Does intellectual monopoly stimulate or stifle innovation?

Chu, Angus C. and Cozzi, Guido and Galli, Silvia

November 2010

Does Intellectual Monopoly Stimulate or Stifle Innovation?

Angus C. Chu, Guido Cozzi, and Silvia Galli

May 2011

Angus Chu: anguscccc@gmail.com. School of Economics, Shanghai University of Finance and Economics, China, Tel: +86-21-65902984, Fax: +86-21-65903688.

Guido Cozzi: guido.cozzi@durham.ac.uk. Durham Business School, Durham University, UK, Tel: +44(0)191 334 5146, Fax: +44 (0)191 334 5201.

Silvia Galli: s.galli@hull.ac.uk. Hull University Business School, University of Hull, UK, Tel: +44 (0) 1482 463 239, Fax: +44(0) 1482 463484.

Abstract

This study develops an R&D-based growth model with vertical and horizontal innovation to shed some light on the current debate on whether patent protection stimulates or stifles innovation. We analyze the effects of patent protection in the form of blocking patents. We show that patent protection changes the direction of innovation by having asymmetric effects on vertical innovation (i.e., quality improvement) and horizontal innovation (i.e., variety expansion). Calibrating the model and simulating the transition dynamics, we find that strengthening the effect of blocking patents stifles vertical innovation and decreases economic growth but increases social welfare due to an increase in horizontal innovation.

In light of this finding, we argue that in order to properly analyze the growth and welfare implications of patents, it is important to consider their often neglected compositional effects on vertical and horizontal innovation.

Keywords: economic growth, innovation, intellectual property rights.

JEL classification: O31, O34, O40.

We would like to thank the seminar participants at Academia Sinica, Chinese University of Hong Kong, ETH Zurich, Kyoto University, National Taiwan University, Royal Economic Society, Shanghai Macroeconomic Workshop, and Taipei International Conference on Growth, Trade and Dynamics for their helpful comments and suggestions. The usual disclaimer applies. The previous version of this paper was circulated under the title "Innovation-Specific Patent Protection".

1 Introduction

Since the early 1980's, the patent system in the US has undergone substantial changes.[1] As a result of this patent reform, the strength of patent protection in the US has increased. For example, Park (2008) provides an index of patent rights on a scale of 0 to 5 (a larger number implies stronger protection) and shows that the strength of patent rights in the US increases from 3.8 in 1975 to 4.9 in 2005.[2] In other words, patentholders can now better protect their inventions against imitation as well as *subsequent innovation*. When a patent protects an invention against subsequent innovation, a blocking patent arises. A classic example of blocking patents is James Watt's patent on his steam engine. Boldrin *et al.* (2008) argue that "[b]y patenting the separate condenser Boulton and Watt, from 1769 to 1800, had almost absolute control on the development of the steam engine. They were able to use the power of their patent and the legal system to frustrate the efforts of engineers such as Jonathan Hornblower to further improve the fuel efficiency of the steam engine." As for the current patent system, economists have become even more concerned about the innovation-stifling effect of blocking patents. For example, Shapiro (2001) argues that "[w]ith cumulative innovation and multiple blocking patents, stronger patent rights can have the perverse effect of stifling, not encouraging, innovation." In this study, we provide a growth-theoretic analysis on the effects of patent protection in the form of blocking patents.

In an environment with cumulative or sequential innovation, blocking patents give rise to overlapping patent rights across sequential innovators and lead to contrasting effects on R&D. On the one hand, the traditional view suggests that stronger patent rights improve the protection for existing inventions and increase their value to the patentholders. On the other hand, the recent argument against patent protection suggests that stronger patent rights stifle

[1] See Gallini (2002), Jaffe (2000) and Jaffe and Lerner (2004) for a detailed discussion on these changes in patent policy.

[2] The index in Park (2008) is an updated version of the index in Ginarte and Park (1997), who examine five categories of patent rights and assign a score from zero to one to each category. These five categories are patent duration, coverage, enforcement mechanisms, restrictions on patent scope, and membership in international treaties.

innovation by giving too much power to existing patentholders, who use this power to extract surplus from subsequent innovators rather than providing more innovation.[3] In this study, we develop a simple growth model to shed some light on this current debate on whether patents stimulate or stifle innovation. We argue that the two seemingly contradictory views of patents are in fact two sides of the same coin. In other words, strengthening existing patentholders' protection against future innovations inevitably decreases subsequent innovators' incentives for R&D and leads to contrasting effects on vertical innovation (i.e., quality improvement within an industry) and horizontal innovation (i.e., variety expansion that gives rise to new industries). In light of this finding, we argue that in order to properly analyze the growth and welfare implications of patents, it is important to consider their often neglected compositional effects on vertical and horizontal innovation.

To analyze the asymmetric effects of patent protection on vertical and horizontal innovation, this study develops an R&D-based growth model that features both quality improvement and variety expansion. Within this framework, we derive the growth and welfare effects of patent protection in the form of blocking patents. A strengthening of blocking patents refers to the case in which a new innovator (e.g., Jonathan Hornblower) has to transfer a larger share of his profit to the previous innovator (e.g., James Watt). We find that there is a tension between maximizing the incentives for vertical innovation and that of horizontal innovation. On the one hand, maximizing the incentives for vertical innovation requires a profit-division rule that allows the new innovator to keep all the profit. On the other hand, maximizing the incentives for horizontal innovation requires a profit-division rule that assigns as much profit to the previous innovator as possible. As a result of these asymmetric effects on vertical and horizontal innovation, strengthening the effect of blocking patents stimulates variety expansion but stifles quality improvement affecting the direction of innovation. This theoretical result is consistent with the empirical finding in Moser (2005), who provides an empirical analysis on how patent protection affects the direction of innovation and finds that

[3] See, for example, Bessen and Meurer (2008), Bodrin and Levine (2008) and Jaffe and Lerner (2004).

the presence of patent laws in a country causes the inventions to be more diversified and directed to a broader set of industries than inventions in countries without patent laws.

Furthermore, strengthening the effect of blocking patents has an additional effect through horizontal innovation on social welfare by increasing the number of varieties, so that there also exists a welfare-maximizing profit-division rule that is generally different from the growth-maximizing rule. Calibrating the model and simulating the transition dynamics, we find that an increase in the effect of blocking patents stifles vertical innovation and decreases the overall growth rate despite the increase in horizontal innovation. This finding is consistent with the recent concerns on the innovation-stifling effects of stronger patent rights. However, we also find that social welfare increases despite the lower growth rate suggesting that a proper welfare analysis should investigate beyond the effects of patent protection on innovation and economic growth.

Nordhaus (1969) is the seminal study on the optimal design of patent protection, and he shows that the optimal patent length should balance between the social benefit of innovation and the social cost of monopolistic distortion. Scotchmer (2004) provides a comprehensive review on the subsequent development in this patent-design literature that is mostly based on partial-equilibrium models. In this literature, an interesting and important policy lever is forward patent protection (i.e., leading patent breadth) that gives rise to the division of profit between sequential innovators.[4] A recent study by Segal and Whinston (2007) analyzes a general antitrust policy lever that has a similar effect as the division of profit between entrants and incumbents. They show that in an infinite-horizon model with leapfrogging, protecting an entrant at the expense of an incumbent has a frontloading effect that potentially increases innovation. However, they also note that their result does not apply to the first firm of a quality ladder because it does not have to share its profit with any incumbent but has the rights to share the next entrant's profit. In the present study, we formalize Segal and Whinston's interesting insight in a dynamic general-equilibrium model and match the model

[4]See, for example, Green and Scotchmer (1995) and Gallini and Scotchmer (2002) for a discussion on the importance of this policy lever.

to the US data in order to provide a quantitative analysis on the division of profit between sequential innovators.

O'Donoghue and Zweimuller (2004) merge the patent-design literature and the R&D-based growth literature by incorporating leading breadth into a quality-ladder growth model with overlapping patent rights across sequential innovators. In their model, for a given rate of innovation, strengthening the effect of blocking patents by reducing the share of profit assigned to the current innovator (i.e., the entrant of a quality ladder) while holding leading breadth constant would decrease the incentives for innovation. Intuitively, along the quality ladder, every innovator is firstly an entrant and then becomes an incumbent whose patent is infringed upon. Therefore, setting aside the issues of profit growth and discounting, every innovator receives the same amount of profit over the lifetime of an invention. Given that the real interest rate is higher than the growth rate in their model, delaying the receipt of profits reduces the present value of the income stream. As a result, the complete frontloading profit-division rule (i.e., allowing the entrant to keep all the profit) tends to maximize the market value of an invention and the incentives for R&D.[5] However, in the present study with both vertical and horizontal innovation, this result no longer holds. In this case, the inventor of a new variety is the first innovator on a quality ladder; therefore, assigning a larger share of profit to the incumbent increases horizontal innovation. Given that quality improvement and variety expansion are both important channels for economic growth, the growth-maximizing profit-division rule should balance between the asymmetric effects of profit division on vertical and horizontal innovation. Furthermore, given that growth maximization does not necessarily give rise to welfare maximization, we characterize both the growth-maximizing and welfare-maximizing profit-division rules.

