Exhaustible natural resources, normal prices and intertemporal equilibrium

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Abstract

This paper proposes an extension of the classical theory of normal prices to an n-commodity economy with exhaustible natural resources. The central idea is developed by two analytical steps. Firstly, it is assumed that a given *flow* of an exhaustible resource in short supply is combined with the co-existence of two methods of production using that resource. Sraffa's equations are reinterpreted by adopting the concept of *effectual supply* of natural resources and avoiding the assumption of perfect foresight. Secondly, in force of the Hotelling rule, some limitations are imposed to the dynamics of normal prices and, by implication, to technical and structural change. A comparison, between such approach and the notion of intertemporal equilibrium with natural resources, introduces the central argument. The final part of the paper presents a critical assessment of recent works in this area. The conclusions are focused on methodological issues.

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EXHAUSTIBLE NATURAL RESOURCES, NORMAL PRICES AND INTERTEMPORAL EQUILIBRIUM[1]

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Introduction

In this paper we propose a revised theory of normal prices for an economy with exhaustible natural resources. Our initial work in this field of analysis (Parrinello 1982-1983) adopted the Sraffian assumption that two methods (processes, techniques), using a natural resource, co-exist in each period. In a recent article (Parrinello 2001) we presented a simple model of an oil-corn economy, in which that assumption is maintained and in addition an exogenous technical change is assumed to satisfy the Hotelling rule, i.e. the rate of appreciation of the exhaustible resource is equal to the rate of interest. This paper clarifies the flow dimension of the resource in short supply and relaxes the scope of the Hotelling rule, but at the same time contends that it is not sufficient to assume that the change in the normal prices, which follows the change in techniques, is just "small" or "once in a while" (*una tantum*) in order to validate the application of the method of long period equilibrium to a non stationary economy. Instead we shall

[1] I would like to thank Christian Bidard, Guido Erreygers and two anonymous referees for their comments, under the usual exemption from responsibility.
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distinguish the *small deviations between the rate of change* in the normal price of an exhaustible resource and the given rate of interest from the "small" *rate of change* in the price of all reproducible commodities.

In Part I we resume the distinction between intertemporal long period equilibrium and classical equilibrium[2]. We compare two notions of equilibrium, which have in common the assumption of a given distributive variable and the presence of an exhaustible natural resource.

Part II contains the central idea of our argument. We first claim that, in the presence of an exhaustible resource in short supply, Sraffa's given quantities must include a *flow* of the resource, instead of its total stock left in the ground. Perfect foresight is not assumed. This amounts to a revised interpretation of Sraffa's price equations in the case of land, at the level of the *general theory* of normal prices. Furthermore, we suggest a model in which certain corridors are superimposed to the change in the normal prices determined by the general theory.

Part III presents a critical assessment of the previous work of the author (Parrinello, 2001) and of the intertemporal approach shared to a different extent by Schefold (1989, 2001), Kurz and Salvadori (1995, 1997, 2001), Foley-Michl (1999) and Bidard-Erreygers (2001, 2002).

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[2] Our distinction between the two notions of equilibrium owes to Schefold (1997, chapter 18). The notion of classical equilibrium coincides with that of long period positions adopted by Garegnani (1976).

I. Two different notions of long period equilibrium with natural resources

I.1. Intertemporal equilibrium

Assume an economy with *n* produced commodities, labour and an exhaustible natural resource in short supply. *Let us call "R" this resource*, without specifying for the moment if it is land or an exhaustible resource. Assume that n-1 *single product industries* produce commodities 1,2,...,n-1 by n-1 distinct methods using commodities 1,...,n-1, n and labour. Let one unit of commodity *i* be produced in period t by the n-vector $\mathbf{a}_{i,t}$ of commodity inputs and $l_{i,t}$ units of labour, $i = 1,...,n-1$. Instead assume that in each period *t* industry *n* produces commodity *n* by two methods α_t, β_t using R beside the other inputs and let g_t^j denote the quantity of R used to produce one unit of commodity *n* in period *t* by method j, $j = \alpha_t, \beta_t$. We take *one unit of labour in period t = 0 as the standard of value*. Let $P_i(t)$, $\rho(t)$, $W(t)$ indicate discounted prices: the price of commodity *i*, the price of R and the wage rate, respectively. The chosen numeraire prescribes $W(0) = 1$. The price equations of competitive intertemporal equilibrium in terms of *discounted* prices:

