Rosa Luxemburg's critique of Marx's schemes of reproduction: a re-evaluation and a possible generalisation

Desai, Meghnad and Veneziani, Roberto

2005

Rosa Luxemburg's critique of Marx's schemes of reproduction: A re-evaluation and a possible generalisation

BY

Meghnad Desai* and Roberto Veneziani†

Abstract:

Rosa Luxemburg's critique of Marx's Schemes of Reproduction is the only genuinely immanent critique of Marxian dynamics. But it needs to be brought up to date. To this end, in this paper it is placed in the context of more modern discussions of dual instability, in order to gain a better understanding of the role of price - as against quantity - adjustment in tackling instability. This allows us to discuss the path of prices as well as of quantities which have been the sole subject of controversy hitherto. Our approach also allows us to tackle the money circuit of capital parallel to the value circuit as outlined in the first part of Capital Vol 2.

JEL classifications: B14, C61, E11, O41

Keywords: Rosa Luxemburg, accumulation, dynamic instability, linear models.

* Department of Economics, London School of Economics and Political Sciences.

† Corresponding author: Department of Economics, Queen Mary University of London, Mile End Road, London E1 4NS. E-mail: r.veneziani@qmul.ac.uk.

1. INTRODUCTION

Rosa Luxemburg's immanent critique of Karl Marx's accumulation schemes is well known. In her *The Accumulation of Capital* (1951), she not only gave an account of the rather spirited, but confused debate which broke out among the European Marxists on the publication of Volume 2 of Capital - especially the last chapter on the Scheme of Expanded Reproduction, - but she offered her own cogent critique and proposed a solution (Luxemburg, 1951; see also Desai, 1979, chapter 15). Marx's Scheme appeared to offer a model of capitalist accumulation, which guaranteed perpetual growth at a steady rate for both sectors (Departments), and indeed has been characterised as the fastest converging two sector balanced growth model by Morishima (1973, chapter 10). Marx's model seemed to go against the vision of a permanently cyclical course for capitalist accumulation outlined in *Capital*, Volume 1, Part VII. There were other inconsistencies between the two volumes' portrayal of capitalist accumulation. In the Scheme for Expanded Reproduction (SER hereafter): (*i*) there is no technical progress and hence the value per unit of physical output in both Departments and the organic composition of capital are constants; (*ii*) the profit rate is computed in value terms rather than in money terms and differs between the two sectors; and (*iii*) the profit rate in each sector is constant and does not decline. Also in terms of Volume 2 itself, while Part I laid out the three circuits of capital in terms of value, money, and physical quantities, the SER is only in value terms. Prices or money play no role in the SER.

The SER generated a long debate. Was it a scheme for abstract study of how gross and net output are reconciled at the aggregate level, hence a pioneering exercise in national income accounting, or was it a realistic picture of how it was possible for a capitalist economy to grow? Was it a scheme for planning as it later proved useful in Fel'dman's model for the Soviet Economy's First Five Year Plan or did it lay down impossibility conditions for a capitalist economy ever to achieve balanced growth if it relied on uncoordinated accumulation plans, as against the accommodative behaviour postulated in the SER?[1]

Rosa Luxemburg criticised Marx's model as inconsistent within its own logic, since the realisation problem is not squarely faced but solved, as it were, by a sleight of hand. The limits to the supply of labour power are assumed away. Since each Department does not have ready demand for its output before it could plan accumulation, where does the money come from - she cogently asked - for each Department to place the order for extra machines which dissolves the problem?

Where does the money come from? This question is loaded with dynamite. Marx had not tackled the circuit of money capital in the SER. Thus, the exchange of goods between the two Departments seemed to occur outside the exchange nexus, in a gigantic book keeping exercise for all capitalists together. If the question of money is posed, then each Department has to sell its output and realise the value and surplus value before making accumulation plans. Absent the issue of money, and the virtual market economy of SER can be swallowed.

