'Just one of us': Consumers playing oligopoly in mixed markets

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Abstract. Consumer cooperatives represent a highly successful example of democratic form of enterprises operating in developed countries. They are usually medium to large-scale companies competing with the profit-maximizing firms in the retail sector. This paper describes this situation as a mixed oligopoly in which consumer cooperatives maximize the utility of consumer-members and, in return, refund them with a share of the profits corresponding to the ratio of their individual spending to the cooperative's total sales. We show that when consumers possess quasi-linear preferences over a bundle of symmetrically differentiated goods, and companies operate using a linear technology, the presence of consumer cooperatives positively affects total industry output, as well as welfare. The effect of cooperatives on welfare proves to be even more significant when goods are either complements or highly differentiated, and when competition is *à la* Cournot rather than *à la* Bertrand.

Keywords: Consumer Cooperatives, Profit-maximizing Firms, Mixed Oligopoly.

JEL codes: L13, L21, L22, L31, L33, L81, P13

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1. Introduction

Since 1844, the idea of cooperation as firstly presented by the Rochdale Society of Equitable Pioneers has spread around the world and, today, more than 700 million co-operators are active throughout 100 countries (ICA 2006). Consumer cooperatives (henceforth Coops) represent one of the most successful examples of democratic and participative forms of enterprises, ably competing with the well-established and conventional for-profit firms. Among the various cooperative forms of enterprises, Coops are firms that operate in the retail sector with the aim of acting on behalf of their consumer-members. These are usually entitled to elect their representatives who participate in assemblies and hire the (professional or non-professional) managers running the firm. In large Coops the assembly elects a board of directors that, on its behalf, supervises the managers.

Throughout their history (see Finch, Trombley & Rabas 1998, for a brief account of the US case) Coops have become well-established in several countries without, in general, holding a dominant market position. There are, however, a few exceptions to this, namely in Switzerland, Finland, Japan and, to a lesser extent, Italy. Coops in Switzerland have a long tradition, with its two main groups (Migros and Coop) accounting for approximately 4.5 million members and a turnover of 27.4 billion euro. In Finland, compared to the population, there are proportionately more cooperative members than in any other country in the world, totalling 6.9 million. Finnish Coops have an estimated turnover of 11 billion euro (EuroCoop 2009). Japan also boasts a very significant consumer cooperative movement with over 25.8 million members, achieving a turnover of approximately 38,365 billion US dollar in 2009 (JCCU, 2009). Meanwhile, today, Italy's largest group of consumer cooperatives successfully competes with the large private retail chains. Among the top 30 Italian retail firms, 9 are consumer cooperatives, with more than 7 million consumer-members and a recorded turnover of about 12.9 billion euro in 2009, corresponding to around 18% of total Italian market share (E-coop 2010).

For Europe as a whole, the European Association of Consumer Cooperatives estimates that there are approximately 3,200 consumer cooperatives (with a total turnover of 70 billion euro), employing 300,000 workers and serving about 25 million consumer-members (Euro-Coop 2008).

Up to now, the economic literature on consumer cooperatives has mainly focused on the behaviour of these firms under monopoly, perfect or monopolistic competition.[1] However, in developed countries, the retail industry is characterized more and more by large-scale firms, e.g. the Cooperative Group in the UK with its wide range of retail and financial services. Therefore, we are seeing that modern Coops are competing increasingly on a oligopolistic level with conventional profit-maximizing firms (henceforth PMFs), giving rise to a specific example of a mixed oligopoly.[2]

[1]See. Bekenstein (1943), Enke (1945), Yamey (1950), Anderson, Porter & Maurice (1979) and (1980), Ireland & Law (1983), Sexton (1983), Sexton & Sexton (1987), Farrell (1985), and more recently, Hart & Moore (1996) and (1998), Kelsey & Milne (2010), Mikami (2003) and (2010).

[2]The term 'mixed oligopoly' is usually adopted to describe a market in which one or more publicly-owned firms compete against PMFs on an oligopolistic level. Publicly-owned firms are thought to maximize social welfare, i.e. the sum of consumer and producer surplus (see De Fraja & Delbono 1990, Corneo & Jeanne 1994). Alternatively, we can understand a publicly-owned firm as one financed directly by all consumers through income tax. As a result, the marginal-cost pricing is only attained in the special case in which the income of the median voter equals the average income (Corneo 1997).

To the best of our knowledge, there is no literature specifically dealing with a mixed oligopoly between Coops and PMFs, with the exception of Kelsey & Milne (2008) and Goering (2008). The former examine the effects of the presence of consumer-shareholders on company decision-making under both a monopoly and oligopoly. They show that the presence of consumers among company stakeholders may be of strategic advantage and may, ultimately, increase company profit. In their model, consumers have non-zero mass and act strategically. In contrast, in our model, Coops compete with pure PMFs in a differentiated oligopoly. Moreover, Coops maximize the utility of a representative consumer, assumed atomistic, and therefore, solely interested in his/her consumer surplus. On the other hand, Goering presents a homogeneous goods duopoly between a PMF and a non-profit company maximizing a parametric combination of profit and consumer surplus, assumed exogenously.[3]

A wide range of related papers deals with labour-managed firms *à la* Ward (1958) and Vanek (1970) assumed to be competing with PMFs under a duopoly with homogenous or differentiated goods (see Law & Stewart 1983, Okuguchi 1986, Cremer & Cremér 1992). Furthermore, some recent contributions model the behaviour of agricultural cooperatives under either an imperfect competition or mixed duopoly, with homogeneous or vertically differentiated goods.[4] In general, in the above models, labour-managed firms and farmer-cooperatives are not assumed to act on behalf of consumers. In the typical labour-managed firm described in the literature, worker-members are assumed to maximize per-worker added value, thus, implying that labour-managed firms set their output more restrictively compared to standard profit-maximizing firms. On the other hand, agriculture cooperatives are generally modelled as firms using the input received from their farmer-members to deliver end goods to consumers. This implies that agriculture cooperatives have an incentive to over-produce, since farmers do not internalize their production externality on the final market price. However, strong similarities with consumer cooperatives arise when agriculture cooperatives purchase input on behalf of their members. This is due to the fact that they are competing with profit-maximizing firms in selling input to farmers, who are acting as consumers. Empirically, the presence of agriculture cooperatives increases sales and reduces prices on input markets, breaking existing monopsonies (Hansmann, 1996). Therefore, in this respect, some of the results of this paper may also be applied to agriculture cooperatives selling input to farmers.

In this paper, we represent a Coop as a company which maximizes the utility of a representative (atomistic) consumer buying goods and receiving a share of the profits proportional to the ratio of his/her individual spending to the cooperative's total sales.[5] As a result, every Coop is shown to set a price equal to its average production cost, hence affecting the equilibrium behaviour of rival PMFs. All firms are assumed to possess a constant return-of-scale technology, and, therefore, in equilibrium, every Coop sets a price equal to its constant marginal cost. The marginal cost pricing rule emerges endogenously in our model. This pricing rule renders our results comparable to those obtained in mixed oligopoly models with state-owned firms and PMFs (Cremèr, Marchand & Thisse 1989, De Fraja & Delbono 1989). Moreover, the constant average cost assumption results in overcoming the traditional

[3]Kopel, Löffler and Marini (2010) explore the effects arising when consumers delegate a manager to maximize a weighted sum of their aggregate utilities and profits.

[4]See Rodhes (1983), Fulton (1989), Sexton (1990), Tennbakk (1992), Albaek & Schultz (1998), Fulton & Giannakas (2001) and Pennerstorfer & Weiss (2007).

[5]In Coops this share takes the form of a 'patronage rebate' applied to consumer-member purchases.

problem of Coop membership instability.[6] At the end of the paper, we briefly consider the effects that may occur assuming increasing marginal costs.

The main purpose of this paper is to present a detailed taxonomy of the results obtained in an oligopoly in which an arbitrary number of PMFs and Coops compete strategically either in quantities or in prices and goods are differentiated. We show that, under consumer quasi-linear preferences, the presence of Coops in the market positively affects both the total output and welfare (and market prices negatively). Under the Cournot oligopoly with homogeneous goods, it can be shown that the presence of Coops pushes all PMFs out of the market or, alternatively, forces them to behave as perfectly competitive firms, thus, maximizing social welfare. When, instead, goods are differentiated, the Coop effect on welfare proves to be more significant when goods are either complements or highly differentiated, and when competition is *à la* Cournot rather than *à la* Bertrand. Based on the above results, we should expect consumer cooperatives to be present more often in markets exhibiting such features.

The paper is organized as follows: Section 2 introduces the model; Sections 3 and 4 present the main results under a mixed oligopoly with quantity and price competition; and Section 5 contains our concluding remarks.