$$\left.\begin{aligned}
&\mathbf{A}_{n-1,t}\mathbf{P}(t) + \mathbf{l}_{n-1,t}W(t) = \mathbf{P}_{n-1}(t+1) \\
&\mathbf{a}_{n.t}^{\alpha_t}\mathbf{P}(t) + l_{n,t}^{\alpha_t}W(t) + g_t^{\alpha_t}\rho(t) = P_n(t+1) \\
&\mathbf{a}_{n.t}^{\beta_t}\mathbf{P}(t) + l_{n,t}^{\beta_t}W(t) + g_t^{\beta_t}\rho(t) = P_n(t+1)
\end{aligned}\right\} \qquad [1]$$

$$\mathbf{P}(t) \geq \mathbf{0}, \quad \rho(t) \geq 0 \quad W(t) \geq 0 \quad \forall t,$$

where **P**(t) is the column n-vector $\left[P_1(t),....,P_{n-1}(t),P_n(t)\right]$ and $\mathbf{P}_{n-1}(t)$ is the column n-1-vector $\left[P_1(t),....,P_{n-1}(t)\right]$. The prices in terms of current labour:

$$p_{i,t} = \frac{P_i(t)}{W(t)} \qquad i = 1,.......,\text{n}; \qquad \rho_t = \frac{\rho(t)}{W(t)} \ .$$

Let $r_{\text{w,t}}$ denote the *own rate of interest on labour* in period t:

$$(1+r_{w,t}) \equiv \frac{W(t)}{W(t+1)} .$$

Assume that $r_{\text{w,t}} = r_{\text{w}}$ is given and constant. The corresponding equations in terms of *current* prices:

$$\left.\begin{array}{l} (1+r_w)\left(\mathbf{A}_{n-1,t}\mathbf{p}_t + \mathbf{l}_{n-1,t}\right) = \mathbf{p}_{n-1,t+1} \\ (1+r_w)\left(\mathbf{a}^{\alpha_t}_{n.t}\mathbf{p}_t + l^{\alpha_t}_{n,t} + g^{\alpha_t}_t\rho_t\right) = p_{n,t+1} \\ (1+r_w)\left(\mathbf{a}^{\beta_t}_{n.t}\mathbf{p}_t + l^{\beta_t}_{n,t} + g^{\beta_t}_t\rho_t\right) = p_{n,t+1} \end{array}\right\} \qquad [2]$$

$$\mathbf{p}_t \geq \mathbf{0}, \quad \rho_{\text{t}} \geq 0 \qquad \forall t .$$

Equations [2] can be re-written:

$$\begin{aligned} 1+r_w &= \frac{p_{i,t+1}}{\mathbf{a}_{i,t}\mathbf{p}_t + l_{i,t}} \equiv \frac{p_{i,t+1}}{p_{i,t}} . \frac{p_{i,t}}{\mathbf{a}_{i,t}\mathbf{p}_t + l_{i,t}} \equiv \\ &\equiv (1+\pi_{\text{i,t}})(1+\text{r}_{\text{i,t}}), \qquad \text{i} = 1,2,...,\text{n - 1} \end{aligned}$$

$$[3]$$

$$\begin{aligned} 1+r_w &= \frac{p_{n,t+1}}{\mathbf{a}^j_{n,t}\mathbf{p}_t + l^j_{n,t} + g^j_t\rho_t} \equiv \frac{p_{n,t+1}}{p_{n,t}} \frac{p_{n,t}}{\mathbf{a}^j_{n,t}\mathbf{p}_t + l^j_{n,t} + g^j_t\rho_t} \equiv \\ &\equiv (1+\pi_{\text{n,t}})(1+\text{r}_{\text{n,t}}), \qquad j = \alpha_t, \beta_t . \end{aligned}$$

where $\pi_{i,t} \equiv \dfrac{p_{i,t+1} - p_{i,t}}{p_{i,t}}$ is the rate of change in the price of commodity i , $i = 1,...,n-1,n$. Instead $\text{r}_{\text{i,t}}$ can be interpreted as a *classical rate of profit* on the supply price of capital good i, $i = 1,...,n-1,n$. Notice that the prices used to calculate $r_{i,t}$ are associated with the *same* time period. [3]