This study also relates to other growth-theoretic studies on patent policy. Judd (1985) provides the seminal dynamic general-equilibrium analysis on patent length, and he finds that an infinite patent length maximizes innovation and welfare. Subsequent studies find that

[5] See also Chu (2009) for a quantitative analysis of the profit-division rule in the O'Donoghue-Zweimuller model.

strengthening patent protection in various forms does not necessarily increase innovation and may even stifle it. Examples include Horowitz and Lai (1996) on patent length, O'Donoghue and Zweimuller (2004) on leading breadth and patentability requirement, Koleda (2004) on patentability requirement, and Furukawa (2007) and Horii and Iwaisako (2007) on patent protection against imitation. The present study differs from these studies by (a) analyzing a different patent-policy lever (i.e., the profit-division rule between sequential innovators) and (b) emphasizing the asymmetric effects of patent protection on vertical and horizontal innovation.[6] In other words, rather than analyzing the effects of patent policy on the *level* of innovation as is common in the literature, we consider a much less explored question that is the effects of patent policy on the *composition* or *direction* of innovation.

Cozzi (2001) analyzes patent protection in the form of intellectual appropriability (i.e., the ability of an innovator to patent her invention in the presence of spying activities) in a quality-ladder model. Cozzi and Spinesi (2006) extend this analysis into a model with both vertical and horizontal innovation. In their model, spying activities are targeted only at quality improvement. Therefore, strengthening intellectual appropriability stimulates vertical innovation (at the expense of horizontal innovation) and increases long-run growth because horizontal innovation only has a level effect in their model for removing scale effects. In contrast, long-run growth depends on both vertical and horizontal innovation in the present study,[7] and hence, the asymmetric effects of profit division on vertical and horizontal innovation give rise to a growth-maximizing profit-division rule.

Acs and Sanders (2009) and Cozzi and Galli (2009) also analyze the division of profit between innovators. Acs and Sanders (2009) analyze the separation between invention and commercialization in a variety-expanding model while Cozzi and Galli (2009) consider basic research and applied research in a quality-ladder model. In these studies, each invention (i.e.,

[6] O'Donoghue and Zweimuller (2004) also consider a model with both vertical and horizontal innovation in their appendix. However, their focus is on the effects of patentability requirement and leading breadth, and they did not analyze the effects of alternative profit-division rules in the presence of vertical and horizontal innovation.

[7] See footnotes (8) and (26) for a discussion on the issue of scale effects in R&D-based growth models.

a new variety or a quality improvement) is created in a two-step innovation process; therefore, there exists a growth-maximizing division of profit that balances between the incentives of the first and second innovators of each invention. The present study differs from these studies by analyzing the division of profit between sequential innovators within the same industry (in which every innovator is firstly an entrant and then becomes an incumbent). Also, we consider a model that features both vertical and horizontal innovation. We find that frontloading (backloading) the income stream along the quality ladder stimulates vertical (horizontal) innovation, and it is the interaction of these two types of innovation that gives rise to a growth-maximizing profit-division rule in this study.

This study also relates to Acemoglu (2009), who shows that under the current patent system, the equilibrium diversity of innovation is insufficient. In other words, innovators have too much incentive to invest in R&D on improving existing products but too little incentive to invest in R&D on developing new products that may become useful in the future. Acemoglu suggests that increasing the diversity of researchers could be a partial remedy against this problem of insufficient diversity. The present study suggests another possible solution that is to increase the share of profit assigned to the pioneering inventor of a product. In this case, there will be a reallocation of research inputs from vertical innovation (i.e., R&D on existing products) to horizontal innovation (i.e., R&D on new products).

The rest of this study is organized as follows. Section 2 describes the model. Section 3 defines the equilibrium and characterizes the equilibrium allocation. Section 4 considers the growth and welfare effects of patent protection. Section 5 calibrates the model and simulates the transition dynamics to provide a quantitative analysis. The final section concludes.

2 The model

To consider both vertical and horizontal innovation in an R&D-based growth model,[8] we modify the Grossman-Helpman (1991) quality-ladder model[9] by endogenizing the number of varieties in the economy. Furthermore, to consider the division of profit between sequential innovators along the quality ladder, we assume that each entrant (i.e., the most recent innovator) infringes the patent of the incumbent (i.e., the previous innovator). As a result of this patent infringement, the entrant has to transfer a share $s \in [0,1]$ of her profit to the incumbent. However, with vertical innovation, every innovator's patent would eventually be infringed by the next innovation, and she can then extract a share s of profit from the next entrant. This formulation of profit division between sequential innovators originates from O'Donoghue and Zweimuller (2004). As for horizontal innovation, the invention of a new variety does not infringe any patent,[10] so that a variety inventor does not have to share her profit but maintains the rights to extract profit from the next entrant. Given that the Grossman-Helpman model is well-studied, we will describe the familiar features briefly to conserve space and discuss new features (i.e., variety expansion and profit division) in details.

2.1 Households

There is a unit continuum of identical households. Their lifetime utility is given by

$$U = \int_0^\infty e^{-\rho t} \ln c_t dt, \tag{1}$$

[8] See, also, Dinopoulos and Thompson (1999a, 1999b), Howitt (1999), Jones (1999), Li (2000), Peretto (1998, 1999), Peretto and Smulders (2002), Segerstrom (2000) and Young (1998). The focus of these studies is on the removal of scale effects in R&D-based growth models. Given that scale effect is not the focus of this study, we normalize the supply of skilled labor to unity to set aside this issue.

[9] See also Aghion and Howitt (1992) and Segerstrom *et al.* (1990) for other pioneering studies on the quality-ladder growth model.

[10] In the main text, we also discuss the alternative case in which a newly invented variety infringes the patents of other existing varieties.

where $\rho > 0$ is discount rate, and c_t is the consumption index at time t. The consumption index is defined as[11]

$$c_t \equiv \exp\left(\int_0^{n_t^*} \ln y_t(i) di\right). \tag{2}$$

(2) shows that the households derive utility by consuming a continuum of products $y_t(i)$. In Grossman and Helpman (1991), there is a unit continuum of these products. In the present study, we endogenize the number of varieties by allowing for horizontal innovation. n_t^* is the number of active varieties that are consumed by households at time t, and its law of motion is given by

$$\dot{n}_t^* = \dot{n}_t - \delta n_t^*. \tag{3}$$

n_t is the total number of varieties that have been invented in the past, and $\dot{n}_t$ is the number of newly invented varieties at time t. We follow Grossman and Lai (2004) to allow for the possibility that an invented variety becomes obsolete at some point. For tractability, we assume that each active variety $i \in [0, n_t^*]$ at time t faces the same probability $\delta > 0$ to become permanently obsolete.[12]

Households maximize (1) subject to

$$\dot{a}_t = r_t a_t + w_{h,t} + w_{l,t} L - \int_0^{n_t^*} p_t(i) y_t(i) di. \tag{4}$$

a_t is the value of assets owned by households, and r_t is the rate of return. To simplify the analysis, we assume that households supply one unit of high-skill labor for R&D and $L > 1$ units of low-skill labor for production.[13] The wage rates for high-skill and low-skill labors

[11] In their appendix, O'Donoghue and Zweimuller (2004) also consider this Cobb-Douglas specification, which is similar to the CES form in Howitt (1999) and Segerstrom (2000) except for the different elasticity of substitution across varieties. In this study, we focus on the Cobb-Douglas aggregator which enables us to compute the consumption index's transition path along which the arrival rate of innovation varies.

[12] Due to the quality distribution across varieties, the model would become considerably more complicated if we allow the obsolescence rate to depend on the age of a variety.

[13] In Grossman and Helpman (1991), a homogenous type of labor is allocated between R&D and production. In reality, R&D engineers and scientists often have a high level of education. Given that this model features two R&D sectors involving the allocation of high-skill labor, we naturally distinguish between high-skill

are $w_{h,t}$ and $w_{l,t}$ respectively. $p_t(i)$ is the price of product i at time t. If we denote ζ_t as the Hamiltonian co-state variable, then households' intratemporal optimality condition is

$$p_t(i)y_t(i) = 1/\zeta_t \tag{5}$$

for $i \in [0, n_t^*]$, and the intertemporal optimality condition is

$$r_t = \rho - \dot{\zeta}_t/\zeta_t. \tag{6}$$

2.2 Production

There is a continuum of active varieties $i \in [0, n_t^*]$ that are consumed by households at time t. The production function for the most recent innovator in industry i is

$$y_t(i) = z^{q_t(i)} l_t(i). \tag{7}$$

The parameter $z > 1$ is the exogenous step size of each productivity improvement. $q_t(i)$ is the number of productivity improvements that have occurred in industry i as of time t. $l_t(i)$ is the number of low-skill production workers employed in industry i. Given $z^{q_t(i)}$, the marginal cost of production for the most recent innovator in industry i is

$$mc_t(i) = w_{l,t}/z^{q_t(i)}. \tag{8}$$

Notice that we here adopt a "cost reducing" view of vertical innovation following Peretto (1998, 1999) and Peretto and Smulders (2002).[14] In each industry that has at least two

labor for R&D and low-skill labor for production. However, it is useful to note that our main result (i.e., an increase in s increases horizontal innovation but decreases vertical innovation) carries over to a setting with homogenous labor that is allocated across production, vertical R&D and horizontal R&D, but the analysis becomes more complicated.

