Reciprocity, inequity-aversion, and oligopolistic competition

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Reciprocity, Inequity Aversion, and Oligopolistic Competition,[†]

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Abstract

This paper extends the Cournot and Bertrand models of strategic interaction between firms by assuming that managers are not only profit maximizers, but also have preferences for reciprocity or are averse to inequity. A reciprocal manager responds to unkind behavior of rivals with unkind actions, while at the same time, it responds to kind behavior of rivals with kind actions. An inequity averse manager likes to reduce the difference between own profits and the rivals' profits. The paper finds that if firms with reciprocal managers compete à la Cournot, then they may be able to sustain "collusive" outcomes under a *constructive reciprocity* equilibrium. By contrast, Stackelberg warfare may emerge under a *destructive reciprocity* equilibrium. If there is Cournot competition between firms and their managers are averse to advantageous (disadvantageous) inequity, then firms are better (worse) off than if managers only care about maximizing profits. If firms compete à la Bertrand, then only under very restrictive conditions will managers' preferences for reciprocity or inequity aversion have an impact on equilibrium outcomes.

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1 Introduction

The assumption that individuals behave as if maximizing their material payoffs, despite its central role in economic analysis, is at odds with a large body of evidence from psychology and from experimental economics. Economic agents often pursue objectives other than actual payoff maximization. Many observed departures from material payoff maximizing behavior arise through actions that favor fairness or reciprocity, or that show concern for relative payoffs.

Motivated by this evidence, this paper studies strategic interactions between firms whose managers not only care about maximizing profits, but also care about the intentions of their rivals or the distribution of profits across firms. The analyzes focuses on the two canonical models of strategic interaction: Cournot and Bertrand competition. I study the economic implications for market output, price, profits and consumer surplus of having managers with these types of preferences making decisions in firms.

The paper starts by incorporating preferences for reciprocity into the Cournot model of quantity competition. I assume that a reciprocal manager cares about maximizing profits but also about the intentions of his rivals. If a reciprocal manager expects his rivals to produce more output than his own perception of their fair output, then he is willing to sacrifice some of his firm's profits to reduce the rivals' profits. By contrast, if a reciprocal manager expects the output of his rivals to fall short of his own perception of their fair output, then he is willing to sacrifice some of his firm's profits to increase his rivals' profits.

I find that if firms with reciprocal managers compete à la Cournot and the managers think that the fair output of their rivals is greater than the equilibrium output the rivals would produce if they only cared about maximizing profits, then firms attain a *constructive reciprocity* equilibrium. In such an equilibrium firms produce less than they would produce if their managers only cared about maximizing profits. This happens because a reciprocal manager wishes to reward his rivals for producing less than his perception of what the fair output of his rivals is. The reciprocal manager does that by reducing firm output.

Taking the perspective of an outside observer who is ignorant about managers' preferences for reciprocity, a constructive reciprocity equilibrium is indistinguishable from a "collusive" outcome.[1] Thus, a constructive reciprocity equilibrium is good for firms since it leads to higher profits than the profits of firms with managers who only care about maximizing profits and in addition it provides managers payoff gains from being treated kindly. A constructive reciprocity equilibrium is bad for consumers since it reduces quantity and raises market price.

However, if firms with reciprocal managers compete à la Cournot and their managers think that the fair output of their rivals is smaller than the equilibrium output the rivals would produce if they only cared about maximizing profits, then firms end up in a *destructive reciprocity* equilibrium. In such an equilibrium

[1] Throughout the paper I consider that collusive outcomes describe situations where firms produce less than the Cournot-Nash quantities of firms whose managers only care about maximizing profits.

firms produce more than they would produce if their managers only cared about maximizing profits. This happens because a reciprocal manager wishes to punish his rivals for producing more than his perception of what the fair output of his rivals is. The reciprocal manager does that by increasing firm output.

A destructive reciprocity equilibrium is bad for firms since it leads to lower profits than the profits of firms with managers who only care about maximizing profits and in addition it makes managers incur payoff loses from being treated unkindly. This equilibrium is good for consumers since it increases quantity and reduces market price. If one ignores managers' preferences for reciprocity, a destructive reciprocity equilibrium is indistinguishable from Stackelberg warfare.[2]

The paper proceeds by studying the impact of inequity aversion on Cournot competition. I assume that an inequity averse manager cares about maximizing profits and, in addition, likes to reduce the difference between his firm's profits and the rivals' profits. More specifically, such a manager is assumed to feel compassion towards his rivals (aversion to advantageous inequity) when the average profits of his rivals are smaller than his firm's profits and envy towards his rivals (aversion to disadvantageous inequity) when the average profits of his rivals are greater than his firm's profits.

The paper shows that the set of Nash equilibria of Cournot competition between firms with inequity averse managers changes monotonically with compassion and envy. If there is quantity competition and managers' degree of envy increases, then the largest Nash equilibria of the Cournot game moves closer to the perfectly competitive outcome.[3] By contrast, if there is quantity competition and managers' degree of compassion increases, then the smallest Nash equilibria of the Cournot game moves closer to the best collusive outcome.

I also find that Fehr and Schmidt's (1999) piecewise linear specification of inequity aversion can change the strategic interaction between firms' choice variables in Cournot competition: quantities become strategic complements over intermediate output levels. I show that this gives rise to a continuum of equilibria. However, as the number of firms grows the impact of piecewise inequity aversion on the set of Nash equilibria vanishes.

Finally, the paper shows that only under very restrictive conditions will preferences for reciprocity or inequity aversion have an impact on Bertrand competition. For example, when two firms with inequity averse managers engage in Bertrand competition and marginal costs are constant, only if both managers are willing to give up more than one dollar of their profit to raise the average profit of their opponents by a dollar, can there be an equilibrium where price is above marginal cost.

This paper is related to a recent strand of literature in economics that studies the consequences of relaxing the assumption of individual greed.[4] The paper is

[2]I consider that Stackelberg warfare describes situations where firms produce more than the Cournot-Nash outputs of firms whose managers only care about maximizing profits.

[3]A similar result has also been found in a different context. Demougin and Fluet (2003) show that in a rank order tournament the principal is better off when agents are envious than when they are compassionate.

[4]Rabin (1993) introduces fairness considerations into game theory. Englmaier and

also related to literature in industrial organization that analyzes how firms will choose prices and product characteristics when managers have certain behavioral biases. An example is Al-Najjar et al. (2006) work on the pricing decision of firms whose managers confound fixed, sunk and variable costs.

The impact of interdependent preferences on strategic interactions between firms in imperfectly competitive markets has not received much attention. The only exception is Bolton and Ockenfels (2000) who find that their model of inequity aversion has no impact on Cournot and Bertrand competition. In this paper I find that both preferences for reciprocity as well as inequity aversion can change the outcome of Cournot competition. These findings stand in contrast to those in Bolton and Ockenfels (2000).[5]

Several papers have explored the implications of interdependent preferences in perfectly competitive markets. For example, Segal and Sobel (2004) show that interdependent preferences have no impact on equilibria of auction-style environments.[6] In this paper I show that only under very restrictive conditions do preferences for reciprocity or inequity aversion change the outcome of Bertrand competition. This is consistent with previous literature.

The paper proceeds as follows. Section 2 discusses ways of modeling reciprocity and inequity aversion. Section 3 studies Cournot competition between reciprocal managers. Section 4 studies Cournot competition between inequity averse managers. Section 5 studies Bertrand price competition with reciprocal and inequity averse managers. Section 6 discusses how inequity aversion can explain behavior in experimental endogenous timing games and how preferences for reciprocity can facilitate collusion in dynamic oligopolies. Section 7 concludes the paper. All proofs are in the Appendix.

2 Interdependent Preferences

Many experiments show that individuals are willing to incur losses to punish those who treat them unkindly or to reward those who treat them kindly. This type of behavior is called preferences for reciprocity.[7] A person with this type of preferences cares about the intentions behind the actions of their opponents but is not bothered by unfair payoff distributions. Experiments also find that many individuals are willing to give up some material payoff to move in the direction of more equitable distributions of payoffs. This type of behavior is

Wambach (2002) study optimal contracts when the agent suffers from being better off or worse off than the principal. Biel (2003) studies how the optimal incentive contract in team production is affected when workers are averse to inequity. Sappington (2004) studies inequity aversion in adverse selection contexts.

[5] Section 5 compares the findings in the two papers.

[6] Bolton and Ockenfels (2000) and Fehr and Schmidt (1999) show how the competitive prediction of the ultimatum game with many proposers and one responder studied by Prasnikar and Roth (1992) continues to hold under the assumption that some individuals in the population care about inequity aversion.

[7] Preferences for reciprocity are also called preferences for process or intentions based fairness.

called inequity aversion.[8]

Preferences for reciprocity were first modeled in the economics literature by Rabin (1993) using the theory of psychological games of Geanakoplos et al. (1989). In Rabin's model the weight a firm places on a rival's monetary profits depends on the interpretation of that rival's intentions which are evaluated using beliefs (and beliefs about beliefs) over strategy choices. This approach provides a model of preferences for reciprocity since a firm places a positive weight on a rivals' profit when the firm thinks that the behavior of the rival is nice and negative if it thinks that the behavior is nasty.

Inequity aversion theories assume that individuals are concerned about their own material payoff but also the consequences of their acts on payoff distributions. An inequity averse person cares about the distribution of payoffs but it does not care about the intentions that lead others to choose certain actions. There are two main theories of inequity aversion: Fehr and Schmidt's (1999) and Bolton and Ockenfels (2000). According to Fehr and Schmidt's (1999) model a player cares about his own payoff and dislikes absolute payoff differences between his own payoff and the payoff of any other player.[9] According to Bolton and Ockenfels's (2000) "Theory of Equity, Reciprocity, and Competition" (henceforth ERC) a player is concerned with both his own payoff and his relative share of the total group payoff.[10]

Preferences for reciprocity and inequity aversion have been shown to explain behavior in bargaining games and in trust games.[11] For example, in ultimatum games offers are usually much more generous than predicted by equilibrium and low offers are often rejected. These offers are consistent with an equilibrium in which players make offers knowing that other players may reject allocations that appear unfair.[12]

Segal and Sobel (1999) provide an axiomatic foundation for interdependent preferences that can reflect reciprocity, inequity aversion, altruism as well spitefulness.[13] They assume that in addition to conventional preferences over outcomes, players in a strategic environment also have preferences over strategy profiles. Their representation theorem shows that the payoff function of a firm

[8] Inequity aversion is sometimes called preference for outcome based fairness.

[9] Neilson (2000) provides an axiomatic characterization of the Fehr and Schmidt (1999) model of inequity aversion.

[10] According to ERC, a player would be equally happy if all players received the same payoff or if some were rich and some were poor as long as he received the average payoff, while according to Fehr and Schmidt (1999) he would clearly prefer that all players get the same.

