Exploitation and productiveness: the generalised commodity exploitation theorem once again

Veneziani, Roberto and Yoshihara, Naoki

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Exploitation and productiveness: the generalised commodity exploitation theorem once again.[1]

Roberto Veneziani[2] and Naoki Yoshihara[3]

Abstract: In a recent contribution on this journal, Matsuo (2009) has provided an interesting argument to refute the Generalised Commodity Exploitation Theorem (GCET), by highlighting a potential asymmetry between labour and other commodities. In this paper, a novel characterisation of the relation between exploitation and productiveness that is at the heart of the GCET is proved. This result is interesting per se, because it is weaker and more general than the standard GCET. But, owing to the rigorous specification of all the relevant conditions, it also clarifies the structure of Matsuo's argument, and its dubious theoretical features. It is also argued that, even if Matsuo's formal argument were deemed convincing, a revised version of the GCET can be proved, which reinstates the symmetry between labour and other commodities in the standard Leontief setting.

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[2] (Corresponding author) Department of Economics, Queen Mary University of London, Mile End Road, London E1 4NS, UK. E-mail: r.veneziani@qmul.ac.uk.

[3] The Institute of Economic Research, Hitotsubashi University, Naka 2-1, Kunitachi, Tokyo 186-8603, Japan. Phone: (81)-42-580-8354, Fax: (81)-42-580-8333. e-mail: yosihara@ier.hit-u.ac.jp.

1 Introduction

In debates on exploitation theory, a prominent role is assigned to the so-called "Fundamental Marxian Theorem" (Okishio, 1963; Morishima, 1973; henceforth, FMT), which is interpreted as showing that positive profits are synonymous with the exploitation of labour. Some authors, however, have argued that the FMT does not prove that the exploitation of labour is the sole source of profits, because the FMT can be generalised to all commodities, and profits simply derive from the productiveness of the economy. The latter result is also known as the "Generalised Commodity Exploitation Theorem" (Bowles and Gintis, 1981; Roemer, 1982; henceforth, GCET), and it proves that positive profits occur if and only if every commodity is 'exploited' - that is, the commodity i value of commodity i itself is less than one - in economies in which the vectors of the values of all commodities are nonnegative.

In a recent contribution on this journal, Matsuo (2009) provides an interesting argument to question the GCET. In a nutshell, Matsuo's core argument is that whereas the relation between the exploitation of labour and positive profits relies on a purely technological condition concerning the productiveness of the economy, the analogous relation between the 'exploitation' of other commodities and positive profits, relies on some conditions that are not purely technical and embody social relations. Therefore, there is an asymmetry between labour and other commodities in the generation of profits, because, argues Matsuo, whereas the former assumption is natural, the latter are less compelling. If only the former assumption is required to hold, it is possible to show that there are situations in which some produced goods are 'exploited,' but labour is not and profits are not positive.

This argument is logically correct and it is theoretically more convincing than other recent attempts to "refute" the GCET.[1] In this paper, however, we argue that it does not conclusively establish the desired result. First, let M denote the augmented input matrix including the workers' consumption bundle.[2] The GCET states that "the productiveness of M is equivalent to the exploitation of any commodity, and so the profitability of the system cannot be 'explained' by labor's exploitation" (Roemer, 1986, p.24). From a mathematical viewpoint, this claim is not falsified by Matsuo and it remains true that an economy can produce a surplus over and above workers' consumption, and the replacement of inputs used up in the production process, if and only if every commodity is exploited in the above sense, and the value vectors are nonnegative. In this paper, a novel version of the GCET is provided, which forcefully highlights the logic of the theorem and rigorously states all the relevant conditions. The result is interesting per se, because it is weaker and more general than the standard versions of the GCET proved in the literature, but it also clarifies the basic structure of Matsuo's argument, and its problematic theoretical implications.

Second, the theoretical relevance of the asymmetry between labour and other commodities derived in Matsuo's main counterexample is unclear. Matsuo shows that there exists one economy which is productive in the sense of being able to produce a surplus over and above the material inputs used up in the production process (but not necessarily workers' consumption), such that (i) labour is not exploited, (ii) there is a commodity i whose commodity i value is positive and lower than one, but (iii) there is no semipositive price vector such that profits are nonnegative in all sectors. In order to derive this example, Matsuo has to violate the condition of productiveness of M, but, as shown below, this implies that the economy cannot reproduce itself. Further, although the commodity i value of commodity i is positive and smaller than one, the vector of commodity i values of all goods is not (and cannot be) nonnegative. Yet, the significance of value vectors with some (or even many) negative entries is highly questionable. Arguably, both features raise serious doubts on the example, and on the theoretical conclusions derived from it.

