Thinking by Analogy, Systematic Risk, and Option Prices

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Abstract

People tend to think by analogies and comparisons. Such way of thinking, termed coarse thinking by Mullainathan et al [Quarterly Journal of Economics, May 2008] is intuitively very appealing. We develop a new option pricing model based on the idea that the market consists of coarse thinkers as well as rational investors when limits to arbitrage (transaction costs) prevent rational investors from profiting at the expense of coarse thinkers. The new formula, which is a closed form solution to the model, is a generalization of the Black-Scholes formula. The new formula potentially provides a unified explanation for various implied volatility puzzles

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Thinking by Analogy and Option Prices

Although, Black and Scholes (1973) formula is the most widely used formula in financial markets, a number of empirical facts have been discovered which cast doubt on its validity: 1) The Black-Scholes implied volatility is not a constant, rather it exhibits a skew/smile when plotted against the striking price. See Rubinstein (1985, 1994), and Dumas, Fleming, and Whaley (1998) among others. 2) In contradiction with Black Scholes formula, systematic risk of the underlying has been found to affect option prices. In fact, both the level of implied volatility as well as the steepness of implied volatility curve increases with systematic risk. See Duan and Wei (2009). 3) Investor sentiment affects option prices. Specifically, implied volatility curve steepens in a bearish market. See Han(2008).4) In contrast with the prediction of the Black Scholes model, implied volatility also exhibits a term structure. That is, it changes with time to expiry of the option. See Derman, Kani and Zhou (1996). Both downward sloping and upward sloping term structures are observed.

In addition to the above, related intriguing empirical facts that require an explanation are: 1) Gold implied volatility curve typically has a shape (a positive skew) which is opposite to the shape of an equity implied volatility curve (a negative skew). See Derman (2003). However, recently traders have been noticing that the gold skew has reversed and has become negative just like an equity option skew.[2] Why does gold typically have a positive skew? Why did the pattern reverse recently? 2) Single stock implied volatility curves are generally flatter than equity index implied volatilities (see Bollen and Whaley (2004), and Duan and Wei (2009)). 3) Single stock implied volatility curves are more symmetrical, hence they look more like a smile compared to index option implied volatility curves which typically decrease monotonically. See Derman (2003). 4) At-the-money implied volatility is generally an unbiased and efficient forecast of ex-post realized volatility (see Christensen and Prabhala (1998)). However, in-the-money call option and out-of-the-money put option implied volatilities are significantly higher than the historical or realized volatilities. 5) Black Scholes formula predicts that the Sharpe-ratio of an option should be equal to the Sharpe-ratio of its underlying instrument. Despite the above mentioned shortcomings of the Black Scholes formula, there is no evidence suggesting that Sharpe-ratios of options differ from

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[2] http://www.bestcashcow.com/articles/negative-gold-skew-7631.

the Sharpe-ratios of their underlying instruments. Hence, any new approach aimed at addressing the above mentioned shortcomings must preserve the equality of Sharpe-ratios between an option and its underlying instrument.

In general, pricing models that have been proposed to overcome some of these shortcomings can be classified into three broad categories: 1) Stochastic volatility and GARCH models (Heston and Nandi (2000), Duan (1995), Heston (1993), Melino and Turnbull (1990), Wiggins (1987), and Hull and White (1987)). 2) Models with jumps in the underlying price process (Amin (1993), Ball and Torous (1985)). 3) Models with stochastic volatility as well as random jumps. See Bakshi, Cao, and Chen (1997) for a discussion of their empirical performance (mixed). Most of these models modify the price process of the underlying. Hence, the focus of these models is on finding the right distributional assumptions that could explain the implied volatility puzzles.

In this article, we put forward a new option pricing model that could provide a unified explanation for the above mentioned empirical facts. Our model is based on the idea that people tend to think by analogies and comparisons. In sharp contrast with other option pricing models (stochastic volatility and jump diffusion models), we do not modify the price process of the underlying. This allows us to provide an economic explanation for the systematic shortcomings of the Black-Scholes model.

Mullainathan, Schwartzstein & Shleifer (2008) formalize "thinking by analogy" in the context of a model of persuasion. Their model is based on the notion that agents use analogies for assigning values to attributes (the attribute valued in their model is "quality"). The idea is that people co-categorize situations that they consider analogous and assessment of attributes in a given situation is affected by other situations in the same category. This way of drawing inferences, which is termed coarse thinking, is in contrast with rational (Bayesian) thinking in which each situation is evaluated logically (often deductively), in isolation, and according to its own merit. Coarse thinking appears to be a natural way of modeling how humans process information. See Kahneman & Tversky (1982), Lakoff (1987), Edelmen (1992), Zaltman (1997), and Carpenter, Glazer, & Nakamoto (1994) among others.

Anecdotal evidence of the role of coarse thinking is all around us. In fact, Mullainathan et al (2008) use the advertising theme of Alberto Culver Natural Silk Shampoo as a motivating example to explain coarse thinking. The shampoo was advertised with a slogan "We put silk in the bottle." The company actually put some silk in the shampoo.

However, as conceded by the company spokesman, silk does not do anything for hair (Carpenter et al (1994)). Then, why did the company put silk in the shampoo? Mullainathan et al (2008) write that the company was relying on the fact that consumers co-categorize shampoo with hair. This co-categorization leads consumers to value "silk" in shampoo because they value "silky" in hair (clearly not a rational response). That is, a positive trait from hair is transferred to shampoo by adding silk to it. Such transfer of the perceived informational content of an attribute across co-categorized situations is termed *transference*.

In this article, we raise and provide an answer to the following question. Given undeniable evidence of the role of coarse thinking or thinking-by-analogy in almost everything we do, what are the implications for options pricing if some investors are prone to thinking-by-analogy (coarse thinking)? Intuitively, an in-the-money call option is similar to its underlying stock. So rather than investing in the underlying outright, some investors prefer to buy a deep in-the-money call option, as a deep in-the-money call offers nearly the same dollar-for-dollar increase or decrease in payoff as the underlying while requiring only a fraction of investment. In fact, a common advice given by investment bankers and brokers to their clients is to replace stocks in their portfolios with in-the-money call options. Their advice is based on thinking by analogy.[3] They emphasize the similarity between a stock and an in-the-money call and highlight the leveraging advantage. However, this leveraging advantage comes at a cost. Leverage is a double edged sword and if the option expires out-of-the-money, the loss is 100% whereas the loss from investing in the stock is lower as long as the stock price does not fall to 0.

An in-the-money call is riskier than the underlying (like any other call option). Hence, a rational risk averse investor should demand a higher expected return than what he demands for holding the underlying. A coarse thinker, on the other hand, due to the similarity between the two (emphasized by professionals, often as free advice, see references in foot note 3), co-categorizes an in-the-money call with the underlying and equates (mistakenly) the expected return of the two. That is, the price he is willing to way is determined in *transference* with the underlying stock by equating the expected returns. In other

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[3] Option traders and investment professionals often advise people to buy in-the-money calls rather than the underlying stocks. As illustrative examples, see the following:
http://ezinearticles.com/?Call-Options-As-an-Alternative-to-Buying-the-Underlying-Security&id=4274772,
http://www.investingblog.org/archives/194/deep-in-the-money-options/,
http://www.triplescreenmethod.com/TradersCorner/TC052705.asp,
http://daytrading.about.com/od/stocks/a/OptionsInvest.htm

words, a coarse thinker is willing to pay a higher price for an in-the-money call option than a rational investor. If market frictions (for example, transaction costs in creating replicating portfolios) prevent rational investors from making arbitrage profits at the expense of coarse thinkers, both types will survive, and the price dynamics of in-the-money call options (and corresponding out-of-the-money put options via put-call parity) will be affected. The very fact that professional advice by investment bankers and brokers is based on reasoning by analogy suggests that coarse thinkers are not getting arbitraged out of the market.