2. The Model

2.1. **Consumer Preferences.** The demand side of the market is represented by a *continuum* of atomistic consumers, $i \in I$, whose mass is normalized to one, i.e. $I = [0,1]$. Every consumer is assumed to possess quasi-linear preferences defined on $(n+1)$ commodities, n symmetrically differentiated goods[7] x_k $(k = 1, ..., n)$ and a *numeraire* y, expressed by the following utility function $U_i : \mathbb{R}_+^{n+1} \rightarrow \mathbb{R}_+$

$$U_i\left(x_1^i, x_2^i, ., x_{k,}^i .., x_n^i, y^i\right) = u_i\left(x_1^i, x_2^i, ., x_{k,}^i .., x_n^i\right) + y^i \tag{2.1}$$

where $x_{k,}^i$and y^i denote the individual consumption of these goods. Let $u_i(.)$ be smooth, increasing and strictly concave in all $x_{k,}^i$.[8]

If the available income of each i-th consumer (denoted by $\overline{y}^i$) is sufficiently high, every individual inverse demand can be obtained from the first-order conditions of the problem maximization (2.1) subject to budget constraint

$$\sum_{k=1}^{n} p_k\left(x_1, ., x_n\right) x_k^i + y^i \leq \overline{y}^i, \tag{2.2}$$

as

$$p_k = \frac{\partial u_i\left(x_1^i, x_2^i, ., x_n^i\right)}{\partial x_k^i}, \text{ for } x_k^i > 0 \text{ and } k = 1, 2, ...n. \tag{2.3}$$

In (2.2) the price of good x_k depends on the profile of quantities $(x_1, ., x_n)$ (the market is a oligopoly) and not on every individual purchase x_k^i of the good.

[6] See Anderson, Maurice & Porter (1979), Sandler & Tschirhart (1981), Sexton (1983) and Sexton & Sexton (11987). In our paper all consumers buy Coop goods and are, therefore, entitled to become members. This assumption is in line with the typical "open door" principle of cooperatives. Moreover, given the constant-return-of-scale technology, Coop efficiency cannot be affected by favouring the entry or the exit of members.

[7] A good here may also be interpreted as a bundle of goods sold by every firm in the market.

[8] The Hessian of U_i is negative semidefinite for all $(x_1^i, x_2^i, ., x_n^i) \in \mathbb{R}_+^n$.

2.2. **Industry.** The retail industry consists of n firms supplying n differentiated goods (or bundles of goods), whose m are supplied by consumer cooperatives and $(n-m)$ by traditional profit-maximizing firms. Let $M \subset N$ denote the set of all Coops and $N \backslash M$ the set of all PMFs. Normally, PMFs are assumed to maximize their profit

$$\pi_k\left(x_1, x_2, .., x_n\right) = p_k\left(x_1, x_2, ...x_n\right) x_k - c_k(x_k). \tag{2.4}$$

In general we will assume linear variable costs and zero fixed costs for all firms. As expected, Coops act on behalf of atomistic consumers, and every consumer is assumed to receive a share of the Coop's net profit proportional to the amount of goods purchased over the Coop's total sales. This can be expressed by the following objective-function for a Coop $j \in M$,[9]

$$\begin{cases} \max\limits_{x_j^i} u_i\left(x_1^i, x_2^i, ., x_{k,}^i .., x_n^i\right) + y^i & \text{s.t} \\ \sum\limits_{k=1}^{n} p_k\left(x_1, ., x_n\right) x_k^i + y^i \leq \overline{y}^i + \sum\limits_{j \in M} \dfrac{x_j^i}{x_j}\left[p_j\left(x_1, ., x_n\right) x_j - c_j\left(x_j\right)\right]. \end{cases} \tag{2.5}$$

The problem (2.5) reduces to

$$\max_{x_j^i}\left\{u_i\left(x_1^i, x_{2,}^i, .., x_n^i\right) + \overline{y}^i - \sum_{j \in M} \frac{c_j\left(x_j\right)}{x_j} x_j^i - \sum_{k \in N \backslash M} p_k\left(x_1, ., x_n\right) x_k^i\right\} \tag{2.6}$$

and, the FOC for interior maximum of (2.6) for every $j \in M$ can be written as

$$\frac{\partial u_i\left(x_1^i, .., x_n^i\right)}{\partial x_j^i} = \frac{c_j(x_j)}{x_j} \text{ for } x_j > 0. \tag{2.7}$$

as long as the price charged by a j-th Coop is sufficiently high to generate non-negative profits, namely, for $p_j\left(x_1, .x_n\right) \geq \frac{c_j(x_j)}{x_j}$. Expression (2.7) indicates that a Coop acting on behalf of atomistic consumers sets its quantity to equate every consumer's willingness to pay for good j at its average cost, so as to distribute the maximum consumer surplus to consumer-members (which are all consumers here).

Once (2.7) is respected for every single consumer, the Coop aggregates it for all consumers $i \in I$, obtaining

$$\frac{\partial u\left(x_1, .., x_n\right)}{\partial x_j} = \frac{c_j(x_j)}{x_j} \text{ for } x_j > 0. \tag{2.8}$$

Since all firms possess a constant-return-of-scale technology, every Coop makes the total consumer willingness to pay for good j equal to marginal cost.[10]

3. Oligopoly with Quantity Competition

In order to study the implications of the simultaneous presence of both PMFs and Coops in an oligopolistic market, let the following utility function represent the preferences of a i-th consumer in the economy:[11]

[9]Note that when prices instead of quantities are firm choice variables, PMF and Coop payoffs can be expressed as a function of a price vector $(p_1, p_2, ..p_n)$.

[10]Coop behaviour would be different if assumed to act on behalf of all consumers together. In this case, consumers could coordinate their actions to affect the prices of all goods in the market.

[11]See Shubik & Levitan (1971), Vives (1984) and Dixit (1983) for further details on this utility specification.

$$U_i\left(x_1, x_{2,}, .., x_n, y\right) = \alpha \sum_{k=1}^{n} x_k^i - (1/2)\left[\sum_{k=1}^{n}\left(x_k^i\right)^2 + \beta \sum_{k=1}^{n} x_k^i \sum_{r\neq k} x_r^i\right] + y^i \tag{3.1}$$

where $\alpha > 0$ and $\beta \in (1/(1-n), 1]$ represents the degree of product differentiation. For $\beta = 0$, goods are independent and for $\beta = 1$ goods are perfect substitutes. Moreover, for $\beta < 0$ goods become complements.[12]

Let also all firms $k = 1, 2, ...n$ possess identical strategy sets $X_k = [0, \infty)$ and identical technology, expressed by a linear cost function, $c_k\left(x_k\right) = cx_k$ with $0 < c < \alpha$.

By (3.1) and (2.3), the following individual linear inverse demand for every good $k = 1, 2, .., n$ is obtained

$$\alpha - x_k^i - \beta \sum_{h\neq k} x_h^i = p_k \text{ for } x_k^i > 0. \tag{3.2}$$

Inverse market demand for one good can simply be obtained by integrating (3.2) over all consumers $i \in I$. Moreover, the FOC of problem (2.6) yields the following FOC for every Coop producing the j-th good

$$\alpha - x_j^i - \beta \sum_{h\neq j} x_h^i = c. \tag{3.3}$$

Expression (3.3) is the FOC of a Coop acting on behalf of one atomistic consumer buying its product. A Coop will decide its own market quantity aggregating (3.3) for all consumers.

3.1. **The Benchmark Case: Oligopoly with all PMFs.** We begin by illustrating the case in which all firms are PMFs and the choice variables are quantities. If firms are PMFs, they simply maximize their profits concerning the quantity of the k-th good,

$$\pi_k\left(x_1, x_2, ..., x_n\right) = (\alpha - x_k - \beta \sum_{r\neq k} x_r)x_k - cx_k. \tag{3.4}$$

Solving this simple maximization problem yields the following best-replies for each k-th PMF,

$$x_k\left(x_{-k}\right) = \frac{1}{2}\left(\alpha - \beta x_{-k} - c\right)$$

where $x_{-k} = \left(x_1, x_2, .., x_{k-1}, x_{k+1}, .., x_n\right)$, and therefore pure-PMF Nash equilibrium quantities $\left(\overline{x}_1, \overline{x}_2, ..., \overline{x}_n\right)$ are easily obtained as

$$\overline{x}_k = \frac{(\alpha - c)}{2 + \beta\left(n-1\right)} \tag{3.5}$$

for $k = 1, 2, .., n$ and prices are given by

$$p_k\left(\overline{x}_1, ., \overline{x}_n\right) = \frac{\alpha + c + \beta c\left(n-1\right)}{\beta\left(n-1\right) + 2}.$$

[12]For $\beta = 1/(1-n)$ most of equilibrium quantities and prices under Cournot and Bertrand competition become indefinite. This is why, in what follows, we assume $\beta \in (1/(1-n), 1]$.

It is made evident by (3.5) that for $\beta = 1$ the usual Cournot solution with homogenous goods ($\overline{x}_k = (\alpha - c) / (n+1)$) occurs, while for $\beta = 0$ goods are independent and all PMFs act monopolistically ($\overline{x}_k = (\alpha - c) / 2$).