[14]It is useful to note that cost reduction is isomorphic to quality improvement in these studies as well as

generations of innovation, the most recent innovator infringes the previous innovator's patent. As a result of this patent infringement, the most recent innovator pays a licensing fee by transferring a share s of her profit to the previous innovator. We follow O'Donoghue and Zweimuller (2004) to consider an exogenous profit-division rule.[15,16] This profit-division rule can be interpreted as the outcome of a bargaining game, in which the bargaining power of each side can be influenced by patent policy.[17] Therefore, it is not an unrealistic assumption to treat s as a policy parameter.

O'Donoghue and Zweimuller (2004) are interested in the effects of leading breadth on R&D and economic growth through the consolidation of market power that enables the most recent innovator and the previous innovator to consolidate their market power and charge a higher markup. We do not adopt this formulation here for three reasons. First, the collusion between innovators may be prohibited by antitrust laws. Second, the licensing agreement only allows the most recent innovator to produce, but it may not prevent the previous innovator from selling her products at a lower price. As a result, the previous innovator may have the incentives to continue selling her products and undercut the markup. Third, we want to focus on the profit-division effect (instead of the markup effect) of patent protection in this study. Given these considerations, we assume that the most recent innovator and the

in the current framework. To see this, the reader could easily reinterpret $y_t(i)$ as the consumption of the latest version, $q_t(i)$, of product i, along the lines of Grossman and Helpman (1991), that is by assuming $\ln c_t \equiv \left(\int_0^{n_t^*} \ln \sum_{j=0}^{q_t(i)} z^j y_t(i) di \right)$, with consumption good i's production function given by $y_t(i) = l_t(i)$. Clearly, the profit function (10) would follow directly from Bertrand competition, instead of the no longer valid (8) and (9).

[15] O'Donoghue and Zweimuller (2004) consider the more general case in which the current innovator may infringe the patents of multiple previous innovators. For the purpose of the present study, it is sufficient to demonstrate the asymmetric effects of the profit-division rule on vertical and horizontal innovation by considering the simple case of profit division between the entrant and the incumbent.

[16] Chu and Pan (2010) analyze the effects of blocking patents under the case of an *endogenous* profit-division rule and an *endogenous* step size of innovation in a quality-ladder model with only vertical innovation. As in the present study, they also find that blocking patents have a non-monotonic effect on economic growth.

[17] In reality, a patentholder enforces her patent rights through the Court, which decides her case of patent infringement against a potential infringer. Therefore, when it becomes more likely for the Court to favor patentees, the bargaining power of patentholders strengthens relative to potential infringers. Of course, this will indirectly affect also the outcomes of potential pre-trial settlements.

previous innovator engage in the usual Bertrand competition as in Grossman and Helpman (1991). The profit-maximizing price for the most recent innovator is a constant markup (given by the step size z) over her own marginal cost in (8).[18]

$$p_t(i) = z(w_{l,t}/z^{q_t(i)}). \tag{9}$$

Given (7) - (9), the monopolistic profit generated by the most recent innovation is

$$\pi_t(i) = (z-1)w_{l,t}l_t(i) = \left(\frac{z-1}{z}\right)\frac{1}{\zeta_t}, \tag{10}$$

where the second equality is obtained by using (5), (7) and (9). Due to profit division, the most recent innovator obtains $(1-s)\pi_t$ while the previous innovator obtains $s\pi_t$. The above discussion implicitly assumes that the most recent innovation and the second-most recent innovation are owned by different firms (i.e., the Arrow replacement effect). In Lemma 1, we show that the Arrow replacement effect is indeed present in this quality-ladder model with profit division.[19]

Lemma 1 *The Arrow replacement effect is present.*

Proof. See the Appendix A. ■

Finally, for a newly invented variety, we make the usual simplifying assumption that the productivity of labor in each new variety[20] is randomly drawn from the existing distribution

[18] Li (2001) considers a CES version of (2) without horizontal innovation. In this case, the monopolistic markup is determined by either the quality step size or the elasticity of substitution depending on whether innovation is drastic or non-drastic. Without loss of generality, we focus on non-drastic innovation as in the original Grossman-Helpman model.

[19] Cozzi (2007) shows that the Arrow effect is not necessarily inconsistent with the empirical observation that incumbents often target innovation at their own industries. Under this interpretation, the incumbents' choice of R&D is simply indeterminate, so that the aggregate economy behaves *as if* innovation is targeted only by entrants. See also Etro (2004, 2008) for an interesting analysis on innovation by incumbents with a first-mover advantage.

[20] Or the quality of each new variety, in the equivalent quality ladder interpretation explained above.

of active products $i \in [0, n_t^*]$. We also assume that a variety inventor can only patent the most advanced technology. Given that the lower-productivity production methods are unpatented, Bertrand competition drives the markup down to z as well.[21] However, because there is no previous patentholder in the newly created industry, the variety inventor obtains the entire π until the next productivity improvement occurs, and then she can extract $s\pi$ from the entrant.

What happens when a variety invention infringes the patents of existing varieties? For example, Hall *et al.* (2001) define an original innovation as "a patent that cites a broad set of technologies or which has a certain percentage of citations given to different patent classes". If we view an original innovation as a horizontal innovation and assume that the probability of patent infringement is increasing in the number of patent citations, then horizontal innovation may in fact be more at risk of patent infringements. Here we discuss the implication of an alternative assumption that a newly invented variety infringes all previous horizontal patents. In this case, the infringed patentholders should all claim a right to share among themselves a fraction of the profits. But this means that the share that will go to each infringed patentholder is zero as a result of the continuum of products (or tending to zero with countable products). Let us assume that each infringed party has to pay a however small, but discrete, legal fee ε in order to sue the infringer. Then, in equilibrium no previous horizontal innovator will ever sue the current horizontal innovator.

2.3 Vertical innovation

Denote $v_{2,t}(i)$ as the value of the patent held by the second-most recent innovator in industry i. Because $\pi_t(i) = \pi_t$ for $i \in [0, n_t^*]$ from (10), $v_{2,t}(i) = v_{2,t}$ in a symmetric equilibrium (i.e., an equal arrival rate of innovation across industries).[22] In this case, the familiar no-arbitrage

[21] In the alternative case of drastic innovation, a new variety inventor and the most recent innovator for an existing variety would also choose the same equilibrium markup that is determined by the elasticity of substitution.

[22] We follow the standard approach in the literature to focus on the symmetric equilibrium. See Cozzi (2005) and Cozzi *et al.* (2007) for a discussion on the symmetric equilibrium in the quality-ladder model.

condition for $v_{2,t}$ is

$$r_t v_{2,t} = s\pi_t + \dot{v}_{2,t} - (\delta + \lambda_t)v_{2,t}. \tag{11}$$

The left-hand side of (11) is the return on this asset. The right-hand side of (11) is the sum of (a) the profit $s\pi_t$ received by the patentholder, (b) the potential capital gain $\dot{v}_{2,t}$, and (c) the expected capital loss due to obsolescence $\delta v_{2,t}$ and creative destruction $\lambda_t v_{2,t}$, where λ_t is the Poisson arrival rate of innovation in the industry. As for the value of the patent held by the most recent innovator, the no-arbitrage condition for $v_{1,t}$ is

$$r_t v_{1,t} = (1-s)\pi_t + \dot{v}_{1,t} - (\delta + \lambda_t)v_{1,t} + \lambda_t v_{2,t}. \tag{12}$$

The intuition behind (12) is the same as (11) except for the addition of the last term. When the next quality improvement occurs, the most recent innovator becomes the second-most recent innovator and hence her net expected capital loss is $\lambda_t(v_{1,t} - v_{2,t})$.

There is a unit continuum of vertical-R&D firms indexed by $j \in [0,1]$ doing research on vertical innovation in each industry i. They hire high-skill labor $h_{q,t}(j)$ to create productivity improvements, and the expected profit of firm j is

$$\pi_{q,t}(j) = v_{1,t}\lambda_t(j) - w_{h,t}h_{q,t}(j). \tag{13}$$

The *firm-level* arrival rate of innovation is

$$\lambda_t(j) = \bar{\varphi}_{q,t} h_{q,t}(j), \tag{14}$$

where $\bar{\varphi}_{q,t}$ is the productivity of vertical R&D at time t. The zero-expected-profit condition for vertical R&D is

$$v_{1,t}\bar{\varphi}_{q,t} = w_{h,t}. \tag{15}$$

We follow Jones and Williams (2000) to assume that $\bar{\varphi}_{q,t} = \varphi_q (h_{q,t})^{\phi_q - 1}$, where $\varphi_q > 0$

is a productivity parameter for vertical R&D and $\phi_q \in (0,1)$ captures the usual negative externality in intratemporal duplication within each industry. In equilibrium, the industry-level arrival rate of innovation equals the aggregate of firm-level arrival rates. Therefore, at the *aggregate* level, the arrival rate of vertical innovation for each variety is $\lambda_t = \varphi_q (h_{q,t})^{\phi_q}$.[23]

2.4 Horizontal innovation

Denote $v_{n,t}$ as the value of inventing a new variety. The no-arbitrage condition for $v_{n,t}$ is

$$r_t v_{n,t} = \pi_t + \dot{v}_{n,t} - (\delta + \lambda_t) v_{n,t} + \lambda_t v_{2,t}. \tag{16}$$

The only difference between (12) and (16) is that a variety inventor captures π_t while a quality innovator captures $(1-s)\pi_t$. There is also a unit continuum of horizontal-R&D firms indexed by $k \in [0,1]$ doing research on creating new varieties. They hire high-skill labor $h_{n,t}(k)$ to create inventions, and the profit of firm k is

$$\pi_{n,t}(k) = v_{n,t} \dot{n}_t(k) - w_{h,t} h_{n,t}(k). \tag{17}$$

The number of inventions created by firm k is[24]

$$\dot{n}_t(k) = \bar{\varphi}_{n,t} h_{n,t}(k), \tag{18}$$

where $\bar{\varphi}_{n,t}$ is the productivity of horizontal R&D at time t. The zero-profit condition for horizontal R&D is

$$v_{n,t} \bar{\varphi}_{n,t} = w_{h,t}. \tag{19}$$

[23] Despite decreasing returns to scale at the aggregate level, we assume constant returns to scale at the firm level in order to be consistent with free entry and zero expected profit in the R&D sector.

