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Abstract

Stock prices often diverge from measures of fundamental value, which simple present-value models fail to explain. This paper tries to find causes for these long-run price movements and their persistence by estimating a STAR model for the price-earnings ratio of the S&P500 index for 1961Q1 - 2009Q3, with a transition function that depends on a wider set of exogenous or predetermined transition variables. Several economic, monetary and financial variables, as well as linear combinations of these, are found to have nonlinear effects on stock prices. A two-step estimation procedure is proposed to select the transition variables and estimate their weights. This STAR model can be interpreted as a heterogeneous agent asset pricing model that makes a distinction between chartists and fundamentalists, where the set of transition variables is included in the agents' information set.

JEL Classification: G12, E44, C22

Keywords: Heterogeneous agents, Regime switching, Stock prices, STAR models.

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1 Introduction

Financial asset prices, stock prices in particular, are historically more volatile than real economic activity. This is a puzzle, since the stock price of a corporation reflects a forecast by the market of future profits and should therefore be smoother than the actual profits. However, over the past century there have been several periods, of sometimes decades, during which stock prices appear to be diverging from reasonable forecasts of future profits. Such periods often end with abrupt market crashes.

This is illustrated in Figure 1, which shows the price-earnings (PE) and price-dividend (PD) ratios of the S&P500 index since 1881[1]. These ratios show the level of the S&P500 index relative to the profits that the stocks are generating. In particular the path of the PE ratio (left) seems stable or mean-reversive in the long run. Even after reaching all-time record levels around the year 2000, the PE ratio recently dropped again below its average value during the credit crisis. This latest peak is comparable in size to earlier episodes, most notably the 1920s.

Despite the long-run mean reversion in stock-market valuation, the market can be almost uniquely increasing or decreasing for intervals as long as twenty years, while cycles of this magnitude are hard to observe in the real economy. In this paper, I try to find causes for these bubbles in the stock market, defined as the deviation in the asset's price from its estimated fundamental value.

Conventional Efficient Market Hypothesis models, where a single rational representative agent prices financial assets as a prediction of discounted future cash flows, often fail to explain this excess volatility (Shiller, 1981). As an alternative to the assumption of a single rational representative agent, one could assume that movements in asset prices reflect some form of market sentiment, which can be represented by a heterogeneous agent model. Many heterogeneous agent models make a distinction between fundamentalists, who believe deviations from the fundamental value should decrease over time, and chartists (or trend-followers) who believe current deviations to be persistent. The growing literature on heterogeneous beliefs includes models that can generate rational bubbles, where chartists are aware of the fundamental value but believe deviations to be

[1] Source: Robert Shiller, http://www.irrationalexuberance.com/index.htm

persistent for now (e.g. Brock and Hommes, 1998), and irrational bubbles which are caused by noise traders or irrational agents who have unrealistic beliefs about the future (e.g. Shleifer and Vishny, 1997). Hommes (2006) gives a survey of such heterogeneous agent models in economics and finance.

Empirically, heterogeneous beliefs can be estimated with regime-switching regression models. The distinct regimes represent the asset pricing processes according to the different types of agents and a regime-switching process determines which type is dominant at each point in time. Switching models can either choose between the regimes (e.g. threshold autoregressive (TAR) or Markov switching models), or estimate a weighted average of regimes (e.g. smooth transition autoregressive (STAR) models). In the latter case, the weights of the regimes can be interpreted as the fractions of agents belonging to each type. Another distinction is that in TAR and STAR models the regime switch is caused by a transition variable chosen by the econometrician, while in Markov switching models the switch depends on an unobservable.

Several studies exist that apply these methods to stock market data, mostly switching between two regimes. Bredin and Hyde (2008) study stock returns in small countries using a STAR model and find that market conditions in larger countries can cause regime changes in Danish and Irish stock markets. Kaliva and Koskinen (2008) analyze the S&P500 index by estimating a mixture autoregressive model (LMARX) that switches between an error correction regime and a bubble regime, following Shleifer and Vishny (1997). They find that regime switching is caused by inflation and lagged stock prices. In van Norden and Shaller (1999) the regime switch is caused by the level of the PD ratio. In Boswijk et al. (2007) a two-type version of the model defined by Brock and Hommes (1998) is applied to the PE and PD ratios of the S&P500 index. In their model the transition function depends on lagged values of the dependent variable. This is justified by the assumed behavior of the agents, who choose regime according to the profitability of both regimes in previous periods. The extreme levels of the stock market in the late 1990s are explained as the trend-following regime dominating the fundamentalist regime for several years.