Apart from the analogy based advice coming from market professionals such as investment bankers and brokers, recent academic research also suggests that reasoning by analogy or coarse thinking matters for investors' willingness to pay in a controlled laboratory experiment. In an interesting paper, Rockenbach (2004) shows for the first time, that in a laboratory experiment, people seem to price an in-the-money call by equating its expected return with the expected return on the underlying. Siddiqi (2011) further extends Rockenbach (2004) and explicitly tests for the coarse thinking hypothesis in a laboratory experiment by varying the similarity between an option and its underlying, and finds that the hypothesis outperforms other hypotheses. Hence, experimental evidence strongly supports the hypothesis of coarse thinking or thinking by analogy.

The literature on limits to arbitrage is extensive. See Gromb & Vayonos (2010) for a survey of state of the art in the limits to arbitrage literature. As they write, explaining anomalies typically requires two ingredients: first, a demand shock which is experienced by investors other than arbitrageurs and second, constraints that prevent arbitrageurs from arbitraging irrational investors out of the market. In our context, the demand shock is experienced by coarse thinkers as they co-categorize an in-the-money call option with the underlying stock. Consequently, they experience a positive demand shock as the amount they are willing to pay for the option goes up. The constraints preventing arbitrageurs from making arbitrage profits are transaction costs involved in creating a replicating portfolio.

Our approach relates to Bollen and Whaley (2004). They argue that, in the presence of limits to arbitrage, net demand pressure could determine the level and the slope of the implied volatility curve. However, they do not identify the sources of such pressures. In this article, we argue that the demand pressures are caused by thinking by analogy or coarse thinking. Identifying the source of demand pressures not only allows us to derive a closed form expression for option pricing but also allows us to provide potential explanations for

the differential pricing structures of equity options and gold options apart from potentially explaining other implied volatility anomalies. Our approach also relates to Duan and Wei (2009) who empirically demonstrate that systematic risk matters in determining both the level as well as the slope of the implied volatility curve. We argue that the demand pressure created by thinking-by-analogy could lead to the empirical findings in Duan and Wei (2009). Hence, our approach provides a bridge that connects Bollen and Whaley (2004) with Duan and Wei (2009).

In this article, we formalize the intuition described above and derive closed form solutions for call and put options. We call these formulae the behavioral option pricing formulae. We then investigate the implications for implied volatility if actual price dynamics are determined according to the behavioral formulae and the Black-Scholes formulae are used to back-out implied volatility. Our findings are consistent with the empirical facts mentioned in the introduction. The behavioral formula is a promising alternative as it is easy to implement (simple closed from solution exists) and is essentially a generalization of the original Black-Scholes formula. Black-Scholes formula is obtained as a limiting case of the behavioral formula.

Coarse thinking or analogy based reasoning is likely to play an important role in understanding financial market behavior. Many researchers have pointed out that there appears to be clear departures from Bayesian thinking (Babcock & Loewenstein (1997), Babcock, Wang, & Loewenstein (1996), Hogarth & Einhorn (1992), Kahneman & Frederick (2002), Kahneman, Slovic, & Tversky (1982)). Such departures from rational thinking have been measured both at the individual as well as the market level (Siddiqi (2009), Kluger & Wyatt (2004)). However, the question of what type of behavior to allow for if non-Bayesian behavior is admitted is a difficult one to address in the absence of an alternative which is amenable to systematic analysis. Coarse thinking may provide such an alternative especially when the intuitive appeal of analogy based reasoning is undeniable.

1. Thinking by Analogy: A Simple Numerical Example

To fix ideas, consider a simple example. Suppose there is stock that has a price of $140 today. Tomorrow, with an equal chance, it can either go up to $200 or go down to $90. Let's call these possibilities Red and Blue states respectively. Suppose there is another asset with

payoff equal to $140 in the Red state or $30 in the Blue state. A coarse thinker needs to decide how much to pay for that asset. He compares the asset with the stock and notices a pattern. He notices that if he subtracts 60 from the stock's payoff, the asset's payoff in the corresponding state is obtained (200-60=140 and 90-60=30). Having established the similarity between the stock and the asset, he proceeds to determine how much to pay for the asset in analogy with the stock: The stock has an expected return of 3.6% ($\frac{0.5(200-140)+0.5(90-140)}{140}$). That is, for every $1 invested in the stock, the expected payoff is $1.036 ($\frac{0.5(200)+0.5(90)}{140}$). He argues that as the asset is similar to the stock, for every $1 investment in the asset, the expected payoff should also be $1.036. The concept of an expected return is an equalizer that allows one to measure the performance of different assets by converting them to the same base of $1. The notion that similar assets should pay similarly when converted to a basis of $1 for comparison, leads a coarse thinker to argue in favor of the equality of expected returns.

That is, the price he is willing to pay for the asset is obtained as follows:

$$\frac{Expected\ payoff}{Prie} = 1.036$$

$$=> Price = \frac{0.5(140)+0.5(30)}{1.036} = \$82.05$$

For comparison, the rational price (based on the principle of no arbitrage) of the asset is $140 - 60e^{-r(T-t)}$. Where r the risk free rate and $(T-t)$ is time to expiry. With $r = 0.01$ per day and $(T-t) = 1$ day, the no-arbitrage price is $80.6.

As can be seen, the price that a coarse thinker is willing to pay is different (higher) from the price that a rational arbitrageur is willing to pay. If there are no transaction costs, then a rational arbitrageur can make money at the expense of coarse thinkers with the following strategy: Short the asset to a coarse thinker for $82.05, and create a portfolio (by using the stock and riskless bonds) that exactly replicates the payoffs from the asset. The cost of such a replicating portfolio is $80.6. The liability from shorting has been perfectly hedged by the replicating portfolio so his net liability is zero. Hence, the difference, $82.05-$80.6=$1.45 is his riskless or arbitrage profit. However, if transaction costs are involved, creating a replicating portfolio becomes costlier by the amount of transaction costs. If the transaction costs are greater than $1.45 then arbitrage profits cannot be made and both types will survive in the market. In this particular example, creating a replicating portfolio requires

buying a stock worth $140 and selling bonds worth $59.4. Even if we assume only 1% brokerage commission, the broker's fee alone is $1.4 for the stock plus $0.594 for the bonds, which amounts to $1.994 in total. Hence, the transaction costs are larger than the potential gain of $1.45 making arbitrage impossible.

In this article, we derive a new option pricing formula, which is applicable if both types of investors simultaneously exist in the market due to limits to arbitrage. Before deriving the new option pricing formula, we first illustrate the main results of this article in the context of a binomial model in the next section.

