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Abstract

This paper presents a new characterization result for competitive allocations in quasilinear economies. This result is informed by the analysis of non-cooperative dynamic search and bargaining games. Such games provide models of decentralized markets with trading frictions. A central objective of this literature is to investigate how equilibrium outcomes depend on the level of the frictions. In particular, does the trading outcome become Walrasian when frictions become small? Existing specifications of such games provide divergent answers. The characterization result is used to investigate what causes these differences and to generalize insights from the analysis of specific search and bargaining games.

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1 Introduction

In a dynamic matching and bargaining game, a large population of traders interacts repeatedly in a decentralized market.[1] Every trading period, traders are *matched* to form small groups where they *bargain* over the terms of trade. If they fail to reach an agreement, they can wait at some cost until the next period to be rematched into a new group. These waiting costs are the *frictions* of trading in the decentralized market. A major question in the literature concerns the trading outcome when frictions become small: Does the outcome become Walrasian? Ideally, one would like not only to find answers for particular trading institutions, but also to gain a general understanding of the conditions under which trading with vanishing frictions has this property and the conditions under which it does not. In this paper I use methods from cooperative and non-cooperative game theory to address this question. Recent contributions that fall into the framework of this paper include work by Moreno and Wooders (2002), Mortensen and Wright (2002), Satterthwaite and Shneyerov (2007), and De Fraja and Sakovics (2001).

The main result modifies a characterization result from cooperative game theory for quasilinear economies by Shapley and Shubik (1971): If an allocation is feasible then it is competitive if and only if it is pairwise efficient (pairwise stable). I weaken the requirement that the outcome must be pairwise efficient for *all* pairs of traders. Instead, I require the outcome to be pairwise efficient for traders who trade with probability less than one. I introduce two new conditions (Monotonicity and No Rent Extraction) to characterize the trading outcome for traders who trade with certainty.

The characterization result is informed by the analysis of non-cooperative dynamic matching and bargaining games. The reason for weakening pairwise efficiency to a subset of traders is that, in such games, traders with types who transact with probability one leave the market quickly. Other traders might therefore not be matched with them and the allocation does not need to be pairwise efficient with respect to these types.[2]

As an illustration of the main result, I use a parameterized class of steady-state search and bargaining games that is similar to the one used by Gale (1987). There is a continuum

[1] The literature on dynamic matching and bargaining games is vast. Osborne and Rubinstein (1990) and Gale (2000) are excellent surveys. Diamond (1971) demonstrated that small frictions can lead to severe distortions. Subsequent work was done by Gale (1986, 1987), Rubinstein and Wolinsky (1985, 1990), McLennan and Sonnenschein (1991) and extended in various directions by DeFraja and Sakovics (2001), Serrano (2002), Mortensen and Wright (2002), Kunimoto and Serrano (2004), Satterthwaite and Shneyerov (2007, 2008), Atakan (2007), and Shneyerov and Wong (2010)

[2] Dagan, Serrano, and Volij (2000) also combine cooperative and non-cooperative elements to analyze dynamic matching and bargaining games. However, they assume that all coalitions can form and they provide a characterization result for a general economy.

of buyers who have unit demand and valuations (types) $v \in [0, 1]$ for an indivisible good, and there is a continuum of sellers who have unit capacity and costs $c \in [0, 1]$. These traders are matched into small groups. In these groups, they bargain, and, if they reach an agreement, they trade. The groups are connected to form a large market by allowing unsuccessful traders to be matched into new groups in the next period. Integration, however, is imperfect because there is a probability (exit rate) that a trader dies while waiting. These are the *frictions* of trading. Finally, at the end of each period, there is an exogenous inflow of new buyers and sellers.

This class of games allows for a variety of specifications of the matching technology and of the bargaining protocol. Regardless of how matching and bargaining is specified, the game and its solution concept will give rise to an *outcome* that consists of (a) probabilities of trading for entering types and (b) expected equilibrium payoffs. An outcome is called *feasible* if it is consistent with an allocation for the quasilinear economy defined by the distribution of buyer's valuations and seller's costs.

Suppose there is some sequence of exit rates ("frictions") that converges to zero. In addition, suppose that for each exit rate an equilibrium outcome of a specific trading game is selected. This defines a sequence of outcomes. I state conditions on this sequence that are jointly necessary and sufficient for convergence to the competitive outcome. The first condition, *Monotonicity*, requires that trading probabilities are monotone—buyers with higher valuations are more likely to trade, while sellers with higher costs are less likely to trade. The second condition, *No Rent Extraction*, requires that traders receive some part of the surplus they generate. Technically, this is a condition on the slope of the payoffs. The third and the fourth conditions are jointly equivalent to pairwise efficiency of types who do not trade with certainty. Specifically, the third condition, *Availability*, requires that traders who do not trade with certainty are available. In the application, a type is available if others are matched frequently with traders having such types. The fourth condition, *Weak Pairwise Efficiency*, requires that for all pairs of buyers and sellers who are both available the sum of their expected payoffs is at least as large as the payoffs they could realize by trading with each other. The *Availability* condition relates to the matching technology, whereas the other conditions relate to the bargaining protocol. The main result (Proposition 1) is essentially this: A sequence of feasible outcomes converges to the Walrasian outcome if and only if the four conditions hold.

I apply this result to the parameterized dynamic matching and bargaining game introduced before. I show how to verify each of the conditions. Importantly, I argue that the conditions often follow from basic equilibrium restrictions onto outcomes. It is not necessary to actually compute the equilibrium outcomes. The main result applies to *all* trading games that map quasilinear economies into trading outcomes. It extends therefore well beyond the parameterized example and its particular form of trading frictions. I comment extensively on how the conditions of the result can be verified for general matching technologies and bargaining protocols.

Whenever convergence fails in some model, at least one of the conditions must be violated, which allows a "classification" of such failures. I show that the failure in Rubinstein and Wolinsky (1990) and in Serrano (2002) can be linked to a failure of Weak Pairwise Efficiency (the fourth condition), the failure in Lauermann (2011) to rent extraction (a failure of the second condition), and the failure in Rubinstein and Wolinsky (1985) and in De Fraja and Sakovics (2001) can be linked to a failure of feasibility; that is, the limit outcome does not correspond to an allocation that is feasible in the benchmark economy. By stating necessary conditions, the main result suggests conditions under which decentralized trading is not well approximated by market clearance.