3.2. **Mixed Cournot Oligopoly.** Let us assume that a group of m firms in the market ($m \leq n$) convert to Coops accepting all consumers as their members. The market, thus, becomes a mixed oligopoly where m Coops compete against $(n-m)$ traditional PMFs.

By aggregating (3.3) for all consumers and differentiating (3.4), the following best-responses are obtained, respectively,

$$x_j\Big(\sum_{h\in N\backslash M} x_h, \sum_{r\in M\backslash\{j\}} x_r\Big) = \alpha - \beta \sum_{h\in N\backslash M} x_h - \beta \sum_{r\in M\backslash\{j\}} x_r - c, \tag{3.6}$$

$\forall j \in M$,

$$x_h\Big(\sum_{j\in M} x_j, \sum_{g\in(N\backslash M)\backslash\{h\}} x_g\Big) = \frac{\alpha - \beta \sum_{j\in M} x_j - \beta \sum_{g\in(N\backslash M)\backslash\{h\}} x_g - c}{2}, \tag{3.7}$$

$\forall h \in N\backslash M$.

Exploiting the symmetry of the m Coop and of the $(n-m)$ PMFs, the following mixed oligopoly Nash equilibrium quantities are obtained for every Coop

$$x_j^* = \frac{(2-\beta)(\alpha - c)}{2 + (n+m-3)\beta - (n-1)\beta^2} \quad \forall j \in M, \tag{3.8}$$

and every PMF

$$x_h^* = \frac{(1-\beta)(\alpha - c)}{2 + (n+m-3)\beta - (n-1)\beta^2} \quad \forall h \in N\backslash M, \tag{3.9}$$

with corresponding equilibrium prices

$$p_j\left(x_1^*, x_2^*, .., x_n^*\right) = c$$

for every Coop and

$$p_h\left(x_1^*, x_2^*, ., x_n^*\right) = \frac{\alpha + c - \beta\left(\alpha\left(m+1\right) - c\left(2m+n-2\right)\right) + \beta^2\left(m\alpha - c\left(n+m-1\right)\right)}{2 + \beta\left(n+m-3\right) - \beta^2(n-1)}$$

for every PMF, respectively.

It can be proved that, in general, if goods are perfect substitutes ($\beta = 1$) the model yields the extreme prediction that the presence of even just one Coop in the market pushes PMFs out of the market.[13] This could, alternatively, be interpreted as if the presence of Coops forces all PMFs to adopt a perfectly competitive behaviour in order to remain in the market. Either way, as the equilibrium price coincides with all the average and marginal costs of the

[13] Alternatively, one could assume that Coops are less efficient than PMFs or that PMFs enjoy some sort of cost advantage. In this case both types of firms can co-exist also when goods are perfectly homogeneous. (see for instance Cremer, Marchand & Thisse, 1998).

firm, each consumer's willingness to pay for the homogeneous good is equal to every firm's marginal production cost, thus, implying welfare maximization (since $u' = c$).

As an additional observation, please note that the total market output under mixed oligopoly $X^* = \sum_{k=1,..,n} x_k^*$ is equal to

$$X^* = mx_j^* + (n-m)x_h^* = \frac{(\alpha - c)\,(n(1-\beta)+m)}{2+(n+m-3)\,\beta-(n-1)\,\beta^2}. \tag{3.10}$$

For $m = 0$ the above expression coincides with pure n-PMF oligopoly

$$X^*(m=0) = \frac{n\,(\alpha-c)}{2+\beta\,(n-1)} \tag{3.11}$$

and for $m = n$ the expression turns into pure n-Coop total quantity, with

$$X^*(m=n) = \frac{n\,(\alpha-c)}{1+\beta\,(n-1)}. \tag{3.12}$$

From (3.11) and (3.12) pure Coop oligopoly clearly yields higher output than pure PMF oligopoly. Moreover, expression (3.10) makes it clear that under a mixed oligopoly the total output increases monotonically with the number of active Coops in the market.

The next proposition compares firm output obtained in pure PMF and mixed oligopolies under competition in quantities.

Proposition 1. *Under a mixed oligopoly in quantities, for* $\beta \in (1/(1-n), 1]$ *Coop output is always greater than PMF output, namely,* $x_j^* > x_h^*$ *for all* $j \in M$ *and* $h \in N\backslash M$*. Moreover, the output obtained by a firm in a pure PMF oligopoly is lower (higher) than the output of a Coop (PMF) in a mixed oligopoly, namely,* $x_j^* > \overline{x}_k \geq x_h^*$*.*

Proof. See the Appendix. □

3.3. **Welfare Analysis: PMFs vs. Mixed Oligopoly.** The analysis of social welfare under a mixed oligopoly with differentiated goods requires a careful calculation of the interacting effects of the Coops' and PMFs' simultaneous presence regarding consumer surplus and profits in all markets. Using the properties of quasi-linear preferences, consumer welfare can be measured by using consumer surplus which, in turn, corresponds to the value of consumer utilities.

Under a pure PMF oligopoly, for all k-th goods produced, total welfare (TW_k) can be computed as the sum of consumer surplus plus firm profits,

$$TW_k^{PMF} = \int_0^1 U_i\left(\overline{x}_1^i(t), ., \overline{x}_n^i(t), \overline{y}^i(t)\right) dt - p_k(\overline{x}_1, ., \overline{x}_n)\,\overline{x}_k + p_k(\overline{x}_1, ., \overline{x}_n)\,\overline{x}_k - c\overline{x}_k =$$
$$= U(\overline{x}_1, \overline{x}_{2,}, .., \overline{x}_n) - c\overline{x}_k + \overline{y}.$$

Adding up the welfare generated in all n markets and using (3.1) to obtain the utility functions aggregated for all consumers, we have

$$TW^{PMF} = (\alpha - c)\sum_{k=1}^n \overline{x}_k - (1/2)\left[\sum_{i=1}^n (\overline{x}_k)^2 + \beta \sum_{k=1}^n \overline{x}_k \sum_{r\neq k} \overline{x}_r\right] + \overline{y},$$

which, by the symmetry of all firms, can be written as

$$TW^{PMF} = (\alpha - c)\,n \cdot \overline{x}_k - (1/2)\left[n\,(\overline{x}_k)^2 + \beta n(n-1)\overline{x}_k^2\right] + \overline{y}. \tag{3.13}$$

In a mixed oligopoly, total welfare generated in all markets managed by a j-th Coop is given by the area below the demand function and above the marginal cost function,

$$TW_j^{COOP} = \int_0^{x_j^*} p_j(\tau)d\tau - cx_j^*, \tag{3.14}$$

which using (3.1), (3.14) can be simply expressed as

$$TW_j^{COOP} = \sum_{j\in M} (\alpha - c)\cdot x_j^* - (1/2)\left[\sum_{j\in M} \left(x_j^*\right)^2 + \beta \sum_{j\in M} x_j^* \textstyle\sum_{r\neq j} x_r^*\right].$$

Finally, total welfare under a mixed oligopoly can be expressed as

$$\begin{aligned}&\sum_{h\in N\setminus M} TW_h^{PMF} + \sum_{j\in M} TW_j^{COOP} = \\ &= \sum_{h\in N\setminus M} (\alpha - c)\, x_h^* - (1/2)\left[\sum_{h\in N\setminus M} (x_h^*)^2 + \beta \sum_{h\in N\setminus M} x_h^* \textstyle\sum_{r\neq h} x_r^*\right] + \\ &+ \sum_{j\in M} (\alpha - c)\, x_j^* - (1/2)\left[\sum_{j\in M} \left(x_j^*\right)^2 + \beta \sum_{j\in M} x_j^* \textstyle\sum_{r\neq j} x_r^*\right] + y^*.\end{aligned}$$

Now, plugging (3.5), (3.8) and (3.9) into the above expressions, we obtain the following values for total welfare (see Appendix),

$$TW^{PMF} = \frac{1}{2}\frac{n(\alpha - c)^2 (3 + \beta(n-1))}{(2 + \beta(n-1))^2} + \overline{y}, \tag{3.15}$$

under pure PMF oligopoly

$$TW^{COOP} = \frac{1}{2}\frac{n(\alpha - c)^2}{(1 + \beta(n-1))} + \widetilde{y} \tag{3.16}$$

pure Coop oligopoly

$$TW^{MO} = \underbrace{\tfrac{1}{2}\tfrac{(n-m)(\alpha-c)^2(1-\beta)\left(3+\beta(n+m-4)-\beta^2(n-1)\right)}{(2+\beta(n+m-3)-\beta^2(n-1))^2}}_{\sum_{h\in N\setminus M} TW_h} + \underbrace{\tfrac{1}{2}\tfrac{m(\alpha-c)^2(2-\beta)}{2+\beta(n+m-3)-\beta^2(n-1)}}_{\sum_{j\in M} TW_j} + y^*. \tag{3.17}$$

and mixed oligopoly with m Coops and $(n-m)$ PMFs, respectively.