[24] Due to the assumption of a continuum of varieties, there is no strategic interaction across varieties. Therefore, we do not need to distinguish between single-product and multi-product firms.

Again, $\bar{\varphi}_{n,t} = \varphi_n (h_{n,t})^{\phi_n - 1}$, where $\varphi_n > 0$ is a productivity parameter for variety-expanding R&D and $\phi_n \in (0,1)$ captures the duplication externality in horizontal innovation. At the *aggregate* level, the total number of inventions created at time t is

$$\dot{n}_t = \varphi_n (h_{n,t})^{\phi_n}. \tag{20}$$

3 Decentralized equilibrium

The equilibrium is a time path $\{y_t(i), l_t, h_{q,t}, h_{n,t}, r_t, p_t(i), w_{l,t}, w_{h,t}, v_{n,t}, v_{1,t}, v_{2,t}\}$, $t \geq 0$. Also, at each instant of time,

- households maximize utility taking $\{r_t, p_t(i), w_{l,t}, w_{h,t}\}$ as given;
- production firms produce $\{y_t(i)\}$ and choose $\{p_t(i)\}$ to maximize profit taking $\{w_{l,t}\}$ as given;
- vertical-innovation firms choose $\{h_{q,t}\}$ to maximize expected profit taking $\{w_{h,t}, v_{1,t}\}$ as given;
- horizontal-innovation firms choose $\{h_{n,t}\}$ to maximize profit taking $\{w_{h,t}, v_{n,t}\}$ as given;
- the low-skill labor market clears such that $n_t^* l_t = L$; and
- the high-skill labor market clears such that $h_{n,t} + n_t^* h_{q,t} = 1$.

3.1 Stationary equilibrium

We focus on a stationary equilibrium, in which the number of active varieties is constant. Substituting (20) into (3) yields $\dot{n}_t^* = \varphi_n (h_{n,t})^{\phi_n} - \delta n_t^*$. Therefore, $\dot{n}_t^* = 0$ implies that

$$n^* = \dot{n}/\delta = \varphi_n (h_n)^{\phi_n}/\delta. \tag{21}$$

The number of production workers per variety is

$$l = \frac{L}{n^*} = \frac{\delta L}{\varphi_n (h_n)^{\phi_n}}. \tag{22}$$

Let us choose low-skill labor as the numeraire (i.e., $w_{l,t} = 1$ for all t). Then, combining (5), (7) and (9) shows that ζ is constant in the stationary equilibrium implying that $r = \rho$ from (6) and $\dot{\pi}_t / \pi_t = 0$ from (10). Applying the stationary equilibrium conditions on (11), (12) and (16) yields

$$v_1 = \frac{(1-s)\pi + \lambda v_2}{\rho + \delta + \lambda} = \frac{\pi}{\rho + \delta + \lambda} \left(1 - s + s \frac{\lambda}{\rho + \delta + \lambda} \right), \tag{23}$$

$$v_n = \frac{\pi + \lambda v_2}{\rho + \delta + \lambda} = \frac{\pi}{\rho + \delta + \lambda} \left(1 + s \frac{\lambda}{\rho + \delta + \lambda} \right). \tag{24}$$

(24) shows that the value of a new variety v_n is increasing in s for a given innovation rate λ because a larger s allows the variety inventor to extract more profit from the next innovator. In contrast, (23) shows that the value of a productivity improvement v_1 is decreasing in s for a given λ because of the backloading effect $\lambda/(\rho + \delta + \lambda) < 1$. In other words, delaying the income stream reduces its expected present value due to discounting ρ and the possibility of obsolescence δ.

Substituting (23) and (24) into $v_1 \bar{\varphi}_q = v_n \bar{\varphi}_n$ from (15) and (19) yields

$$(h_n)^{1-\phi_n} = \left(\frac{\varphi_n}{\varphi_q} \frac{\rho + \delta + (1+s)\varphi_q (h_q)^{\phi_q}}{(1-s)(\rho + \delta) + \varphi_q (h_q)^{\phi_q}} \right) (h_q)^{1-\phi_q}. \tag{25}$$

We will refer to (25) as the *arbitrage condition*. To close the model, we manipulate $h_{n,t} + n_t^* h_{q,t} = 1$ to derive

$$\frac{\delta(1-h_n)}{\varphi_n (h_n)^{\phi_n}} = h_q. \tag{26}$$

We will refer to (26) as the *resource constraint*. The equilibrium allocation of high-skill labor is implicitly determined by solving (25) and (26). Taking the total differentials of (26) yields

$$\frac{dh_n}{dh_q} = -\left(\frac{1-h_n}{h_n + \phi_n(1-h_n)}\right)\frac{h_n}{h_q} < 0. \tag{27}$$

In other words, the resource constraint describes a negative relationship between h_n and h_q. As for the arbitrage condition in (25), h_q has opposing effects on the arbitrage condition. On the one hand, an increase in h_q decreases $\bar{\varphi}_q$. For a given value of v_n/v_1, h_n must rise and $\bar{\varphi}_n$ must fall to balance $v_1\bar{\varphi}_q = v_n\bar{\varphi}_n$. On the other hand, a larger h_q increases λ and decreases v_n/v_1 when $s > 0$. If this latter effect is strong enough, it may lead to a decrease in h_n. Taking the total differentials of (25) yields

$$\frac{dh_n}{dh_q} = \frac{1}{1-\phi_n}\left(1 - \phi_q - \phi_q \frac{s^2(\rho+\delta)}{\rho+\delta+(1+s)\varphi_q(h_q)^{\phi_q}} \frac{\varphi_q(h_q)^{\phi_q}}{(1-s)(\rho+\delta)+\varphi_q(h_q)^{\phi_q}}\right)\frac{h_n}{h_q}. \tag{28}$$

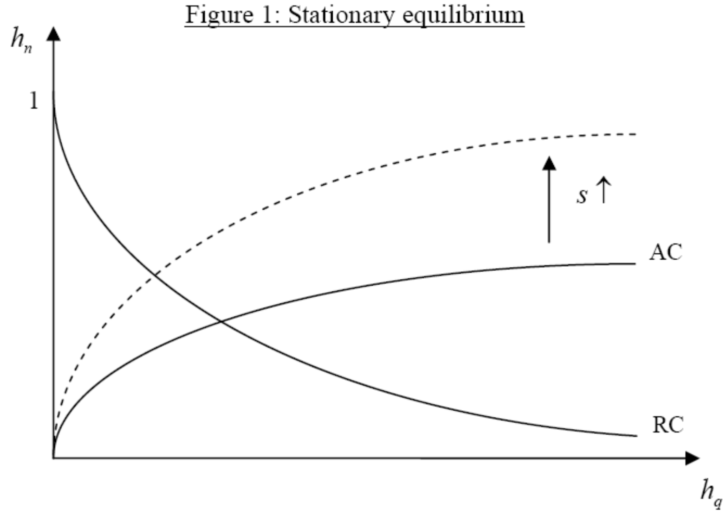
(28) shows that dh_n/dh_q must be positive when h_q equals zero or becomes sufficiently large. However, at intermediate values of h_q, it is possible for dh_n/dh_q to be negative. In this case, there may be multiple equilibria. To rule out multiple equilibrium, which is not the focus of this study, Lemma 2 derives the parameter condition under which (28) is always positive, which is sufficient to ensure that the stationary equilibrium is unique. Let's define a parameter threshold $\bar{\phi}_q \equiv [1 - 0.5s^2/(1+\sqrt{1-s^2})] \in [0.5, 1]$.

Lemma 2 *If $\phi_q < \bar{\phi}_q$, then $dh_n/dh_q > 0$ in (28) $\forall h_q > 0$.*

Proof. See the Appendix A. ■

Figure 1 plots (25) and (26) in the (h_q, h_n) space. The resource constraint (RC) is negatively sloped while the arbitrage condition (AC) is positively sloped given the parameter condition in Lemma 2.

Figure 1: Stationary equilibrium



Therefore, if an equilibrium exists, it must be unique. Also, a larger s increases the market value of a new variety and decreases that of a quality improvement; consequently, horizontal R&D h_n rises and vertical R&D h_q falls. Given this intuitive result (summarized in Proposition 1), the next section uses the growth-theoretic framework to analyze the effects of the profit-division rule on economic growth and social welfare.

