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# **Practical approach to estimating cost of capital**

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# Practical approach to estimating cost of capital

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## ABSTRACT

The recent as well as precedent market crashes has increased a number of already existing biases when estimating a forward looking cost of capital for company's stakeholders. With cost of capital being essential in corporate valuation and decision making the following paper analyzes the research carried out by numerous academics up to date and provides a comprehensive overview on the appropriate choices of inputs and methods for estimating cost of capital. The paper draws the necessary attention to the times of crises. An additional study shows how different preferences can result in variation in cost of equity capital and terminal value of a company.

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# 1 INTRODUCTION

The following paper provides a comprehensive overview of the major debates over estimating the inputs for cost of capital used in the enterprise valuation and corporate valuation. Hundreds of research academics and practitioners over the recent decades have argued concerning methodology used for estimating every single variable in cost of capital. These debates not only made the choices for estimating cost of capital a complex subject, but combined with constantly evolving market characteristics they have made cost of capital and valuation an art.

So far most of practitioners have chosen methods that have been framed by major services, simplicity, data availability, common sense and overall consensus. This paper tries to factor in all of that, in addition, providing a critical opinion at some instances and incorporating major considerations relevant to using cost of capital in the distressed environment.

The objective of the paper is to overview and to compare different methods proposed by various authors and services for estimating elements of cost of capital. The paper aims to analyze as well as conform with or refute widely accepted choices in cost of capital calculations. In the concluding case study we question how the choices of methods lead to varying estimates of cost of equity capital and how that impacts the value of a company.

The paper is structured as follows. Section 2 provides brief outlook on the importance of cost of capital in the corporate decisions making and its impact on the value of the company. Section 3 continues by discussing the main inputs used in cost of equity capital calculations with a particular focus on the Capital Asset Pricing Model. We discuss the implications of using different proxies for risk-free rate, market portfolio, beta estimation methodologies and adjustments as well as separate considerations to be noted when using cost of capital in the distressed environment. These mainly include market distortions, treatment of tax shields and positive debt betas. Section 4 extends discussion on cost of equity by looking at the recent research carried out on equity risk premiums. Here we also analyze the failure of the CAPM and provide with our point of view on applying additional risk premiums suggested by various authors. Section 5 provides with considerations related to estimating after-tax cost of debt, with a focus on synthetic ratings and marginal tax rates. Section 6 puts together the elements of cost of capital, presents a classical framework for Weighted Average of Cost of Capital and provides with the main caveats surrounding capital weights. We end our discussion with a case study in Section 7 where

we illustrate a variety of choices discussed in the previous sections and their impact on cost of equity estimation for Kraft Foods.

Finally, we conclude the paper with the summary and our recommendations on the preferred inputs and methods for estimating cost of capital as well as areas of further research.

## 2 COST OF CAPITAL OVERVIEW

Cost of capital is central to corporate decision making and valuation. Reducing cost of financing is a fundamental determinant of the value creation as much as freeing cash-flows and sustaining healthy growth rates. It enters one of the most vital performance measurements employed by management and analysts, an economic value added. Perhaps one of the simplest ways to illustrate that is the notion of economic profit which measures value created by a company over time:

$$\text{Economic Profit} = \text{Invested Capital} \times (\text{ROIC}^1 - \text{WACC})$$

As you can see positive or negative economic profit that adds or destroys value in any single period is essentially defined by the spread between return on invested capital and cost of capital<sup>2</sup>. In any single period that cost of capital exceeds return on invested capital, negative economic profit will reduce the value of company.

Cost of capital also enters key value driver formulas or for those who are familiar more, a well-established cash-flow perpetuity formula used to present simple pro-forma constant growth valuation framework:

$$\text{Value} = \frac{\text{FCF}}{\text{WACC} - g} \quad (2.1)$$

where:

*FCF* – free cash flows to equity

*WACC* – weighted average of cost of capital

*g* – growth rate

---

<sup>1</sup> ROIC (Return on Invested Capital) – return earned on every invested dollar in the company, on all capital or on marginal capital.

<sup>2</sup> When we talk about cost of capital we usually refer to weighted average cost of capital, an average required return of all investors participating in the financing of a company. Terms such as cost of capital, WACC, required rate of return or discount rate are used interchangeably.

Key value driver formulas are more than often used in DCF Enterprise Valuation method by practitioners to come up with the terminate value of a company. This term accounts for a chunky part of the total present value of a company and is extremely sensitive to the estimations of cost of capital.

As a result, understanding WACC is central for both corporate decision making as well as valuation.

As a final remark, we would like to point out that cost of capital should only take into consideration returns required by investors on securities held in the company. To avoid double counting, it should not gauge operating liabilities such as cost of financing that are already captured in the free cash flows. Finally, it should be based on the market value of the assets this way allowing it to reflect expectations of the investors.

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### 3 COST OF EQUITY: CAPITAL ASSET PRICING MODEL

Before we begin the discussion on more debatable issues on estimation of cost of capital, we would first like to give a comprehensive overview on asset pricing model that has gained widest acceptance over the years by both academics and practitioners – the capital asset pricing model.

Basic CAPM gives a well founded linear relationship between risk and return that is easy to grasp as well as adjust to company-specific risk characteristics. As most of the readers are well aware of, simple CAPM provides an expected return defined as a risk premium over a riskless rate. The risk premium is adjusted by the factor  $\beta$  – henceforth “beta” - that captures subject firm’s relative operating and financial leverage as well as business cyclicity.

$$R_e = \beta(MRP) + R_f \quad (3.1)$$

where:

$R_e$  – expected return on any asset

$\beta$  – beta

$MRP$  – market risk premium

$R_f$  – riskless rate of return

Given the relationship, CAPM shows compensation on an asset required by investors if they added the asset to a well diversified portfolio, hence CAPM measures market risk only.

In the current chapter we will provide a comprehensive overview and the related debates on the elements of the capital asset pricing model used to estimate cost of equity.

#### 3.1 RISK-FREE RATE

To estimate risk-free rate practitioners typically use government bonds that best match forecast period of the cash flows. Government bonds are assumed to be risk-free under assumption that government will always be able to meet the payments – at least in nominal terms - by printing more currency. As a result, returns on such security has no covariance with the market as represented by zero beta in the CAPM.



Ideally, each forecasted cashflow should be discounted using yield of a bond whose duration coincides with the duration of the forecasted cashflows<sup>3</sup>. Term structure of interest rates is rarely flat and using single bond yield to discount cashflows in different periods might slightly understate or overstate required return on equity. However, as suggested by Damodaran, if long-term rates are higher no more than 2-3%, there should be insignificant effect on the present value of a company. Should that not hold, one should work his way through to match different yields with the respective cash flows for more precision.

Most valuation practitioners approximate that and for the sake of simplicity employ long-term government bonds, such as US-treasury bonds or German bunds, whose characteristics allow for low credit risk, high liquidity as well as help them match the maturity and the forecast period best. They use zero coupon bonds or STRIPS<sup>4</sup> since they do not make any interim payments, therefore imply no reinvestment risk and prevent from shorter effective time to maturity. Having said all that, a widely accepted choice of preference is a 10-year zero. Longer maturity than that, such as one of 30 year bonds might match the cash flows better (especially bearing in mind that most investments in companies exhibit reinvestment risk that is also common in long term government bonds). However, it would likely require us to account for illiquidity on such bonds, while shorter maturity, such as one of t-bills would understate risk-free rate given the fact that the yield curve is mostly upward sloping as suggested before. Furthermore, short-term rates are more volatile and susceptible to central bank actions and therefore might warp valuation. In corporate investment analysis, on the other hand, shorter maturities can be used should the duration of project's cash flows be less than 10 years.

Choosing risk free rate also has important implications for risk premiums. Should an analyst apply historical market risk premiums they should be calculated over the same maturities as the risk-free proxy used in deriving required return on equity. Thus, if we use yields on 10 year bonds as a risk-free proxy, we should calculate historical stock market premium in excess of 10 year bond as well. As word caution, Morningstar/Ibbotson Associates which provide data service for equity risk premia estimate long-term premium over 20-year bonds. If an analyst decides to use these data service market premiums, she should use the 20-year government bond yields as a risk-free rate accordingly.

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<sup>3</sup>  $D = \frac{\sum_{j=1}^T \frac{T_j C_j}{(1+i)^{T_j}}}{\sum_{j=1}^T \frac{C_j}{(1+i)^{T_j}}}$ , where D – duration, T-number of cashflow periods, C – cashflow and i – required yield

<sup>4</sup> "Separate Trading of Registered Interest and Principal of Securities (STRIPS) program that allows US investors to trade coupon payments and the principal of treasuries separately.

Finally, risk free proxy should be consistent with the cash flows. Should the growth include inflationary effect, i.e. cash flows stated in nominal terms, the risk free rate should also be stated in nominal terms<sup>5</sup>. Moreover, the bond that is used in deducing risk free rate should be denominated in the same currency as the cash flows. Damodaran (2007) proposes that given the purchasing power parity, one could theoretically value a foreign company in local currency using local bond yields. However, it is not really used in practice. Perhaps a reason for that is that foreign exchange rates and interest rates as well as inflationary processes rarely follow parities as they are described in economic textbooks<sup>6</sup> and given the noise that exists in foreign exchange markets that would distort valuation.

All in all, we believe that the choice for risk-free rate should firstly be consistent with the market risk premium used. However, we believe that 10-year government bonds fit the notion of risk-free rate best for the valuation purposes.

### 3.2 BETA ESTIMATION

Most conventional way of finding CAPM's beta is through the means of statistical regression using historical data. Beta of an asset measures how much it comoves with the market. In essence, it shows how many times a stock amplifies the movements of the market. Beta is obtained by regressing total returns of a company on total returns of a market portfolio. Though it practically makes no difference, few data service providers such as Morningstar regress excess returns instead of total returns.

$$R = \alpha + \beta \times R_m + \varepsilon \quad (3.2)$$

where:

$R$  – stock return

$\beta$  – beta of stock

$R_m$  – market return

$\alpha$  – excess return

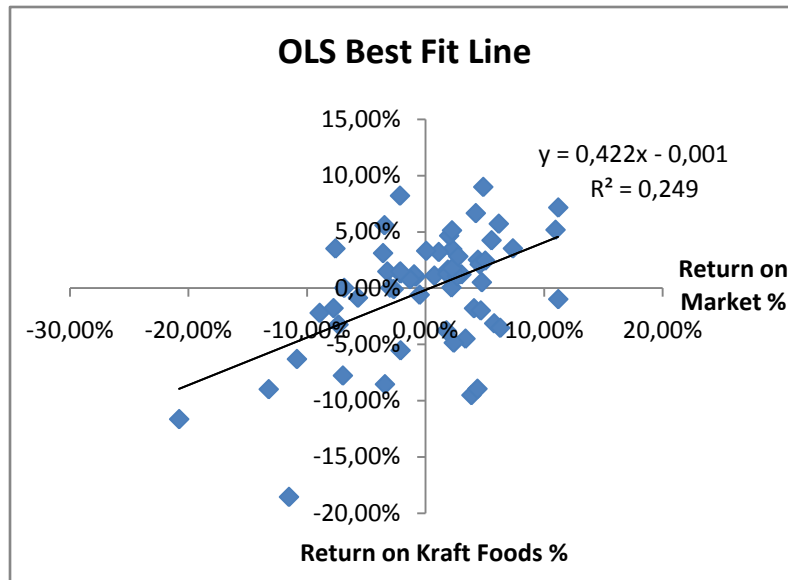
$\varepsilon$  – error term

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<sup>5</sup> Exceptional case is when we are valuing companies that operate in countries of unpredictable hyperinflation. We might then want to project cash-flows and estimate cost of capital and related inputs in real terms.

<sup>6</sup> See Sarno, L. (2005). Viewpoint: Towards a Solution to the Puzzles in Exchange Rate Economics: Where do we Stand?, Canadian Journal of Economics 38(3): 673 – 708

**FIGURE 3.1: REGRESSION BETA**



Since market risk premium and risk free rate are constant throughout the market in any single period, beta is the sole driver of stock returns in the original CAPM framework. As a result, it is crucial to understand the caveats of the main methods and considerations used to estimate beta in practice.

Main issues related to estimating betas turn around sample size of returns, frequency of returns, choice of proxy for market portfolio, methods used to make forward looking adjustments, i.e. smoothing techniques of betas.

Sample size of returns has no accepted standards since large data service providers use different measurement periods. Bloomberg, for instance, calculates raw beta based on weekly return sample of 2 years. Other data providers such as Standard & Poor's or Morningstar use 5 year period based on monthly returns (see Appendix for more details). Longer period allows for smaller standard error in the regression, but it exposes the results to biases resulting from changing risk characteristics of a company. Using longer period might undermine recent changes in business mix or capital structure, common for emerging firms or periods of corporate restructuring.

Studies of the CAPM in the 70s and 80s showed that sample periods of monthly data of around 5 years give the most unbiased results.<sup>7</sup>

There are exceptions to the rules as one might expect. One is that of extreme market re-pricing.

A study carried out by McKinsey<sup>8</sup> shows that dot-com bubble generated artificially low betas for more mature industries. Before the surge in tech stock prices, mature industries represented significant portion of the U.S. market cap. After the tech surge, instead U.S. market became heavily weighted on tech stocks that drove returns for U.S. indices (see next page). These returns were uncorrelated with mature industries and biased the betas of these industries downwards. The results pertained several years after the burst as beta estimates are based on a look-back period. Indeed, the same has happened during the recent 2007-2009 financial crisis when highly leveraged companies and financials were driving the U.S. market down. Companies that had low leverage saw their betas diminish with respect to S&P500 as their returns were less correlated with the market than before. It might be useful to check the structural changes in the market in advance.

To inspect beta estimates for deviations, one can build a time series analysis by rolling a 5 year beta sample period of monthly data as done in classical studies of Fama&French<sup>9</sup> or simply by plotting index returns against company returns. This consequently should allow us to examine the changes in beta as well as the underlying corporate or market factors that would otherwise be covered by outdated returns that can potentially dominate the sample as portrayed by the low R<sup>2</sup> and low historical beta of Kraft Foods in 2006.

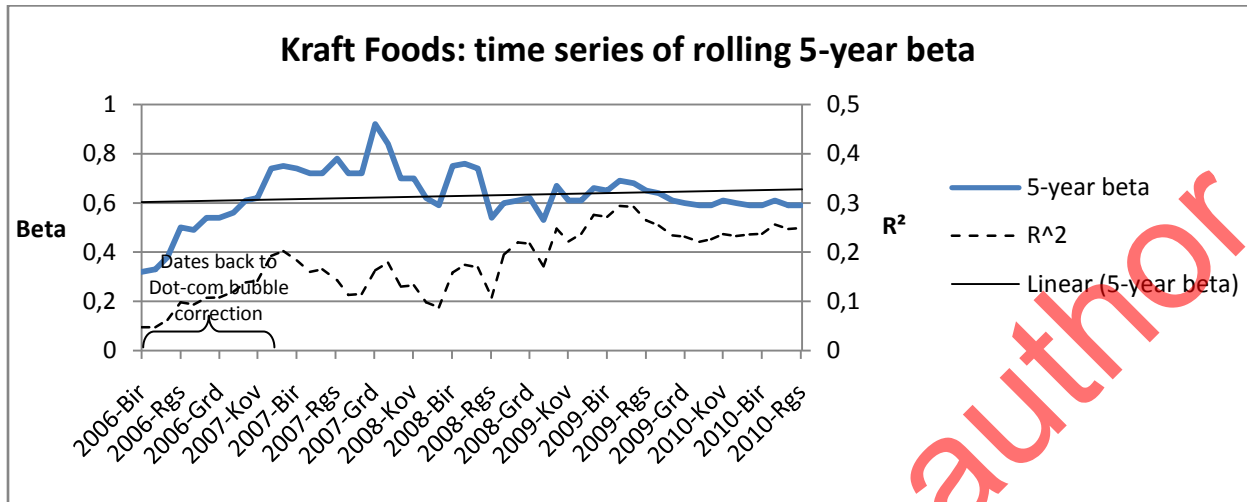
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<sup>7</sup> Black, Scholes and Jensen (1972), "The Capital Asset Pricing Model: Some Empirical Tests"  
Alexander and Chervany (1980), "On the Estimation and Stability of Beta"

<sup>8</sup> Annema and Goedhart, "Better Betas", McKinsey on Finance (Winter 2003)

<sup>9</sup> See Fama and MacBeth (1973), "Risk return and equilibrium empirical tests", Journal of Political Economy 81 (1973), pp. 607–636; Fama and French (1992), "The cross section of expected stock returns"

**FIGURE 3.2: TIME SERIES OF ROLLING BETA**



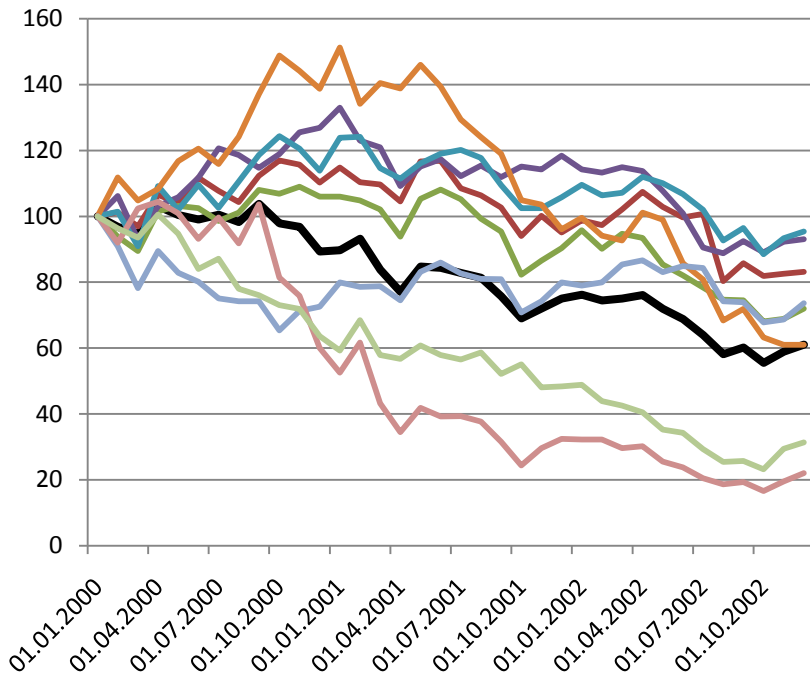
Source: Author's calculation based on Datastream.

Should one find that turbulence in the markets over the 5 year sample make beta estimates misleading for the future, it would be reasonable to reduce the sample size to 12 months while increasing the frequency of returns. If there is a consensus that the last 12 months represent a forward looking equilibrium prices better than 5 years, beta estimates based and 1 year observation will give more precise results.