[11] Camerer (2003) and Sobel (2005) provide excellent reviews of this literature.

[12] Sobel (2005) argues convincingly that models of interdependent preferences such as reciprocity can provide clearer and more intuitive explanations of interesting economic phenomena.

[13] Some individuals also display altruism and others spitefulness. An altruistic person is willing to increase the payoff of his opponents at a personal cost to himself, irrespective of the payoff distribution and irrespective of the behavior of the opponents. A spiteful person is willing to decrease the payoff of his opponents at a personal cost to himself, irrespective of the payoff distribution and irrespective of the behavior of the opponents. I will not study the impact of altruism and spitefulness on oligopolistic competition.

with such preferences is of the form

$$U_i(O(s_i, s^*_{-i})) = u_i(O(s_i, s^*_{-i})) + \sum_{j \neq i} w_{ij}(s_i, s^*_{-i}) u_j(O(s_i, s^*_{-i})), \tag{1}$$

where s_i is the strategy of player i, s^*_{-i} is the strategy that the rest of the players are playing, u_i is the utility from outcomes of player i, u_j is the utility from outcomes of player $j \neq i$, and w_{ij} is a coefficient that measures the weight player i gives to player j's utility, which is a function of the entire strategy profile. Positive values of the coefficient w_{ij} mean that player i is willing to sacrifice his payoff from outcomes in order to increase the payoff of player j. Negative values mean that player i is willing to sacrifice his payoff from outcomes in order to lower player j's payoff. Since the coefficient w_{ij} depends on the strategy chosen by player j, there is scope to model reciprocity.[14]

In this paper I apply Segal and Sobel's (1999) approach since using psychological games would complicate the analysis substantially without providing additional insights into the problem.

3 Reciprocity and Cournot Competition

Let $N = \{1, 2, \dots, n\}$ denote the set of firms. Let price be determined according to the inverse demand function $P(Q)$, where $Q = \sum q_i$. I make the standard assumption that $P(Q)$ is strictly positive on some bounded interval $(0, \bar{Q})$ with $P(Q) = 0$ for $Q \geq \bar{Q}$. I also assume that $P(Q)$ is strictly decreasing in the interval for which $P(Q) > 0$. Firms have costs of production given by $C_i(q_i)$. Firms costs of production are assumed to be increasing. To incorporate preferences for reciprocity I assume that the payoff function of the manager of firm i is given by

$$U_i(q_i, Q_{-i}) = \pi_i(q_i, Q_{-i}) + w_i\left(Q_{-i}, Q^F_{-i}\right) \sum_{j \neq i} \pi_j(q_i, Q_{-i}),$$

where $\pi_i(q_i, Q_{-i})$ is profits of firm i and $w_i\left(Q_{-i}, Q^F_{-i}\right)$ is the weight that manager i places on its rivals aggregate profits $\sum_{j \neq i} \pi_j(q_i, Q_{-i})$. As usual, firm i's profits depend on firm i's output, q_i, and on the aggregate output of its rivals, Q_{-i}, such that

$$\pi_i(q_i, Q_{-i}) = R_i\left(q_i, Q_{-i}\right) - C_i(q_i),$$

where $R_i(q_i, Q_{-i}) = P\left(Q\right) q_i$ is revenue. I assume that the weight manager i places on its rivals aggregate profits depends on his perception of the fair aggregate output of his rivals, Q^F_{-i}, and on the aggregate output of his rivals. Furthermore, I assume that

$$w_i(Q_{-i}, Q^F_{-i}) \begin{cases} > 0 \text{ if } Q_{-i} < Q^F_{-i}, \\ = 0 \text{ if } Q_{-i} = Q^F_{-i} \\ < 0 \text{ otherwise} \end{cases}, \tag{2}$$

[14] The underlying preferences in (1) are defined over outcomes. If an outcome specifies a material payoff to each player, then it is permissible for u_i to depend on other players' material payoffs. Thus, this approach also generalizes the inequity aversion approach.

that is, the manager of firm i places a positive weight on his rivals aggregate profits when the rivals produce less than Q_{-i}^F, he places no weight on his rivals profits when the rivals produce Q_{-i}^F, and he places a negative weight on his rivals profits when the rivals produce more than Q_{-i}^F. These conditions capture the fact that a reciprocal manager cares about the intentions of his rivals.

The first condition expresses constructive reciprocity. If manager i expects the aggregate output of his rivals to fall short of his own perception of the fair aggregate output of the rivals, then manager i is willing to sacrifice some of firm i's profits to increase the rivals' profits. The third condition expresses destructive reciprocity. When manager i expects his rivals to produce more than his perception of the fair aggregate output of the rivals, then manager i is willing to sacrifice some of firm i's profits to reduce the rivals' profits.[15]

I assume throughout that managers' preferences for reciprocity as well as perceptions of the fair aggregate of the rivals are common knowledge. The problem of manager i is to maximize its payoff function taking the quantities produced by its rivals as given, that is, manager i solves the following problem

$$\max_{q_i} U_i(q_i, Q_{-i}) = \pi_i(q_i, Q_{-i}) + w_i\left(Q_{-i}, Q_{-i}^F\right) \sum_{j \neq i} \pi_j(q_i, Q_{-i}).$$

The best reply to Q_{-i} is given by

$$r_i^R(Q_{-i}) = \arg_{q_i} \max P\left(Q\right) q_i - C_i(q_i) + w_i(Q_{-i}, Q_{-i}^F) \sum_{j \neq i} \left[P\left(Q\right) q_j - C_j(q_j)\right]. \tag{3}$$

Let $q^F = (Q_{-1}^F, Q_{-2}^F, \ldots, Q_{-n}^F)$ denote the vector of managers' perceptions of the fair aggregate output of their rivals. Let the n-firm Cournot oligopoly with reciprocal managers be denoted by $\Gamma^R(U, w, q^F)$. To begin the analysis I need to guarantee existence of equilibrium of $\Gamma^R(U, w, q^F)$.

There are four types of existence results which may apply to the Cournot model. The first type of result uses the standard existence theorem due to Nash and shows that every n-firm Cournot oligopoly has a Nash equilibrium if each firm's payoff is quasiconcave in q_i.[16]

The second type of result, due to Bamon and Frayssé (1985) and Novshek (1985), shows that every n-firm Cournot oligopoly has a Nash equilibrium if each firm's payoff depends on other firms' outputs only via their sum and marginal revenue is a decreasing function of the aggregate output of all other firms.

The third type of result deals with cases in which the Cournot game is a supermodular game. Here there are two different types of results, one for $n = 2$ and another one for $n \geq 2$. Milgrom and Roberts (1990) show that if the natural order on of one of the firms' action sets is reversed, then the Cournot duopoly is a supermodular game.[17] Amir (1996) provides conditions

[15] Weighting functions that satisfy condition (2) arise naturally. For example, $w_i(Q_{-i}, Q_{-i}^F) = \alpha(Q_{-i}^F - Q_{-i})$, $w_i(Q_{-i}, Q_{-i}^F) = \alpha(Q_{-i}^F - Q_{-i})^3$, or $w_i(Q_{-i}, Q_{-i}^F) = \alpha\left(Q_{-i}^F / Q_{-i} - 1\right)$, with $\alpha > 0$.

[16] This existence result is quite restrictive. See Ch. 4 in Vives (2001).

[17] This argument breaks down when there are three or more firms.

under which the n-firm Cournot oligopoly is a log-supermodular game. However, under these conditions, best replies are increasing which is not considered to be the "normal" case in Cournot games. Finally, Tarsky (1955), McManus (1962, 1964), and Roberts and Sonnenschein (1977), show that every n-firm symmetric Cournot oligopoly has a Nash equilibrium if cost functions are convex.

My goal is not only to prove existence of equilibria for the Cournot game with reciprocal managers but also to state comparative static results. The assumptions required to state each of the four existence results imply different trade-offs between generality in existence versus generality in comparative static results. I decide to focus on the Cournot duopoly case and treat it as a supermodular game. However, to provide intuition for some of the results I will often use the n-firm smooth version of the Cournot oligopoly game with quasiconcave and differentiable payoff functions.

My first result guarantees that the Cournot duopoly game with reciprocal managers is a supermodular game.

Lemma 1: *If $n = 2$ and U_i has decreasing differences in (q_i, Q_{-i}), then $\Gamma^R(U, w, q^F)$ is a supermodular game.*

The assumption that the payoff function has decreasing differences in (q_i, Q_{-i}) means that the marginal returns to a manager from increasing output are lower if the rivals produce a higher output. Note that if managers care about profits, then the requirement that π_i has decreasing differences in (q_i, Q_{-i}) boils down to the assumption that the revenue of firm i has decreasing differences in (q_i, Q_{-i}). However, if managers have preferences for reciprocity, then the requirement that U_i has decreasing differences in (q_i, Q_{-i}) also implies that the weight that manager i places on the payoff from reciprocity can not be too large by comparison to the weight he places on firm i's profits.

The best way to illustrate this point is to refer to a smooth version of the n-firm Cournot oligopoly game with reciprocal managers.[18] In that game the condition that U_i has decreasing differences in (q_i, Q_{-i}) is equivalent to requiring that

$$\frac{\partial^2 U_i}{\partial q_i \partial Q_{-i}} = P'(Q) + P''(Q)\, q_i + \partial \left\{ w_i \left(Q_{-i}, Q_{-i}^F \right) P'(Q)\, Q_{-i} \right\} / \partial Q_{-i} < 0.$$

This condition is satisfied if the decreasing marginal revenue property holds, that is, $P'(Q) + P''(Q)\, q_i < 0$, and if the impact of a change in rivals' output on manager i's marginal payoff from reciprocity is relatively small by comparison with its impact on marginal revenue, that is $\partial \left\{ w_i \left(Q_{-i}, Q_{-i}^F \right) P'(Q)\, Q_{-i} \right\} / \partial Q_{-i} < |P'(Q) + P''(Q)\, q_i|$.

[18] In the smooth version of the n-firm Cournot oligoply game $P(Q)$ is twice continuously differentiable with $P'(Q) < 0$ (in the interval for which $P(Q) > 0$) and that the decreasing marginal revenue property holds, that is, $P'(Q) + P''(Q)\, q_i \leq 0$. Firms costs of production are assumed to twice continuously differentiable with $C_i' \geq 0$. The function $w_i(Q_{-i}, Q_{-i}^F)$ is assumed to be differentiable in both arguments with $\partial w_i / \partial Q_{-i} < 0$ and $\partial w_i / \partial Q_{-i}^F > 0$.

Thus, if preferences for reciprocity are very important relative to profits, then quantities may be strategic complements over some output ranges and strategic substitutes over others. If that happens, then I can no longer use the theory of supermodular games to state general results that characterize the impact of reciprocity on Cournot competition. Lemma 1 rules out this possibility.

