "The theory of circulation and money" (Part 6). We discuss these three different parts of the *Elements*. But first, one shall recall Walras' peculiar vocabulary.

Walras replaced J-B Say and the classics' 'trinity' of land, capital and labor, by the landed capital (i.e. land), capital (capital goods) and the personal capital (i.e. persons) that produce the services of land, the services of capital and the services of personal capital (labor). That is to say that he distinguished, among production factors, between stocks and flows. First, Walras distinguished between fixed capital and circulating capital. Fixed capital (or 'capital') refers to durable goods that are not immediately destroyed during the production process; circulating capital (or 'income') refers to fungible goods that are immediately consumed. Thus, pieces of furniture, buildings, machines etc. are capital goods (fixed capital); bread, meat, seeds, etc. are incomes that may be used to satisfy final consumption or as raw material for agriculture and industry. According to their use, some goods may be capital goods or incomes (like trees for instance). 'Incomes'[35] are classified as consumable services or as producing services. Consumable services are the incomes absorbed by private and public final consumption; producing services are the incomes which are transformed to produce other goods (capital or incomes). In Walras' vocabulary, there are no factors of production but producing services. Thus, incomes of land, labor and capital, are called respectively service of land, service of labor (personal services) and, service of capital.

In order to avoid misunderstandings we will take into consideration only Walras' general equilibrium model with constant technical coefficients and no theory of marginal productivity[36]. Moreover we do not take into consideration dynamic or temporal elements as far as the closed model of Leontief which is our benchmark is a static model without time; hence, we only deal with static general equilibrium without time[37].

General equilibrium is described as the coordination of two elements: equilibrium in exchange and equilibrium in production. According to Walras, general equilibrium has three characteristics, the two first ones are exchange equilibrium conditions and the third one is the production equilibrium condition (see Walras [1874] 1977, 224):

[35] The income is the service of a fixed or circulating capital. Indeed, "it is the essence of capital to give rise to income" (Walras [1874] 1977, 213).

[36] We put aside Walras' (late) introduction of variable technical coefficients (i.e. substitutions), which is not relevant for a comparison with Leontief's models. Moreover, Leontief defended against George Stigler's interpretation of Neoclassical theory Walras' theory of production with constant technical coefficients.

[37] Our work shall be furthered by a comparison of the dynamic model of Leontief (developed in 1953) and the circular view of production in a temporal or intertemporal general equilibrium.

1. Markets of productive services: effective demand and supply of productive services are equal, and there are stationary current prices; the same for the market of new capital goods;
2. Market of products: effective demand and supply of products are equal, and there are stationary current prices;
3. Equilibrium prices: selling prices of products equal the costs of the productive services that enter into them[38];

This view of the economy as a two-sphere model, the sphere of exchanges and the sphere of production, fits well with Leontief's 1928 exchange-production model. However, *a priori*, Walras' description of general equilibrium supposes that "productive services" are transformed into "products", which is typically a one-way avenue view of production. Now, in what case general equilibrium would represent production as a circular flow? In the case products are transformed into products –excepted land, which is not produced of course. Did Walras implement such a circular flow view of production?

Walras' theory of production and capitalization: production as a one-way avenue?

We first examine the "Theory of production" (Part 4 of the *Elements*). Scrutiny of Walras' "theory of production" shows that his analysis represents production as a one-way avenue from the primary factors 'land, labour and capital' to final goods. As a consequence, in this model, the service of land (T), the service of capital (K) and the service of personal capital (P) produce (final) goods A, B, C, D… The "theory of production" represents then a linear process, a one-way avenue.