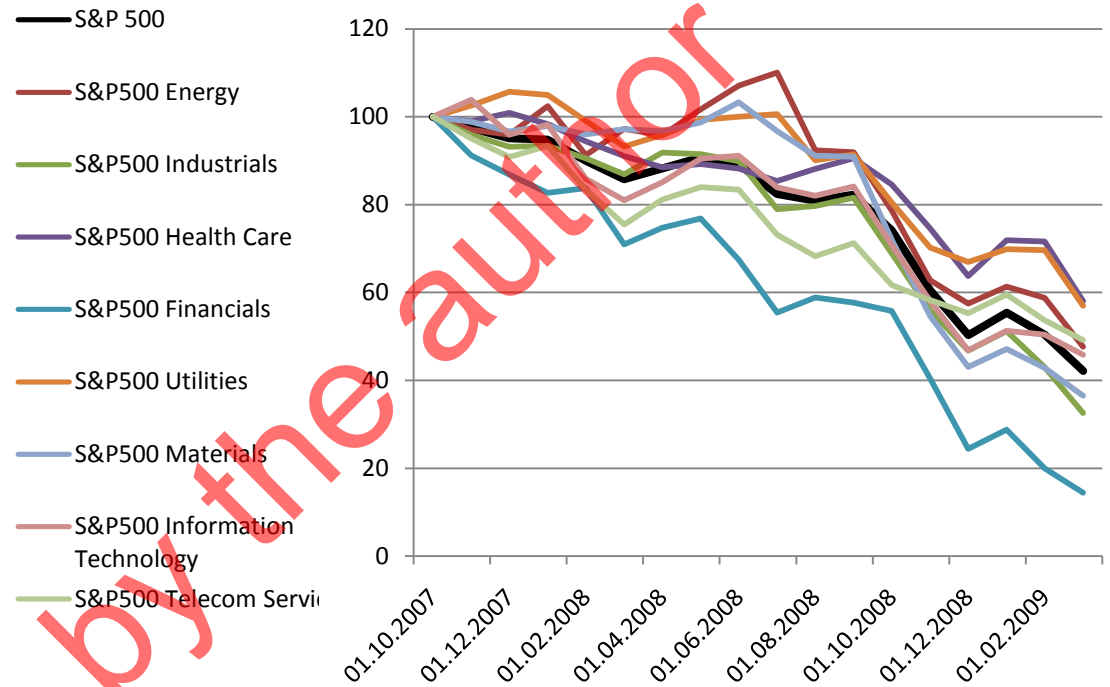
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FIGURE 3.3: PERFORMANCE AND WEIGHTS OF S&P FAMILY INDICES AND S&P 500 COMPOSITE DURING THE CRISES

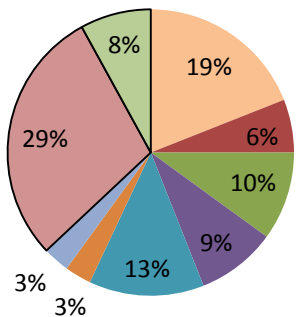
### Dot-com bubble



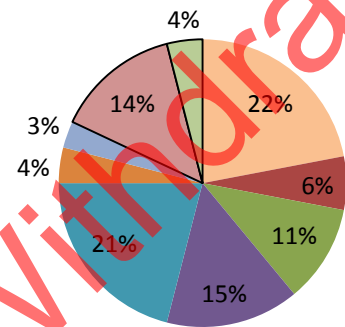
### Financial Crisis



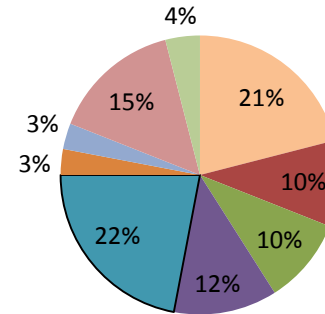
### Jan 1999



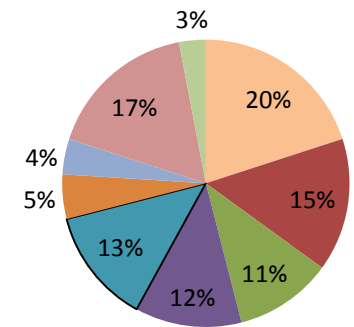
### Jan 2002



### Jan 2006



### Aug 2007



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Source: Author's calculations based on Datastream

Frequency of returns is another issue that relates to estimation of beta. Though high frequency data provide better estimates of covariance, however it might just as well overlook the illiquidity of certain stocks. Should a stock be less liquid and not trade on particular days, the correlation with the market that is actively trading on those days will be low resulting a downward bias of beta estimate.<sup>10</sup>

To go around illiquidity problem for stocks that trade infrequently even on monthly basis, one could employ a lagged-beta model (see further: Sum Beta) where stock excess returns are regressed on both excess market returns of the same as well as of the prior period.

There has also been research carried out to check viability of betas measured on ultra high frequency basis, such as 5 min trading intervals. The authors base the research on the fact that bid/ask spreads and non-synchronous trading (also referred to as the market microstructure in the literature) produce autocorrelations which effectively deny efficient market hypothesis. They claim that if you measure returns on high-frequency basis (say on 1 minute intervals), these autocorrelations are reduced because asset prices do not make it on time to reach equilibriums<sup>11</sup>. However, we speculate that by measuring returns on such high frequency, one takes into account only the most active market participants, such as day traders and active proprietary traders; the returns induced by other (less responsive) types of market participants are excluded. We assume that it is the actual delay of other participants to incorporate market news into prices that most likely cause autocorrelations. The question then arises if beta estimated on minute-by-minute prices that are most likely moved only by active prop traders is a good estimate of systematic risk. We believe it is not.

The market and the stock have to be extremely liquid to estimate covariance between the market proxy and underlying asset using, say, 1 day time frame of 5 min returns. Furthermore, beta estimated on such high frequency relies on timely and precise execution of trades which are subject to human error and errors in trading systems.

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<sup>10</sup> Trading delays and price adjustment delays have been first discussed to bias beta estimates by Fisher (1966), and later grounded by Scholes and Williams (1977), Dimson (1979) and other authors in analytical and empirical research to provide evidence that betas are downward (upward) biased for stocks which trade less (more) frequently than the index used in the regression.

<sup>11</sup> Tsay R.S., Yeh J.H., "Non-synchronous Trading and High-Frequency Beta" (2003); Andersen T.G., Bollerslev T., Diebold F.X., Wu J., "Realized Beta: Persistence and Predictability" (2004)

We believe that, next to liquidity, yet another problem with high-frequency betas is that it already enters the field of behavioral finance which is still a very young science. Until we can model human behavior and major services start providing high-frequency betas we believe it is not really a viable methodology. As a result, we believe that practical application of ultra high-frequency betas is yet decades away. High frequency betas might be useful in high frequency transactions and trading strategies, but not in corporate finance.

To sum up, there are no established standards on the choices for inputs of calculating regression beta. However, we recommend adjusting the sample returns for non-recurring events and using the sample size which, most importantly, would exclude periods of market turbulence that distort a forward looking relationship between the stock and the market. Other choices, such as frequency, should be arbitrary based on market and stock liquidity.

### 3.2.1 MARKET PORTFOLIO

Though CAPM suggests that beta should be regressed based on market portfolio that includes all possible assets including human capital, it is rather impossible to construct such market portfolio. A common solution is to use a broad and well-diversified market index, such as S&P 500 or MSCI World Index. Analysts in the U.S. often rely on S&P 500 as a proxy for market portfolio, whereas finance professionals outside U.S. use MSCI World or regional MSCI indices.

MSCI Barra Research claims that with diminishing barriers and market imperfections already in 80s a lot of investors started using *All Country World Index* as a proxy for market portfolio. *AC World Index* currently includes 23 developed markets that comprise *The World Index* as well as additional 22 emerging markets. Furthermore, with the appetite for international small caps MSCI *All Country World Investable Market Index* that covers small caps was introduced. MSCI Barra Claims that *AC World Index IMI* covers 99% of global investment universe and is widely used now by investors as a proxy of market portfolio.



**TABLE 3.1: CORRELATION BETWEEN MARKET PROXY INDICES**

	THE WORLD INDEX Standard (Large+Mid Cap)	THE WORLD INDEX IMI (Large+Mid+ Small Cap)	AC WORLD INDEX Standard (Large+Mid Cap)	AC WORLD INDEX IMI (Large+Mid+ Small Cap)	S&P 500
THE WORLD INDEX Standard (Large+Mid Cap)	1				
THE WORLD INDEX IMI (Large+Mid+Small Cap)	0.999	1			
AC WORLD INDEX Standard (Large+Mid Cap)	0.997	0.996	1		
AC WORLD INDEX IMI (Large+Mid+Small Cap)	0.996	0.997	0.999	1	
S&P 500	0.876	0.882	0.850	0.855	1

Source: Author's calculations based on MSCI Barra and Standard & Poor's data  
All correlations based on USD returns

The performance of these indices is publicly available on the websites of data providers tracking them. As you can see from the correlation matrix above (Table 3.1) there is no radical difference in the choice for proxy of market portfolio. The correlation, especially in between global MSCI family indices is approximately 100%. Lower correlation of S&P 500 with MSCI World family indices might arise because of lower diversification of S&P 500 due to regional as well as large cap stock concentration comprising the index.

**TABLE 3.2: MARKET PROXY CHARACTERISTICS**

	Number of assets	Weight of top 10 companies (%)	Asset selection Risk (% Std Dev)	Asset Selection Risk Contribution (% Total Risk)
S&P 500	500	19.74	-	-
MSCI World	1,655	9.5	1.52	0.25
MSCI ACWI	2,397	8.4	1.37	0.2
MSCI ACWI IMI	8,531	7.4	1.21	0.15

Source: MSCI Barra. Data as of June 2009

The index volatility and performance results indicate that AC World Index IMI exhibits best risk/return characteristics measured over the past 4 years when the markets tumbled and were extremely volatile. This is likely due to high diversification common to a market portfolio.

**TABLE 3.3: MARKET PROXY PERFORMANCE**

Index	Monthly Volatility	Return
THE WORLD INDEX Standard (Large+Mid Cap)	27.13%	-23.27%
THE WORLD INDEX IMI (Large+Mid+Small Cap)	27.11%	-22.03%
AC WORLD INDEX Standard (Large+Mid Cap)	27.13%	-20.43%
AC WORLD INDEX IMI (Large+Mid+Small Cap)	27.02%	-19.09%
S&P 500	32.30%	-22.21%

Source: Author's calculations based on MSCI Barra and Standard & Poor's

During periods of market volatility and asset bubbles MSCI World family indices will likely be a better proxy for market portfolio. As discussed previously, certain industries during extreme re-pricing distort the correlations in the market and drive the betas of companies in other industries to artificially low levels. Thus, we believe that using a broader index should help us reduce the effect of industry or country specific asset price discrepancies on beta estimates.

It is also important not to estimate beta of a company based on a local index because it will not be a sensible measure for an international investor that has access to global markets. Beta will fail to represent systematic risk. Local index can be dominated by several large companies or industries and regressing on a local index will give a beta that measures company's co variation with an industry or simply with itself. For instance, a mega cap company dominating a market would simply yield a beta of around 1 since it often moves the local market alone.

Quite often no matter how precise and considerate you will be in deriving regression betas standard error will remain high while R<sup>2</sup> low. A conventional solution to increase precision of beta estimates is to use industry averages. Using a number of comparable companies and averaging their betas significantly reduces interval within which beta estimates are likely to fall<sup>12</sup>. Furthermore, using industry averages allows us to account for recent or even future changes in capital structure or business mix in

<sup>12</sup> As long as standard errors of beta estimates of different companies in the industry are uncorrelated, using bigger sample of betas to find average will reduce standard error since overestimates and underestimates of individual betas of companies in same industry tend to cancel out. Statistically,

$$\text{Standard Error}_{\text{industry}} = \frac{\text{Average Standard Error}_{\text{comparable firm}}}{\sqrt{n}}$$

estimating a company's beta<sup>13</sup>. As most of the readers are aware of, betas capture risks related to operating and financial leverage as well as cyclicalities of a business. Building bottom-up betas will help us make better judgment on these underlying risks (see further for bottom-up approach).

All in all, our choice of market proxy would be one of the Morgan Stanley Capital International family indices since these indices exhibit properties of market portfolio best. However, the choice for market proxy should always be consistent with the equity risk premium if the latter is obtained through services.

### 3.2.2 ADJUSTING FOR FINANCIAL LEVERAGE

Using a simple regression beta would disregard a forward-looking target capital structure if the company does not have optimal levels of leverage at the time when regression beta is calculated. Likewise, when calculating bottom-up beta, averaging betas of individual companies would ignore the leverage effect on individual businesses across industry. Most practitioners use the Hamada formula<sup>14</sup> based on the famous theories of Miller and Modigliani on capital structure to account for leverage in the company:

$$\beta_L = \beta_u + \frac{D}{E}(\beta_u - \beta_D) - \frac{V_{txa}}{E}(\beta_u - \beta_{txa}) \quad (3.3)$$

where:

$\beta_L$  - levered (equity) beta of the firm

$\beta_u$  - unlevered (asset) beta of a firm

$\beta_D$  - beta of debt

$\beta_{txa}$  - beta of tax shields

$V_{txa}$  - value of company's tax shields

Practitioners, however, simplify the formula at few instances. First, assuming that debt claim always has priority over equity holders' claim on assets, beta of debt is very low, presumably zero. Second, if a company maintains a constant capital structure, value of tax shields will move in line with operating assets, therefore risk of the tax shields will be similar to that of operating cash flows, thus beta of the tax shields will be equal to unlevered/asset beta ( $\beta_u = \beta_{txa}$ ). Taking into account these simplifications, the Hamada formula can further be simplified (see Practitioner's formula further).

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<sup>13</sup> Damodaran A., "Investment Valuation: Tools and Techniques for Determining Value of Any Asset", 2<sup>nd</sup> Edition, John, Wiley and Sons (2002)

<sup>14</sup> R. Hamada (1972), "The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks"

Different authors have suggested different formulae on relevering and unlevering betas (see appendix for summary). These formulas generally capture differently the risks of realizing tax savings resulting from the tax shields of debt in the capital structure. For instance, if a company has been losing money in the previous period, and it allocates its losses to the current accounting period to reduce the tax liability (which is a usual practice under GAAP), it will not realize tax savings from interest payments in the current period. As a result, the cost of debt will be greater by the loss/deferral of these tax savings (Grabowski, Pratt, 2008).

Most of the formulas assume that there is no negative effect on operations from the amount of leverage, only interest cost.

According to Grabowski and Pratt (2008), Hamada formula implies that total risk constitutes from mostly business rather than financial risk. To be more precise, it assumes that the debt is constant absolute value in the capital structure and therefore it understates the benefits of tax shields of highly rated debt of a public company:

$$\beta_L = \beta_U \left( 1 + (1 - t) \frac{D}{E} \right) \quad (3.4)$$

where:

$\beta_L$  - levered/asset beta

$\beta_U$  - unlevered/equity beta

D - market value of debt capital

E - market value of equity capital

t - tax rate

Most companies, however, manage their leverage to target debt-to-equity ratio.

Practitioners' formula, on the other hand, assumes the least benefit from the tax shields. In other words, it assigns a considerable financial risk to the leverage:

$$\beta_L = \beta_U \left( 1 + \frac{D}{E} \right) \quad (3.5)$$

where:

$\beta_L$  - levered/asset beta

$\beta_U$  - unlevered/equity beta

D - market value of debt capital

E - market value of equity capital

The choice of unlevering and relevering formulas has significant impact on beta and, as a result, to the overall cost of equity. Basically, they treat the underlying risks of realizing tax shields differently due to tax-loss carryforwards mentioned earlier, and the more they assume that tax savings are unlikely, the higher the overall cost of equity and cost of capital will be. If for example some of the peer companies (discussed later) have high debt, Hamada formula will overestimate the unlevered beta. Also because it assumes constant debt whereas debt often changes during the five year estimation period of beta the formula would distort the true picture unless you account for capital structure changes during the period (say, by averaging capital structure over the 5 years).

A lot of academics have concluded though that Milles-Ezzel formula reflects best the fact that firms maintain constant debt-to-equity ratio based on market values<sup>15</sup>. Fernandez formula is applicable best if firm maintains a fixed book value leverage ratio<sup>16</sup>.

Milles-Ezzell formula is especially relevant in a distressed environment when debt betas are higher than before. Disregarding positive debt betas will likely lead to understating cost of capital:

$$\beta_L = \beta_U + \frac{D}{E}(\beta_U - \beta_d) \left[ 1 - \frac{(t \times k_{d(pt)})}{(1 + k_{d(pt)})} \right] \quad (3.6)$$

where:

$\beta_L$  - levered/asset beta

$\beta_U$  - unlevered/equity beta

$\beta_d$  - beta of debt

$k_{d(pt)}$  - cost of debt prior to tax effect

D – market value of debt capital

E – market value of equity capital

t – tax rate

All in all, since companies typically manage their debt to target D/E ratios going forward, debt fluctuates in line with operating assets. As a result, tax shields also carry the risk of operating assets (or operating cashflows). The formula that captures that is the Harris-Pringle formula:

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<sup>15</sup> Andre Farber, Roland Gillet, and Ariane Szafarz, "A General Formula for the WACC," *International Journal of Business* (Spring, 2006): 211-218; Enrique R. Arzac and Lawrence R. Glosten, "A Reconsideration of Tax Shield Valuation," *European Financial Management* (2005):458, Roger Grabowski, "Problems with Cost of Capital Estimation in the Current Environment" (2009).

<sup>16</sup> Pablo Fernandez, "Levered and Unlevered Beta," Working paper, April 20, 2006.

$$\beta_L = B_U + \frac{D}{E}(B_U - B_d) = B_U \left(1 + \frac{D}{E}\right) - B_d \frac{D}{E} \quad (3.7)$$

where:

$\beta_L$  - levered/asset beta

$\beta_U$  - unlevered/equity beta

$\beta_d$  - beta of debt

D – market value of debt capital

E – market value of equity capital

t – tax rate

Theoretically, in a distressed environment Milles-Ezzell formula should work best because tax shields will be more closely linked to the value of debt which in turn fluctuates a lot during times of distress. It treats tax shields for one period as if they carried the risk of debt and in the following periods as if they carried the risk of operating assets. It essentially captures the fact that for one period a company will be in distress, but it will be profitable afterwards and it will be able to use the tax shields again. However, Grabowski (2008) shows in his calculations that betas relevered with Harris-Pringle and Milles-Ezzell formulas yield results that are virtually the same. Having said that, our recommendation is to use the Harris-Pringle formula as it both accounts for varying beta of debt capital and has a form that is rather familiar.

### 3.2.3 ADJUSTING FOR OPERATING LEVERAGE

After having accounted for different capital structure, one can adjust asset beta for different operating leverage (proportion of fixed cost in the total cost structure) as an intermediate step between unlevering and relevering company's equity beta. Companies that operate under high fixed cost run higher risks especially when there are changes in revenues. As a result, one can observe that cyclical businesses have lower fixed cost than those with stable stream of revenues or those in the mature industries.

Removing the effect of fixed cost from the cost structure works rather the same way as removing financial leverage from capital structure.

To account for differences in operating leverage, Damodaran (2002) suggests adjusting the unlevered equity beta from average operating leverage in the industry to the company's level of operating leverage:

$$\beta_{business} = \frac{\beta_{U_{ind}}}{1 + \left(\frac{FC}{VC}\right)_{ind}} \rightarrow \beta_{U_{business}} = \beta_{business} \left(1 + \left(\frac{FC}{VC}\right)_{business}\right) = \beta_{asset} \quad (3.8)$$

where:

$\beta_{business}$  – unlevered beta of equity

$\beta_{U_{business}}$  – unlevered beta of equity after accounting for operating leverage

$\beta_{U_{ind}}$  – unlevered beta of equity, industry standard

$FC$  – fixed Cost

$VC$  – variable Cost

This would remove the implicit (erroneous) assumption that all companies in the industry have same operating leverage. Once you have concluded on the cost structure of the business in question, you can relever the business beta to the current or target operating leverage to arrive at a better estimate of asset beta.

### 3.2.4 ADJUSTING FOR CASH

Companies often hold significant amounts of cash or cash equivalents for operating or other purposes. Cash holdings often vary across the industries and require additional adjustments because of their different underlying risks than those of the business itself. If cash and financial investments comprise a small portion of total assets, this adjustment can be incorporated in the calculations of beta.

Unless analyst decides upon valuation of liquid securities separately from cashflows derived from operations (by reducing net income by the revenue arising from financial investments), it is necessary to increase/decrease the unlevered beta accordingly if cash investments carry higher/lower risk than operating assets. To do so one can calculate the weighted average of unlevered betas of business (i.e. non-cash operating assets) and cash equivalents.

$$\beta_U = \beta_{U_{adjusted\ for\ cash}} \left(\frac{Noncash\ assets}{Total\ assets}\right) + \beta_{cash} \left(\frac{Cash\ \&\ Eq.}{Total\ assets}\right) \quad (3.9)$$

If cash in the company is invested in extremely safe and liquid investments such as t-bills or commercial paper, these holdings do not carry systematic risk and their beta is zero in turn.

$$\beta_U = \beta_{U_{adjusted\ for\ cash}} \left(\frac{Noncash\ assets}{Total\ assets}\right) = \beta_{U_{adjusted\ for\ cash}} \left(\frac{Noncash\ assets}{D+E}\right) \quad (3.10)$$

$$\beta_{U_{adjusted\ for\ cash}} = \frac{\beta_U}{\left(1 - \frac{Cash\ \&\ Eq.}{D+E}\right)} \quad (3.11)$$

In case one computes bottom-up beta (see further), an analyst can reduce debt value by the amount of cash holdings and relevel the betas to the resulting *net* debt to equity ratio<sup>17</sup>. This will have a similar effect as the formula (3.11). Lower D/E ratio will result in lower betas and lower cost of equity. However, when calculating cost of capital more weight will fall on cost of equity which will at least partly offset lower COEC.