2. Thinking by Analogy: The Binomial Case

The main results in this article can be illustrated in the context of a binomial situation. Consider a simple two state world. The equally likely states are Red, and Blue. There is a stock with payoffs X_1, and X_2 corresponding to states Red, and Blue respectively. The state realization takes place at time T. The current time is time t. We denote the risk free discount rate by r. The current price of the stock is S. There is another asset, which is a call option on the stock. By definition, the payoffs from the call option in the two states are:

$$C_1 = max\{(X_1 - K), 0\}, C_2 = max\{(X_2 - K), 0\} \tag{1}$$

Where K is the striking price, and C_1, and C_2, are the payoffs from the call option corresponding to Red, and Blue states respectively.

As can be seen, the payoffs in the two states depend on the payoffs from the stock in corresponding states. Furthermore, by appropriately changing the striking price, the call option can be made more or less similar to the underlying stock with the similarity becoming exact as K approaches zero (all payoffs are constrained to be non-negative). As our focus is on in-the-money call options, we assume:

$X_1 - K > 0$, and $X_2 - K > 0$.

How much is a coarse thinker willing to pay for this call option?

A coarse thinker co-categorizes this call option with the underlying and values it in *transference* with the underlying stock. In other words, a coarse thinker values the option in such a way

so as to equate the expected return on the call option with the expected return on the underlying.

We denote the return on an asset by $q \in Q$, where Q is some subset of $\mathfrak{R}$ (the set of real numbers). In calculating, the return of the call option, a coarse thinker faces two similar, but not identical, observable situations, $s \in \{0,1\}$. In $s = 0$, "return demanded on the call option" is the attribute of interest and in $s = 1$, "actual return available on the underlying stock" is the attribute of interest. The coarse thinker has access to all the information described above. We denote this public information by I.

The actual expected return available on the underlying stock is given by,

$$E[q \mid I, s = 1] = \frac{\{X_1 - S\} + \{X_2 - S\}}{2 \times S} \tag{2}$$

For a coarse thinker, the expected return demanded on the call option is:

$$E[q|I, s = 0] = E[q|I, s = 1] = \frac{\{X_1 - S\} + \{X_2 - S\}}{2 \times S} \tag{3}$$

So, the coarse thinker infers the price of the call option, P_c, from:

$$\frac{\{C_1 - P_c\} + \{C_2 - P_c\}}{2 \times P_c} = \frac{\{X_1 - S\} + \{X_2 - S\}}{2 \times S} \tag{4}$$

It follows,

$$P_c = \frac{C_1 + C_2}{X_1 + X_2} \times S$$
$$\Rightarrow P_c = \left(1 - \frac{2K}{X_1 + X_2}\right) S \tag{5}$$

Given co-categorization of the call option with the underlying stock, coarse thinkers choose a price for the option that equates the expected return on the option with the expected

return on the underlying stock (*transference*). A coarse thinker prices the call option in analogy with the underlying stock. The underlying stock has a certain link between the payoffs and price, which is captured by the concept of expected return. While pricing with analogy, the same link is transferred to the asset being priced. The concept of an expected return is an equalizer that allows one to measure the performance of different assets by converting them to the same base of $1. The notion that similar assets should pay similarly when converted to a basis of $1 for comparison, leads a coarse thinker to argue in favor of the equality of expected returns.

The rational price P_r can be determined (from the principle of no-arbitrage):

$$P_r = S - Ke^{-r(T-t)} \tag{6}$$

If limits to arbitrage prevent rational arbitrageurs from making riskless profits at the expense of coarse thinkers, both types will survive in the market. If α is the fraction of rational investors in the market, the market price of the call option is:

$$P_c^M = \alpha\left(S - Ke^{-r(T-t)}\right) + (1 - \alpha)\left\{\left(1 - \frac{2K}{X_1 + X_2}\right)S\right\} \tag{7}$$

Proposition 1 *The price of a call option in the presence of coarse thinkers ($\alpha < 1$) is always larger than the price in the absence of coarse thinkers ($\alpha = 1$) as long as the underlying stock price reflects a positive risk premium. Specifically, the difference between the two prices is $(1 - \alpha)\left(Ke^{-r(T-t)} - Ke^{-(r+\delta)(T-t)}\right)$ where δ is the risk premium reflected in the price of the underlying stock.*

Proof.

Subtracting equation (6) from equation (7) yields: $(1 - \alpha)\left(Ke^{-r(T-t)} - \frac{K}{\frac{X_1+X_2}{2}}S\right)$

As $S = e^{-(r+\delta)(T-t)}\left(\frac{X_1+X_2}{2}\right)$, the desired expression follows which is always greater than zero as long as $\delta > 0$.

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From this point onwards, we assume that $\alpha = {}^K\!/_S$ if $S > K$. Otherwise, $\alpha = 1$. As a call option becomes more in-the-money, its similarity with the underlying increases. Arguably, that

makes α a function of K/S. The simplest such function is the assumed linear function, which is equal to K/S. As thinking-by-analogy is only relevant when there is an analogy, that is, when a call option is in-the-money, we assume $\alpha = 1$ when the stock price is less than or equal to K. That is, coarse thinking does not happen when the analogy between an option and its underlying disappears. That is, when the option is not in-the-money.

Even though proposition 1 does not derive a general option pricing formula (which is derived in the next section), and considers a binomial case only, it leads to some interesting realizations which are helpful in developing intuition. Higher the value of risk premium, higher will be the price of the call option and consequently the level of implied volatility. Hence, proposition 1 potentially provides a theoretical explanation for the empirical findings reported in Duan and Wei (2009). The differences between different individual equity option implied volatilities could be explained by appealing to the differences in systematic risks leading to different risk premia across stocks. Thinking by analogy causes the risk premium (which is a reward for systematic risk) of the underlying to affect option prices.

The central prediction of asset pricing theory is that the risk premium on an asset depends on the covariance between the asset's return and a measure of aggregate risk. Specifically, $\delta = E[Asset's\ Return - r] = -covarince\left(\frac{A}{E[A]}, Asset's\ Return\right)$ where A measures aggregate risk. In the CAPM, A is a linear transformation of the return on the market portfolio. In Lucas's consumption based model, A is the marginal rate of substitution of consumption. In Rubinstein's model A is equal to the inverse of the return on the market portfolio. Essentially, the equation says that an asset that pays well when times are bad (aggregate risk is high) has a negative risk premium because such an asset is in high demand as it provides insurance against consumption risk. Generally, assets pay well when times are good (aggregate risk is low), hence most assets have a positive risk premium. However, there is one asset that pays well when times are bad. That asset is gold. Hence, gold should have a negative risk premium. The negative value of δ implies that the price of a call option in the presence of coarse thinkers is lower than the price in their absence. Consequently, implied volatility of in-the-money call options on gold is lower than the implied volatility of at-the-money and out-of-the-money call options as coarse thinkers are only present when a call

option is in-the-money. Hence, thinking-by-analogy could explain the positive skew in implied volatility curves obtained from gold options.

Proposition 2 shows that the presence of coarse thinkers does not change the Sharpe-ratio of options. This shows that the mispricing of options with respect to the underlying due to the presence of coarse thinkers is not reflected in the Sharpe-ratio as the change in expected excess return is offset by the change in variance.

Proposition 2 *The Sharpe-ratio of an option remains unchanged regardless of whether the coarse thinkers are present or not. Specifically, the Sharpe-ratio remains equal to the Sharpe-ratio of the underlying regardless of the presence of coarse thinkers.*

Proof.