I.2. Intertemporal long period equilibrium

An intertemporal long period equilibrium can be conceived as an asymptotic state of the economy in which, flukes apart, technical coefficients are constant, relative prices and relative quantities are constant and the own rates of interest are equalized. In terms of current prices:

$$\mathbf{p}_{t+1} = \mathbf{p}_t = \mathbf{p}, \qquad [4a]$$

$$\rho_{t+1} = \rho_t = \rho. \qquad [4b]$$

By dropping the time index on the technical coefficients, the equations of intertemporal long period equilibrium:

$$\left.\begin{array}{l} (1+r_w)(\mathbf{A}_{n-1}\mathbf{p} + \mathbf{l}_{n-1}) = \mathbf{p}_{n-1} \\ (1+r_w)(\mathbf{a}_n^{\alpha}\mathbf{p} + l_n^{\alpha} + g^{\alpha}\rho) = p_n \\ (1+r_w)(\mathbf{a}_n^{\beta}\mathbf{p} + l_n^{\beta} + g^{\beta}\rho) = p_n \end{array}\right\} \qquad [5]$$

$$\mathbf{p} \geq \mathbf{0}, \qquad \rho \geq 0.$$

Therefore the prices $\mathbf{p}$, ρ that solve equations [5] reflect, on the side of values**,** a strictly stationary economy. Such asymptotic state of the economy does not exist if R is an exhaustible resource in short supply. Long period intertemporal equilibrium cannot exist in this important case. Its existence is admitted only if R is Ricardian land.

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[3] This interpretation was first adopted by Schefold 1997.

I.3. Classical equilibrium

Within the classical approach, the technical coefficients are allowed to change "slowly" or "once in a while" over time. In particular their change should be slow enough to justify the assumption that investment decisions in a certain period depend on the normal prices ruling in that period so that the determination of normal prices of each period is self-contained within the same period. The equations of normal prices with a uniform and constant rate of profit *r*:

$$\left.\begin{aligned} (1+r)\left(\mathbf{A}_{n-1,t}\mathbf{p}_t + \mathbf{l}_{n-1,t}\right) &= \mathbf{p}_{n-1,t} \\ (1+r)\left(\mathbf{a}^{\alpha_t}_{n,t}\mathbf{p}_t + l^{\alpha_t}_{n,t} + g^{\alpha_t}_t \rho_t\right) &= p_{n,t} \\ (1+r)\left(\mathbf{a}^{\beta_t}_{n.t}\mathbf{p}_t + l^{\beta_t}_{n,t} + g^{\beta_t}_t \rho_t\right) &= p_{n,t} \end{aligned}\right\} \qquad [6]$$

$$\mathbf{p}_t \geq \mathbf{0}, \quad \rho_{\mathrm{t}} \geq 0 \qquad \forall t \ .$$

From a purely formal point of view equations [6] can be derived from [3] by setting r $= r_{\mathrm{w}}$ and assuming that the change in the prices of all *n* produced commodities is negligible: $\pi_{i,t} = 0$, $i = 1,...,n-1,n$.

If R is an exhaustible resource, equations [6] cannot apply to an indefinite number of periods and to a uniform quality of R (unless in each t the inputs of R used by methods α_t, β_t are infinitely small), simply because the resource will run down. As a consequence "$\forall t$" in [6] must be qualified by the proviso that the quality of R occasionally changes, still maintaining the assumption that two methods α_t, β_t using a uniform quality of R co-exist in each period. Such a state of the economy does not rule out the simultaneous use of a third method γ_t, implementing a certain amount of a superior quality of R (in particular its residual amount in the period of complete exhaustion and substitution), which receives a differential rent.

Despite the fact that a classical equilibrium is not necessarily associated with a stationary economy, it has to be explained whether it can cope with the existence of an exhaustible R. Before dealing with this problem in part II, in the next two sections we shall clarify some features of the choice of the numeraire and the meaning of the Hotelling rule.

I.4 The choice of the numeraire

An important difference between the notions of intertemporal equilibrium and classical equilibrium concerns the choice of the numeraire.

Equations [5], in which the prices are independent of time, are supposed to be the stationary result of a process of adjustment of an economy under complete forward markets and/or perfect foresight. Therefore the system of production represented by the coefficients of [5] is the result of an intertemporal choice among alternative techniques and the choice of a *unique numeraire*, (in our case w(0) = 1), is necessary in order to compare the values of dated commodities. Instead the equations [6] do not derive from such assumptions. Any sequence of [6], t = 1,2,...,T, includes T independent systems of equations. As a consequence, the determination of the prices $\mathbf{p}_t$, ρ_t, is *self-contained* in each period, given the techniques ruling in that period and given the uniform rate of profit *r*. The choice of *distinct numeraires* for each period (for example w(t) = 1, t = 0,1,....) is a meaningful feature of system [6], because the techniques used in each period are supposed to be profit-*maximizing at the normal prices of* *that period*. Instead the choice of distinct numeraires can be misleading in other models of prices with exhaustible resources, which apparently possess classical features but where the determination of prices is not self-contained in each period.

