# Estimates of the Steady State Growth Rates for Ireland

Casadio, Paolo and Paradiso, Antonio and Rao, B. Bhaskara

22 April 2011

# Estimates of the Steady State Growth Rates for Ireland

Paolo Casadio
paolo.casadio@yahoo.it
Intesa Sanpaolo Bank Group, Risk Management, Rome (Italy)

Antonio Paradiso
anto_paradiso@hotmail.com
Department of Economics, University of Rome La Sapienza, Rome (Italy)

B. Bhaskara Rao
raob123@bigpond.com
School of Economics and Finance, University of Western Sydney, Sydney (Australia)

Abstract

This paper estimates the steady state growth rate for Ireland with an extended version of the Solow (1956) growth model. We show that the education and trade openness have played an important role to improve the long-run growth rate. Policies to further improve the long-run growth rate are suggested.

Keywords: SSGR, Economic Growth, Trade Openness, Education, Ireland.

JEL Classification: C22, O52, O40.

## 1. Introduction

The Irish economy during the second part of 1990s was one of the most successful in the OECD. In the period 1995-2005 GDP grew at an average rate of 7.7%, well far ahead of many other industrial economies. Ireland relies heavily on trade and foreign investment, with the combined value of import and exports equivalent to above 140% of GDP. Using the index of trade openness (import + exports as percentage of GDP) Ireland in 2010 was one of the most open economies in the OECD area after Belgium, Hungary, Luxembourg, and Slovak Republic. The time-series data reveals a growing trend toward openness after the 1970s and since its membership in the European Economic Community. Another characteristic of Ireland is the steep rise in the educational attainment since the 1990s, which brought Ireland in line with countries with high historical level of education. According to the Barro and Lee (2010) dataset, Ireland is a leading country in education, in line with schooling levels in Sweden, Germany and the Republic of Korea.

These considerations suggest that these variables have played an important role in explaining the long-term growth rate of Ireland and its higher performance. We investigate this aspect with an extended version of Solow (1956) growth model incorporating education and trade openness as key variables of its long-run growth. Our approach broadly follows the specification and methodology in Rao (2010) and Paradiso and Rao (2011).

## 2. Specification of the model

The starting point is the well-known Cobb-Douglas production function:

$$Y_t = A_t K^{\alpha} L_t^{1-\alpha} \tag{1}$$

where Y is for output, K for stock of capital and A is the stock of knowledge. Following Rao (2010) and Paradiso and Rao (2011) we assume the following general evolution for *A*, as a simple function of two variables S and Z for schooling and/or openness.

$$A_t = A_0 e^{\gamma_i Z_{it} \cdot T + \phi_1 S_t + \phi_2 S_t^2} \tag{2}$$

Transforming (1) into its intensive form, substituting (2) for $A_t$, and taking its logs gives:

$$\ln y_t = \ln A_0 + \gamma_i Z_{it} \cdot T + \phi_1 S_t + \phi_2 S_t^2 + \alpha \ln k_t \tag{3}$$

where $y = (Y / L)$ and $k = (K / L)$. In the steady state, when $\Delta \ln k \to 0$, the Steady State Growth Rate (SSGR) is equal to the growth rate of the stock of knowledge ($\Delta \ln A$). There are two ways to measure the SSGR. One restrictive method considers all the changes in the variables as zero, whereas in a less restrictive version only $\Delta \ln k = 0$. The first one may be treated as the very long run growth rate and the second as the growth rate in the intermediate period. We call the first as SSGR1 and the second as SSGR2 and are as follows.

$$SSGR1 = \gamma_i Z_{it} \tag{4}$$

$$SSGR2 = \gamma_i \Delta Z_{it} \times T + \gamma_i Z_{it} + \phi_1 \Delta S_t + 2\phi_2 \Delta S_t \times S_t \tag{5}$$

We make use of both of these measures of SSGR and try to understand the potential factors influencing the SSGRs and how policy can improve them.

**3. Empirical Estimates**

Yearly data from 1960 to 2010 are used to estimate the long-run relationship in equation (3). Our selected growth-enhancing variables are: trade openness (TRADE) and human capital index (HKI).[1] Definitions of variables and sources of data are in the appendix. Three estimations techniques are used, which are the Fully Modified OLS (FMOLS), Canonical Cointegrating Regression (CCR) and Dynamic OLS (DOLS). These estimators deal with the problem of second-order asymptotic bias arising from serial correlation and endogeneity and they are asymptotically equivalent and efficient. Two dummies are added in the estimations and are discussed in appendix.