2 The Model

I consider a trading environment that consists of buyers and sellers who want to trade an indivisible good. This trading environment, together with the feasibility condition, defines the general model. The *traders* (or *agents*) have quasilinear preferences and maximize expected payoffs. The sellers each have one unit of the good, and their costs of trading are given by $c \in [0, 1]$. The buyers each want to buy one unit of the good, and their valuations of the good are given by $v \in [0, 1]$. If a seller trades with a buyer at a price p, the payoffs are $p - c$ and $v - p$, respectively. An abstract economy is characterized by two functions $G^S(c)$ and $G^B(v)$ that map the unit interval into itself. The functions are zero at zero, and they are strictly increasing and continuously differentiable.

The functions $G^S(c)$ and $G^B(v)$ are interpreted as defining a large, static economy with transferable utility (quasilinear preferences). $G^S(c)$ is the mass of sellers with costs below c, and $G^B(v)$ is the mass of buyers with valuations below v.[3] (In Section 4, G^S and G^B define a constant exogenous inflow of new traders into a *dynamic* economy.) Let p^w be defined as the Walrasian price such that the mass of sellers having costs below p^w is equal to the mass of buyers having a valuation above p^w, $G^S(p^w) = G^B(1) - G^B(p^w)$. Since G^S and G^B are strictly increasing and continuous functions, the market clearing

[3]In general, $G^S(1)$ and $G^B(1)$ do not need to be one, which allows modelling large economies with a different mass of agents on each side.

price exists and is unique.

A trading outcome is a vector $A = \left[V^S, V^B, Q^S, Q^B\right]$, where $V^S(c)$ and $V^B(v)$ are the expected payoffs, and $Q^S(c)$ and $Q^B(v)$ are the trading probabilities of the sellers and buyers. An outcome does not explicitly specify the transfers that are made between buyers and sellers. However, in a quasilinear environment with risk-neutral agents, the difference between the expected consumption value $vQ^B(v)$ and the expected payoff $V^B(v)$ is equal to the transfer made by the buyer, and the sum of the expected cost $cQ^S(c)$. Similarly, the expected payoff $V^S(c)$ is equal to the transfer received by the seller. I do not include transfers explicitly in the outcome because the previous discussion implies that transfers would be redundant. Let Σ denote the set of measurable functions $f : [0, 1] \rightarrow [0, 1]$. Any element of Σ^4 constitutes an outcome.

An outcome defines a feasible allocation for an economy given by $G^S(c)$ and $G^B(v)$ if the following two statements are true. First, the total mass of the buyers who trade equals the total mass of the sellers who trade, that is, $\int_0^1 Q^S(c)\, dG^S(c) = \int_0^1 Q^B(v)\, dG^B(v)$. Second, the total amount of transfers collectively made by buyers equal the total amount of transfers received by sellers, $\int_0^1 \left(v\, Q^B(v) - V^B(v)\right) dG^B(v) = \int_0^1 \left(V^S(c) + c\, Q^S(c)\right) dG^S(c)$. An outcome that meets these two requirements satisfies the *feasibility* condition**.**

Given an outcome, the trading surplus is defined as $S(A) \equiv \int_0^1 V^B(v)\, dG^B(v) + \int_0^1 V^S(c)\, dG^S(c)$. The surplus coincides with the ex-ante expected payoffs. The object of interest is the maximal surplus that can be realized subject to the feasibility constraint. The maximal surplus is denoted by S^*. If the outcomes are feasible, the transfers cancel, and the surplus is solely determined by the allocation of the indivisible good given by the trading probabilities $Q = \left[Q^S, Q^B\right]$. Denote the set of Walrasian allocations by Q^W (it is a set because Q^S and Q^B are not determined at the point p^w). It is straightforward to verify that an outcome is efficient if and only if the allocation of the good is Walrasian (this is the analogue of the First and Second Welfare Theorem for a quasilinear economy).[4]

Lemma 1 *For all outcomes that satisfy feasibility: $S(A) = S^*$ if and only if $Q \in \left[Q^W\right]$, where Q^W is the set of functions such that for sellers, $Q^S(c) = 1$ if $c < p^w$ and $Q^S(c) = 0$ if $c > p^w$ and for buyers, $Q^B(v) = 1$ if $v > p^w$ and $Q^B(v) = 0$ if $v < p^w$.*

Suppose that for any pair of types c and v the sum of their interim expected payoffs $V^S(c) + V^B(v)$ is weakly larger than their private surplus $v - c$. Intuitively, all gains

[4] The surplus does not change if a zero measure of traders has trading probabilities different from Q^W. Therefore, I state the lemma for the equivalence class of the set Q^W, which is denoted by $\left[Q^W\right]$. Two functions are equivalent if the integral of their difference is zero.

from trade are exhausted. Following Feldman (1973), such an outcome is called pairwise efficient. Pairwise efficiency is equivalent to pairwise stability. Let V^W denote Walrasian payoffs, where $V^W = \left(V^S, V^B\right)$, with $V^S = \max\{p^w - c, 0\}$, and $V^B = \max\{v - p^w, 0\}$. Let A^W denote the set of Walrasian outcomes with $Q \in Q^W$ and $V = V^W$. The following Lemma restates a well-known result by Shapley and Shubik (1971), which is readily extended to the continuum case considered here.

Lemma 2 *Suppose an outcome A satisfies Feasibility. Then,*

$$A \in \left[A^W\right] \quad \Leftrightarrow \quad V^S(c) + V^B(v) \geq v - c \qquad \forall v, c.$$

3 The Main Result

3.1 Summary and Conditions

Let $\{A_k\}_{k=1}^{\infty}$ be a sequence of outcomes. In Section 4, I obtain such a sequence as the sequence of outcomes of equilibria of a dynamic matching and bargaining game when the exit rate converges to zero. I define conditions onto such sequences. Because I want to state conditions that are necessary for convergence to a Walrasian limit, these conditions are stated directly onto limits. The main result is that a sequence of outcomes that has uniformly bounded variation and satisfies feasibility becomes Walrasian if and only if these conditions hold.

The assumption that the sequence has uniformly bounded variation[5] ensures that a pointwise convergent subsequence exists (by Helley's selection theorem; see Kolmogorov and Fomin, 1970).[6] A sufficient condition for a set of functions to have uniformly bounded variation is that the functions are monotone (see the discussion following Corollary 1). Let $\bar{A}$ be the limit of some convergent subsequence, $\bar{A} = \left(\bar{V}^S, \bar{V}^B, \bar{Q}^S, \bar{Q}^B\right)$. The following conditions are with respect to $\bar{A}$.

The first two conditions are both requirements with respect to the slope of the elements of limit outcomes. A sequence of outcomes satisfies *Monotonicity* (Condition 1) if, for any limit outcome, $\bar{Q}^S$ is nonincreasing and $\bar{Q}^B$ is nondecreasing.