To sum up, typically cash in contrast to other assets in the company carries no or little systematic risk. As a result, it is necessary to adjust unlevered beta by increasing it relative to the size of cash holding to avoid understating the true beta of operating assets.

### 3.2.5 MODIFYING BETA

If the betas used were somewhat static and not forward looking the application of the CAPM would be rather restricted. There have been research carried out that betas tend to converge to market and industry averages. The two proposed ideas were to adjust beta to the industry norm, a technique called Vasicek's shrinkage<sup>18</sup>, and to the market norm, what is known as the Blume's adjustment<sup>19</sup>.

Vasicek suggested that betas with high standard error tend to converge to industry norm more than those with low standard error. As high betas are likely to be those with high standard error, they by rule tend to industry averages more. The technique finds weighted average between peer group beta and company beta, giving more weight to peer group beta if the standard error is high.

A better known adjustment is the Blume's adjustment which is also used by Bloomberg and Value Line. It involves multiplying beta by one third and adding two thirds to obtain a beta that is more forward looking. This is based on the assumption that betas tend to average market beta of 1 over time.

$$\beta_{adj} = (0.33) + (0.67)\beta_{raw} \quad (3.12)$$

where:

$\beta_{adj}$  – adjusted beta

$\beta_{raw}$  – raw beta

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<sup>17</sup> Aswath Damodaran, "Dealing with Cash, Cross Holdings and Other Non-Operating Assets: Approaches and Implications", (September 2005)

<sup>18</sup> Oldrich A. Vasicek, A note on Using Cross-Sectional Information in Bayesian Estimation of Security Prices," *Journal of Finance* (1973)

<sup>19</sup> Marshall Blume, "Portfolio Theory: A Step Towards Its Practical Application," *Journal of Business* (April 1970).



With the idea behind similar to that of Vasizek's industry shrinkage, a more advanced market smoothing technique has been proposed by Koller, Goedhart and Wessels (2004). Whenever the standard error of beta is high, the following technique used by McKinsey & Company will tend beta to market average:

$$\beta_{adj} = \frac{\sigma_{\varepsilon}^2}{\sigma_{\varepsilon}^2 + \sigma_b^2} (1) + \left(1 - \frac{\sigma_{\varepsilon}^2}{\sigma_{\varepsilon}^2 + \sigma_b^2}\right) \beta_{raw} \quad (3.13)$$

where:

$\beta_{adj}$  – adjusted beta

$\beta_{raw}$  - raw beta

$\sigma_b$  – cross-sectional deviation of all betas

$\sigma_{\varepsilon}$  – standard error of the regression beta

Other adjustment, called the Sum Beta<sup>20</sup>, tries to capture delay with which a stock price reflects market information. It is especially persistent in midsize and smaller companies. The adjustment tries to reduce this lag effect by adding two independent regression coefficients: first, stock's excess returns on market's excess returns, and second, company's excess returns on previous period's market excess returns. All excess returns are calculated over 30 day T-bill rate given the one month sample frequency of the regression accordingly. This supposedly captures the lag of comovement between the stock and the market and enables beta to reflect systematic risk better. When price reactions to the stock market are non-synchronous, the correlation between the stock returns and the market returns is lower, and as a result, traditional OLS betas are biased downwards for companies other than the largest ones<sup>21</sup>. Though this understatement of systematic risk is often captured as excess return of smaller companies over CAPM predicted returns in a small firm premium, the precision of it is as debatable as the small firm premium itself. The 2006 SBBI Valuation Edition<sup>22</sup> shows that excess returns over CAPM (i.e. deviation of actual data from the model) calculated with sum beta are significantly lower than excess returns over CAPM calculated with simple OLS beta. This might explain partly the failure of CAPM to correctly estimate returns for smaller companies, and perhaps, suggest improved accuracy of Sum Betas over OLS Betas. Some analysts prefer to calculate Sum Betas and make smaller adjustment for the size effect in the CAPM. Finally, sum betas are specifically useful in the current distressed environment as

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<sup>20</sup> Roger G. Ibbotson, Paul D. Kaplan, and James D. Peterson, "Estimates of Small-Stock Betas Are Much Too Low," *Journal of Portfolio Management* (Summer 1997): 104-111.

<sup>21</sup> Scholes and Williams (1977), "Estimating Betas for Nonsynchronous Data"

<sup>22</sup> SBBI Valuation Edition 2006 Yearbook (Chicago: Morningstar, 2006), Table 7-10, 143.

market capitalization of a lot of companies has shrunk pushing them to midcap to smallcap size and making them less responsive to market changes.

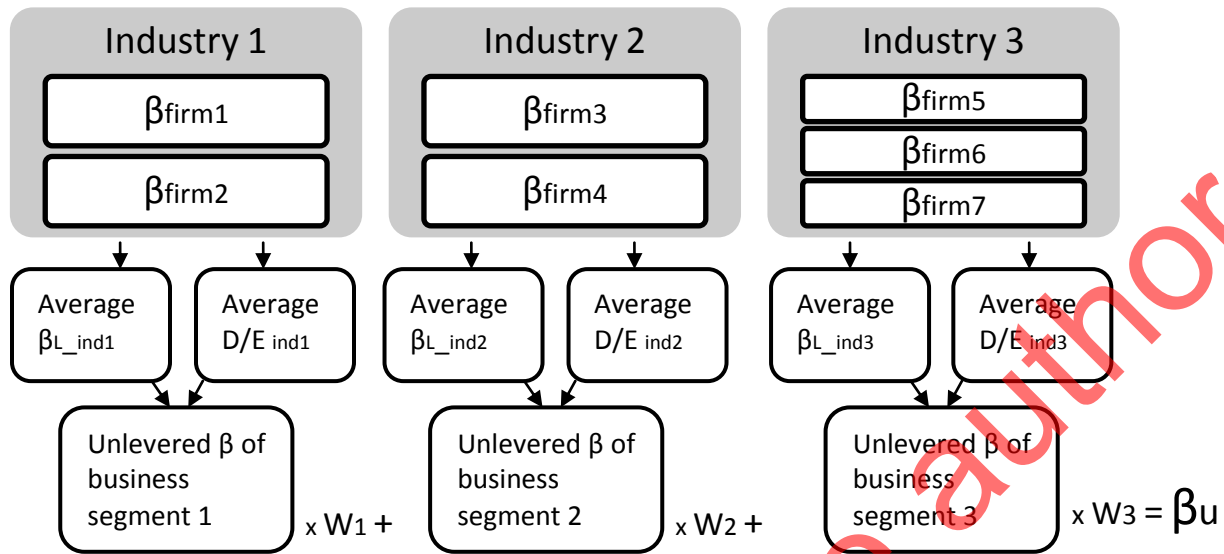
All things considered, Blume's adjustment has gained more ground over the years than other smoothing techniques. This is likely due to a wide application of the adjustment by the major services. Finally, sum betas offer a promising way to account for market anomalies and delays in trading of certain stocks when calculating betas. Therefore, one should consider using the method when calculating a solo regression beta.

### 3.2.6 BOTTOM-UP BETA

When the stock data is noisy, the prices are experiencing large corrections during times of distress or simply are unavailable as it happens for the privately held companies, the traditional top-down approach of estimating beta of a company from regressing excess returns is often no good (i.e. low  $R^2$ , high standard error). Instead, one can use bottom-up approach by deriving beta based on the peer group and fundamentals. Damodaran (2002) suggests breaking the beta risks into underlying components, notably financial and operating leverage based on peer companies in the same industry or business segment. He claims further that because betas of different assets can be averaged by using their market weights, company's beta is essentially a weighted average of betas of businesses that it operates in. As a result, to find bottom-up beta based on the peer group, we can use regression betas of listed companies that operate in the same industries as the subject company. The levered betas should then be averaged within every group of companies (see Figure 3.4). Then using average industry D/E ratios, unlevered betas should be found for every business/industry group that the subject company is in. Though it might seem reasonable to unlever every peer company separately and then average the betas, Damodaran argues that this will likely compound standard errors of peer betas.

Finally, these unlevered betas should be weighed against the proportion of value or - if the latter is unavailable - income/revenue that is derived from every business/industry that the subject company operates in. Summing up these weighted betas would give unlevered beta of the subject. The unlevered beta can be afterwards relevered to the target D/E ratio.

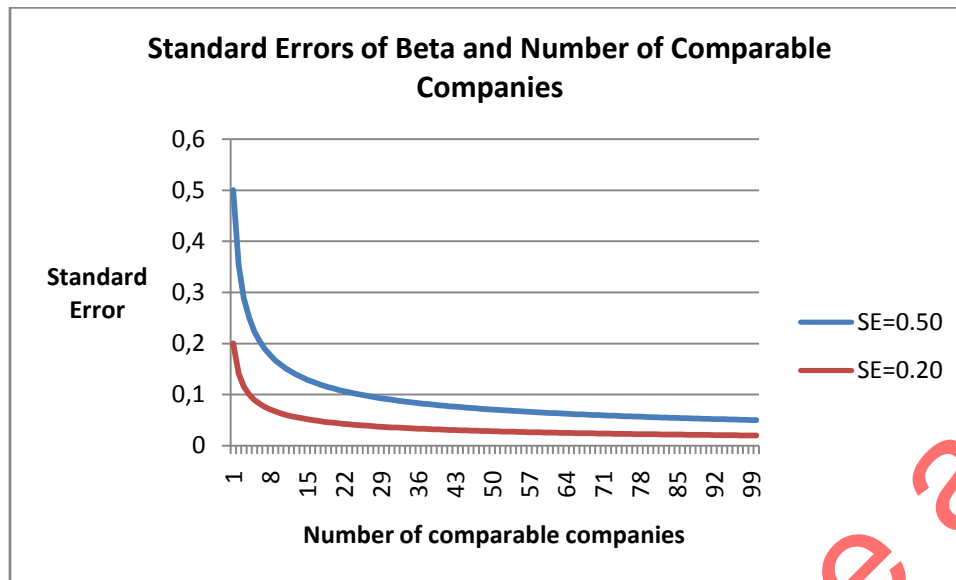
FIGURE 3.4: CALCULATION OF BOTTOM-UP BETA



Using comparable firms to derive company beta allows us to avoid one major flaw of regression betas – large standard error in estimation. By using a larger number of peer companies it is possible to reduce this interval within which the true beta falls. If the standard errors of betas are uncorrelated across the peer group, averaging their betas would reduce standard error exponentially if compared to the standalone regression beta of the subject company<sup>23</sup>. In the Figure 3.5, you can see how two illustrative standard errors of beta shrink as the number of comparable companies used increases.

<sup>23</sup>  $Standard\ Error_{bottom-up\ \beta} = \frac{Average\ Standard\ Error_{Comparable\ firms}}{\sqrt{number\ of\ comparables}}$

FIGURE 3.5: STANDARD ERROR OF BOTTOM-UP BETA



Quite obviously, the biggest disadvantage of including numerous comparable companies in the calculation of beta of the subject company is the computational cost. Not only is it time consuming to run regressions for betas of a number of peer companies; often it is hard to define the comparable companies.

To avoid running regressions manually for every peer group company, one can use service betas such as Bloomberg. The returns should be regressed against a well diversified global equity index. On the other hand, if the returns are regressed against a local index the sample peer group should be large enough so that estimation errors would average out.

The number of comparable companies in the peer group should be around 20, as a rule of thumb. While the larger number would reduce the standard error as shown previously, the benefits would be very marginal (a sample of 20 comparable companies reduces SE by around 80 percent, whereas a sample of 100, by 90 percent). As a matter of fact, one should define the comparable companies according to the number of players in the industry. If there is a large number of companies, one could narrow it by industry segment, revenue size etc. However, if there are only few players, you might want to consider a broader picture.

There are couple of extra points to be noted.

Firstly, industry raw betas should be averaged using simple average instead of a weighted one. Simple average would not undermine smaller companies in the calculation and, as a result, would help minimize standard error of estimation. The same applies for using median in this case. Though a general practice of central tendency in finance is often median, to reduce standard error for bottom-up betas one should use mean instead of median.

Secondly, even though the underlying cost structure in comparable companies is similar, if big differences in operating leverage exist, one should remove it using the formula noted earlier.

All in all, using comparable firms to derive beta enables us to account for changes in business mix, major divestitures, acquisitions or restructurings. All these changes can be factored in by adjusting peer groups accordingly. Say, if a company is divesting a unit, we exclude the respective industry for estimating beta; or on the contrary, if a company has plans to enter a new market, we include the comparable companies that operate in it for estimating beta. Finally, using comparable companies to estimate beta allows us to overcome such issue as lack of quality data often common for newly listed or private companies.

### 3.2.7 DEBT BETA

During times of distress debt often carries systematic risk that tends to be ignored in conventional calculations of cost of equity. The risk of debt capital is measured by the beta of debt which is calculated in an analogical way to equity beta (regressing market returns on debt returns). The debt betas account for the risk that interest is paid when due as economy or market proxy changes. As a result, ignoring significantly positive debt betas would provide us with incorrect estimations of equity betas and cost of capital thereof. Debt betas are positively correlated with credit ratings and, as research suggests<sup>24</sup>, in the long-run have been in the range between 0.30 and 0.40.

Generally, there are two proposed approaches in estimating debt betas.

The first one is to use a general regression formula to come up with debt betas:<sup>25</sup>

$$R_d = \alpha + \beta_d (R_m - R_f(1 - t)) + \varepsilon \quad (3.14)$$

where:

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<sup>24</sup> Tim Koller, Marc Goedhart, and David Wessels, *Valuation: Measuring and Managing the Value of Companies*, 4<sup>th</sup> ed. (John Wiley & Sons, 2005), 32.

<sup>25</sup> Simon Benninga, *Financial Modeling*, 2<sup>nd</sup> ed. (Cambridge, 2000), 414.

$R_d$  – return on subject debt  
 $\beta_d$  – debt beta  
 $R_m$  – historical market return  
 $R_f$  – risk-free rate  
 $t$  – marginal tax rate

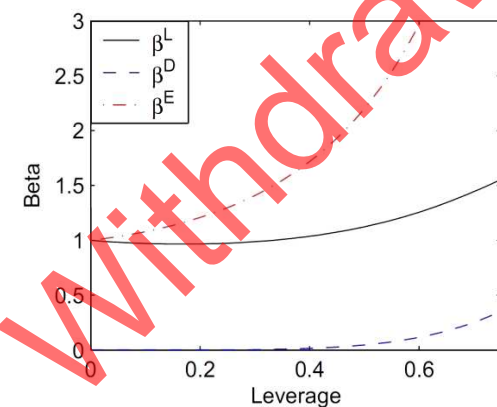
The second approach is to calculate the implied debt beta using market risk premium and default spread (actual or synthetic) over the risk-free rate. The calculation then relies on the assumption of how much risk captured by the default spread is attributable to the market risk.

$$\text{Implied } \beta_d = \frac{\text{Spread}}{\text{MRP}} \times \text{Perc. MarketRisk} \quad (3.15)$$

Schaefer and Strebulaev (2006) that attempted to predict debt betas concluded that a rather small proportion of overall yield spread reflects credit risk<sup>26</sup>. Damodaran (2002) in his calculations assumes half of default spread attributable to the market risk.

Debt betas have been increasing as financial distress has continued over the past several years. As equity values have been wiped out, debt to equity ratio, hence relative financial leverage has been increasing. Figure 3.6 depicts this relationship between the levels of leverage and betas of equity and debt.

**FIGURE 3.6: LEVERAGE AND ITS IMPACT ON BETAS**



Source: Korteweg A., *The Cost of Financial Distress* (September, 2007), working paper

<sup>26</sup> Schaefer S., Strebulaev I., "Risk in Capital Structure Arbitrage"

Duff&Phelps also provide with evidence of increasing debt betas over the period of recent financial crisis (see table 3.4). Having said that, it is crucial to account for positive debt betas in a distressed environment.

**TABLE 3.4: RECENT DEBT BETAS BY CREDIT RATING**

Rating	Dec 2008	May 2009	August 2009
Aaa	0.12	0.20	0.22
Aa	0.17	0.21	0.24
A	0.35	0.33	0.36
Baa	0.42	0.38	0.41
Ba	0.68	0.55	0.58
B	0.77	0.66	0.69
Caa	1.11	1.00	1.03
Ca-D	1.50	1.49	1.49

*Source: calculations by Duff&Phelps LLC; the regression method*

To simplify the calculation of debt beta, we check the validity of Damodaran's implied beta formula. We use Duff&Phelps regression debt betas, and assumption that 50% of risk captured by the default spread is attributable to the market risk. This way we deduce the MRP implied in the formula:

**TABLE 3.5: MARKET RISK PREMIUM IMPLIED BY DEBT BETAS**

Rating	Date	Bd	Spread	MRP	%Market Risk
Aaa	Dec-08	0.12	4.70%	19.58%	50%
	May-09	0.2	5.36%	13.40%	50%
	Aug-09	0.22	5.12%	11.64%	50%
Baa	Dec-08	0.42	8.07%	9.61%	50%
	May-09	0.38	7.76%	10.21%	50%
	Aug-09	0.41	6.38%	7.78%	50%

Table 3.5 above shows that attributing 50% of spread to the market risk significantly overstates the market risk premium which historically did not exceed 6%. If the relationship between the variables holds correctly, we can estimate ourselves the percentage of market risk captured by the spreads by inputting market risk premia:

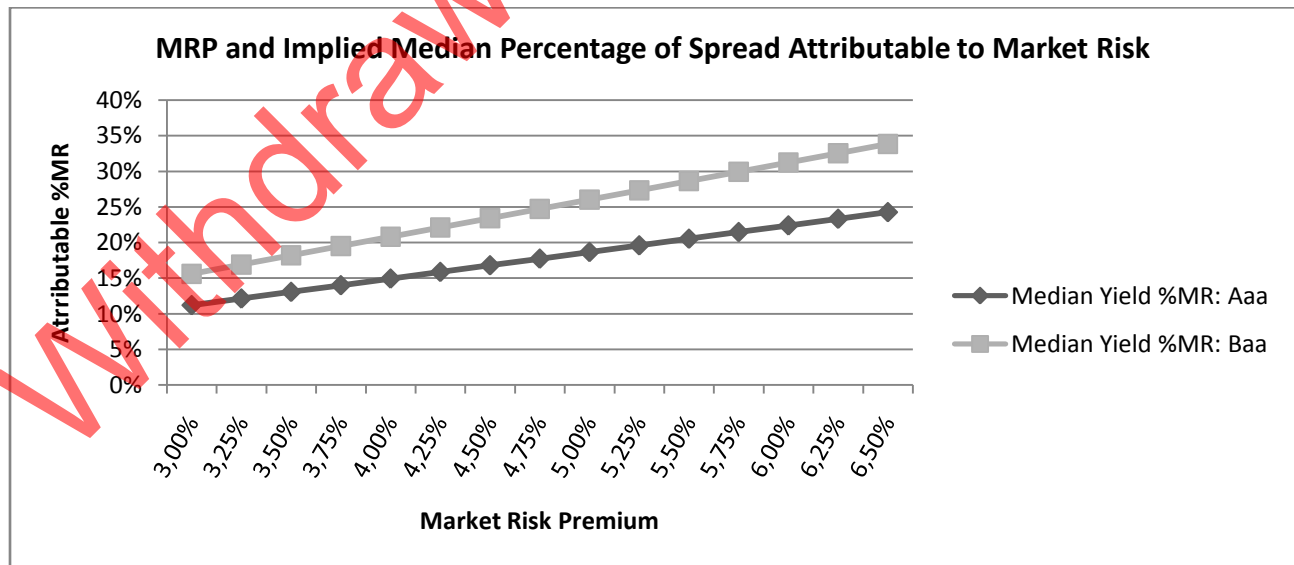
**TABLE 3.6: PERCENTAGE OF MARKET RISK ATTRIBUTABLE TO THE CREDIT SPREADS USING IMPLIED MRP**

Rating	Date	Bd	Spread	Implied MRP*	%Market Risk
Aaa	Dec-08	0.12	4.70%	6.43%	16%
	May-09	0.2	5.36%	5.94%	22%
	Aug-09	0.22	5.12%	5.30%	23%
Baa	Dec-08	0.42	8.07%	6.43%	33%
	May-09	0.38	7.76%	5.94%	29%
	Aug-09	0.41	6.38%	5.30%	34%

\*from Damodaran.com

As you can see from Table 3.6 above, the percentage of market risk we should attribute to spreads highly depends on the bond grade. High grade bonds (Aaa) are less susceptible to systematic risk (around 20% yield spread compensating for the market risk) whereas investment grade bonds (Baa) compensate more for the systematic risk relative to the spread (around 30%). Similarly, below in Figure 3.7 we plot the choice of historical MRP and the resulting *implied median perc. of yield spreads attributable to the market risk* of both high grade and investment grade bonds using debt betas on the three related dates. Effectively, one could use these conditional percentages to find the debt beta for the two bond grades at any given time using the formula as a rule of thumb.