Expression (3.17) illustrates that social welfare in a mixed oligopoly accounts for the sum of welfare yielded in $(n-m)$ markets in which PMFs produce plus welfare yielded in m markets in which Coops are, in turn, active.

The analysis of (3.17) shows that the presence of Coops can be relatively more beneficial in some circumstances than in others and, in particular, for specific levels of product differentiation. Figure 1 reveals that in terms of total welfare, a pure Coop duopoly (continuous line) out-performs both a pure PMF duopoly and a mixed duopoly for any degree of goods differentiation which is obvious, considering that a pure Coop basically acts as a welfare maximizer.

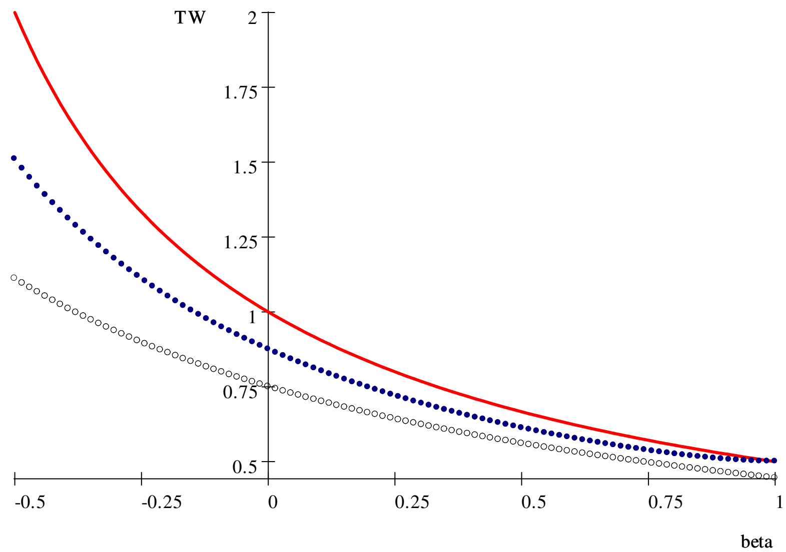

Figure 1 - Cournot competition: pure PMF (circled line), pure Co-op (continuous line) and mixed duopoly total welfare (dotted line), for $(\alpha - c) = 1$ and $\beta = [-0.5, 1]$.

Under a mixed duopoly (dotted line) for $\beta = 1$ (homogeneous goods), only the Coop remains in the market and welfare is, therefore, maximized. Moreover, we can note that the relative efficiency of a mixed market versus a pure PMF market (circled line) is higher when goods are either complements ($\beta < 0$) or highly differentiated. When goods become increasingly homogeneous, the welfare loss determined in a pure PMF versus a mixed duopoly or a pure Coop duopoly decreases progressively but never disappears. Similarly, a mixed duopoly increasingly approximates maximum social welfare for increasingly substitute goods.

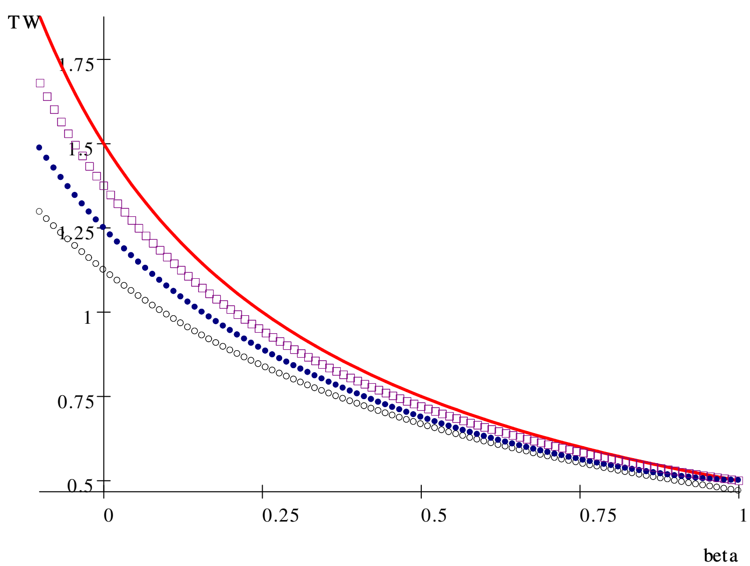

Figure 2 - Cournot competition: pure PMF (circled line), pure Co-op (continuous line) and mixed triopoly total welfare with $m = 1$ (dotted line), $m = 2$ (squared line), for $(\alpha - c) = 1$ and $\beta = [-0.1, 1]$.

The results illustrated in figure 1 still hold for more than two firms that compete à la Cournot (see figure 2). Moreover, it can be proven that the entry of new Coops in the market is always an advantage for social welfare.

Proposition 2. *Social welfare under mixed oligopoly increases with the number of m Coops regardless of the number of n firms active in the market.*

Proof. See the Appendix. □

The positive effect of Coops on welfare still holds true when the total number of firms in the market increases. Figure 3 illustrates that the entry of new firms, boosting competition, always exerts a favourable impact on market welfare. Consequently, if the new entrants are Coops, this impact is even stronger. Consumers should therefore exert pressure on respective Coops to set up new selling units, thus, increasing competition and welfare.[14]

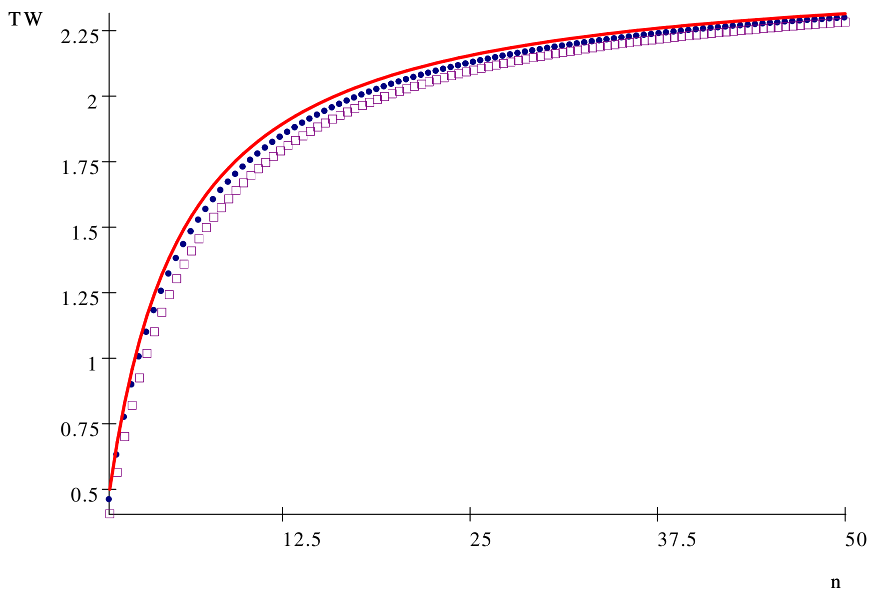

Fig. 3 - Cournot Oligopoly: Mixed Oligopoly welfare for $m = \frac{n}{4}$ (squared line), $m = \frac{2}{3}n$ (dotted line), and $m = n$ (optimum) (continuous line) for $\beta = 0.2$, $(a - c) = 1$, $n = 1, 2, .., 50$.

However, a simple comparison shows that when goods are substitutes ($\beta > 0$), the welfare raised by a pure Coop oligopoly becomes less and less advantageous compared to a pure PMF oligopoly when both n and β increase. When competition is high (which happens for high n and β) the different forms of market do not perform so differently and, thus,welfare is not so dissimilar. See next proposition.

Proposition 3. *When the total number of firms in the market increases (higher n) and goods become increasingly substitute (higher β), the welfare ratio $\rho(\beta, n) = TW_{COOP}/TW_{PMF}$ decreases monotonically within the interval* $(1, 4/3]$.

Proof. See the Appendix. □

[14]It would be interesting to model the entry of different types of firms (Coops and PMFs) in an endogenous timing model in which firms can decide to take an earlier or later move to enter the market (Hamilton & Slutsky, 1990). In this case, we could assess which specific timing configuration would maximize market welfare.

Figure 4 shows that when the number of firms increases and goods become increasingly homogeneous, the ratio measuring the relative advantage yielded by Coops vs. PMFs in terms of welfare, progressively falls, approaching its lower bound. Therefore, if Coops aspire to match consumer needs, we should see these types of firms more frequently in highly monopolistic markets in which goods are either highly differentiated or complements.