Proposition 1 *Given $\phi_q < \bar{\phi}_q$, there exists a unique equilibrium (h_q, h_n). The equilibrium $h_n(s)$ is increasing in s whereas $h_q(s)$ is decreasing in s.*

Proof. At $h_q = 0$, $h_n = 0$ in (25) and $h_n = 1$ in (26). As h_q approaches infinity, h_n in (26) approaches zero. Therefore, (25) and (26) must cross exactly once given Lemma 2. An increase in s shifts up (25) in the (h_q, h_n) space leading to an increase in h_n and a decrease in h_q. See Figure 1. ■

4 Growth and welfare effects of blocking patents

In this section, we analyze the effects of blocking patents on economic growth and social welfare. We first derive the growth-maximizing profit-division rule and then the welfare-maximizing rule. Finally, we compare them and characterize the condition under which one is above the other.

4.1 The growth-maximizing profit-division rule

To derive the balanced growth rate of the consumption index, we substitute (7) into (2) to obtain

$$\ln c_t = \int_0^{n^*} [q_t(i) \ln z + \ln l(i)] di = \left(n^* \int_0^t \lambda_\tau d\tau \right) \ln z + n^* \ln l. \tag{29}$$

The second equality of (29) is obtained by (a) applying symmetry $l(i) = l$ from (10), (b) normalizing $q_0(i) = 0$ for all i, and (c) using the law of large numbers that implies $\int_0^{n^*} q_t(i)di = n^* \int_0^t \lambda_\tau d\tau$.[25] Differentiating (29) with respect to time yields the balanced growth rate of the consumption index given by

$$g \equiv \frac{\dot{c}_t}{c_t} = n^* \lambda \ln z, \tag{30}$$

where the steady-state number of varieties is $n^* = \varphi_n (h_n)^{\phi_n} / \delta$, and the arrival rate of productivity improvement in each industry is $\lambda = \varphi_q (h_q)^{\phi_q}$.

Corollary 1 *n^* is increasing in s whereas λ is decreasing in s.*

Proof. Recall that $n^* = \varphi_n (h_n)^{\phi_n} / \delta$ and $\lambda = \varphi_q (h_q)^{\phi_q}$. Then, from Proposition 1, h_n is increasing in s whereas h_q is decreasing in s. ■

[25] Note that at each instant of time, the average quality of new varieties is the same as the average quality of obsolete varieties because they are drawn from the same quality distribution. In Appendix B, we derive an expression for $\ln c_t$ when n_t^* varies over time.

To see why the equilibrium growth rate depends on the number of varieties, let's consider the symmetric case of (2) given by $\ln c_t = n^* \ln y_t(i)$. Differentiating $\ln c_t$ with respect to time yields $g = n^* \dot{y}_t(i)/y_t(i)$. In other words, for a given quality growth rate of each variety, increasing the number of varieties causes the aggregate consumption index to grow at a higher rate.[26] Given that increasing s has a positive effect on n^* and a negative effect on λ, there is generally a growth-maximizing profit-division rule. Differentiating the log of (30) with respect to s yields

$$\frac{1}{g}\frac{\partial g}{\partial s} = \frac{\phi_n}{h_n}\frac{\partial h_n}{\partial s} + \frac{\phi_q}{h_q}\frac{\partial h_q}{\partial s}, \tag{31}$$

where $\partial h_n/\partial s > 0$ and $\partial h_q/\partial s < 0$ from Proposition 1. From (27), we can derive

$$\frac{1}{h_n}\frac{dh_n}{ds} = -\frac{1}{h_q}\left(\frac{1-h_n}{h_n + \phi_n(1-h_n)}\right)\frac{dh_q}{ds}. \tag{32}$$

Substituting (32) into (31) yields

$$\frac{1}{g}\frac{\partial g}{\partial s} = -\frac{1}{h_q}\left(\frac{\phi_n(1-h_n)}{h_n + \phi_n(1-h_n)} - \phi_q\right)\frac{dh_q}{ds}. \tag{33}$$

Therefore,

$$\frac{\partial g}{\partial s} > 0 \Leftrightarrow h_n(s) < \Phi \equiv \frac{\phi_n(1-\phi_q)}{\phi_q + \phi_n(1-\phi_q)}. \tag{34}$$

To gain a better understanding of (34), we maximize (30) by directly choosing h_n and h_q subject to (26). Substituting $\lambda = \varphi_q (h_q)^{\phi_q}$ and $h_q = (1-h_n)/n^*$ into (30) yields $g = (n^*)^{1-\phi_q}(1-h_n)^{\phi_q}\varphi_q \ln z$, where $n^* = \varphi_n (h_n)^{\phi_n}/\delta$ from (21). It is easy to show that the growth-maximizing h_n is given by Φ, which is increasing in ϕ_n and decreasing in ϕ_q. In other words, as horizontal R&D exhibits a smaller degree of negative duplication externality (i.e., a larger ϕ_n) or as vertical R&D exhibits a larger degree of duplication externality (i.e., a

[26] It is useful to note that this result of horizontal innovation affecting long-run growth does not rely on a stationary number of varieties. In the case of a growing number of varieties, horizontal innovation would still have an effect on long-run growth if the long-run variety growth rate is endogenous. However, it is common for studies on R&D-based growth models with vertical and horizontal innovation to assume a setup in which the long-run variety growth rate is equal to the exogenous population growth rate for the purpose of eliminating scale effects.

smaller ϕ_q), the economy should allocate more research labor to horizontal R&D to maximize economic growth. Therefore, the growth-maximizing profit-division rule $s_g \equiv \arg\max g(s)$ is characterized by moving the equilibrium $h_n(s_g)$ to as close to Φ as possible.

Proposition 2 *If an interior growth-maximizing profit-division rule s_g exists, it is implicitly defined by $h_n(s_g) = \Phi$. If $h_n(0) > \Phi$, then $s_g = 0$. If $h_n(1) < \Phi$, then $s_g = 1$.*

Proof. Note (33) and (34). Also, recall that $h_n(s)$ is increasing in s. ■

4.2 The welfare-maximizing profit-division rule

To derive the steady-state welfare,[27] we normalize the time index such that time 0 is the instant when the economy reaches the stationary equilibrium. In this case, (1) becomes[28]

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g}{\rho}\right) = \frac{1}{\rho}\left(n^* \ln l + \frac{n^* \lambda \ln z}{\rho}\right), \tag{35}$$

where $l = L/n^*$ is decreasing in s. In other words, social welfare is determined by the growth rate g as well as the initial level of consumption $\ln c_0$. Because of this additional level effect, the welfare-maximizing profit-division rule is generally different from the growth-maximizing rule. When s increases, it creates a positive effect as well as a negative effect on $\ln c_0 = n^* \ln l$. By increasing h_n and n^*, a larger s increases the number of varieties available for consumption on the one hand and decreases output per variety on the other. Differentiating $\ln c_0$ with respect to s yields

$$\frac{\partial \ln c_0}{\partial s} = (\ln l - 1)\,\frac{\partial n^*}{\partial s}, \tag{36}$$

[27] In this section, we restrict our attention to steady-state welfare. A more complete welfare analysis would take into account the evolution of households' utility during the transitional path from the initial state to the steady state, and we will perform this analysis numerically in the next section. However, such an analysis is analytically much more complicated. Therefore, we firstly follow the usual treatment in the literature to derive the optimal patent policy that maximizes steady-state welfare. See, for example, Acemoglu and Akcigit (2009), Futagami and Iwaisako (2003, 2007) and Grossman and Lai (2004).

[28] Equation (35) is based on the normalization that $q_0(i) = 0$ for all i. If we modify this normalization to $q_0(i) = q > 0$ for all i, then there will be an extra term $n^* q \ln z$ inside the bracket in (35). It can be shown that $q > 0$ has the same effect as a larger L on steady-state welfare.

where $n^* = \varphi_n (h_n)^{\phi_n}/\delta$ so that $\partial n^*/\partial s > 0$. Therefore,

$$\frac{\partial \ln c_0}{\partial s} > 0 \Leftrightarrow h_n(s) < \Delta \equiv \left(\frac{\delta L}{\varphi_n e}\right)^{1/\phi_n}, \tag{37}$$

where $e = \exp(1)$. In other words, the level of h_n that maximizes initial consumption is given by Δ. Equation (22) shows that for a given $(h_n)^{\phi_n}$, a larger $\delta L/\varphi_n$ increases l, so that h_n can be larger while initial consumption still rises.

Differentiating (35) with respect to s yields

$$\frac{\partial U}{\partial s} = \frac{1}{\rho}\left(\frac{\partial \ln c_0}{\partial s} + \frac{1}{\rho}\frac{\partial g}{\partial s}\right). \tag{38}$$

Denote the welfare-maximizing profit-division rule by $s_u \equiv \arg\max U(s)$.[29] In Proposition 3, we show that

$$s_u \geq s_g \Leftrightarrow \Delta \geq \Phi. \tag{39}$$

Intuitively, the welfare-maximizing h_n balances between the growth effect and the initial-level effect on welfare. Therefore, it is a weighted average of Δ and Φ. If $\Delta \geq \Phi$, then the welfare-maximizing h_n is above the growth-maximizing h_n, and vice versa. Given that $h_n(s)$ is increasing in s, $\Delta \geq \Phi$ would also imply $s_u \geq s_g$.