**FIGURE 3.7: RELATIONSHIP BETWEEN MARKET RISK PREMIUM AND THE PERCENTAGE OF MARKET RISK ATTRIBUTABLE TO THE CREDIT SPREADS**



Note that this way of deducing implied betas falls short on the fact that the market risk premium that you use in the formula might not be consistent with the market risk premium that is contained in the



spread. Nonetheless, if we use mid-range market risk premium which is very likely to be the one that the bond traders assume, it should yield approximately correct debt beta.

All in all, we believe though that the regression betas should be superior as they measure directly the covariance of bond yields with the market returns. Thus, if one requires these estimates without significant computational effort, she could use bond index data provided by the services and major investment banks and the related debt beta estimates.

## 4 EQUITY RISK PREMIUMS

### 4.1 MARKET RISK PREMIUM

Equity risk premiums are by far some of the most important elements in estimating cost of equity and the overall cost of capital. Studies have found that estimations of equity risk premiums lead to the highest errors in estimation of cost of capital<sup>27</sup>. Another study suggested that different estimations of ERP on average impact cost of equity at around 2 percentage points and can be as high as 4 percentage points<sup>28</sup>.

As with regards to market risk premium, a lot of debate turns around not only about specific details of methodology used, but also about fundamental approach on how to find market risk premium. Though the most common practice so far has been estimating MRP from historical premium data, not always can it be extrapolated as historical data is not necessarily representative of the future expectations.

Three different approaches that most academics and practitioners present to estimate equity risk premium are to survey the expectations and sentiments of analysts, managers or academics; obtain ERP from historical premium of past equity returns relative to the riskless investment and, finally, estimate a forward-looking premium based on the market rates or prices on traded assets today.

One of the most extensive surveys carried out to date available publicly is by Fernandez<sup>29</sup>. He has examined opinions of analysts, company managers, academics and corporate finance textbook authors.

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<sup>27</sup> Wayne Ferson and Dennis Locke, "Estimating the Cost of Capital through Time: An Analysis of the Sources of Error," *Management Science* (April 1998): 485-500.

<sup>28</sup> Seth Armitage, *The Cost of Capital: Intermediate Theory* (Cambridge: Cambridge University Press, 2005), 319-320.

<sup>29</sup> See Pablo Fernandez, "Market Risk Premium used in 2010 by Analysts and Companies: a survey with 2,400 answers", (May 2010)

While analysts and managers present opinions that are closer to realized equity premiums, it is interesting to find that academics are very inconsistent in their opinions with premiums on average higher by around 2% than those of practitioners. Fernandez also finds that in over 150 valuation textbooks MRP has declined from 8.9% in 1988 to 5.7% in 2009.

While survey premiums have been tracked for over a decade now and have become more accessible, they are rarely applied in practice for computations since premiums obtained from investor sentiment are overly responsive to recent stock price movements. Furthermore, there are more psychological elements that were found to play a role in the results, including the way a question is framed. Finally, suffice it to say, as in every investor survey there is a risk that responders might have incentives to introduce a bias in their opinions.

The most widely used approach of estimating market risk premium remains historical premium approach, where long-term returns on stocks are compared with long term default-free (government) securities. The difference provides us with a realized historical risk premium.

Though there is consensus that historical data is the most representative of actual premium, many different sources<sup>30</sup> indicate that it is yet not a solid figure and can range anywhere from zero or even negative figures to 12% depending on the methodology used to calculate it. The reasons behind the differences mainly lie in the choice of risk-free rate as well as market proxy, averaging method and historical time span.

The two different schools often disagree about the length of period for estimation of MRP. On one side, there is a big support for shorter recent periods that are thought to represent a more updated view on the future. On the other side, there is a prevalent opinion that longer time spans should gauge the reality better as history tends to repeat itself. Nonetheless, each of the approaches has its pitfalls. Using shorter period for estimation leads to higher standard error (ex. Given 20% standard deviation of stock returns, standard error would go up three times shrinking a sample from 80 to 10 years). On the other hand, using longer periods of time might forego the fact that trading was infrequent in the past and fundamental characteristics of economy and the market have changed. In essence, one might find a discussion on how much data to extrapolate analogical to the debate between the fundamental and technical analysis, which, in fact, never ends. An analyst is then left to find the golden mean (or choose a

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<sup>30</sup> See Ibbotson Associates, Duff&Phelps, Aswath Damodaran

better between the two evils). Either way, should the analyst pursue to calculate equity risk premium instead of using service data, non-recurring events should be removed from the sample in order to improve its predictive power. In fact, an interesting observation made by Damodaran is that including the period with the recent financial turmoil in calculation of ERP would downgrade long-term ERP by 1%. This eventually would lead to a flawed conclusion that one of the worst crises in history (and the flight-to-quality that followed) has made equities a safer investment.

**TABLE 4.1: EQUITY RISK PREMIUMS UNDER DIFFERENT ASSUMPTIONS**

	ERP: Stocks minus T.Bills		ERP: Stocks minus T.Bonds	
	Arithmetic	Geometric	Arithmetic	Geometric
1928-2009	7.53%	6.03%	5.56%	<b>4.29%</b>
1967-2009	5.48%	3.78%	4.09%	2.74%
1997-2009	-1.59%	-5.47%	-3.68%	-7.22%

Source: *Equity Risk Premiums (ERP): Determinants, Estimation and Implications*, Damodaran (2010)

Despite the large variation in estimates, Damodaran<sup>31</sup> suggests that using long-term geometric average premium over the long-term rate as the risk premium should yield the most unbiased result (here, 4.29%). To be more precise, one should consider that using t-bill over t-bonds would undermine the term-structure of interest rates and the fact that it is long-term government bonds that are often used as risk-free rate in corporate finance and valuation (see earlier discussion). Therefore, one should stick to t-bonds for the ERP estimates. As pointed out previously, he also suggests that taking into account longer period for estimations would reduce significantly the noise and standard error relative to the actual equity premium size. Finally, using geometric mean over arithmetic mean would prevent from overstating premiums because returns tend to mean revert<sup>32</sup>.

Grabowski (2008) suggests that the optimal approach to estimating the equity risk premium is taking arithmetic average of 50 years of excess returns. He claims that 50 year sample excludes events that are unlikely to repeat (World War 2, the Great Depression etc.) and extreme stock market volatility that dominated 30s through 50s. Furthermore, taking a sample larger than 50 years does not account for

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<sup>31</sup> Aswath Damodaran, *Equity Risk Premiums (ERP): Determinants, Estimation and Implications* (February 2010)

<sup>32</sup> Recent research by Fama and French (1988a, 1988b), Poterba and Summers (1988), and Bekaert and Hodrick (1992) finds significant autocorrelation between stock returns in different periods over the long run. They speculate that this is because of a stationary component in stock prices which implies that stock returns should mean revert over time.

limited opportunities for international diversification. Studies have shown<sup>33</sup> that international diversification lowers volatility which results in lower market risk premium. Finally, taking a shorter period than that, increases standard error of estimation as well as might bias results due to the fact that there was a persistent downward movement of interest rates since 1981. From 1950s to 1981, however, interest rates have been hiking so taking a 50 year sample captures the full cycle of interest rates<sup>34</sup>.

Either way, market risk premium derived using 80 year sample and geometric average as suggested by Damodaran is rather similar to ERP found using 50 year sample and arithmetic average as suggested by Grabowski, both slightly above 4%. Note that geometric average of the same data sample nearly always gives lower ERP than arithmetic average. One study<sup>35</sup> concluded though that market risk premium should lie somewhere in between arithmetic and geometric average, being closer to geometric average as data sample increases and autocorrelations of market returns become more negative.

Finally, a word of caution is that historical risk premiums should be applied only to mature markets as volatility and only a small sample of reliable data featured in emerging markets would lead to standard errors that would otherwise make ERP inapplicable.

As discussed earlier, financial turmoil can rather distort the estimates of ERP. In line with the calculations of Damodaran, Credit Suisse has shown a staggering fact that shorter, i.e. recent ten-year equity premiums have become negative on a global scale, while 50-year premiums are just around 1%:

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<sup>33</sup> Gikas Hourdouvelis, Dimitrios Malliartopulos, and Richard Priestley, "The Impact of Globalization on the Equity Cost of Capital", Working paper, May 2004

<sup>34</sup> Booth, "Estimating Equity Risk Premium and Equity Costs"

<sup>35</sup> Daniel C. Indro and Wayne Y.Lee, "Biases in Arithmetic and Geometric Averages as Estimates of Long-run Expected Returns and Risk Premia," *Financial Management* (Winter 1997), 81-90.

**TABLE 4.2: EQUITY RISK PREMIUMS IN COUNTRIES UNDER DIFFERENT TIME SAMPLE PERIODS**

Country	2000-2009	1960-2009	1900-2009
Australia	1.00%	3.50%	6.00%
Belgium	-5.70%	1.00%	2.60%
Canada	-2.00%	1.50%	3.70%
Denmark	-0.10%	0.90%	1.80%
Finland	-10.2%	4.10%	4.60%
France	-6.50%	-0.90%	3.30%
Germany	-6.90%	0.40%	5.40%
Ireland	-8.20%	3.50%	2.60%
Italy	-7.20%	-1.50%	3.80%
Japan	-7.80%	-0.80%	5.10%
Netherlands	-8.60%	3.30%	3.50%
New Zealand	-0.90%	2.80%	2.40%
Norway	1.90%	2.00%	2.00%
South Africa	3.30%	6.60%	5.40%
Spain	0.50%	3.70%	2.40%
Sweden	-3.90%	4.40%	3.60%
Switzerland	-3.40%	2.80%	2.10%
UK	-3.10%	3.30%	3.90%
US	-7.40%	2.30%	4.20%
World	-6.60%	0.90%	3.70%
World ex US	-5.20%	0.60%	3.80%
Europe	-5.70%	1.30%	3.90%

Source: Credit Suisse Global Investment Returns Yearbook, 2010, data by Morningstar Inc

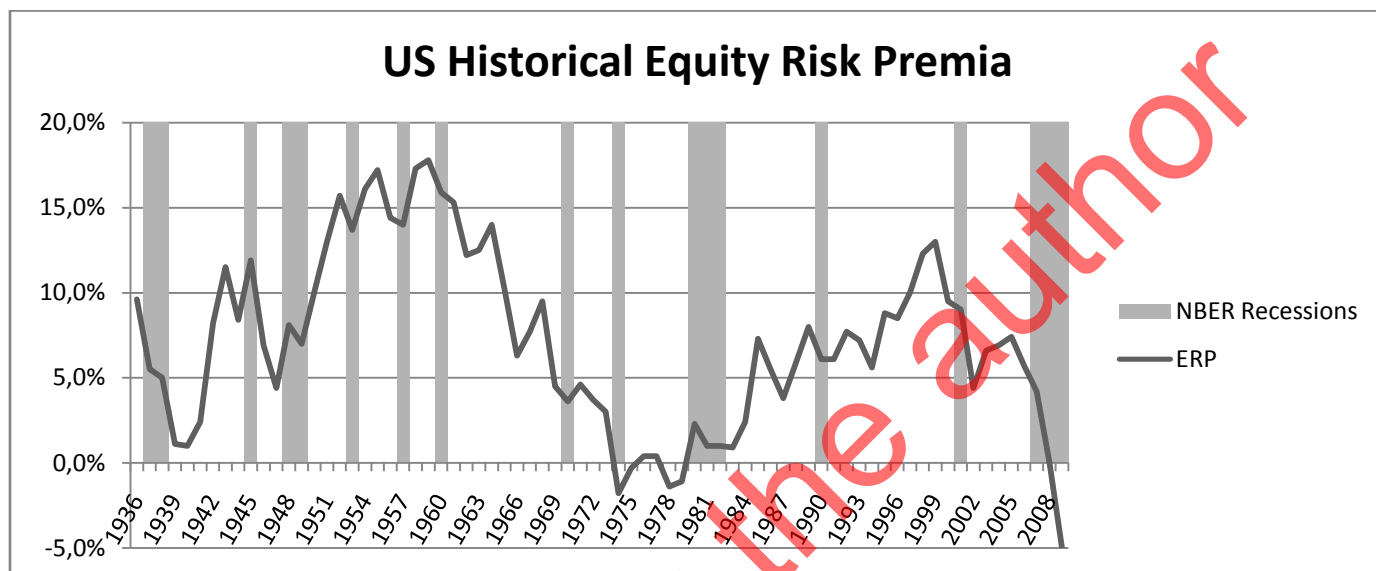
The finding again reminds the fact that recent events might not provide us with data representative of the future. Furthermore, one can notice that premiums on a global scale are often much lower than regional equity premiums. Damodaran speculates that averaging regional equity premiums (arithmetic mean) reduces survivor bias that is present in calculating local equity premiums. Lower global ERP might be no surprise, however, bearing in mind that a choice for market proxy (ex. S&P500 vs MSCI World) should have a similar impact on calculating ERP due to the levels of diversification underlying local and global indices.

What drives risk aversion of the investors and the resulting ERP? Lettau, Ludvigson and Wachter (2008)<sup>36</sup> argue that the primary source is the volatility in the economy. In particular, lower real economic growth and unstable macro fundamentals lead to lower appetite for equities and a higher ERP thereof. Damodaran (2010) further adds that quality and reliability of information available to investors plays a vital role. The absence of quality information observed in emerging markets is one of the key reasons for higher equity risk premiums in such markets.

<sup>36</sup> Lettau, M., S.C. Ludvigson and J.A. Wachter, 2008. The Declining Equity Risk Premium: What role does macroeconomic risk play? Review of Financial Studies, v21, 1653-1687.

Clearly the macro fundamentals have a significant impact on the risk-aversion of the investors. However, as you can see from the graph, ERP is not entirely dependent on economic conditions.

**FIGURE 4.1: TIME SERIES OF THE HISTORICAL EQUITY RISK PREMIUM IN THE UNITED STATES**



*Source: Author's calculation based on NBER and Morningstar data; time-series rolling 10 year arithmetic averages*

Despite the comprehensiveness of historical data, estimates of historical premium are backward looking, however. Estimated MRP, on the other hand, should be conditional to the market situation as of the date of valuation. A well grounded solution offered by Damadoran to overcome the problem and obtain a more updated and forward-looking premium is to calculate an implied equity risk premium. One of the most elaborate yet intuitive ways to do that is to build a DCF model based on current market prices of market proxy and expected cash flows from its constituents.

To put it straight forward, Damadoran (2010) uses dividends along with share buybacks to calculate free cash flow to equity (FCFE) which are later discounted with required return on equity to arrive at present value of equity index. The argument for including share repurchases is that over the last decade companies have paid only half of their FCFE as dividends whereas they have used the remaining cash balances that they built up over time to fund their stock buybacks. Therefore, taking the aggregate of dividends and stock buybacks should allow us to estimate total cash flows to equity more precisely. The model advantage comes from the fact that it can be split into several growth phases accounting for changes in the growth of earnings and dividends in the short and long term.

Having said that, the value of equity is derived as follows:

$$\text{Value of Equity} = \sum_{t=1}^{t=N} \frac{E(\text{FCFE}_t)}{(1+k_e)^t} + \frac{E(\text{FCFE}_{N+1})}{(k_e-g_N)(1+k_e)^N} \quad (4.1)$$

Where:

$E(\text{FCFE}_t)$  – expected free cash flows to equity at time  $t$

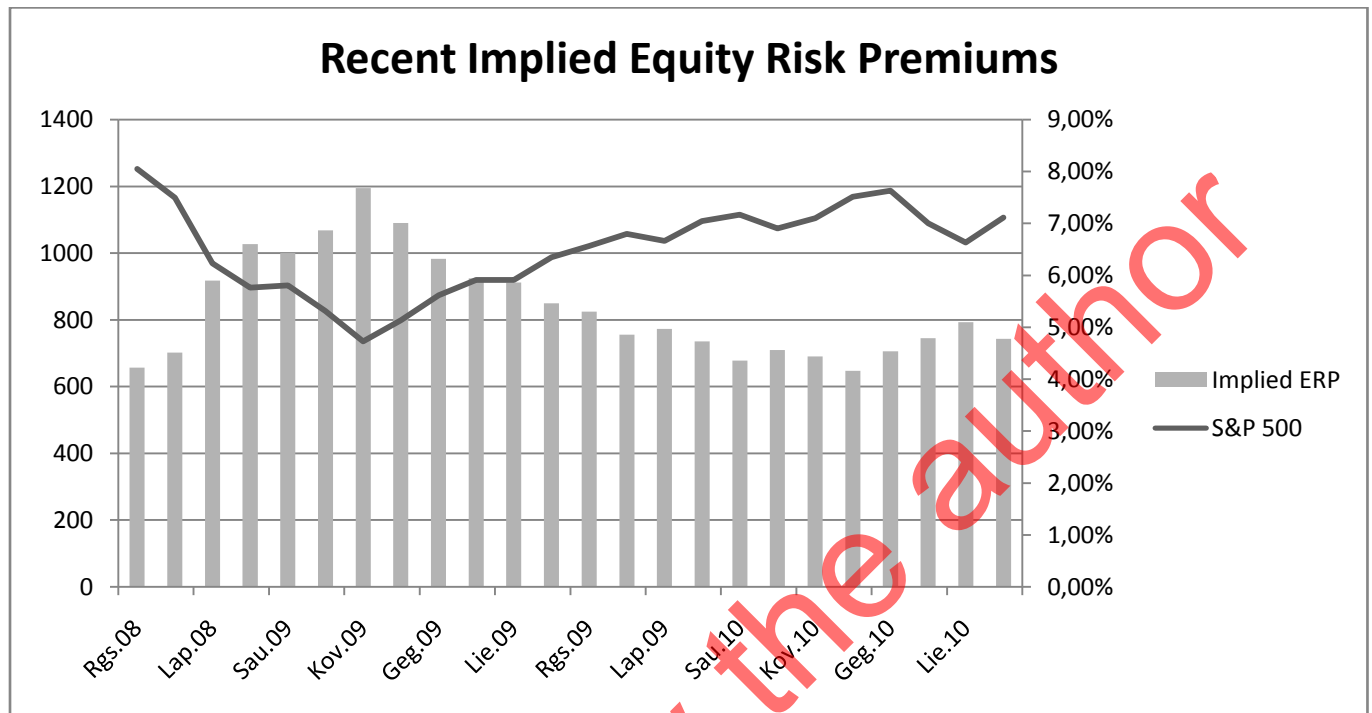
$k_e$  – cost of equity or required return on market portfolio by investors

$g_N$  – expected growth rate after year  $N$

The equation shows that equity will generate expected free cash flows (potential dividend) that will grow with the earnings until year  $N$  and will exhibit stable growth rate  $g_N$  after year  $N$ . Given the current value of the equity and potential dividend (and stock buyback) yield, we can solve for the rate of return on equity  $k_e$  required by investors. Subtracting risk-free rate from the required return on equity, we will arrive at what Damodaran claims to be “a more realistic” estimate of equity risk premium.

The two methods suggested have their own pros and cons. If one believes in efficient markets, current implied equity risk premium estimated from current level of index is the best choice. Contrary, if you believe that markets can be undervalued or overvalued, you should use average historical or average implied equity premium. Finally, depending on the purpose of analysis, an analyst might choose to use one or another. Implied equity risk premium should give a better snapshot of company value at the time in acquisition (using different historical ERP figures would implicitly assume that the market is undervalued or overvalued). On the other hand, historical averages come in handy when deriving cost of capital for calculations of long-term company investments.