The model in this paper is similar to that in Boswijk et al. (2007). The main difference is that the agents choose their regime based on a wider information set including macroeconomic, monetary and financial variables. In practice, this means I estimate a STAR model, where the transition function depends on a linear combination of exogenous or predetermined variables. There are not many earlier applications of STAR models where the transition function depends on a linear combination of transition variables. Examples include Medeiros and Veiga (2005) and Becker and Osborn (2007), although both these papers address univariate models, where the transition function depends on a combination of lags of the dependent variable. In this paper, I propose a procedure based on linearity tests, to select the transition variables from a large set of macroeconomic and financial variables and to estimate their weights. The transition variables that are found by this procedure can be interpreted as the variables that have the most influence on the behaviour of the agents.

This paper proceeds as follows. In the next section I describe the asset pricing model and the STAR specification in more detail. Data descriptions, linearity tests, estimation results and diagnostic checks are presented in section three. Section four concludes.

2 The model

In this section, a simple present value asset pricing model, consistent with the efficient market hypothesis, is presented. It can be shown that this model fails to explain some stylized facts of the S&P500 index over the past century, which gives the motivation to relax the assumption of a single representative agent and introduce heterogeneous agents.

In a simple present-value asset pricing model, the price of a financial asset equals the discounted sum of the expected price of the asset one period ahead and any expected cash flows (dividends) paid out on the asset in the coming period. Iterating forward, the price represents a infinite sum of discounted expected dividends:

$$P_t = E_t\left[\frac{P_{t+1}+D_{t+1}}{1+r_{t+1}}\right] = E_t\left[\sum_{i=1}^{\infty}\frac{D_{t+i}}{\prod\limits_{j=1}^{i}(1+r_{t+j})}\right] = E_t\left[\sum_{i=1}^{\infty}\left(\prod_{j=1}^{i}\frac{1+g_{t+j}}{1+r_{t+j}}\right)D_t\right]$$

The discount factor is given by $(1+r_t)^{-1}$ and g_t is defined as the dividend growth rate such that $D_{t+1} = (1+g_t)D_t$. This equation can be rewritten as:

$$\frac{P_t}{D_t} = E_t\left[\sum_{i=1}^{\infty}\left(\prod_{j=1}^{i}\frac{1+g_{t+j}}{1+r_{t+j}}\right)\right] \tag{1}$$

This model is known as the Gordon growth model (Gordon, 1962). Under the assumption of constant dividend growth and a constant discount factor the PD ratio is constant over time:

$$\frac{P_t}{D_t} = \frac{1+g}{r-g}$$

A constant PD ratio can be shown to hold as well for more general asset pricing models that do not necessarily need constant growth rates and discount factors as long as certain restrictions on the dividend growth rate and discount factor apply (Munk, 2010). However, as figure 1 shows, the PD ratio is far from constant over time.

According to equation (1), under constant discount rates, movements in the PD ratio can be caused only by changing expectations on the dividend growth rate. However, it can be shown that neither the PD nor the PE ratio are good predictors for future dividend growth rates. In fact, both valuation ratios work well as a predictor for the future direction of the S&P500 index. High valuation ratios seem to predict decreasing stock prices, while low ratios predict increasing prices. (Campbell and Shiller, 2001).

Gordon's assumption of a constant discount factor is very restrictive. Indeed, in modern asset pricing models it is common to apply a Stochastic Discount Factor (SDF) that represents the time-varying risk aversion of the agents (e.g. Cochrane, 2001 or Munk, 2010). However, given the lack of observed correlation between valuation ratios and future dividend growth rates, the idea that investors price assets solely as a discounted stream of expected dividends seems incomplete. This can be solved by introducing heterogeneous beliefs.