Proposition 3 *The welfare-maximizing profit-division rule s_u is below (above) the growth-maximizing profit-division rule s_g if Δ is smaller (larger) than Φ.*

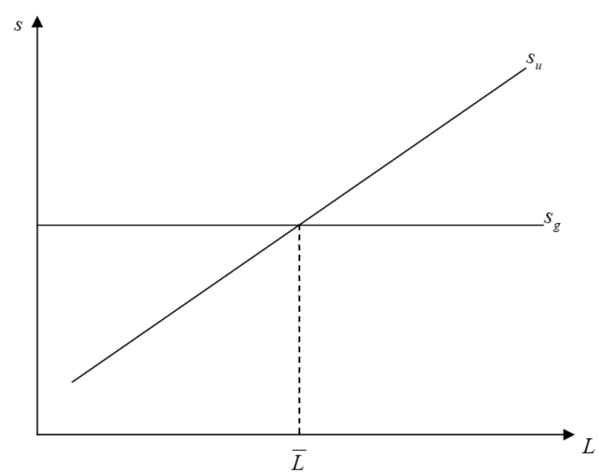
Proof. From (34), we know that $\partial g/\partial s = 0$ at $h_n(s) = \Phi$. From (37), we know that $\partial \ln c_0/\partial s = 0$ at $h_n(s) = \Delta$. Suppose $\Delta = \Phi$. Then, (38) shows that $s_u = s_g$. If $\Delta \geq (\leq)\Phi$, then $s_u \geq (\leq) s_g$ because $h_n(s)$ is increasing in s. ■

Finally, we discuss how the supply of unskilled labor L affects the welfare-maximizing profit-division rule. From (25) and (26), we see that neither the arbitrage condition nor the

[29] It is useful to note that as in the case of the growth-maximizing profit-division rule, the welfare-maximizing profit-division rule can be a corner solution (i.e., $s_u \to 0$ or $s_u \to 1$).

resource constraint depend on L. Therefore, the supply of unskilled labor has no effect on the growth-maximizing profit-division rule. Furthermore, given that Δ is increasing in L, it must be the case that s_u is increasing in L. Intuitively, a larger supply of unskilled labor increases output per variety and magnifies the positive effect of n^* on the initial level of consumption $\ln c_0 = n^* \ln L - n^* \ln n^*$ through the term $n^* \ln L$. Given that the welfare-maximizing s_u is increasing in L while the growth-maximizing s_g is independent of L, we have the following result illustrated in Figure 2.

Figure 2: Growth-maximizing and welfare-maximizing profit-division rules



Let's firstly define a threshold value of L given by $\overline{L} \equiv \varphi_n \Phi^{\phi_n} e/\delta$.

Corollary 2 *If L is smaller (larger) than $\overline{L}$, then s_u is below (above) s_g.*

Proof. This result follows from Proposition 3 because $L \leq \overline{L} \equiv \varphi_n \Phi^{\phi_n} e/\delta$ is equivalent to $\Delta \leq \Phi$. ■

5 Quantitative analysis

In this section, we calibrate the model to illustrate quantitatively the growth and welfare effects of strengthening blocking patents (i.e., increasing s). First, we evaluate the effects of increasing s from 0 to 1 on steady-state welfare. Then, we simulate the transition dynamics to compute the complete welfare changes. Specifically, we consider two types of policy reform: (a) an immediate increase in s, and (b) a gradual increase in s.

5.1 Steady-state welfare

For the structural parameters, we either consider conventional parameter values or calibrate their values by using empirical moments in the US before the patent-policy reform in 1982. For the discount rate ρ, we set it to 0.03.[30] For the R&D externality parameters ϕ_q and ϕ_n, we consider the symmetric case of $\phi = \phi_q = \phi_n$ and follow Jones and Williams (2000) to consider a value of $\phi = 0.5$.[31] Similarly, we consider the symmetric case of $\varphi = \varphi_q = \varphi_n$ for R&D productivity as in Gersbach *et al.* (2009).[32] To calibrate the values of the remaining structural parameters φ, δ, z and L, we use the following four empirical moments (i) the arrival rate of vertical innovation, (ii) the average growth rate of total factor productivity, (iii) R&D as a share of GDP, and (iv) the ratio of R&D scientists and engineers to labor force. For (i), we follow Acemoglu and Akcigit (2009) to consider an innovation-arrival rate of $\lambda = 0.33$. For (ii), we consider a value of $g = 1.5\%$. For (iii), we use a value of

[30] We have also considered a higher discount rate of 0.05. Although the welfare gains become smaller, the qualitative implication of our results remains unchanged.

[31] While Kortum's (1992) estimated value for a parameter similar to ϕ is 0.2, Jones and Williams (2000) use the empirical estimates of the social return to R&D to show that a lower bound for ϕ is 0.5. Therefore, we use $\phi = 0.5$ as our benchmark.

[32] In this calibration exercise, we consider the benchmark case of symmetric R&D parameters because a more detailed calibration requires disaggregate data on vertical and horizontal R&D. Unfortunately, we do not know of such data. However, if we follow the interpretation of Aghion and Howitt (1996) to treat horizontal R&D mainly as basic research and vertical R&D as applied research, then we can consider the data on basic R&D as a benchmark. According to OECD: Main Science and Technology Indicators, basic R&D is about 0.33% of US GDP in 1982. In our model's calibration, about 26% of high-skill labor is allocated to horizontal R&D implying that horizontal R&D as a share of GDP is about 0.39%. Therefore, the calibration based on symmetric R&D parameters is roughly in line with the data.

$R\&D/GDP = w_h/(w_h + w_l L + n^* \pi) = 1.5\%$. For (iv), there were 711.8 thousands full-time equivalent R&D scientists and engineers in the US in 1982,[33] and there were 110.2 millions people in the US labor force in 1982. Given these empirical moments, we have the following calibrated values $\{\varphi, \delta, z, L\} = \{0.64, 0.12, 1.02, 153.8\}$.

Table 1: Effects of s on growth and welfare

s	0.0	0.2	0.4	0.6	0.8	1.0
λ	0.33	0.30	0.27	0.25	0.22	0.18
g	1.500%	1.513%	1.505%	1.474%	1.413%	1.301%
U	388.1	417.4	445.5	473.0	500.8	530.1

Table 1 shows that an increase in s would stifle vertical innovation by decreasing the arrival rate of productivity improvements. Despite the increase in horizontal innovation, the overall growth rate eventually decreases. This finding is consistent with the recent concerns about patent protection stifling the innovation process. However, Table 1 also suggests an interesting possibility that despite the lower growth rate, steady-state welfare U in (35) increases due to the higher rate of horizontal innovation.[34,35] This illustrative exercise suggests the importance of taking into consideration the stimulating effect of s on horizontal innovation for a proper welfare analysis.

[33] This data is obtained from National Science Foundation. See the number of full-time equivalent R&D scientists and engineers in the US.

[34] It is useful to note that this finding of a welfare gain is robust to the normalization of $q_0(i) = 0$ for all i. In the case of $q_0(i) = q > 0$ for all i, the welfare gain would have been more substantial because $q > 0$ has the same effect as a larger L as discussed before.

[35] We have also considered a hypothetical value of $s = 1.1$ and find that welfare continues to increase in s. This result also applies to the subsequent results with transition dynamics. However, a potential problem with $s > 1$ is that if patent infringment occurs only when an entrant launches her product in the market (rather than when she comes up with the innovation), she may not have the incentives to launch her high-quality product to avoid paying the penality to the incumbent. If every subsequent entrant acts in this way, then vertical innovation would come to a halt.

5.2 Immediate patent reform

In the previous section, we evaluated the effects of an increase in s on steady-state welfare. However, such an analysis neglects the welfare changes during the transition path. Therefore, in this section, we simulate the transition dynamics of the model.[36] Given the transition path of the consumption index, we can then evaluate the complete welfare effects of an immediate increase in s from $s = 0$ to $s \in \{0.2, 0.4, 0.6, 0.8, 1.0\}$. Comparing Tables 1 and 2, we see that increasing s would improve welfare even taking into consideration transition dynamics. However, the magnitude of the welfare improvement is smaller than in the case of steady-state welfare.

Table 2: Welfare effects of an immediate increase in s

s	0.0	0.2	0.4	0.6	0.8	1.0
$U(transition)$	388.1	411.8	434.4	456.3	478.0	500.4

5.3 Gradual patent reform

In the previous section, we evaluated the welfare effects of an immediate increase in s. However, in the US, the patent reform may be more accurately described as a gradual reform. For example, in 1982, the US Congress established the Court of Appeals for the Federal Circuit (CAFC) as a centralized appellate court for patent cases. "Over the next decade, in case after case, the court significantly broadened and strengthened the rights of patent holders."[37] Also, the Ginarte-Park index (described in Section 1) shows that the strength of patent protection in the US *gradually* increases from 3.8 in 1975 to 4.9 in 1995.[38]

[36] See Appendix B for a description of the dynamic system and the numerical algorithm.

[37] Jaffe and Lerner (2004, p. 9-10).