In this paper, we want to pose the question of the money circuit and of prices explicitly. This is done not just for exegetical reasons but because the stability of the growth path in terms of value can be misleading, since it can very well be that in the realm of prices instability may reign. This is not just idle speculation. There is a well established literature on dual instability in Leontieff systems (see, e.g., Sargan, 1958; Jorgenson, 1960A, 1960B). This literature has a basic theorem, which states that: if the quantity system is stable, then the price system will be unstable, and vice versa. The reason for this is mathematically trivial; the roots of one system are the reciprocal of the roots of the other system. Extending this conjecture to the SER (which is an early example of the Leontieff input-output system) is at the heart of our inquiry.

In what follows, the next section will lay out the general outlines of the SER as first proposed by Marx. Marx's peculiar investment rule for the two sectors is then explained since it is the root cause for the super stable behaviour of the model. Most authors, including Luxemburg, did not question this rule. But it is at the heart of the problem: no capitalist economy can have a cooperative rule whereby one sector's capitalists are willing to mop up the unsold output of another sector. But once Marx's rule is dropped, there are many possibilities open as to how accumulation behaviour will be determined. In his treatment of SER, Morishima (1973) puts forward one such rule and analyses the roots of the resulting system. As a first option, we lay out the price system of Marx's SER and then Morishima's (1973) modification. This reveals the underlying instability of the Marxian model. This then shows the scope for future developments of the

Marxian model in a full three circuits of capital logic, where stability and instability are dialectically and mathematically intertwined.

2. THE MODEL

Marx's SER has a familiar structure. Its elements are as follows:

(a) There are two Departments wherein Department I makes machines which are consumed during the production process as constant capital, while Department II produces wage goods which are consumed by workers as well as capitalists;

(b) The rate of exploitation is identical across the two Departments and constant over time as is the organic composition of capital in each Department;

(c) Department I produces more units (measured in labour values) than is required for the replacement of consumed constant capital and by corollary Department II produces less than the sum of variable capital and surplus value;

(d) The imbalance is resolved (*i*) by Department I accumulating ½ of its own surplus value and putting it in production at the same rate organic composition of capital as initially used; and (*ii*) by Department II buying up all the output of Department I surplus to the requirement of Department I and again keeping the same organic composition of capital as it started with.

As is well known, the result of this process is that after the first year where the two Departments grow unevenly, every subsequent period they grow at constant rates. Accumulation in both Departments is at a constant rate.

This is a powerful result and in the general context of Marx's critique of political economy, it is surprising. Its interpretation remains contested and indeed became the subject of a long and intense controversy in the 25 years following the publication of Volume 2 in 1884. Today, we understand the structure of Marx's SER much better, but Rosa Luxemburg mounted an immanent critique of Marx's scheme in as much as it rules out any realisation problem or any problem of shortage of labour power. She put the model in the larger context of a world with rich and poor countries wherein the rich countries solved their realisation problem by selling to the poor countries and buying cheap food stuffs from them to keep the value of the wage low.

For our purposes the crucial insight that Luxemburg offers is to ask how Department I finds money to accumulate when it cannot sell its output and realise costs, much less profits. Where does the money come from? This points to a need to place the SER in the three circuits of capital framework where in addition to the C-C' (value capital) circuit we have the M-M' (money capital) and the Q-Q (physical capital) circuits.[2] This means prices have to be solved for explicitly; the money sums advanced and realised have to be calculated. Since productivity is constant by assumption in the SER, the C-C' and Q-Q circuits are exact images of each other. Thus out of the three circuits at least two need to be developed . The missing circuit is the money capital circuit.