The results of Campbell and Shiller (2001) suggest to estimate mispricings in the market as the PD or PE ratio in deviation from its long-run average:

$$Y_t = \frac{P_t}{D_t} - \frac{1+\bar{g}}{\bar{r}-\bar{g}} \rightarrow \ P_t = \frac{1+\bar{g}}{\bar{r}-\bar{g}} D_t + Y_t D_t$$

In this case, $\frac{1+\bar{g}}{\bar{r}-\bar{g}}$ is the long-run average of the PD or PE ratio and represents an estimate of its fundamental value. Y_t gives the size of the bubble in the market, which can be both negative or positive. We call the asset over-valued if $Y_t > 0$ and under-valued if $Y_t < 0$. The price of the asset P_t can then be decomposed in an estimated fundamental value $\frac{1+\bar{g}}{\bar{r}-\bar{g}} D_t$ and a bubble $Y_t D_t$.

In the model, investors are classified either as fundamentalists, who believe that prices should return to its fundamental value (mean reversion), or chartists (trend-followers) who believe the current mispricings to continue. (Brock and Hommes, 1998). Hence, fundamentalists try to buy stocks when the market is undervalued and sell when the market is overvalued. They believe that mispricings in the market disappear over time: $E_t^F [Y_{t+1}] = \alpha^F Y_t$, with $\alpha^F < 1$.

Chartists, on the other hand, speculate that the stock market diverges further away from the fundamental value: $E_t^C [Y_{t+1}] = \alpha^C Y_t$, with $\alpha^C \geqslant 1$. This does not necessarily imply that chartists are irrational. They might be aware of the mispricings, but believe deviations to persist under current market conditions, at least in the short run.

By allowing the fraction of chartists and fundamentalists to change over time, the asset pricing process can be described by a smooth transition autoregressive (STAR) process.

$$Y_t = \alpha_1 Y_{t-1}(1 - G(s_{t-1})) + \alpha_2 Y_{t-1} G(s_{t-1}) + \varepsilon_t \quad \begin{array}{l} \alpha_1 < 1 \\ \alpha_2 \geqslant 1 \end{array} \tag{2}$$

$G(s_{t-1})$ is the fraction of chartists in the market. Agents choose regime according to a transition variable or set of transition variables s_{t-1}. Boswijk et al. (2007) estimate a variant of this model for annual S&P500 index PD and PE ratios in deviation from their mean for the period 1871 to 2003. Agents base their regime choice on the profitability of each regime in the previous period. The transition function therefore depends on lagged values of the dependent variable. The model in this paper is similar to that in Boswijk et al. (2007) with the difference that in this paper the transition function is driven by a set of predetermined variables (more precisely; a linear combination of predetermined variables), which can be interpreted as an extension of the agents' information set. Of interest is to estimate which factors (e.g. monetary policy, the business cycle or market volatility) are determining market sentiment and therefore asset price changes.

The transition function $G(s_{t-1})$ is assumed to be a logistic function, as in the standard LSTAR model (Teräsvirta, 1994):

$$G(s_{t-1}) = (1 + exp[-\gamma(s_{t-1} - c)])^{-1} \tag{3}$$

The transition variable s_t is usually a lagged value or lagged difference of the dependent variable. It can, however, also be a different (predetermined or exogenous) variable, or a linear combination of variables: $s_t = X_t'\beta$. In the latter case the transition function becomes:

$$G(s_{t-1}) = \Big(1 + exp[-\gamma(X_{t-1}'\beta - c)]\Big)^{-1} \tag{4}$$

The vector β represents the weights of the different variables. For this model, there is an identification problem since γ, c and β can not be all identified. This can be solved by placing a restriction on β. In this paper the elements of β sum to one, so that the elements of β represent the weights of the transition variables.

3 Estimation results

The dependent variable in the model is the quarterly S&P500 PE ratio in logs, in deviation from its average value. I consider in this paper the PE ratio rather than the PD ratio as the more appropriate measure of stock-market valuation, mainly because in recent decades corporations tend to distribute an increasing part of their profits amongst shareholders by buying shares rather than paying dividends. Dividends have therefore been relatively low recently, leading to high PD ratios (See Figure 1). This issue is further discussed by Fama and French (2001). I follow the convention to smooth earnings over a period of ten years, creating the so-called cyclically adjusted PE ratio.

According to the Bayesian Information Criterion (BIC), the best fitting linear AR(q) process for this time series is the AR(1) process. Following the modeling cycle proposed by Teräsvirta (1994), the STAR model will therefore be estimated with an autoregressive structure of only one lag, as in equation (2). Autocorrelation tests and other misspecification tests at the end of this section confirm that this lag length is sufficient.