Sraffa, in his theory of normal prices, suggests that the uniform rate of profit is determined by a given rate of interest on money. If it is assumed that a forward market exists for money (although it is not assumed that other forward markets exist), the own rate of interest on money, instead of r_w, the own rate of interest on labour, will govern the uniform rate of profit. This assumption introduces an important link among the normal prices of different periods, although these prices are not determined simultaneously.

I.5 The Hotelling rule

The *Hotelling rule* is the extension of the condition of a uniform rate of return to the conservation process of an exhaustible natural resource. It can be formulated in different equivalent forms; some versions are more transparent than others. We defined $\rho(t)$ the discounted price of R and $\rho_t = \dfrac{\rho(t)}{W(t)}$ the current price of R. Then $r_{\rho,t} \equiv \dfrac{\rho(t)}{\rho(t+1)} - 1$ is the own rate of interest on R in period t, t+1.

The Hotelling rule in terms of the discounted price of R:

$$\rho(t+1) = \rho(t)\,. \qquad [7a]$$

In terms of the own rate of interest on R:

$$r_{\rho,t} = 0\,. \qquad [7b]$$

In terms of the current price of R and of the own rate of interest on the current standard (labour):

$$\rho_{t+1} = (1 + r_w)\rho_t . \qquad [7c]$$

Equation [7a] states that the discounted price of R must be constant. Equation [7c] is the usual formulation of the Hotelling rule: the current price of R must appreciate at a rate equal to the prevailing interest rate. Notice that the rate of interest r_w, to which the rate of appreciation of R must be equal, is the own rate of interest on the good or basket of goods in terms of which the current value ρ_t is defined (in our case it is the own rate of interest on labour in period t, a constant). Equation [7b] states that the own rate of interest on R must be zero in each period. This condition on a pure number per unit of time is, in a sense, more encompassing. If [7b] is satisfied, then [7c] is satisfied for whatever choice of the standard in terms of which the price ρ_t is defined.[4]

The Hotelling rule, $\rho_{t+1} = (1 + r_w)\rho_t$, violates condition [4b] which characterizes a long period intertemporal equilibrium. We shall explore whether

[4] If we assume that the money rate of interest, instead of the own rate of interest on labour, is *given and constant* over time, the Hotelling rule, under the assumption that money at date *t* is chosen as the standard of value in period t takes the form $\rho_{m,t+1} = (1 + r_m)\rho_{m,t}$, where $r_m = \dfrac{P_m(t)}{P_m(t+1)} - 1$ is the given own rate of interest on money (the money rate of interest), $P_m(t)$ is the nominal price of money at time t, and $\rho_{m,t} = \dfrac{\rho_m(t)}{P_m(t)}$ is the current money price of R. If a perfect financial market exists, the own rate of interest on money held as a store of value should be equal to zero.

this rule can be applied without contradiction for the determination of a classical equilibrium in the presence of an exhaustible R.

II Normal prices and exhaustible R.

We intend to argue that the classical theory of normal prices, represented by the same mathematical form [6], can deal with an exhaustible R, but it must be reinterpreted. Furthermore we contend that the theory of normal prices must face the Hotelling rule not at a level of a general theory, but by the formulation of models, which add specific constraints to the changes in the normal prices, determined by that theory. We shall develop our argument in two analytical steps (sections II.1 and II.3), which will lead us to a model of normal prices. These steps will be interposed by a discussion of the scope of the theory of normal prices (section II.2).

I1.1 The effectual supply of R

For the sake of our argument, let us first consider some hypothetical cases in which a given constant flow of R is available in each period t =1,...,T. A fixed flow of R can be attributed to various circumstances. It may happen that it is fixed because of natural constraints and no technique is known to change its rate. In addition, suppose that the flow of R cannot be stored. This case can be easily absorbed as case of intensive land cultivation and the last two equations of system [6] can be interpreted correspondingly. Like in the case of land of uniform quality, two methods α_t, β_t using a uniform R co-exist over a sequence of periods. The difference with respect to land is that *a flow of R in short supply* plays the role of a given total amount of land

and that the flow will not last forever. The co-existence of α_t, β_t reveals *the scarcity of R* and the price equations [6] determine the price of n produced commodities and ρ_t *as if the latter were an intensive rent*.