Initially, assume that coarse thinkers are not present. Let x be the number of units of the underlying stock needed and let B be the dollar amount invested in a risk-free bond to create a portfolio that replicates the call option. It follows,

$$P_c = Sx + B \tag{I}$$
$$X_1 - K = xX_1 + B(1 + r) \tag{II}$$

Substitute B from (I) into (II) and re-arrange to get:

$$X_1 - K - P_c = x(X_1 - S) - xSr + P_c r$$
$$\Rightarrow \Delta C = x\Delta S - xSr + P_c r$$
$$\Rightarrow \Delta C = xS\left(\frac{\Delta S}{S} - r\right) + P_c r$$
$$\Rightarrow \frac{\Delta C}{P_c} = x\frac{S}{P_c}\left(\frac{\Delta S}{S} - r\right) + r$$
$$\Rightarrow R_c = \Omega(R_s - r) + r$$

Where Ω is the elasticity of option's price with respect to the stock price, R_c and R_s are returns on call and stock respectively. It follows,

$$E[R_c] = \Omega(E[R_s - r]) + r \tag{III}$$
$$Var[R_c] = \Omega^2 Var[R_s]$$
$$\Rightarrow \sigma_c^2 = \Omega^2 \sigma_s^2 \tag{IV}$$

$$Sharpe - ratio\ of\ Call\ without\ coarse\ thinkers = \frac{E[R_c] - r}{\sigma_c}$$

$$=\frac{\Omega(E[R_s]-r)+r-r}{\Omega\sigma_s}=\frac{E[R_s]-r}{\sigma_s}=Sharpe-ratio\ of\ the\ underlying\ stock$$

If coarse thinkers are also present, it follows,

$$E[R_c]=\alpha\{\Omega(E[R_s]-r)+r\}+(1-\alpha)E[R_s] \tag{V}$$

$$\sigma_c^2=\{\alpha\Omega+(1-\alpha)\}^2\sigma_s^2 \tag{VI}$$

$$Sharpe-ratio\ of\ Call\ with\ coarse\ thinkers=\frac{\alpha\{\Omega(E[R_s-r]+r)\}+(1-\alpha)E[R_s]-r}{\{\alpha\Omega+(1-\alpha)\}\sigma_s}$$

$$=\frac{\{\alpha\Omega+(1-\alpha)\}E[R_s]-r\{\alpha\Omega+(1-\alpha)\}}{\{\alpha\Omega+(1-\alpha)\}\sigma_s}=\frac{E[R_s]-r}{\sigma_s}=Sharpe-ratio\ of\ the\ underlying\ stock$$

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Proposition 2 shows that the mispricing caused by the presence of coarse thinkers does not change the Sharpe-ratio. It remains equal to the Sharpe-ratio of the underlying. Hence, the fact that the empirical Sharpe-ratios of options and underlying stocks do not differ cannot be used to argue that there is no mispricing in options with respect to the underlying.

Proposition 3 shows the condition under which rational arbitrageurs cannot make arbitrage profits at the expense of coarse thinkers. Consequently, both types will survive in the market.

Proposition 3 *Coarse thinkers cannot be arbitraged out of the market if*
$(1-\alpha)\{Ke^{-r(T-t)}-Ke^{-(r+\delta)(T-t)}\}<c$ *where c is the transaction cost involved in the arbitrage scheme and δ>0*

Proof.

The presence of coarse thinkers increases the price of an in-the-money call option beyond its rational price. A rational arbitrageur interested in profiting from this situation should do the following: Write a call option and create a replicating portfolio. If there are no transaction costs involved then he would pocket the difference between the rational price and the market price without creating any liability for him when the option expires. As proposition 1 shows, the difference is $(1-\alpha)\{Ke^{-r(T-t)}-Ke^{-(r+\delta)(T-t)}\}$. However, if there are transaction costs involved then he would follow the strategy only if the benefit is greater than the cost. Otherwise, arbitrage profits cannot be made.

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In this section, we considered a world with only two states (call option can take only two possible values). However, in reality, a call option can take a large number of possible values making the creation of a replicating portfolio quite costly due to significant transaction costs. The benefit of arbitrage strategy does not change as the number of states increases. It remains equal to $(1 - \alpha)\{Ke^{-r(T-t)} - Ke^{-(r+\delta)(T-t)}\}$. However, transaction costs surely increase as more assets are needed in the replicating portfolio. Hence, making arbitrage profits at the expense of coarse thinkers is highly unlikely.

In the next section, we generalize the results obtained in this section, and derive new option pricing formulae for call and put options.

3. Thinking by Analogy: The New Option Pricing Formulae

In this section, we consider the following question: What are the new option pricing formulae if rational investors and coarse thinkers are both present in the market due to limited arbitrage? Before formally deriving the formulae, it is helpful to discuss the intuition: As proposition 1 shows the effect of coarse thinkers is to increase the price of an in-the-money call option by $(1 - \alpha)\{Ke^{-r(T-t)} - Ke^{-(r+\delta)(T-t)}\}$. Hence, the new formula for the price of an in-the-money call option should approximately be equal to the old formula plus $(1 - \alpha)\{Ke^{-r(T-t)}e^{-(r+\delta)(T-t)}\}$ (not exactly equal due to a shift from a binomial distribution to a normal distribution). Similarly, the price of a corresponding out-of-the-money put option can be obtained via put-call parity, which should also be approximately higher by $(1 - \alpha)\{Ke^{-r(T-t)} - Ke^{-(r+\delta)(T-t)}\}$.

Formally, propositions 4 shows the intermediate steps needed in deriving the new option pricing formulae. Proposition 5 derives the new formulae by using the results in proposition 4.

Dividends are assumed to be zero throughout this article for simplicity. All options are European. As in the Black-Scholes model, we assume that the price of the underlying follows a geometric Brownian motion:

$$dS = \mu S dt + \sigma S dZ \tag{8}$$

where S is the stock price, μ is a constant denoting the expected return on the underlying stock, σ is a constant denoting the standard deviation of return, and dZ is a random variable which is an accumulation of a large number of independent random effects over an interval dt. dZ has a mean of zero. It can be shown that variance of dZ scales with the length of the time interval under consideration.

That is,

$$Var[dZ] \propto dt$$
$$\Rightarrow \sqrt{Var[dZ]} \propto \sqrt{dt}$$

It follows,

$$dZ \sim n\sqrt{dt}$$

where n is a standard normal variable with a mean equal to zero and a standard deviation equal to one.

The price of a European call option (C) is then considered as a function of the underlying stock price (S) and time (t), that is, $C = f(S,t)$. Ito's lemma leads to

$$dC = \{\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}\mu S + 1/2 \times \frac{\partial^2 C}{\partial S^2}\sigma^2 S^2\}dt + \{\frac{\partial C}{\partial S}\sigma S\}dZ \tag{9}$$

Proposition 4 In the absence of coarse thinkers, equation (9) can also be expressed as follows: $dC = \left\{\frac{SN(d_1)}{C}(\mu - r) + r\right\}Cdt + \left\{\frac{SN(d_1)}{C}\right\}\sigma CdZ$

Where $N(.)$ is the standard normal distribution, r is the risk free rate, and

$$d_1 = \frac{ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$ **with K being the striking price of the option.**

Proof.

