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Abstract

This paper focuses on the evolution of child labour, fertility and human capital in an economy with two production sectors and two types of workers endowed with two different levels of human capital. Adults allocate their time endowment between work and child rearing and choose the time allocation of children between schooling and work. The heterogeneity between low and high skilled workers allows for an endogenous analysis of inequality generated by child labour. We show that the persistence of child labour can be explained through the competition between children and low-skilled workers. This persistence, in turn, can easily induce an increase in the inequality and an average impoverishment within the country.

JEL classification: J13; J24; J82; K31.

Keywords: Child Labor, Fertility, Human capital, Inequality.

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1 Introduction

According to the ILO (International Labour Organization) more than 200 million children in the world today are involved in child labour. Child labour persists even if it has been declared illegal at both the national and international levels. As is apparent from Figure 1 the number of children who aren't enrolled in primary or secondary school decreases with per capita income. We use the data on children not attending school as a proxy of child labour since the shortage of data on child labour.

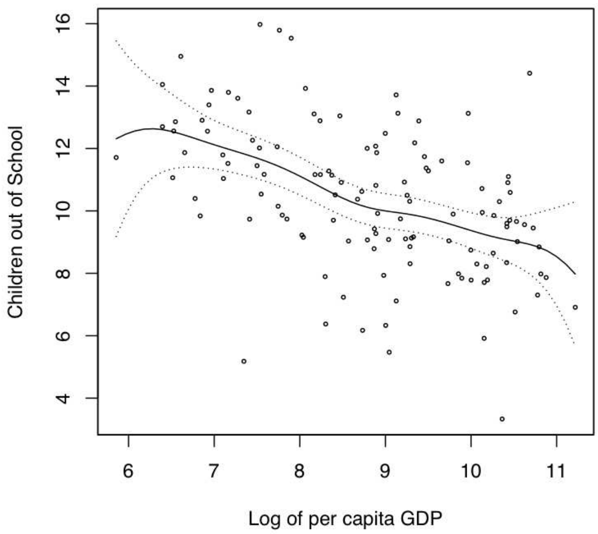


Figure 1: Children out of school, primary. Nonparametric kernel smoother, year 2007. Data are from World Development Indicators (2010).

A large body of the literature has developed theoretical and empirical models to study the causes of child labour persistence. The benchmark framework is based on two main axioms: the luxury axiom and the substitution axiom (Basu and Van, 1998). The luxury axiom implies that parents send children to work if their income is below a certain threshold. The substitution axiom implies that adult labour and child labour are substitutes. These axioms lead to multiple equilibria in the labour market, with one equilibrium where adult wage is low and children work and another where adult wage is high and children do not work.

This framework has been recently extended by Hazan and Berdugo (2002) and Doepke and Zilibotti (2005) which introduce endogenous fertility choices. They analyze the relationship between child labour, fertility and human capital showing the existence of multiple development path. In early stages of development, the economy is in a development trap where child labour is abundant, fertility is high and output per capita is low. Technological progress, allows a take-off from the underdevelopment trap because it gradually increases the wage differential between parental and child labour and hence the return of investment in education.

However these contributions do not consider the presence of inequality, i.e. the economy can follow different paths of growth that are characterized – in equilibrium – by a unique level human capital. We extend this framework taking into account two groups of individuals with two different levels of human capital. In this respect our work is related to the literature on inequality, differential fertility, and economic growth. In particular, Moav (2005) develops a theory of fertility that offers an explanation for the persistence of poverty within and across countries. The basic idea is that the cost of child quantity increases with parent's human capital since the opportunity cost of time is high. High-income families choose low fertility rates and high investment in education. This implies that high income persist in the dynasty. On the other hand, poor households choose relatively high fertility rates with relatively low investment in their offspring's education. Therefore, their offspring are poor as well. De la Croix and Doepke (2003) argue that a higher inequality, by increasing fertility differential between rich and poor families, lowers average education and, therefore, growth. The motivation for this result is that a large fertility differential implies more weight on children with little education.

Our paper focus on the evolution of child labour, fertility and human capital in an economy with two production sectors - traditional and modern - and two types of workers - low and high skilled. In particular, traditional sector employs unskilled labour while the modern sector uses unskilled and skilled workers. According to the existent literature, we assume that child labour is a perfect substitute for unskilled adult but children are relatively less productive.

Adults allocate their time endowment between work and child raring. They choose the time allocation of children between schooling and work. Hence, households can have two, possible, sources of income: income by parents and child income. Parents preferences are defined over the consumption of the two goods, the number of children, and the children's future level of human capital. Human capital of children is an increasing, strictly concave function of the time devoted to school. If child's time is devoted entirely to work they enter the adult age with a minimum level of human capital (see Galor and Weil, 2000; De la Croix and Doepke, 2004; Hazan and Berdugo, 2002).