If $\Gamma^R(U, w, q^F)$ is a supermodular game, then it follows from Topkis (1979), that the equilibrium set is non-empty and has a smallest and a largest pure-strategy Cournot-Nash equilibrium.[19] The next result shows how managers' perceptions of the fair output of their rivals change the outcome of Cournot competition.

Proposition 1 *If $n = 2$, $\Gamma^R(U, w, q^F)$ is a supermodular game, and U_i has decreasing differences in $\left(q_i, Q^F_{-i}\right)$, then the smallest and the largest Cournot-Nash equilibria of $\Gamma^R(U, w, q^F)$ are nonincreasing functions of q^F.*

This result tells us that if the weight that managers place on reciprocity is relatively small by comparison to the weight they place on profits and the marginal returns from increasing output are decreasing with managers' perceptions of the fair output of their rivals, then the higher are managers' perceptions of the fair output of their rivals the lower is the set of Cournot-Nash equilibria.[20]

The intuition behind this result is straightforward. The assumption that the manager's payoff function has decreasing differences in (q_i, Q^F_{-i}) means that the larger a reciprocal manager perceives the fair output of their rivals to be, the smaller are the marginal returns from increasing production.[21] Thus, an increase in Q^F_{-i} shifts the best reply of a reciprocal manager i towards the origin. In other words, the more manager i perceives the fair output of his rivals to be high, the more he is willing to produce a smaller output level for any output level of the rivals. If this happens for every manager in every firm, then the higher are managers' perceptions of the fair output of their rivals the lower will be the set of Cournot-Nash equilibria.

Proposition 1 is a comparative statics result that characterizes the impact that mangers' perceptions of the fair output of their rivals have on equilibrium quantities of Cournot competition. I am also interested in comparing the outcome of Cournot competition among firms with reciprocal managers to that of Cournot competition among firms with managers who only care about maxi-

[19] Thes assumption that U_i has decreasing differences in (q_i, Q_{-i}) guarantees that best replies are decreasing and this implies existence of equilibrium.

[20] Note that this result does not imply that all Nash equilibria of $\Gamma(U, w, q^F)$ are nonincreasing functions of q^F. In fact we may have that a Nash equilibrium in the interior of the set of Nash equilibria of $\Gamma(U, w, \bar{q}^F)$ may be higher than the correspondent Nash equilibrium in the interior of the set of Nash equilibria of $\Gamma(U, w, \hat{q}^F)$ with $\hat{q}^F$ higher than $\bar{q}^F$. Still, a decrease in equilibrium output can be justified by a coordination argument since the smallest Cournot-Nash equilibrium is the most preferred equilibrium for firms whereas the largest equilibrium is the less preferred one.

[21] In the smooth version of the n-firm Cournot oligopoly game with reciprocal managers the condition that U_i has decreasing differences in (q_i, Q^F_{-i}) is equivalent to the requirement that $\partial^2 U_i / \partial q_i \partial Q^F_{-i} < 0$. In that game we have that $\partial^2 U_i / \partial q_i \partial Q^F_{-i} = \left(\partial w_i / \partial Q^F_{-i}\right) P'(Q) Q_{-i}$. Since $P'(Q) < 0$ and $Q_{-i} > 0$ the condition holds if $\partial w_i / \partial Q^F_{-i} > 0$.

mizing profits. To do that I compare the equilibria of game $\Gamma^S(\pi)$, the standard supermodular Cournot game with managers who only care about profits, to the equilibria of $\Gamma^R(U, w, q^F)$, the supermodular Cournot game with reciprocal managers. I assume that these two games are identical in all respects (market demand, costs, and number of firms) with the exception of managers' preferences. However, allowing for multiple equilibria makes the comparison cumbersome. Thus, I assume that the game $\Gamma^S(\pi)$ has decreasing differences in (q_i, Q_{-i}), and that best replies have a slope greater than -1.[22] It is a well known result that these two conditions guarantee that $\Gamma^S(\pi)$ has a unique equilibrium. Lemma 2 provides conditions under which the game $\Gamma^R(U, w, q^F)$ also has a unique equilibrium.

Lemma 2: *If $n = 2$, $\Gamma^R(U, w, q^F)$ is a supermodular game, and the managers' best replies have a slope greater than -1, then there exists an unique equilibrium of $\Gamma^R(U, w, q^F)$.*

This result guarantees that the supermodular Cournot game with reciprocal managers has a unique equilibrium. The condition that drives the result is the assumption that best replies have a slope strictly between $(-1, 0)$.[23] I am now ready to state the first result that compares the outcome of Cournot competition with reciprocal managers to that of Cournot competition with managers who only care about profits.

Proposition 2: *If $n = 2$, $\Gamma^S(\pi)$ is a supermodular game such that best replies have a slope greater than -1, $\Gamma^R(U, w, q^F)$ is a supermodular game such that*
(i) $U_i = \pi_i + w_i \sum_{j \neq i} \pi_j$,
(ii) U_i has decreasing differences in (q_i, Q^F_{-i}),

[22] In the smooth version of the standard n-firm Cournot oligopoly game these assumptions are satisfied if the decreasing marginal revenue property holds, marginal cost is increasing, and $P'(Q) - C_i''(q_i) < 0$, $i = 1, \dots, n$. Under these conditions the profit of firm i is strictly concave in q_i.This follows since $\partial^2 \pi_i / \partial q_i^2 = P'(Q) + P''(Q) q_i + P'(Q) - C_i''(q_i) < 0$. We also have that $\partial^2 \pi_i / \partial q_i \partial Q_{-i} = P'(Q) + P''(Q) q_i < 0$. It also follows that the best reply function of firm i is has its slope is in the interval $(-1, 0)$:

$$r_i'(Q_{-i}) = -\frac{\partial^2 \pi_i / \partial q_i \partial Q_{-i}}{\partial^2 \pi_i / \partial q_i^2} = -\frac{P'(Q) + P''(Q) q_i}{P'(Q) + P''(Q) q_i + P'(Q) - C_i''(q_i)}.$$

Theorem 2.8 in Vives (2001) shows that these conditions imply that the smooth version of the standard n-firm Cournot oligopoly game has a unique equilibrium.

[23] In the smooth version of the n-firm Cournot oligopoly game with reciprocal managers the slope of the best reply of firm i is given by $r_i'(Q_{-i}) = -\frac{\partial^2 U_i / \partial q_i \partial Q_{-i}}{\partial^2 U_i / \partial q_i^2}$, where

$$r_i'(Q_{-i}) = \frac{P'(Q) + P''(Q)\, q_i + \partial \left\{ w_i(Q_{-i}, Q^F_{-i}) P'(Q)\, Q_{-i} \right\} / \partial Q_{-i}}{P'(Q) + P''(Q)\, q_i + P'(Q) - C''(q_i) + w_i(Q_{-i}, Q^F_{-i}) P''(Q)\, Q_{-i},}.$$

The slope is strictly above -1 if

$$\left| \partial \left\{ w_i(Q_{-i}, Q^F_{-i}) P'(Q)\, Q_{-i} \right\} / \partial Q_{-i} \right| < \left| P'(Q) - C''(q_i) + w_i(Q_{-i}, Q^F_{-i}) P''(Q)\, Q_{-i} \right|.$$

This condition implies that the game has a unique equilibrium by Theorem 2.8 in Vives (2001).

(iii) the managers' best replies have a slope greater than -1, *and*
(iv) $Q^F_{-i} = Q^{NS}_{-i}$ *for all* i,
then the Nash equilibrium of $\Gamma^S(\pi)$ *coincides with that of* $\Gamma^R(U, w, q^F)$.

Proposition 2 shows that if firms with reciprocal managers compete à la Cournot and managers perceive the fair output of their rivals to be equal to the output that the rivals would produce if they only cared about profits, then they will produce the same quantities as the ones produced by managers who only care about profits. In this case preferences for reciprocity just pivot managers' best replies around the Cournot-Nash outcome of the game played between managers who only care about profits and so the equilibrium is left unchanged. In this case market output, consumer welfare, and profits are the same with reciprocal managers or with managers who only care about profits.

To clarify the intuition Proposition 2 I refer to the smooth n-firm Cournot oligopoly game with reciprocal managers. In that game the best reply of manager i to Q_{-i} is implicitly defined by the first-order condition

$$\frac{\partial U_i}{\partial q_i} = P(Q) + P'(Q)\, q_i - C_i'(q_i) + w_i(Q_{-i}, Q^F_{-i}) P'(Q)\, Q_{-i} = 0. \tag{4}$$

It is straightforward to interpret this condition. The term $P(Q) + P'(Q)\, q_i$ represents marginal revenue and the term $C_i'(q_i)$ marginal cost. These two terms represent the impact that a change in q_i has on firm i's profit.[24] The novelty here is the term $w_i(Q_{-i}, Q^F_{-i}) P'(Q)\, Q_{-i}$. This term represents the impact that a change in q_i has on manager i's payoff from reciprocity.

It follows from (4) that the best reply of a reciprocal manager i intercepts the best reply of a manager who only cares about profits at $Q_{-i} = Q^F_{-i}$. This happens because $Q_{-i} = Q^F_{-i}$ implies $w_i(Q_{-i}, Q^F_{-i}) = 0$ and (4) reduces to $MR_i = MC_i$ which implies that market output is the same with reciprocal managers or with managers who only care about maximizing profits.

Proposition 2 tells us that a critical condition for the Cournot-Nash equilibrium of the game with reciprocal managers to differ from the Cournot-Nash equilibrium of the game with managers who only care about profits is that reciprocal managers' perceptions of the fair output of their rivals are different from the equilibrium output of the rivals when managers only care about profits. The next result explores the implications of this possibility.

Proposition 3: *If* $n = 2$, $\Gamma^S(\pi)$ *is a supermodular game such that best replies have a slope greater than* -1, $\Gamma^R(U, w, q^F)$ *is a supermodular game such that*
(i) $U_i = \pi_i + w_i \sum_{j \neq i} \pi_j$,
(ii) U_i *has decreasing differences in* (q_i, Q^F_{-i}),
(iii) the managers' best replies have a slope greater than -1, *and*
(iv) $Q^F_{-i} > (<) Q^{NS}_{-i}$ *for all* i,
then the Nash equilibrium of $\Gamma^S(\pi)$ *is greater (smaller) than that of* $\Gamma^R(U, w, q^F)$.

[24] In the standard smooth n-firm Cournot oligopoly game the best reply of firm i to Q_{-i} is the unique solution to the first-order condition $\partial \pi_i / \partial q_i = P(Q) + P'(Q)\, q_i - C_i'(q_i) = 0$.

Proposition 3 tells us that if reciprocal managers perceive the fair output of their rivals to be greater than the equilibrium output that the rivals would produce if all managers only cared about profits, then reciprocal managers will produce less than managers who only care about profits. This is the constructive reciprocity equilibrium. On the other hand, if reciprocal managers perceive the fair output of their rivals to be smaller than the equilibrium output that the rivals would produce if all managers only cared about profits, then reciprocal managers will produce more than managers who only care about profits. This is the destructive reciprocity equilibrium.