Assume now that the whole amount of R becomes physically available for use without costs of extraction and it is divided among many competitive proprietors. I contend that also in this case system [6] represents the equations of normal prices, provided that ρ_t is interpreted as a royalty on a unit flow of R during period *t*. Assuming that two methods using R coexist in each period, before the period of complete exhaustion, is as much as plausible as in the previous case. In fact, the economic scarcity of the resource is perceived before its complete exhaustion and its owners can be assumed to distribute over different periods their endowments between *a flow in effectual supply* and the residual stock left in the ground. I call "effectual" this supply because it is symmetrical to the notion of "effectual" demand in Adam Smith. It is a quantity supplied at the current long period prices, instead of a supply *function*, and it depends on long-term expectations. Moreover, if we would assume that the total stock of R in the ground is known, the effectual (flow) supply of R would be the difference between the existing stock and *the stock of R in demand*. Only the *physical* availability of R in excess to its current use makes this assumption seemingly different from the assumption of a given amount of land in the theory of rent: indeed, an effectual supply of land also exists but it simply coincides with its existing total amount.

II.2 Should we narrow the scope of the theory of normal prices?

Sraffa presents a theory of *rent,* but he does not mention the *price* of land (or of mines) in his book. In this regard a problem of interpretation of the method of long period equilibrium already exists if R is assumed to be land. Suppose that ρ_t^* is the intensive rent which is determined as a solution to the price equations equations [6]. Can $\frac{\rho_t^*}{r}$, the present value of the rent as a fixed annuity *ad infinitum*[5], be *interpreted as the normal price of land in period t*? If this interpretation should be accepted, the normal price of a not yet cultivated land would be zero, because its rent is zero, and then, in the period in which it starts to be used and receives a rent, it suddenly changes to a positive level with a huge increase in the total value of that land. A frequent change in the methods of production on different qualities of R, associated with a gradual increase in demand, would lead to frequent and large wealth effects. This result is not plausible because it implies a high degree of instability associated with fluctuations of normal prices.

A similar consequence can be obtained in the case in which R is exhaustible and the total deposit of R is perfectly known. However, the problem becomes subtler. In the absence of extraction costs, a unit of the flow of R on the ground and used in a period, and a unit of R left in the ground in the same period are homogenous (a part from the flow and the stock dimensions) and then both units must receive the same price on a perfect market. Can ρ_t^* be interpreted as a normal price, i.e. a centre of

[5] This interpretation was considered in passing in Parrinello (1983 p. 190) and in Schefold (1989 p.199).

gravity of market royalties in period t? If the answer to this question is affirmative, whereas we reject the previous idea that $\frac{\rho_t^*}{r}$ is the normal price of land because of the instability argument, we should conclude that the same theory of normal prices is not a theory of all prices. The price of R (land or oil in the ground) must be left to a less general theory, compared with the theory of normal prices of produced commodities and rents. I am not saying that the same theory determines two different normal prices for the same commodity, but that equations [6] determine only one normal price ρ_t^* - the rent on land or a royalty on a flow in short supply of an exhaustible resource - and offers, as such, no ready made theory of a centre of gravity for the market prices of the total amount of R in the ground at time t. This argument would recommend a separate theory for the normal price of the total stock of R in the ground, although it must be admitted that this price has to be related to the values of ρ_t^* over a sequence of periods.

Instead of pursuing this negative approach, that confines the scope of the traditional theory of normal prices, we shall explore a different route to maintain a unified theory of normal prices for the n commodities and R.

II.3 A model of normal prices

We present a model to clarify the need for a formal qualification of the "small changes over time" argument in order to apply the classical method of long period equilibrium to a specific non stationary economy, under the progressive exhaustion of natural resources. We suppose that the path of normal prices, determined by the equations [6], satisfy certain constraints that impose different corridors to the path of the prices, $\mathbf{p}_t$, of the n commodities and to the path of the price ρ_t of R. The latter kind of corridor reflects the Hotelling rule, whereas the former imposes a limit to the amplitude of the capital gains and losses in the n industries.

