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Abstract By assuming that grandparents take care of grandchildren, in this paper we aim at studying the effects of longevity on economic growth in the basic OLG model with endogenous fertility. We show that a rise in longevity can actually reduce long-run growth. Moreover, we also find that an increasing longevity (*i*) increases the supply of labour by the young parents, and (*ii*) causes fertility either to increase of decrease depending on the size of the grandparental child rearing time.

Keywords Longevity; OLG model

JEL Classification J13; J22; O41

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1. Introduction

A major recent stylised fact is the decline in adult mortality experienced in several countries around the world (indeed, the rise in life expectancy in developed countries resulted in a doubling of the ratio of life cycle years lived after 65 to years lived 20 to 64, see Livi-Bacci, 2006, and it is expected to increase further on in the near future, see, e.g., the time evolution of the old-age dependency ratio in countries such as Italy, Japan and Spain as reported in Sinn, 2007, Figure 6, p. 10). However, the sharp increase in longevity has also augmented the share of leisure time that the elderly can devote (even as a quasi-complete substitute for parental care) to take care of grandchildren.

So far the economic growth literature, at least those framed in the basic overlapping generations (OLG) model of growth by Diamond (1965), has retained that a positive relationship between longevity and economic growth exists (see, e.g., Ehrlich and Lui, 1991), essentially because of the increase in savings and capital accumulation that a longer life span causes. Indeed, Fanti (2009) shows in a Diamond's growth model augmented with endogenous fertility motivated by the so-called "weak form of altruism" (see Zhang and Zhang, 1998), that a rise in the life span increases savings and reduces the demand for children, so that capital accumulation increases more than when fertility is not and economic decision variable. Moreover, such a positive relationship seems be robust in the economic theoretical literature holding also when the basic Diamond's OLG model is extended for endogenous fertility motivated by altruistic reasons with voluntary inter-generational bequests, social security and education (see Zhang et al., 2001),[1] or when adult mortality is endogenously determined by public investments in health (see Chakraborty, 2004, which states "Yet health plays a role quite unlike any other human capital: by increasing lifespans it makes

[1] For instance, Zhang et al. (2001, p. 485) claim: "A rise in longevity has direct effects on fertility, human capital investment, and growth, as well as indirect effects through increasing unfunded social security contributions… The net effects of rising longevity on fertility tend to be negative, but positive on human capital investment and growth."

individuals effectively more patient and willing to invest, and by reducing mortality risks, it raises the return on investment… when life expectancy is low, individuals discount the future more heavily and are less inclined to save and invest.", p. 120).[2]

However, some empirical works have shown that the relationship between longevity and economic growth may be hump-shaped, namely when life expectancy is fairly low (high), a increase in it causes economic growth to raise (fall), see, e.g., Maddison (1992), Kelley and Schmidt (1995) and Barro and Sala-i-Martin (2004). To the best of our knowledge, from the theoretical grounds the sole model that accords with these empirical facts is Zhang et al. (2003), which departs from the basic Diamond's model by extending Zhang et al. (2001) with accidental bequests. Indeed, it is just through the interaction between education and accidental bequests that a rise in longevity can actually reduce the rate of economic growth.

It should be noted that the above mentioned theoretical literature abstracts from another stylised fact recently evidenced: the role that grandparents have in raising grandchildren. As an example, ISFOL (2011) reveals that in Italy 36 per cent of women that belongs to the active population decide to have a child only whether some forms of child care assistance inside the home, provided by non-cohabitant family members, e.g., grandparents, exist. This phenomenon can actually produce macroeconomic effects through the reduction in the opportunity cost of children by parents and the rise in both the labour supply by women and the fertility rate.

The aim of this paper is to investigate how the existence of an exogenous provision of grandparental child care activity inside the home affects the relationship between longevity and long-run economic growth. To this purpose we consider a general equilibrium OLG model of neoclassical growth with exogenous longevity and endogenous fertility. By assuming a perfect annuities market and taking into account the evidence of the importance of the child care assistance by grandparents (Minkler and Fuller-Thomson, 1999; Hayslip and Kaminski, 2005; Hank and Buber, 2009), as a substitute for parental child care inside the home, we show that an increasing

[2] See also Blackburn and Cipriani (2002).

longevity (*i*) increases the supply of labour by the young parents, (*ii*) causes fertility either to increase of decrease depending on the size of the grandparental child rearing time, and (*iii*) can actually work as a growth-reducing device in a simple OLG model à la Diamond (1965). Moreover, we also find that a rise in the time devotes by grandparents to take care of grandchildren does not affect the labour supply of the young parents.

The value added of the present paper essentially lies in the provision of another explanation of the hump-shaped relationship between longevity and long-run growth, as evidenced by the empirical literature above mentioned.

The rest of the paper is organised as follows. Section 2 presents the model and discusses the results. Section 3 concludes.