We show that when the relative wage between adult labour and child labour is below a certain threshold parents have a high number of children and send them to work, while when income is above this threshold fertility decreases and parents find optimal educate their children. This framework allows for an endogenous analysis of inequality generated by child labour. The model shows a strong persistence of child labour which is mainly due to the fertility choices of the two groups. In this respect we find a result similar to the De la Croix and Doepke (2003), but differently from them we show that the presence of the child labour increases the persistence of inequality. Indeed, low-skilled workers do not find convenient to change the children time allocation between schooling and work if unskilled wage increases. We show that inequality persists and increases in the long run.

Section 2 introduces the basic structure of the model. Section 3 describes the optimal individual choices. In Section 4, we investigate the general equilibrium configurations of the model. In Section 5, analysis the long run dynamics of the model. Section 6 concludes.

2 The Model

We analyze an overlapping-generation economy that consists of two sectors: an agricultural and a modern sector. In every period t, the economy is populated by N_t individuals. Each of them is endowed with a level of human capital, h_t^i. This level is endogenously determined by parent's choice on the children's time allocation between labour and schooling. Adults can supply skilled or unskilled labour, while children can only supply unskilled labour.

2.1 Production

Production in the agricultural sector occurs within a period according to a linear production technology using unskilled labour as inputs. The output produced at time t, $Y_{A,t}$, is:

$$Y_{A,t} = \eta L_{A,t}, \tag{1}$$

where $L_{A,t}$ is the amount of unskilled labour employed in the production of the agricultural good in period t, and $\eta > 0$ is a technological parameter representing the average and marginal productivity of labour.

Production in the modern sector occurs within a period according to a neoclassical, constant return to scale, Cobb–Douglas production technology using unskilled and skilled labour as inputs. The output produced at time t, $Y_{M,t}$, is:

$$Y_{M,t} = \psi(H_t)^{\mu}(L_{M,t})^{1-\mu} = \psi(h_{M,t})^{\mu}L_{M,t}; \qquad 0 < \mu < 1, \tag{2}$$

where $h_{M,t} \equiv H_t/L_{M,t}$ is the ratio of skilled H_t to unskilled labor $L_{M,t}$ employed in the production of the modern good in period t, $\psi > 0$ is the technological level in the modern sector.

Assuming the modern good as numeraire, i.e. $p_{M,t} = 1$, the wage of unskilled workers in the agricultural sectors is:

$$w_t^u = p_{A,t}\eta, \tag{3}$$

where $p_{A,t}$ is the price of agricultural good.

In the modern sector, the wage of unskilled workers, i.e. w_t^u, and the wage rate per efficiency unit w_t^s are:

$$w_t^u = \psi(1-\mu)\left(h_{M,t}\right)^{\mu}, \tag{4}$$

and

$$w_t^s = \psi\mu\left(h_{M,t}\right)^{\mu-1}. \tag{5}$$

Free mobility of unskilled workers between the modern and agricultural sectors leads to the equalization of wages of unskilled labour in both sectors. Hence, from equations (3) and (4) we get:

$$p_{A,t} = \frac{\psi(1-\mu)\left(h_{M,t}\right)^{\mu}}{\eta}.$$

2.2 Preferences

Members of generation t live for two periods: childhood and adulthood. In the first period of their lives, individuals may either work, go to school or both. In the second period of their lives, agents supply unskilled or skilled labour. Individual's preferences are defined over the consumption of agricultural good, i.e. $c^i_{A,t}$, the consumption of modern good, i.e. $c^i_{M,t}$, the number of children n^i_t, and the human capital of children h^i_{t+1}.[1] The utility function of an agent i of generation t is given by:

[1] As it is clear from equation (6), we assume that parents are aware of the human capital of their children rather than their income. Although the results of the model are not crucially affected by this choice, we believe that this is a more realistic assumption, see for instance De la Croix and Doepke (2003) for discussion on this point.

$$U_t^i = \alpha \ln[\big(c_{A,t}^i)^\gamma (c_{M,t}^i\big)^{1-\gamma}] + (1-\alpha)\ln(n_t^i h_{t+1}^i), \tag{6}$$

where $\alpha \in (0,1)$ is the altruism factor and $\gamma \in (0,1)$ is a parameter of preference between consumption goods.