In a constructive reciprocity equilibrium market output is smaller than the one of Cournot competition with managers who only care about profits. Thus, consumers are worse off if reciprocal managers' perceptions of fairness lead to a constructive reciprocity equilibrium than if managers only care about maximizing profits. The opposite happens in a destructive reciprocity equilibrium: market output is larger than that in the equilibrium of the Cournot game with managers who only care about profits and consumers are better off.

The intuition behind the constructive reciprocity equilibrium can be illustrated by (4). If a reciprocal manager i expects his rivals to produce an equilibrium output smaller than Q_{-i}^{F}, then his best reply is to produce a smaller amount than the one he would produce if he only cared about profits. This happens because if $Q_{-i} < Q_{-i}^{F}$, then manager i places a positive weight on his rivals profits and this implies that $w_i(Q_{-i}, Q_{-i}^{F})P'(Q)\,Q_{-i} < 0$. In this case, if manager i produces less than the best reply of a manager who only cares about profits, then he has a first-order gain in payoff from constructive reciprocity (he increases the profits of his opponents) and a second-order loss in profits (he reduces the profits of his firm). Manager i will reduce production until the difference between marginal revenue and marginal cost equals the marginal payoff from constructive reciprocity.

The intuition behind the destructive reciprocity equilibrium can also be illustrated by (4). If a reciprocal manager i expects his rivals to produce an equilibrium output greater than Q_{-i}^{F}, then his best reply is to produce a larger amount than the one he would produce if he only cared about profits. This happens because if $Q_{-i} > Q_{-i}^{F}$, then manager i places a negative weight on his rivals profits, that is, $w_i(Q_{-i}, Q_{-i}^{F}) < 0$. This in turn implies that $w_i(Q_{-i}, Q_{-i}^{F})P'(Q)\,Q_{-i} > 0$. If this is the case, then (4) is not satisfied if manager i would produce the best reply of a manager who only cares about profits since then we would have $MR_i - MC_i = 0$ but $w_i(Q_{-i}, Q_{-i}^{F})P'(Q)\,Q_{-i} > 0$. In fact, if manager i produces slightly more than the best reply of a manager who only cares about profits he has a first-order gain in payoff from destructive reciprocity (he reduces the profits of his rivals) and a second-order loss in material payoff (he reduces the profits of his firm). Manager i will increase production until the difference between marginal revenue and marginal cost equals the marginal payoff from destructive reciprocity.

4 Inequity Aversion and Cournot Competition

Another important type of interdependent preferences is inequity aversion. To study the impact of inequity aversion on Cournot competition I assume that manager i's payoff function is additively separable in firm i's profits and the profits of his rivals, that is

$$U_i(\pi_i, \pi_{-i}) = \pi_i + \sum_{j \neq i} \lambda_{ij}(\pi_j - \pi_i),$$

where λ_{ij} is a function that measures how differences in profits between firm j and firm i have an impact on the weight that manager i puts on firm j's profits.[25] Furthermore, I assume that

$$\lambda_{ij}(\pi_j - \pi_i) \begin{cases} > 0 \text{ if } \pi_j < \pi_i \\ = 0 \text{ if } \pi_j = \pi_i \\ < 0 \text{ otherwise} \end{cases}, \tag{5}$$

that is, manager i places a positive weight on firm j's profits when j's profits are smaller than those of firm i, he places no weight on j's profits when j's profits are equals to those of firm i, and he places a negative weight on j's profits when j's profits are greater than those of firm i. These conditions capture the fact that an inequity averse manager cares about the distribution of profits. The first condition expresses aversion to advantageous inequity. If firm i's profits are greater than those of firm j then manager i is willing to sacrifice some of firm i's profits to increase firm j's profits. The last third condition expresses aversion to disadvantageous inequity. If firm i's profits are smaller than those of firm j then manager i is willing to sacrifice some of firm i's profits to reduce firm j's profits.

The problem of manager i is to maximize his payoff function taking the quantities produced by the other firms as given and taking into consideration the impact of its output choice on the distribution of profits, that is

$$\max_{q_i} U_i(q_i, Q_{-i}) = \pi_i(q_i, Q_{-i}) + \sum_{j \neq i} \lambda_{ij}(\pi_j(q_i, Q_{-i}) - \pi_i(q_i, Q_{-i})).$$

The best reply of an inequity averse manager i to Q_{-i} is given by

$$r_i(Q_{-i}) = \arg_{q_i} \max \pi_i(q_i, Q_{-i}) + \sum_{j \neq i} \lambda_{ij}(\pi_j(q_i, Q_{-i}) - \pi_i(q_i, Q_{-i})).$$

I assume that the game is smooth and symmetric.[26] Furthermore, I start the

[25] Neilson (2006) offers a full axiomatic characterization of this payoff function.

[26] It is hard to state general results that characterize the impact of inequity aversion on Cournot competition for asymmetric games. In those games firms have different costs of production or different weight functions. If firms have different costs, then the most efficient firms will produce more output and have higher profits and the less efficient firms will produce less output and have lower profits. This implies that the most efficient firms will fell compassion toward the less efficient firms and the less efficient firms will feel envy toward the most efficient firms. This may lead the most efficient firms to produce less than in a game with selfish firms and the less efficient firms to produce more. Thus, it is not clear how aggregate output will change in an asymmetric Cournot oligopoly game when firms are averse to inequality in payoffs.

analysis by assuming that λ_{ij} is twice differentiable. If that is the case, then I can write the first-order condition to manager i's optimization problem as

$$\frac{\partial U_i}{\partial q_i} = \frac{\partial \pi_i}{\partial q_i} + \sum_{j \neq i} \lambda'_{ij}(\pi_j - \pi_i)\left(\frac{\partial \pi_j}{\partial q_i} - \frac{\partial \pi_i}{\partial q_i}\right) = 0, \tag{6}$$

where $\partial \pi_i / \partial q_i = P(Q) + P'(Q)\, q_i - C'_i(q_i)$ and $\partial \pi_j / \partial q_i = P'(Q)\, q_j$, for all $j \neq i$. To guarantee that the first-order condition is the solution to firm i's the problem I also assume that the payoff function is strictly concave in q_i.

Lemma 3: *If*

$$\left|\sum_{j \neq i} \lambda''_{ij}(\pi_j - \pi_i)\left(\frac{\partial \pi_j}{\partial q_i} - \frac{\partial \pi_i}{\partial q_i}\right)^2 + \sum_{j \neq i} \lambda'_{ij}(\pi_j - \pi_i)\left(\frac{\partial^2 \pi_j}{\partial q_i^2} - \frac{\partial^2 \pi_i}{\partial q_i^2}\right)\right| \leq \left|\frac{\partial^2 \pi_i}{\partial q_i^2}\right|, \tag{7}$$

then there exists an equilibrium of the symmetric n-firm symmetric Cournot game with inequity averse managers and the equilibrium is unique.

Condition (7) guarantees that the payoff function of firm i is strictly concave in q_i. This guarantees existence of equilibrium. The assumption that the game is symmetric together with condition (7) imply that the equilibrium is unique. Let $q^{NI} = (q_1^{NI}, \ldots, q_n^{NI})$ denote the Nash equilibrium strategy profile of the n-firm Cournot game with inequity averse firms. I can now state the following result.

Proposition 4: *In the n-firm smooth and symmetric Cournot game with inequity averse managers if*
(i) $\lambda_{ij}(\pi_j - \pi_i)$ satisfies (5) and $\lambda'_{ij}(0) = 0$ for all i and j, then $q^{NI} = q^{NS}$;
(ii) $\lambda_{ij}(\pi_j - \pi_i)$ satisfies (5) and $\sum_{j \neq i} \lambda'_{ij}(0) < 0$ for all i, then $q^{NI} > q^{NS}$.

Proposition 4 provides conditions under which differentiable specifications of inequity aversion will or will not change the equilibrium outcome of smooth and symmetric Cournot games. Part (i) shows that the equilibrium strategy profile of the symmetric Cournot game with inequity averse managers coincides with that of the Cournot game with managers who only care about profits if the weighting function satisfies condition (5) and $\lambda'_{ij}(0) = 0$ for all i and j.[27] The intuition behind this result is as follows. The fact that the game is symmetric together with the assumption that $\lambda'_{ij}(0) = 0$, for all i and j, imply inequity aversion pivots the best reply of each manager around the Cournot-Nash equilibrium output of the game with managers who only care about profits. This type of inequity aversion changes the best reply functions of firms but does not change the equilibrium outcome of Cournot competition. Thus, when

[27] For example, the weighting function

$$\lambda_{ij}(\pi_j - \pi_i) = \begin{cases} -\alpha_{ij}(\pi_j - \pi_i)^2, & \text{if } \pi_j \geq \pi_i \\ \alpha_{ij}(\pi_j - \pi_i)^2, & \text{otherwise} \end{cases},$$

with $\alpha_{ij} > 0$, satisfies condition (5) $\lambda'_{ij}(0) = 0$ for all i and j.

$\lambda'_{ij}(0) = 0$ the market output with inequity averse managers is equal to the market output with manages who only care about profits.

Part (ii) shows that the equilibrium strategy profile of the symmetric Cournot game with inequity averse managers is greater than that of the game with managers who only care about profits if the weighting function satisfies condition (5) and $\sum_{j\neq i} \lambda'_{ij}(0) < 0$ for all i.[28] The fact that the game is symmetric together with the assumption that $\sum_{j\neq i} \lambda'_{ij}(0) < 0$ for all i imply inequity aversion pivots the best reply of manager i around the point $q^p \in r_i^S(Q_{-i})$ such that $\sum_{j\neq i} \lambda'_{ij}(\pi_j - \pi_i)\left(q_j^p - q_i^p\right) < 0$.[29] This type of inequity aversion changes both the best reply functions of firms as well as the equilibrium outcome of Cournot competition. We see that if $\sum_{j\neq i} \lambda'_{ij}(0) < 0$ for all i, then the market output with inequity averse managers is strictly greater than the market output with managers who only care about profits.[30]

Bolton and Ockenfels's (2000) were the first to study the impact of inequity aversion on equilibrium outcomes in oligopolistic markets. According to Bolton and Ockenfels's specification of inequity aversion the payoff function takes the form

$$U_i(\pi) = v\left(\pi_i, \frac{\pi_i}{\sum_{j=1}^n \pi_j}\right),$$

where the function v is assumed to be globally non-decreasing and concave in the first argument, to be strictly concave in the second argument (relative payoff), and to satisfy $v_2(\pi_i, 1/n) = 0$ for all π_i. Bolton and Ockenfels shows that this type of inequity aversion has no impact on equilibrium outcomes in symmetric Cournot games. Proposition 4 shows that Bolton and Ockenfels's result is driven by the assumption that $v_2(\pi_i, 1/n) = 0$ for all π_i.