2. The model

Consider a general equilibrium OLG closed economy populated by perfectly rational and identical individuals. Life of each person within every generation is divided between childhood and adulthood. Economic decisions are exclusively made in the latter period of life, which is in turn divided between youth (working period) and old age (retirement period). As an adult, each individual has preferences towards material consumption and the number of children, as in Eckstein and Wolpin (1985), Eckstein et al. (1988) and Galor and Weil (1996), which are assumed to be a normal good. This is the so-called weak form of altruism towards children (see Zhang and Zhang, 1998), because parents directly derive utility from the number of children they have but do not enjoy from the utility drawn by their offspring. Young agents of generation t (N_t) have an endowment of one unit of time, a fraction of which ($0 < h_t < 1$) is supplied to firms, while earning the wage w_t per unit of time, and the remaining part ($0 < q < 1$) is spent to care for n_t descendants, with q being the exogenous fraction of the time endowment that each parent devotes to raise a child, that is the bearing of children implies an opportunity cost for parents (see, e.g., Morand, 1999).[3] We assume that each young individual dies at the onset of old age with an exogenous probability $1-\pi$, that is $0<\pi\le 1$ is the probability of surviving at the end of youth. Moreover, we also assume that each grandparent (i.e., those belonging to generation $t-1$) devotes a constant fraction $0<z<1$ of her time endowment to take care of each grandchild, that is the provision of child care assistance inside the home by the old reduces the opportunity cost for parents to rear children. Since only πN_{t-1} old individuals are alive at time t, the labour supply of the young of generation t can definitely be written as

$$h_t = 1 - n_t(q - z\pi), \tag{1}$$

where $z < q/\pi := z_1$ must hold as a technical condition to ensure the existence of a positive number of children,[4] and their working period budget constraint reads therefore as:

$$c_{1,t} + s_t = w_t h_t = w_t[1 - n_t(q - z\pi)], \tag{2}$$

that is, the labour income is divided between consumption ($c_{1,t}$) and saving (s_t).

Old individuals retiree and live with the amount of resources saved when young plus the expected interest accrued from period t to period $t+1$ at the rate r^e_{t+1}. Moreover, the existence of a perfect annuities market (where savings are intermediated through mutual funds) implies that old survivors will benefit not only from their own past saving plus interest, but also from the saving

[3] Note that in order to take into account the negative substitution effect – on fertility – of the female labour earnings due to the potential increase of women's labour force participation (see Mincer, 1963), a possible extension of the present paper is to introduce a gender gap to differentiate the child bearing technology on the basis of the ability of male and female to raise children. Indeed, it seems widely accepted that the low female labour participation exerts a depressing role on economic growth, while also being a reason of the inverse relationship between fertility and growth, because of the high opportunity cost (wage) of raising children in developed countries, which, in turn, increases with the growth rate.

[4] Note also that $h_t > 0$ implies $n_t < 1/(q - z\pi) := \overline{n}$, which is the maximum number of children that a young individual can give birth to. Indeed, the higher the time spent raising a child by the young (old), the lower (higher) $\overline{n}$.

plus interest of those who have deceased.[5] Therefore, the budget constraint at time $t+1$ of the typical old individual started working at time t is expressed as:

$$c_{2,t+1} = \frac{1+r^{e}_{t+1}}{\pi} s_t , \tag{3}$$

where $c_{2,t+1}$ is old age consumption.

The typical individual of generation t chooses fertility and saving to maximise the expected lifetime utility function

$$U_t = \ln(c_{1,t}) + \pi \ln(c_{2,t+1}) + \gamma \ln(n_t), \tag{4}$$

subject to Eqs. (2) and (3) where $\gamma > 0$ is the parents' relative taste for children. Therefore, fertility and saving are respectively determined as:

$$n_t = n = \frac{\gamma}{(q - z\pi)(1+\pi+\gamma)}, \tag{5}$$

$$s_t = \frac{\pi w_t}{1+\pi+\gamma}. \tag{6}$$

Through Eqs. (1) and (5), the labour supply is constant and given by:

$$h_t = h = \frac{1+\pi}{1+\pi+\gamma}. \tag{7}$$

From Eq. (7) we have the following proposition.

Proposition 1. *(Labour supply). (1) A rise in life expectancy (π) monotonically increases the labour supply. (2) A rise in the child-rearing time by the old (z) does not affect the labour supply.*

Proof. Since $\partial h / \partial \pi = \gamma / (1+\pi+\gamma)^2 > 0$ and $\partial h / \partial z = 0$, then Proposition 2 follows. **Q.E.D.**

[5] This hypothesis is indeed reasonable to characterise developed economies where markets of annuities are largely adopted. It should be noted, however, that the results of the present paper hold even under the hypothesis of accidental bequests, as in Abel (1985).