It may be objected that the example does depict a logically possible scenario, and that in general it is legitimate to construct economies in which reproducibility and nonnegativity restrictions on value vectors are violated. This argument seems rather objectionable on methodological grounds, but even assuming, *for the sake of argument*, Matsuo's example to be robust from the methodological viewpoint, this paper shows that in any case the asymmetry between labour and other commodities in the generation of profits is not convincingly established in standard Leontief economies. For it is possible to prove that for each commodity k, there exists an economy such that (i) labour is exploited, but (ii) profits are nonpositive for some price vector and (iii) commodity k is not exploited. In this sense, *even if* Matsuo's example is considered methodologically unobjectionable, it does not establish the desired asymmetry because it can be extended in such a way as to lead to a generalisation of the GCET itself, rather than to its refutation, and the distinction between purely technical productiveness conditions and 'pseudo' productiveness conditions seems theoretically doubtful.

The rest of the paper is organised as follows. In the next section, a new version of the GCET is proved and the theoretical relevance of Matsuo's example is questioned. In section 3, it is argued that, even if Matsuo's example is considered compelling, in any case the alleged asymmetry between labour and other commodities does not follow, and a generalised version of the GCET can be proved. Section 4 concludes.

2 The GCET and Matsuo's critique

Consider a standard Leontief economy. Let A be a $n \times n$ non-negative input matrix, let L be a $1 \times n$ semi-positive vector of direct labour inputs, and let b be a $n \times 1$ semi-positive vector of workers' wage goods. Let A be a $(n+1) \times (n+1)$ augmented input matrix

$$\mathsf{A} = \begin{bmatrix} A & b \\ L & 0 \end{bmatrix}.$$

Let $\mathsf{A}_{(k)}$ denote the $n \times n$ matrix obtained from A by removing the k-th column and the k-th row. Thus, $\mathsf{A}_{(n+1)} = A$. Let x be a $n \times 1$ vector of activity levels, let p be a $1 \times n$ vector of prices, let λ be a $1 \times n$ vector of labour values, and let $\lambda^{(k)}$ be a $1 \times (n+1)$ vector of commodity k values, where $k \neq n+1$, describing the amount of commodity k directly and indirectly necessary to produce every commodity. Formally, λ and $\lambda^{(k)}$ are defined by the following standard formulas: $\lambda = \lambda A + L$ and $\lambda^{(k)} = \lambda_{[k]}^{(k)} \mathsf{A}$, where $\lambda_{[k]}^{(k)} = (\lambda_1^{(k)}, ..., \lambda_{k-1}^{(k)}, 1, \lambda_{k+1}^{(k)}, ..., \lambda_{n+1}^{(k)})$. The (labour) value of labour power is given by λb. [3]

For all vectors $z=(z_1,\ldots,z_p)$ and $y=(y_1,\ldots,y_p)\in \mathbb{R}^p$, let the following notation hold: $z\geq y$ if and only if $z_i\geq y_i$ $(i=1,\ldots,p)$; $z>y$ if and only if $z\geq y$ and $z\neq y$; $z\gg y$ if and only if $z_i>y_i$ $(i=1,\ldots,p)$. Further, for every $n\times 1$ vector z and $1\times n$ vector y, let $z_{-\{m+1,n\}}$ and $y_{-\{m+1,n\}}$ be the $m\times 1$ and the $1\times m$ vectors respectively obtained from z and y by deleting all i-th components, for $i=m+1,\ldots,n$, where $m<n$. If $m=n$, then $z_{-\{m+1,n\}}=z$ and $y_{-\{m+1,n\}}=y$. Moreover, let $z_{\{m+1,n\}}$ and $y_{\{m+1,n\}}$ be the $(n-m)\times 1$ and the $1\times(n-m)$ vectors respectively obtained from z and y by deleting all i-th components, for $i=1,\ldots,m$, where $m<n$.

Let $M\equiv A+bL$. For each commodity $k=1,\ldots,n$, let M_k be the k-th row vector of the matrix M, so that $M_k=(m_{kj})_{j=1,\ldots,n}$, where m_{kj} represents the amount of commodity k that must be invested to produce one unit of commodity j. Let $M^{(m)}$ be the $m\times m$ square matrix, where $m\leq n$, which is obtained from M by deleting all the i-th rows and columns of M, for $i=m+1,\ldots,n$. In general, through appropriate permutations of its rows and columns, any *decomposable* matrix M can be represented as $M=\begin{bmatrix} M_{11} & M_{12} \\ \mathbf{0} & M_{22}\end{bmatrix}$ with $M_{11}=M^{(m)}$ being indecomposable. This follows noting that a decomposable matrix may always be reduced to the canonical form

$$M = \begin{bmatrix} M'_{11} & M'_{12} & \ldots & M'_{1s} \\ \mathbf{0} & M'_{22} & \ldots & M'_{2s} \\ \ldots & \ldots & \ldots & \ldots \\ \mathbf{0} & \mathbf{0} & \ldots & M'_{ss} \end{bmatrix}$$

where $M'_{11}, M'_{22}, \ldots, M'_{ss}$ are indecomposable square matrices not necessarily of the same order (see, e.g., Kurz and Salvadori, 1995, p.516). The desired result then follows by setting $M_{11} = M'_{11}$, $M_{12} = \begin{bmatrix} M'_{12} & M'_{13} & \ldots & M'_{1s} \end{bmatrix}$, and so on. With a slight abuse of notation, we assume that the latter representation holds also if M is *indecomposable*, letting $m = n$ and noting that in this case $M = M_{11}$.