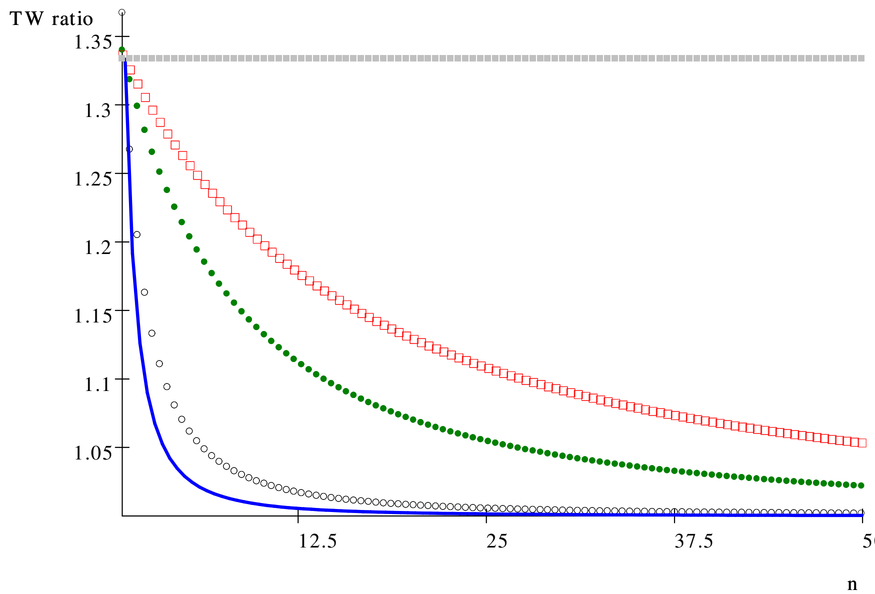

Fig. 4 - Values of $\rho(\beta, n)$ for $\beta = 1$ (contnuous thin line), $\beta = 0.5$ (circled line), $\beta = 0.1$ (dotted line), $\beta = 0.05$ (squared line), $\beta = 0$ (thick dashed line), for $(a - c) = 1$ and $n = 1, 2, ..50$.

In the next section, we will consider the case of price competition.

4. Price Competition

It would be interesting to compare the case of quantity competition to that of price competition so as to verify whether differences arise. An obvious difference is that, when goods are perfectly homogeneous, Bertrand competition exhibits the extreme prediction that firm set prices equal to marginal cost, regardless of the objective functions of firms competing in the market.

4.1. **Oligopoly with all PMFs.** When all firms are PMFs, we first obtain the direct demand for each k-th good as a price function,

$$x_k\left(p_1, p_2, .., p_n\right) = \frac{\alpha\left(1-\beta\right) - p_k - (n-2)\beta p_k + \beta \sum_{h \neq k} p_h}{\left(1-\beta\right)\left((n-1)\beta + 1\right)}$$

for $k = 1, 2, .., n$ and $\beta \neq 1$.[15]

As a result, all PMF profit function can be written as

$$\pi_k\left(p_1, ., p_n\right) = \left(p_k - c\right) x_k\left(p_1, p_2, .., p_n\right). \tag{4.1}$$

[15]Since demands are not defined for $\beta = 1$, output level under homogeneous goods are simply defined as firm direct demands for prices equal to marginal costs.

Differentiating (4.1) with respect to p_k yields the best-response of every k-th PMF as

$$p_k(p_{-k}) = \frac{1}{2}\frac{\alpha(1-\beta) + c(n-2)\beta + c + \beta\mathbf{p}_{-k}}{\beta(n-2)+1}$$

where $p_{-k} = (p_1, p_2, .., p_{k-1}, p_{k+1}, .., p_n)$.

By symmetry, the Nash equilibrium price of every k-th PMF can be obtained as

$$\begin{cases} \overline{p}_k = \dfrac{(\alpha(1-\beta) + \beta c(n-2) + c)}{\beta(n-3)+2} & \text{for } \beta \neq 1 \\ \\ \overline{p}_k = c \quad \text{for } \beta = 1, \end{cases} \tag{4.2}$$

with associated quantities:

$$\begin{cases} x_k(\overline{p}_1, \overline{p}_2, .., \overline{p}_n) = \dfrac{(\alpha - c)(1+\beta(n-2))}{(1+\beta(n-1))(2+\beta(n-3))} & \text{for } \beta \neq 1 \\ \\ x_k(\overline{p}_1, \overline{p}_2, .., \overline{p}_n) = \dfrac{(\alpha-c)}{n} \quad \text{for } \beta = 1. \end{cases} \tag{4.3}$$

4.2. **Mixed Oligopoly with Price Competition.** Again we assume that $m \leq n$ firms start behaving as Coops. By (3.1) and (3.4), we obtain the following direct demands for a PMF $h \in N \backslash M$, given the price charged by other firms,

$$x_h(p_1, ., p_n) = \frac{\alpha(1-\beta) - p_h - \beta(n-2)p_h + \beta \sum\limits_{r \in (N \backslash M) \backslash h} p_r + m\beta c}{(1-\beta)(1+\beta(n-1))} \tag{4.4}$$

and the price charged by a Coop $j \in M$

$$x_j(p_1, ., p_n) = \frac{\alpha(1-\beta) - c - (n-m-1)\beta c + \beta \sum\limits_{h \in N \backslash M} p_h}{(1-\beta)(1+\beta(n-1))} \tag{4.5}$$

for $\beta \neq 1$.

By (4.4) we can put the profit-function of a PMF as a function of prices,

$$\pi_h(p_1, ., p_n) = (p_h - c)\, x_h(p_1, ., p_n)$$

and, after straightforward calculations, the following mixed oligopoly equilibrium prices are obtained

$$\begin{cases} p_h^* = \dfrac{\alpha(1-\beta) + c(1+\beta(n+m-2))}{2+\beta(n+m-3)} & \text{for } \beta \neq 1 \\ \\ p_h^* = c \qquad \text{for } \beta = 1 \end{cases} \tag{4.6}$$

and

$$p_j^* = c$$

with associated quantities

$$x_h\left(p_1^*, p_2^*, .., p_n^*\right) = \frac{(\alpha - c)\left(1 + \beta\left(n - 2\right)\right)}{\left(1 + \beta\left(n - 1\right)\right)\left(2 + \beta\left(n + m - 3\right)\right)} \tag{4.7}$$

for every PMF and

$$x_j\left(p_1^*, p_2^*, .., p_n^*\right) = \frac{(\alpha - c)\left(2 + \beta\left(2n - 3\right)\right)}{\left(1 + \beta\left(n - 1\right)\right)\left(2 + \beta\left(n + m - 3\right)\right)}, \tag{4.8}$$

for every Coop, respectively, for $\beta \neq 1$ and

$$x_h\left(p_1^*, ., p_n^*\right) = x_j\left(p_1^*, ., p_n^*\right) = \frac{(\alpha - c)}{n}$$

for $\beta = 1$.

All results regarding prices and outputs under Bertrand competition are condensed into the next two propositions.

Proposition 4. *Under price competition: (i) for $\beta \in [0,1]$, mixed oligopoly prices are for all firms either lower than or equal to pure PMF oligopoly prices, namely, $\overline{p}_k \geq p_h^* \geq p_j^*$ for every $j \in M$, $h \in N \backslash M$ and $k = 1, 2, ..n$. (ii) For $\beta \in (1/(1-n), 0)$, pure PMF oligopoly prices are higher (lower) than Coop (PMF) mixed oligopoly prices, namely, $p_h^* > \overline{p}_k > p_j^*$.*

Proof. See the Appendix. □

Proposition 5. *Under price competition: (i) for $\beta \in [0,1]$, Coop (PMF) mixed oligopoly output is higher (lower) than or equal to pure PMF oligopoly output, namely, $x_j\left(p^*\right) \geq x_k\left(\overline{p}\right) \geq x_h\left(p^*\right)$. (ii) For $\beta \in (1/(1-n), 0)$, PMF mixed oligopoly output is higher (lower) than pure PMF oligopoly (Coop mixed oligopoly) output, namely, $x_j\left(p^*\right) > x_h\left(p^*\right) > x_k\left(\overline{p}\right)$.*

Proof. See the Appendix □

Under price competition, the positive effect of Coops on output is even more marked than under quantity competition. This is particularly true when goods are complements, as also PMFs expand their output more than under a pure PMF oligopoly. (see proposition 5, point (ii)).

4.3. **Welfare Comparison under Price Competition.** For the sake of briefness, all calculations for total welfare under price competition can be found in the Appendix. The results of these calculations, which are not so different to those obtained in the case of quantity competition, are reported here. Total welfare under a mixed oligopoly with an arbitrary number of PMFs and Coops competing in prices is obtained as

$$TW^{MO} = \underbrace{\frac{1}{2}\frac{(n-m)(\alpha-c)^2(3+\beta(n+m-4))(1+\beta(n-2))}{(2+\beta(n+m-3))^2(1+\beta(n-1))}}_{\sum_{h\in N\backslash M} TW_h} + \underbrace{\frac{1}{2}\frac{m(\alpha-c)^2(2+\beta(2n-3))}{(1+\beta(n-1))(2+\beta(n+m-3))}}_{\sum_{j\in M} TW_j} \tag{4.9}$$

Setting $m = 0$ in (4.9) we can obtain pure PMF oligopoly total welfare as

$$TW^{PMF} = \frac{1}{2}\frac{n\left(\alpha - c\right)^2\left(1 + \beta\left(n - 2\right)\right)\left(3 + \beta\left(n - 4\right)\right)}{\left(1 + \beta\left(n - 1\right)\right)\left(2 + \beta\left(n - 3\right)\right)^2},$$

while, by setting $n = m$, we have the pure Coop total welfare as

$$TW^{COOP} = \frac{1}{2}\frac{(\alpha - c)^2 n}{(1 + \beta(n-1))^2}.$$

The pure Coop oligopoly always yields optimum social welfare, under both Cournot and Bertrand competition. When plotting social welfare under price competition, no particular differences emerge regarding the case of quantity competition, except that, when goods are perfectly homogeneous (i.e., $\beta = 1$), all firms behave in exactly the same way by setting their prices equal to their marginal costs.