Now, in a market economy, the law of supply and demand determines prices, that is to say that under competitive conditions equilibrium prices clear the market. Moreover, Walras expected that under competitive conditions, the prices of final goods are equal to the costs of the services, that is to say only land services, personal services and capital services. Walras wrote the so-called production equations[39]:

$$a_t p_t + a_p p_p + a_k p_k + ... = 1$$
$$b_t p_t + b_p p_p + b_k p_k + ... = p_b$$
$$c_t p_t + c_p p_p + c_k p_k + ... = p_c$$
$$d_t p_t + d_p p_p + d_k p_k + ... = p_d$$
$$.....$$

Let p_i be the price of product *i*, j_i is the coefficient of production (the quantity of *i* used to produce one unity of *j*) and, the commodity A is the *numéraire*. As the quantities of T, P, K are momentarily given at this stage of the *Elements* (Part 4), capital goods are primary factors. Given initial endowments of capital (T, P, K), services of land, of personal capital and of capital are transformed into final goods. The "trinity" quoted by Leontief appears clearly here. This is the same view than in Cassel's mathematical general equilibrium model. In this case, the Sraffa distinction between the "one-way avenue" view and the "circular flow" view of production is again perfectly functional. This Walrasian model of production is the benchmark taken into consideration by most of Neo-Ricardian authors. In this model there are only primary factors of production and intermediate inputs –which are at the core of Leontief's analysis– are eliminated. For instance, Pasinetti rightly noticed that in Walras'

[38] In his recent article, Mark Blaug (2009) chose to ignore the intimate relation between production equations, technical coefficients matrices and supply and demand equations systems.

[39] Production equations are not enough to solve the model. One needs first of all demand and supply equations. See Walras ([1874] 1977, 240).

"theory of production", intermediate inputs (raw material, noted M) are reduced to their original factors of production (services of land, personal capital and capital). This procedure Walras employed is a kind of "vertical integration": the vertically integrated sector "is a very simple one as it eliminates all intermediate goods and resolves each final commodity into its ultimate constituent elements: a (flow) quantity of labour and a (stock) quantity of capital goods. It may be interesting to recall that this procedure was used by Léon Walras himself in his *Elements*, although in a more rudimentary way [see *Elements*, Jaffé, p.241]" (Pasinetti 1981, 110)[40]. Hence, in Pasinetti's view of Marginalist theory, production is like a veil. Indeed, because production is analyzed through "production factors" considered as primary factors (like land), production is symmetrical to pure exchange: "demand for the final commodities is transformed into demand for the scarce resources" (*Ibid.*, 16). Hence, "the model had, so to speak, eliminated the process of production from the analysis, in the sense that all conclusions drawn at the stage of the scarce resources could be carried over, directly and without the slightest modification, to final commodities" (*Ibid.*). We then understand that the representation of production as a one-way avenue is a way not to take into consideration the production process: production is only a matter of demand of scarce resources (primary factors). This is the fundamental theoretical stakes of the Sraffa's Guillotine distinction.

However, Sraffa's distinction seems a little bit less convincing since Walras considered capital as given only as a provisory assumption for the sake of simplicity[41]. Indeed, in Walras' "theory of capitalization and credit" (Part 5) of the *Elements of pure economics*, capital is produced like the products A, B, C, D, … Put under very simple terms[42], in competitive equilibrium conditions, the selling prices of capital goods of different kind, K, K', K'' … noted P_k, $P_{k'}$, $P_{k''}$… are equal to their cost of production[43] (expressed in terms of services of capital, of land and of labor)[44]:

$$
\begin{aligned}
& k_t p_t + \ldots + k_p p_p + \ldots + k_k p_k + k_{k'} p_{k'} + k_{k''} p_{k''} + \ldots = P_k, \\
& k'_t p_t + \ldots + k'_p p_p + \ldots + k'_k p_k + k'_{k'} p_{k'} + k'_{k''} p_{k''} + \ldots = P_{k'}, \\
& k''_t p_t + \ldots + k''_p p_p + \ldots + k''_k p_k + k''_{k'} p_{k'} + k''_{k''} p_{k''} + \ldots = P_{k''}, \\
& \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}
$$

In this case, given an initial endowment of vintage capital, capital goods K, K', K'' … are produced and are used (and combined with productive services) to make products A, B, C, D, …[45] Should we consider this model of production as a one-way avenue process? No: it now looks like a circular flow, however this might be misleading, as we should see.

[40] Indeed, Walras wrote: "We shall now proceed […] to reduce the case of the application of raw materials to the case of the direct combination of productive services alone. Indeed, we must proceed in this manner, because raw materials are themselves products which are obtained either by the exclusive application of productive services alone [services of labor, land and capital] or by the application of these services to other raw materials, which were in turn obtained by similar application to still prior raw materials, and so on" ([1874] 1977, 241). Hence, by substitutions, prices of raw materials may be eliminated from the costs equations.