Keeping this representation of M in mind, a weaker and more general version of the GCET can now be proved.

Theorem 1 (*Weak GCET*): *Let the economy* (A, L, b) *be such that* $A > \mathbf{0}$, $L > 0$, $b > 0$, $M = A + bL$ *is represented by* $M = \begin{bmatrix} M_{11} & M_{12} \\ \mathbf{0} & M_{22} \end{bmatrix}$ *with* $M_{11} = M^{(m)}$, *for some* $0 < m \leq n$, *and* M_{11} *is indecomposable with* $M_{k'} \gg 0$ *for some* $k' \in \{1, \ldots, m\}$. *Moreover, let there exist* $k'' \in \{1, \ldots, m\}$ *such that* $b_{k''} L_{-\{m+1,n\}} > 0$. *Then, for any commodity* $k \in \{1, \ldots, m\}$, *the following statements are equivalent*:

(*i*) *there exists no* $p \gg 0$ *s.t.* $p[I - A - bL] \leq 0$;

(*ii*) $\lambda b < 1$ *for some* λ *with* $\lambda_{-\{m+1,n\}} \gg 0$;

(*iii*) $\lambda_k^{(k)} < 1$ *for some* $\lambda^{(k)}$ *with* $\lambda_{-\{m+1,n\}}^{(k)} \gg 0$.

Proof. First of all, note that $M_{12} \neq \mathbf{0}$, since there exists $k' \in \{1,\ldots,m\}$ such that $M_{k'} \gg 0$. Also, the indecomposable square matrix $M^{(m)}$ always contains at least those $k \in \{1,\ldots,n\}$ with $M_k \gg 0$. Second, since $k'' \in \{1,\ldots,m\}$ such that $b_{k''}L_{-\{m+1,n\}} > 0$, $b_{k''} > 0$ and $L_{-\{m+1,n\}} \neq 0$ hold.

Let us take any $k \in \{1,\ldots,m\}$ which constitutes the indecomposable matrix $M^{(m)}$. Then, since $M^{(m)}$ is indecomposable, $M_k^{(m)} \neq 0$. Let us show the statements (i), (ii), and (iii) are equivalent for this k.

1. $(i) \Rightarrow (ii)$. If Theorem 1-(i) holds, then by [Nikaido (1970; Corollary 30.2)], there exists $x > 0$ such that $[I - A - bL]x > 0$, which implies that M is semiproductive. Moreover, since $M^{(m)}$ is indecomposable, it follows from [Nikaido (1970; Theorem 20.2)] that $M^{(m)}$ is productive. Since $M_{21} = \mathbf{0}$, $\lambda_{-\{m+1,n\}}$ can be determined solely based on $M^{(m)}$, independently of M_{12} and M_{22}. Thus, $\lambda_{-\{m+1,n\}} = \lambda_{-\{m+1,n\}}M^{(m)} + (1-\lambda b)L_{-\{m+1,n\}}$. Since $M^{(m)}$ is productive and indecomposable, it follows from [Nikaido (1970; Theorem 20.2)] that $\left[I - M^{(m)}\right]^{-1} \gg \mathbf{0}$ exists. Thus, $\lambda_{-\{m+1,n\}} = (1-\lambda b)L_{-\{m+1,n\}}\left[I - M^{(m)}\right]^{-1}$ holds, and if $1-\lambda b \leq 0$, then it follows that $\lambda_{-\{m+1,n\}} \leq 0$. However, $1-\lambda b \leq 0$ cannot hold. To see this, note that $b_{\{m+1,n\}} = 0$ follows from $M_{21} = \mathbf{0}$ and $L_{-\{m+1,n\}} > 0$, because $M_{21} \geq b_{\{m+1,n\}}L_{-\{m+1,n\}}$. However, $b_{\{m+1,n\}} = 0$ implies that $1-\lambda b = 1-\lambda_{-\{m+1,n\}}b_{-\{m+1,n\}}$, and therefore, $1-\lambda b > 0$ and $\lambda_{-\{m+1,n\}} \gg 0$ solely hold, since $L_{-\{m+1,n\}} > 0$.