We suppose that children born with some basic human capital a which can be increased by attending school. In particular, human capital of children in period $t+1$ is an increasing, strictly concave function of the time devoted to school (see Galor and Weil, 2000; De la Croix and Doepke, 2004; Hazan and Berdugo, 2002):

$$h_{t+1}^i = a(b + e_t^i)^\beta, \tag{7}$$

where $a, b > 0$ and $\beta \in (0,1)$.

Parents allocate their income between the consumption of the two goods, i.e $c_{A,t}^i$ and $c_{M,t}^i$ and child rearing. In particular, raising each born child takes a fraction $z \in (0,1)$ of an adult's income. In addition, parents allocates the time endowment of children between schooling, $e_t^i \in [0,1]$, and labour force participation $(1-e_t^i) \in [0,1]$. We assume that, each child can offer only $\theta \in [0,1]$ units of unskilled labour, that is children are substitutes for unskilled adult workers but relatively less productive. Therefore, each household can have two, possible, sources of income: i) income by parents $I_t^i = \max w_t^s h_t^i, w_t^u$ and, ii) child income $(1-e_t^i)\theta w_t^u$. While children can work only as unskilled workers, and their income would be θw_t^u, parents will choose to work in the sector that guarantees them a higher income. Hence, the budget constraint is

$$p_{A,t} c_{A,t}^i + c_{M,t}^i \leq (1 - z n_t^i) I_t^i + (1 - e_t^i)\theta w_t^u n_t^i. \tag{8}$$

Assumption 2.1. *We assume that the opportunity cost of raising a child must be higher than the wage she/he gets in the labour market:*

$$z > \theta \tag{9}$$

Assumption (2.1) implies that the relative return of child labour when each child just works is $\theta/z < 1$.

As investigation strategy, we first analyze the optimal individual's choices when the wage ratio, w_t^s/w_t^u, is fixed. This analysis allows us to show the presence of multiple equilibria, coherently with the literature on endogenous fertility and accumulation of human capital. The second step is to consider two types of individuals characterized by two different level of human capital, high and low. Under this assumption we determine the wage ratio which guarantees, in each period, the market clearing. Finally we investigate the long run dynamics of this equilibria taking together the individual accumulation of human capital and the equilibrium conditions.

3 Individual choices

Each household has to choose $c^i_{A,t}$, $c^i_{M,t}$, n^i_t and e^i_t so as to maximize the utility function (6) subject to the budget constraint (8). Given the relative wage, under assumption 2.1, the optimal consumption of the two goods, the optimal schooling and the optimal number of children chosen by member i of generation t are given by:

$$c^i_{A,t} = \frac{\gamma\alpha I^i_t}{p_{A,t}}, \tag{10}$$

$$c^i_{M,t} = (1-\gamma)\alpha I^i_t, \tag{11}$$

$$e^i_t = \begin{cases} 0 & \text{if } r^i_t \le \frac{\theta(\beta+b)}{\beta z}, \\ \frac{r^i_t\beta z - \theta(\beta+b)}{\theta(1-\beta)} & \text{if } \frac{\theta(\beta+b)}{\beta z} \le r^i_t \le \frac{\theta(1+b)}{\beta z}, \\ 1 & \text{if } r^i_t \ge \frac{\theta(1+b)}{\beta z}. \end{cases} \tag{12}$$

and:

$$n^i_t = \begin{cases} \frac{(1-\alpha)r^i_t}{zr^i_t-\theta} & \text{if } r^i_t \le \frac{\theta(\beta+b)}{\beta z}, \\ \frac{(1-\alpha)(1-\beta)r^i_t}{zr^i_t-\theta(1+b)} & \text{if } \frac{\theta(\beta+b)}{\beta z} \le r^i_t \le \frac{\theta(1+b)}{\beta z}, \\ \frac{1-\alpha}{z} & \text{if } r^i_t \ge \frac{\theta(1+b)}{\beta z}. \end{cases} \tag{13}$$

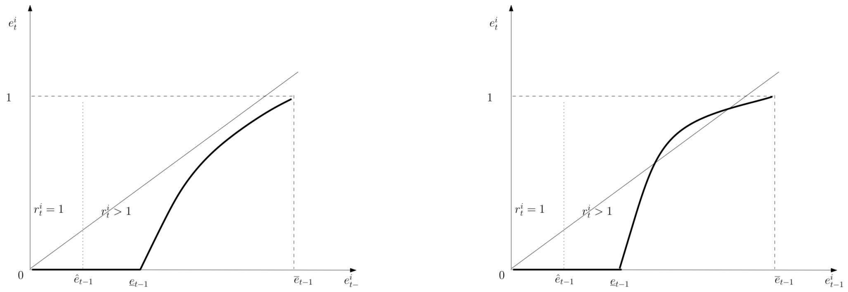
where $r^i_t \equiv I^i_t/w^u_t$.