[38] The Ginarte-Park index is an aggregate measure of patent rights rather than a direct measure of the profit-division rule. Although an empirical measure of "s" is not available, the anecdotal evidence from Jaffe and Lerner (2004) seems to suggest that it increases gradually in the US rather than once and for all in the early 1980's.

Table 3: Index of patent rights from Park (2008)

Year	1975	1980	1985	1990	1995	2000	2005
United States	3.83	4.35	4.68	4.68	4.88	4.88	4.88

Therefore, in this section, we evaluate the welfare effects of a gradual increase in s from $s = 0$ to $\overline{s} \in \{0.2, 0.4, 0.6, 0.8, 1.0\}$. Following Cozzi and Galli (2009), we consider a law of motion for s_t given by

$$\dot{s}_t = \psi(\overline{s} - s_t), \tag{40}$$

where the parameter $\psi \in (0, 1)$ determines the speed of the patent reform. In the numerical exercise, we consider $\psi = 0.05$ for illustrative purposes. Table 4 shows that a gradual increase in s would improve social welfare but by a smaller magnitude than an immediate increase in s. Furthermore, the welfare gain is increasing in ψ (i.e., increasing in the speed of reform). As ψ approaches one, the welfare gain becomes the same as in Section 5.2.

Table 4: Welfare effects of a gradual increase in s

s	0.0	0.2	0.4	0.6	0.8	1.0
$U(\psi = 0.05)$	388.1	404.7	420.8	436.3	451.6	467.1

6 Conclusion

This study develops a simple growth model to shed some light on an often debated question that is whether patent protection stimulates or stifles innovation. We show that both sides of the argument are valid. Specifically, protecting incumbents at the expense of entrants would stimulate horizontal innovation but stifle vertical innovation, and the opposite occurs when entrants are protected against incumbents. Although the distinction between vertical and horizontal innovation is blurred in reality, our point is still valid in the sense that patent protection has asymmetric effects on different types of innovation that have different chances of patent infringements, and hence, the traditional tradeoff of optimal patent protection needs to be modified to take into account this asymmetric effect of patent policy. In other words, the optimal patent policy should be innovation-specific. If vertical (horizontal) innovation is crucial to social welfare, then a more frontloading (backloading) profit-division rule should be implemented. Furthermore, if we follow Aghion and Howitt (1996) to treat horizontal R&D as basic research and vertical R&D as applied research, then our finding implies that a gradual increase in the bargaining power of the basic researchers could be welfare-improving, and this finding is consistent with the two-stage R&D analysis in Cozzi and Galli (2009), who consider a transition to more upstream bargaining power.

Finally, in this study, we have considered a stylized growth model for analytical tractability, and the numerical exercises are for illustrative purposes. Therefore, it would be interesting for future studies to develop a more general dynamic general-equilibrium model to obtain more precise quantitative implications of strengthening patent protection.

References

[1] Acemoglu, D., 2009. A note on diversity and technological progress. manuscript.

[2] Acemoglu, D., and Akcigit, U., 2009. State-dependent intellectual property rights policy. manuscript.

[3] Acs, Z., and Sanders, M., 2009. Intellectual property rights and the knowledge spillover theory of entrepreneurship. Jena Economic Research Papers No. 2008-069.

[4] Aghion, P., and Howitt, P., 1992. A model of growth through creative destruction. *Econometrica* 60, 323-351.

[5] Aghion, P., and Howitt, P., 1996. Research and development in the growth process. *Journal of Economic Growth* 1, 49-73.

[6] Bessen, J., and Meurer, M., 2008. Patent failure: How judges, bureaucrats, and lawyers put innovators at risk. Princeton University Press.

[7] Boldrin, M., and Levine, D., 2008. Against intellectual monopoly. Cambridge University Press.

[8] Boldrin, M., Levine, D., and Nuvolari, A., 2008. Do patents encourage or hinder innovation? The case of the steam engine. *The Freeman: Ideas on Liberty*, Volume 58, Issue 10.

[9] Chu, A., 2009. Effects of blocking patents on R&D: A quantitative DGE analysis. *Journal of Economic Growth* 14, 55-78.

[10] Chu, A., and Pan, S., 2010. The escape-infringement effect of blocking patents on innovation and economic growth. MPRA Working Papers No. 27233.

[11] Cozzi, G., 2001. Inventing or spying? Implications for growth. *Journal of Economic Growth* 6, 55-77.

[12] Cozzi, G., 2005. Animal spirits and the composition of innovation. *European Economic Review* 49, 627-637.

[13] Cozzi, G., 2007. The Arrow effect under competitive R&D. *The B.E. Journal of Macroeconomics*: Vol. 7: Iss. 1 (Contributions), Article 2.

[14] Cozzi, G., and Galli, S., 2009. Upstream innovation protection: Common law evolution and the dynamics of wage inequality. *University of Glasgow* Working Papers No. 2009-20.

[15] Cozzi, G., Giordani, P., and Zamparelli, L., 2007. The refoundation of the symmetric equilibrium in Schumpeterian growth models. *Journal of Economic Theory* 136, 788-797.

[16] Cozzi, G., and Spinesi, L., 2006. Intellectual appropriability, product differentiation, and growth. *Macroeconomic Dynamics* 10, 39-55.

[17] Dinopoulos, E., and Thompson, P., 1999a. Schumpeterian growth without scale effects. *Journal of Economic Growth* 3, 313-335.

[18] Dinopoulos, E., and Thompson, P., 1999b. Scale effects in Schumpeterian models of economic growth. *Journal of Evolutionary Economics* 9, 157-185.

[19] Etro, F., 2004. Innovation by leaders. *Economic Journal* 114, 281-303.

[20] Etro, F., 2008. Stackelberg competition with endogenous entry. *Economic Journal* 118, 1670-1697.

[21] Furukawa, Y., 2007. The protection of intellectual property rights and endogenous growth: Is stronger always better? *Journal of Economic Dynamics and Control* 31, 3644-3670.

[22] Futagami, K., and Iwaisako, T., 2003. Patent policy in an endogenous growth model. *Journal of Economics* 78, 239-258.

[23] Futagami, K., and Iwaisako, T., 2007. Dynamic analysis of patent policy in an endogenous growth model. *Journal of Economic Theory* 132, 306-334.

[24] Gallini, N., 2002. The economics of patents: Lessons from recent U.S. patent reform. *Journal of Economic Perspectives* 16, 131-154.

[25] Gallini, N., and Scotchmer, S., 2002. Intellectual property: When is it the best incentive system? In A. Jaffe, J. Lerner and S. Stern (eds), *Innovation Policy and the Economy*, vol. 2, pp. 51-78. The MIT Press.

[26] Gersbach, H., Sorger, G., and Amon, C., 2009. Hierarchical growth: Basic and applied research. *CER-ETH Economics* Working Paper No. 09/118.

[27] Ginarte, J., and Park, W., 1997. Determinants of patent rights: A cross-national study. *Research Policy* 26, 283-301.

[28] Green, J., and Scotchmer, S., 1995. On the division of profit in sequential innovations. *RAND Journal of Economics* 26, 20-33.

[29] Grossman, G., and Helpman, E., 1991. Quality ladders in the theory of growth. *Review of Economic Studies* 58, 43-61.

[30] Grossman, G., and Lai, E., 2004. International protection of intellectual property. *American Economic Review* 94, 1635-1653.

[31] Hall, B., Jaffe, A., and Trajtenberg, M., 2001. The NBER patent citations data file: Lessons, insights and methodological issues. NBER Working Papers No. 8498.

[32] Horii, R., and Iwaisako, T., 2007. Economic growth with imperfect protection of intellectual property rights. *Journal of Economics* 90, 45-85.

[33] Horowitz, A., and Lai, E., 1996. Patent length and the rate of innovation. *International Economic Review* 37, 785-801.

[34] Howitt, P., 1999. Steady endogenous growth with population and R&D inputs growing. *Journal of Political Economy* 107, 715-730.

[35] Jaffe, A., 2000. The U.S. patent system in transition: Policy innovation and the innovation process. *Research Policy* 29, 531-557.

[36] Jaffe, A., and Lerner, J., 2004. Innovation and its discontents: How our broken system is endangering innovation and progress, and what to do about it. Princeton, NJ: Princeton University Press.

[37] Jones, C., 1999. Growth: With or without scale effects. *American Economic Review* 89, 139-144.

[38] Jones, C., and Williams, J., 2000. Too much of a good thing? The economics of investment in R&D. *Journal of Economic Growth* 5, 65-85.

[39] Judd, K., 1985. On the performance of patents. *Econometrica* 53, 567-586.

[40] Koleda, G., 2004. Patents' novelty requirement and endogenous growth. *Revue d'economie politique* 114, 201-221.

[41] Kortum, S., 1992. Inventions, R&D and industry growth. Ph.D. Dissertation, Yale University.

[42] Li, C.-W., 2000. Endogenous vs. semi-endogenous growth in a two-R&D-sector model. *Economic Journal* 110, C109-C122.

[43] Li, C.-W., 2001. On the policy implications of endogenous technological progress. *Economic Journal* 111, C164-C179.

[44] Moser, P., 2005. How do patent laws influence innovation? Evidence from nineteenth-century World's Fairs. *American Economic Review* 95, 1214-1236.