In order to describe the production structure of the SER, let A be the 2×2 matrix of input coefficients, where a_{ij} is the quantity of good i used in the production of good j; let $A_{i\cdot}$ and $A_{\cdot j}$ denote the i-th row and j-th column of A, respectively. Sector 1 produces capital goods, whereas sector 2 produces wage goods. Let Y be the 2×1 vector of activity levels; let e denote the uniform rate of exploitation. The familiar SER tables can be written as

$$Y_1 = a_{11}\, Y_1 + a_{21}\, (1 + e)\, Y_1, \tag{1a}$$

$$Y_2 = a_{12}\, Y_2 + a_{22}\, (1 + e)\, Y_2. \tag{1b}$$

The first equation is about the output of Department I which uses a_{11} of its own output and a_{21} of Department II's output as inputs. The same logic applies to (1b) which gives us the output of Department II. In addition we begin with a disequilibrium condition that

$$Y_1 > a_{11}\, Y_1 + a_{12}\, Y_2, \tag{2a}$$

$$Y_2 < a_{21}(1 + e)Y_1 + a_{22}\, (1 + e)Y_2. \tag{2b}$$

Marx's resolution of the disequilibrium is that Department I invests half (or generally a portion β) of its surplus value which equals $e\ a_{21}Y_1$. A portion of it goes to Department I itself as extra demand. This is given by the *organic composition of capital* which is $k_i = a_{1i}/(a_{1i} + a_{2i})$, $i = 1, 2$. Once this demand for its own product has been decided then Department II steps in to buy the rest of the available surplus output of Department I. So given (2a), at all t we have

$$Y_{1t} - (a_{11}Y_{1t} + a_{12}Y_{2t}) - (\beta\, e\ a_{21}Y_{1t})k_1 = \theta_t\ (e\ a_{22}Y_{2t})k_2. \tag{3}$$

In (3), θ_t is whatever number will equate the two sides at t. The result of steady growth follows from the next period onwards.

Since this is an arbitrary behavioural rule, it is better to formulate the SER more generally. Morishima (1973) has done this. It is useful for our purposes to take his approach. He postulates a constant propensity to accumulate by all capitalists and the possibility for all capitalists to invest in either sector – given profit rate equalisation, – but retains the assumption of a constant organic composition of capital. This allows him to set up the demand and supply equations for the two Departments in the form of a pair of difference equations. Thus, he has

$$Y_{1t} = a_{11}Y_{1t} + a_{12}Y_{2t} + a_{11}\,\Delta Y_{1t} + a_{12}\,\Delta Y_{2t}, \tag{4a}$$

$$Y_{2t} = a_{21}\,Y_{1t} + a_{22}\,Y_{2t} + a_{21}\,\Delta Y_{1t} + a_{22}\,\Delta Y_{2t} + b(s_1 Y_{1t} + s_2\,Y_{2t}), \tag{4b}$$

where $s_i = a_{2i}\,e$ is the rate of surplus value in Department i, i = I, II, and b is the capitalist's propensity to consume, which is one minus the propensity to accumulate, and is assumed to be equal for all capitalists. This pair of difference equations can be written in matrix form as:

$$Y = AY + A\,\Delta Y + BY, \tag{4}$$

where B is a matrix with zeros in the upper row and $[bs_1,\, bs_2]$ in the lower row. The latter expression can be rearranged as

$$[I - A - B]\;Y = A\,\Delta Y. \tag{5}$$

Since Δ denotes a forward difference, we can write down the basic difference equation of the SER as in Morishima (1973, p.123).

$$[\mathrm{I} - B]^{-1}A\;Y_{t+1} = R\;Y_{t+1} = Y_t, \tag{6}$$

where $R_{1i} = a_{1i}$ and $R_{2i} = (b\;s_i\;a_{1i} + a_{2i})/(1 - ba_{22})$, i = 1, 2. Hence, all the R_{ij}'s are positive. Solving the characteristic equation of the 2 x 2 matrix R, we can examine the roots, then the dynamic path of output will be determined by the reciprocal of the two roots of R. Of the two roots, μ_1 is positive, smaller than one, but greater than μ_2 - which is positive if and only if $a_{12}/a_{22} < a_{11}/a_{21}$, that is if and only if

Department II is less capital intensive than Department I, as in Marx's SER.[3] Morishima shows that unless the economy starts out with just the right proportions, it will have unstable balanced growth: "an economy starting from an initial point away from the balanced growth path will diverge from it as time goes on" (Morishima, 1973, p.125). This result is dramatically different from the fast convergence to balanced growth of the original Marxian SER.