[5] A family Φ of functions $f : [0,1] \to [0,1]$ has a uniformly bounded variation if there is some constant C s.t. $\sum_{k=1}^{n} |f(x_k) - f(x_{k-1})| \leq C$ for every finite partition $0 = x_1 < x_2 < \cdots < x_n = 1$.

[6] Uniformly bounded variation is a technical condition that ensures that the set of outcomes is sequentially compact. I would not need this condition (the condition would be trivial) if I would work with a finite set of N types. With a finite set of types, the set of outcomes would be $[0,1]^{2N}$ (payoffs and trading probabilities for each type), which is compact.

No Rent Extraction (Condition 2) requires that, first, $a - b \leq \bar{V}^S(b) - \bar{V}^S(a) \leq 0$ if $a \leq b$, and, second, whenever $\bar{Q}^S(c_x) = 1$ for some c_x, then $\bar{V}^S(c) \geq \bar{V}^S(c_x) + (c_x - c)$ for all c. Thus, the sellers' payoffs must be nonincreasing in costs, and the difference between the payoffs of any two types must be no more than the difference between the types. Furthermore, whenever some type c_x trades with probability one, then all other sellers must receive at least the same payoff plus the difference in costs $(c_x - c)$. Similarly, for buyers, first, $0 \leq \bar{V}^B(b) - \bar{V}^B(a) \leq b - a$ if $a \leq b$, and, second, whenever $\bar{Q}^B(v_x) = 1$, then $\bar{V}^B(v) \geq \bar{V}^B(v_x) + (v - v_x)$.

A sequence of outcomes satisfies *Pairwise Efficiency of Available Types*, if, for any pair of types c_x and v_x for which it is true that $\bar{Q}^B(v) < 1$ for all v below v_x and $\bar{Q}^S(c) < 1$ for all c above c_x, then payoffs are $\bar{V}^S(c) + \bar{V}^B(v) \geq v - c$ for all $v < v_x$ and $c > c_x$. Thus, payoffs are pairwise efficient for those types who do not trade with probability one.

For interpretations and applications, this condition is split into two parts. For this split, I use two sequences of functions which act as indicator functions. Let $L_k^j : [0,1] \times \Sigma^4 \to [0,1]$ with $j \in \{B, S\}$. Given the sequence, let the limits be $\bar{L}^B(v) \equiv \liminf L_k^B(v, A_k)$ and $\bar{L}^S(c) \equiv \liminf L_k^S(c, A_k)$. The interest in these functions is in whether their limit is equal to one or not. The desired interpretation of these indicator functions in search and bargaining games is as matching probabilities: $\bar{L}^B(v)$ is the probability to be matched with a buyer having valuation above v and $\bar{L}^S(c)$ is the probability to be matched with a seller having costs below c. Given this interpretation, each of the following conditions has a distinct economic meaning, and each condition can fail independently.

A sequence of outcomes satisfies *Availability* (Condition 3) relative to a pair of sequences of functions L_k^B and L_k^S if $\bar{Q}^B(v) < 1$ for all v below some v_x implies that $\bar{L}^B(v) = 1$ for all $v < v_x$ and if $\bar{Q}^S(c) < 1$ for all c above some c_x implies that $\bar{L}^S(c) = 1$ for all $c > c_x$. Thus, traders who do not trade with certainty are *available* in the limit, with availability defined as L^B or L^S converging to one.

A sequence of outcomes satisfies *Weak Pairwise Efficiency* (Condition 4) relative to a pair of functions L_k^B and L_k^S if for any pair of types c and v for which $\bar{L}^S(c) = 1$ and $\bar{L}^B(v) = 1$, the sum $\bar{V}^S(c) + \bar{V}^B(v) \geq v - c$. Thus, for all pairs of traders v and c who are available, the sum of the expected payoffs exceeds the private surplus between the types. By construction, *Availability* and *Weak Pairwise Efficiency* hold if and only if *Pairwise Efficiency of Available Types* is true.

3.2 Main Result

In this section, I state and prove my main result. A sequence of outcomes converges is said to converge pointwise to the set A^W if it converges pointwise everywhere, except possibly at p^w. The result weakens the pairwise efficiency requirement from Lemma 2 and requires pairwise efficiency only for a subset of "available" types. The No Rent Extraction condition is then used to extend pairwise efficiency to all pairs.

Proposition 1 *Suppose some sequence $\{A_k\}_{k=1}^{\infty}$ satisfies feasibility and has uniformly bounded variation. Then, the sequence converges pointwise to the Walrasian outcome A^W if and only if the sequence satisfies Monotonicity, No Rent Extraction, and Weak Pairwise Efficiency of Available Types.*

Proof of Proposition 1: Every sequence of functions with uniformly bounded variation has a pointwise convergent subsequence by Helley's selection theorem, see above. Therefore, I can work with the limit of some convergent subsequence, $\bar{A}$.

Given the limit $\bar{A}$, define cutoff types c_x and v_x as the highest cost and lowest valuation such that traders with these types trade with certainty: $c_x \equiv \sup\left\{c|\bar{Q}^S(c) = 1\right\}$ if there is some c such that $\bar{Q}^S(c) = 1$ and $c_x = 0$ otherwise. Similarly, $v_x \equiv \inf\left\{v|\bar{Q}^B(v) = 1\right\}$ if $\bar{Q}^B(v) = 1$ for some v and $v_x = 1$ otherwise. First, I show that the No Rent Extraction condition implies

$$\begin{aligned} & \bar{V}^S(c) \geq \bar{V}^S(c_x) + (c_x - c) \quad \text{for all } c, \\ \text{and} \quad & \bar{V}^B(v) \geq \bar{V}^B(v_x) + (v - v_x) \quad \text{for all } v. \end{aligned}$$

For all types $c \in [c_x, 1]$, the first inequality follows directly by the No Rent Extraction condition: if $c_x < c$, No Rent Extraction requires that $(c_x - c) \leq \bar{V}^S(c) - \bar{V}^S(c_x)$. For types $c \in [0, c_x]$, the inequality is trivially true if $c_x = 0$; if $c_x > 0$, one can choose $\varepsilon \geq 0$ arbitrarily close to zero with $\bar{Q}^S(c_x - \varepsilon) = 1$ by definition of c_x. Hence, for all $c \leq c_x - \varepsilon$, the No Rent Extraction condition implies that $\bar{V}^S(c) \geq \bar{V}^S(c_x - \varepsilon) + (c_x - c) - \varepsilon$. Because the No Rent Extraction condition implies (Lipschitz-)continuity of the payoffs and because ε is arbitrary, $\bar{V}^S(c) \geq \bar{V}^S(c_x) + (c_x - c)$. So, the first inequality holds for all $c \in [0, 1]$. The second inequality follows for buyers by symmetric reasoning. Adding the two inequalities yields a lower bound on the sum of the payoffs of all types c and v:

$$\bar{V}^S(c) + \bar{V}^B(v) \geq v - c + \bar{V}^S(c_x) + \bar{V}^B(v_x) - (v_x - c_x). \tag{1}$$