Given w^s_t/w^u_t, agents, according to their level of human capital h^i_t, will choose to work as unskilled if, and only if, $w^s_t h^i_t < w^u_t$, otherwise they work as skilled. Since $h^i_t = a(b + e^i_{t-1})^\beta$, the ratio r^i_t is a function of the level of education at the period $t-1$, that is:

$$r^i_t = \begin{cases} 1 & \text{if } e^i_{t-1} \le \hat{e}_{t-1}, \\ \frac{w^s_t h^i_t}{w^u_t} & \text{if } e^i_{t-1} \ge \hat{e}_{t-1}, \end{cases}$$

where $\hat{e}_{t-1} = (w^u_t/w^s_t)^{1/\beta} - b$.

In other words, if parents find convenient to work as unskilled, their choices on fertility and education do not depend on income, since $r^i_t = 1$ – see equations (12) and (13). This result is a consequence of the perfect substitutability



(a) (b)

Figure 2: The dynamics of education when $\frac{\theta}{z} \geq \frac{\beta}{\beta+b}$.

between child and unskilled adult labour. Hence, if parent's education is below the threshold level $\hat{e}_{t-1}$, their choices are given by the relation between some parameters values of the model. In particular, we have that all the adults working as unskilled do not invest in education ($e_t = 0$) if

$$\frac{\theta}{z} \geq \frac{\beta}{\beta + b}, \tag{14}$$

that is, if the ratio between the income of child labour and the cost of rising children, i.e. the relative return of child labour, is sufficiently high, then unskilled parents, i.e. $e^i_{t-1} \leq \hat{e}_{t-1}$, choose to send children to work and do not invest in education. When $e^i_{t-1} \geq \hat{e}_{t-1}$ skilled parents send children to schooling only when e^i_{t-1} is sufficiently high, that is $e^i_{t-1} \geq \underline{e}_{t-1}$ (see figure 3). Therefore:

$$e^i_t = \begin{cases} 0 & \text{if } e^i_{t-1} \leq \underline{e}_{t-1}, \\ \frac{w^s_t}{w^u_t} \frac{\beta z}{\theta(1-\beta)} a(b + e^i_{t-1})^\beta - \frac{\theta(\beta+b)}{\theta(1-\beta)} & \text{if } \underline{e}_{t-1} \leq e^i_{t-1} \leq \bar{e}_{t-1}, \\ 1 & \text{if } \bar{e}_{t-1} \leq e^i_{t-1} \leq 1. \end{cases} \tag{15}$$

where:

$$\underline{e}_{t-1} = \left[\frac{\theta(\beta + b)}{a\beta z} \frac{w^u_t}{w^s_t} \right]^{1/\beta} - b,$$

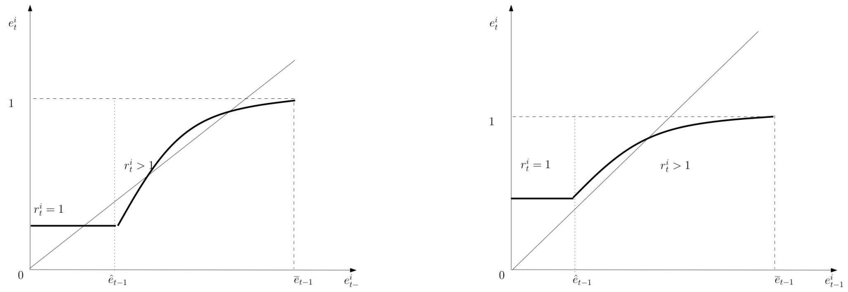
and:

$$\bar{e}_{t-1} = \left[\frac{\theta(1 + b)}{a\beta z} \frac{w^u_t}{w^s_t} \right]^{1/\beta} - b.$$

Simple calculations show that $\bar{e}_{t-1} > \underline{e}_{t-1} > \hat{e}_{t-1}$.