See Appendix A.

∎

The expression derived in proposition 4 is useful because it allows one to express the expected return and the standard deviation of call's return as a function of underlying's expected return and standard deviation. Specifically, from proposition 4, one can see that the

expected return and the standard deviation of call's return over an interval dt are as follows:

$\frac{E[dC]}{C} = \left\{ \frac{SN(d_1)}{C} (\mu - r) + r \right\}$ and $\sigma_{call} \left(\frac{dC}{C} \right) = \left\{ \frac{SN(d_1)}{C} \right\} \sigma$.

If both coarse thinkers and rational investors are present with fractions equal to $(1 - \alpha)$ and α respectively, the expected return on the call option becomes:

$\frac{E[dC^M]}{C^M} = \alpha \left\{ \frac{SN(d_1)}{C} (\mu - r) + r \right\} + (1 - \alpha)\mu$ where C^M denotes the market price of a call option when coarse thinkers are also present.

It follows,

$$Var \left(\frac{E[dC^M]}{C^M} \right) = Var \left(\alpha \left\{ \frac{SN(d_1)}{C} (\mu - r) + r \right\} + (1 - \alpha)\mu \right) = \left\{ (1 - \alpha) + \alpha \frac{SN(d_1)}{C} \right\}^2 \sigma^2$$

Hence,

$$\sigma \left(\frac{dC^M}{C^M} \right) = \left\{ (1 - \alpha) + \alpha \frac{SN(d_1)}{C} \right\} \sigma$$

  With the help of the above results, the new option pricing formulae can be derived as shown in proposition 5.

Proposition 5 If coarse thinkers are present, the price of a European call option is:

$$\boldsymbol{Call_E = SN(d_1) - Ke^{-r(T-t)}N(d_2) \left\{ \frac{K}{S} + \left(1 - \frac{K}{S} \right) \left(e^{-\delta(T-t)} \right) \right\}} \quad \boldsymbol{if \quad S > K}$$

$$\boldsymbol{Call_E = SN(d_1) - Ke^{-r(T-t)}N(d_2)} \quad \boldsymbol{if \quad S \le K}$$

where $\boldsymbol{d_1 = \frac{ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2} \right)(T-t)}{\sigma\sqrt{T-t}}, d_2 = \frac{ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2} \right)(T-t)}{\sigma\sqrt{T-t}}}$,

$\boldsymbol{\delta = risk\ premium\ on\ the\ underlying = E[Return\ on\ the\ underlying] - r > 0}$ **and**

$\boldsymbol{(T - t)\ is\ time\ to\ expiy}$.

If coarse thinkers are present, the price of a European put option is:

$$\boldsymbol{Put_E = Ke^{-r(T-t)} \left\{ 1 - N(d_2) \left\{ \frac{K}{S} + \left(1 - \frac{K}{S} \right) \left(e^{-\delta(T-t)} \right) \right\} \right\} - SN(-d_1)\ if \quad S > K}$$

$$\boldsymbol{Put_E = Ke^{-r(T-t)}N(-d_2) - SN(-d_1) \quad if\ S \le K}$$

Proof.

See Appendix B.

■

Proposition 5 shows the pricing formulae for European call and put options. As can be seen, when a call is in-the-money, its price is equal to the Black Scholes price of call plus $(1 - \alpha)N(d_2)\{Ke^{-r(T-t)} - Ke^{-(r+\delta)(T-t)}\}$. Similarly, the price of an out-of-the-money put is equal to the Black-Scholes price of put plus $(1 - \alpha)N(d_2)\{Ke^{-r(T-t)} - Ke^{-(r+\delta)(T-t)}\}$. Hence, the formulae are consistent with the intuition discussed earlier. As can be seen, when $S \leq K$ *or equivalently* $\alpha = 1$, the behavioral formulae converge to the Black-Scholes formulae. Hence, the Black-Scholes model is a special case of the behavioral model.

3.1 In-the-money Put Option vs. Short-Selling

Apart from the analogy between an in-the-money call and its underlying, another analogy exists if there is short-selling interest in the underlying. The analogy is clearly reflected in the arguments made by some experienced investment bankers and brokers who urge their clients to buy an in-the-money put option on the underlying instead of short-selling the underlying. They argue that an in-the-money put offers nearly the same dollar-for-dollar gain as short-selling, while limiting the maximum loss to the amount of premium paid.[4]

Proposition 6 shows the options pricing formulae when short-selling demand also exists and some (bearish) coarse thinkers form an analogy with in-the-money puts.

Proposition 6 **If bullish coarse thinkers (who form an analogy between investing in the underlying and buying in-the-money calls) are present in the market for in-the-money calls and bearish coarse thinkers (who form an analogy between short selling the underlying and buying in-the-money puts) are present in the market for in-the-money puts, the price of a European put option is given by:**

$$Put_E = Ke^{-r(T-t)}N(-d_2)\left\{\frac{S}{K} + \left(1 - \frac{S}{K}\right)e^{-\delta_d(T-t)}\right\} - SN(-d_1) \qquad if\ K > S$$

$$Put_E = Ke^{-r(T-t)}N(-d_2) - SN(-d_1) \qquad\qquad\qquad if\ K = S$$

$$Put_E = Ke^{-r(T-t)}\left\{1 - N(d_2)\left\{\frac{K}{S} + \left(1 - \frac{K}{S}\right)\left(e^{-\delta_u(T-t)}\right)\right\}\right\} - SN(-d_1) \qquad if\ K < S$$

where $\delta_d = E[Return\ on\ the\ underlying] - r < 0$. **That is, it is the expected excess discount in the price of the underlying. In other words, $-\delta_d$ is the expected return from short-selling the underlying as expected by bearish coarse thinkers.**
$\delta_u = E[Return\ on\ the\ underlying] - r > 0$ **is the expected premium on the underlying as expected by bullish coarse thinkers.** $d_1 = \frac{ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$, $d_2 = \frac{ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$ **and** $(T - t)$ **is time to expiy.**

And the price of a European call option is given by:

$$Call_E = SN(d_1) - Ke^{-r(T-t)}\left\{1 - N(-d_2)\left(\frac{S}{K} + \left(1 - \frac{S}{K}\right)e^{-\delta_d(T-t)}\right)\right\} \qquad if\ K > S$$

$$Call_E = SN(d_1) - Ke^{-r(T-t)}N(d_2) \qquad\qquad\qquad\qquad\qquad if\ K = S$$

$$Call_E = SN(d_1) - Ke^{-r(T-t)}N(d_2)\left\{\frac{K}{S} + \left(1 - \frac{K}{S}\right)e^{-\delta_u(T-t)}\right\} \qquad if\ K < S$$

Proof.

See Appendix C.

■

Proposition 5 derives the formulae which are applicable when short-selling market in the underlying does not exist, whereas proposition 6 derives formulae which are applicable when short-selling market in the underlying exists. It is unlikely that there is much short-selling interest in indices such as S&P 500 or Dow Jones Industrial Average (DJIA). Indices are calculated as weighted average of a large number of component stocks. When indices fall in value, it is due to a fall in some of the component stocks. Short-sellers are better-off if they short-sell the falling stocks directly rather than trying to short sell the entire index. Besides, short-selling an index is not feasible as it would require borrowing a large number of different stocks in the right proportions (transaction costs alone are prohibitive in such a scheme). Hence, active short-selling market does not exist in indices. Consequently, the formulae in proposition 5 can be considered applicable to index options and formulae in proposition 6 are applicable to single stock options.