2. $(i) \Rightarrow (iii)$. Suppose Theorem 1-(*i*) holds. Since $M_{21} = \mathbf{0}$, $\lambda_{-\{m+1,n\}}^{(k)}$ can be determined solely based on $M^{(m)}$, independently of M_{12} and M_{22}. Thus, $\lambda_{-\{m+1,n\}}^{(k)} = \lambda_{-\{m+1,n\}}^{(k)} M^{(m)} + \left(1 - \lambda_k^{(k)}\right) M_k^{(m)}$. Note that, since $M^{(m)}$ is productive and indecomposable, it follows from [Nikaido (1970; Theorem 20.2)] that $\left[I - M^{(m)}\right]^{-1} \gg \mathbf{0}$ exists. Hence, given $M_k^{(m)} \neq 0$, if $1 - \lambda_k^{(k)} \leq 0$, then $\lambda_{-\{m+1,n\}}^{(k)} = \left(1 - \lambda_k^{(k)}\right) M_k^{(m)} \left[I - M^{(m)}\right]^{-1} \leq 0$, but then $1 - \lambda_k^{(k)} > 0$, which is a contradiction. Thus, $1 - \lambda_k^{(k)} > 0$ and $\lambda_{-\{m+1,n\}}^{(k)} \gg 0$ hold.

3. $(iii) \Rightarrow (i)$. If Theorem 1-(*iii*) holds, then taking $M_{21} = \mathbf{0}$, there exists $\lambda_{-\{m+1,n\}}^{(k)} \gg 0$ such that $\lambda_{-\{m+1,n\}}^{(k)} = \lambda_{-\{m+1,n\}}^{(k)} M^{(m)} + \left(1 - \lambda_k^{(k)}\right) M_k^{(m)}$, so that by $M^{(m)}$ being indecomposable and [Nikaido (1970; Theorem 20.2)], $M^{(m)}$ is productive. Then, there exists $x > 0$ with $x_{-\{m+1,n\}} > 0$ and $x_{\{m+1,n\}} = 0$ such that $[I - A - bL]x > 0$ with $\left([I - A - bL]x\right)_{-\{m+1,n\}} \gg 0$. Thus, by [Nikaido (1970; Corollary 30.2)], Theorem 1-(*i*) holds.

4. $(ii) \Rightarrow (i)$. If Theorem 1-(*ii*) holds, then, taking $M_{21} = \mathbf{0}$, there exists $\lambda_{-\{m+1,n\}} \gg 0$ such that $\lambda_{-\{m+1,n\}} = \lambda_{-\{m+1,n\}} M^{(m)} + (1 - \lambda b) L_{-\{m+1,n\}}$. Again, by $M^{(m)}$ being indecomposable and [Nikaido (1970; Theorem 20.2)], $M^{(m)}$ is productive, thus there exists $x > 0$ with $x_{-\{m+1,n\}} > 0$ and $x_{\{m+1,n\}} = 0$ such that $[I - A - bL]x > 0$ with $\left([I - A - bL]x\right)_{-\{m+1,n\}} \gg 0$. Thus, Theorem 1-(i) holds. ■

Theorem 1 provides a novel characterisation of the relation between exploitation and the productiveness of an economy that is at the heart of the GCET. As a preliminary step, let us say that commodity i is *directly or indirectly invested for producing* commodity j if there is a sequence $\{k_0, k_1, \ldots, k_r\}$ such that $k_0 = i$, $k_r = j$, and for each $k_l \in \{k_0, k_1, \ldots, k_{r-1}\}$, $m_{k_l k_{l+1}} > 0$. Note that if $r = 1$, then $m_{ij} > 0$, which implies that commodity i is *directly invested for producing* commodity j. Then, let commodity j be called a *commodity of a basic sector* if it is directly or indirectly invested for producing every commodity. In Theorem 1, each commodity in the set $\{1, \ldots, m\}$, which constitutes the indecomposable matrix $M^{(m)}$ with $M_{k'} \gneq 0$ for some $k' \in \{1, \ldots, m\}$, is a commodity of a basic sector.[4] In contrast, each commodity in the set $\{m+1, \ldots, n\}$ is a *commodity of a non-basic sector*, since each j in $\{m+1, \ldots, n\}$ is directly or indirectly invested for producing at most commodites in $\{m+1, \ldots, n\}$. Then, for any commodity k of a basic sector, Theorem 1 proves that there exists no strictly positive price vector such that profits are (weakly) negative if and only if both labour and commodity k are exploited. Theorem 1 is weaker and more general than the standard characterisations of the GCET (e.g., Bowles and Gintis, 1981; Roemer, 1982) because part (*i*) is required to hold only for *strictly positive* (and not just weakly positive) price vectors, which implies that the matrix M is only required to be *semiproductive*.[5] Consequently, the nonnegativity restrictions on the value vectors in parts (*ii*) and (*iii*) hold only for the first m entries, and not necessarily for all goods. Furthermore, although this