Our estimation strategy is the following. We estimate the long-run relationship with these three methods and if all the results are similar and plausible, we verify the existence of the cointegrating relationship with the Engle-Granger (EG) residual test. If the tests confirm the existence of the long-run relationship, we construct an Error Correction Model (ECM). Then we study the factor loading and test for correct specifications i.e., we test for the normality, absence of autocorrelation, and absence of heteroskedasticity in the residuals.

In the first model, shown in Table 1, equation (3) is estimated with $Z_{1t} = HKI_t$ = $S_t$. The results are good except for the coefficient of capital $\alpha$, which is not statistically significant in the estimates

[1] We have also included investment ratio (IRAT) as an additional determinant of SSGR but its coefficient was insignificant. This is to be expected because in the Solow model IRAT affects only the level of the steady state income and not the SSGR. Results with IRAT are not reported to conserve space.

with FMOLS and DOLS, and very low compared to its stylized value of one third in the growth accounting exercises. Therefore, we estimated model 2 in which HKI is replaced with *ln*TRADE as the determinant of SSGR. That is, $Z_{1t} = \ln TRADE_t$, and $S_t = HKI_t$. The results are in Table 2 and it can be seen that all estimates are significant but the coefficient for capital ( $\alpha$ ) is above unity in DOLS estimation and this is implausible.

**Table 1: Results of Model 1**

$$\ln y_t = Intercept + \gamma_1 HKI_t \cdot T + \phi_1 HKI_t + \phi_2 HKI_t^2 + \alpha \ln k_t$$

| | FMOLS | DOLS | CCR |
|---|---|---|---|
| *Intercept* | -10.246***<br>(0.672) | -11.048***<br>(0.849) | -9.965***<br>(0.756) |
| $\ln k$ | 0.065<br>(0.045) | 0.039<br>(0.104) | 0.093*<br>(0.053) |
| $HKI \cdot T$ | 0.003***<br>(0.000) | 0.003***<br>(0.000) | 0.003***<br>(0.000) |
| *HKI* | 1.365***<br>(0.112) | 1.499***<br>(0.300) | 1.330***<br>(0.116) |
| $HKI^2$ | -0.074***<br>(0.006) | -0.081***<br>(0.016) | -0.073***<br>(0.005) |
| $\lambda$ | -0.806***<br>(0.135) | | |
| EG residual test | -5.027** | | |
| LM(1) test (p-value) | 0.775 | | |
| LM(2) test (p-value) | 0.800 | | |
| LM(4) test (p-value) | 0.661 | | |
| JB test (p-value) | 0.478 | | |
| BPG test (p-value) | 0.636 | | |

Notes: Regressand = $\ln Y / L$ . Time period 1960 to 2010. Standard errors are reported in ( ) brackets. *, **, *** denotes significance at 10%, 5%, and 1%, respectively. FMOLS = Fully Modified Ordinary Least Squares; DOLS = Dynamic Ordinary Least Squares; CCR = Canonical Cointegrating Relationship. EG = Engle-Granger *t*-test for cointegration. $\lambda$ , factor loading in the ECM. BPG, Breusch-Pagan-Godfrey heteroskedasticiy test; JB, Jarque-Bera normality test, LM, Bresuch-Godfrey serial correlation LM test. FMOLS uses Newey-West automatic bandwidth selection in computing the long-run variance matrix. In the DOLS leads and lags are selected according to SIC criteria. The standard errors for the DOLS estimation are calculated using the Newey-West correction. A dummy for 2008 financial crisis is added also in ECM formulation.

| **Table 2: Results of Model 2** | | | |
|---|---|---|---|
| $\ln y_t = Intercept + \gamma_1 \ln TRADE_t \cdot T + \phi_1 HKI_t + \phi_2 HKI_t^2 + \alpha \ln k_t$ | | | |
| | FMOLS | DOLS | CCR |
| *Intercept* | -7.210***<br>(0.931) | -4.768***<br>(0.849) | -7.404***<br>(0.983) |
| $\ln k$ | 0.763***<br>(0.068) | 1.058***<br>(0.102) | 0.746***<br>(0.078) |
| $\ln TRADE \cdot T$ | 0.017***<br>(0.001) | 0.018***<br>(0.001) | 0.017***<br>(0.001) |
| *HKI* | 1.149***<br>(0.157) | 0.991***<br>(0.113) | 1.175***<br>(0.154) |
| $HKI^2$ | -0.059***<br>(0.008) | -0.061***<br>(0.006) | -0.060***<br>(0.007) |
| $\lambda$ | -0.434***<br>(0.096) | | |
| EG residual test | -5.163** | | |
| LM(1) test (p-value) | 0.378 | | |
| LM(2) test (p-value) | 0.544 | | |
| LM(4) test (p-value) | 0.788 | | |
| JB test (p-value) | 0.762 | | |
| BPG test (p-value) | 0.314 | | |
| See note for Table 1. | | | |