3. Dual Instability

The more interesting task, however, is to trace out the circuit of money capital M-M'. We take it that the money initially advanced by the capitalists at the beginning of t, M_t, buys the initial inputs $(a_{11} + a_{21})Y_{1t}$ for Department I, and the initial inputs $(a_{12} + a_{22})Y_{2t}$ for Department II. At the end of the M-M' circuit, all the output is sold at the new price and the difference between M' and M is profits or m as Marx labels it or,

$$M'_{t+1} = M_t + m_{(t/t+1)}. \tag{7}$$

The subscript to m is written as it is because it emerges within the production period but is realised at the end. Marx solves the realisation problem by using the equations for accumulation. This relates to the way in which M_{t+1} will enter into production again. But since the realisation has to be before the re-advancing of the realised capital, Marx's solution is questioned by Luxemburg. However, we can investigate this issue in the more general framework posited by Morishima.

First, let p_{t+1} denote the 1×2 vector of prices prevailing at the end of t – beginning of $t + 1$. By pre-multiplying (4) by p_{t+1}, we obtain

$$p_{t+1}\, Y_t = p_{t+1}AY_t + p_{t+1}\, A\Delta Y_t + p_{t+1}BY_t. \tag{8}$$

Equation (8) looks simple but contains a lot of information. The term $p_{t+1}Y_t$ on the left hand side is output at the end of t - beginning of $t + 1$, valued at prices which will prevail in $t + 1$. On the right hand side, first, we have expenditure to replace inputs consumed during production, $p_{t+1}AY_t$. Then, is the buying of the inputs for next period's production which is the term $p_{t+1}A\Delta Y_t$. Since the inputs are used in the next period they are priced at next period's prices. The last item is the consumption expenditure of the capitalists which is also bought at the end of period at the new prices.

Indeed, equation (8) shows the theoretical and analytical differences with the literature on dual instability, which arise from the class perspective incorporated in the system. In Solow (1959) and Jorgenson (1960A, 1960B), there is no class dimension, and thus the consumption of all agents is equally treated as part of the reproducibility requirements of the system, and is incorporated into the Leontieff matrix A. Here, we distinguish between worker and capitalist consumption, whereby the former is included in A and is advanced by the capitalists, whereas the latter is included in B and is paid out of revenues.

Absent a theory of prices, however, there is not much else we can say about the M-M' circuit. Hence, we shall add some more structure to Morishima's (1973) model. We assume that at the beginning of each period, capitalists can decide either to enter production buying all necessary inputs (including wage goods) using their wealth (which is thus transformed into productive capital). Alternatively, they can lend their wealth on the financial market and receive the competitive interest rate, thus becoming financial capitalists. Let r_t denote the competitive interest rate at t. If they choose to produce one unit of good i, capitalists need to advance an amount of money equal to $p_t A._i$ and receive a price equal to p_{it+1} at the end of the production process. By investing the same amount of money in the financial market, they earn a gross return of $(1 + r_t)\, p_t A._i$. Hence, assuming that competitive conditions prevail, both investments must yield the same return and the no arbitrage conditions can be written as follows.[4]

$$p_{t+1} = (1 + r_t)\, p_t A. \qquad (9)$$

Equation (9) describes the dynamic path of prices in a rather convenient form and it allows us to exploit the results of the literature on dual instability, conveniently modified to fit our specific framework. In fact, Morishima's analysis of R can be applied noting that if B is equal to the null matrix then $R = A$, and that nothing in Morishima's analysis depends on $I - B \neq I$: assuming that department I is more capital intensive, A has two positive roots smaller than one. Hence, whereas the quantity system has an unstable balanced growth path, the price system has a (asymptotically) stable balanced 'growth' path, with prices vanishing in the limit, provided the interest rate is equal to zero.