I now consider the impact that Fehr and Schmidt's (1999) specification of inequity aversion has on equilibrium outcomes of Cournot competition.[31] Recall that under the assumptions made in this paper the n-firm smooth Cournot game has best reply functions with a negative slope, that is, quantities are strategic substitutes. I will now show that Fehr and Schmidt's (1999) form of inequity aversion makes quantities become strategic complements over intermediate output levels. I will also show that if managers with this type of preferences play Cournot games, then there can be a continuum of symmetric equilibria.

According to Fehr and Schmidt's (1999) specification manager i's payoff

[28] The weighting function $\lambda_{ij}(\pi_j - \pi_i) = -\alpha_{ij}\left[(\pi_j - \pi_i)^3 + (\pi_j - \pi_i)\right]$, with $\alpha_{ij} > 0$, satisfies condition (5) and $\lambda'_{ij}(0) = -\alpha_{ij} < 0$.

[29] We have that $-\sum_{j\neq i} \lambda'_{ij}(\pi_j - \pi_i)\left[P(Q) - C'_i(q_i)\right] = P'(Q)\sum_{j\neq i} \lambda'_{ij}(\pi_j - \pi_i)\,(q_j - q_i)$, with $\lambda'_{ij}(\pi_j - \pi_i) < 0$ for all π_j and π_i. Since $P(Q) - C'_i(q_i) > 0$ and $P'(Q) < 0$ the equality is satisfied if $\sum\limits_{j\neq i} \lambda'_{ij}(\pi_j - \pi_i)\,(q_j - q_i) < 0$.

[30] Assumption (5) and differentiability rule out the case where $\lambda'_{ij}(0) > 0$.

[31] Feher and Schmidt's specification is applied frequently to study the impact of inequity aversion on economic behavior.

function is given by

$$U_i(\pi_i, \pi_{-i}) = \pi_i - \left[\frac{\alpha_i}{n-1} \sum_{j \neq i} \max(\pi_j - \pi_i, 0) + \frac{\beta_i}{n-1} \max \sum_{j \neq i} (\pi_i - \pi_j, 0) \right]. \tag{8}$$

The terms in the square bracket are the payoff effects of compassion and envy, respectively. We see that if firm i's profits are greater than the average profits of its rivals then manager i feels compassion towards its rivals. However, if firm i's profits are smaller than the average profits of its rivals then manager i feels envious of his rivals.[32] This model of inequity aversion has piecewise linear indifference curves over a firm's own profits and its rivals' profits.

Manager i's inequity aversion towards its rivals is characterized by the pair of parameters (α_i, β_i), $i = 1, 2, \ldots, n$.[33] Manager i exhibits strict inequity aversion when both α_i and β_i are strictly greater than zero. Manager i only cares about maximizing profits when $\alpha_i = \beta_i = 0$. In all other cases manager is (weakly) averse to inequity. I assume that α_i and β_i, $i = 1, \ldots, n$, are common knowledge. Let $\alpha = (\alpha_1, \ldots, \alpha_n)$ and $\beta = (\beta_1, \ldots, \beta_n)$. My next result characterizes the best reply of a manager with piecewise linear inequity aversion.

Proposition 5: *The best reply of manager i in the n-firm Cournot game with piecewise linear inequity aversion is defined by*

$$r_i(Q_{-i}) = \begin{cases} s_i(Q_{-i}), & 0 \leq \frac{1}{n-1} Q_{-i} \leq q(\beta_i) \\ \frac{1}{n-1} Q_{-i}, & q(\beta_i) \leq \frac{1}{n-1} Q_{-i} \leq q(\alpha_i) \\ t_i(Q_{-i}), & q(\alpha_i) \leq \frac{1}{n-1} Q_{-i} \end{cases},$$

where

$$\begin{aligned} s_i(Q_{-i}) &= \arg\max_{q_i} (1 - \beta_i) \pi_i(q_i, Q_{-i}) + \frac{\beta_i}{n-1} \sum_{j \neq i} \pi_j(q_i, Q_{-i}), \\ t_i(Q_{-i}) &= \arg\max_{q_i} (1 + \alpha_i) \pi_i(q_i, Q_{-i}) - \frac{\alpha_i}{n-1} \sum_{j \neq i} \pi_j(q_i, Q_{-i}), \end{aligned}$$

$q(\beta_i)$ is the solution to $(1 - \beta_i)[P(nq) - C_i'(q)] + P'(nq)q = 0$, and $q(\alpha_i)$ is the solution to $(1 + \alpha_i)[P(nq) - C_i'(q)] + P'(nq)q = 0$.

[32] When there are only two firms in the market manager i's payoff function becomes

$$U_i(\pi_i, \pi_j) = \pi_i - [\alpha_i \max(\pi_j - \pi_i, 0) + \beta_i \max(\pi_i - \pi_j, 0)], \; i \neq j = 1, 2.$$

Fehr and Schmidt assume that the dislike of disadvantageous inequity is stronger than that of advantageous inequity, i.e. $\alpha_i > \beta_i$ and that β_i is smaller than 1. We make no assumptions about the relation between α_i and β_i but we assume, like Fehr and Schmidt, that β_i is smaller than 1.

[33] Alternatively, I could have assumed that manager i has different feelings of compassion and envy towards each rival. In this case we would have two inequity aversion parameters for each rival of each firm, that is, we would have α_{ij} and β_{ij} for $i \neq j = 1, \ldots, n$. I assume, like Ferh and Schmidt, that manager i feels the same degree of compassion and envy towards all rivals.

Proposition 5 characterizes the impact of piecewise linear inequity aversion on a manager's optimal output choice for any output levels of its rivals. It tells us that a manager's best reply is continuous like in the standard Cournot game However, the best reply of a manager with piecewise linear inequity aversion is no longer monotonic.

With piecewise linear inequity aversion the best reply of the manager has three different segments. When a firm's rivals produce low output levels the best response of an inequity averse manager has a negative slope and consists of a smaller output level than the output level of a manager who only cares about the profits of his firm. However, when a firm's rivals produce intermediate output levels the best response of an inequity averse manager has a positive slope and consists in producing the average output level of the rivals. Finally, when a firm's rivals produce high output levels the best response of an inequity averse manager has a negative slope and consists of a larger output level than the output level a manager who only cares about his firm's profits.

I am now ready to characterize the set of Nash equilibria of the n-firm symmetric Cournot oligopoly game when managers are averse to inequity in the sense of Fehr and Schmidt (1999). I do that in the next two results.

Proposition 6: *The unique Nash equilibrium of the n-firm symmetric Cournot game with managers who only care about the profits of their own firms is always an equilibrium of the n-firm symmetric Cournot game with piecewise linear inequity averse managers.*

Recall that, under the assumptions made, there is a unique equilibrium of the symmetric Cournot game with managers who only care about the profits of their own firms. In that equilibrium firms produce the same amount and the market price is between the perfectly competitive market price and the monopoly price. Proposition 6 shows that this equilibrium always belongs to the set of equilibria of the Cournot game with piecewise linear inequity aversion managers.

Proposition 7: *The set of Nash equilibria of the n-firm symmetric Cournot game with piecewise linear inequity averse managers is given by*

$$N^{IA} = \{(q_1, \ldots, q_n) : q_i = q_j,\ \forall i \neq j,\ \text{and}\ q(\beta) \leq q_i \leq q(\alpha),\ i = 1, \ldots, n\},$$

where $q(\beta) = \max\left[q\left(\beta_1\right), \ldots, q(\beta_n)\right]$, *and* $q(\alpha) = \min\left[q(\alpha_1), \ldots, q(\alpha_n)\right]$.

Proposition 7 tells us that if all managers are strictly averse to inequity, then there is a continuum of equilibria in the n-firm symmetric Cournot game with inequity averse managers.[34] It follows that in some of the equilibria of the Cournot game with piecewise linear inequity aversion, the market price may be lower than the equilibrium market price in the Cournot game with managers who only care about the profits of their own firms whereas in other

[34] The continuum of equilibria also exists when the game is not too asymmetric. However, when there are large cost asymmetries between firms there is a unique asymmetric equilibrium.

equilibria the market price may be higher. Thus, it is not clear whether piecewise linear inequity aversion is generally good or bad for consumers (or for firms).[35] However, we can state conditions under which piecewise linear inequity aversion is good or bad for consumers and firms. To do that I look at the impact of changes in the managers' degree of compassion and envy.

Proposition 8: *The largest Nash equilibria of the n-firm symmetric Cournot game with piecewise linear inequity averse managers is a nondecreasing function of α. The smallest Nash equilibria is a nonincreasing function of β.*

This welfare result characterizes the impact of compassion and envy on the set of Nash equilibria of the Cournot game with piecewise linear inequity averse managers. It tells us that there is a weak complementarity between the managers' degree of compassion and equilibrium output, that is, an increase in envy increases the market output produced in the largest Nash equilibria of the Cournot model with inequity averse managers. If that is the case, then an increase in the degree of envy reduces firms' profits and increases consumer surplus.

On the other hand, Proposition 8 tells us that an increase in compassion reduces the market output produced in the smallest Nash equilibria of the Cournot model with piecewise linear inequity averse managers. If that is the case, then an increase in the degree of compassion increases firms' profits and decreases consumer surplus.[36]

The next result studies the implications of an increase in the number of firms when there is quantity competition in markets where managers have piecewise linear inequity aversion. To state this result I assume that α_i and β_i, $i = 1, \ldots, n$, are drawn from a uniform distribution with support on $[0, 1]$.

Proposition 9: *As the number of firms increases the set of Nash equilibria of the n-firm symmetric Cournot game with piecewise linear inequity averse managers converges to the unique Nash equilibrium of n-firm symmetric Cournot game with manages who only care about the profits of their own firms.*

This result shows that increasing the number of firms reduces the impact of piecewise linear inequity aversion on the set of Nash equilibria of the n-firm Cournot game. This happens because when there are n firms, the smallest Nash equilibria of the game is determined by the firm that has the manager with the

[35] Proposition 7 also shows that if there is at least one manager who is not averse to inequity, then there is a unique equilibrium of the symmetric Cournot game with piecewise linear inequity aversion: the equilibrium of the symmetric Cournot game with managers who only care about profits of their own firms. This point has been made before in papers that study the implications of interdependent preferences in ultimatum games and in perfectly competitive markets. See Bolton and Ockenfels (2000), Fehr and Schmidt (1999), and Segal and Sobel (2004).