In model 3 we also include both HKI and lnTRADE as determinants of SSGR. In this case $S_t = HKI_t, Z_{1t} = HKI_t, Z_{2t} = \ln TRADE_t$ such that equation (3) becomes:

$$\ln y_t = \ln A_0 + \gamma_1 HKI_t \cdot T + \gamma_2 \ln TRADE_t + \phi_1 HKI_t + \phi_2 HKI_t^2 + \alpha \ln k_t \qquad (6)$$

Note that there is an additional term for education ( $HKI \times T$ ) compared to the specification in model 2. This implies that some of the non-linear effects of HKI are offset by the underlying trend in HKI, which may be due to improvements in the quality of education over time. The results are in Table 3 and are impressive. All the coefficients are statistically significant, the coefficient of capital is close to one third, the residual tests (EG) confirm the existence of a long-run relationship and ECM is satisfactory. This is our preferred estimate and we use this model to compute the dynamics

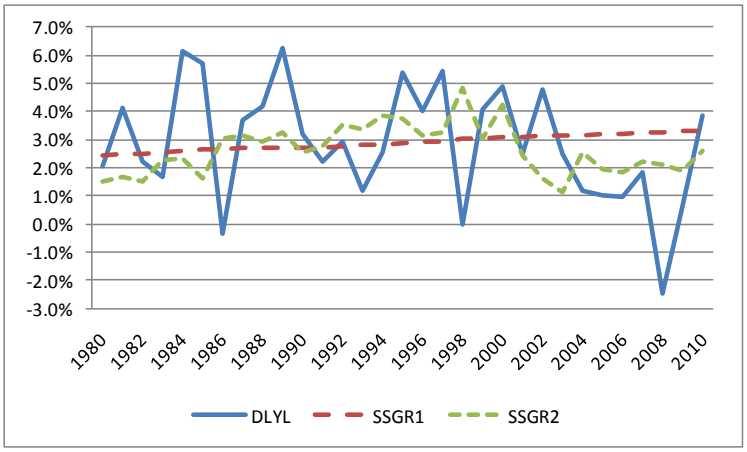
of SSGR in equations (4) and (5). The plots of the two measures of SSGR for model 3 and the actual growth of output per worker (*DLYL*) are in Figure 1. SSGR1 is the very long run growth rate when changes in all variables are zero. It is smooth and shows a mild upward trend. SSGR2 is the steady state growth rate in the intermediate period, when net investment is zero i.e., $\Delta \ln k = 0,$ but changes in HKI and TRADE are not zero. SSGR2 is close to *DLYL*. More importantly, SSGR2 shows the 2000s weakness of the economy although the actual growth rate is still over 2% in 2000-2002 period.

**Table 3: Results of Model 3**

$$\ln y_t = Intercept + \gamma_1 HKI_t \cdot T + \gamma_2 \ln TRADE_t + \phi_1 HKI_t + \phi_2 HKI_t^2 + \alpha \ln k_t$$

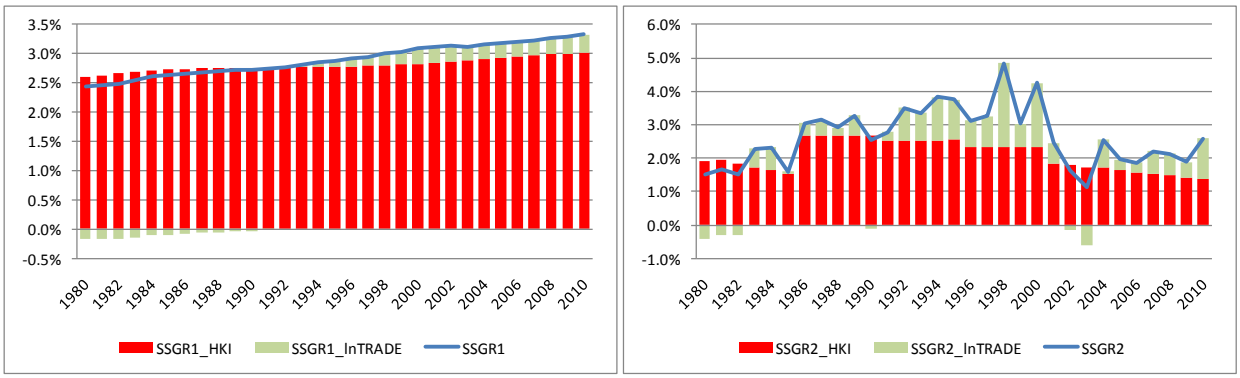
| | FMOLS | DOLS | CCR |
|---|---|---|---|
| *Intercept* | -9.663***<br>(0.503) | -9.489***<br>(2.149) | -9.292***<br>(0.581) |
| $\ln k$ | 0.242***<br>(0.052) | 0.363***<br>(0.175) | 0.291***<br>(0.067) |
| $HKI \cdot T$ | 0.003***<br>(0.000) | 0.003***<br>(0.000) | 0.002***<br>(0.000) |
| $\ln TRADE \cdot T$ | 0.004***<br>(0.001) | 0.007***<br>(0.002) | 0.005***<br>(0.001) |
| *HKI* | 1.355***<br>(0.079) | 1.464***<br>(0.291) | 1.316***<br>(0.082) |
| $HKI^2$ | -0.073***<br>(0.004) | -0.079***<br>(0.014) | -0.071***<br>(0.004) |
| $\lambda$ | -0.932***<br>(0.131) | | |
| EG residual test | -5.812*** | | |
| LM(1) test (p-value) | 0.460 | | |
| LM(2) test (p-value) | 0.555 | | |
| LM(4) test (p-value) | 0.662 | | |
| JB test (p-value) | 0.288 | | |
| BPG test (p-value) | 0.716 | | |
| See note for Table 1. | | | |