[41] In Leontief's static models (1937) (1944), there is no capital (like in a short term macroeconomic model).

[42] We do not examine in details the relationships between prices of the capital goods and their services, neither the solution of the capitalization problem by Walras which are beyond the scope of this paper.

[43] Costs depend on technical coefficients k_i, which the quantity of commodity *i* necessary to produce one unit of output K.

[44] Prices of services of capital are in small letters (p) and prices of capital goods are in capital letters (P). See Walras ([1874] 1977, 280).

[45] It is interesting to underline the fact that in Walras' view capital is not measured in monetary terms, capital is simply the set of goods that are not immediately consumed and are used in production of other goods.

Circulating Capital in a Bermuda Triangle

However, among all the Neo-Ricardian authors we quoted, only Bidard considered that the capital model of Walras is a circular flow model. His conversion is very recent; in his 1991 French book –already quoted– he still considered Walras' models as one-way avenue ones (see above); but in the English version of his book (2004), *Prices, Reproduction, Scarcity*, he finally chose to consider the production of capital in general equilibrium models as a sufficient condition for being a circular flow production representation: "The *conception of production*. It is in the opposition between two conceptions of production that Sraffa situates the core of the distinction between classical and marginalist approaches as well as the root of the marginalist's mistakes. […] The neoclassical theory of production is usually formulated in the framework described by Sraffa: production is conceived as 'a one-way avenue that leads from "Factors of production" to "Consumption goods" […], whence the role of the initial endowments and the interpretation of prices as scarcity indicators. However the neoclassical theory cannot be reduced to that representation. Thus all growth models, including that of Solow (1956), are centered on the reproduction of capital: this observation is independent of the critiques that may otherwise be made of the conception of capital" (Bidard 2004, 304). As Bidard noticed, "the *Eléments d'Economie Politique Pure* proceed by successive deepenings and, far from contrasting two views of production, Walras considered the circular conception as an extension of the 'one-way' approach. Walras' theory of accumulation assumes the production of capital goods" (2004, 254). In other words, in a Walrasian capital-model, production is a circular flow as far as capital goods and labor produce final commodities and capital goods. Hence Bidard changed his mind and he showed that Neo-Ricardians' interpretations were misleading. According to his definition of production, even a RBC model is a circular flow one. Nevertheless, one feels that there might be some relevance in Sraffa's Guillotine…

Indeed, is the introduction of the production of fixed capital enough to transform the Walrasian representation of production into a circular flow model? Is it enough for transforming a pumpkin into a coach? *Stricto sensu* one may admit that it is. But a Walrasian model without capital, like the model of Leontief, would then be a "one-way" avenue model… yet the 1937 static model of Leontief doesn't deal with capital and is still a circular flow model. Why? Because in such a model, there is circulating capital, that is to say commodities producing commodities like oil that one can use as a final good to have a pleasant journey in Cambridge or as a circulating capital to deliver mails to smart rhetoricians. In a model of Leontief, there is no need for fixed capital to have a circular flow model. It makes a difference with Bidard's interpretation. And, on this point, we rejoin Kurz and Salvadori's interpretation of Walras' equations of production of capital goods: "if [in Walras's theory of production and capitalization] there is only one labor service, (…) if land services are not in short supply (…), and *if "fixed" capital is actually circulating capital*[46] (…) then the set of [production] equations [of capital goods] is nothing else than the set of equations investigated in Chapter 4" which is the Neo-Ricardian model with any number of commodities (the circular flow model) (Kurz and Salvadori 1995, 25, italics added). So, where is circulating capital? In a note, Kurz and Salvadori indicated that in Walras' *Elements* "it is also assumed that there are no circulating capital goods, for example, raw materials. Circulating capital goods are taken into account in the fourth edition of the *Eléments* in a way which is analogous to the one in which fixed capital is treated (cf. Walras, [1879] 1954, §205)" (23). What is the §205? Nothing else but the paragraph already quoted by Pasinetti about the vertical integration of raw materials M… that is to say the reduction of raw