As explanatory variables, I consider a number of business cycle indicators (the Aruoba-Diebold-Scotti Business Conditions (ADS) Index, industrial production and consumer sentiment), monetary variables (consumer credit growth, M2 growth and the federal fund rate), government bond yields (three-month, one-year and ten-year term US treasury bonds and their term spreads) corporate bond yields (triple-A, ten-year term and their yield spread with treasury bonds), stock market volatility, stock market returns, and consumer price inflation[2]. The end-of-quarter values for all these variables are considered, but also their first differences and average values (moving averages) over lengths of one to four quarters. These data are not available for the full period of S&P500 data, so the model is estimated for 1961Q1-2009Q3. To make estimation numerically more manageable, all variables are demeaned and re-scaled so that they have a similar range. All explanatory variables are lagged one period with respect to the dependent variable and are therefore predetermined.

To determine which explanatory variables are valid transition variables in the STAR model they are submitted to a linearity test, following Luukkonen et. al (1988). First, a univariate transition function ($s_t = x_t$) is considered. A third-order Taylor approximation of (2) with a univariate transition function (3) around $\gamma = 0$ gives:

[2] Sources: FRED® (Federal Reserve Economic Data) and http://finance.yahoo.com/

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \sum_{i=1}^{3} \phi_{1+i} Y_{t-1} x_{t-1}^{i} + \epsilon_t \tag{5}$$

Linearity can now be tested by estimating this Taylor approximation by OLS and testing $H_o : \phi_2 = \phi_3 = \phi_4 = 0$. When the null hypothesis is rejected, x_t is a potential transition variable. Results of the linearity tests are in Table 1 , which shows the test statistics and its corresponding p-values. The test statistic is asymptotically *F(n,T-k-n-1)* distributed under the null, with $T = 194$ (observations), $k = 2$ (unrestricted parameters) and $n = 3$ (restricted parameters). Table 1 also includes the lagged dependent variable Y_{t-1}, for which linearity is not rejected. This result does not imply that Y is not a regime-switching process, but rather that the level of Y_{t-1} does not contain much information about the probability of a regime switch. In fact, the results in Table 1 show that for several explanatory variables, linearity is rejected with very convincing p-values. I also find that for macroeconomic indicators, the rejection of linearity is the strongest when the average value over three months is considered, while for financial and monetary indicators, the strongest effects come from end-of-quarter levels. This can be interpreted as business cycle developments having an effect on investors' behavior only after some persistence, while financial developments are faster observed and considered.

Following Teräsvirta (1994), the model is estimated with the transition variables for which rejection of linearity is the strongest. As the test results for some potential transition variables are very close, it is advisable to estimate more than one model. I estimate the model with three different transition variables: The quarterly average ADS index (A measure of business conditions in the US, based on a number of real-time macroeconomic variables (Aruoba et al., 2009)), the industrial production growth rate and stock market returns. After selection of the transition variable the parameters are estimated with nonlinear least squares. Starting values are found with a two-dimensional grid search for γ and c.

Parameter estimates are given in Table 2. For all three STAR models, two distinct regimes are estimated, with $\alpha_1 < 1$ and $\alpha_2 \geqslant 1$. The ADS index, industrial production and stock returns have a positive effect on the fraction of fundamentalists in the economy. The table also shows the sum of squared residuals, AIC and BIC for both the STAR model and for an alternative linear model $Y_t = \omega_1 Y_{t-1} + \omega_2 x_{t-1}$, which includes the same explanatory variable. For comparison, an AR(1) and a random walk process give an AIC of respectively -976.1 and -978.4. The three nonlinear models therefore fit the data better than linear and univariate alternatives, even when accounting for the extra parameters to estimate. Although the models outperform their linear alternatives, the specifications are far from perfect. In the first model, the fundamentalist regime is clearly an outlier regime: The estimate for α_1 is very small and imprecise (Not significantly smaller than the chartist regime). In the last two models, the parameter estimate for γ is very large. The model therefore turns in practice into a threshold autoregressive (TAR) model. Without the smooth transition, the market is either completely fundamentalist, or completely chartist. This would be a unrealistic conclusion. These issues can be solved by introducing a multivariate transition function, as in equation (4).