**FIGURE 4.2: RECENT IMPLIED EQUITY RISK PREMIUMS IN THE UNITED STATES**



Source: Damodaran.com

The implied premiums, however, should be treated with care; lack of market wide consensus estimates for growth that would truly be incorporated in the market prices as of the day of valuation, absence of historical consensus growth estimates (say, dating back to 60s) that would allow us to check the validity of model by comparing the implied results with the realized premiums - all of these loose ends make the approach questionable. Those who argue for implied premium, claim that nowadays information moves faster, investors are more sophisticated and markets are deeper. However, in such case risk associated with holding equity investments should be lower than in the previous decades. As a result, implied equity risk premium should be lower than the realized historical premia too. However, if you track implied equity risk premia by Damodaran, you will find that it is mostly higher (even than the historical premium estimated by Damodaran).

Nonetheless, it is the most forward looking model developed so far that is also widely preferred by the investment banks. And even though it rests on human capacity to predict future, in our opinion, it is still the best cross-check when choosing a number out of the range of historical realized equity risk premiums.



Having said that, with all the research carried out, we believe that the conditional equity risk premium should fall in the range between 4% and 5%, being on the higher end during economic down cycle (hence, risk-averse investors demanding higher premium for risky investment) and on the lower end during economic up cycle when the markets are bullish.

## 4.2 FAILURE OF CAPM: ADDITIONAL PREMIUMS

### 4.2.1 SMALL FIRM PREMIUM

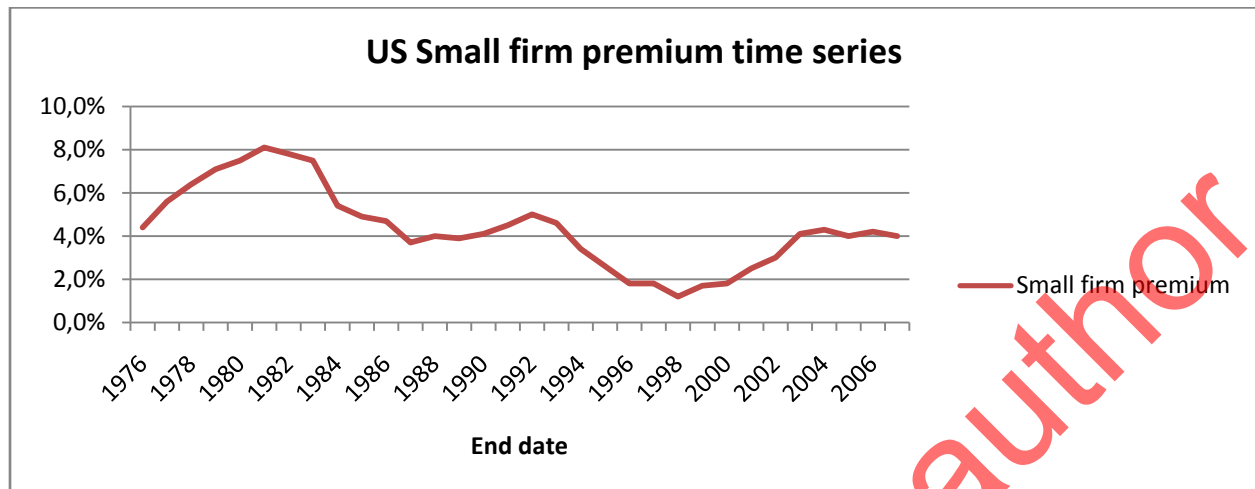
Studies since early 1980s have found that small firms earn returns in excess of those estimated by CAPM<sup>37</sup>. Many studies have pursued the suit in search of small firm premium to come up with diverse results. Some argued that small firm premium is truly a temporary phenomenon as it has disappeared at some points in time. Others claim that it can be not the market cap but other factors playing the role, for instance, illiquidity of such stocks or lack of quality information available on them. Small firm premium has been approached as a market anomaly by some. However, those that oppose this theory argue that market participants would have exploited it for the profit and it would not persist while, in fact, studies show that small firms still earn excess returns at present day.

A more conventional way to approach small firm premium is to assume failure of CAPM betas to fully gauge the systematic risk present in smaller companies. We would then look for returns that small firms would produce in excess of CAPM estimates.

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<sup>37</sup> Branz (1981), Reingunum (1981), Dimson (1986), Bergstorm, Frashure and Chislom (1991), Chan, Hamao and Lakonishok (1991)

**FIGURE 4.3: TIME SERIES OF SMALL FIRM PREMIUM IN THE UNITED STATES**



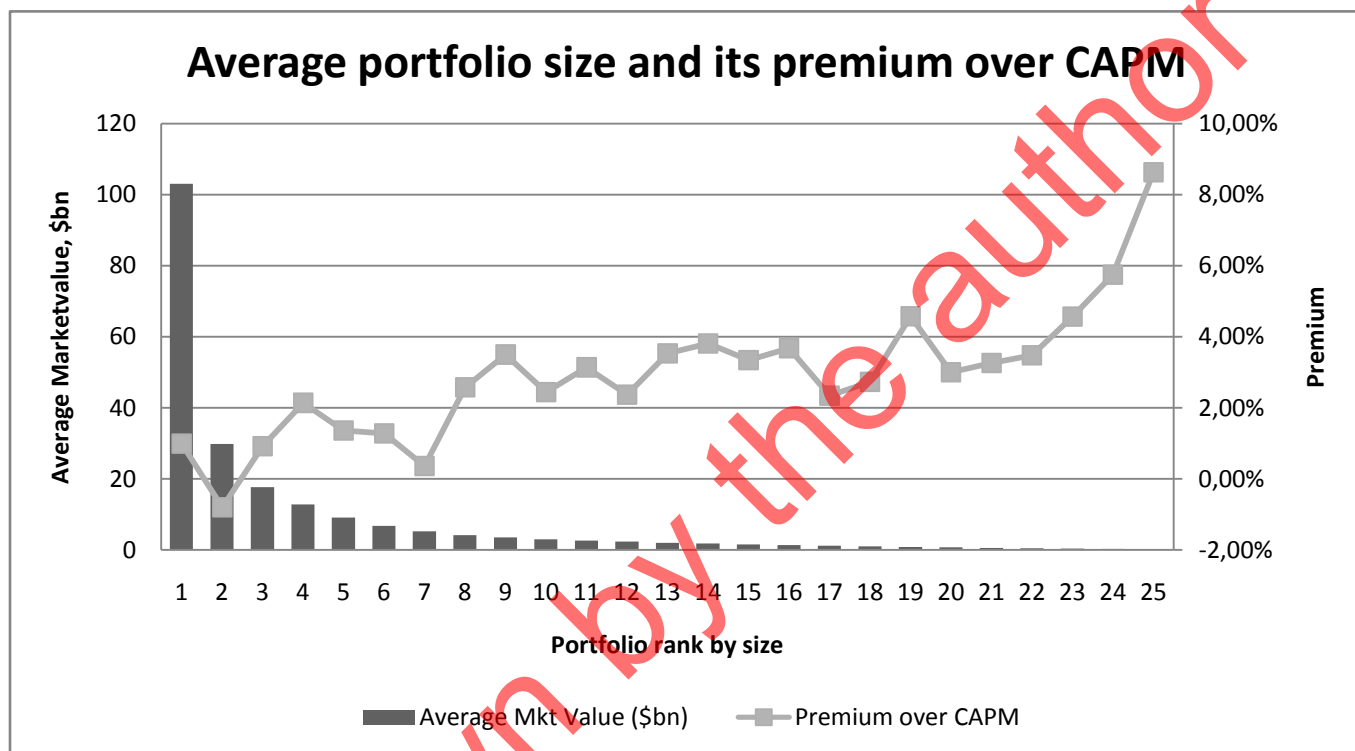
Source: Author's calculation based on 2008 Ibbotson SBBI Valuation Yearbook

In the Figure 4.3 we present time series of US small firm premium. The figure represents a 50-year moving average of micro-cap<sup>38</sup> stock excess returns over CAPM. We believe that the present day small firm premium should be estimated dating back 50 years as this would be more reflective of current market situation as well as allow us to exclude non-recurrent events that preceded the data sample (WWII, the Great Depression etc.). As you can see from the figure, beginning of the series captures the first half of 1900s which is why we believe the small firm premium is more pronounced. The estimates are quite consistent with a widespread presumption that small firm premium lies in the range between 2 to 6 percent. Other calculations by Damodaran, Morningstar and Duff&Phelps indicate that the small firm premium lies on the upper end of the scale, i.e. starting at around 5% at the lowest decile<sup>39</sup>. The difference, we assume, comes from the fact that it is based on the lowest decile of ranked portfolios instead of the lowest pentile portrayed in the time series. Again, small firm premium depends on how we define a small firm and how small we think the company is going to be in the future.

<sup>38</sup> Lowest market cap size pentile (bottom 20%) in NYSE/AMEX/NASDAQ

<sup>39</sup> See SBBI Valuation Yearbook and Duff&Phelps Risk Premium Report for detailed data and calculations

**FIGURE 4.4: EXCESS RETURNS OVER CAPM RANKED BY DIFFERENT PORTFOLIO SIZES**



Source: data from Duff & Phelps Risk Premium Report 2010 (see appendix 2)

There are several key drawbacks to be noted before applying small firm premium. Firstly, small firm premium estimates often come with large standard errors that make the application rather dubious. Secondly, small caps happen to become large caps over time (Microsoft employed 11 people in 1978). Our suggestion is to be very considerate when applying small firm premium together with aggressive growth assumption (in excess of inflation) in calculations of terminate value of a company. Thirdly, recent developments in beta calculation techniques such as the sum beta have shown to partly correct for the failure of CAPM to capture extra risks of small firms. Moreover, size premium is exposed to same calculation biases discussed earlier as the market risk premium (historical sample size, averaging method etc.) although these computational choices should be made according to estimation methodology of ERP. Finally, just as mentioned earlier, one should make sure that the small firm premium is not overlapping with other company specific risk factors such as illiquidity.

#### 4.2.2 ILLIQUIDITY PREMIUM

It is already a widely accepted axiom that illiquid assets with same characteristics are less valuable than liquid ones. The question though is what defines the illiquidity and how to measure or account for in valuing assets.

A lot of studies have been carried out under assumption that all the assets are illiquid to certain degree and that this illiquidity can be measured in bid-ask spreads of the traded assets and turnover ratios as key factors.

Many studies since 1970 have attempted to incorporate illiquidity into asset pricing models with rather mixed results. A study by Amihud and Mendelson (1989)<sup>40</sup> has shown that every 1% increase in bid-ask spread led to a quarter percent higher annual return. Furthermore, Datar, Nair and Radcliffe (1998)<sup>41</sup> show that every 1% increase in turnover ratio reduces excess annual returns of illiquid stocks by around half percent. Numerous other studies have also found consistently higher returns on less liquid stocks using turnover ratios and bid-ask spreads as proxies for illiquidity<sup>42</sup>. Even though nearly all of the studies accounted for factors such as firm size or market-to-book value, the application of illiquidity discount in practice still remains a misty subject.

One recent study<sup>43</sup> has found that illiquid stocks return on average 1.1% higher annual return and that 80% of this premium can be explained by the illiquidity of the stock market itself. Having that in mind, we believe that individual stock illiquidity might be more a matter of general market risk premium estimates during bearish times in that case.

The most recent research<sup>44</sup> on the topic carried out by Roger Ibbotson and Zhiwu Chen (2009) also presents an evidence that illiquid basket of stocks outperforms a liquid one. They claim that illiquidity, in fact, explains excess returns better than size. However, the results are very questionable considering the absence of calculation methodology.

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<sup>40</sup> Amihud, Y. and . Mendelson, 1989, the Effects of Beta, Bid-Ask Spread, Residual Risk and Size on Stock Returns, *Journal of Finance*, v 44, 479-486.

<sup>41</sup> Datar, V.T., N. Y. Naik and R. Radcliffe, 1998, "Liquidity and stock returns: An alternative test," *Journal of Financial Markets* 1, 203-219.

<sup>42</sup> see Brennan and Subrahmanyam (1996), Eleswarapu (1997), Nguyen, Mishra and Prakash (2005)

<sup>43</sup> Acharya, V. and L.H. Pedersen, 2005, Asset Pricing with Liquidity Risk, *Journal of Financial Economics*, v77, 375-410.

<sup>44</sup> <http://advisor.morningstar.com/articles/article.asp?docId=16609>  
<http://www.morningstar.com/cover/vidocenter.aspx?id=348365>

Rephael, Kadan and Wohl (2008) find that liquidity premium on trading volume-basis has declined over the past four decades<sup>45</sup>. They measure it in all firm sizes and find that the results are statistically unrelated to size effect. They argue that with the rise of ETFs and index funds illiquidity premium has been virtually erased on volume-basis and very low on other liquidity measures because anyone now can have exposure to illiquid stocks through these vehicles without experiencing any trading delay.

Grabowski (2008) argues that all the excess returns that are featured in less liquid stocks can be captured by the small firm premium. He argues that bid-ask spreads are often quoted on exchanges but are not realized for most of illiquid stocks while bid-ask spread is further mitigated by the fact that it is measured on higher frequency basis, whereas excess returns for SFP, on monthly basis. He further adds that transaction cost which a lot of academics claim to be higher for illiquid securities is also partly captured in SFP. Either way, it has been acknowledged that small firm premium overlaps with illiquidity premium as, naturally, those are the small stocks that trade thinly. The issue however is that most academics focus on explaining excess returns either based on size or liquidity without trying to establish a firm relationship between the excess returns based on the two measures.

There has been a huge debate going on whether small firm premium or illiquidity premium (discount) has more explanatory power in measuring excess returns. In the world of finance, however, illiquidity premium has not been accepted as much as small firm premium. Perhaps the reason behind is that it is rather hard to define what truly makes an asset illiquid and to control for other factors (ex. size of a traded blocks of stock has an impact on bid/ask quote for any company size). Furthermore, we believe that application of illiquidity premium would also complicate estimation of cost of equity for private companies as they are not traded and could not be cross-referenced to their traded peers on the basis of bid/ask spread or turnover ratio. Using size premium, on the other hand, is not hard to look up for returns in excess of CAPM for comparable size traded companies. Also note that private companies are valued using specific illiquidity discounts for lack of marketability<sup>46</sup> and applying illiquidity discounts twice at several levels rather stands against a common sense.

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<sup>45</sup> Azi Be-Rephael, Ohad Kadan, and Avi Wohl, "The Diminishing Liquidity Premium" (June 2008)

<sup>46</sup> Discount for lack of marketability is already a common practice in valuing private companies. The discount is applied at enterprise value level of a company and as studies show is around 30% (see Pratt, Shannon P. and Alina V. Niculita, "Chapter 17: Discounts for Illiquidity and Lack of Marketability," *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 5<sup>th</sup> edition, New York, 2008, p. 431.)

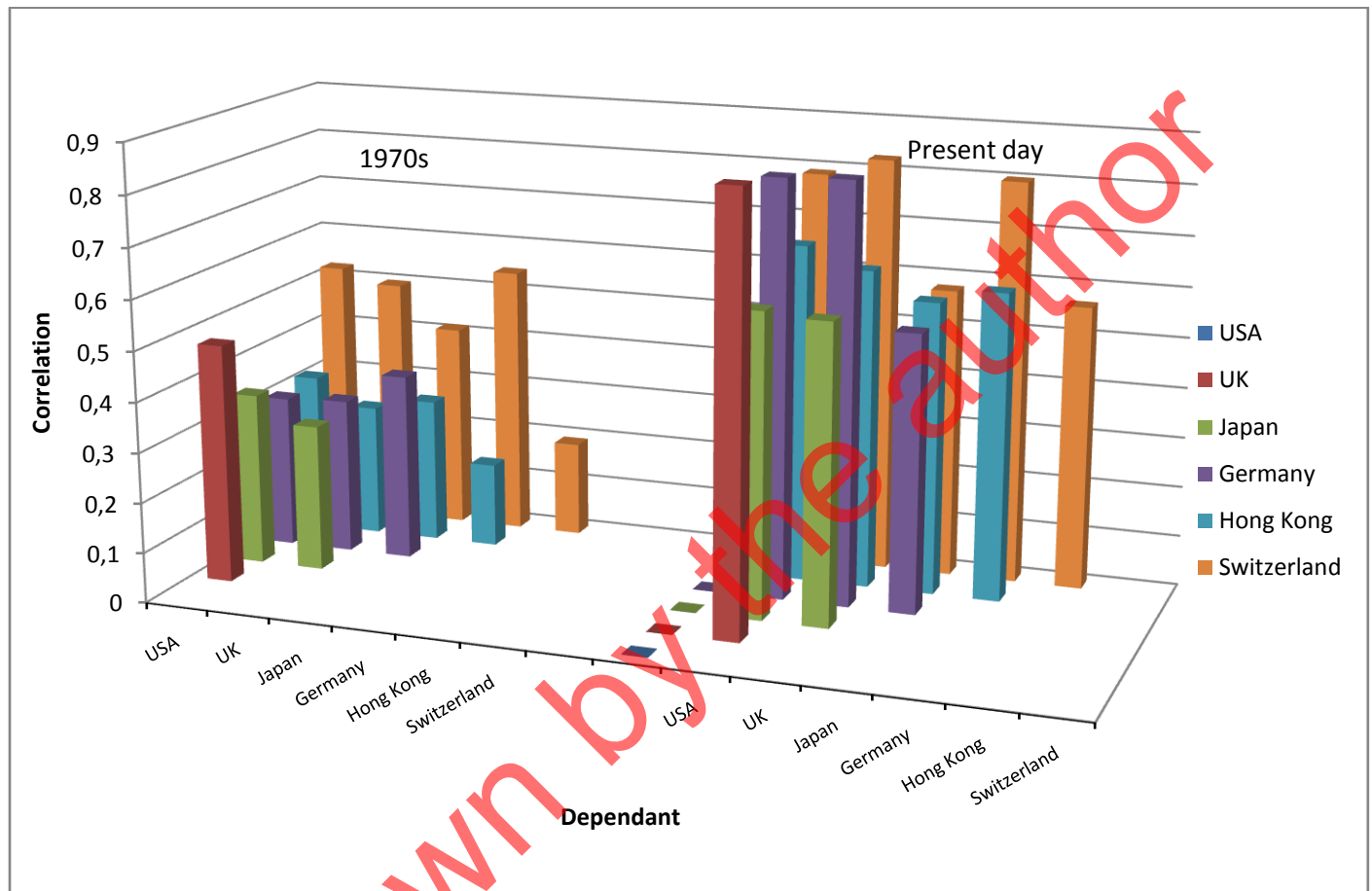
Liquidity certainly matters for many other classes of assets as well, such as fixed income securities or collectibles. However, when it comes to stocks, our recommendation is to stick to small firm premium when checking for excess returns over CAPM of smaller and less liquid companies.

#### 4.2.3 COUNTRY RISK PREMIUM

The country risk premium arises from the fact that over the last several decades economies around the globe have become very closely linked. Correlations between international stock markets have been increasing and possibility for international diversification – diminishing. During times of distress these correlations can become excessively high as seen in the recent contagion of stock market crashes worldwide.

Given the fact that all investors should be rewarded for any non-diversifiable or systematic risk, CAPM betas theoretically should also explain why global investors investing in two stocks with same underlying business/financial risks in two different countries demand different returns. However, CAPM and its variants (ex. Global CAPM) fails to do so despite the fact that the risk of investing in what investors perceive as a more risky country qualifies as systematic risk: it is non-diversifiable (or extremely limited due to high correlations between international stock markets) while barriers for cross-border investments (significantly higher transaction cost etc.) have been virtually erased with developments of technology. In fact, Damodaran shows that betas in riskier emerging market not only fail to gauge these extra risks, but are lower if regressed against a proxy of global market portfolio such as MSCI World Index instead of local index.

**FIGURE 4.5: STOCK MARKET CORRELATIONS ACROSS REGIONS AND TIME**



Source: Author's calculations based on MSCI Barra regional index data. Correlations based on ten-year average monthly returns in local currencies.

Damodaran suggests that some analysts also calculate country risk premium by measuring relative (higher) volatility of emerging market over mature market (say, Brazil over US). They then multiply equity premium (US) by the relative volatility factor to obtain a higher equity risk premium:

$$CRP = MRP_{US} \frac{\sigma_{EM}}{\sigma_{US}} - MRP_{US} \quad (4.2)$$

where **CRP** – country risk premium, **MRP<sub>US</sub>** – market risk premium in the developed market (e.g.US), **σ<sub>EM</sub>** – stock market volatility in the emerging market, **σ<sub>US</sub>** – stock market volatility in the developed market.

This might work for a country like Brazil that has relatively high trading volumes. However, in a country like Bulgaria volatility of local market might be lower than in a mature market like US simply due to lower trading volumes which eventually will result in a negative country risk premium.