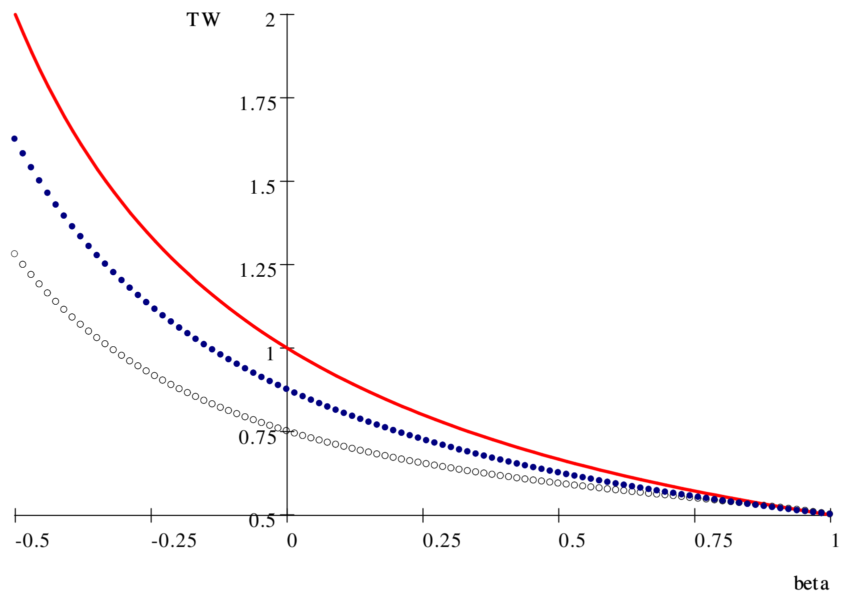

Figure 5- Bertrand competition: pure PMF (circled line), pure Co-op (continuous line) and mixed duopoly total welfare (dotted line), for $(\alpha - c) = 1$ and $\beta = [-0.5, 1]$.

Under both quantity competition and price competition, social welfare increases with the number of Coops. A proposition analogue to proposition 2 is presented below.

Proposition 6. *Social welfare under mixed oligopoly and price competition increases with the number of m Coops regardless of the number of n firms active on the market.*

Proof. See the Appendix. □

An important difference between Bertrand and Cournot competition emerges in terms of welfare loss for a pure PMF oligopoly versus a pure Coop oligopoly. As shown in Figure 6, the loss is definitively larger for quantity compared to price competition and the difference is particularly high when goods are highly homogeneous. This is the case where the presence of at least one Coop in the market is definitively more beneficial under Cournot than under Bertrand competition. Additional welfare comparisons between Cournot and Bertrand oligopolies are provided in the Appendix.

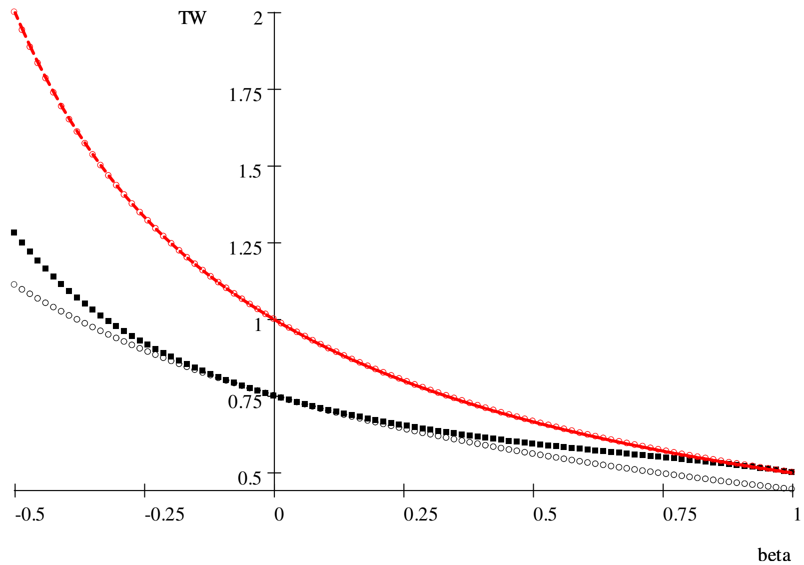

Fig.6- Total welfare in a pure PMF duopoly under Cournot (circled line) and Bertrand competion (dotted line) compared to a pure Co-op market (continuous line) for $(a-c)=1$ and $\beta \in [-0.5, 1]$.

5. Concluding Remarks

Although consumer cooperatives are, in general, well-established in several countries, their behaviour is still largely unknown and requires additional research, notably to identify the effects of the strategic interaction between consumer cooperatives and traditional profit-maximizing firms in oligopolistic markets. This paper has attempted to take a first step in this direction, showing the main effects arising in a mixed oligopoly with profit-maximizing firms and consumer cooperatives competing either *à la* Cournot or *à la* Bertrand in markets with heterogeneous goods. We have shown that the presence of Coops is particularly beneficial for industry output and social welfare in mainly two cases. The first is under Cournot competition and homogeneous goods. In this case, Coops expand their output and grow to the extent to push PMFs out of the market, or if interpreted differently, force them to behave as perfectly competitive firms setting a price equal to the marginal cost and making zero profit as a result. Instead, the second case arises when market competition is relatively weak, namely, when goods are either complements or highly differentiated and the presence of Coops appears to be particularly valuable, considerably increasing output and welfare. In this paper, we have also shown that Coops affect total welfare more under Cournot rather than under Bertrand competition. Therefore, according to our model, consumer cooperatives are likely to behave not so differently to traditional profit-maximizing firms in all the retail markets in which goods are highly (but not fully) homogeneous and competition occurs mostly in price. As a reaction to these market forces, Coops may attempt to propose genuinely differentiated goods to their customers and, consequently, enhance consumer welfare.

Some of the results of this paper call for further analysis. First of all, throughout the paper we have assumed a constant return-of-scale technology for firms. Some of the recent literature on mixed oligopoly has assumed increasing returns-of-scale, thus, implying increasing marginal costs. In this case, a Coop, with its typical output expanding behaviour, could prove endogenously less efficient than a PMF, and so impose negative externality on the society it operates in. This effect would be overturned if a Coop were managed jointly by all consumers or by someone acting on their behalf. In such a case, consumers would no longer be atomistic and could promote welfare-enhancing pricing strategies, affecting, to consumer advantage, the rival PMFs pricing policies. Developing a consumer cooperative model in which consumers are organized into coalitions to act strategically in their interests, may well be a topic of great interest for future research.

6. Appendix

Proof of Proposition 1. The first result can be easily checked by the direct inspection of expressions (3.8) and (3.9). Note that for $\beta = 1$, it occurs that

$$\begin{cases} x_j^*(\beta = 1) = \dfrac{\alpha - c}{m} \\ x_h^*(\beta = 1) = 0, \end{cases} \tag{6.1}$$

implying that when goods are homogeneous, only Coops remain active in a mixed oligopoly market. In this case, the economy total output is given by

$$\sum_{j \in M} x_j^* + \sum_{h \in N \backslash M} x_h^* = m\frac{\alpha - c}{m} + 0 = (\alpha - c) > \sum_k \overline{x}_k = n(\alpha - c)/(n+1).$$

The second result can be proved by noting that, for all $j \in M$ and $k \in N$,

$$x_j^* - \overline{x}_k = \frac{(\beta(n-m-1)+2)(\alpha - c)}{(\beta(n+m-3) - \beta^2(n-1)+2)(\beta(n-1)+2)} \tag{6.2}$$

and expression (6.2) is always strictly positive for $\beta \in (1/(1-n), 1]$ and $n \geq 2$.