In the same spirit as the variable selection procedure for the univariate transition function, I now propose a similar procedure allowing for the possibility that the transition function depends on a linear combination of variables: $s_t = X_t'\beta$ To keep computation simple, a first-order Taylor approximation of (2) with a multivariate transition function (4) around $\gamma = 0$ is considered:

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-1}(X_{t-1}'\beta) + \epsilon_t$$

Or:

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \sum_{i=1}^{p} \theta_i Y_{t-1} X_{i,t-1} + \epsilon_t \qquad (6)$$

Such that $\theta_i = \phi_2 \beta_i$ and p is the number of variables in X. The restriction $\phi_2 = 0$ is identical to the restriction $\theta_1 = ... = \theta_p = 0$. We can therefore estimate (6) for several sets of explanatory variables, test for linearity ($H_o : \theta_1 = ... = \theta_p = 0$) and choose the set of variables that leads to the strongest rejection of the null hypothesis (i.e. the smallest p-value). As sets of transition variables I apply all possible sets of one to six variables, which never include more than one variable of the same "group": 1) business cycle indicators, 2) monetary variables and interest rates, 3) term spreads and yield spreads, 4) stock market volatility, 5) stock market returns and 6) consumer price inflation.

The test-statistic is again asymptotically *F(n,T-k-n-1)* distributed under the null, with $n = p$. After choosing the set of variables for which rejection of linearity is the strongest, a first estimate of β can be derived from the OLS estimates of (6), using the initial condition $\sum_{i=1}^{p}\beta_i = 1$:

$$\widetilde{\beta}_j = \widehat{\theta}_j\left(\sum_{i=1}^{p}\widehat{\theta}_i\right)^{-1}$$

Keeping $\widetilde{\beta}$ fixed, γ, c and α can be estimated with NLS in the same manner as for the univariate case described above. As a final stage, the model is estimated once more, with the previous estimates as starting values, fixing none of the parameters (except $\sum_{i=1}^{p}\beta_i = 1$). I find in practice that the parameter estimates do not change much in the final stage, suggesting the Taylor approximation provides reasonable estimates of β.

The optimal combination of variables found by this procedure is a set of three variables, including industrial production growth, the first difference in the yield on ten-year treasury bonds and the yield spread between triple-A corporate bonds and treasury bonds. In the test based on equation (6) these variables result in a p-value of 4.6×10^{-4}, with $\widetilde{\beta} = (0.58, 0.72, -0.30)'$. Substituting $x_t = X_t'\widetilde{\beta}$ into the third order Taylor approximation (5) and running the original linearity test, gives a p-value of 2.0×10^{-10}; a stronger rejection of linearity than for any variable in Table 1.

This optimal linear combination is closely followed by a the same set of variables excluding the yield spread. This two-variable set gives a p-value of 6.2×10^{-4}, with $\widetilde{\beta} = (0.45, 0.55)'$. Substituting $x_t = X_t'\widetilde{\beta}$ into (5) and running the original linearity test, gives a p-value of 2.2×10^{-9}. I estimate the model with both of these linear combinations of transition variables. Parameter estimates and goodness of fit measures are presented in Table 3.

In both estimated models, two distinct regimes are identified. α_1 represents the fundamentalist regime and is significantly smaller than one, while α_2, representing the chartist regime, is significantly greater than one. The estimates of β show the effects of the different transition variables on the agents' choice of regime. Industrial production growth has a positive coefficient, implying in this case it supports the chartist regime: An increase in industrial production causes an increase in the fraction of chartists in the economy. One can interpret this result as investors being more willing to believe in persistence of deviations in the stock market during times of positive macroeconomic news. During an overvalued market, this result implies that positive macroeconomic conditions have an inflationary effect on stock prices.

The first difference in the yield on ten-year treasury bonds also has a positive coefficient. A decrease in this interest rate causes an increase in the fraction of fundamentalists. This result is somewhat remarkable, because this is in contrast with the idea that low interest rates would fuel a credit-driven stock market bubble. However, a possible explanation is that a sudden drop in the yield on governments bonds is a signal for financial stress, because investors turn to the relatively safe treasury bills. For the same reason, the yield spread has an estimated negative coefficient and is supporting the fundamentalist regime, as high yield spreads are a sign that investors prefer safer treasury bonds to corporate bonds. The role of interest rates and spreads as indicators of financial stress is discussed by Bernanke (1990) and Hakkio and Keeton (2009) amongst others. These signals could be seen as warnings that mispricings in the stock markets are not sustainable, resulting in a switch to the fundamentalist regime.