Damodaran further suggests that one could combine relative local equity market volatility with default spread. You first obtain relative emerging stock market volatility with respect to the volatility of country's sovereign debt (EM stock market volatility/EM sovereign bond volatility). You then multiply it with country default spread. This presumably captures both equity market volatility and default risk associated with the country. However, given the fact that this relative volatility factor measures stock market volatility in local terms, one should question if this method does not defeat the purpose of calculating country risk in the first place. After all, we are trying to bring in a perspective of a global investor when adding extra country risk premiums in the CAPM.

In the end, the most straightforward and widely accepted method for estimating country risk premium is (i) using standalone default spread of country's sovereign debt; or (ii) regressing a country's sovereign debt returns against risk-less debt returns. Both methods essentially should yield similar results. Perhaps the latter would be more responsive to recent changes in the economy and stock market behaviour as ratings often lag the markets but it would then have to be adjusted for non-recurrent events as these would distort a forward looking view.

There is couple of problems related to extracting country risk premium from default spreads. First, country's sovereign debt might not be denominated in the same currency as the riskless debt making it hard to compare with risk-free rate. Second, country's sovereign debt might not be rated.

To overcome the first problem, one could cross-reference two identical ratings of emerging market sovereign bonds and use the one that has sovereign debt denominated in major currencies.

If country's sovereign debt is not rated, hence the second problem, one could look up for a comparable country that has a similar country risk score in the Economist or other similar services that ranks countries based on fundamental economic and political risks. We could then attach a bond rating of this comparable country and derive a sovereign default spread thereof. The problem with such rankings though is that they are not linear (rank 20 does not implicate that a country is twice as risky as that which is ranked 10<sup>th</sup>).



In practice, country risk premiums mostly are added on top of equity risk premium of a mature market such as US. As a result, beta that is larger than 1 essentially amplifies country risk, while beta that is lower than 1 diminishes it. This way we assume that the subject company is exposed to country risk as much as to the market risk<sup>47</sup>:

$$A.1) k_{e_{USD}} = \beta(MRP_{US} + CRP) + Rf_{US} \quad (4.3)$$

where  $\beta$  - stock beta,  $MRP_{US}$  – market risk premium in the developed market (e.g.US),  $CRP$  – country risk premium, and  $Rf_{US}$  – risk free rate in the developed market.

Because inputs used for calculation of cost of equity are quoted in USD terms, one has to account for inflation rate differentials between US and the emerging market:

$$A.2) k_e = (1 + k_{e_{USD}}) \left( \frac{1 + \pi_{EM}}{1 + \pi_{USD}} \right) \quad (4.4)$$

where  $\beta$  - stock beta,  $k_e$  – cost of equity in the emerging market in local currency,  $k_{e_{USD}}$  – cost of equity in the emerging market in USD terms,  $\pi_{EM}$  - inflation rate in the emerging market and  $\pi_{USD}$  - inflation rate in the US (or equivalent).

Another method offered by practitioners is to use local risk free rate over developed market risk premium in calculations of cost of equity in emerging market:

$$B.1) k_e = \beta(MRP_{US} + Rf_{EM}) + Rf_{EM} \quad (4.5)$$

where  $\beta$  - stock beta,  $k_e$  – cost of equity in the emerging market in local currency,  $MRP_{US}$  – market risk premium in the developed market (e.g.US),  $Rf_{EM}$  - risk free rate over the developed market risk premium.

To account for the effect of local currency, one can use currency swap spread which in turn is considered to incorporate both country risk as well as currency risk.

$$B.2) Rf_{EM} = Rf_{USD} + FXS \quad (4.6)$$

$Rf_{EM}$  - risk free rate over the developed market risk premium,  $Rf_{US}$  – risk free rate in the developed market,  $FXS$  – currency swap spread.

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<sup>47</sup> If we assume that every company in the country is exposed to country risk equally, the country risk is incorporate the following way:  $K_e = \beta(MRP) + Rf + \text{Country risk}$

There are couple of points to question regarding the currently used methodology in practice. First, are country risk premiums really sufficient to compensate for lower betas regressed against a global index?

Damodaran estimates that an average beta in an emerging market is lower when regressed against a global index than regressed against a local index. For instance, he estimates that an average beta in Brazil is 0.98 on local basis and 0.81 on global basis. Assume 10 year US t-bond rate is 3.65% and Brazilian 5.25%. Equity risk premium in the US is 4.3%. Given the latter, the cost of equity for an average company in Brazil estimated on a global basis would be:

$$K_e = \beta_{global}(MRP_{US} + CRP) + Rf_{global} = 0.81 (4.3\% + 1.6\%) + 3.65\% = 8.43\%$$

However, assuming that businesses in Brazil are less risky on a global scale than on a local scale stands against common sense<sup>48</sup>. If we have used beta regressed on local market cost of equity for an average Brazilian company would be:

$$K_e = \beta_{local}(MRP_{US} + CountryRisk) + Rf_{global} = 0.98 (4.3\% + 1.6\%) + 3.65\% = 9.43\%$$

Let alone x% out of country risk premium would then make up for the failure of betas to capture all systematic business risks measured on a global scale:

$$0.81 (4.3\% + 1.6\% + x) + 3.65\% = 9.43\% \\ x = 1.23\%$$

It is rather striking to find that if betas were lower, country risk premium should be higher by more than 1% over the estimated 1.6%.

The second aspect that raises questions and should wary anyone intending to apply the country risk premium relates to credit default swap spreads for sovereign debt as a proxy for country risk. Damodaran shows that CDS spreads are only marginally different from sovereign debt default spreads on average and virtually the same at some points over time. Having that in mind, he suggests that CDS spreads could be a measure of country risk premium. At the time of the study, 14 Feb 2010, CDS market yielded 1.58% for Brazilian sovereign long term debt, while default spread on the same Brazilian debt was 1.6%. However, CDS spread on US t-bonds was 49bp. If we assume that US t-bonds are riskless,

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<sup>48</sup> Since betas measure both directional change and magnitude,  $\beta = \rho (\sigma / \sigma_m)$ , we assume that the lower betas of emerging markets regressed on a global index result from: (i) lower volatility of EM stocks due to lower trading volumes or (ii) lower correlation of EM stocks with global index due to developed market stocks comprising the index.

country risk premium on Brazil should then be naturally lower than approximately 1.6%. In fact, just a month later US government 10-year swap spread turned negative, implying that for the first time in history the cost of borrowing for corporations (as measured by the 10 year swap rate) was lower than the cost of borrowing for US government. Not surprisingly, high quality companies had seen their debt offering lower yields than US treasuries.

Some analysts prefer using corresponding US corporate bond spreads as proxies for country risk given much higher participation and liquidity in US corporate debt market as compared to foreign sovereign debt. Bearing in mind the discussion earlier, we believe that corporate debt spreads might be a better proxy for country risk premium as well.

PriceWaterhouseCoopers have a slightly different approach to country risk premium. They have developed a uniform model that establishes a synthetic rating based on multiple rating agencies. They take the following steps to come up with a country risk premium.

- (i) Obtain direct bond market default spreads for a sample of countries where bonds are comparable, i.e. denominated in foreign developed market currency (e.g. USD), have similar duration and liquidity;
- (ii) Obtain sovereign credit scores from rating agencies such as Standard and Poor's, Moody's, Euromoney and EIU;
- (iii) Convert algebraic ratings into numerical percentages representing risk levels. The 20 bands of Moody's and Standard and Poor's ratings are converted into numerical percentages using a linear relationship. Euromoney and EIU provide the risk assessment already in numerical percentage form.
- (iv) Regress numerical percentages against sovereign default spreads of the sample countries where there is direct sovereign bond information.
- (v) Obtain statistical relationship between numerical percentages and default spreads to derive a predictive model of country risk. PwC finds a strong exponential relationship with R squared over 90% between the numerical risk level percentages and sovereign default spreads. They build this exponential relationship for all 4 rating agencies, obtain a risk premium based on sovereign debt default spread for each and then average all available country risk premiums to arrive at a single measure of country risk premium for an emerging country. They then apply the country risk premium for both cost of equity and cost of debt on top of the risk free rate:

$$Rf_{EM} = (1 + Rf_{US})(1 + CRP) \quad (4.7)$$

where  $Rf_{EM}$  – risk free rate in the emerging market,  $Rf_{US}$  – risk free rate in developed market (e.g. US) and  $CRP$  – country risk premium (in this case sovereign debt spread).

Kruschwitz, Lofflery and Mandl (2010) argue that using CDS and sovereign debt spreads as suggested by Damodaran and used by others is an intrinsically wrong method and that country risk premium generally can only be applied in multi-factor asset pricing models<sup>49</sup>. Revoltella, Mucci and Mihaljek (2010) point out that CDS spreads tend to overshoot 10-20 times during times of distress making them a loose proxy for country risk premium<sup>50</sup>. They suggest that in order to price country risk using CDS spreads it is necessary to remove the market sentiment.

Finally, one should not forget that emerging markets that have country risk premiums attached mature over time. This would technically demand for adjustment going forward. However, bearing in mind that country risk premium has been applied only for a limited period of time, there is no in-depth research carried out up to date regarding the adjustment.

All in all, country risk premium estimates have been largely based on bond spreads and credit default spreads with only minor suggested improvements. Although Damodaran also proposes calculating country risk premium using implied equity premia, due to the biases that implied equity risk premiums remain exposed to, especially in the emerging markets, we do not recommend this approach. Until then, we suggest to use CDS and bond spreads as proxies for country risk premium.

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<sup>49</sup> Kruschwitz L. , Lofflery A., Mandl G. (2010), “Damodaran’s Country Risk Premium: A Serious Critique”, on Social Sciences Research Network

<sup>50</sup> D. Revoltella, F. Mucci and D. Mihaljek (2010), “Properly pricing country risk: a model for pricing long-term fundamental risk applied to central and eastern European countries”, Financial Theory and Practice vol. 34 (3) p. 219-245

#### 4.2.4 COMPANY-SPECIFIC RISK PREMIUM

There have been attempts to account for extra-risks in the CAPM that relate to company-specific risk factors. Grabowski (2008) argues for company-specific risk adjustments based on the fact that typically investors do not hold diversified portfolios as assumed by CAPM and seek for advice on portfolio diversification. This would be a good argument if diversification proved either extremely costly or unobtainable as in the case of country risk premium. However, large institutional investors have been long diversified, while index funds have made diversification more feasible than ever before even for the smaller investors. Thus, we believe that adding company-specific risk premium looks more like a desire to tweak CAPM just to match required returns of undiversified investors than to correct for a failure in the general CAPM framework. As Brealey, Meyers and Allen explain, company specific risk factors should be instead incorporated in the cashflow forecast. In our opinion, this would not only fit the general CAPM framework better, but also assure that company specific risks are not double-counted.

For instance, valuing firms in distress, an analyst should calculate expected values of future cashflows that incorporate upside and downside scenarios, such as bankruptcy, rather than attaching additional idiosyncratic risk premiums and then extrapolating historical cashflows. On the other hand, the systematic risk arising in distressed environment would then be captured by equity<sup>51</sup> as well as positive debt betas in the calculations of cost of capital.

With regards to capital structure, firms in distress often deviate from its target capital structure for several periods. We believe that adjusting expected value of cashflows should also offset the cost of capital levered for target D/E ratio that is consistent only with a healthy business environment.

Note that whatever the adjustments are, it is important to treat risk factors based on the underlying assumptions of the asset pricing models.

All in all, we recommend avoid using any additional company-specific risk premiums as these risk should be captured in the variation of the expected cashflows.

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<sup>51</sup> As pointed out before, during times of market downturn and distress correlations between the troubled stocks and the market tend to 1. If we used shorter look-back period and higher data frequency to estimate equity betas of distressed companies, they should be higher, hence, reflect any systematic risk of distress. Otherwise, one could use bottom-up industry betas and relever them to higher D/E ratios arising from depressed equity values of a distressed company.

The former, however, is valid for distressed firms only. Healthy stable firms such as those in consumer industries often see correlations diverge when market plummets. Therefore, one should avoid using same sample period of estimating beta for stable companies as they might then be artificially low.

## 5 COST OF DEBT

Calculating cost of debt is a rather straightforward process as compared to cost of equity. Basically speaking, it is expected yield to maturity on its debt. However, the problem is that expected values are not available and only the promised rate of return based on promised coupon payments of company's bonds can be implied. The promised YTM is found by solving for internal rate of return of company's long term bond payments (e.g. using goal seek or solver).

The bonds of choice for calculating cost of debt should be long-term because of the same underlying reasons as the long-term risk-free rate used in the cost of equity calculations. It should also be liquid enough so that prices (and therefore YTM) would be responsive to market conditions. Furthermore, it should not have options attached since they affect price without having any impact on coupon payments which in turn distort yield to maturity and our proxy for cost of debt. Finally, coupon rate can be used as proxy for cost of debt only at the time the bond is issued and only if it is issued at par value.

If the bonds traded do not meet the criteria above, one can either infer the cost of debt by using credit spreads provided by the rating agencies or calculate synthetic spreads.

If the bond is rated by a rating agency such as S&P and Moody's, cost of debt is easily obtained by adding the related bond spread to a risk-free rate (US Treasuries).

For companies with high-yield debt only YTM would be a bad proxy for cost of debt. Because high yield debt has disproportionately larger yields than those of investment grade bonds due to different underlying probabilities of default as well as recovery rates, cost of debt would be excessively high using YTM. A solution then is to use a simple CAMP and high-yield debt betas to estimate cost of debt for such companies.

Debt betas can be obtained using the methods outlined previously in the related section.

If the company is not rated, Damodaran suggests checking the recent bank borrowing history or estimating a synthetic rating using interest coverage ratios. Once an interest coverage ratio is established, one can reference it to the relevant credit rating to estimate an implicit spread:

**TABLE 5.1: INTEREST COVERAGE RATIOS AND CREDIT RATINGS**

Interest Coverate Ratio: Small Market Cap (<\$5 billion)	Interest Coverate Ratio: Large Market Cap (>\$5 billion)	Rating	Typical Default Rate
<12.5	>8.5	AAA	1.25%
9.50-12.50	6.5-8.5	AA	1.75%
7.50-9.50	5.5-6.5	A+	2.25%
6.00-7.50	4.25-5.5	A	2.50%
4.50-6.00	3-4.25	A-	3.00%
4.00-4.50	2.5-3.0	BBB	3.50%
3.50-4.00	2.00-2.25	BB+	4.25%
3.00-3.50	2.00-2.25	BB	5.00%
2.50-3.00	1.75-2.0	B+	6.00%
2.00-2.50	1.5-1.75	B	7.25%
1.50-2.00	1.25-1.5	B-	8.50%
1.25-1.50	0.8-1.25	CCC	10.00%
0.80-1.25	0.65-0.8	CC	12.00%
0.50-0.80	0.2-0.65	C	15.00%
<0.65	<0.2	D	20.00%

Source: Compustat and Bondsonline.com. Calculations by Damodaran (2009).

This is, however, a very rough approximation since rating agencies use multiple ratios and even qualitative information such as interviewing top management regarding the future plans of a company to set up a rating. Table 5.2 below by Moody's presents multiple dimensions over which synthetic rating could be established.

**TABLE 5.2: FINANCIAL RATIOS AND CREDIT RATINGS**

Universe of companies	AAA	AA	A	BBB	BB	B	CCC
EBITA/Average Assets	15.3%	15.6%	12.5%	10.1%	9.6%	7.3%	2.0%
Operating Margin	14.9%	17.0%	13.8%	12.6%	12.2%	8.5%	1.6%
EBITA margin	14.8%	17.5%	15.2%	13.9%	13.4%	9.4%	2.4%
EBITA / Interest	17.00x	13.70x	8.20x	5.10x	3.40x	1.50x	0.30x
[FFO + interest expense] / Interest expense	15.5x	15.5x	9.6x	6.6x	4.7x	2.6x	1.5x
Total Debt/EBITDA	0.90x	1.00x	1.70x	2.40x	3.30x	5.00x	6.30x
Total Debt / [Total Debt + Equity + Minorities]	22.7%	32.5%	39.1%	44.8%	53.5%	70.2%	92.5%
FFO / Debt	117.3%	68.4%	43.8%	29.2%	21.8%	12.0%	4.3%
[FFO - Dividends Paid] / Net Debt	96.3%	38.4%	38.7%	27.7%	20.0%	11.7%	4.6%
CAPEX / Depreciation	1.60x	1.40x	1.30x	1.20x	1.20x	1.10x	0.90x
Number of companies	6	35	176	354	436	442	56

Source: Moody december 2007, global universe including NA, EMEA and AsiaPacific

For emerging market companies Damodaran suggests restating ratios in dollar terms to account for extra risks that these companies might exhibit. Moreover, it has already become a general practice to add a country risk premium on top of company default spread while calculating cost of debt of an

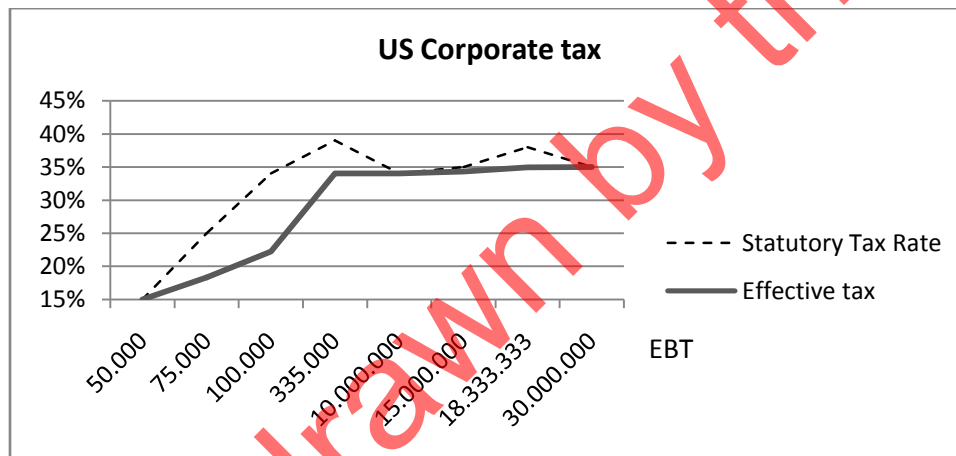
emerging market company. Nevertheless, Damodaran argues that this might be correct only if the company is riskier than the country it operates in and only if the revenues are generated in the emerging market.

## 5.1 TAXES

Since interest is tax deductible and reduces company's earnings in the income statement before the corporate tax is applied, thus providing an interest tax-shield, analysts use cost of debt on after-tax basis by multiplying it with  $(1 - \text{Tax Rate})$  in the enterprise valuation.

For valuation purposes, most analysts use marginal tax rate which sometimes differs from the effective tax rate obtained by dividing taxes due by the taxable income. Marginal tax rate, on the other hand, measures the rate at which an extra dollar would be taxed, hence the marginal.

**FIGURE 5.1: CORPORATE TAX RATES IN THE UNITED STATES**



Source: Author's calculations

For example, in the US marginal tax rate can vary depending on the current taxable income and the incremental income that a company is expected to receive in the near future. Because there are statutory tax bubbles in the tax brackets, companies might see marginal income taxed at higher rates than the effective tax rate on the total taxable future income. However, as taxable income grows beyond the highest bracket of \$18,333,333, marginal tax rate converges to effective tax rate which is flat at 35%. Generally, marginal tax rate can be defined as:

$$MTR = \frac{\text{Change in PV of Tax liability}}{\text{Change in PV of Taxable Income}} \quad (5.1)$$



Some practitioners define marginal taxes as those that the company would pay if the financing or non-operating items were removed<sup>52</sup>. Foreign operations or debt might, however, complicate the calculation of marginal tax rate.

Notwithstanding, the main problem is in the legal environment that most academics and practitioners identify while trying to determine the marginal tax rate. Graham in his research<sup>53</sup> provides with a simulation where he shows that most of the companies will not realize tax shields at statutory rates because of tax-loss carryforwards and backwards. This means that companies after recovering after losses would have to operate profitably for several years until they can enjoy tax deductions again. Eventually, this would lead to lower marginal tax rate, which according to simulation, is lower than statutory rate by 5% for an average US company. As a result, if a historical effective tax rate is lower than marginal tax rate, it might be reasonable to assume a lower rate. On the other hand, for healthy strong companies marginal tax rate would be as high as statutory rate in the last tax bracket, i.e. 35% for US.