In the next section, we show that the new formulae are consistent with the empirical facts regarding implied volatility. Hence, they could provide a unified explanation for the implied volatility puzzles.

4. Implications for Implied Volatility

Table 1 shows the behavioral price as well as the rational price (Black Scholes price) of a call option whose moneyness varies from at-the-money (no coarse thinkers) to various levels of in-the-money (coarse thinkers present). Table 1 reports implied volatility by assuming that the actual price is determined by the behavioral formula (shown in Proposition 5) whereas the Black-Scholes formula is used to back-out implied volatility.

Table 1 Behavioral Price Vs. Rational Price r=1%, K=100, δ=6%, Expiry= 1 month, Actual Volatility of the Underlying=20%				
K/S	Behavioral Price (1)	Rational Price (2)	The Difference (1)-(2)	Implied Volatility
1	2.3275	2.3275	0	20%
0.95	5.7338	5.7103	0.0235	20.29%
0.91	10.2431	10.1985	0.0466	21.39%
0.87	15.1605	15.0964	0.0641	25.33%
0.83	20.1653	20.0833	0.082	31.83%
0.8	25.1805	25.0822	0.0983	38.60%
0.77	30.1956	30.0822	0.1134	45.10%

As table 1 shows, the difference between the behavioral price and the rational price arguably remains smaller than the transaction costs involved in creating a replicating portfolio. In table 1, the maximum value of the difference occurs at the moneyness level of 0.77 and it is only equal to 11.34 cents. In any replicating portfolio, the broker's commission on the underlying stock alone will be higher than this value, illustrating that limits to arbitrage could prevent rational investors from profiting at the expense of coarse thinkers.

The volatility skew, on the other hand, is quite steep, as shown in figure 1.
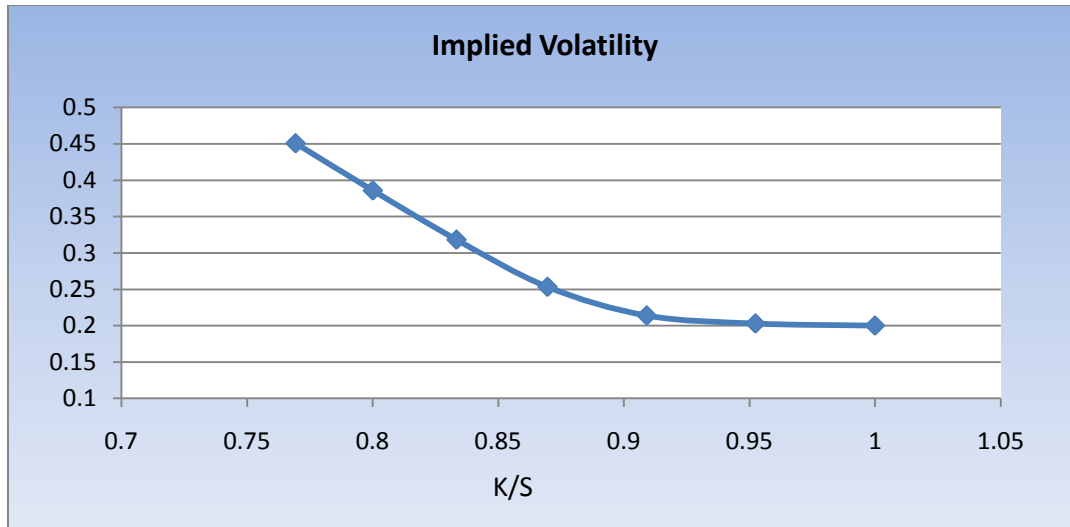
Figure 1

4.1 Implied Volatility and Systematic Risk

Empirical work on derivatives has uncovered an intriguing feature. Duan and Wei (2009) show that systematic risk affects both the level as well as the steepness of the implied volatility curves of individual equity options. Specifically, a higher amount of systematic risk leads to a higher level of implied volatility and a steeper slope of the implied volatility curve. This finding is in perfect agreement with the new option pricing formulae derived in this article. Hence, thinking by analogy could explain the empirical findings in Duan and Wei (2009).

Figure 2 plots the implied volatility curves when risk premia on the underlying stock are 6% and 3%. Other parameters are: K=100, r=1%, time to expiry= 1 month, actual volatility of the underlying = 20%. As can be seen, the level as well as the steepness of the implied volatility curve increases as risk premium on the underlying increases. Hence, the differential pricing structure of individual equity options can be understood in terms of different systematic risks resulting in different risk premia. Coarse thinkers co-categorize an in-the-money call with its underlying stock. Risk premium affects how the stock is priced. Co-categorization with the underlying causes the price of an in-the-money call to respond to the risk premium on the underlying.
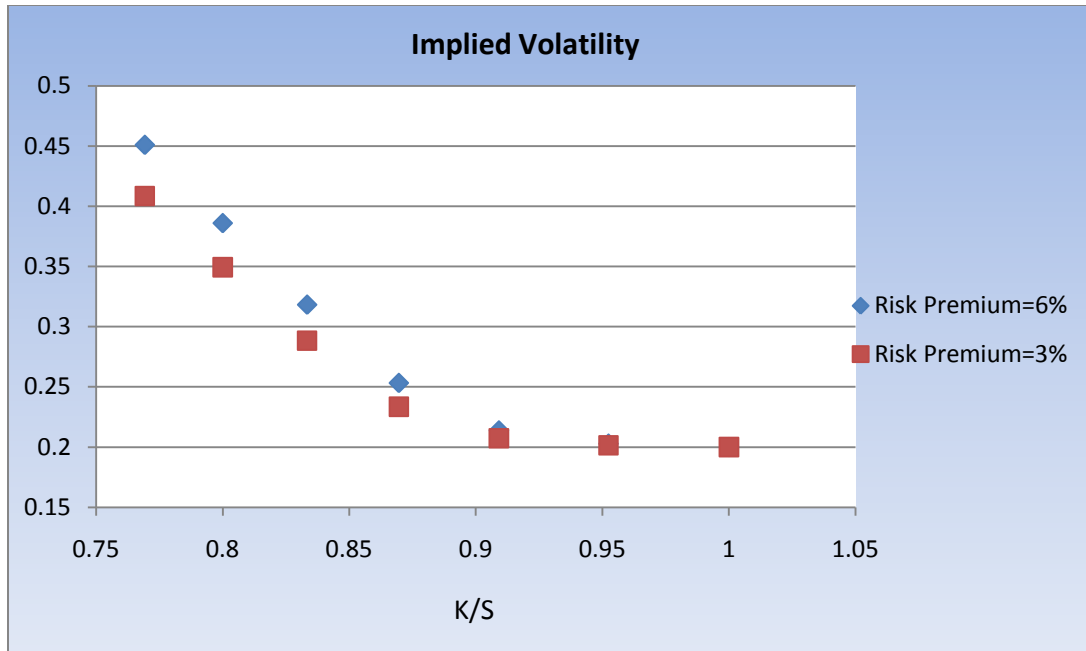
Figure 2

4.2 Investor Sentiment and Option Prices

Han (2008) shows that investor sentiment matters in determining the slope of the implied volatility curve. Specifically, Han (2008) shows that in a bearish market, implied volatility curves of S&P 500 options steepen.