Finally, for all $h \in N \backslash M$

$$\overline{x}_k - x_h^* = \frac{(\alpha - c)}{2 + \beta(n-1)} - \frac{(\alpha - c)(1-\beta)}{2 + \beta(n+m) - \beta^2(n-1) - 3\beta}$$

is equal to zero for $\beta = 0$, since $\overline{x}_k(\beta = 0) = x_h^*(\beta = 0) = (\alpha - c)/2$. Straightforward manipulations show that for $\beta \neq 0$

$$\overline{x}_k - x_h^* = \frac{(\alpha - c)m\beta}{(\beta(n+m-3) + \beta^2(1-n) + 2)(n(\beta - 1) + 2)} > 0$$

if

$$\big(\beta(n+m-3) + \beta^2(1-n) + 2\big) > 0$$

which is always satisfied for $\beta \in (1/1-n, 0)$ and $\beta \in (0, 1]$. ■

Proof of Proposition 2. By inspecting (3.17), it can be observed that the welfare raised by a Coop is higher than the welfare raised by a PMF whenever

$$\begin{aligned} &\frac{(2-\beta)(2+\beta(n+m-4)-\beta^2(n-1))}{(2+\beta(n+m-3)-\beta^2(n-1))^2} \\ &> \frac{(1-\beta)(3+\beta(n+m-4)-\beta^2(n-1))}{(2+\beta(n+m-3)-\beta^2(n-1))^2} \end{aligned}$$

which implies

$$(2-\beta)\left(2+\beta\left(n+m-3\right)-\beta^2\left(n-1\right)\right)>$$
$$>(1-\beta)\left(3+\beta\left(n+m-4\right)-\beta^2\left(n-1\right)\right),$$

and then

$$(1-\beta)^2+(2-\beta)\left(\beta\left(n+m-3\right)-\beta^2\left(n-1\right)\right)>$$
$$>(1-\beta)\left(\beta\left(n+m-3\right)-\beta^2\left(n-1\right)\right)$$

which always holds for $m\leq n$ and $\beta\in(1/(1-n),1]$. ■

Proof of Proposition 3. Straightforward manipulations of (3.15) and (3.16) show that

$$\rho(n,\beta)=\frac{TW^{COOP}}{TW^{PMF}}=\frac{\left(2+\beta\left(n-1\right)\right)^2}{\left(3+\beta\left(n-1\right)\right)\left(1+\beta\left(n-1\right)\right)}$$

and for $\beta\in[0,1]$ the above ratio decreases monotonically both in n (for $n\geq1$) and in β (for $\beta\geq0$) within the interval $(1,4/3]$. For $\beta=0$, it reaches the value of $\rho\left(n,0\right)=4/3$, that it is also obtained under monopoly ($n=1$). For $\beta=1$, ρ reaches the value of $\rho\left(n,1\right)=\frac{(n+3)^2}{(n+5)(n+1)}$, which is lower than $4/3$ for $n>1$. Regardless of the degree of product differentiation, the ratio $\rho(n,\beta)$ always converges to 1 for $n\rightarrow+\infty$:

$$\lim_{n\rightarrow+\infty}\rho(n,\beta)=1.$$

■

Proof of Proposition 4. (i)-(ii) By expressions (4.2), (4.6) and by Bertrand equilibrium property, when goods are homogeneous ($\beta=1$) no difference occurs between mixed and pure oligopoly equilibrium prices, since $\overline{p}_k=p_j^*=p_h^*=c$. When goods are independent ($\beta=0$) all PMFs behave as monopolists under both pure and mixed oligopolies, with $p_h^*=\overline{p}_k=\dfrac{a+c}{2}$ whereas, also in this case, Coops behave as perfectly competitive firms, setting $p_j^*=c$. Moreover, for $\beta\in(0,1)$

$$(\overline{p}_k-p_h^*)=\frac{\left(\alpha-c\right)\left(1-\beta\right)m\beta}{\left(2+\beta\left(n+m-3\right)\right)\left(2+\beta\left(n-3\right)\right)},\tag{6.3}$$

which is zero for $m=0$ and monotonically increasing in the number of Coops, since

$$\frac{d\left(\overline{p}_k-p_h^*\right)}{dm}=\frac{\left(1-\beta\right)\left(\alpha-c\right)\beta}{\left(2+\beta\left(n+m-3\right)\right)^2}>0$$

for $n\geq1$. For $\beta\in(1/1-n,0)$, (6.3) becomes negative and result (ii) is thus established.

Proof of Proposition 5. (i)-(ii) Note that, for $\beta=0$

$$x_k\left(\overline{p}\right)=x_h\left(p^*\right)=\frac{1}{2}\left(\alpha-c\right)$$

and, for every j-th Coop,

$$x_j\left(p^*,\beta=0\right)=\left(\alpha-c\right)\tag{6.4}$$

and therefore

$$x_j\left(p^*,\beta=0\right)>x_h\left(p^*,\beta=0\right)=x_k\left(\overline{p},\beta=0\right).$$

Moreover, for $\beta = 1$ in all types of oligopoly the same quantities are chosen with

$$x_k(\overline{p}, \beta = 1) = x_h(p^*, \beta = 1) = x_j(p^*, \beta = 1) = \frac{(\alpha - c)}{n}.$$

When $\beta \in (0, 1)$, a simple inspection of (**??**) and (4.7) shows that, for $m \geq 1$,

$$x_k(\overline{p}) > x_h(p^*),$$

while for $\beta \in (1/1 - n, 0)$ the opposite holds and

$$x_h(p^*) > x_k(\overline{p}).$$

Finally, we see that

$$x_j(p^*) - x_k(\overline{p}) = \frac{(\beta(3n - m - 5) + \beta^2(2m - 4n + 3 - mn + n^2) + 2)(\alpha - c)}{(2 + \beta(n + m - 3))(1 + \beta(n - 1))(2 + \beta(n - 3))}$$

whose both numerator and denominator are strictly positive for $\beta \in (1/1 - n, 1)$. ■

Proof of Proposition 6. By (4.9), the welfare raised by a Coop is higher than that raised by a PMF whenever

$$\frac{(2 + \beta(2n - 3))}{(2 + \beta(n + m - 3))} > \frac{(3 + \beta(n + m - 4))(1 + \beta(n - 2))}{(2 + \beta(n + m - 3))^2}$$

implying

$$(2 + \beta(2n - 3))(2 + \beta(n + m - 3)) > (1 + \beta(n - 2))(3 + \beta(n + m - 4))$$

which holds if $m \leq n$ and $\beta \in (1/(1 - n), 1]$. ■

Welfare under Cournot Competition

By symmetry of all j-th Coop and all h-th PMF, the welfare raised in a mixed Cournot oligopoly can be expressed as

$$TW^{MO} = (n - m)\left[(\alpha - c)x_h^* - \tfrac{1}{2}\left(x_h^{*2} + \beta m x_j^* x_h^* + \beta(n - m - 1)x_h^{*2}\right)\right] + \\ + m\left[(\alpha - c)x_j^* - \tfrac{1}{2}\left(x_j^{*2} + \beta(n - m)x_j^* x_h^* + \beta(m - 1)x_j^{*2}\right)\right] + y^*.$$

Plugging (3.8) and (3.9) into the above expression, mixed oligopoly welfare is obtained as in (3.17). For $m = 0$ (3.17) becomes

$$TW^{PMF} = \frac{1}{2}\frac{n(\alpha - c)^2(3 + \beta(n - 1))}{(2 + \beta(n - 1))^2} + \overline{y},$$

which is pure PMF oligopoly welfare. For $m = n$, (3.17) turns into

$$TW^{COOP} = \frac{1}{2}\frac{n(\alpha - c)^2}{1 + \beta(n - 1)} + \widetilde{y},$$

i.e., pure Coop oligopoly welfare. In the specific case in which goods are homogeneous ($\beta = 1$), the above expression becomes:

$$TW^{COOP}(\beta = 1) = \frac{1}{2}(\alpha - c)^2 + \widetilde{y},$$

which is also the maximum welfare obtainable in the market for $\beta = 1$.

Welfare under Bertrand Competition

The welfare under a mixed Bertrand oligopoly can be obtained as

$$\begin{aligned}TW_p^{MO} &= (n-m)\left[(\alpha-c)\,x_h(p^*) - \tfrac{1}{2}x_h^2(p^*) + \beta m x_j(p^*)\,x_h(p^*) + \right.\\ &\left. + \beta(n-m-1)x_h^2(p^*)\right] + m\left[(\alpha-c)\,x_j(p^*) - \right.\\ &\left. - \tfrac{1}{2}\left(x_j^2(p^*) + \beta\,(n-m)\,x_j(p^*)\,x_h(p^*) + \beta(m-1)x_j^2(p^*)\right)\right] + y(p^*),\end{aligned}$$

which, using (4.7) and (4.8), yields

$$TW_p^{MO} = \frac{1}{2}\frac{(n-m)(a-c)^2(3+\beta(n+m-4))(1+\beta(n-2))}{(2+\beta(n+m-3))^2(1+\beta(n-1))} + \frac{1}{2}\frac{m(\alpha-c)^2(2+\beta(2n-3))}{(1+\beta(n-1))(2+\beta(n+m-3))} + y(p^*). \tag{6.5}$$