Weak GCET holds for any M, it is immediate to show that if M is indecomposable, Theorem 1 reduces to the standard GCET. More precisely, when M is indecomposable, the nonnegativity restrictions in Theorem 1-(*ii*)-(*iii*) apply to all n goods and Theorem 1-(*i*) is equivalent to the standard condition requiring the existence of a weakly positive price vector such that profits are strictly positive (i.e., $\exists p > 0$ s.t. $p[I - A - bL] \gg 0$).[6]

There are a number of points that should be stressed about Theorem 1. First, it is worth noting that the Weak GCET does not necessarily hold for a commodity of a non-basic sector. To identify the commodities in $\{1, \ldots, m\}$ as ones of *basic* sectors, the existence of $k' \in \{1, \ldots, m\}$ with $M_{k'} \gg 0$ is indispensable. In fact, the following example shows a case that $M^{(m)}$ is indecomposable, where $\{1, \ldots, m\} = \{1\}$, but $M_{k'} \gg 0$ does not hold for $k' = 1$, and so this commodity 1 is *not* of a basic sector. Then, the example also shows that the Weak GCET does not hold for such a commodity.

Example 1: Let $n = 2$, $0 < a_{11} < 1$, $a_{12} = a_{21} = a_{22} = 0$, $b_1 = 0$, $b_2 L_1 > 0$, and $b_2 L_2 > 1$. Thus, $M_1 > 0$. By applying the standard argument for the vector of commodity k values to the case $k = 1$, we obtain:

$$\frac{\lambda^{(1)}}{1 - \lambda_1^{(1)}} = (a_{11}, 0)[I - M]^{-1} = \left(\frac{a_{11}}{1 - a_{11}}, 0 \right),$$

$$\text{where } [I-M]^{-1} = \begin{bmatrix} \frac{1}{1-a_{11}} & 0 \\ \frac{b_2 L_1}{(1-a_{11})(1-b_2 L_2)} & \frac{1}{1-b_2 L_2} \end{bmatrix}.$$

Since $\lambda^{(1)} = (a_{11}, 0)$ and $1-\lambda_1^{(1)} = 1-a_{11} > 0$, Theorem 1-(*iii*) holds for $k=1$. However, in this economy, Theorem 1-(*i*) is violated. This is because $b_2 L_2 > 1$, so that there is a positive price $p = (p_1, p_2) \gg 0$, where p_1 is small enough relative to p_2, such that $p < pM$. Thus, the equivalence between Theorem 1-(*i*) and Theorem 1-(*iii*) does not hold for $k=1$. This implies that the Weak GCET does not hold for $k=1$ in this economy. Note that $k=1$ is a commodity of a non-basic sector, since it is neither directly nor indirectly invested for producing commodity 2. ■

Second, the nonnegativity restrictions in parts (*ii*) and (*iii*) of Theorem 1 are important, even though they are often overlooked: the possibility of nonnegative profits (part (*i*)) is equivalent to the exploitation of labour *for a strictly positive vector* of labour values of the m commodities of basic sectors in the economy. Similarly, part (*iii*) states that commodity k is 'exploited' for a given vector of commodity k values whose 'core' m components are *positive*. We shall go back to these nonnegativity restrictions on value vectors below, when we discuss Matsuo's example.

Third, in order to discuss Matsuo's argument, it is important to note that the GCET has a counterpart focusing on the production side of the economy. Part (*i*) is equivalent to the existence of an activity vector $x > 0$ with $x_{-\{m+1,n\}} > 0$ and

$x_{\{m+1,n\}} = 0$ such that $[I - A - bL]x > 0$ with $([I - A - bL]x)_{-\{m+1,n\}} \gg 0$. If M is indecomposable, or if part (*i*) is strengthened to require that there exists $p \gg 0$ such that $p[I - A - bL] \gg 0$, then Theorem 1-(*i*) is equivalent to the Hawkins-Simon condition for $I - M$, and therefore to the existence of an activity vector $x \gg 0$ such that $[I - A - bL]x \gg 0$. In the latter case, it can be shown that $\lambda_{-\{m+1,n\}} \gg 0$ of condition (*ii*) is strengthened to $\lambda \gg 0$ if and only if $|I - A| > 0$, and $\lambda^{(k)}_{-\{m+1,n\}} \gg 0$ of condition (*iii*) is strengthened to $\lambda^{(k)} \gg 0$ if and only if $|I - \mathsf{A}_{(k)}| > 0$.[7]