To sum up, calculating cost of debt is not as complex task as calculating cost of equity. However, there are several caveats, notably a reasonable forward looking marginal tax rate as well as synthetic rating that demand extra diligence when calculating the after-tax cost of debt.

## 6 WEIGHTED AVERAGE OF COST OF CAPITAL

Weighted average cost of capital is mainly used to value the whole enterprise or the equity thereof. The most appropriate framework portraying WACC valuation is the following:

Value of a Levered Firm = Value of Levered Assets = Value of Debt – Value of Tax Shields + Value of Equity

In such case an after-tax cost of capital is applied to net after-tax cashflows of a firm assuming tax deductibility of interest expense:

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<sup>52</sup> See Tim Koller, Marc Goedhart, and David Wessels, *Valuation: Measuring and Managing the Value of Companies*, 4<sup>th</sup> ed. (John Wiley & Sons, 2005), 177.

<sup>53</sup> John Graham, "Debt and Marginal Tax Rate", "Proxies for the Corporate Marginal Tax Rate" (1996), "Using Tax Return Data to Simulate Corporate Marginal Tax Rates" (2007)

$$WACC = (k_e \times W_e) + (1 - t)(k_d \times W_d) + (k_h \times W_h) \quad (6.1)$$

where:

$WACC$  – weighted average of cost of capital

$k_e$  – cost of equity capital

$k_d$  - cost of debt capital

$k_h$  - cost of hybrid securities

$W_e$  - weight of equity capital in the capital structure

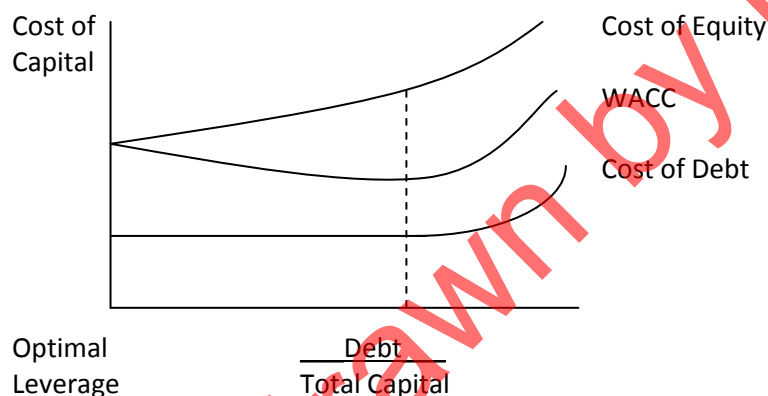
$W_d$ - weight of debt capital in the capital structure

$W_h$  - weight of hybrid securities in the capital structure

In traditional WACC framework a company should use capital structure that minimizes its cost of capital therefore maximizing present value of future cashflows it generates and the company value thereof.

WACC as a function of total leverage can be depicted with the classical WACC “smile” that plots the relationship between leverage, overall cost of capital and its elements:

**FIGURE 6.1: RELATIONSHIP BETWEEN WACC, COST OF EQUITY AND COST OF DEBT**



The shape of the WACC smile is also consistent with the recent research<sup>54</sup> which concludes that excess leverage has a more negative impact for a company than insufficient leverage (you can observe WACC curve increasing faster from the point of optimal leverage when extra debt is levied, not when it marginally decreases).

Amounts of debt that companies carry are often related to profitability and cyclicity of business.

<sup>54</sup> Jules Van Binsbergen, John Graham, and Jie Yang, “ The Cost of Debt”, Working paper (Sep 2007)

When a company is more profitable, it can afford more leverage without running a risk of falling behind its interest payments. On the other hand, when a company is less profitable and there is a risk that it will not be able to meet its debt obligations, adding extra leverage will increase the risk of default, and debt and equity holders will demand extra compensation for such risk.

Businesses that are mature and less susceptible to economic shifts, i.e. low beta firms, such as those in consumer goods, often carry significant amounts of debt. High-tech companies often are financed mainly by equity as they carry higher risks of falling short in cash since they use every marginal dollar for new product developments.

Though levels of leverage vary quite extensively among industries, researchers found that they might vary even more within an industry<sup>55</sup>.

To estimate WACC one should either use current market values of debt and equity outstanding or target (industry) levels assuming that a company operates under suboptimal capital structure temporarily. Our suggestion is to apply target weights as in volatile markets outstanding values of equity and debt might not be reflective of future long-term periods. If there are significant changes in the capital structure, one should use varying WACC adjusted for both different weights of cost of debt and equity. Under such circumstances though, most practitioners suggest using APV valuation method.

When choosing industry D/E ratios, we advise to estimate median values instead of using averages. As Savage (2009) recalls, one should always remember a statistician who drowned crossing a river that is on average 3 feet deep<sup>56</sup>. In other words, one should not make a judgement about a central value in the sample without knowing the distribution. Even if you do know the distribution and you notice that several points in the sample are extreme values, mean is very likely to understate/overstate a true forward-looking central value of the sample. Generally, Savage argues that decisions based on build-up methods with underlying inputs as *averages* are on average wrong. There has also been research carried out that shows simple mean values to consistently overstate valuation multiples<sup>57</sup>. All in all, it is a generally accepted truth in statistical-financial sampling to use medians. However, as an exception to

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<sup>55</sup> Peter MacKay and Gordon Phillips, "How Does Industry Affect Firm Financial Structure?", *The Review of Financial Studies*, Issue 4 (2005)

<sup>56</sup> Savage S., "The Flaw of Averages: Why We Underestimate Risk in the Face of Uncertainty", Wiley (2009)

<sup>57</sup> See Ruback R.S. and Baker M., "Estimating Industry Multiples" (1999). The authors find that the best measure of central tendency for multiples is harmonic mean that lies in between arithmetic and geometric means. Harmonic mean does not suffer from misweighting the data in the sample. However, further empirical research on application of harmonic mean in corporate finance has not been carried out so far.

that, we suggest to use mean value if the data sample is really small without significant outliers. Median basically eliminates information from data points around it and if the sample is small we believe we cannot afford that.

Returning to D/E ratios, should one decide to use current market values (say, for acquisition purposes), it is important to include the value of debt equivalent claims such as operating lease.

The value of operating lease liabilities can be estimated using the following formula (6.2) below. Note that debt equivalent claims should be incorporated in total debt only if one plans to adjust free cash flows for these claims later on as well.

$$\text{Lease Value}_{t-1} = \frac{\text{Rental Expense}_t}{k_d + \frac{1}{\text{Asset Life}}} \quad (6.2)$$

If debt and equity values cannot be extracted from market information, e.g. as it happens for private companies, one can look up for book value of debt, although it is recommended to avoid using book values for distressed companies as market value of such liabilities might be significantly lower. Equity values can be either deduced using multiples or running an iterative DCF. Iterative DCF is performed assuming reasonable capital structure; it is then repeated using the resulting debt to enterprise values until valuation does not yield significantly different results.

In summary, up till now we have outlined the process of calculating cost of capital. Employing the methods described in the Chapter 3 and Chapter 4 one should be able to arrive at the estimate of cost of equity backed up by extensive considerations. Chapter 5 has provided a brief yet detailed overview on both how to estimate the cost of debt for rated and non-rated companies as well as why an analyst should treat marginal tax rate with care for after-tax cost of debt calculations. Finally, in the current chapter we put the main elements of weighted average cost of capital together, introduce several important considerations on capital structure and close the overview of cost of capital.

In the following section, we look at how sensitive to the underlying assumptions is one of the building blocks of cost of capital, namely cost of equity.

## 7 COST OF EQUITY CAPITAL SENSITIVITY TO BETA ESTIMATION TECHNIQUES

Cost of equity is undoubtedly the most widely debated element of overall cost of capital among academics and practitioners. Equity premiums have been researched since the dawn of CAPM, and yet, there is no single consensus on how to best estimate these premiums. Betas, however, have been surrounded less by this ambiguity. This is partly because of a wide acceptance of few major services that have been proposing their methodologies and providing their estimates of systematic risk measures for at least several decades now. Nonetheless, many users of CAPM often forget how crucial the choices made are in calculating betas.

The following research provides a good insight on how different methods in calculating beta from the returns of a single stock can impact the cost of equity capital. To illustrate the varying betas and COEC, we analyze returns of Kraft Foods Inc. on a number of dimensions often proposed in COEC estimations.

### 7.1 METHODOLOGY

To check the results that different beta estimation methodologies yield, we run 16 different regressions based on a combination of choice of index, data sample period and frequency, possible adjustment for non-recurring events in the sample and, finally, calculation technique of beta itself. The two beta regression techniques used are Ordinary Least Squares and Sum-Beta. Sum-beta is estimated adding two independent regression coefficients: first, company's excess returns over market's excess returns, and second, company's excess returns over previous period's market excess returns. Excess returns are calculated over 30-day Treasury bill yields corresponding to the dates of observations.

The 16 resulting betas are unlevered using three different methodologies to result in a total of 48 different betas, which are then rounded to 2 decimals. The methodologies used are Hamada, Harris-Pringle as well as additional methodology often applied by practitioners. We unlever the raw betas using debt-to-equity ratios provided by Thompson Reuters. Despite rather significantly varying D/E estimates for Kraft Foods Inc. by the author and other services, Thompson Reuters is chosen to remain consistent given the fact that it also provides corresponding industry and sector D/E ratios, which are later used to relever betas. We use this company debt to equity ratio as well as the latest book values to total debt (inclusive of all interest bearing debt and lease obligations) to estimate the value of debt and equity all together. Using the latest book value of cash and short-term investments, we adjust the unlevered beta

for cash and estimate the beta of operating assets. The betas are then relevered using sector D/E ratios assuming that a median company in the sector operates under efficient capital structure. Finally, the relevered betas are smoothed using the Blume's adjustment.

The frequencies and sample periods used are 5 year sample using monthly observations and 2 year sample using weekly observations. We select these settings under assumption that it is the most widely accepted choice by both services and by practitioners.

To adjust for non-recurring events we inspect for extreme trading volumes and low correlations. We have determined a period of recently lower correlations between the stock and the market during FY2009 Q3 through FY2010 Q2. We exclude this period based on the assumption that merger negotiations between Kraft and Cadbury that took place recently distort a forward looking relationship between the Kraft's stock and the market. To make up for excluded observations, we extend the look back period to match the size of data sample across regressions.

The marginal tax rate used to unlever/relever betas using Hamada formula is 35%. This is above the effective tax rate of 29.37% for FY2009. However, we use the highest future marginal statutory tax rate expecting a significant increase in Kraft's taxable income going forward. After paying down the debt, we believe that an already global dominant position of Kraft in food production and retailing strengthened by the merger with Cadbury will further add to growth in Kraft's EBT. Kraft ranks #53 on Fortune 500 list.

The debt beta used to unlever/relever betas using Harris-Pringle formula is 0.2. This an average estimate of debt betas calculated between 1963-2009 by Duff & Phelps on companies listed in NYSE, AMEX and NASDAQ. We believe the estimate is a representative of medium-investment grade credit rating that Kraft Foods have.

The overall 48 resulting betas are then referenced, ranked by size and included in a table measuring Kraft's COEC sensitivity to the choice of market risk premium and beta.

**TABLE 7.1: METHOD OF CHOICE AND THE RESULTING BETAS FOR KRAFT FOODS INC.**

Index choice	Sample period/frequency	Data	Regression	Results:Raw Beta	Unlevering formula	Results: Unlevered Beta	Adjusted for Cash	Relevered to sector	Smoothed (Blume's adjustment)	Reference number
S&P 500	5 years/ monthly	Recent Sample	Sum-Beta	0.74	Hamada	0.46	0.49	0.62	0.74	(1.)
					Harris-Pringle	0.48	0.51	0.63	0.75	(2.)
					Practitioners	0.39	0.41	0.58	0.72	(3.)
			OLS	0.59	Hamada	0.37	0.39	0.50	0.66	(4.)
					Harris-Pringle	0.41	0.43	0.52	0.68	(5.)
					Practitioners	0.31	0.33	0.46	0.64	(6.)
		Ex.non-recurring events	Sum-Beta	0.89	Hamada	0.56	0.59	0.75	0.83	(7.)
					Harris-Pringle	0.56	0.59	0.75	0.84	(8.)
					Practitioners	0.47	0.49	0.70	0.80	(9.)
			OLS	0.67	Hamada	0.42	0.44	0.56	0.71	(10.)
					Harris-Pringle	0.44	0.47	0.58	0.72	(11.)
					Practitioners	0.35	0.37	0.52	0.68	(12.)
	2 years/ weekly	Recent Sample	Sum-Beta	0.59	Hamada	0.37	0.39	0.50	0.66	(13.)
					Harris-Pringle	0.41	0.43	0.52	0.68	(14.)
					Practitioners	0.31	0.33	0.46	0.64	(15.)
			OLS	0.56	Hamada	0.35	0.37	0.47	0.64	(16.)
					Harris-Pringle	0.39	0.41	0.49	0.66	(17.)
					Practitioners	0.29	0.31	0.43	0.62	(18.)
		Ex.non-recurring events	Sum-Beta	0.59	Hamada	0.37	0.39	0.50	0.66	(19.)
					Harris-Pringle	0.40	0.43	0.52	0.68	(20.)
					Practitioners	0.31	0.33	0.46	0.64	(21.)
			OLS	0.61	Hamada	0.39	0.41	0.52	0.68	(22.)
					Harris-Pringle	0.42	0.44	0.54	0.69	(23.)
					Practitioners	0.32	0.34	0.48	0.65	(24.)
MSCI World	5 years/ monthly	Recent Sample	Sum-Beta	0.65	Hamada	0.41	0.43	0.55	0.70	(25.)
					Harris-Pringle	0.44	0.46	0.57	0.71	(26.)
					Practitioners	0.34	0.36	0.51	0.67	(27.)
			OLS	0.54	Hamada	0.34	0.36	0.45	0.63	(28.)
					Harris-Pringle	0.38	0.40	0.48	0.65	(29.)
					Practitioners	0.28	0.30	0.42	0.61	(30.)
		Ex.non-recurring events	Sum-Beta	0.77	Hamada	0.49	0.51	0.65	0.77	(31.)
					Harris-Pringle	0.50	0.53	0.66	0.77	(32.)
					Practitioners	0.41	0.43	0.60	0.73	(33.)
			OLS	0.60	Hamada	0.38	0.40	0.51	0.67	(34.)
					Harris-Pringle	0.41	0.43	0.53	0.69	(35.)
					Practitioners	0.32	0.33	0.47	0.65	(36.)
	2 years/ weekly	Recent Sample	Sum-Beta	0.53	Hamada	0.33	0.35	0.44	0.63	(37.)
					Harris-Pringle	0.37	0.39	0.47	0.65	(38.)
					Practitioners	0.28	0.29	0.41	0.61	(39.)
			OLS	0.51	Hamada	0.32	0.34	0.43	0.62	(40.)
					Harris-Pringle	0.36	0.38	0.45	0.63	(41.)
					Practitioners	0.27	0.28	0.40	0.60	(42.)
		Ex.non-recurring events	Sum-Beta	0.53	Hamada	0.33	0.35	0.44	0.63	(43.)
					Harris-Pringle	0.37	0.39	0.47	0.65	(44.)
					Practitioners	0.28	0.29	0.41	0.61	(45.)
			OLS	0.55	Hamada	0.35	0.37	0.47	0.64	(46.)
					Harris-Pringle	0.39	0.41	0.49	0.66	(47.)
					Practitioners	0.29	0.31	0.43	0.62	(48.)

## 7.2 RESULTS

The estimated betas proved to provide the most explanatory power (i.e. highest  $R^2$ ) with the lowest standard error using 2 months data of weekly returns ( $R^2$  above 40% with S.E. below 3%). S&P 500 also proved to explain the relationship between the index and the stock best. This is no surprise, however, bearing in mind that Kraft Foods is both listed in the US where S&P500 is based as well as is part of the index. As a result, in line with the findings of Damodaran, betas regressed on a local index are higher than regressed on a global index.

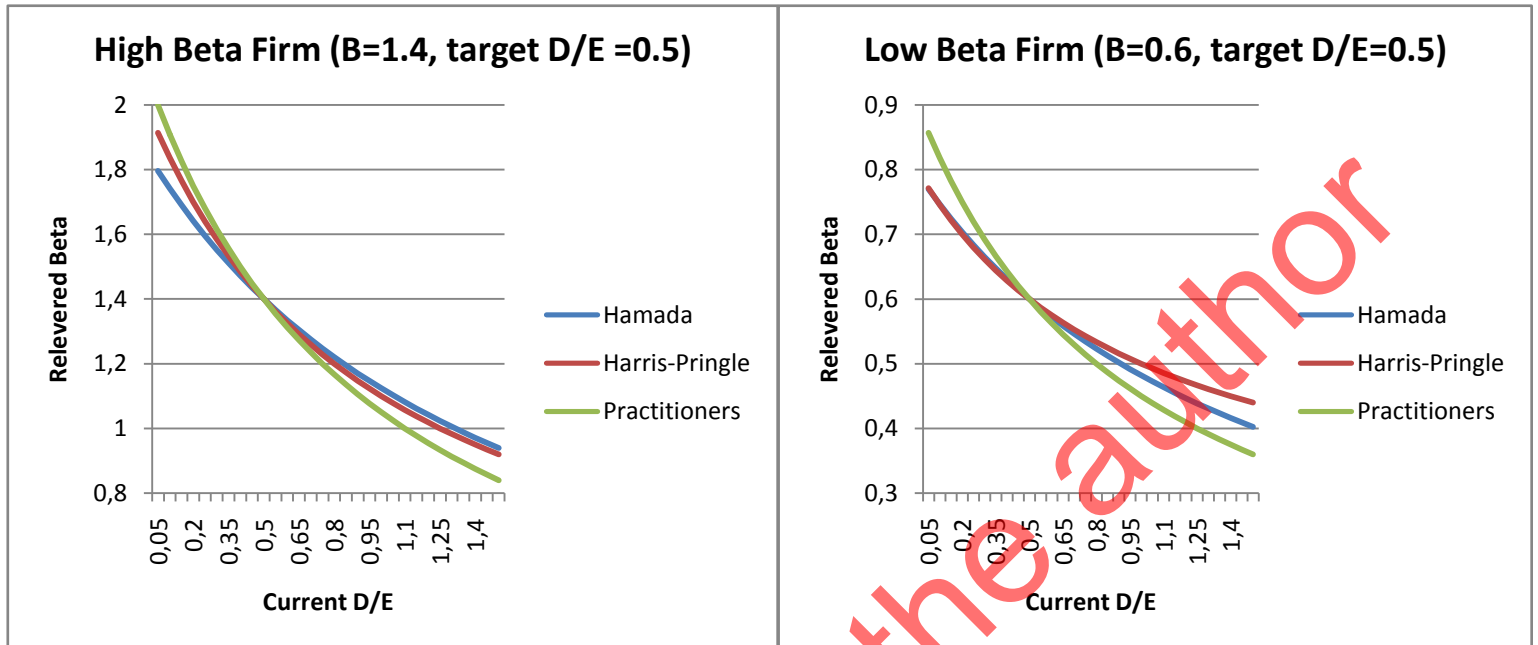
Out of those regressed using two months of weekly data with S&P 500, the most precise betas appeared to be those estimated excluding non-recurrent events from the sample. Both OLS and Sum-Beta yielded  $R^2=0.48$  with S.E.=0.028. Sum-Beta was slightly higher at 0.61 than OLS beta which was estimated to be 0.59.

The highest standard error with lowest  $R^2$  was registered using unadjusted sample of recent 5 years of monthly returns ( $R^2\approx 0.25$  with S.E=0.053).

Betas relevered using Harris-Pringle formula are higher than Hamada or Practitioner's formula across the whole sample of 16 betas, while Practitioner's formula have yielded the lowest relevered betas. This is an interesting contradicting observation to Grabowski's argument that Practitioner's formula treats the risk of realizing tax shields of interest payments higher than any other formula. Most obviously, it does depend whether the company is in a high or low beta industry as well as underleveraged or overleveraged. Figure 7.1 portrays the relationship between the beta relevered using different formulas and the deviation from the target capital structure assuming optimal debt-to-equity ratio of 0.5. One can notice that the patterns are quite different for a sample high beta and low beta firms.