Let ε_t^R, $(i = 1,..,n-1,n)$, be a given positive number, which is small compared to r and let $\pi_t^R \equiv \dfrac{\rho_{t+1} - \rho_t}{\rho_t}$ indicate the rate of appreciation of R. Then $r - \varepsilon_t^R \leq \pi_t^R \leq r + \varepsilon_t^R$ represents a corridor for the path of the price ρ_t, which solves equations [6]. Similarly, let $\pi_{i,t} \equiv \dfrac{p_{i,t+1} - p_{i,t}}{p_{i,t}}$ be the rate of change in the normal price of commodity i and let $\varepsilon_{i,t}$ be a positive small number. Then $-\varepsilon_{i,t} \leq \pi_{i,t} \leq \varepsilon_{i,t}$ sets a corridor for the path of price $p_{i,t}$ with $i = 1,..,n-1,n$. The assumption underlying these constraints is that 1) if $r - \varepsilon_t^R \leq \pi_t^R \leq r + \varepsilon_t^R$ is satisfied, capitalists do not perceive a sufficient incentive to cause net movements of capital between the store of value (asset), represented by the amount of R left in the ground, and the n industries; 2) if $-\varepsilon_{i,t} \leq \pi_{i,t} \leq \varepsilon_{i,t}$ is satisfied for each i, capital would not tend to move among the n industries. We obtain the following model of normal prices:

$$\left.\begin{array}{l}(1+r)\left(\mathbf{A}_{n-1,t}\mathbf{p}_t + \mathbf{l}_{n-1,t}\right) = \mathbf{p}_{n-1,t} \\ (1+r)\left(\mathbf{a}_{n,t}^{\alpha_t}\mathbf{p}_t + l_{n,t}^{\alpha_t} + g_t^{\alpha_t}\rho_t\right) = p_{n,t} \\ (1+r)\left(\mathbf{a}_{n.t}^{\beta_t}\mathbf{p}_t + l_{n,t}^{\beta_t} + g_t^{\beta_t}\rho_t\right) = p_{n,t}\end{array}\right\} \qquad [8]$$

$$-\varepsilon_{i,t} \le \pi_{i,t} \le \varepsilon_{i,t} \qquad i = 1,..,n \qquad [9]$$

$$\text{where } \pi_{i,t} \equiv \frac{p_{i,t+1} - p_{i,t}}{p_{i,t}} \qquad i = 1,..,n$$

$$r - \varepsilon_t^R \le \pi_t^R \le r + \varepsilon_t^R \qquad [10]$$

$$\text{where } \pi_t^R \equiv \frac{\rho_{t+1} - \rho_t}{\rho_t}$$

$$\mathbf{p}_t \ge \mathbf{0}, \quad \rho_t \ge 0 .$$

Relations [8] - [10] stand as a specification of Sraffa's price equations for the case of an exhaustible natural resource. We submit two comments on the general features of model [8] - [10] and four comments on specific features of the same model.

II.4 General features

1) The dated technical coefficients of equations [8] represent for each *t* the given quantities of a system of production in long period equilibrium[6]. The outputs of the $n+1$ processes are given quantities in effectual demand; the inputs are given quantities in derived demand. The total amount of R, used by processes α_t, β_t , is equal to the given effectual flow supply of the natural resource in period t. The effectual demand and the effectual supply are supposed to embed the relevant expectations about the future prices.

[6] Such a system of production is not a snapshot of the system of production observed in period *t* ; but it finds an empirical correlate in the observed systems.

2) Sraffa's given quantities represent the observed technology; i.e. the system of production in use. Even in the presence of a stationary economy such a system would be given, but not arbitrarily given like a neoclassical blueprint of techniques. The system of production is assumed to be "square" (i.e. the number of methods of production is equal to the number of commodities) and viable in each period. In the presence of a non stationary economy with an exhaustible natural resource, we need also the constraints [9] and [10] in order to guarantee a sufficiently uniform rate of return on capital goods and on the natural asset R. [9] and [10] *imply certain constraints on the given sequence of systems of production*. Such constraints can be derived by substituting the prices $\mathbf{p}_t, \rho_t$, that solve equations [8], into inequalities [9] and [10]. The normal prices are still determined by the techniques in use in each period, but the given dynamics of the system of production is not arbitrarily given. We are *assuming that* the same competitive mechanism, underlying the "squareness" of techniques, brings about a technical change which is *biased* in the sense illustrated above.

II.5 Specific features

1. The technical coefficients of each equation [8], corresponding to a certain process, are calculated by dividing the absolute quantities of inputs and of output by the absolute output of the process itself. This calculation is independent from any assumption on returns to scale.

2. Constraint [10] imposes a definite corridor in which the gradual changes in the price of R must be contained. Therefore the control of the money rate of interest is bounded not only by the technical admissible range of the rate of profit *r*, but also by the corridors [9], [10].
3. Many constraints of form [10] should hold in the general case of heterogeneous exhaustible resources. More importantly, a price $\rho_{k,t}$ for a quality *k* of R, which is not yet used in period t, may be positive and increasing with t; but it does not appear in the model. The price $\rho_{k,t}$ cannot be determined as a normal price because it does not depend only by the observable system of production in use and the given r.
4. It is not assumed that the total amount of R in the ground is perfectly known. As a consequence, although the solution to the price equations determines the normal price $\rho_t = \rho_t^*$, the normal value of the total deposit of R may not be determined by a simple multiplication of ρ_t^* times a known quantity.