The estimated coefficient for the yield spread is not statistically significant different from zero. Indeed, Table 3 shows that the second model, excluding the yield spread, is the preferred model according to the selection criteria. The goodness-of-fit measures in Table 3 moreover show that these two models outperform their linear alternatives as well as the STAR models with univariate transition functions in Table 2.

An intuitive interpretation of the results is found by giving (2) the alternative formulation of an AR(1) process with a time-varying parameter: $Y_t = \delta_t Y_{t-1} + \varepsilon_t$, where $\delta_t = \alpha_1(1 - G(s_{t-1})) + \alpha_2 G(s_{t-1})$, which can be interpreted as an indicator of market sentiment. When $\delta_t \geqslant 1$ the PE ratio is further diverging from its mean, implying that the chartist regime is dominant, while the PE ratio is reverting to its mean when $\delta_t < 1$. Plots of δ_t over time as well as plots of the transition functions against the linear combination of transition variables are shown in Figure 2 for both multivariate models. The charts for both models are very similar. For the majority of time, a significant proportion of the market speculates on further inflation of the bubble, with δ_t fluctuating around one. On several occasions, however, the market turns almost completely to the fundamentalist regime. The end of the "dot-com" era in 2001 and the credit crunch in 2008 are recent examples of such episodes.

Finally, the empirical results presented in Table 3 are evaluated with tests for autocorrelation, parameter constancy and remaining non-linearity. Technical details on all three tests can be found in Eitrheim and Teräsvirta (1996). The test results (test statistics and p-values) are presented in Table 4.

The first test is a generalized case of the test for qth order serial correlation in linear models proposed by Godfrey (1988). For a fourth order autocorrelation test, the resulting test statistic is asymptotically $F(4,\ T\text{-}8)$ distributed under the null, with T=194 (sample size). Table 4 shows that for both models, there is no reason the reject the null hypothesis of no serial correlation. This means the autoregressive structure of only one lag is sufficient and longer lags do not need to be considered.

The test for remaining non-linearity checks whether any variables in addition to the specified transition function have significant non-linear effects on Y. This could be the case when a transition variable is omitted, or when there are in reality more than two different regimes, in which case a Multiple Regime STAR model should be estimated (Franses & van Dijk, 1999). The test statistic is asymptotically $F(3,\ T\text{-}6)$ distributed under the null. I repeat this test for all potential transition variables considered in this paper. Table 4 shows the test statistics for the same variables as in Table 1. For the majority of the variables, there is no reason to reject the null hypothesis of no remaining non-linearity, with comfortably high p-values. For a few cases, the p-values are lower than ten or even five percent, although this does not need to imply misspecification. Since the test is repeated for many variables, it is not unlikely that Type I errors are made.

The test for parameter constancy is a generalization of the test for structural breaks in a linear model. The null hypothesis is that the parameters in (2) and (4) are constant over time. The test statistic is asymptotically $F(3,\ T\text{-}6)$ distributed under the null. Table 4 shows also for this test high p-values. The model therefore seems well specified. The diagnostic tests give no reason to change the specification.

4 Concluding remarks

This paper presents an estimation method and empirical results for a heterogeneous agent asset pricing model. Two types of investors are identified: Fundamentalists, who buy undervalued shares and sell overvalued shares, and chartists, who have a shorter-term strategy and speculate on the persistence of mispricings in the market. The existence of chartists in the economy can explain the creation of bubbles in asset prices.

To estimate the effects of different variables on the behavior of investors, a STAR model with a multivariate transition function is proposed. The empirical results outperform simple AR models as well as multivariate linear models in terms of goodness-of-fit.

It is found that the fraction of chartist in the economy increases during periods of good macroeconomic conditions. Signals of financial instability, on the other hand, support the fundamentalist regime. This can be interpreted as investors losing belief in the persistence of bubbles during periods of financial instability. No evidence is found that supports the idea of a credit-driven bubble, inflated by too expansive monetary policy (i.e. low interest rates).