[45] Nordhaus, W., 1969. Invention, growth, and welfare. Cambridge, Mass.: The MIT Press.

[46] O'Donoghue, T., and Zweimuller, J., 2004. Patents in a model of endogenous growth. *Journal of Economic Growth* 9, 81-123.

[47] Park, W., 2008. International patent protection: 1960-2005. *Research Policy* 37, 761-766.

[48] Peretto, P., 1998. Technological change and population growth. *Journal of Economic Growth* 3, 283-311.

[49] Peretto, P., 1999. Cost reduction, entry, and the interdependence of market structure and economic growth. *Journal of Monetary Economics* 43, 173-195.

[50] Peretto, P., and Smulders, S., 2002. Technological distance, growth and scale effects. *Economic Journal* 112, 603-624.

[51] Scotchmer, S., 2004. Innovation and incentives. Cambridge, Mass.: The MIT Press.

[52] Segal, I., and Whinston, M., 2007. Antitrust in innovative industries. *American Economic Review* 97, 1703-1730.

[53] Segerstrom, P., 2000. The long-run growth effects of R&D subsidies. *Journal of Economic Growth* 5, 277-305.

[54] Segerstrom, P., Anant, T., and Dinopoulos, E., 1990. A Schumpeterian model of the product life cycle. *American Economic Review* 80, 1077-1092.

[55] Shapiro, C., 2001. Navigating the patent thicket: Cross licenses, patent pools, and standard setting. In A. Jaffe, J. Lerner, S. Stern (eds), *Innovation Policy and the Economy*, vol. 1, pp. 119-150. The MIT Press.

[56] Young, A., 1998. Growth without scale effects. *Journal of Political Economy* 106, 41-63.

Appendix A: Proofs

Proof of Lemma 1. From (23), the value of a quality improvement is $v_1 = \frac{\pi}{\rho+\delta+\lambda}\left(1 - s + s\frac{\lambda}{\rho+\delta+\lambda}\right)$ for a firm that does not own the previous innovation. For an incumbent (i.e., a firm that owns the previous innovation), the incremental value of a quality improvement is $v_I = \frac{\pi}{\rho+\delta+\lambda}\left(1 + s\frac{\lambda}{\rho+\delta+\lambda}\right) - v_2$.[39] The first term in v_I reflects that the firm's new product infringes its own patent and hence it does not have to pay any licensing fee. The second term (i.e., $-v_2$) reflects that the incumbent's old invention loses the opportunity to extract profit from the new entrant. Substituting $v_2 = \frac{\pi}{\rho+\delta+\lambda}s$ into v_I yields $v_I = v_1$ for $s \in [0,1]$, so that the incumbent is indifferent as to where to target innovation. As a result, all the aggregate variables behave as if quality improvement is targeted only by the entrants (i.e., the Arrow replacement effect).[40] ■

Proof of Lemma 2. Let's firstly define a new variable $x \equiv \varphi_q (h_q)^{\phi_q}$ and a new function

$$f(x) \equiv \frac{1}{\rho + \delta + (1+s)x}\left(\frac{x}{(1-s)(\rho+\delta)+x}\right). \tag{A1}$$

Simple differentiation yields

$$\arg\max f(x) = (\rho+\delta)\sqrt{\frac{1-s}{1+s}}. \tag{A2}$$

Given that dh_n/dh_q in (28) is decreasing in $f(x)$, maximizing $f(x)$ is equivalent to minimizing

[39] To be consistent with the assumption of no market-power consolidation, an upper bound of z is imposed on the markup, so that π is the same in v_1 and v_I. In the case of market-power consolidation, the markup would be given by z^2 regardless of whether or not the two generations of quality improvement are owned by the same firm, so that π would be the same in v_1 and v_I as well.

[40] This new interpretation of the Arrow effect is developed by Cozzi (2007), who shows that the incumbent's current invention faces the same probability of being displaced regardless of whether or not an incumbent targets innovation at her own industry. Under the traditional interpretation (i.e., when an incumbent obtains a new invention, she loses the value of the old invention), it should be v_1 (instead of v_2) that is substracted from v_I. In this case, $v_I = \frac{\pi}{\rho+\delta+\lambda}\left(1 + s\frac{\lambda}{\rho+\delta+\lambda}\right) - v_1 = \frac{\pi}{\rho+\delta+\lambda}s$, and hence $v_I < v_1 \iff s < \widehat{s} \equiv \frac{\rho+\delta+\lambda}{2(\rho+\delta)+\lambda} \in [0.5, 1]$. Therefore, when $s < \widehat{s}$, quality improvement is targeted by entrants only, so that the Arrow replacement effect is again present.

the bracketed term in (28). Substituting (A2) into (28) yields

$$\frac{dh_n}{dh_q} = \frac{1}{1-\phi_n}\left(1-\phi_q-\phi_q\frac{s^2}{2-s^2+2\sqrt{1-s^2}}\right)\frac{h_n}{h_q}. \tag{A3}$$

Manipulating (A3) shows that $\phi_q < [1-0.5s^2/(1+\sqrt{1-s^2})] \in [0.5, 1]$ implies $dh_n/dh_q > 0$ in (28) for any value of $h_q > 0$. ■

Appendix B: Transition dynamics

The system of equations that characterizes the dynamics of the model is as follows.

$$\dot{n}_t^* = \varphi_n (h_{n,t})^{\phi_n} - \delta n_t^* \tag{B1}$$

$$\dot{\zeta}_t / \zeta_t = \rho - r_t \tag{B2}$$

$$\dot{v}_{2,t} = (r_t + \lambda_t + \delta) v_{2,t} - s\pi_t \tag{B3}$$

$$\dot{v}_{1,t} = (r_t + \lambda_t + \delta) v_{1,t} - \lambda_t v_{2,t} - (1-s)\pi_t \tag{B4}$$

$$\dot{v}_{n,t} = (r_t + \lambda_t + \delta) v_{n,t} - \lambda_t v_{2,t} - \pi_t \tag{B5}$$

$$\pi_t = \left(\frac{z-1}{z}\right) \frac{1}{\zeta_t} \tag{B6}$$

$$\lambda_t = \varphi_q (h_{q,t})^{\phi_q} \tag{B7}$$

$$v_{1,t} \varphi_q (h_{q,t})^{\phi_q - 1} = v_{n,t} \varphi_n (h_{n,t})^{\phi_n - 1} \tag{B8}$$

$$h_{n,t} + n_t^* h_{q,t} = 1 \tag{B9}$$

$$n_t^* l_t = L \tag{B10}$$

$$\pi_t = (z-1) w_{l,t} l_t = \left(\frac{z-1}{z}\right) \frac{1}{\zeta_t} \Longrightarrow z w_{l,t} l_t = \frac{1}{\zeta_t} \tag{B11}$$

Finally, we choose l_t as the numeraire by setting $w_{l,t} = 1$. The endogenous variables in this system are $\{n_t^*, \zeta_t, v_{2,t}, v_{1,t}, v_{n,t}, \pi_t, \lambda_t,\ h_{q,t}, h_{n,t}, l_t, r_t\}$.

In all our numerical simulations, in order to simulate the dynamic transition from one steady state to another, we first compute the initial steady state and the final steady state, associated with the initial and final level of s; then we discretize all the differential equations in system (B1)-(B11), and plug them as well as the remaining equation restrictions in a .mod file, which allows Dynare to apply its deterministic routines, needed to compute the dynamic rational expectations equilibrium transition from the initial to the final steady state.

Since Dynare also analyses the eigenvalues of the Jacobian matrix at the final steady state, while simulating the transitional path we always make sure that in all our simulations the conditions for the determinacy of the steady state are satisfied, that is the number of stable eigenvalues is equal to the number of predetermined variables. Hence, all the transitional paths we have obtained are along the unique equilibrium of the economy analyzed.

In order to calculate the complete change in welfare, we need to keep track of the evolution of the consumption index.

$$\ln c_t = \int_0^{n_t^*} (q_t(i) \ln z + \ln l_t(i)) di = \left(\int_0^{n_t^*} q_t(i) di \right) \ln z + n_t^* \ln l_t. \tag{B12}$$

Normalizing $q_0(i) = 0$ for all i, we can re-express the level of aggregate technology as

$$\int_0^{n_t^*} q_t(i) di = \int_0^t n_\tau^* \lambda_\tau d\tau + \int_0^t \dot{n}_\tau^* \left(\int_0^\tau \lambda_\upsilon d\upsilon \right) d\tau. \tag{B13}$$

The first term on the right hand side of (B13) is the accumulated number of productivity improvements that have occurred from time 0 to time t. The second term on the right hand side of (B13) is the change in aggregate technology due to the introduction of new varieties net of obsolescence. Using the data generated by Dynare, we could then compute the discretized version of the welfare integral, which allowed the welfare experiments reported in the tables of Section 5.

Notice that by normalizing $q_0(i) = 0$ for all i, in light of (B13), we are minimizing the effect of $\dot{n}_t^*$ on welfare. This proves the robustness of the welfare comparisons in Tables 2 and 4. Given that n_t^* increases from the initial steady state to the new steady state in our numerical exercises, any alternative positive level of the $q_0(i)$'s would imply a higher transitional welfare effect of an increase in s.