If relative return of child rearing when each child just works is below a certain level but above the elasticity of human capital with respect to education, when education is to its maximum level $e = 1$, that is:

$$\frac{\beta}{1 + b} \leq \frac{\theta}{z} \leq \frac{\beta}{\beta + b}, \tag{16}$$



(a) (b)

Figure 3: The dynamics of education when $\frac{\beta}{1+b} \leq \frac{\theta}{z} \leq \frac{\beta}{\beta+b}$.

the optimal education choice is always positive regardless the level of e^i_{t-1} (see figure 3). In particular:

$$e^i_t = \begin{cases} \frac{\beta z - \theta(\beta+b)}{\theta(1-\beta)} & \text{if } e^i_{t-1} \leq \hat{e}_{t-1}, \\ \frac{w^s_t}{w^u_t}\frac{\beta z}{\theta(1-\beta)} a(b+e^i_{t-1})^\beta - \frac{\theta(\beta+b)}{\theta(1-\beta)} & \text{if } \hat{e}_{t-1} \leq e^i_{t-1} \leq \bar{e}_{t-1}, \\ 1 & \text{if } \bar{e}_{t-1} \leq e^i_{t-1} \leq 1. \end{cases}$$

Finally when the relative return of child rearing when each child just works is below the elasticity of human capital with respect to education, that is :

$$\frac{\theta}{z} \leq \frac{\beta}{1+b}, \tag{17}$$

the optimal education choice is always to its maximum level, that is $e^i_t = 1$, regardless the level of e^i_{t-1}.

This analysis highlights the emergence of three different cases. In the first, when inequality (14) holds, adults that work as unskilled do not invest in children's education. In this situation it is also possible that high-skilled workers find convenient to choose $e^i_t = 0$. In the second case, when inequality (16) holds, all the population allocates a positive fraction of children's time in education. Finally in the third case, all the adults prefers to send their children to school independently by the parent's human capital, in this case the new generation is characterized by the same and maximum level of education. We do not analyze the third case since the result is obviously trivial. More interesting, as it can be intuitively understood from Figures 2, 3, 4, 5, two groups of adults emerge. The first characterized by the same low level of education, the second by a higher one. For this reason in the following section we assume that at the beginning there are only two groups of people characterized by low and high skilled.

4 General Equilibrium

The results of previous section highlight that the ratio w^u/w^s is the crucial factor which determines individual choices. Since choices on fertility and on children time allocation between labour and schooling affect individual and aggregate labour supply, in every period t, these choices also affect the ratio w^u/w^s. For the sake of the argument we assume that in the initial period, $t = 0$, population is divided in two groups which are endowed with two different levels of human capital.

Since the endowment of human capital determines the income of each household, the economy is characterized by two classes of income. Thus, the two groups make different fertility and schooling decisions. In the long run, different fertility choices change the relative size of the two groups, and, as we will analyze below, this process deeply affect our result.

In each period, the aggregate demand of the two goods are[2]

$$D_A = c_A^u N^u + c_A^s N^s, \tag{18}$$

and

$$D_M = c_M^u N^u + c_M^s N^s. \tag{19}$$

The income of the two groups depends on the level of h^s and h^u. Since the demand in the modern sector is positive, there must be some adults that work as skilled. This implies that at least the high-skilled workers must find convenient to work as skilled, that is $I^s = w^s h^s \geq w^u = I^u$. In what follows we assume that in the first period this inequality is strict, that is, the two groups have different income. This assumption allow us to have initial inequality. In order to derive the equilibrium outcomes and in particular the labour supply, we must analyze two different cases: i) $w_t^s h_t^u < w_t^u$, and ii) $w_t^s h_t^u = w_t^u$.

Case 1: $w^s h^u < w^u$

In this case, all the low-skilled adults would choose to work as unskilled. Hence, the supply of unskilled labour, L is given by the labour supplied by low-skilled adults plus the labour supply of children. At equilibrium this supply must be equal to the total demand of unskilled labour. Thus,

$$L_A + L_M = (1 - zn^u)N^u + \theta[(1 - e^u)n^u N^u + (1 - e^s)n^s N^s]. \tag{20}$$

[2]Since all the variables refer to period t, time index is avoided for simplicity.

At the same time, the supply of skilled labour, given by the fertility choices of high-skilled population must be equal to the demand of skilled labour, that is

$$H = (1 - zn^s)h^s N^s. \tag{21}$$

The equilibrium in the goods market implies that total demand of both goods should be equal to the their total supply, that is

$$Y_A = D_A = c_A^u N^u + c_A^s N^s, \tag{22}$$

and

$$Y_M = D_M = c_M^u N^u + c_M^s N^s \tag{23}$$

From equations (2), (4), (5), (11) and (23), it holds

$$L_M = (1-\gamma)\alpha \frac{(1-\mu)h_M N^u + \mu h^s N^s}{h_M}. \tag{24}$$

The ratio between equations (21) and (24) defines the equilibrium level of h_M, that is,

$$h_M^* = \frac{h^s}{1-\mu}\left[\frac{1-zn^s}{(1-\gamma)\alpha} - \mu\right]\frac{N^s}{N^u}. \tag{25}$$

Note that h_M^* depends only on n^s. The other variables N^s, N^u and h^s are given at period t. In order to understand the relation between h_M^* and n^s we must rewrite r^s given equation (25), that is,

$$r_t^s = \frac{w_t^s h_t^s}{w_t^u} = \frac{\mu\alpha(1-\gamma)}{(1-zn^s) - \mu\alpha(1-\gamma)}\frac{N^u}{N^s}. \tag{26}$$

Therefore we can have different values of h_M^* and r_t^s according to the values of fertility of skilled workers at time t. In order to derive the possible equilibria in any configuration we must solve the system given by equations (13) and (26).