Weak Pairwise Efficiency of Available Types implies that the right-hand side is at least $(v - c)$. Consider two cases for the ordering of c_x and v_x. First, suppose $v_x - c_x > 0$. By the definition of c_x and v_x, the trading probabilities $\bar{Q}^S(c) < 1$ for all $c > c_x$ and $\bar{Q}^B(v) < 1$ for all $v < v_x$. Therefore, by Weak Pairwise Efficiency of Available Types and by continuity of payoffs, the sum of the expected payoffs $V^S(c_x) + \bar{V}^B(v_x) \geq v_x - c_x$ in the first case. Now, consider the second case, $v_x - c_x \leq 0$. This case is trivial: Since $(v_x - c_x)$ is non-positive and payoffs are non-negative, the sum $\bar{V}^S(c_x) + \bar{V}^B(v_x) \geq v_x - c_x$. So, for both possible orderings of c_x and v_x, the sum of the last three terms from equation (1) is positive; hence, for all v and for all c, $\bar{V}^S(c) + \bar{V}^B(v) \geq v - c$ (payoffs are pairwise efficient).

By continuity of the integral operator, feasibility of each A_k implies feasibility of the limit. According to Lemma 2, feasibility and pairwise efficiency of the limit outcome implies $\bar{A} \in [A^W]$. By Monotonicity of the limit functions, the limit trading probabilities must be exactly in Q^W.[7] Similarly, continuity of payoffs implies that the limit payoffs must be exactly in V^W. (Otherwise, if for some type c, $\bar{V}^S(c) > p^w - c$, the continuity of $\bar{V}^S$ would imply that payoffs are higher than $p^w - c$ for an open set of sellers' types, implying that $S(\bar{A}) > S^*$, a contradiction.) Thus, I have proven that the limit of every convergent subsequence is A^W, which implies that $\lim_{k\to\infty} A_k = A^W$ for the original sequence.

Necessity of the conditions is shown as follows. Suppose the sequence $\{A_k\}$ becomes Walrasian. *Monotonicity*: The limit trading probabilities of sellers are monotone because those sellers with costs below p^w trade with probability one, while those with costs above p^w trade with probability zero. (Trading probabilities at p^w are not determined.) A symmetric observation applies to buyers. *No Rent extraction*: Sellers' payoffs are decreasing at a slope equal to minus one if $c < p^w$ and payoffs have a slope of zero for all $c > p^w$. So, the slope is bounded within $[-1, 0]$, and the slope is equal to -1 if $\bar{Q}^S(c) = 1$. Again, a symmetric observation applies to buyers. *Weak Pairwise Efficiency of Available Types:* Weak Pairwise Efficiency holds for all types because the Walrasian outcome is such that $\bar{V}^S(c) + \bar{V}^B(v) \geq (v - p^w) + (p^w - c)$ for all v and all c. **QED**.

Intuition. The proof starts by defining cut-off types c_x and v_x such that types above c_x and below v_x trade with probability less than one. The outcome for intermediate types with valuations and costs below v_x and above c_x is pairwise efficient by Weak Pairwise Efficiency of Available Types. The No Rent Extraction allows extending this efficiency result to the extreme types (buyers with valuations above v_x and sellers with costs below

[7] This is the only place where monotonicity is used.

c_x) who might not be available. This implies pairwise efficiency for all types. Lemma 2 implies that the outcome must be equivalent to the competitive outcome. Monotonicity and No Rent Extraction imply that the limit outcome is exactly Walrasian.

Independence of Conditions. Given Feasibility and Uniformly Bounded Variation, the No Rent Extraction condition and the Weak Pairwise Efficiency condition jointly imply that every limit outcome is equivalent to the Walrasian outcome. Monotonicity is therefore essentially implied by the other conditions. Monotonicity is stated as a separate condition because the cost of requiring it in applications is low and because it ensures exact convergence to the Walrasian outcome.

No Rent Extraction and Weak Pairwise Efficiency of Available Types are independent, either conditional on Feasibility and Uniformly Bounded Variation or not. An example where the No Rent Extraction fails but not Weak Pairwise Efficiency is discussed on p.19. Examples where No Rent Extraction may hold but Weak Pairwise Efficiency fails, are outcomes from models with entry, see Section 5.1, and models where bargaining is according to a simultaneous double auction, see p.22.

Feasibility and Uniformly Bounded Variation of the elements of the sequence are independent of the three conditions onto the limit. Feasibility is not implied by any combination of the other conditions. Examples of a sequence of outcomes satisfying all conditions except feasibility are outcomes from models with exogenous stocks, discussed in Section 5.2. Feasibility implies neither No Rent Extraction nor Weak Pairwise Efficiency. All failures of convergence to the competitive outcomes discussed in Section 4 are examples.

4 An Application of the Main Proposition

4.1 A Parameterized Class of Games

I introduce a parameterized example of a steady-state dynamic matching and bargaining game that illustrates how Proposition 1 can be applied. The parameterized example is a simplification of models by Gale (1987), Mortensen and Wright, and by Satterthwaite and Shneyerov (2008).

Traders interact repeatedly in a stationary market over infinitely many periods. At the beginning of each period, there is a *stock* of traders. This stock is characterized by the distribution of the types. $\Phi^S(c)$ is the mass of sellers in the stock with costs below c, and $\Phi^B(v)$ is the mass of buyers with valuations below v. The stock is endogenously determined. Within each period, the interaction of traders is as follows:

1. Matching. Buyers and sellers from the stock are randomly matched into groups consisting of either one buyer and one seller or one buyer and two sellers, depending on a parameter ζ. The probability that a buyer is matched with one or two sellers is $(1-\zeta)\frac{\Phi^S(1)}{M}$ and $\frac{\zeta}{2}\frac{\Phi^S(1)}{M}$, respectively, with $M = \max\left\{(1-\zeta)\Phi^S(1) + \frac{\zeta}{2}\Phi^S(1), \Phi^B(1)\right\}$. The probability that a seller is matched either alone with a buyer or together with another seller and a buyer is $(1-\zeta)\frac{\Phi^B(1)}{M}$ and $\zeta\frac{\Phi^B(1)}{M}$, respectively. If $\zeta = 0$, all matches are in pairs of one buyer and one seller, and if $\zeta = 1$ all matches are between one buyer and two sellers. The parameter ζ measures the degree of direct competition between sellers.[8] Matching is independent of the types so that the type of any given trader in a match is distributed according to the distribution of types in the stock. The matching technology is similar to De Fraja and Sakovics (2001) and allows me to capture one-to-one, and two("many")-to-one matching. Sellers do not observe whether they have a competitor.