[36] This result is quite intuitive. In fact, Fehr and Schmidt's payoff function implies that if manager i has a higher monetary payoff than the average payoff of his opponents and $\beta_i = 1/2$, then manager i is just as willing to keep one dollar to himself as to give it to his rivals. Now, suppose that all managers have the same preferences as manager i. In this case managers are acting as if they are maximizing their joint profit, $\sum \pi_i$. So, if $\beta_i = 1/2$, with $i = 1, \ldots, n$, then compassion leads to the best collusive outcome.

lowest degree of compassion. Similarly, the largest Nash equilibria of the game is determined by the firm that has the manager with the lowest degree of envy.[37] If the levels of compassion and envy of each manager in each firm are drawn from a uniform distribution with support on $[0,1]$, then an increase in the number of firms makes it more likely that the lowest level of compassion as well as the lowest level of envy are both very close to zero. Thus, as the number of firms increases the smallest and the largest Nash equilibria of the n-firm symmetric Cournot game with inequity averse managers converges to that of the Cournot game with managers who only care about the profits of their own firms.[38]

5 Bertrand Competition

In the standard model of Bertrand competition firms select independently the price for the product and every firm has the commitment to supply whatever demand is forthcoming at the price it sets. Demand is strictly downward-sloping when positive, cutting both axes, and firms have increasing cost functions $C_i(q_i)$. Firms that set the lowest price split the demand and the remaining firms do not sell anything. That is, given a vector of prices $(p_i)_{i\in N}$ the sales of firm i are

$$q_i = \begin{cases} \frac{D(p_i)}{l}, & \text{if } \; p_j \geq p_i, \forall j \in N \\ 0, & \text{otherwise} \end{cases},$$

where $l = \#\{j \in N : p_j = p_i\}$. It is a well know result that equilibrium outcomes of Bertrand competition depend on the shape of the cost function. If marginal costs are constant and identical, then the only equilibrium is one where all firms set price equal to marginal cost, have zero profits, and split the market demand equally.[39]

Here I will focus on the impact of reciprocity and inequity aversion on Bertrand competition with constant and identical marginal costs. I start by extending the model by allowing managers to have preferences for reciprocity. I assume that the payoff of manager i becomes

$$U_i(p_i, p_{-i}) = \pi_i(p_i, p_{-i}) + \sum_{j\neq i} w_{ij}\left(p_j, p^F\right)\pi_j(p_i, p_{-i}),$$

where

$$w_{ij}(p_j, p^f) \begin{cases} > 0 \text{ if } p_j > p^f \\ = 0 \text{ if } p_j = p^f \\ < 0 \text{ otherwise} \end{cases}, \tag{9}$$

[37] The same intuition is present in the first model in Fehr and Schmidt (1999).

[38] Huck et al. (2004) review of the evidence on experimental Cournot markets. They find that evidence from experimental Cournot games shows that when there are only two firms in the market collusive outcomes are frequent. However, as the number of firms increases output approaches the Nash-equilibrium.

[39] Dastidar (1995) shows that in symmetric Bertrand competition with increasing marginal costs (decreasing returns), there is a continuum of symmetric equilibria where firms set a price in the interval $[p_L, p_H]$, and this interval contains the perfectly competitive price.

that is, manager i places a positive weight on the profits of a rival that sets a price above the fair price, p^f, he places no weight on the profits of a rival that sets price equal to p^f, and he places a negative weight on the profits of a rival that sets price below p^f. I assume that the fair price is equal to or above marginal cost, c, and equal to or lower than the monopoly price, p^m, that is $p^f \in [c, p^m]$, with $p^m = \max_p (p-c)D(p)$. In the symmetric equilibrium of the n-firm symmetric Bertrand game with constant marginal costs and preferences for reciprocity firms will charge a price $p \in \left[p^f, p^m\right]$ that is higher than marginal cost if and only if

$$\frac{1}{n}pD(p) + \frac{n-1}{n}w(p,p^f)pD(p) \geq pD(p)$$

or $w(p,p^f) \geq 1$. This condition says that managers have to place more weight on a rival's profit than on their own profit for price to be above marginal cost. Such a high weight to preferences for reciprocity is not plausible. Thus, this result tells us that preferences for reciprocity should have no impact on Bertrand competition.

The last result in the paper characterizes the equilibrium of the n-firm symmetric Bertrand game when managers have piecewise linear inequity aversion and constant marginal costs.[40]

Proposition 10: *The set of Nash equilibria of the n-firm symmetric Bertrand game with piecewise linear inequity averse managers and constant marginal costs is given by*

$$p_i = \begin{cases} a, & \text{if } 1 - \frac{1}{n} \leq \min(\beta_1, \dots, \beta_n) \\ c, & \text{otherwise} \end{cases},$$

where $a \in (c, \bar{p}]$, with $\bar{p}$ being the choke-off price for demand.

This result shows that if marginal cost are constant and there is at least one manager with a degree of compassion smaller than $1 - 1/n$, then the only equilibrium is for all firms to charge price equal to marginal cost. By contrast, if marginal costs are constant and all managers have a degree of compassion greater than $1 - 1/n$, then there is a continuum of symmetric equilibria where firms charge a price between marginal cost and the price that leads to zero market demand.

There are two interpretations of Proposition 10. For a fixed number of firms, this result tells us that piecewise linear inequity aversion can only raise price above marginal cost in Bertrand competition between firms with constant marginal costs when all managers have a very high level of compassion.[41] For a fixed level of compassion, say β, with $\beta \in (1/2, 1)$, this result tells us that an increase in the number of firms makes it is harder for piecewise linear inequity aversion to lead firms to set price above marginal cost. Of course, if we assume

[40] The result extends to other forms of inequity aversion.

[41] Recall that if $\beta = 1/2$ implies that a manager is just indifferent between keeping one dollar to heself and giving this dollar to her competitors.

that β_i, $i = 1, \dots, n$, has a uniform distribution on $[0, 1]$, then an increase in n raises $1 - \frac{1}{n}$ and reduces $\min(\beta_1, \dots, \beta_n)$ which makes it even harder to satisfy the condition that allows firms to charge price above marginal cost.

6 Endogenous Timing and Dynamic Oligopoly

This section discusses two other market games where introducing interdependent preferences may provide new insights on economic behavior.

The literature on endogenous timing market games tries to identify factors that might lead to the endogenous emergence of sequential or simultaneous play in oligopolistic markets. The prediction of asymmetric equilibria with Stackelberg outcomes is clearly the most frequent result in this literature. Several experiments have tried to validate this prediction, but failed to find support for it. By contrast, the experiments find that simultaneous-move symmetric outcomes are modal. Santos-Pinto (2006) shows that inequity aversion is able to organize most of the experimental evidence on endogenous timing games since it makes symmetric outcomes more attractive to players than asymmetric ones.

Santos-Pinto (2007) studies the impact of preferences for reciprocity in the infinitely repeated versions of the Cournot and Bertrand games. He finds that, for plausible perceptions of fairness, preferences for reciprocity facilitate collusion in both types of games. Introducing preferences for reciprocity in the infinitely repeated Cournot game implies that the punishment outcome becomes a destructive reciprocity state whereas the collusive outcome becomes a constructive reciprocity state. These two effects imply that the critical discount rate at wish collusion can be sustained in the infinitely repeated Cournot game tends to be lower when firms have preferences for reciprocity than when firms only care about profits. In the infinitely repeated Bertrand game preferences for reciprocity do not alter the payoff of the punishment outcome nor that of the one period deviation. However, they raise the payoff of collusion when firms perceptions of the fair market price are strictly below the monopoly price.

7 Conclusion

This paper studies the impact of interdependent preferences (reciprocity and inequity aversion) on Cournot and Bertrand competition. The results obtained shows that interdependent preferences may lead to more or less competitive outcomes in the static Cournot model of strategic interaction between firms. This depends critically on firms' perceptions of fairness. The paper also shows that interdependent preferences should have no impact on Bertrand competition.

There are many interesting avenues for research on this topic that are beyond the scope of this paper. For example, what happens to equilibrium outcomes when some managers have interdependent preferences and others do not. If firms can select managers with different preferences which ones should be chosen? Do market forces eliminate managers with interdependent preferences?

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8 Appendix

Proof of Lemma 1: It is a well known result that a Cournot duopoly game with decreasing best replies, when one firm's strategy set is given the reverse order, is a supermodular game–see pp. 34 in Vives (2001). If U_i has decreasing differences in (q_i, q_j), then managers' best replies are decreasing and so $\Gamma^R(U, w, q^F)$ is a supermodular game. *Q.E.D.*

Proof of Proposition 1: Theorem 6 in Milgrom and Roberts (1990). *Q.E.D.*

Proof of Lemma 2: Theorem 2.8 in Vives (2001). *Q.E.D.*

Proof of Proposition 2: We know by Lemma 2 that $\Gamma^R(U, w, q^F)$ has a unique equilibrium. Let $q^{NR} = (q_1^{NR}, \ldots, q_n^{NR})$ denote the unique Nash equilibrium of $\Gamma^R(U, w, q^F)$. Let $q^{NS} = (q_1^{NS}, \ldots, q_n^{NS})$ denote the unique Nash equilibrium of $\Gamma^S(\pi)$. I wish to show that if $Q^F_{-i} = Q^{NS}_{-i}$, then $q_i^{NR} = q_i^{NS}$, with $i = 1, 2$. To do that I only need show that a reciprocal manager i has no incentive to deviate from $q_i^{NR} = q_i^{NS}$ when his rival plays $Q^{NR}_{-i} = Q^{NS}_{-i} = Q^F_{-i}$. But, if $Q^F_{-i} = Q^{NS}_{-i} = Q^{NR}_{-i}$ then $w_i(Q_{-i}, Q^F_{-i}) = 0$. If that is the case, then the best reply of manager i to Q^{NR}_{-i} is indeed $q_i^{NR} = q_i^{NS}$. *Q.E.D.*

Proof of Proposition 3: We know from Proposition 1 that if $\hat{q}^F = q^{NS}$, then $q^{NR} = q^{NS}$. If $\tilde{q}^F > (<) \hat{q}^F = q^{NS}$, then Proposition 1 implies that the unique Cournot-Nash equilibrium of $\Gamma^R(U, w, \tilde{q}^F)$ is smaller (greater) than the unique Cournot-Nash equilibrium of $\Gamma^R(U, w, \hat{q}^F)$ *Q.E.D.*

Proof of Lemma 3: To prove existence of equilibrium we only need to show that condition (7) implies that the payoff function of manager i is strictly concave in q_i. The second derivative of the payoff function of manager i is given by

$$\frac{\partial^2 U_i}{\partial q_i^2} = \frac{\partial^2 \pi_i}{\partial q_i^2} + \sum_{j \neq i} \lambda''_{ij}(\pi_j - \pi_i) \left(\frac{\partial \pi_j}{\partial q_i} - \frac{\partial \pi_i}{\partial q_i} \right)^2 + \sum_{j \neq i} \lambda'_{ij}(\pi_j - \pi_i) \left(\frac{\partial^2 \pi_j}{\partial q_i^2} - \frac{\partial^2 \pi_i}{\partial q_i^2} \right),$$

where $\partial^2 \pi_i / \partial q_i^2 = 2P'(Q) + P''(Q)\, q_i - C''(q_i)$ and $\partial^2 \pi_j / \partial q_i^2 = P''(Q)\, q_j$, for all $j \neq i$. Manager i's payoff function is strictly concave in q_i if $\partial^2 U_i / \partial q_i^2 < 0$. It is easy to check that condition (7) implies $\partial^2 U_i / \partial q_i^2 < 0$. The assumption that the game is symmetric implies that there is a unique equilibrium. *Q.E.D.*