The latter two conditions provide the foundations for Matsuo's critique of the GCET. In fact, he argues that the condition $|I - A| > 0$ on productiveness is a purely technical and weak assumption, whereas the condition $|I - \mathsf{A}_{(k)}| > 0$ embodies a social relation, because the vector b enters $\mathsf{A}_{(k)}$, for $k \neq n+1$, and b does not contain purely technical data. The essential logic of Matsuo's argument can be illustrated as follows: focusing on an indecomposable matrix M, it aims to find an example with $|I - \mathsf{A}_{(k)}| < 0$ for some k, by suitably choosing (L, b), such that $0 < \lambda^{(k)}_k < 1$, so that commodity k is exploited, but $\lambda b > 1$ with $\lambda \gg 0$, so that labour is not exploited, and negative profits can occur, that is $p[I - A - bL] \leq 0$ for some vector of prices $p > 0$. Matsuo (2009, Section 4) obtains the desired result for $k = 1$ by choosing the following specification:

$$\mathsf{A} = \begin{bmatrix} 0.5 & 0.2 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}.$$

Theorem 1 forcefully exposes the rather peculiar and strong assumptions underlying the latter example, which make its interpretation and its theoretical implications for the debate on labour exploitation and profits rather unclear. First of all, both the generality and theoretical relevance of the example are objectionable, because the economy cannot be in a reproducible solution, or in a sustainable path, or in equilibrium, *whatever* the notion of equilibrium adopted. On the one hand, it is worth noting in passing that, given the values of the parameters chosen by Matsuo, in the given example it is not just the case that condition (*i*) in Theorem 1 is violated for *some* vector of strictly positive prices $p \gg 0$: there exists no (semipositive) price of the consumption good such that the two sectors are (weakly) profitable. On the other hand, and perhaps more importantly, using a result proved by Gale (1960, Theorem 2.10), it is not difficult to prove that $p[I-A-bL] \ll 0$ holds for some $p \geq 0$ if and only if there is no $x > 0$ such that $[I-A-bL]x \geq 0$. Thus, in Matsuo's example, the economy simply cannot reproduce itself.

Another way of looking at this problem is to focus on wages. The economy cannot be reproducible, given that workers' aggregate consumption exceeds net output: the maximal net output of the consumption good per unit of labour expended is $\max_{Lx=1, x>0}([I-A]x)_2 = 0.5$, but the aggregate consumption of the workers per unit of labour is $b_2 Lx = 1$ under $Lx = 1$. From a theoretical viewpoint, the problem seems to be the definition of net output which, according to Matsuo, only involves the replacement of the physical inputs of production and not workers' consumption. Theoretically, it is true, as Matsuo suggests, that the bundle b is not a purely technological variable. It is also true, however, that there are physical (and not just social) constraints on b, given by total net output, that must be satisfied for the economy to be reproducible (if not viable), and these constraints are violated in his example. If one requires that the system be able to reproduce itself in the sense that $[I - A - bL]x > 0$ holds for some $x > 0$, then the GCET continues to hold.

Secondly, there is a fundamental conceptual issue that Matsuo does not address adequately, relegating it to a footnote (Matsuo, 2009, fn.7). In order to have negative profits, non-exploitation of labour, and the 'exploitation' of some commodity k, the augmented matrix M cannot be productive if it is indecomposable, and thus the vector of embodied values in the commodity k numeraire cannot be nonnegative. But then, this raises the issue of the significance of such vector of values, since the latter are meant to represent amounts of commodity k directly and indirectly inputted in the production process of each commodity, which are naturally supposed to be nonnegative. In Matsuo's (2009) example - in which M is indecomposable, - the commodity 1 value of commodity 1, $\lambda_1^{(1)}$, is positive and smaller than one, but both $\lambda_2^{(1)}$ and $\lambda_3^{(1)}$ are actually negative. Focusing only on $\lambda_1^{(1)}$ is arguably misleading because $\left(\lambda_1^{(1)}, \lambda_2^{(1)}, \lambda_3^{(1)}\right)$ represents the solution vector of a system of equations. Therefore if solution vectors containing negative values are theoretically doubtful, then the interpretation of the inequality $\lambda_1^{(1)} < 1$ - and in general of Matsuo's example - is unclear.

3 The revised GCET

The arguments developed in the previous section raise serious doubts about the significance of Matsuo's example, and on the theoretical conclusions derived from it. It may even be argued that an economic system characterised by negative values and a non-viable input-output matrix (augmented for workers consumption) would simply collapse and exploitation itself would be impossible.[8] From this perspective, non-negativity constraints are just part of the definition of logically possible solutions. Some authors may object, though, that this type of examples do depict logically possible (albeit temporary or disequilibrium) scenarios, and in general it is legitimate to analyse economies in which reproducibility and nonnegativity restrictions on value vectors are violated. Although this contention seems rather uncompelling on methodological grounds, in this section it is argued that *even if* the latter objections are set aside, for the sake of argument, and examples can be constructed without taking into account equilibrium, or reproducibility, and negative values are considered to be meaningful magnitudes, it is still unclear that Matsuo's argument convincingly establishes the desired claim. For a similar logic can be applied to reinstate the symmetry between labour and other goods, albeit in a less rigorous way.