2a. Bargaining: Observation. Within each group, the buyer observes the type(s) of the seller(s). Seller(s) observe a signal, $\hat{v} = (1-\eta)v + \eta\varepsilon$: The parameter $\eta \in [0,1]$ measures how noisy the signal is, with noise ε being distributed according to the standard normal. If $\eta = 0$, the type is perfectly observed and bargaining is with symmetric information. If $\eta = 1$, nothing about the type is observed and bargaining is with asymmetric information. Past actions are private, that is, a trader's history is private information.

2b. Bargaining: Offers. Having observed types and signals, one market side is chosen to be the proposer of a price offer; the other side is chosen to be the responder. With probability β, the buyer makes a price offer, and with probability $(1-\beta)$, the seller(s) make(s) a price offer. The other market side can either accept or reject the offer. If the buyer is chosen to propose and if there are two sellers and both accept the offer, each seller gets to trade with probability $\frac{1}{2}$. If there are two sellers and they are chosen to propose, the buyer can accept the lower of the two prices. The parameter β measures the bargaining power of the buyer.

3. Exit and Entry. After the bargaining stage, traders exit and enter the market. Those pairs of traders who reached an agreement leave the market and consume the good. Of those traders who did not reach an agreement, a share δ exits ("dies") and looses the possibility of trading. A share $(1-\delta)$ of these traders remains for the next period. Finally, there is entry by a mass $G^B(1)$ of buyers and a mass $G^S(1)$ of sellers with types distributed according to the functions G^S and G^B, defined in Section 2.

The endogenous objects in this market are the distributions of types, Φ^S and Φ^B, and the actions in the bargaining stage. The actions are denoted by $a^S = \left[p^S(c,\hat{v}), r^S(c)\right]$

[8] These matching probabilities arise if first a share ζ of sellers are bound into pairs. The resulting mass of individual sellers and pairs of sellers is $(1-\zeta)\Phi^S(1) + \frac{\zeta}{2}\Phi^S(1)$. Then, all individuals and pairs of the short side of the market are matched randomly with the long side, so the longer side is rationed.

and $a^B = \left[p^B(v, c_1, c_2), r^B(v)\right]$, where p and r denote the price-offer and acceptance (reservation-price) strategies. For example, $p^B(v, c_1, c_2)$ is the offer of type v when facing two sellers with costs c_1 and c_2. (The price offer to a single seller is encoded by setting $c_2 = 2$ as $p^B(v, c_1, 2)$.) The buyer accepts a price offer p if and only if $p \leq r^B(v)$. I collect these endogenous objects in the *market constellation* $\sigma = \left[\Phi^S, \Phi^B, a^S, a^B\right]$. Note that the exit rate acts similar to a discount rate on the individual level of the agents.[9] There is no explicit discounting. On the aggregate level, the exit rate ensures that a unique steady state exists for all strategy profiles; see Nöldeke and Tröger (2009).

Each market constellation σ determines payoffs for the traders. I denote by $q^S(c, a)$ the per-period trading probability of a seller with cost c who uses action a given σ. I denote by $Q^S(c, a)$ the probability to trade at some time (rather than exiting), the so-called lifetime trading probability. Let $P(c, a)$ denote the expected price conditional on trading. A seller's expected payoff from taking action a is denoted by $U^S(c, a)$. Payoffs are equal to the expected trading probability times the profit conditional on trade, $U^S(c, a) = Q^S(c, a)(P(c, a) - c)$. (If a seller does not trade, the profit is zero.) I define q^B, Q^B, P, and $U^B(v, a)$ symmetrically for the buyer. Given a constellation σ, maximized payoffs are denoted by $V^B(v) = sup_a U^B(v, a)$ and $V^S(c) = sup_a U^S(c, a)$.

Steady State. The stock of buyers at the beginning of a period is characterized by Φ^B. The mass of buyers at the end of the period is the sum of the entering buyers and the initial buyers who neither traded nor died. Φ^B is a steady-state stock if and only if the stock at the end of a period is the same as the stock in the beginning; that is,

$$G^B(v) + (1 - \delta) \int_0^v \left(1 - q^B(\tau, a(\tau))\right) d\Phi^B(\tau) = \Phi^B(v).$$

A similar condition has to hold for the distribution of sellers' types.

Steady-State Equilibrium. A market constellation σ^* constitutes an equilibrium if (a) the steady-state conditions hold, if (b) the actions are mutually optimal, and if (c) the acceptance decision is such that an offer is accepted if and only if it makes the receiver better off than continuation, $r^*(c) = (1 - \delta) V^S(c) + c$ and $r^*(v) = v - (1 - \delta) V^B(v)$. This latter requirement is a refinement that captures sequential rationality. Without this refinement, traders would be free to reject any off-equilibrium price offer.

[9] Butters (1979) and McAfee (1993) introduced the usage of an exit rate.

4.2 Example: Unavailable Traders

I discuss a simple example in which the limit outcome is not pairwise efficient. The example illustrates that traders who trade quickly are not available. The specification of the parameterized model is as follows: Information is symmetric (sellers observe the valuation of the buyer), matching is pairwise, and buyers have no bargaining power (only sellers make price offers), $\eta, \zeta, \beta = (0, 0, 0)$. This specification allows a straightforward equilibrium characterization. The economy is very simple: All sellers have costs of zero and there are only two types of buyers, one high valuation type, $v = 1$, and one low valuation type, $v = 0.1$. The mass of sellers is two, and the mass of each type of buyer is one.

When the exit rate is sufficiently low, it is shown that it is an equilibrium that sellers offer a price equal to one to all buyers. The equilibrium payoffs of sellers are $V^S(0) = 1/2$ while the payoffs to buyers are $V^B(v) = 0$ for both types. Sellers trade with probability one-half, buyers having a low valuation do not trade at all, and buyers having high valuations trade with probability one.

Observation: Let $\eta, \zeta, \beta = (0, 0, 0)$. Then the following is an equilibrium for all $\delta \leq 4/5$: The bargaining profile is $p^S(0, \hat{v}) = 1$, $r^S(0) = (1 - \delta) 0.5$ and $p^B(v, c_1, c_2) = 1$ and $r^B(v) = v$, $v \in \{0.1, 1\}$. The stocks are $\Phi^S(c) \equiv 1 + 1/\delta$ for all c, $\Phi^B(v) = 0$ if $v < 0.1$, $\Phi^B(v) = 1/\delta$ if $0.1 \leq v < 1$, $G^B(1) = 1/\delta + 1$. The equilibrium outcome is unique and given by $V^S(0) = 0.5$, $V^B(v) = 0$, $Q^S(0) = 0.5$, $Q^B(0.1) = 0$, $Q^B(1) = 1$.