However, the existence of a positive interest rate enriches the dynamic possibilities of this economy. Assume for simplicity that the interest rate is constant. (At most, it will fluctuate in the neighbourhood of the constant profit rate, which is implied by the linear structure.) Let λ_1 and λ_2 denote the two latent roots of the characteristic equation associated with A; we can write $1 > \lambda_1 > \lambda_2 > 0$.[5] The solution to (9) will be as follows.

$$p_{1t} = \chi_1\, n_{11}\, (1 + r)^t\, (\lambda_1)^t + \chi_2\, n_{21}\, (1 + r)^t\, (\lambda_2)^t, \qquad (10)$$

$$p_{2t} = \chi_1\, n_{12}\, (1 + r)^t\, (\lambda_1)^t + \chi_2\, n_{22}\, (1 + r)^t\, (\lambda_2)^t, \tag{11}$$

where χ_1 and χ_2 are constants determined by initial prices $p_0 = (p_{10}, p_{20})$, and the ns are obtained from the two sets of equations

$$(a_{11} - \lambda_i)\, n_{i1} + a_{12}\, n_{i2} = 0, \tag{12a}$$

$$a_{21}\, n_{i1} + (a_{22} - \lambda_i)\, n_{i2} = 0. \tag{12b}$$

From (10)-(12), we can derive our main result concerning price dynamics. *THEOREM 1. (Dual Instability) In the SER economy with competitive pricing and a positive, constant interest rate r:*

(i) *If $1/(1 + r) \geq \lambda_1$ and initial prices do not correspond to the stationary values, then prices tend to the latter. Depending on initial conditions, decrease can take place along a balanced or an unbalanced path, but the balanced path is stable.*

(ii) *If $1/(1 + r) > \lambda_1$, then the only stationary prices are equal to zero, whereas if $1/(1 + r) = \lambda_1$ then there exists a strictly positive vector of stationary prices.*

Proof. Part (i). First, convergence follows from (10) and (11) noting that $1 > \lambda_1 > \lambda_2$. Second, initial conditions determine the value of χ_1 and χ_2 and thus whether prices will move at the same rate; but given that $\lambda_i(1 + r) \leq 1$, $i = 1, 2$, then if prices are not on a balanced path, they will converge to it.

Part (ii). Finally, if $(1 + r) = 1/\lambda_1$, then by the Perron-Frobenius theorem there is a strictly positive vector of constant prices p^F satisfying (9). If $(1 + r)$

$< 1/\lambda_1$ - and thus $(1 + r) < 1/\lambda_2$ - then the only constant vector prices satisfying (9) is the zero vector. *Q.E.D.*

Theorem 1 describes the form in which dual instability manifests itself in the Marxian SER schema, if one assumes competitive dynamic pricing. But we can also give an economic interpretation. Given the linear structure of the system, the reciprocal of λ_1 is (one plus) the profit rate of the system. Thus if $\lambda_1 > 1/(1 + r)$, i.e. rate of interest exceeds the profit rate, dual instability would be present: although the price system, too, would have at least one unstable root the balanced growth path would be asymptotically stable. Yet, arguably, this is the economically uninteresting case because the gross interest rate would persistently exceed the profitability of the system. The results above concern the case where either the profit rate exceeds the gross interest rate or they are just equal.