Suppose, for the sake of argument, that Matsuo's example describes an 'exceptional, but logically possible scenario,' and that the analysis of such 'exceptional, but logically possible scenarios' is considered methodologically appropriate. Then, one can also construct legitimate counterexamples of economies in which the condition $|I-A|>0$ does not hold - for example, owing to a random, transitory negative shock on productiveness - such that $\lambda b<1$, $p\left[I-A-bL\right]\square\ 0$ for some $p\square\ 0$, and $\lambda_k^{(k)}\geq 1$ with $\lambda^{(k)}\square\ 0$ for some k.

Example 2: Consider the following economy:

$$\mathsf{A}=\begin{bmatrix} a_{11} & a_{12} & b_1 \\ 0 & a_{22} & b_2 \\ L_1 & L_2 & 0 \end{bmatrix}.$$

Then, the vector of commodity 1 values, $\lambda^{(1)}$, is given by:

$$\left(\lambda_1^{(1)},\lambda_2^{(1)},\lambda_3^{(1)}\right)=\left(1,\lambda_2^{(1)},\lambda_3^{(1)}\right)\begin{bmatrix} a_{11} & a_{12} & b_1 \\ 0 & a_{22} & b_2 \\ L_1 & L_2 & 0 \end{bmatrix},$$

so that

$$\lambda_1^{(1)}=\frac{\left(a_{12}+b_1L_2\right)b_2L_1}{1-a_{22}-b_2L_2}+b_1L_1+a_{11};\ \lambda_2^{(1)}=\frac{\left(a_{12}+b_1L_2\right)}{1-a_{22}-b_2L_2};\ \lambda_3^{(1)}=\frac{\left(a_{12}+b_1L_2\right)b_2}{1-a_{22}-b_2L_2}+b_1.$$

In contrast, the vector of labour values λ is given by:

$$\left(\lambda_1,\lambda_2,\lambda_3\right)=\left(\lambda_1,\lambda_2,1\right)\begin{bmatrix} a_{11} & a_{12} & b_1 \\ 0 & a_{22} & b_2 \\ L_1 & L_2 & 0 \end{bmatrix},$$

so that

$$\lambda_1=\frac{L_1}{1-a_{11}};\ \lambda_2=\frac{L_2}{1-a_{22}}+\frac{L_1a_{12}}{\left(1-a_{11}\right)\left(1-a_{22}\right)};\ \lambda_3=\lambda_1b_1+\lambda_2b_2.$$

Suppose that, due to a temporary negative technological shock, $a_{11}>1$, but a_{12} and a_{22} are zero, and $0<b_1L_1<b_2L_2<b_1L_2<1$. Then, there is a sufficiently small L_1

such that $0 < \lambda b < 1$, but $p[I - A - bL] \square 0$ for some $p \square 0$, and $\lambda_1^{(1)} > 1$ with $\lambda^{(1)} \square 0$. ■

The argument in Example 2 can be easily extended to other goods and, together with Matsuo's example, it proves the following version of the GCET.

Theorem 2 (Revised GCET): *Consider the set of economies $\mathcal{E} = < A, L, b >$ in which n commodities are produced according to the technology (A, L) and in which the wage bundle is b.*

1. *Let the economy $(A, L, b) \in \mathcal{E}$ be such that $A > \mathbf{0}$, $L > 0$, $b > 0$, $M = A + bL$ is represented by $M = \begin{bmatrix} M_{11} & M_{12} \\ \mathbf{0} & M_{22} \end{bmatrix}$ with $M_{11} = M^{(m)}$, for some $0 < m \leq n$, and M_{11} is indecomposable with $M_{k'} \square 0$ for some $k' \in \{1, \ldots, m\}$. Moreover, let there exist $k'' \in \{1, \ldots, m\}$ such that $b_{k''} L_{-\{m+1,n\}} > 0$. Then, for any commodity $k \in \{1, \ldots, m\}$, the following statements are equivalent*:

   (*i*) *there exists no* $p \square 0$ *s.t.* $p[I - A - bL] \leq 0$;

   (*ii*) $\lambda b < 1$ *for some* λ *with* $\lambda_{-\{m+1,n\}} \square 0$;

   (*iii*) $\lambda_k^{(k)} < 1$ *for some* $\lambda^{(k)}$ *with* $\lambda_{-\{m+1,n\}}^{(k)} \square 0$.

2. *For each good $k = 1, ..., n$, there exists an economy $(A, L, b) \in \mathcal{E}$ such that $\lambda b < 1$, $p[I - A - bL] \leq 0$ for some $p \square 0$, and $\lambda_k^{(k)} \geq 1$ with $\lambda^{(k)} \square 0$. Furthermore, there exists another economy $(A', L', b') \in \mathcal{E}$ such that $\lambda_k'^{(k)} < 1$,*

$\lambda' b' > 1$, *and* $p'[I - A' - b'L'] \leq 0$ *for some* $p' \gg 0$.