Proof: Step 1. The stock satisfies the steady-state conditions given the bargaining profile.

A buyer and a seller trade if and only if the buyer's valuation is high. There is an equal mass of buyers and sellers in the stock. Therefore, high valuation buyers trade with probability one in any given period, $q^B(1, a) = 1$. Low valuation buyers trade with probability zero. The per-period trading probability of sellers is $q^S = 1/(1/\delta + 1)$. The steady-state conditions are easily verified. For example, the steady-state condition for buyers requires that at $v = 0.1$, $1 + (1 - \delta)(1 - 0) 1/\delta = 1/\delta$, which holds. Intuitively, if the mass of low valuation buyers in the stock is $1/\delta$, then in any period the mass of buyers who exit (die) is $(1/\delta)\delta$, which is equal to the mass of such buyers who enter.

Step 2. The expected payoffs are $V^S(0) = 1/2$ and $V^B(0.1) = V^B(1) = 0$.

This is immediate for buyers, since sellers always offer a price equal to one. For sellers, note that their lifetime trading probability is recursively defined as $Q^S = q^S + (1 - \delta)\left(1 - q^S\right) Q^S$, and, hence, $Q^S = \frac{q^S}{q^S + \delta - \delta q^S} = 0.5$, using $q^S = 1/(1/\delta + 1)$. Intuitively, a mass one of buyers with high valuations enter the market and trade. Therefore, the mass of sellers who end up trading has to be equal to one, too. Since the total mass of entering sellers is two, this implies $Q^S = 0.5$. Finally, $V^S(0) = Q^S(p - c) = 0.5(1 - 0)$ implies the claim.

Step 3. The bargaining profile constitutes an equilibrium.

Given payoffs characterized before, reservation prices satisfy the equilibrium conditions. The buyers' price offers are trivially optimal because buyers never make offers (offers need to be only ex-ante optimal). Sellers' price offers are optimal if and only if there is no incentive to decrease prices to trade with low valuation buyers. There will be no such incentive if the low valuation is below the sellers' continuation payoffs, that is, $0.1 \leq (1 - \delta)\, 0.5$, which holds if and only if $\delta \leq 4/5$.

Step 4. The outcome is unique when $\delta \leq 4/5$.

First, note that buyers' payoffs are zero in every equilibrium because sellers have all the bargaining power. Second, every equilibrium in which sellers trade only with high valuation buyers is outcome equivalent to the one described before. Third, if in some equilibrium sellers also trade with buyers having low valuations, then the share of low valuation buyers in the stock is smaller. This implies that sellers have a higher probability of trading with high valuation buyers, which implies that their continuation payoffs are higher than $(1 - \delta)\, 0.5$. Thus, offering a price equal to 0.1 would not be optimal if $\delta \leq 4/5$. Contraction. **QED.**

The limit outcome is not pairwise efficient: For $v = 1$ and $c = 0$, $\bar{V}^B(1) + \bar{V}^S(0) = 0.5 < 1 - 0$. Thus, there are "unrealized gains from trade" between those types. This is an equilibrium even when $\delta \to 0$ is because buyers with high valuations trade immediately and are not available. Intuitively, only a fraction of sellers can be successfully matched with buyers having a high type, since there are more sellers than buyers with such types who come to the market. Thus, the fact that buyers having high valuations are not available is driven by feasibility constraints.

4.3 Verification and Interpretation of the Conditions

The previous example illustrates that it is not immediate that outcomes are pairwise efficient in the limit. I now discuss how Proposition 1 can be applied to the class of dynamic matching and bargaining games introduced before.

Take a vanishing sequence of exit rates $\{\delta_k\}$ with $\delta_k \to 0$. Assume that there exists at least one equilibrium for each δ_k. Pick one equilibrium for each k, and denote the corresponding outcome by A_k. This gives a sequence of outcomes $\{A_k\}$. In the following, I denote equilibrium magnitudes corresponding to δ_k by subscripts k, such as p_k^S, p_k^B,... I argue in the remaining subsections that this sequence satisfies the conditions of the proposition and that the limit of the sequence is therefore competitive in two cases which are similar to the settings of Gale (1987) and Satterthwaite and Shneyerov (2008).[10]

The main purpose of the proof of the following Corollary is to demonstrate how the conditions are applied in a particular game. I also comment extensively on how the conditions can be verified for general matching technologies and bargaining games. The central observations are that (i) Availability follows if the matching technology is such that there is a positive probability to be matched with any set of traders from the other market side who make up a positive share of the stock, (ii) No Rent Extraction and Monotonicity hold in games where preferences (valuations and costs) are private information, (iii) Weak Pairwise Efficiency holds if the bargaining game is "not too inefficient," in a sense to be made precise.

Corollary 1 *If $\{A_k\}$ is a sequence of outcomes generated by equilibria for a vanishing sequence of exit rates $\{\delta_k\}$, then the sequence converges to the competitive outcome A^W if (i) information is asymmetric, $\beta = 0$, $\eta = 1$, $\zeta \in [0, 1]$ or if (ii) information is symmetric $\eta = 0$, matching is pairwise, $\zeta = 0$, and the buyer has bargaining power, $\beta \in (0, 1)$.*

In the following sections, I verify the four conditions. For the result above to be indeed a corollary to Proposition 1, it is necessary to show that A_k has a uniformly bounded variation and that it satisfies Feasibility. Feasibility follows immediately from the steady-state conditions. The fact that the sequence has a uniformly bounded variation is verified together with Monotonicity and No Rent Extraction.

Remark. Let me discuss the remaining cases. Without providing a proof, I conjecture that the limit is competitive whenever the distribution of bargaining power is interior, $\beta \in (0, 1)$, for all η and ζ. If sellers have all the bargaining power, $\beta = 0$, the outcome converges if either information is asymmetric, $\eta > 0$, or if there is competition among sellers with some probability, $\zeta > 0$. If $\beta = 0$ (sellers have all the bargaining power) and both, $\eta = 0$ (no noise) and $\zeta = 0$ (no competition), convergence fails, see the discussion in Section 4.5. If buyers have all the bargaining power, $\beta = 1$, convergence fails for all η and ζ, by the same argument as for $\beta = 0$.

[10] An important difference is the existence of an entry stage in these models, see Section 5.1. In addition, the set of types is discrete in the model by Gale (1987), rather than a continuum. An analogous statement of Proposition 1 for an economy with a discrete set of types would imply that the limit outcome becomes close to the competitive outcome when the set of types becomes dense in the unit interval.