**Figure 1: SSGR from Model 3**




The contribution of *HKI* and *TRADE* to SSGR1 and SSGR2 are in Figure 2. In both cases the steady state growth rate is mainly explained by *HKI*. It is interesting to note that the contribution of TRADE is slightly negative in the early 1980s but later became positive with the exception for 2003 for SSGR2. This may be due to the high protectionist tariffs, which were phased out and replaced after the mid 1980s with the more moderate tariffs of the European Union (EU). Ireland benefited both from free trade within the EU and from lower EU tariffs. In SSGR2 the contribution of *HKI* to the long run growth rate has been decreasing since 1990, offset by an increasing contribution of *TRADE*. Only after 2000 both contributions become smaller, reducing the rate of growth of the economy in the following years.

**Figure 2: Contributions of SSGR1 and SSGR2 to Growth**



Although SSGR is still very high in Ireland, our estimates suggest that further improvements can be made by improving education through job training schemes or by increasing the openness of the markets. An increase of 1% in both HKI and TRADE will produce an increase of 0.6% in SSGR1. SSGR2 is less sensitive to HKI and TRADE. An increase of TRADE by 5% will increase SSGR2 by only 0.1%.

## 4. Conclusions

This paper used an extended Solow (1956) growth model to estimate the long run and medium term growth rates for Ireland. It is found that these two growth rates depend on the openness of the economy and education and the latter seems to be the dominant determinant of the two growth rates. The long run growth rate has shown a mild upward trend increasing from about 2.25% in the early 1980 to 3% by 2010. The medium term growth rate showed some fluctuations and is below the long term growth rate since the early 2000s. If the Irish economy is made more open and the education levels are increased, both the long run and medium term growth rates will be closer and are likely to exceed 3% or even 3.5 %.

**Data Appendix**

$Y$ = Real GDP; $L$ = Employment (Total economy); *HKI* = Human Capital Index measured as average years of education; *IRAT* = Ratio of investment to GDP; *TRADE* = Ratio of imports plus exports to GDP.

All data, excluding *HKI*, are taken and constructed from AMECO-EUROSTAT database. *HKI* is taken from Barro-Lee (2010) database.

**Dummy variables**

Dummy for the second half of 1960s and dummy for the 2008-2009 financial crisis. The dummy for 1960s corresponds to a dummy for the years 1965-1966. These years corresponds to a brake in the high growth rates of 1960s (in the period 1965-1966 the average growth rate was 1.4% against an average of 4% of previous 5 years).

**References**

Barro, R., and Lee, J. (2010) A new data set of education attainment in the world, 1950-2010, NBER Working Paper No. 15902. Available at http://www.barrolee.com/data/dataexp.htm (Accessed 29 April 2011).

Paradiso, A., and Rao, B. B. (2011) How to offset the negative trend growth rate in the Italian economy?, *Applied Economics Letters*, iFirst, DOI: 10.1080/13504851.2010.54306.

Rao, B. B. (2010) Time-series econometrics of growth-models: A guide for applied economists, *Applied Economics*, **42**, 73-86.

Solow, R. M. (1956) A contribution to the theory of economic growth, *Quarterly Journal of Economics*, **70**, 65-94.