In order to simplify the notation we denote: $A \equiv \alpha\mu(1-\gamma)$ and $x_t \equiv N_t^u/N_t^s$. We obtain the following equilibria for n^s and r^s:

$$n_t^s = \begin{cases} \frac{\theta(1-A) - zAx_t + \sqrt{\Delta_1(x_t)}}{2\theta z} & x_0 \leq x_t \leq x_2 \\ \frac{\theta(1+b)(1-A) - zAx_t + \sqrt{\Delta_2(x_t)}}{2\theta z} & x_2 \leq x_t \leq x_3 \\ \frac{(1-\alpha)}{z} & x_t \geq x_3 \end{cases} \tag{27}$$

and:

$$r_t^s = \begin{cases} \frac{2\theta A x_t}{\theta(1-A) + zAx_t - \sqrt{\Delta_1(x_t)}} & x_0 \leq x_t \leq x_2 \\ \frac{2\theta(1+b)Ax_t}{\theta(1+b)(1-A) + zAx_t - \sqrt{\Delta_2(x_t)}} & x_2 \leq x_t \leq x_3 \\ \frac{Ax_t}{\alpha - A} & x_t \geq x_3 \end{cases} \tag{28}$$

where $\Delta_1(x_t) = [zAx_t - \theta(1-A)]^2 + 4\theta(1-\alpha)zAx_t$ and $\Delta_2(x_t) = [zAx_t - \theta(1+b)(1-A)]^2 + 4\theta(1-\beta)(1-\alpha)zAx_t$.
In the Appendix we obtain the values for x_0, x_2 and x_3 and we show how they may change according to the values of parameters. Note that for any value of x_t there is only one value for the fertility of high skilled workers which is an equilibrium.

Given the equilibrium values of fertility choices, we obtain all the other variables of the model as a function of x_t: in particular, children time allocation between work and schooling, the accumulation of human capital for high-skilled adults, the wage ratio.

Note that while in the first case presented in Section 3 – when inequality (14) holds – x_t can lie in one of the three intervals highlighted in equation (27), in the second case – when inequality (16) holds – the first interval does not exist since the high skilled workers always invest in education.

Case 2: $w^s h^u = w^u$

In this case, the ratio $r_t^s = h^s/h^u$. Hence from equation (12) we can directly characterize the dynamic of human capital as follows:

$$h_{t+1}^s = \begin{cases} ab^\beta & \text{if } h_t^s \leq \frac{\theta(\beta+b)}{\beta z} h_t^u, \\ a\left[\frac{\frac{h^s}{h^u}\beta z - \theta(1+b)}{\theta(1-\beta)}\right]^\beta & \text{if } \frac{\theta(\beta+b)}{\beta z} h^u \leq h^s \leq \frac{\theta(1+b)}{\beta z} h^u, \\ a(1+b)^\beta & \text{if } h^s \geq \frac{\theta(1+b)}{\beta z} h^u. \end{cases} \tag{29}$$

The fact that r_t does not depend on other variables strictly depend on the maximization of profits in production.

In order to investigate the relation between the two cases presented above and the possible long run equilibria, next section focuses on the dynamics of the model.

5 Long Run

Fertility choices of the two groups affect the relative size of high and low skilled labour. This relation is crucial in determining the wage ratio, and hence the dynamics of human capital.

Since $N_{t+1}^u = n_t^u N_t^u$ and $N_{t+1}^s = n_t^s N_t^s$, the population dynamics is given by

$$N_{t+1} = n_t^u N_t^u + n_t^s N_t^s. \tag{30}$$

At the same time it is possible to determine the dynamics of $x_t \equiv N_t^u/N_t^s$. Indeed we have that

$$x_{t+1} = \frac{n_t^u}{n_t^s} x_t \tag{31}$$

The long run equilibria imply that the choices of individuals do not change from one period to the next. Since the fertility choices between the two groups are different, the ratio x_t changes over time. In particular starting from $w_s h^u < w_u$, we have that the number of children of low-skilled workers is higher than the one of high-skilled workers. Hence x_t tends to increase over time. This process leads to a continuous increase in the ratio w_s/w_u. Hence, the population dynamics generates an increase in the return of human capital, which is completely caught by the high-skilled workers. Indeed, the low-skilled workers are trapped in a low-skilled equilibrium. This process generates a continuous increase in the inequality and an increase in child labour, since generation by generation the number of low-skilled workers increases with respect to the high-skilled.