4.4 Availability

The function $L_k^B(v, A_k)$ is interpreted as the probability that a seller in the stock is matched at least once during his lifetime with a buyer having a type larger than v before being forced to exit, given the exit rate δ_k and the outcome A_k. Similarly, $L_k^S(c, A_k)$ is the probability that a buyer is matched at least once with a seller having costs below c. With this interpretation, Availability is a property of the *matching technology.* Availability holds for all parameter choices of ζ, β, and η, and it holds for *all* action profiles (not just equilibrium profiles). For this, I show first that the steady-state conditions imply that types who do not trade with probability one must make up a positive share of the stock. Then, I show that the matching technology implies that, whenever a set of types makes up a positive share of the stock, the probability to match with such a type is strictly positive and non-vanishing; this implies Availability. The Availability condition is easily violated when there is an entry stage. Entry is discussed in Section 5.1.

The basic observation is the following: Traders who are less likely to trade, stay in the stock for a longer period of time and make up a larger share of it. The steady-state condition can be rewritten to show that:[11]

$$\Phi^B(v') - \Phi^B(v'') = \frac{1}{\delta}\int_{v''}^{v'} \left(1 - Q^B(\tau) + \delta Q^B(\tau)\right) dG^B(\tau). \tag{2}$$

The mass of any given type in the stock is proportional to the probability of *not* being able to trade, which is $\left(1 - Q^B(\tau)\right)$, and the mass in the inflow, dG^B. This implies, in particular, that buyers who do not trade with probability one make up a positive, non-vanishing share of the stock of traders: By equation (2), the mass of these buyers is proportional to $\delta^{-1}\left(1 - Q^B\right)\left(G^B(v') - G^B(v'')\right)$, while the total mass of all buyers (and sellers) is at most $\delta^{-1}G^B(1)$ (by taking the integral from 0 to 1 at $Q^B = 0$). The relation between the probability of not trading and the share in the stock is independent of the specific matching technology and follows mechanically from the steady-state conditions.

The probability to be matched in any given period with a buyer with type at least as large as v is denoted by $X^B(v)$, and the probability to be matched with a seller with type at most as high as c is denoted by $X^S(c)$. For example, the probability of being matched with a buyer from the set $[v, 1]$ is $X^B(v) = \left(\Phi^B(1) - \Phi^B(v)\right) M^{-1}$. The probability for

[11] Evaluating the steady conditions for $\Phi^B(v'') - \Phi^B(v')$ and reordering terms implies that $\int_{v'}^{v''} \left(1 - (1-\delta)\left(1 - q^B(\tau, a(\tau))\right)\right) d\Phi^B(\tau) = G^B(v'') - G^B(v')$. Multiplying both sides of the identity pointwise by $\left(1 - (1-\delta)\left(1 - q^B\right)\right)^{-1}$, yields $\Phi^B(v') - \Phi^B(v'') = \int_{v'}^{v''} \left(\frac{1}{q^B + \delta - q^B\delta}\right) dG^B$. Rewriting further by using $Q^B = \frac{q^B}{q^B + \delta - q^B\delta}$ implies the claim.

a seller to be matched at least once during his lifetime with a buyer who has a type at least as large as v is denoted by L^B,[12]

$$L^B(v) = X^B(v) + \left(1 - X^B(v)\right)(1-\delta) L^B(v). \tag{3}$$

As apparent from the definition, when $\delta_k \to 0$, the probability $L_k^B(v)$ converges to one if and only if the per-period matching probability is large relative to the exit rate,

$$L_k^B(v) \to 1 \quad \Leftrightarrow \quad \frac{X_k^B(v)}{\delta_k} \to \infty. \tag{4}$$

I show that the prior observations imply that the Availability condition holds. Suppose there is some v_x such that the limit trading probability is smaller than one for all types below. Take any v'' and v' below v_x to define an interval $[v'', v']$ below v_x for which the probability of not trading is strictly positive. Using equation (2), I have argued that the share of these types in the stock must be strictly positive and non-vanishing in the limit. The probability that a seller is matched in any given period with a buyer who has valuation at least v'' is equal to the share of buyers with these types in the stock. Therefore, the previous observation that the share of these types is strictly positive and non-vanishing in the limit implies that $\liminf X_k^B(v'') > 0$. Hence, the ratio $X_k^B(v)/\delta_k$ diverges to infinity, and, by observation (4), types $v \geq v''$ become available, $L_k^B(v'') \to 1$. Thus, I have now demonstrated that the Availability condition holds in the model relative to L^B as defined before, for all parameters ζ, β, and η. The Availability condition for the seller's side relative to an analogously defined function L^S follows from the same logic. I have not used any assumption on the bargaining profile. Availability is indeed a property of the matching technology only. Except for the steady state conditions, no further equilibrium conditions are used.

One can extend the arguments from before to other matching technologies beyond the parameterized example. First, the relation between the trading probability and the share of types in the stock, documented in equation (2), follows solely from the steady-state conditions. Therefore, it is sufficient for Availability that the matching technology is such that there is a strictly positive, non-vanishing probability to be matched with any set of types that make up a positive share of the stock. Importantly, it is not necessary to calculate an equilibrium to check whether or not a given matching technology implies

[12] Note that by (2), the trading probabilities Q_k^B and the exit rate δ_k uniquely determine the stock Φ^B and, therefore, the matching rate $X_k^B(v)$. Thus, for given δ_k, (2) allows to define $L_k(v, A_k)$ as a function of A_k only, without reference to Φ^B, as required for the application of the conditions.

a positive matching probability with types having a positive share in the stock. In the current example, this property follows immediately from the matching function.

Let me discuss the requirement that the trading probabilities are below one for all types that are above c_x or below v_x, respectively. The condition could be relaxed to require only that there is some arbitrarily small ε such that for all types in $(c_x, c_x + \varepsilon)$ or $(v_x - \varepsilon, v_x)$ trading probabilities are below one. What is needed is that the set of such type has positive mass. In particular, if the set of types were discrete rather than a continuum, an equivalent condition would require availability of any single type who trades with probability less than one. It is because of this reason that I summarize this condition as requiring availability of those types who do not trade with certainty, despite the fact that the statement of the condition imposes the much stronger requirement that all types above (below) do not trade with certainty, too.

Failure of Availability with Entry. Availability does not hold in models with an entry stage: Agents who do not enter are not available even though they trade with probability zero. Entry is discussed in Section 5.1.

4.5 Monotonicity and No Rent Extraction

The Monotonicity and the No Rent Extraction conditions are immediate whenever bargaining takes place under asymmetric information. With asymmetric information, the trading probability and the expected price paid by an agent depend only on the action that is chosen in the bargaining game but not on the type. In such games, Monotonicity and No Rent Extraction follow from incentive compatibility conditions.