To be sure, it is well-known that, when $\mathcal{A}$ (or M) is indecomposable, the productiveness of A is indispensable for the GCET (and in general for the relation between labour exploitation and profits) to hold, and in this respect Theorem 2-(2) (and Example 2) is not a completely surprising result. Further, it might be objected that the whole point of Matsuo's paper is to emphasise that the violation of the condition on the productiveness of A is essentially different from the violations of the conditions on the productiveness of the matrices $\mathcal{A}_{(k)}$. Theorem 2 and the previous arguments, however, do suggest that such essential asymmetry is not obvious. The symmetry remains true in the sense that, as argued above, both types of violations are equally logically possible, and they can both only be transitory. Further, from a theoretical viewpoint, it is arguably unsatisfactory to treat A as a purely technological object. For it represents the outcome of past and present decisions of capitalists concerning choice of techniques and technical progress, and these decisions clearly reflect social relations. To treat A as something fundamentally different from $\mathcal{A}_{(k)}$, for $k \neq n+1$, and completely abstracted from social relations is to fetishise commodities and technical data.

4 Concluding remarks

The Generalised Commodity Exploitation Theorem states that labour is not the only source of profits in standard linear economies: the productiveness of the economy, and ultimately its profitability, is equivalent to the exploitation of *any* commodity. According to Matsuo (2009), the latter result hides a deep asymmetry between labour and other commodities: the equivalence between the exploitation of labour and positive profits derives from a purely technical condition concerning productiveness, whereas the equivalence between the exploitation of commodity k and positive profits requires an assumption on social relations. This paper suggests that the latter argument is not entirely compelling from a theoretical viewpoint. For the violation of the conditions of the GCET imply that the economy is not reproducible and the value vectors are not nonnegative. Both characteristics seems theoretically doubtful. But even if Matsuo's argument is considered compelling, in any case the differences between labour and other commodities in the generation of exploitation and profits are not obvious, and a generalised version of the GCET can be proved which reinstates the symmetry between labour and other commodities, along the lines of Matsuo's own argument.

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[1] See, for example, Fujimoto and Fujita (2008) and Fujimoto and Opocher (2009). As shown in Yoshihara and Veneziani (2009), the latter contributions do not provide an explicit, rigorous definition of exploitation. Moreover, a generalised version of the GCET immediately follows in the analytical framework adopted in these papers, whose main feature is *precisely* the symmetrical treatment of all goods and types of labour.

[2] In the standard notation: $M = A + bL.$ See section 2 below for a detailed explanation.

[3] Alternatively, one could define $\lambda^{(k)} = \lambda^{(k)}_{[k]}\mathsf{A}$, where $\lambda^{(k)}_{[k]} = (\lambda^{(k)}_1, ..., \lambda^{(k)}_{k-1}, 1, \lambda^{(k)}_{k+1}, ..., \lambda^{(k)}_{n+1})$, for all k, $1 \leq k \leq n+1$. In this case, $\lambda^{(n+1)} = \left(\lambda, \lambda^{(n+1)}_{n+1}\right)$, where $\lambda^{(n+1)}_{n+1} = \lambda b$. This notation is adopted, e.g., in Fujimoto and Fujita (2008) and Fujimoto and Opocher (2009).

[4] This is because, firstly, every $j \in \{1, \ldots, m\}$ is directly or indirectly invested for producing any commodity in $\{1, \ldots, m\}$, which follows from $M^{(m)}$ being indecomposable. Secondly, since $M_{k'} \gg 0$, $k' \in \{1, \ldots, m\}$ is directly invested for producing every commodity. Thus, it follows that every $j \in \{1, \ldots, m\}$ is directly or indirectly invested for producing every commodity, since j is directly or indirectly invested for producing k'.

[5] Matrix M is semiproductive if there exists a vector $x \geq 0$ such that $x > Mx$.

[6] It follows from [Nikaido (1970; Theorem 20.2)] that if M is indecomposable, it can be proved that M is productive if and only if it is semiproductive. From the last equivalence relation, it also follows that, if M is indecomposable, the condition that k is of a basic sector is no longer indispensable in showing the standard GCET.

[7] Thus, though Matsuo (2009) presents the standard GCET by adopting the conditions of $|I - A| > 0$ and $\left|I - \mathsf{A}_{(k)}\right| > 0$ instead of the non-negativity of k-commodity value vector and labour value vector, these Hawkins-Simon type conditions are unnecessary for the GCET in general: they are necessary only in the case of indecomposable M or A.

[8] This argument has been suggested to us by an anonymous referee.