The continuous increase in the wage ratio induces to satisfy, at a certain time, $\tilde{t}$, the equality $w_{\tilde{t}}^s h^u = w_{\tilde{t}}^u$.

Before $\tilde{t}$, from equations (27) and (31), we obtain that the dynamics of population ratio is

$$x_{t+1} = \begin{cases} \frac{2z\theta n^u x_t}{\theta(1-A) - zAx_t + \sqrt{\Delta_1(x_t)}} & x_0 \leq x_t \leq x_2 \\ \frac{2z\theta(1+b) n^u x_t}{\theta(1+b)(1-A) - zAx_t + \sqrt{\Delta_2(x_t)}} & x_2 \leq x_t \leq x_3 \\ \frac{zn^u x_t}{(1-\alpha)} & x_t \geq x_3 \end{cases} \tag{32}$$

where n^u is given by

$$n_t^u = \begin{cases} \frac{(1-\alpha)}{z-\theta} & \text{if } 1 \leq \frac{\theta(\beta+b)}{\beta z}, \\ \frac{(1-\alpha)(1-\beta)}{z-\theta(1+b)} & \text{if } \frac{\theta(\beta+b)}{\beta z} \leq 1 \leq \frac{\theta(1+b)}{\beta z}, \\ \frac{1-\alpha}{z} & \text{if } 1 \geq \frac{\theta(1+b)}{\beta z}. \end{cases} \tag{33}$$

that replicates the three cases underlined in Section 3.

From equation (32) we are able to characterize all the possible long term equilibria of the model.

Case a. When low skilled workers do not invest in education, i. e. $1 \leq \frac{\theta(\beta+b)}{\beta z}$, we could have the following results.

i. if $x_t \leq x_2$, high skilled workers send their children to work, then in the following period the inequality in the economy disappears and all the population will be characterized with the minimum level of human capital;

ii. if $x_2 \leq x_t \leq x_3$, high skilled workers allocate the children time between schooling and labour. Generation by generation the income of skilled adults increases. If this process reach $e_t^s = 1$ for $t < \tilde{t}$, then the high skilled workers reach the maximum level of capital, but their size in the economy reduces continuously. Although the choice of education and fertility are fixed, until period $\tilde{t}$ the income of high skilled increases. After period $\tilde{t}$ all the variable of the model are at equilibrium. If instead the economy reach $\tilde{t}$ before $e_t = 1$ the accumulation of human capital of high skilled workers would change according to equation (29).

iii. if $x_t \geq x_3$ the high skilled workers would choose the maximum level of education and their income increases until period $\tilde{t}$.

Case b. When low skilled workers allocate children time between schooling and labour, i.e. $\frac{\theta(\beta+b)}{\beta z} \leq 1 \leq \frac{\theta(1+b)}{\beta z}$, the low-skilled workers have an higher human capital with respect to the previous case. This implies that the condition $w_t^s h_t^u = w_t^u$ can be easily reached. In particular we have that

i. the high skilled workers cannot send their children to work since they get a wage higher than the low skilled workers.

ii. if $x_2 \leq x_t \leq x_3$, high skilled workers allocate the children time between schooling and labour, but the fraction of time spent to education is greater than that of the low skilled workers. It is more likely that the increase in the wage ratio reach $t = \tilde{t}$ before $e_t^s = 1$. Hence this economy would be characterized by a lower inequality even if most of the low-skilled workers continuous to work as unskilled.

iii. if $x_t \geq x_3$ the high skilled workers would choose the maximum level of education and their income increases until period $\tilde{t}$.

The transition can go on for several generation. During that transition the inequality in the economy increases and there will be an increase in child labour, since the unskilled population would increase more than the skilled one. The fact that fertility is endogenous to the model determine such a result. The two groups have different choices of fertility and the high-skilled group become smaller with respect to the total population, implying that they will get a higher wage.

6 Final Remarks

This paper is built on the idea that the persistence of child labour is strictly linked to the presence of inequality within the country. For this reason we present a model where the population is divided in two groups endowed with two different level of human capital. We study how this initial heterogeneity affect the distribution of income in the long run. The crucial result of this analysis is that the increase in the return of human capital is not sufficient to induce a transition to a high-skilled economy. The presence of two groups, with different levels of initial human capital, generates a continuous increase in the income of the high skilled workers with respect to those endowed with a low level of human capital. This, in turn, implies that inequality in the economy will increase. The presence of endogenous fertility induces low income group to make an higher number of children. Thus, child labour will increase. The substitutability between adult and child labour increases the resilience of this result: the economy is trapped in an equilibrium with a high fraction of the population with low income and low human capital.