The model is in this paper applied to demeaned PE ratios, but can be applied to any asset price in deviation from some measure of fundamental value. Examples could include exchange rate movements not driven by interest rate differentials, or government bond spreads not explained by fiscal developments. Also international stock market indices other than the S&P500 index could be analyzed, although few stock markets exist for which this long-term time series are available.

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Tables and charts

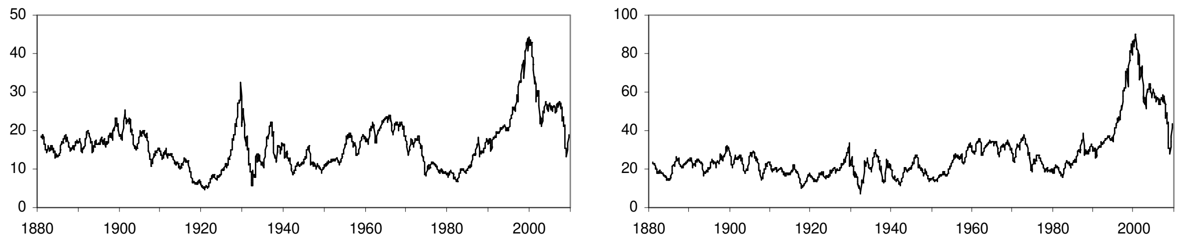

Figure 1: S&P 500 index 1881-2009: Price-Earnings ratio (left) and Price-Dividend ratio (right).

TABLE 1. Linearity test results

Transition Variable x_{t-1}	test-statistic	p-value
Y	0.774	0.51
ADS-index, 3-month average	12.244	2.4×10^{-7}
Industrial production, quarterly growth rate	10.654	1.7×10^{-6}
M2 growth rate	1.876	0.14
Federal fund rate	3.925	0.01
$\triangle$(10-year treasury bond yield)	9.225	1.0×10^{-5}
Yield Spread (AAA corporate vs. 10-year treasury yield)	7.784	6.3×10^{-5}
Term Spread (10-year vs. 3-month treasury yield)	5.204	1.8×10^{-5}
Rate of return on S&P500	10.037	3.6×10^{-6}
Consumer price inflation	7.542	8.6×10^{-5}

Notes: Test statistics and corresponding p-values for $H_o : \phi_2 = \phi_3 = \phi_4 = 0$ in equation (5). Transition variables refer either to end-of-quarter levels or to growth rates / differences in the last month of the quarter, unless denoted otherwise.

TABLE 2. Parameter estimates: Univariate transition function

x_{t-1}	α_1	α_2	γ	c	SSR	AIC	BIC
ADS-index, 3-month average	0.35	1.02	3.26	-1.12	1.16	-983.8	-967.5
	(1.09)	(0.02)	(2.21)	(0.94)	{1.22}	{-974.7}	{-961.6}
Industrial production,	0.83	1.00	269.1	-0.31	1.14	-986.1	-969.8
quarterly growth rate	(0.03)	(0.01)	(268.9)	(0.01)	{0.97}	{-974.2}	{-961.1}
Rate of return on S&P500	0.93	1.00	6160.5	-0.11	1.18	-979.2	-962.8
	(0.02)	(0.01)	(-)	(0.45)	{1.22)	{-975.0}	{-961.9}

Notes: NLS parameter estimates for model (2). Standard errors in parenthesis. Sum of Squared Residuals, Aikaike Information Criterion and Bayesian (Schwarz) Information Criterion are reported for the STAR model and the Linear model $Y_t = \omega_1 Y_{t-1} + \omega_2 x_{t-1}$. The latter are reported in accolades. All estimated models include a constant, which are not significantly different from zero and are therefore not reported.

TABLE 3. Parameter estimates: Multivariate transition function

	α_1	α_2	γ	c	β_1	β_2	β_3	SSR	AIC	BIC
$X = (x_1, x_2, x_3)$	0.82	1.04	14.00	-0.14	0.44	0.70	-0.14	1.10	-989.6	-966.7
	(0.07)	(0.02)	(10.65)	(0.09)	(0.08)	(0.08)	(0.10)	{1.14}	{-985.3}	{-965.6}
$X = (x_1, x_2)$	0.81	1.03	18.30	-0.12	0.41	0.59	.	1.11	-989.7	-970.1
	(0.06)	(0.02)	(10.40)	(0.06)	(0.05)	(0.05)		{1.15}	{-984.6}	{-968.2}

Notes: NLS parameter estimates for model (2). Standard errors in parenthesis. x_1 =Industrial production, quarterly growth rate. x_2 =$\triangle$(10-year treasury bond yield). x_3 =Yield Spread (AAA corporate vs. 10-year treasury yield). Sum of Squared Residuals, Aikaike Information Criterion and Bayesian (Schwarz) Information Criterion are reported for the STAR model and the Linear model $Y_t = \omega_1 Y_{t-1} + \omega_2' X_{t-1}$. The latter are reported in accolades. All estimated models include a constant, which are not significantly different from zero and are therefore not reported.