Proof of Proposition 4: The symmetry assumption implies that $q_i = q_j$, for all $i \neq j$, and this implies that $\pi_i = \pi_j$, for all $i \neq j$. If that is the case, then

(6) is given by

$$\frac{\partial \pi_i}{\partial q_i} + \sum_{j \neq i} \lambda'_{ij}(0) \left(\frac{\partial \pi_j}{\partial q_i} - \left. \frac{\partial \pi_i}{\partial q_i} \right|_{q_i = q_j} \right) = 0,$$

with

$$\begin{aligned} \frac{\partial \pi_j}{\partial q_i} - \left. \frac{\partial \pi_i}{\partial q_i} \right|_{q_i = q_j} &= P'(Q) q_j - \left. [P'(Q) q_i + P(Q) - C'(q_i)] \right|_{q_i = q_j} \\ &= -\left[P(Q) - C'(q_i) \right]. \end{aligned} \tag{10}$$

Using (10) the first-order condition becomes

$$\frac{\partial U_i}{\partial q_i} = P'(Q) q_i + [P(Q) - C'(q_i)] \left[1 - \sum_{j \neq i} \lambda'_{ij}(0) \right] = 0. \tag{11}$$

If $\lambda'_{ij}(0) = 0$ for all i and j, then (11) reduces to $\partial \pi_i / \partial q_i = 0$ and this implies $q^{NI} = q^{NS}$. This proves part (i). If $\sum_{j \neq i} \lambda'_{ij}(0) < 0$ for all i, then $1 - \sum_{j \neq i} \lambda'_{ij}(0) > 1$ in (11). Suppose, by contradiction that $q^{NI} = q^{NS}$. If that is the case, then $1 - \sum_{j \neq i} \lambda'_{ij}(0) > 1$ in (11) together with $P(Q) - C'(q_i) > 0$ and $P'(Q) < 0$ imply that the left-hand side of (11) is positive. Thus, $q^{NI} = q^{NS}$ is not an equilibrium when $\sum_{j \neq i} \lambda'_{ij}(0) < 0$ for all i. The assumption that marginal revenue is decreasing, that $C''_i(q_i) - P'(Q) > 0$ together with $\sum_{j \neq i} \lambda'_{ij}(0) < 0$ imply that $\partial^2 U_i / \partial q_i^2 < 0$. This in turn implies that $q^{NI} > q^{NS}$. This proves part (ii). *Q.E.D.*

Proof of Proposition 5: To prove this result I will start by showing that $q(\alpha_i)$ is an increasing function of α_i and that $q(\beta_i)$ is a decreasing function of β_i for $i = 1, \ldots, n$. Let

$$\begin{aligned} h(q, \alpha_i) &= (1 + \alpha_i) \left[P(nq) - C'_i(q) \right] + P'(nq) q = 0, \\ g(q, \beta_i) &= (1 - \beta_i) \left[P(nq) - C'_i(q) \right] + P'(nq) q = 0, \end{aligned}$$

which imply

$$\begin{aligned} \frac{\partial q}{\partial \alpha_i} &= -\frac{\partial h / \partial \alpha_i}{\partial h / \partial q} = -\frac{P(Q) - C'_i(q)}{(1 + n(1 + \alpha_i)) P'(Q) + n P''(Q) q - C''_i(q)} > 0, \\ \frac{\partial q}{\partial \beta_i} &= -\frac{\partial g / \partial \beta_i}{\partial g / \partial q} = -\frac{-\left[P(Q) - C'_i(q) \right]}{(1 + n(1 - \beta_i)) P'(Q) + n P''(Q) q - C''_i(q)} < 0, \end{aligned}$$

since $P'(Q) < 0$, $P'(Q) \leq 0$, and $C''_i(q_i) \geq 0$.
I will now show that $q_i = \frac{1}{n-1} \sum_{j \neq i} q_j$ is a best response for manager i when the rivals produce

$$q_i^N \leq \bar{q}_j \leq q(\alpha_i), \tag{12}$$

where $\bar{q}_j = \frac{1}{n-1}\sum_{j\neq i} q_j$. To do that I will show that manager i can not gain from deviating from $q_i = \bar{q}_j$ when (12) holds. Suppose, that (12) holds and that firm i produces $q_i = \bar{q}_j + \varepsilon$, with $\varepsilon > 0$. In this case manager i's payoff is

$$U_i = (1-\beta_i)\left[P(Q)\,q_i - C_i(q_i)\right] + \frac{\beta_i}{n-1}\sum_{j\neq i}\left[P(Q)\,q_j - C_j(q_j)\right]$$

and the change in manager i's payoff from producing $q_i = \bar{q}_j + \varepsilon$, $\varepsilon > 0$, instead of $\bar{q}_j$ is approximately equal to

$$\begin{aligned} dU_i &\approx (1-\beta_i)\left[P'(Q)\,q_i + P(Q) - C_i'(q_i)\right] + \frac{\beta_i}{n-1}\sum_{j\neq i} P'(Q)\,q_j\Bigg|_{q_i=\bar{q}_j} (\varepsilon) \\ &= \left[\left(P'(n\bar{q}_j)\,\bar{q}_j + P(n\bar{q}_j) - C_i'(\bar{q}_j)\right) - \beta_i\left(P(n\bar{q}_j) - C_i'(\bar{q}_j)\right)\right]\varepsilon. \end{aligned}$$

The square brackets are negative since $q_i = \bar{q}_j > \arg\max\left[P(Q)\,q_i - C_i(q_i)\right]$ and $P(n\bar{q}_j) - C_i'(\bar{q}_j) > 0$. So, when (12) holds, manager i can not gain by producing more than $\bar{q}_j$. Now, suppose that (12) holds and that firm i produces $q_i = \bar{q}_j + \varepsilon$, with $\varepsilon < 0$. In this case manager i's payoff is

$$U_i = (1+\alpha_i)\left[P(Q)\,q_i - C_i(q_i)\right] - \frac{\alpha_i}{n-1}\sum_{j\neq i}\left[P(Q)\,q_j - C_j(q_j)\right],$$

and the change in manager i's payoff from producing $q_i = \bar{q}_j + \varepsilon$, $\varepsilon < 0$, instead of $\bar{q}_j$ is approximately equal to

$$\begin{aligned} dU_i &\approx (1+\alpha_i)\left[P'(Q)\,q_i + P(Q) - C_i'(q_i)\right] - \frac{\alpha_i}{n-1}\sum_{j\neq i} P'(Q)\,q_j\Bigg|_{q_i=\bar{q}_j} (\varepsilon) \\ &= \left[(1+\alpha_i)\left[P(n\bar{q}_j) - C_i'(\bar{q}_j)\right] + P'(n\bar{q}_j)\bar{q}_j\right]\varepsilon = \left.h(q,\alpha_i)\right|_{q=\bar{q}_j}(\varepsilon). \end{aligned}$$

Since $\varepsilon < 0$, we have that $sign\ dU_i = -sign\ \left.h(q,\alpha_i)\right|_{q=\bar{q}_j}$. If $\bar{q}_j = q(\alpha_i)$ we have that $sign\ dU_i = 0$. If $q_i^N \leq \bar{q}_j < q(\alpha_i)$, the fact $h(q,\alpha_i)$ is a decreasing function of q implies that $\left.h(q,\alpha_i)\right|_{q=\bar{q}_j} > 0$, which in turn implies that $sign\ dU_i < 0$. So, when (12) holds, manager i can not gain by producing less than $\bar{q}_j$. From this result is follows immediately that if firm i's rivals produce $q(\alpha_i) < \frac{1}{n-1}\sum_{j\neq i} q_j$, then the best response of manager i is given by $t_i(q_{-i})$.

I will now show that $q_i = \frac{1}{n-1}\sum_{j\neq i} q_j$ is a best response for manager i when the rivals produce

$$q(\beta_i) \leq \bar{q}_j \leq q_i^N, \tag{13}$$

To do that I will show that manager i can not gain from deviating from $q_i = \bar{q}_j$ when (13) holds. Suppose, that (13) holds and that firm i produces $q_i = \bar{q}_j + \varepsilon$, with $\varepsilon < 0$. In this case manager i's payoff is given by

$$U_i = (1+\alpha_i)\left[P(Q)\,q_i - C_i(q_i)\right] - \frac{\alpha_i}{n-1}\sum_{j\neq i}\left[P(Q)\,q_j - C_j(q_j)\right],$$

and the change in manager i's payoff from producing $q_i = \bar{q}_j + \varepsilon$, $\varepsilon < 0$, instead of $\bar{q}_j$ is approximately equal to

$$\begin{aligned} dU_i &\approx \left.(1+\alpha_i)\left[P'(Q)\, q_i + P(Q) - C_i'(q_i)\right] - \frac{\alpha_i}{n-1}\sum_{j\neq i} P'(Q)\, q_j\right|_{q_i=\bar{q}_j} (\varepsilon) \\ &= \left[(1+\alpha_i)\left[P'(n\bar{q}_j)\bar{q}_j + P(n\bar{q}_j) - C_i'(\bar{q}_j)\right] - \alpha_i P'(n\bar{q}_j)\bar{q}_j\right]\varepsilon. \end{aligned}$$

The square brackets are positive since $q_i = \bar{q}_j < \arg\max\left[P(Q)\, q_i - C_i(q_i)\right]$ and $P'(n\bar{q}_j) < 0$. So, when (13) holds, manager i can not gain by producing less than $\bar{q}_j$. Now, suppose that (13) holds and that firm i produces $q_i = \bar{q}_j + \varepsilon$, with $\varepsilon > 0$. In this case manager i's payoff is given by

$$U_i = (1-\beta_i)\left[P(Q)\, q_i - C_i(q_i)\right] + \frac{\beta_i}{n-1}\sum_{j\neq i}\left[P(Q)\, q_j - C_j(q_j)\right]$$

and the change in manager i's payoff from producing $q_i = \bar{q}_j + \varepsilon$, $\varepsilon > 0$, instead of $\bar{q}_j$ is approximately equal to

$$\begin{aligned} dU_i &\approx \left.(1-\beta_i)\left[P'(Q)\, q_i + P(Q) - C_i'(q_i)\right] + \frac{\beta_i}{n-1}\sum_{j\neq i} P'(Q)\, q_j\right|_{q_i=\bar{q}_j} (\varepsilon) \\ &= \left[(1-\beta_i)\left[P(n\bar{q}_j) - C_i'(\bar{q}_j)\right] + P'(n\bar{q}_j)\,\bar{q}_j\right]\varepsilon = \left. g(q,\beta_i)\right|_{q=\bar{q}_j}(\varepsilon). \end{aligned}$$