There is strong evidence suggesting that the Sharpe-ratio is countercyclical. See Lettau and Ludvigson (2010) and Lustig and Verdelhan (2010) among others. This suggests that either the quantity of perceived risk or the price of risk (risk aversion) or both are higher in bad times when compared with good times. Also, markets are more bearish in bad times when compared with good times. As risk version or the quantity of perceived risk increases, the risk premium on the underlying goes up. So, in bad times (when markets are more bearish), risk premium on the underlying is higher. Thinking by analogy causes the risk premium in the underlying to affect option prices. Specifically, higher the risk premium, higher is the level and the slope of the implied volatility curve as figure 2 shows. Hence, the findings reported in Han (2008) are consistent with the approach developed in this article.

4.3 The Term Structure of Implied Volatility

Implied volatility also varies with expiry. That is, it has a term structure. See Derman, Kani and Zhou (1996). Both upward sloping and downward sloping term structures are observed. The difference between the behavioral price (C^M) and the Black-Scholes price(C^R) is approximately:

$$C^M - C^R \approx (1 - \alpha)\{Ke^{-r(T-t)} - Ke^{-(r+\delta)(T-t)}\} \tag{10}$$

If the difference increases with expiry, an upward sloping term structure results, and if the difference decreases with expiry, a downward sloping term structure follows.

$$\frac{\partial(C^M - C^R)}{\partial(T-t)} = (1 - \alpha)Ke^{-r(T-t)}\{(r+\delta)e^{-\delta(T-t)} - r\} - \{Ke^{-r(T-t)} - Ke^{-(r+\delta)(T-t)}\}\frac{\partial\alpha}{\partial(T-t)} \tag{11}$$

An in-the-money call converts to the underlying on expiry, hence, the closer an in-the-money call is to expiry, stronger is the analogy with the underlying. As time to expiry increases, the analogy between an in-the-money call and its underlying weakens. That is, $\frac{\partial\alpha}{\partial(T-t)} > 0$. If the first term in (11) is smaller than the second term, a downward sloping term-structure will be observed. However, if the first term in (11) is bigger than the second term and $\delta > re^{\delta(T-t)} - r$, an upward sloping term structure will be observed.

4.4 Single Stock Smile vs. Equity Index Skew

An interesting feature is that even though the implied volatility of equity index options exhibits a skew (figures 1 and 2), the implied volatility of single stock options resembles a symmetric smile. See Derman (2003). Why do we observe this difference?

It is unlikely that there is much short-selling interest in indices such as S&P 500 or Dow Jones Industrial Average (DJIA). Indices are calculated as weighted average of a large number of component stocks. When indices fall in value, it is due to a fall in some of the component stocks. Short-sellers are better-off if they short-sell the falling stocks directly rather than trying to short sell the entire index. Besides, short-selling an index is not feasible as it would require borrowing a large number of different stocks in the right proportions (transaction costs alone are prohibitive in such a scheme). Hence, active short-selling market

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does not exist in indices. Consequently, the formulae in proposition 5 can be considered applicable to index options and formulae in proposition 6 are applicable to single stock options.

Figure 3 shows the implied volatility curve if the actual price is determined in accordance with the behavioral formulae in proposition 6 whereas the Black-Scholes formula is used to back-out implied volatility. Other parameters are K=100, r=5%, time to expiry= 1 month, actual volatility of the underlying = 20%, $abs(\delta_d) = \delta_u = 6\%$. As can be seen, the implied volatility resembles a symmetric smile.
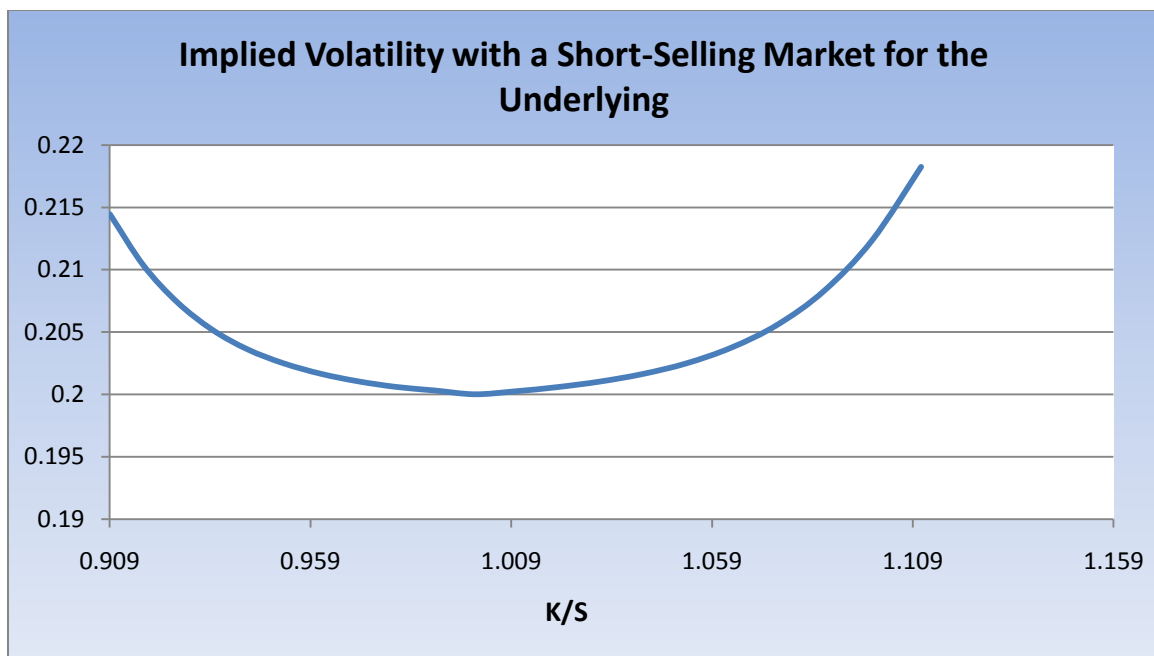
Implied Volatility with a Short-Selling Market for the Underlying

Figure 3

4.4 Implied Volatility of Gold Options

As figures 1 and 2 show, the implied volatility on equity option has a negative skew. An intriguing empirical fact is that in sharp contrast with equity options, implied volatility of gold options has a positive skew. That is, the implied volatility of in-the-money call options on gold is lower than the implied volatility of at-the-money options.

A typical asset pays well when times are good. Gold is special because it pays well when times are bad. Hence, in normal times, gold could provide consumption insurance. As discussed earlier, a central prediction of asset pricing theory is that assets that pay well when

times are bad have a negative risk premium because they are in high demand as they provide consumption insurance. If the risk premium parameter in the behavioral formula takes a negative value (which is presumably the case with gold) , then implied volatility of in-the-money call options would be lower than the implied volatility of at-the-money options potentially providing an explanation for the peculiar skew in gold options.

The recent reversal in the implied volatility pattern of gold from positive skew to negative skew can also be understood. As recession gets prolonged and gold price continues to climb, it attracts speculators who are betting on a further rise in gold prices. The presence of speculators expecting a further rise in gold prices could make the expected return on gold positive, reversing the skew temporarily.