III. A critical appraisal of alternative models

III.1 Technical change constrained by the Hotelling rule.

Suppose that the rate of profit, r_t, can gradually change with t. If we assume that both the rate of appreciation $r_{R,t}$ and the rate of profit r_t are given, subject to [10], the system of price equations becomes overdetermined, unless the technology is supposed to play the role of an adjustment factor. A similar result obtains if we assume as given $r_{R,t}$ and the real wage rate in terms of a given basket of goods, instead of the rate of profit. We indeed assume that the economy possesses a certain flexibility on the side of the technology, especially if we abandon the sharp neoclassical distinction between induced inventions and substitution of production processes within a given blueprint of techniques.

Parrinello (2001) assumes that for any observation of the economy over a sequence of periods t = 1,2,....,T, the technical coefficients and r_t change in such a way that a path of positive prices exist as a solution to the price equations and satisfies the Hotelling rule $r_{R,t} = r_t$. Following the same procedure and assuming that equation $r_{R,t} = r_t$ displaces the corridor constraint [10], the model [8]- [10] for any finite number of periods, t = 1,2,....,T, can be transformed into the equation **Ax** = **b**, which represents in compact form a number of equations higher than the number of unknowns. The elements of matrix **A** and of vector **b** are expressions of the technical coefficients and of $r_{R,t}$, r_t; instead **x** is a vector of prices $\mathbf{p}_t, \rho_t$ t = 1,2,....,T . Formally **Ax** = **b**. has a solution if and only if the rank of **A** is equal to the rank of the augmented matrix (**A**, **b**). The technology (and the rate of profit or both) must change so that the rank condition can be satisfied.

However, perfect foresight with *dynamic consistency* of choices requires that a similar rank condition be satisfied for *any* sequence of periods under observation. Bidard and Erreygers (2002) have recently put forward this argument as a criticism of inconsistency addressed to the oil-corn model (Parrinello 2001). The authors claim that two flaws undermine the model: the lack of realism of the rank condition hypothesis and inconsistency. Let me spend a few words about the inconsistency charge, before admitting that my assumption is not suitable to deal with normal prices in the presence of an exhaustible R, although it does not bring about logical inconsistency.

The rank condition does not imply inconsistency if, following Debreu, we assume one shot decisions taken at the start of the initial period t = 1 of observation and then not revised anymore in periods t = 1, 2,...,T. Instead it implies dynamic (not logical) inconsistency in the sense of Strotz (1955-56), because the fulfilment of the rank condition over a sequence t=1,...,T of observed states of the economy, under the assumption of one shot decision, is only a necessary but not sufficient condition of dynamic consistency. In fact - we accept Bidard and Erreygers objection - it does not guarantee the fulfilment of the rank condition in other subsequent or partially overlapping sequences. The relevant issue is whether the rank condition can represent an ideal case to be used as a useful benchmark or not; the problem of logical over-determinacy is not at issue.[7]

Both the assumption of the rank condition adopted in Parrinello (2001), although not because of logical inconsistency, and the assumption of perfect foresight combined with that of a backstop technique, which we are going to assess in the next section, should not be used to extend the theory

[7] Incidentally, Bidard and Erreygers (2002) address to Parrinello (2001) a criticism that is similar to that put forward by the latter (Parrinello, unpublished paper, 1967) to Pasinetti's multi-sectoral model (Pasinetti 1965, 1993). In fact, this model imposes a rank condition that guarantees the persistence of full employment in the presence of an exogenous technical progress. I still maintain my previous critical assessment of the model from a theoretical point of view: normal prices *are not* associated with persistent full employment. However, the criticism is not methodologically acceptable: we cannot charge of inconsistency a model because its system of equations becomes overdetermined in the absence of some meaningful constraint which its builder explicitly imposes on the parameters.

of normal prices. Trouble shared is not trouble halved. Therefore in this paper we propose a different approach to face the same problem

III. 2 A criticism of the backstop models

The special issue of Metroeconomica (vol.52, 2001) collects some models of economies with exhaustible resources, which contrast with the approach adopted by the present author. These models, called here *backstop models* for reasons which will be explained shortly, follow a mixed classical-neoclassical approach. In fact, the authors assume that a distributive variable (either the rate of profit or the real wage rate) is given and they adopt a partial intertemporal equilibrium analysis associated with perfect foresight. I will focus on Bidard & Erreygers (2001) backstop model, because it is more visible the distance between their approach and mine.[8]