Although the dynamic behaviour of prices is interesting, our aim is to characterise the behaviour of money aggregates, that is, the circuit of money capital. Let ω_t denote the 2×1 vector of aggregate endowments of both goods in the economy at the beginning of t. Since workers do not save, all assets in the economy are owned by capitalists. Hence, the money capital circuit will start off with $M_t = p_t\omega_t$. Assuming full employment of capital, under (9) it will be true that $M_t = p_t\omega_t = p_tAY_t$, all t. Furthermore, if ω_{t+1} denotes physical assets accumulated up to $t + 1$, and available for production, in the circuit of money capital we have $M_{t+1} = p_{t+1}\omega_{t+1}$. In order to derive a precise expression describing the dynamics of M, note that (8) is equivalent to

$$p_{t+1}\, Y_t = p_{t+1}AY_{t+1} + p_{t+1}BY_t, \tag{8'}$$

where the first term on the right hand side is just the monetary value of assets at the beginning of $t + 1$, that is, $p_{t+1}AY_{t+1} = p_{t+1}\omega_{t+1}$. Hence, in terms of Marx's description of the money capital circuit, we have $M_t = p_tAY_t$, $M'_{t+1} = p_{t+1}Y_t$, and $M_{t+1} = p_{t+1}AY_{t+1}$. Therefore, by (7) and (8') we can derive the following equation.

$$M'_{t+1} = M_t + m_{(t/t+1)} = M_{t+1} + p_{t+1}BY_t. \tag{7'}$$

From (7'), it follows that $M'_{t+1} \geq M_t$ as long as profits are nonnegative and $M_{t+1} \geq M_t$ as long as the monetary value of consumption is not higher than money profits. However, in order to characterise the dynamics of money capital more precisely, we need a more explicit expression in terms of M_t and M_{t+1}, which is actually given by (8') and (9), using the fact that $M_t = p_t\omega_t = p_tAy_t$, all t.

$$p_{t+1}\omega_{t+1} = (1 + r)\, p_t\omega_t - (1 + r)\, p_t\, ABY_t. \tag{13}$$

THEOREM 2. *(The M-M' circuit.) In the SER economy with competitive pricing and a constant, positive interest rate r such that $1/(1 + r) \geq \lambda_1$, there is a positive number b* such that money capital grows without bounds for all $b \in [0, b^*)$ along the BGP.*

Proof. 1. If capitalists do not consume, then $b = 0$ and $B = \mathbf{0}$, where $\mathbf{0}$ is a matrix with all zeros and by (7') $M'_{t+1} = M_{t+1}$, all t. The result follows by noting that money capital then moves according to the following expression.

$$p_{t+1}\omega_{t+1} = (1 + r)\, p_t\omega_t. \tag{13'}$$

2. Consider now the case with $b > 0$ and $B \neq \mathbf{0}$. By (8') and using the competitive pricing equation (9), we write

$$M_{t+1} = p_{t+1}AY_{t+1} = (1 + r)p_tAY_t - (1 + r)p_tABY_t, \qquad (14)$$

which describes the dynamics of M_t. Factoring terms out and using (9), the latter expression can be written as

$$M_{t+1} = (1 + r)^{t+1}p_0A^{t+1}[\mathrm{I} - B]Y_t,$$

or, using the diagonalisation $A = V\Lambda V^{-1}$,

$$M_{t+1} = (1 + r)^{t+1}p_0\, V\, \Lambda^{t+1}V^{-1}\, [\mathrm{I} - B]Y_t. \qquad (14')$$

Next, recall that Morishima's (1973) matrix R is defined as $R = [\mathrm{I} - B]^{-1}A$ so that $Y_t = [R^{-1}]^tY_0$, or using the diagonalisation $R^{-1} = E\, Z\, E^{-1}$,

$$Y_t = E\, Z^t\, E^{-1}Y_0, \qquad (15)$$

where Z is the inverse of the diagonal matrix of the eigenvalues of R. Thus

$$M_{t+1} = (1 + r)^{t+1}p_0\, V\, \Lambda^{t+1}V^{-1}AE\, Z^{t+1}\, E^{-1}Y_0. \qquad (14'')$$