4.5 Flatter Single Stock Implied Volatility vs. Steeper Index Implied Volatility

Individual stock implied volatility curves are typically flatter than equity index implied volatility curves. Descriptive statistics on beta values indicate that a majority of single stocks have beta values lower than 1 (which is the beta value of a benchmark index). As one example, Reidl and Serafaim (2010), report descriptive statistics on beta values obtained from a sample of US financial firms (table 2 in their article). They report a median beta value of 0.815. As beta value is a measure of systematic risk, it follows that typically single stocks have lower systematic risks when compared with an equity index. Consequently, single stock risk premia are typically lower than equity index risk premia, potentially explaining the flatter implied volatility curves of single stock options as risk premia and steepness are directly linked in our model (figure 2).

5. Conclusions

People have a tendency to think by analogies and comparisons. This way of thinking, termed coarse thinking by Mullainathan et al (2008), is intuitively very appealing. In this article, we raise and provide an answer to the following question: What are the implications for option pricing if coarse thinkers and rational thinkers co-exist due to limited arbitrage? We derive closed form solutions for new option pricing formulae for European call and put options. We show that the new formulae, which are generalizations of the original Black-Scholes

formulae, could potentially provide a unified explanation for the intriguing implied volatility puzzles in option pricing.

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Appendix A

In this appendix, we show that, in the absence of coarse thinkers, the option pricing stochastic differential equation can be expressed as:

$$dC = \left\{ \frac{SN(d_1)}{C}(\mu - r) + r \right\} C dt + \left\{ \frac{SN(d_1)}{C} \right\} \sigma C dZ \qquad (A1)$$

Starting from Ito's lemma:

$$dC = \left(\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}\mu S + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} \right) dt + \left(\frac{\partial C}{\partial S}\sigma S \right) dZ \qquad (A2)$$

If there are no coarse thinkers then we are in a world described by the Black-Scholes model. The values of various partial derivatives in the Black-Scholes world are:

$$\frac{\partial C}{\partial S} = N(d_1)$$

$$\frac{\partial^2 C}{\partial S^2} = \frac{N'(d_1)}{S\sigma\sqrt{T-t}}$$

$$\frac{\partial C}{\partial t} = -\frac{S\sigma N'(d_1)}{2\sqrt{T-t}} - rKe^{-r(T-t)}N(d_2)$$

Substituting the above values of partial derivatives in equation A2, and re-arranging leads to equation A1.

Appendix B

First, we derive the analogous expression to A1 when coarse thinkers are also present. Let C^M denote the market price of a call option when both coarse thinkers and rational investors are present. It follows,

$$E\left[\frac{dC^M}{C^M}\right] = \left\{ \alpha \left(\frac{SN(d_1)}{C}(\mu - r) + r \right) + (1-\alpha)\mu \right\} \qquad (B1)$$

$$Variance\left\{ E\left[\frac{dC^M}{C^M}\right] \right\} = Variance\left\{ \alpha \left(\frac{SN(d_1)}{C}(\mu - r) + r \right) + (1-\alpha)\mu \right\}$$

$$\Rightarrow \sigma^2\left(\frac{dC^M}{C^M}\right) = (1-\alpha)^2\sigma_s^2 + \left\{\alpha\frac{SN(d_1)}{C}\right\}^2\sigma_s^2 + 2Cov\left[(1-\alpha)\mu,\left\{\alpha\left(\frac{SN(d_1)}{C}\right)(\mu-r)+r\right)\right\}\right]$$

$$\Rightarrow \sigma^2\left(\frac{dC^M}{C^M}\right) = \left\{(1-\alpha)+\alpha\frac{SN(d_1)}{C}\right\}^2\sigma_s^2$$

$$\Rightarrow \sigma\left(\frac{dC^M}{C^M}\right) = \left\{(1-\alpha)+\alpha\frac{SN(d_1)}{C}\right\}\sigma_s \tag{B2}$$

When coarse thinkers are also present, the stochastic process followed by the price a call option is:

$$dC^M = E\left[\frac{dC^M}{C^M}\right]C^M dt + \sigma\left(\frac{dC^M}{C^M}\right)C^M dZ \tag{B3}$$

Substituting from B1 and B2 into B3 leads to:

$$dC^M = \left[(1-\alpha)\mu + \alpha\left\{\frac{SN(d_1)}{C}(\mu-r)+r\right\}\right]C^M dt + \left\{(1-\alpha)+\alpha\frac{SN(d_1)}{C}\right\}\sigma_s C^M dZ \quad if\ S > K \tag{B4}$$

From Ito's lemma:

$$dC^M = \left(\frac{\partial C^M}{\partial t}+\frac{\partial C^M}{\partial S}\mu S+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 C^M}{\partial S^2}\right)dt + \left(\frac{\partial C^M}{\partial S}\sigma_s S\right)dZ \tag{B5}$$

Comparison of (B4) and (B5) leads to:

$$\frac{\partial C^M}{\partial t}+\frac{\partial C^M}{\partial S}\mu S+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 C^M}{\partial S^2} = \left[(1-\alpha)\mu + \alpha\left\{\frac{SN(d_1)}{C}(\mu-r)+r\right\}\right]C^M \quad if\ S > K \tag{B6}$$

$$\frac{\partial C^M}{\partial S}\sigma_s S = \left\{(1-\alpha)+\alpha\frac{SN(d_1)}{C}\right\}\sigma_s C^M \qquad if\ S > K \tag{B7}$$

where $C = SN(d_1) - Ke^{-r(T-t)}N(d_2)$.

Simultaneously solving partial differential equations (B6) and (B7) with the boundary condition: $when\ (T-t)=0, C^M = C = max\{S-K, 0\}$, results in the behavioral option pricing

formula for a European call. The formula for a European put is then obtained via put-call parity. The case $S \leq K$ is the same as in the Black-Scholes model.

Appendix C

Let P^M denote the market price of a put option when both coarse thinkers and rational investors are present. Following a similar logic as in Appendix B:

$$dP^M = \left[(1-\alpha)\mu + \alpha\left\{\frac{-SN(-d_1)}{P}\delta_d + r\right\}\right]P^M dt + \left\{(1-\alpha) + \alpha\frac{SN(-d_1)}{P}\right\}\sigma_s P^M dZ \quad if\ K > S \quad (C1)$$

Where $\alpha = \frac{S}{K}$ and $P = Ke^{-r(T-t)}N(-d_2) - SN(-d_1)$.

It follows,

$$\frac{\partial P^M}{\partial t} + \frac{\partial P^M}{\partial S}\mu S + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 P^M}{\partial S^2} = \left[(1-\alpha)\mu + \alpha\left\{\frac{-SN(-d_1)}{P}\delta_d + r\right\}\right]P^M \qquad if\ K > S \qquad (C2)$$

$$\frac{\partial P^M}{\partial S}\sigma_s S = \left\{(1-\alpha) + \alpha\frac{SN(-d_1)}{P}\right\}\sigma_s P^M \qquad if\ K > S \qquad\qquad (C3)$$

Simultaneously solving partial differential equations (C2) and (C3) with the boundary condition: $when\ (T-t) = 0, P^M = P = max\{K - S, 0\}$, results in the behavioral option pricing formula for a European put. The formula for a European call is then obtained via put-call parity. The case $S > K$ is the same as in Appendix B. The case $S = K$ is the same as in the Black-Scholes model.