A welfare comparison between Bertrand (6.5) and Cournot welfare (3.17) for $(a-c)=1$, yields the following expression:

$$TW_p^{MO} - TW_q^{MO} = \frac{1}{2}\frac{\left(6\beta-2m\beta-2n\beta-\beta^2+n\beta^2-4\right)(n-m)(n-1)(1+\beta(m-1))(\beta-1)\beta^2}{(2+\beta(n+m-3))^2(m\beta-3\beta+n\beta+\beta^2-n\beta^2+2)^2(1+\beta(n-1))},$$

which, under duopoly $(n=2)$ becomes

$$TW_p^{MO} - TW_q^{MO} = \frac{\left(4-\beta^2-2\beta\right)\beta^2}{(\beta+2)^2(\beta-2)^2(\beta+1)}, \tag{6.6}$$

when $m=0$ (pure PMF duopoly) and

$$TW_p^{MO} - TW_q^{MO} = \frac{1}{8}\frac{(\beta+2)(\beta-2)(\beta-1)\beta^2}{(\beta^2-2)^2(\beta+1)} \tag{6.7}$$

when $m=1$ (mixed duopoly). Firstly it is worth noticing that both expressions (6.6) and (6.7) are not monotonic in β. Moreover, welfare differences between price and quantity competition are generally larger under a pure PMF duopoly than under a mixed duopoly. In both cases, such a difference is high when goods are complements. When goods are substitutes, in a pure PMF duopoly the welfare difference between Bertand and Cournot increases with β, and only when β is close to one does it start to decrease. Conversely, in a mixed duopoly such a difference first increases and then decreases to eventually disappear for $\beta=1$. These qualitative results also hold for $n>2$.

References

[1] Albaek S. and C. Schultz (1998), "On the Relative Advantage of Cooperatives", *Economics Letters*, 59, 397-401.

[2] Anderson R.K., S. C. Maurice and Porter P K (1979) "The Economics of Consumer-Managed Firms", *Southern Economic Journal*, July, 119-130.

[3] Anderson R.K., S. C. Maurice and P. K. Porter (1980) "Factor Usage by Consumer-Managed Firms", *Southern Economic Journal,* 522-528.

[4] Bekenstein A. L. (1943) "A Theoretical Analysis of Consumer Co-operatives: The Managerial Problem", *The Journal of Political Economy*, 51, 251-257.

[5] Bulow J., J. Geneakoplos and P. Klemperer (1985) "Multimarket Oligopoly: Strategic Substitutes and Complements", *Journal of Public Economics*, 93, 488–511.

[6] Corneo, G. (1997), "Taxpayer-consumers and Public Pricing", *Economics Letters*, 57, 235–240.

[7] Corneo G. and O. Jeanne (1994) "Oligopole Mixte dans un Marché Commun" *Annales d'Economie et de Statistique*, 33, 73-90.

[8] Cremer, H. and J. Cremer (1992), "Duopoly with Employee-controlled and Profit-maximizingFirms: Bertrand vs Cournot Competition", *Journal of Comparative Economics*, 16, 2, 241-258.

[9] Cremer, H., M. Marchand, and J. F. Thisse (1989), "The Public Firm as an Instrument for Regulating an Oligopolistic Market", *Oxford Economic Papers*, 41, 2, 283-301.

[10] De Fraja, G. and F. Delbono (1989), "Alternative Strategies of a Public Enterprise in Oligopoly," *Oxford Economic Papers*, 41, 2, 302-11.

[11] De Fraja, G. and F. Delbono (1990), "Game Theoretic Models of Mixed Oligopoly",. *Journal of Economic Surveys* 4, 1–17.

[12] Dixit, A. (1979), "A Model of Duopoly Suggesting a Theory of Entry Barriers", *Bell Journal of Economics*, 14, 383-394.

[13] Enke, S. (1945), "Consumer Cooperatives and Economic Efficiency", *American Economic Review*, March, 148-155.

[14] E-Coop (2010), *Sesto Rapporto Sociale Nazionale della Cooperazione di Consumatori*. Coop Italia, Bologna.

[15] EuroCoop (2008, 2009), *Activity Reports*, European Association of Consumer Cooperatives, Bruxelles.

[16] Farrell, J. (1985), "Owner-Consumers and Efficiency", *Economics Letters*, 19, 303-306.

[17] Finch, J. E., C.M. Trombley and B. J. Rabas (1998), "The Role of Multiple Consumption Values in Consumer Cooperative Patronage: an Application of the Theory of Market Choice Behavior", *Journal of Marketing Management*, 8, 44-56

[18] Fulton, M. (1989), "Cooperatives in Oligopolistic Industries: The Western Canadian Fertilizer Industry", *Journal of Agricultural Cooperation*, 4, 1-19.

[19] Fulton, M. and K. Giannakas (2001), "Organizational Commitment in a Mixed Oligopoly: Agricultural Cooperatives and Investor-Owned Firms", *American Journal of Agricultural Economics*, 83, 5, 1258-1265.

[20] Goering, G. E. (2008), "Welfare impacts of a non-profit firm in mixed commercial markets", *Economic Systems*, 32, 326–334.

[21] Hansmann, H. (1996), *The Ownership of Enterprise*. Harvard University Press, Cambridge, Mass.

[22] Hamilton, J. and S. Slutsky (1990), "Endogenous timing in duopoly games: Stackelberg or Cournot equilibria", *Games and Economic Behavior* 2, 29–46.

[23] Hart, O. and J. Moore (1996), "The Governance of Exchanges: Members' Cooperatives versus Outside Ownership", *Oxford Review of Economic Policy*, 12, 4, 53-69.

[24] Hart, O. and J. Moore (1998), "Cooperatives vs. Outside Ownership", *Harvard Institute of Economic Research Working Papers* 1816, Harvard, Institute of Economic Research.

[25] Ireland N. and P. J. Law (1983), "A Cournot-Nash Model of the Consumer Cooperative", *Southern Economic Journal,* 49, 706-716.

[26] ICA (2006), *ICA Annual Report*. International Cooperative Alliance, Geneva.

[27] Kelsey, D. and F. Milne (2008), "Imperfect Competition and Corporate Governance", *Journal of Public Economic Theory*, 10, 1115-1141.

[28] Kelsey, D. and F. Milne (2010), "Takeovers and Cooperatives", *Journal of Economics*, 99,193-209.

[29] Kopel, M., C. Löffler and M. Marini (2010), "Manager Delegation and Consumer Cooperatives under Mixed Oligopoly", *mimeo*, University of Graz, October 2010.

[30] Law, P.J. and G. Stewart (1983), "Stackelberg Duopoly with an Illyrian and Profit-maximising Firm". *Recherches Economiques de Louvain*, 49, 207-212.

[31] Mikami, K. (2003), "Market Power and the Form of Enterprise: Capitalist Firms, Worker-owned Firms and Consumer Cooperatives", *Journal of Economic Behaviour and Organization*, 52, 4, December, 533-552.

[32] Mikami, K. (2010) "Capital Procurement of a Consumer Cooperative: Role of the Membership Market", *Economic Systems*, 34, 2, 178-197.

[33] Okuguchi, K. (1986), "Labor-Managed Bertrand and Cournot Oligopolies", *Journal of Economics*, 46, 2, 115–122.

[34] Pennerstorfer D. and C.R.Weiss (2007), "Do Cooperatives Offer High Quality Products?", *mimeo*, Vienna University of Economics and Business.

[35] Rhodes, V. J. (1983), "The Large Agricultural Cooperative as a Competitor", *American Journal of Agricultural Economics*, 65, 1090-95.

[36] Sandler, T. and J. Tschirhart (1981), "On the Number and Membership size of Consumer Managed Firms", *Southern Economic Journal*, 1086-1091.

[37] Sexton, R. (1983), "Economic Considerations in Forming Cooperatives and Establishing Pricing and Financing Policies", *The Journal of Consumer Affairs*, 17, 2, 290-314.

[38] Sexton, R. I. and A. Sexton (1987), "Cooperatives as Entrants", *Rand Journal of Economics*, 18, 581-95.

[39] Sexton, R. J. (1990), "Imperfect Competition in Agricultural Markets and the Role of Cooperatives: a Spatial Analysis", *American Journal of Agricultural Economics*, 72, 709-20.

[40] Shubik, M. and R. Levitan (1971), *Market Structure and Behaviour*. Cambridge, Harvard University Press.

[41] Tennbakk, B. (1992) "The Case of Cooperative Mixed Duopoly", Institute ofor Research on Labor and Employment, UC Berkeley, *Working Paper Series*, 06-01-1992.

[42] Yamey, B.S. (1950) "The Price Policy of Cooperative Societies", *Economica*, February, 23-42.

[43] Vanek, J. (1970), *The General Theory of Labor-Managed Economy*, Cornell University Press, Ithaca, NY.

[44] Vives, X. (1984), "Duopoly Information: Cournot and Bertrand", *Journal of Economic Theory*, 34, 71-94.

[45] JCCU (2009), *Co-op 2009, Facts & Figures*. Japanese Consumers' Co-operative Union, Tokio.

[46] Ward B. (1958), "The Firm in Illyria: Market Syndacalism", *American Economic Review*, 48, 4, 1958, 566-89.