Now, let $z_2 := q/(1+\gamma)$ and $z_3 := q/(3+\gamma)$ be two threshold values of z, where $z_3 < z_2 < z_1$. Then, from Eq. (5) the following proposition holds:

Proposition 2. *(Fertility). (1) Let* $0 < z < z_3$ *hold. Then, the rate of fertility monotonically decreases with the rate of longevity. (2) Let* $z_3 < z < z_2$ *hold. Then, the rate of fertility is a U-shaped function of the rate of longevity. (3) Let* $z_2 < z < z_1$ *hold. Then, the rate of fertility monotonically increases with the rate of longevity.*

Proof. The proof uses the following derivative:

$$\frac{\partial n}{\partial \pi} = \frac{\gamma[z(1+\gamma) - q + 2z\pi]}{(q - z\pi)^2 (1+\pi+\gamma)^2} \lesseqgtr 0 \Leftrightarrow \pi \lesseqgtr \pi_n := \frac{q - z(1+\gamma)}{2z}. \tag{8}$$

If $0 < z < z_3$, then $\pi_n > 1$ and $\frac{\partial n}{\partial \pi} < 0$ for every $0 < \pi < 1$. If $z_3 < z < z_2$, then $0 < \pi_n < 1$ and $\frac{\partial n}{\partial \pi} \lesseqgtr 0$ if and only if $\pi \lesseqgtr \pi_n$, where π_n is the fertility-minimising rate of longevity. If $z_2 < z < z_1$, then $\pi_n < 0$ and $\frac{\partial n}{\partial \pi} > 0$ for every $0 < \pi < 1$. **Q.E.D.**

Propositions 1 and 2 reveal that it is possible to jointly increase both the labour supply and fertility when longevity increases, provided that the time devoted to the child care assistance by grandparents is fairly high. Indeed, a trade-off between labour supply and fertility exists when grandparents do not spend enough time for caring their grandchildren. However, the rise in the child care inside the home (grandparental effect) together with observed increase in longevity in developed countries, may represent a possible explanation of the existence of a positive relationship between labour supply and fertility (see Apps and Rees, 2004).

Firms are identical and markets are competitive. The (aggregate) constant returns to scale Cobb-Douglas technology is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where Y_t, K_t and $L_t = h_t N_t$ are output, capital and the time-t labour input, respectively, $A > 0$ and $0 < \alpha < 1$. Therefore, output per young ($y_t := Y_t / N_t$) is obtained as $y_t = Ak_t^{\alpha} h_t^{1-\alpha}$, where $k_t := K_t / N_t$ is the stock of capital per young person. Assuming that capital fully depreciates at the end of every period and output is sold at unit price, profit maximisation implies:

$$r_t = \alpha A (k_t / h_t)^{\alpha - 1} - 1, \tag{9}$$

$$w_t = (1-\alpha) A (k_t / h_t)^{\alpha}, \tag{10}$$

Knowing that $N_{t+1} = n_t N_t$, the equilibrium on the capital market is

$$n_t k_{t+1} = s_t . \tag{11}$$

Combining Eqs. (5), (6), (7), (10) and (11) we get:

$$k_{t+1} = \frac{\pi}{\gamma}(1-\alpha) A k_t^{\alpha} (q - z\pi)[h(\pi)]^{-\alpha} . \tag{12}$$

Steady-states of the time map Eq. (10) are defined as $k_{t+1} = k_t = k$. Then the following proposition holds.

Proposition 3. *A rise in life expectancy ambiguously affects the long-run stock of capital.*

Proof. From Eq. (12) it can easily be seen that

$$k = k[\pi, h(\pi)]. \tag{13}$$

Totally differentiation Eq. (13) with respect to k gives:

$$\frac{dk}{d\pi} = \overbrace{\frac{\partial k}{\partial \pi}}^{+/-} + \overbrace{\frac{\partial k}{\partial h}}^{-} \cdot \overbrace{\frac{\partial h}{\partial \pi}}^{+}, \tag{14}$$

and Proposition 3 follows. **Q.E.D.**

Indeed, a reduction in adult mortality affects capital accumulation through a twofold channel which goes through (*i*) saving and fertility, and (*ii*) the labour supply. First, a rise in life expectancy directly increases saving and capital accumulation but, depending on the relative size of the time spent by grandparents to care for grandchildren (see Proposition 2), it ambiguously affects fertility. Moreover, it increases the labour supply (see Proposition 1), and this in turn tends to reduce the wage per unit of labour earned by the young parents and then capital accumulation reduces through this channel. Second, there exists a direct negative effect on capital accumulation due to the rise in the labour supply.

As regards the output per young person in the long run, we find that

$$y = y\{k[\pi, h(\pi)], h(\pi)\}, \tag{15}$$

and the following proposition holds.