An essential feature of Sraffa' theory of prices derives from the fact that the determination of normal prices is self-contained in each period. In fact, the determinant factors are the system of production in use and a given value of a distributive variable in the same period. *Long-term expectations, associated with imperfect foresight, are embedded in the given absolute quantities observed in each period.* The reason why the backstop model excludes that R receives a price ρ_t as an intensive rent, which is determined by the co-existence of two methods of production using R, is revealing.

Bidard & Erreygers (2001) assume that the period T of exhaustion of R is known and given; then that a substitute technique (a backstop technique

[8] Schefold (1989, 2001), Kurz & Salvadori (1995, 1997, 2001) and Foley-Michl (1999) models share some features of such a mixed approach.

using no exhaustible resource) starts to be implemented in that period. Hence, the price ρ_t paid for R in period *t*, $t < T$, is determined by backward induction from the prior determination of ρ_T in the period in which two methods co-exist and given the Hotelling rule. Next, in each period t, $t < T$, the producers are assumed to choose a single process using R under the assumption that ρ_t has been already determined. In the end, a single process is chosen on the basis of the usual criterion of choice of techniques: the maximization of *current profits calculated at the current normal prices*.

Firstly, we notice that in each period *t*, $t < T$, the choice of a single process is a result of two assumptions: 1) the producers take the price ρ_t as if it would be fixed on a world market*, instead of facing a flow of R in short supply* and 2), "the demand for guano [the exhaustible resource exemplified in the model] in each period is more or less given" (Bidard-Erreygers 2001, p.248). We notice that, if the producers are capable of perfect foresight, they should not be so myopic: in each period they should choose the techniques (and the critical switch period T as well) that maximize *the present value of a stream of future profits.*

Secondly, the backstop models assume that the backstop technique does not use another "inferior" kind of R. Otherwise a second period of exhaustion T' with a second backstop technique would be necessary for the determination of prices and then a third period T'' ... *ad infinitum*. As a consequence, also the assumption of perfect foresight adopted in the backstop models ought to satisfy a test of dynamic consistency in the presence of repeated choices, if the authors aim to represent an economic process in real time (a feature of the classical approach).

Therefore it is only a special combination of assumptions that rules out the co-existence of two processes using R in each period in the backstop models. If this co-existence is admitted, we are back to the Sraffian notion of scarcity in the case of intensive land cultivation and we must face the problems illustrated in this paper.

Conclusion

Our revision of the theory of normal prices rests on two analytical steps. Firstly, we assume a given *flow* of R in short supply in each period. Secondly, we impose some limitations to the dynamics of normal prices and, therefore, *an implicit constraint on technical change*. We conclude this paper with two methodological remarks.

1. Formally equations [8] are Sraffa's price equations with land of uniform quality. Our reinterpretation of the given quantity of R, as a given flow in effectual supply, is a revision of Sraffa's analysis and extends the *theory* of normal prices to include the prices of exhaustible natural resources. *The revised theory rests on the notions of effectual demand for products and effectual supply of natural resources*. Still, the theory remains fundamentally different from a theory of general equilibrium, because it does not presuppose the existence of demand and supply *functions* for the determination of normal prices. In particular, it does not rest on demand and supply functions of labour and capital; therefore the equilibrium values,

which satisfy equations [8], are compatible with persistent involuntary unemployment.

2. Adding the corridor constraints [9], [10] to the price equations [8] transforms the general theory of normal prices *into a model of prices*. This second analytical step trespasses the field of observable determinants of prices, which characterizes Sraffa's methodology. In fact, the coefficients $\varepsilon_{i,t}, \varepsilon_t^R$ in inequalities [9] [10] are not observable. They stand to set the limits within which the solution to the price equations [8] can be accepted as a plausible explanation of market prices in terms of average values. It should be stressed the fact that $\varepsilon_{i,t}, \varepsilon_t^R$ are left to a different theory, suitable to the specific circumstances in which the theory of normal prices is applied. It might happen that the theoretical prices determined as a solution to [8] predict quite well the market prices in the given circumstances, but their change over time violates [9], [10]. In this case we must admit that such theoretical prices are not *normal*, because they do not allow us to *understand* the market prices, although the latter are *explained* by the former, from the point of view of methodological instrumentalism.

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