If $b = 0$, then (14'') reduces to $M_{t+1} = (1 + r)^{t+1}p_0\, AY_0$, as in part 1. If $b > 0$ consider the BGP of Morishima (1973). First, note that the BGP rate, $1 + g$, is equal to the inverse of the Frobenius-Perron eigenvalue of R, μ_1; that is, $(1 + g) = (\mu_1)^{-1}$. At the BGP, then (14'') can be written as:

$$M_{t+1} = (1 + r)^{t+1}p_0\, V\, \Lambda^{t+1}V^{-1}A(\mu_1)^{-t+1}Y_0. \qquad (14'')$$

Second, by the Perron-Frobenius theorem, μ_1 is a strictly increasing, continuous function of the entries of R. Hence, since R_{21} and R_{22} are strictly increasing in b, a fortiori μ_1 is a strictly increasing, continuous function of b such that $\mu_1 = \lambda_1$ if $b = 0$ and $\mu_1 = 1$ if $b = 1$. Finally, note that $1 \geq \lambda_1(1 + r) > \lambda_2(1 + r)$ and let b^* be the value of b such that $\lambda_1(1 + r) = \mu_1$. Then we can conclude that for all $b \in [0, b^*)$ money capital grows without bounds. *Q.E.D.*

Remarks. The assumption that $1/(1 + r) \geq \lambda_1$ plays no role if $b = 0$: if capitalists do not consume, then a positive interest rate is sufficient for money capital to increase exponentially. Another way to put this is that a profit rate above the rate of interest allows some room for capitalists' consumption without necessarily affecting accumulation. But if capitalists consumption is zero , then accumulation is better secured.

4. Conclusions

We have re-examined Marx's SER which was subjected to an immanent critique by Rosa Luxemburg. However along with her contemporary Marxist critics she took Marx's investment rule as given. The difficulty with this rule is that it is arbitrary and variable enough to ensure superstability but as a piece of legerdemain. We extend Morishima's more systematic analysis to embrace prices and money circuit of capital. We find a relationship between the rate of interest and the rate of profit and explore the limits to capitalist consumption as it affects accumulation. There are still issues which remain. Thus, ideally we require an ex-ante investment rule to see if that will lead to cycles and crises. We hope to take up such issues in future work.

REFERENCES

Desai, M. (1979). *Marxian Economics*, Oxford: Blackwell.

Goodwin R.M. and L. Punzo (1989). *The Dynamics of a Capitalist Economy*, London: Polity.

Jorgenson, D.W. (1960A). "A Dual Stability Theorem", *Econometrica* **28**, 892-899.

Jorgenson, D.W. (1960B). "Growth and Fluctuations: A Causal Interpretation", *Quarterly Journal of Economics* **74**, 416-436.

Luxemburg, R. (1951). *The Accumulation of Capital*, London: Routledge.

Morishima, M. (1958). "Prices, Interest and Profits in a Dynamic Leontief System", *Econometrica* **26**, 358-380.

Morishima, M. (1973). *Marx's Economics: A Dual Theory of Value and Growth*, Cambridge: Cambridge University Press.

Sargan, J.D. (1958). "The Instability of the Leontief Dynamic Model", *Econometrica* **26**, 381-392.

Solow, R.M. (1959). "Competitive Valuation in a Dynamic Input-Output System", *Econometrica* **27**, 30-53.

Uzawa, H. (1961). "On a Two-Sector Model of Economic Growth", *Review of Economic Studies* **29**, 40-47.

[1] On Fel'dman's model see Desai (1979, chapter 17).

[2] This notation is adapted from Desai (1979, chapter 5).

[3] It is interesting to note that this was the stability condition in Uzawa's paper on two sector growth models which revived interest in this literature (Uzawa, 1961).

[4] Equation (9) is derived under the implicit assumption of no fixed capital.

[5] This can also be derived from the Perron-Frobenius theorem given that A is a nonnegative, productive, indecomposable matrix.