[46] In order to implement this assumption – "if fixed capital is actually circulating capital"– Kurz and Salvadori simply consider the depreciation charge of capital good i, d_i, as equal to the price of i, that is $d_i = P_i$ (instead of putting $d_i = h_i P_i$, with h_i a proportionality factor, $0 < h_i < 1$).

materials M to their original factors of production (services of land, personal capital, capital). Hence, Kurz and Salvadori's last assertion is wrong, because in Walras' theory, capital is not reduced to original factors of production, unlike raw materials in the "Theory of production" §205. Following Kurz and Salvadori, we lost once more the track of circulating capital. Once again, circulating capital disappeared in a kind of theoretical Bermuda Triangle.

One should say, at last, how Walras deals with circulating capital, that is to say the commodities that might be used either as final goods or intermediate commodities. Are they eliminated as raw materials (M) were eliminated in the "theory of production"? Or can we find in Walras' theory circulating capital? Moreover, are some commodities employed as means of production and also as final goods? These goods make the essence of the representation of production as a circular flow process: if Walras' theory makes room for such goods, then it represents production as a circular flow process.

The "theory of circulation"

We now turn to Walras' "theory of circulation and money" (Part 6). In this part, Walras introduced the money and dealt with the production of circulating capital that was put aside until then. The treatment of money has always been very difficult in general equilibrium theory. Introduction of money in the *Elements* deeply changed from one edition to another. In the first edition, this part was "Section III. Du Numéraire et de la monnaie", and it is only in the fourth edition that it became the "Theory of circulation and money". As noticed William Jaffé, in the last edition, treatment of money and of circulating capital is intimately linked: "in this final version Walras identified cash balances with circulating capital yielding services of availability; and this enabled him to link the value of money to utility functions in the same way that the values of other categories of circulating capital goods were linked to these functions" (Jaffé *in* Walras [1874] 1977, 601). As unsatisfactory as Walras' treatment of money may be, Part VI shows that circulating capital, and production in general, is not a veil in general equilibrium theory but its very meaning. This is, according to Walras, the complete version of general equilibrium[47].

Now there are goods A, B, C, D, …, money U, services of capital goods K, K', K'', services of fixed capital T and P (of different kinds), M (raw material) and the products A', B', C',D' … What is new is the latter set of commodities: A', B', C',D', … that are the same goods as A, B, C, D … but used as circulating capital. That is to say that now, A, B, C, D, … K, K', K", … M, … T and P produce A, B, C, D, … K, K', K", … M, … P. Of course land is not a reproducible good. Hence, under competitive equilibrium conditions, the new equations of production are written by Walras[48]:

[47] Bridel (1997) underlined that Part 6 appeared to most commentators as quite obscure: "Jaffé finds Walras's exposition 'skeletal' (1981, p. 362); Lange calls it 'somewhat obscure' (1938b, p. 179); Kuenne describes it as replete with 'ambiguities, carelessness, and sheer error' (1963, p. 337); Patinkin asserts that Walras 'failed to clothe his elaborate mathematical framework with adequate economic meaning' and considers that his monetary theory 'has not been thought through to its logical end' (1965, pp. 570-71); van Daal and Jolink regard it as 'a nearly incomprehensible mass of formulae' (1993, p. 105); Marget is the only critic to consider reassuringly that Walras' presentation of the theory of money appears much more complicated than it really is" (113).

[48] Walras ([1874] 1977, 323).

$$a_t p_t + a_p p_p + a_k p_k + ... + a_{a'} p_{a'} + a_{b'} p_{b'} + ... + a_m p_{m'} + ... + a_u p_{u'} = 1,$$
$$b_t p_t + b_p p_p + b_k p_k + ... + b_{a'} p_{a'} + b_{b'} p_{b'} + ... + b_m p_{m'} + ... + b_u p_{u'} = p_b,$$
$$.....$$
$$m_t p_t + m_p p_p + m_k p_k + ... + m_{a'} p_{a'} + m_{b'} p_{b'} + ... + m_m p_{m'} + ... + m_u p_{u'} = p_m,$$
$$.....$$
$$k_t p_t + k_p p_p + k_k p_k + ... + k_{a'} p_{a'} + k_{b'} p_{b'} + ... + k_m p_{m'} + ... + k_u p_{u'} = P_k,$$
$$.....$$