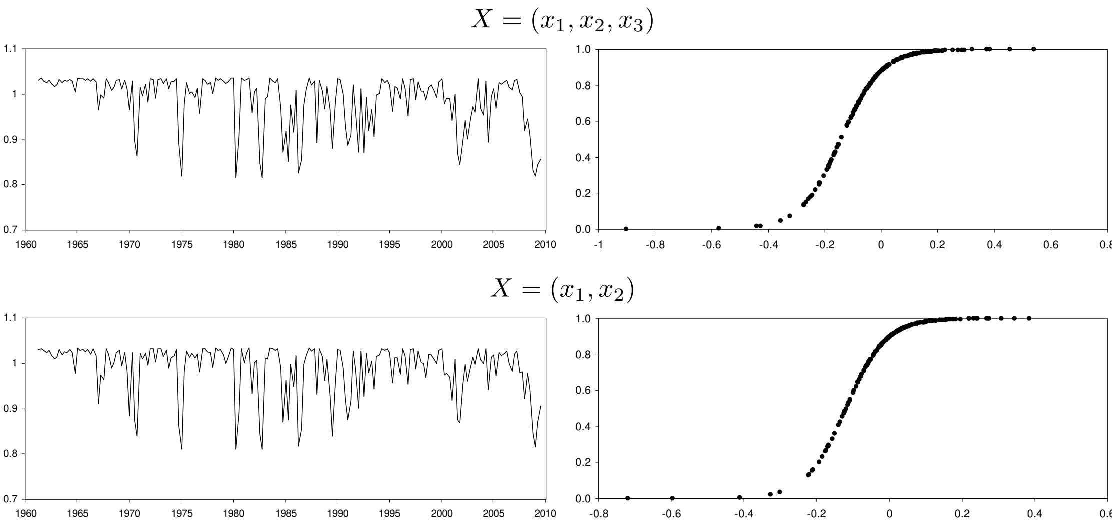

Figure 2 :Regression results: $\widehat{\alpha}_1(1 - G(X'_{t-1}\widehat{\beta})) + \widehat{\alpha_2}G(X'_{t-1}\widehat{\beta})$ over time (left) and $G(X'_{t-1}\widehat{\beta})$ against $X'_{t-1}\widehat{\beta}$ (right)

TABLE 4. Diagnostic tests

	$X = (x_1, x_2, x_3)$		$X = (x_1, x_2)$	
Test	test-statistic	p-value	test-statistic	p-value
Autocorrelation	0.932	0.45	1.022	0.40
Parameter constancy	1.710	0.17	1.5671	0.20
Remaining non-linearity: Transition variable:				
Y	0.358	0.78	0.447	0.72
ADS-index, 3-month average	2.201	0.09	1.957	0.12
Industrial production, quarterly growth rate	0.975	0.41	1.018	0.39
M2 growth rate	0.548	0.65	0.628	0.60
Federal fund rate	1.146	0.33	1.305	0.27
$\triangle$(10-year treasury bond yield)	1.827	0.14	1.622	0.19
Yield spread (AAA corporate vs. 10-year treasury yield)	0.584	0.63	1.412	0.24
Term spread (10-year vs. 3-month treasury bond yield)	0.403	0.75	1.153	0.33
Rate of return on S&P 500	3.171	0.03	3.153	0.03
Consumer price inflation	1.479	0.22	1.492	0.22

Notes: Test statistics and corresponding p-values for fourth order Autocorrelation, Parameter constancy and Remaining Non-Linearity (Eitrheim and Teräsvirta, 1996) Transition variables refer either to end-of-quarter levels or growth rates / differences in the last month of the quarter, unless denoted otherwise.