FIGURE 7.1: DEVIATION FROM THE TARGET LEVERAGE AND RELEVATED BETA

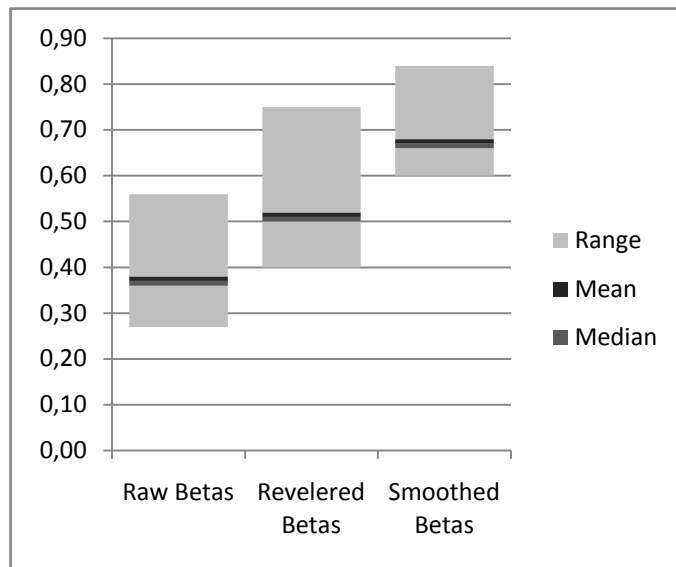


Turning back to the regression results, as you can see from the Figure 7.2, the betas obtain quite a high range of possible values. How does the choice of beta influence the cost of equity for Kraft Foods? You will find in the sensitivity table (Table 7.2) that the method of choice for calculating beta leads to cost of equity capital which varies by 1% depending on the choice of market risk premium used. The sensitivity table also provides a reference to the closest estimates of market risk premiums by academics and services.

1% might seem marginal. However, our subject company operates in a low beta industry which reduces the range of possible regression betas as well as high standard errors. Furthermore, it is one of the largest companies in the world, therefore it experiences little trading delay or investor aversion for small size. For this reason, in our opinion, sum-betas do not yield results significantly different from OLS betas. Furthermore, given the size, company's stock is quite correlated with both local and global index leading to similar regression estimates on local and global basis.

Relevering methods contribute to an increased range of possible long term beta. Nonetheless, smoothing techniques partly correct for the differences in beta estimation methodologies. This is portrayed by a lower range of smoothed betas as compared to raw and relevered betas in Figure 7.2.

**FIGURE 7.2: RANGE OF ESTIMATED BETAS**



What is the impact of beta calculations on the value of Kraft Foods? Let us look into it using 2 betas out of our sample of 48, and compare the differences in enterprise value of a company using simplifying assumptions.

Let us assume we value a company using a beta obtained by regressing an unadjusted sample of 2 years of weekly returns on MSCI World Index. We then relever the beta to sector's D/E ratio using Practitioner's formula. This results in a relevered beta of 0.60 (Ref.no.42). Given the market risk premium of 4.50%, and risk free rate of 2.58%, we calculate cost of equity of 5.28% using CAPM:

$$K_e = 0.60 \times 0.045 + 0.0258 = 5.28\%$$

Further, we roughly estimate cost of debt from the BBB rating given to Kraft Foods:

$$K_d = 0.0258 + spread_{BBB} = 0.0258 + 0.0559 = 8.17\%$$

Assuming D/E ratio of 0.9 and no hybrid securities this gives us WACC of:

$$WACC = W_e K_e + W_d K_d (1 - t) = 52.6\% \times 5.28\% + 47.4\% \times 8.17\% \times (1 - 0.35) = 5.29\%$$

On the other hand, using Sum-Beta of 5 year data sample of monthly excess returns without non-recurring events relevered to target capital structure using Harris-Pringle formula (ref.nr. 8) *ceteris paribus* gives us WACC of 5.86%. Because free cash flow estimates of Kraft and Cadbury are not available on a consolidated basis, we establish a simple proxy to check sensitivity of enterprise value to WACC. Let

us assume a nominal growth for a company in consumer industry of 2% (slightly above average long-term inflation) and free cash flow<sup>58</sup> of \$7bn. The resulting enterprise value using simple perpetuity growth model is:

$$a) EV = \frac{FCF}{WACC - g} = \frac{\$7bn}{5.29\% - 2\%} = \$213bn$$

$$b) EV = \frac{FCF}{WACC - g} = \frac{\$7bn}{5.86\% - 2\%} = \$181bn$$

Although the actual free cash flow estimates are beyond the scope of this research, you can see how large the deviation in enterprise value can be just by using different betas.

### 7.3 CONCLUSION

Which is “the most righteous” beta out of the whole sample? If someone required a strict choice in between 48 different betas, we would quite obviously choose the one with the highest R<sup>2</sup> and lowest S.E. This would be S&P500 based Sum-Betas with weekly frequency and 2 years of sample data exclusive of non-recurrent events. We would prefer Harris-Pringle formula for relevering to target capital structure because it accounts for positive debt betas given BBB rating of Kraft’s stock.

Naturally, there is no single best estimate as well as single best methodology. In the preceding sections of the paper you will find a number of compelling arguments for using different methodologies. Furthermore, the topic is still under scrutiny of many academics. However, we believe that it is a good solution to check consensus as well as sector wide beta estimates before applying a single methodology. After all, it is the many opinions that make up the market and market is always (nearly) right.

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<sup>58</sup> This is just an arbitrary number from separate FCF estimates of Cadbury and Kraft ending FY2009.

**TABLE 7.2: SENSITIVITY OF KRAFT FOOD'S COEC TO THE UNDERLYING INPUTS**

Ref.	Beta	Market Risk Premium									
		3.00%	3.50%	4.00%	4.25%	4.50%	5.00%	5.50%	6.00%	6.50%	7.00%
(42.)	<b>0.60</b>	4.38%	4.68%	4.98%	5.13%	5.28%	5.58%	5.88%	6.18%	6.48%	6.78%
(30.) (39.) (45.)	<b>0.61</b>	4.41%	4.72%	5.02%	5.17%	5.33%	5.63%	5.94%	6.24%	6.55%	6.85%
(18.) (40.) (48.)	<b>0.62</b>	4.44%	4.75%	5.06%	5.22%	5.37%	5.68%	5.99%	6.30%	6.61%	6.92%
(28.) (37.) (41.) (43.)	<b>0.63</b>	4.47%	4.79%	5.10%	5.26%	5.42%	5.73%	6.05%	6.36%	6.68%	6.99%
(6.) (15.) (16.) (21.) (46.)	<b>0.64</b>	4.50%	4.82%	5.14%	5.30%	5.46%	5.78%	6.10%	6.42%	6.74%	7.06%
(24.) (29.) (36.) (38.) (44.)	<b>0.65</b>	4.53%	4.86%	5.18%	5.34%	5.51%	5.83%	6.16%	6.48%	6.81%	7.13%
(4.) (13.) (17.) (19.) (47.)	<b>0.66</b>	4.56%	4.89%	5.22%	5.39%	5.55%	5.88%	6.21%	6.54%	6.87%	7.20%
(27.) (34.)	<b>0.67</b>	4.59%	4.93%	5.26%	5.43%	5.60%	5.93%	6.27%	6.60%	6.94%	7.27%
(5.) (12.) (14.) (20.) (22.)	<b>0.68</b>	4.62%	4.96%	5.30%	5.47%	5.64%	5.98%	6.32%	6.66%	7.00%	7.34%
(23.) (35.)	<b>0.69</b>	4.65%	5.00%	5.34%	5.51%	5.69%	6.03%	6.38%	6.72%	7.07%	7.41%
(25.)	<b>0.70</b>	4.68%	5.03%	5.38%	5.56%	5.73%	6.08%	6.43%	6.78%	7.13%	7.48%
(10.) (26.)	<b>0.71</b>	4.71%	5.07%	5.42%	5.60%	5.78%	6.13%	6.49%	6.84%	7.20%	7.55%
(3.) (11.)	<b>0.72</b>	4.74%	5.10%	5.46%	5.64%	5.82%	6.18%	6.54%	6.90%	7.26%	7.62%
(33.)	<b>0.73</b>	4.77%	5.14%	5.50%	5.68%	5.87%	6.23%	6.60%	6.96%	7.33%	7.69%
(1.)	<b>0.74</b>	4.80%	5.17%	5.54%	5.73%	5.91%	6.28%	6.65%	7.02%	7.39%	7.76%
(2.)	<b>0.75</b>	4.83%	5.21%	5.58%	5.77%	5.96%	6.33%	6.71%	7.08%	7.46%	7.83%
(31.) (32.)	<b>0.77</b>	4.89%	5.28%	5.66%	5.85%	6.05%	6.43%	6.82%	7.20%	7.59%	7.97%
(9.)	<b>0.80</b>	4.98%	5.38%	5.78%	5.98%	6.18%	6.58%	6.98%	7.38%	7.78%	8.18%
(7.)	<b>0.83</b>	5.07%	5.49%	5.90%	6.11%	6.32%	6.73%	7.15%	7.56%	7.98%	8.39%
(8.)	<b>0.84</b>	5.10%	5.52%	5.94%	6.15%	6.36%	6.78%	7.20%	7.62%	8.04%	8.46%
	<b>0.68</b>	Mean									
	<b>0.66</b>	Median									
		Yield on 10 year treasuries (1/9/2010)									
		0.0258									

## 8 SUMMARY AND RECOMMENDATIONS

Cost of capital estimation posed with a challenge virtually everyone who tried to put it in a clear-cut framework, be it academics or corporate finance practitioners. Lack of widely accepted standards continues to make cost of capital as well as valuation an art. The paper attempts to establish guidelines, present with the caveats and uncover some of the biases related to cost of capital estimation.

Since most of the biases arise in estimating cost of equity, the paper is largely focused on that part. Our recommendation on estimating beta can be summarized as follows: we suggest using Sum-Beta for obtaining raw beta from the regression. It is superior to OLS beta as it takes into account autocorrelations of a stock over 2 successive periods and reduces the downward bias arising from delay under which the market news are incorporated in the stock price. Consistent with the results of our research on Kraft Foods, we suggest using 104 weekly returns to populate the data sample. The sample should exclude non-recurring events influencing both the individual stock prices and the overall market prices. A larger adjusted sample size of 104 weekly returns (2 years) should lead to a lower standard error than 60 monthly returns (5 years) and reflect a more forward looking view of company's systematic risks. One should consider, however, if using weekly frequency would not impede consistency with other inputs estimated on monthly basis. Furthermore, though ultra-high frequency betas provide promising methods of excluding market inefficiencies in beta estimates, we believe these betas still have a long way to go until they will gain credibility in corporate finance. Finally, if one is ready to dedicate more time to obtain higher beta precision, we recommend bottom-up /peer group approach.

Our choice of relevering formula would be Harris-Pringle formula. It treats tax shields as if they tracked the risk of operating assets, accounts for positive debt betas and assumes that absolute amount of debt is fluctuating as company manages its capital to target levels, all being crucial to the distressed environment.

When unlevering beta, one should use market D/E ratios (at least for the equity) as equity values are very likely to be eroded during bearish markets. Median target D/E ratios should be used to relever the beta as average values might bias a forward looking central value of industry D/E ratios, especially if the sample is large enough and contains outliers.

Unlevered betas should be adjusted for cash. As debt and equity values shrink in distressed environment cash might carry much higher value relative to the overall debt and equity. However, cash does not exhibit the same systematic risk as the operating assets.

Despite the general preference among analysts of choosing a local index as a market proxy, we recommend to stick to a global index as it is much better diversified and less susceptible to volatilities in single constituents. Bearing in mind the number of assets contained in a global index such as MSCI World, it also matches the original definition of a market portfolio better under the CAPM. Our choice of risk-free rate would be ten year government treasury bonds with the lowest debt beta. These are US t-bonds or the bonds of the most stable and mature countries. We would use 10 year bonds, preferably STRIPS or zero coupons. They provide the best trade-off between required duration, liquidity and reinvestment risk. However, if equity risk premium of choice is estimated using other long-term risk-free proxy, such as the 20-year bonds, to stay consistent, we would use to the latter. Further to that, we recommend choosing a market proxy which would be consistent with ERP as well.

Though implied equity risk premium has become a well promoted method for estimating the extra reward required by the equity investors, we believe that it is too sensitive to inputs such as consensus estimates of future growth. One thing is hearing what analysts and traders have to say about where the market is going, yet another thing is seeing what they actually realize. We would rather stick to what they pay than what they say. However, implied equity risk premium is a good guideline tool for choosing among the variety of historical equity risk premiums estimated using different methods.

Additional premiums should be added in the CAPM if necessary. We argue for the country risk premium and the small firm premium to be put in application since CAPM has failed to capture these extra risks as markets evolved over the last four decades. However, we suggest ignoring liquidity premium as it overlaps with the size premium, and does not bring in any consistency when valuing private firms. Furthermore, company specific premiums should not be applied as these idiosyncratic risks should be captured in the expected value of the future cashflows.

In line with a general practice, when estimating cost of debt we suggest calculating promised yield to maturity of company's debt or using yield spreads provided by the rating agencies. However, when it comes to estimating the cost of non-traded debt, one should consider estimating a synthetic rating based on multiple dimensions instead of a single ratio such as the interest-coverage ratio.

We would neither recommend an effective tax rate nor statutory tax rate for estimating after-tax cost of debt unless the company's taxable income is beyond the highest taxable income bracket going forward. The tax rate at which the marginal income is taxed, hence the marginal tax, often lies in between the effective and statutory rate. Furthermore, statutory rates exhibit bubbles and might significantly overstate the actual marginal tax rate. Instead, we suggest using arbitrary tax rates based on company's profitability, size and future prospects. Simulation models from the recent research provide some good insights on what these arbitrary marginal tax rates could be.

We conclude our research by illustrating how the choices for estimating cost of equity influence the overall cost of capital in the case of Kraft Foods Inc. It is (not) surprising to find how large the range of possible estimates can be and how this could lead to differing enterprise values. Though the combination of choices lead to 48 different beta outputs, we reckon that just by including few extra options of target D/E ratio or sample size and frequency can lead to over 500 different estimates. Even tossing a coin would make it a time consuming way to choose among possible alternatives then.

The paper attempted to establish clear-cut methods and rules for estimating cost of capital. However, we have stumbled upon certain inputs for cost of capital for which we found that further research was necessary. In particular, we believe that further research should contribute towards establishing stronger empirical evidence on the relationship between excess returns based on size and liquidity measures. Furthermore, now that consensus market growth estimates are available, future research should contribute towards analyzing the difference and the relationship between realized and implied equity premiums. Finally, relevant to our study, further research could aim to conclude on the best choices for beta estimates for a sample portfolio using realized returns and different historical equity risk premiums.

All in all, WACC valuation can certainly be inferior to other methods such as APV when times are turbulent. However, bearing in mind that many analysts still use it during the periods of crises, different inputs of WACC are worthwhile paying attention to. One can, of course, always use solver to find WACC. Until then, we highly recommend being considerate when choosing a practical approach to calculating cost of capital.

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Withdrawn by the author

## APPENDIX 1

Beta unlevering and relevering formulas

Method	Formula	Explanation
<i>Hamada</i>	$\beta_L = B_U \left(1 + (1 - t) \frac{D}{E}\right)$	$\beta_L$ - levered/asset beta $\beta_U$ - unlevered/equity beta
<i>Milles-Ezzell</i>	$\beta_L = \beta_U + \frac{D}{E} (\beta_U - \beta_d) \left[1 - \frac{(t \times k_{d(pt)})}{(1 + k_{d(pt)})}\right]$	$\beta_d$ - beta of debt $k_{d(pt)}$ - cost of debt prior to tax effect
<i>Harris-Pringle</i>	$\beta_L = B_U + \frac{D}{E} (B_U - B_d) = B_U \left(1 + \frac{D}{E}\right) - B_d \frac{D}{E}$	D - market value of debt capital E - market value of equity capital t - tax rate
<i>Practitioner's (modified Hamada)</i>	$\beta_L = B_U \left(1 + \frac{D}{E}\right)$	

Service betas and their calculation methodology

Service	Time horizon	Data frequency	Adjustment	Underlying index
<b>Alcar</b>	5 years	Monthly	Bayesian	S&P 500
<b>Barra</b>	Variable	Monthly	Barra risk model (forward looking)	Variable
<b>Bloomberg</b>	Variable (2 year default)	Variable (weekly default)	Blume's	Variable (S&P 500 default)
<b>Capital IQ</b>	Variable (2 year default)	Weekly/Monthly	None	8 domestic (S&P 500 default)
<b>Compustat</b>	5 years	Monthly	None	S&P 500
<b>Damodaran</b>	5 years	Monthly	None	Local (NYSE for US)
<b>Datastream</b>	2½ years	Monthly	Bayesian	Datastream total market
<b>Ibbotson</b>	5 years	Monthly	Towards peer group, weighted by statistical significance	S&P 500
<b>Merrill Lynch</b>	5 years	Monthly	Bayesian*	S&P 500
<b>Reuters</b>	5 years (2 minimum)	Monthly	N/A	S&P 500
<b>S&amp;P</b>	5 years	Monthly	None	S&P 500
<b>Value Line</b>	5 years	Weekly	Blume's**	NYSE composite

\*Includes weighted average of unadjusted prior period betas

\*\*modified:  $0.35 + (0.67 \times \text{unadjusted beta})$

## APPENDIX 2

Companies Ranked by Market Value of Equity  
 Historical Equity Risk Premium: Average Since 1963  
 Data for Year Ending December 31, 2009

Portfolio Rank By Size	Average Mkt Value (\$mils.)	Log of Size	Beta (SumBeta) Since '63	Arithmetics Average Return	Arithmetic Average Risk Premium	Indicated CAPM Premium	Premium over CAPM	Smoothed Premium over CAPM
1	103,041	5.01	0.84	11.53%	4.57%	3.58%	0.99%	-0.83%
2	29,763	4.47	0.94	10.16%	3.20%	4.01%	-0.80%	0.37%
3	17,592	4.25	0.9	11.73%	4.77%	3.84%	0.92%	0.88%
4	12,761	4.11	0.95	13.15%	6.19%	4.05%	2.14%	1.19%
5	9,104	3.96	0.97	12.45%	5.49%	4.13%	1.36%	1.52%
6	6,756	3.83	1.01	12.55%	5.59%	4.31%	1.28%	1.81%
7	5,218	3.72	1	11.59%	4.63%	4.26%	0.36%	2.06%
8	4,160	3.62	1.08	14.11%	7.15%	4.57%	2.58%	2.28%
9	3,481	3.54	1.1	15.12%	8.16%	4.66%	3.50%	2.45%
10	2,965	3.47	1.06	13.93%	6.97%	4.53%	2.44%	2.61%
11	2,594	3.41	1.1	14.78%	7.82%	4.67%	3.14%	2.74%
12	2,281	3.36	1.15	14.22%	7.26%	4.88%	2.37%	2.86%
13	1,992	3.3	1.04	14.90%	7.94%	4.41%	3.53%	2.99%
14	1,741	3.24	1.11	15.49%	8.53%	4.72%	3.81%	3.12%
15	1,523	3.18	1.14	15.15%	8.19%	4.85%	3.34%	3.25%
16	1,311	3.12	1.15	15.54%	8.58%	4.90%	3.68%	3.40%
17	1,127	3.05	1.19	14.35%	7.39%	5.05%	2.34%	3.55%
18	954	2.98	1.21	14.82%	7.86%	5.13%	2.73%	3.71%
19	799	2.9	1.22	16.71%	9.75%	5.17%	4.58%	3.88%
20	664	2.82	1.22	15.15%	8.19%	5.19%	3.00%	4.06%
21	534	2.73	1.21	15.35%	8.39%	5.13%	3.26%	4.27%
22	411	2.61	1.23	15.66%	8.70%	5.22%	3.48%	4.52%
23	315	2.5	1.27	16.91%	9.95%	5.38%	4.57%	4.78%
24	212	2.33	1.26	18.06%	11.10%	5.35%	5.75%	5.17%
25	61	1.79	1.27	20.99%	14.03%	5.40%	8.63%	6.37%

Large Stocks (Ibbotson SBBI data)

11.21% 4.25%

Small Stocks (Ibbotson SBBI data)

16.22% 9.26%

Long-Term T-bonds (Ibbotson SBBI data) 6.96%

Source: Duff & Phelps Risk Premium Report 2010