These equations show that the products are also means of production: A, B, C, D, … are no more pure final goods but also circulating capital. Hence, it is even possible to consider that there are no pure final goods at all[49]. Now, the idea of a one-way avenue from factors of production to final goods looses sense. Moreover Walras didn't give a specific status to labor and land like Austrian marginalists did: they are not "original factors" from which value is imputed. Indeed, according to Walras, what really matters is not land or labor but the production process of circulating capital and proper capital. As noticed Walras, "every hour, nay, every minute, portions of [the] different classes of circulating capital are disappearing and reappearing. Personal capital, capital goods proper and money also disappear and reappear, in a similar manner, but much more slowly. *Only* landed capital escapes this process of renewal" (we stress [1974] 1977, 380).

Hence, Walras' representation of production is as much a circular flow process as Leontief's. According to proposition **P1** and **P3**, representation of production as a circular flow was a necessary and sufficient condition to characterize Leontief's models as Classical models incompatible with marginalism. Consequence of our demonstrations is that Neo-Ricardian propositions **P1** and **P3** are wrong, and Leontief's representation of production is compatible with that of Walras. The complete development of the idea of general interdependence by Walras, in the classical fashion, has been noticed by Leontief and seems to have impressed him; Leontief saw a continuous line between the Classics and Walras: "It is true that the notion of general interdependence was inherent in the classical tradition to which [Walras] was heir, while the concept of measurable subjective utility clearly reflected the new psychologism which invaded all social sciences in the last third of the nineteenth century. But looking forward one observes that the idea of general equilibrium came to occupy a central, dominant role in modern economics, while the once so much discussed problem of value turns out to be one of its outlying, not to say peripheral, branches" (Leontief 1955, 250). The presence of a circular flow representation in Walras' analysis finds an explanation as the *Tableau Economique* of Quesnay directly inspired Walras' view of the process of production. Walras characterized his practical description of the 'dynamic' market economy (the continuous market) as follow: "Any picture such as we have sketched in Lesson 35 describing the economic life of people in terms of concrete numbers is called an economic table (*Tableau économique*). At least, there is one *Tableau Economique*, analogous to our own, which is quite celebrated in the history of economic literature. This is Dr. Quesnay's, which was published at Versailles in 1758…" ([1974] 1977, 393). As noticed Bridel (1997), Walras' (tortured) care for realism made him use Quesnay's simple model of economic flows (36-37). Walras knew that relevance of his general equilibrium theory for 'real world' issues (especially financial and monetary crises) rested upon the treatment of circulating capital, money and time (for a study of the relations between those three elements see Bridel 1997).

[49] If, for instance, consumption goods were also used as intermediate commodities. However, even Sraffa introduced pure final goods (non fundamental commodities).

As a conclusion, it seems that Sraffa's distinction between the two views of production is not functional concerning Walras' theory. In front of the complete model of Walras, Sraffa's Guillotine is not sharp-edged enough. Arguments of Neo-Ricardians considering Walras' model as incompatible with Leontief's model of the circular flow are not operational. As far as Walras' view of production was similar to Leontief's one, concepts and tools Walras shaped became available for the making of Leontief's classical model of general interdependence. In contrast with Sraffa, what primarily mattered to Leontief was not the theory of value but business cycles and the moving circular flow. Indeed, Leontief's PhD dissertation, and his work on interindustrial relations as well, clearly show that, although he was aware of the problem of value and took many similar theoretical decision than Sraffa, Leontief concentrated his effort on the question of quantitative development and business cycles. In sharp contrast with Sraffa who took in consideration "no changes in output" ([1960] 1975, *v*), Leontief explored in his 1928 essay numerous kinds of economic changes like "steady changes", "irregular change" or "oscillations". Like numerous economists of his time, Leontief tried to catch the general interdependence of the economic system. Technical coefficients could map the move of the economic system in a Walras-Schumpeter way, that is to say in considering changes in technical coefficients, e.g. changes in combinations of the elements of the system. Disaggregation was the rule: "The lower the level at which the new combination appears, the finer the net of technical coefficients must cast in order to capture the identity" (Leontief [1928] 1991, 184). Hence, according to Leontief, "the principal merit of general equilibrium theory lies in the fact that it enables us to take account of the highly complex network of mutual interrelationships which transmits the impulse of any local primary change into the remotest corners of the existing economic system" (1937, 110).