Since $\varepsilon > 0$, we have that $sign\ dU_i = sign\ \left. g(q,\beta_i)\right|_{q=\bar{q}_j}$. If $\bar{q}_j = q(\beta_i)$ we have that $sign\ dU_i = 0$. If $q(\beta_i) < \bar{q}_j \leq q_i^N$, the fact $g(q,\beta_i)$ is a decreasing function of q implies that $\left. g(q,\beta_i)\right|_{q=\bar{q}_j} < 0$, which in turn implies that $sign\ dU_i < 0$. So, when (13) holds, firm i can not gain by producing more than $\bar{q}_j$. From this result is follows immediately that if firm i's rivals produce $0 \leq \frac{1}{n-1}\sum_{j\neq i} q_j < q(\beta_i)$, then the best response of manager i is given by $s_i(q_{-i})$. *Q.E.D.*

Proof of Proposition 6: I need to show that $q_i = q_i^N$ is the best response to $q_{-i}^N = \left(q_1^N, \ldots, q_{i-1}^N, q_{i+1}^N, \ldots q_n^N\right)$ in the n-firm symmetric Cournot game with piecewise linear inequity averse managers. The welfare of manager 1 under outcome q^N is given by $\pi_1(q^N) = \left[P\left(nq_i^N\right) - C_i(q_i^N)\right] q_i^N$, where $q_i^N = \arg_{q_1}\max\left[P\left(q_i + \sum_{j\neq i} q_j^N\right) - C_i(q_i)\right] q_i$.
If firm i produces $q_i^N + \varepsilon$, with $\varepsilon > 0$, and all other firms produce q_{-i}^N, then the change in firm i's profit is approximately equal to

$$\begin{aligned} d\pi_i &\approx \varepsilon\, \partial\pi_i/\partial q_i\big|_{q_i=q_i^N} + \frac{1}{2}\varepsilon^2\ \partial^2\pi_i/\partial q_i^2\big|_{q_i=q_i^N} \\ &= \frac{1}{2}\varepsilon^2\left[2P'(Q^N) + P''(Q^N)q_i^N - C''(q_i^N)\right]. \end{aligned} \tag{14}$$

The assumption that $P' < 0$, $P'' \leq 0$, and $C'' \geq 0$ imply that $d\pi_i < 0$. The change in the profit of one of firm i's rivals, say j, is approximately equal to

$$\begin{aligned} d\pi_j &\approx \varepsilon\, \partial\pi_j/\partial q_i\big|_{q_i=q_i^N} + \frac{1}{2}\varepsilon^2\, \partial^2\pi_j/\partial q_i^2\big|_{q_i=q_i^N} \\ &= \varepsilon P'(Q^N)q_j^N + \frac{1}{2}\varepsilon^2 P''(Q^N)q_j^N. \end{aligned}$$

Note that the change in the average profit of firm i's rivals is the same as the change in the profit of a single rival since

$$\begin{aligned} \frac{1}{n-1}\sum_{j\neq i} d\pi_j &\approx \frac{1}{n-1}\varepsilon P'(Q^N)\sum_{j\neq i} q_j^N + \frac{1}{2}\varepsilon^2 P''(Q^N)\sum_{j\neq i} q_j^N \\ &= \varepsilon P'(Q^N)q_j^N + \frac{1}{2}\varepsilon^2 P''(Q^N)q_j^N. \end{aligned} \tag{15}$$

The assumption that $P' < 0$ and $P'' \leq 0$ imply that $\frac{1}{n-1}\sum_{j\neq i} d\pi_j < 0$. We see from (14) and (15) that if firm i produces $q_i^N + \varepsilon$, with $\varepsilon > 0$, and all other firms produce q_{-i}^N, then there is a first order decrease in profits of firm i and a second order decrease in the average profit of firm i's rivals. Thus, if firm i produces $q_i^N + \varepsilon$, with $\varepsilon > 0$, it suffers a loss in profits and also a loss from an increase in inequity aversion given that the average profit of the rivals becomes smaller than firm i's profit. If that is the case, then firm i can not gain by producing $q_i^N + \varepsilon$, with $\varepsilon > 0$, instead of producing q_i^N.
If firm i produces $q_i^N + \varepsilon$, with $\varepsilon < 0$, and all other firms produce q_{-i}^N, then the change in firm i's profit is given by (14) and we have that $d\pi_i < 0$. The change in the average profit of firm i's rivals is given by (15) and we have that $\frac{1}{n-1}\sum_{j\neq i} d\pi_j > 0$ since $\varepsilon < 0$ and the first term is of first order while the second term is of second order. Thus, if firm i produces $q_i^N + \varepsilon$, with $\varepsilon < 0$, it suffers a loss in profits and also a loss from an increase in inequity aversion given that the average profit of the rivals becomes greater than firm i's profit. If that is the case, then firm i can not gain by producing $q_i^N + \varepsilon$, with $\varepsilon < 0$, instead of producing q_i^N.
This proves that $q_i = q_i^N$ is the best reply to $q_{-i}^N = \left(q_1^N, \ldots, q_{i-1}^N, q_{i+1}^N, \ldots q_n^N\right)$ and so q^N is a Nash equilibrium of the n-firm symmetric Cournot game with piecewise linear inequity averse managers. *Q.E.D.*

Proof of Proposition 7: We know that the set N^{IA} is non-empty since it contains at least the Nash equilibrium of the standard n-firm symmetric Cournot game. I need to show that if all managers are strictly averse to inequity, then $q(\beta) < q(\alpha)$, that is, N^{IA} is an interval. We know that $q(\alpha_i)$ is an increasing function of α_i and that $q(\beta_i)$ is a decreasing function of β_i for $i = 1, \ldots, n$. Note that if at least one manager does not feel inequity aversion then $q(\beta) = q(\alpha)$, and N^{IA} is a singleton. To see this suppose that manager i is not inequity averse, that is, $\alpha_i = \beta_i = 0$. If that is the case, then $h(q, \alpha_i) = 0$ and $g(q, \beta_i) = 0$ imply that $q(0) = q^N$. If $q(\alpha_i)$ is an increasing function of α_i and $q(0) = q^N$, then $q(\alpha) = q^N$. Similarly, if $q(\beta_i)$ is a decreasing function of β_i and $q(0) = q^N$, then $q(\beta) = q^N$. So, if at least one manager feels aversion to inequity we have that $q(\beta) = q(\alpha) = q^N = N^{IA}$. I will now show that if all managers are strictly averse to inequity, then $q(\beta) < q(\alpha)$, that is, N^{IA} is an interval. If all managers are strictly averse to inequity, $q(\alpha_i)$ is an increasing function of α_i and $q(0) = q^N$, then $q(\alpha) > q^N = q(0)$. Also, if all managers are strictly inequity averse, $q(\alpha_i)$ is an decreasing function of β_i and $q(0) = q^N$, then $q(\beta) < q^N = q(0)$. This shows that $q(\beta) < q(\alpha)$ when all managers are strictly inequity averse, that is the set N^{IA} is an interval. All outcomes in the set N^{IA} are equilibria of the symmetric Cournot game with inequity aversion since for any profile of quantities, q_{-i}, the quantity q_i belongs to the best response of firm i, $i = 1, \ldots n$. *Q.E.D.*

Proof of Proposition 8: The quantity produced by each firm in the largest Nash equilibria of N^{IA} is given by $q(\alpha) = \min\left[q(\alpha_1), \ldots, q(\alpha_n)\right]$. The largest Nash equilibria of N^{IA} is nondecreasing in α since $\min\left[q(\alpha_1), \ldots, q(\alpha_n)\right]$ is nondecreasing in α. Similarly, the quantity produced by each firm in the smallest Nash equilibria of N^{IA} is given by $q(\beta) = \max\left[q(\beta_1), \ldots, q(\beta_n)\right]$. The smallest Nash equilibria of N^{IA} is nonincreasing in β since $\max\left[q(\beta_1), \ldots, q(\beta_n)\right]$ is nonincreasing in β. *Q.E.D.*

Proof of Proposition 9: When all managers are strictly averse to inequity it follows that $q(\beta) < q^N < q(\alpha)$. Since α_i is drawn from a uniform distribution with support on $[0, 1]$, the larger is n the most likely it becomes that $\min\left(\alpha_1, \ldots \alpha_n\right)$ is closer to zero, this implies that the larger is n the most likely is that $N(\alpha)$ is closer to q^N. Similarly, since β_i is drawn from a uniform distribution with support on $[0, 1]$, the larger is n the most likely it becomes that $\min\left(\beta_1, \ldots, \beta_n\right)$ is closer to zero, this implies that the larger is n the most likely is that $N(\beta)$ is closer to q^N. *Q.E.D.*

Proof of Proposition 10: If marginal costs are constant, then we have $C_i(q_i) = cq_i$, $i = 1, \ldots, n$. The payoff of manager i in the presence of piecewise linear inequity aversion is given by

$$U_i\left(p_i, p_{-i}\right) = \begin{cases} \left(1-\beta_i\right)\left(p_i - c\right) D(p_i), & \text{if } \ p_i < p_j^{\min} \\ \left(1 - \beta_i + \beta_i \frac{l-1}{n-1}\right) \frac{(p_i - c) D(p_i)}{l}, & \text{if } \ p_j \geq p_i, \ \ \forall j \in N \\ -\alpha_i \left(p_j^{\min} - c\right) D(p_j^{\min}), & \text{if } \ p_i > p_j^{\min} \end{cases},$$

where $p_j^{\min} = \min\left(p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n\right)$ and $l = \#\left\{j \in N : p_j = p_i\right\}$. For manager i not to deviate from an equilibrium where firm i plus $l-1$ firms charge $p \in (c, \bar{p}]$ and the remaining firms charge a higher price than p it must be that

$$\left(1-\beta_i\right)\left(p_i - c\right) D(p_i) \leq \left(1 - \beta_i + \beta_i \frac{l-1}{n-1}\right) \frac{(p_i - c) D(p_i)}{l}$$

or $1 - \frac{1}{n} \leq \beta_i$. For all managers not to deviate, the case when $l = n$, from such an equilibrium we need that $1 - \frac{1}{n} \leq \min\left(\beta_1, \ldots, \beta_n\right)$. If this condition does not hold, then there is at least one manager that is always willing to undercut a price $p \in (c, \bar{p}]$. If that is the case, then the only equilibrium is for all firms to charge price equal to marginal cost. *Q.E.D.*