Proposition 4. *A rise in life expectancy ambiguously affects the long-run neoclassical economic growth.*

Proof. Totally differentiation Eq. (13) with respect to k gives:

$$\frac{dy}{d\pi} = \overbrace{\frac{\partial y}{\partial k}}^{+} \cdot \left(\overbrace{\frac{\partial k}{\partial \pi}}^{+/-} + \overbrace{\frac{\partial k}{\partial h}}^{-} \cdot \overbrace{\frac{\partial h}{\partial \pi}}^{+} \right) + \overbrace{\frac{\partial y}{\partial h}}^{+} \cdot \overbrace{\frac{\partial h}{\partial \pi}}^{+}, \tag{16}$$

and Proposition 4 follows. **Q.E.D.**

Proposition 4 shows that a rise in life expectancy still remains ambiguous on the long-run economic growth even if the negative effect of it is weakened, as compared to the effect capital accumulation, by the positive effect on output per capital induced by the increase in the labour supply.

We now resort to numerical simulations (see Table 1) to show Proposition 3 and 4.[6] The parameter set is the following: $A = 10$ (simply a scale parameter in the Cobb-Douglas production function), $\alpha = 0.33$ (see Gollin, 2002; Kehoe and Perri, 2002) and $q = z = \gamma = 0.3$.

Table 1. Long-run capital stock, output, labour supply, fertility and saving when longevity increases.

π	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
k	0.529	1.238	1.843	2.235	2.362	2.211	1.803	1.196	0.505	0.194
y	6.896	9.24	10.648	11.45	11.753	11.58	10.894	9.569	7.235	5.291
h	0.785	0.8	0.812	0.823	0.833	0.842	0.85	0.857	0.863	0.866
n	0.793	0.833	0.892	0.98	1.111	1.315	1.666	2.38	4.545	8.888
s	0.42	1.031	1.646	2.191	2.624	2.909	3.005	2.849	2.296	1.727

Therefore, the following result holds.

Result 1. *When the time spent by grandparents to care for grandchildren if fairly high, both capital accumulation and output per young person are inverted U-shaped functions of the rate of longevity.*

The inverse relationship between longevity and long-run growth described here accords with the empirical evidence presented by Maddison (1992), Kelley and Schmidt (1995) and Barro and Sala-i-Martin (2004). Indeed, this unexpected net effect gets through four channels in this simple general equilibrium economy: (*i*) the labour supply, (*ii*) fertility (*iii*) saving and (*iv*) capital accumulation. When grandparents devote a relatively large amount of their time endowment to the child care assistance inside the home, a rise in longevity causes an increase in both the supply of labour and fertility (see Proposition 1 and 2, respectively). Saving instead first increases, because individuals expect to live longer, but then decreases because the rise in the labour supply reduces the wage. Due

[6] Table 1 also shows the corresponding values of both the labour supply and fertility rate. Note also that with this parameter set $z_2 = 0.23$ and $z_3 = 0.09$, while $\pi_k = 0.495$ and $\pi_y = 0.514$ which represent the capital- and output-maximising longevity rates, respectively.

to the reduction in saving and the rise in fertility, capital accumulation becomes lower when longevity is large enough. Though the increase in the supply of labour causes a positive direct effect on output per young, the negative effect of the reduced capital accumulation dominates when longevity is high, and this definitely causes a reduction in economic growth in the long run.

3. Conclusions

We studied the effects of longevity on the long-run economic growth in an OLG model à la Diamond (1965) with endogenous fertility. As recent empirical works argued, it is natural to presume that grandparents help parents in the caring of their children. Until now the existing theoretical literature abstracted from including this phenomenon in macroeconomic models. The model presented in this paper represents a first attempt to fill this gap by allowing for the grandchildren care assistance inside the home by grandparents. In particular, by assuming perfect annuities market and grandparents that devotes an exogenous fraction of their time endowment to take care of grandchildren, by reducing the opportunity cost of children for the young parents, we showed the existence of an inverse relationship between longevity and economic growth.

We abstracted from including endogenous labour-leisure choices in the second period of life, the former being favoured by the reduced adult mortality (indeed, several governments are aiming at implementing policies to lengthen the age of retirement and/or promote the employment of the elderly). A natural extension of the present paper therefore can be to introduce an endogenous time allocation for the old-aged (for instance, through the choice of how much time devote either to working activities or child care assistance), or, alternatively, include male and female labour participation. Indeed, several government around the world are trying to implement policies aiming at increasing the female labour participation rate as a stimulus to economic growth. The existence of the child care activity by grandparents, by alleviating the time-cost of children by women that belong to the active population, seems to be to go in that direction. However, we conjecture that the final effect on the (neoclassical) per capita economic growth may be ambiguous because of the increase in fertility that it can give rise. These extensions are the object of our future research agenda.

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