IV. Conclusion

The classical concept of circular flow encloses three interrelated elements: *i.* the idea of reproduction, *ii.* the analysis of prices according to physical costs and the share of surplus and, *iii.* the representation of the production process as a circular flow.

Neo-Ricardians are right to say that Leontief's 1928 and 1937-1941 models are Classical, but for wrong reasons. 'Sraffa's Guillotine', that is to say the straight distinction between Classical and Marginalist theories based on the representation of production as a circular flow or as a one-way avenue, is not a powerful criteria unless Walras' general equilibrium model is a Classical one. The myth of a Walrasian model based on a one-way avenue production model has endured enough. Ultimately, what seems to draw the line between Leontief's work and the Walrasian device is the status of demand. However, because of Samuelson's interpretations of non-substitutions theorems and the role of demand in models of Leontief, a classical economist may try to find another argument, some more radical one. However, the models of Leontief should not be considered as Classical or Neoclassical, but a kind of check-point at the border: a duck-rabbit picture. Indeed, saying that Marx's schemas of reproduction are the theoretical basis of Leontief's models doesn't mean that his reference to Walras was accidental: it is instrumental from a historical point of view (Leontief actually used Walras' theory) and from a theoretical point of view.

Indeed, in contrast with Böhm-Bawerk and Cassel, Walras and Leontief shared a close view of the economy as a circular process. The idea that Walras' theory of production rests on production as a one-way avenue is a myth that have endured for generations of economists. Another legend concerns the hypothetical relationship between Bortkiewicz and Leontief. Bortkiewicz was not only the author of a new formulation of the transformation problem but an admirer of Walras' general equilibrium theory. He may have been one of the links between

Ricardo, Marx, Walras and Leontief. As a matter of fact, Leontief's 1928 classical theory of prices is quite peculiar as far as he based the usual classical 'law of costs' on an unusual theory of exchange between producers in a free market economy. Producers play simultaneously the roles of workers and owners. This theory is a peculiar way to present the Classical theory of value. Moreover, Leontief didn't explicitly introduce real wages and profits rates in his prices equations, a way which led to his 1937 model where equilibrium prices are independent of the surplus distribution. As a matter of fact, surplus sharing analysis was not at the first stage of his analysis, in contrast with his detailed study of the moving production process.

Despite Leontief's clear rejection of the concepts of utility and demand, and because of the theoretical similarities between Leontief's view of production and the one of Walras, the formulation of the theory underlying input-output analysis could make an intensive use of the "powerful tools of general equilibrium theory" (Leontief 1949, 213). These tools were primarily used in order to study discrete or marginal changes in technical coefficients. For instance, Leontief (1937) analyzed the effects of a marginal change of productivity coefficients on prices and outputs, using derivative calculus. Leontief's ability to use general equilibrium concepts and instruments was easier as his rhetoric was not a rhetoric of conflict, like Samuelson or Sraffa's ones, but a rhetoric of consensus like that of Tugan-Baranowski, Dmitriev or Bortkiewicz who tried to put together Classical and Neoclassical thoughts. Leontief was first of all looking for tools in order to calculate the effects of exogenous variations on the variables of the economic system; this is why he simplified many theoretical problems –like most of his contemporary fellow macroeconomists. It makes a serious difference with Sraffa's assumption of no changes in outputs. Leontief's approach had more to do with the works of Schumpeter, Ernst Wagemann, Fernand Grünig, or the "Kiel school", than those of Sraffa. History of models of Leontief should not be separated from history of business cycles studies, of non-Keynesian macroeconomics or national accounting's early developments in Russia and Western Europe.

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