This framework can be easily extended to evaluate the issues currently discussed in the literature. For instance, further research is needed to analyze the role of technical progress and international labour standards. With respect to the first issue, preliminary results seem to reject the hypothesis that technical progress can by itself induce the low-skilled group to invest in children's education. Another interesting application of the model is to evaluate public policies that through taxation on high-skilled individual may subsidies the low-skilled labour inducing them to invest in education. This policy may generate together a reduction of inequality and the disappearance of child labour.

A Appendix

In order to obtain the threshold level x_0, x_2 and x_3, we can proceed as follows. First, let us consider the case $D \equiv \frac{\theta(\beta+b)}{\beta z} \geq 1$. This implies that low skilled workers choose the highest fertility and zero investment in education. In order to get an equilibrium in which the high skilled workers invest zero in education, that is in terms of fertility

$$n_t^s = \frac{(1-\alpha)r^{s_t}}{zr^{s_t-\theta}}, \tag{34}$$

the income ratio between skilled and unskilled should be less than D, i.e. $r_t^s \in [1, D]$. If this condition is satisfy from equations (26) and (34) we get

$$n_1^{s*} = \frac{\theta(1-A) - zAx_t + \sqrt{\Delta_1}}{2\theta z}, \tag{35}$$

and

$$r_1^{s*} = \frac{2\theta A x_t}{\theta(1-A) + zAx_t - \sqrt{\Delta_1}}, \tag{36}$$

where $\Delta_1 = [zAx_t - \theta(1-A)]^2 + 4\theta(1-\alpha)zAx_t$. From equation (44), we obtain that $r_t^s \geq 1$ if and only if:

$$x_t \geq x_0 \equiv \begin{cases} \frac{\theta(1-A)}{A(2\theta-z)} & \text{if } z \leq 2\theta, \\ \frac{z(\alpha-A)-\theta(1-A)}{A(z-\theta)} & \text{otherwise} \end{cases} \tag{37}$$

and, $r_t^s \leq D$ if and only if

$$x_t \leq x_2 = \frac{\theta(b+\beta)[b(\alpha-A) - \beta(1-\alpha)]}{zA\beta b}. \tag{38}$$

The two thresholds, x_0 and x_2, determine the interval of the first line of equation (27).

In order to get an equilibrium for

$$n_t^s = \frac{(1-\alpha)(1-\beta)r^{s_t}}{zr^{s_t-\theta(1+b)}}, \tag{39}$$

that is where high skilled workers allocate the children time between schooling and work, the income ratio between skilled and unskilled should satisfy $r_t^s \in [\max 1, D, E]$, where $E = \frac{\theta(1+b)}{\beta z}$. While the equilibrium n_1^{s*} can be realized if and only if $D \geq 1$, n_2^{s*} does not have this restriction. If $r_t^s \in [\max 1, D, E]$ is satisfy from equations (26) and (34) we get

$$n_2^{s*} = \frac{\theta(1+b)(1-A) - zAx_t + \sqrt{\Delta_2}}{2\theta z}, \tag{40}$$

and

$$r_2^{s*} = \frac{2\theta(1+b)Ax_t}{\theta(1+b)(1-A) + zAx_t - \sqrt{\Delta_2}}, \tag{41}$$

where $\Delta_2(x_t) = [zAx_t - \theta(1+b)(1-A)]^2 + 4\theta(1-\beta)(1-\alpha)zAx_t$. From equation (**??**), we obtain that $r_t^s \geq \max 1, D$ if and only if $x_t \geq x_2$. It is not surprising that $r_1^{s*} = r_2^{s*}$ when $x_t = x_2$; hence, the function is continuous. Furthermore, $r_t^s \leq E$ if and only if $x_t \leq x_3$, where

$$x_3 = \frac{\theta(\alpha-A)(1+b)}{A\beta z}. \tag{42}$$

By applying the same procedure, we get that the high skilled workers do not send their children to work if and only if

$$n_3^{s*} = \frac{(1-\alpha)}{z}, \tag{43}$$

and

$$r_3^{s*} = \frac{Ax_t}{\alpha - A}, \tag{44}$$

if and only if $r_t^s \geq E$, that is if $x_t \geq x_3$.

Hence we obtain equations (27) and (28).

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