Bargaining is said to be under asymmetric information if the sellers' signals about the buyers' willingness to pay are not informative, $\eta = 1$, and if the buyers never make offers, $\beta = 0$.[13] I discuss the sellers' side (the buyer's side is analogous). In equilibrium, the optimality condition requires that the action that is chosen by a type c, $a(c)$, maximizes expected payoffs, $a(c) \in \arg\max U^S(c, a)$, and the equilibrium payoff is given by $V^S(c) = \max_a Q^S(a, c)(P(c, a) - c)$. If $\eta = 1$ and $\beta = 0$, the trading probability and the expected price do not depend on the type but only on the action, that is, $Q^S(a, c') = Q^S(a, c'')$ and $P(c', a) = P(c'', a)$ for all actions and for all types c' and c''. Now, the desired properties follow from standard reasoning about Bayesian incentive compatibility when expected utility is linear in the type (see, e.g., Mas-Colell, Whinston, and Green, 1995, Proposition

[13]Buyers should never make offers, because, by assumption, buyers observe the types of the sellers. This assumption is made to keep the notation simple and, if buyers make offers but do not observe the sellers' types, symmetric arguments apply.

23.D.2). Optimality of equilibrium actions requires that for any two types c' and c'',

$$\begin{aligned} V^S(c') = Q^S(a(c'), c') \ \ (P(c', a(c')) - c') &\geq Q^S(a(c''), c'') \ \ (P(c'', a(c'')) - c') \\ &= V^S(c'') + Q^S(a(c''), c'')(c'' - c'). \end{aligned}$$

Using a similar revealed preference argument for $V^S(c'')$ implies the following bound on the payoff difference:

$$Q^S(a(c'), c')(c'' - c') \geq V^S(c') - V^S(c'') \geq Q^S(a(c''), c'')(c'' - c'). \tag{5}$$

This bound holds for every exit rate. Given a sequence of equilibrium trading probabilities and payoffs with pointwise limits $\bar{Q}^S$ and $\bar{V}^S$, it must be true that

$$\bar{Q}^S(c')(c'' - c') \geq \bar{V}^S(c') - \bar{V}^S(c'') \geq \bar{Q}^S(c'')(c'' - c'). \tag{6}$$

Suppose that $c'' > c'$. The inequalities (6) imply that the equilibrium trading probabilities are monotone non-increasing, $\bar{Q}^S(c') \geq \bar{Q}^S(c'')$. Thus, trading probabilities satisfy the Monotonicity condition. Moreover, the No Rent Extraction condition holds. Suppose that $c' > c''$. The first part of the condition is immediate since $\bar{Q}^S(c')(c'' - c') \leq 0$ and $\bar{Q}^S(c'') \leq 1$ together imply that the slope of the payoffs is bounded between zero and minus one, $0 \geq \bar{V}^S(c') - \bar{V}^S(c'') \geq (c'' - c')$ for $c' > c''$, as required. For the second part of the condition, suppose that $\bar{Q}^S(c'') = 1$. Then, the second inequality from (6) implies that $\bar{V}^S(c') \geq \bar{V}^S(c'') + (c'' - c')$, as required.

Bounded Variation. With asymmetric information, every sequence of equilibrium outcomes has uniformly bounded variation. The inequalities (5) imply that the equilibrium trading probabilities and the equilibrium payoffs must be monotone functions for *all* exit rates. Thus, if $\{A_k\}$ is a sequence of equilibrium outcomes, all of the elements of the sequence are monotone functions. Moreover, by definition, the trading probabilities and the payoffs are uniformly bounded by zero and one. For a family of uniformly bounded functions, monotonicity is a sufficient condition for uniformly bounded variation (see Kolmogorov and Fomin, 1970); so, the claim follows.

With asymmetric information, it is not necessary to fully characterize equilibrium in order to check whether or not it is true that the Monotonicity and the No Rent Extraction conditions hold. Instead, with asymmetric information, the Monotonicity and the No Rent Extraction condition follow from the fact that equilibrium outcomes must satisfy standard incentive compatibility constraints.

Failure of No Rent Extraction with Symmetric Information. According to the discussion before, the No Rent Extraction condition is most likely to fail in a situation with *symmetric* information. Consider the basic example with symmetric information in which only sellers make price offers ($\beta = 0, \eta = 0, \zeta = 0$). This case is analyzed in Lauermann (2011).[14] It is shown that the limit outcome is not the Walrasian outcome. The reason for the failure of convergence is the failure of the No Rent Extraction. Since sellers have all the bargaining power, they receive the whole trading surplus. Consequently, buyers' payoffs are zero, independent of their type; that is, the *rent* of the buyers is *extracted* (this is what motivated the name of the condition). Because it can be shown that there must be some interior type of buyer who trades with probability one in the limit, the fact that the buyers' payoffs are constant at zero implies that the No Rent Extraction condition is violated. The other three conditions continue to hold.

Interior Bargaining Power. In Gale's (1987) original model, buyers can make offers as well; that is, β is strictly positive. In this case, the basic example with symmetric information has a property that makes it similar to a game with asymmetric information. Consider, again, the basic example with symmetric information and pairwise matching. But, in contrast to the case considered in the previous paragraph, suppose buyers have some bargaining power ($\beta \in (0,1), \eta = 0, \zeta = 0$). Although it is still true that a trader of type v does not need to **receive** the same offers as a trader of type v_x, such a type can **make** the same offers when chosen to be the proposer. Importantly, in equilibrium, payoffs depend only on the offers made when chosen to be the proposer. (If a trader is chosen to be the responder, the offer is such that the responder is just indifferent between accepting and rejecting.) Therefore, a buyer of type v can mimic the strategy of another type v_x just as the buyer can mimic the actions of another type with asymmetric information. This is sufficient to restore No Rent Extraction.[15] Since the other conditions hold as well, this implies that the bargaining game with symmetric information is Walrasian. Somewhat surprisingly, it turns out that interior bargaining power and private information play a similar role, namely, ensuring that the No Rent Extraction condition is satisfied.

The Role of Information. Intuition derived solely from the Myerson-Satterthwaite

[14] Lauermann (2011) considers homogenous sellers only. However, the analysis extends to sellers having costs distributed according to a smooth distribution on the unit interval.

[15] Given some equilibrium σ^*, let $P^P(v)$ and $Q^P(v)$ be the expected price and trading probabilities of a buyer having type v who rejects all offers when chosen to respond, but who makes optimal offers when chosen to propose. By the reasoning in the text, equilibrium payoffs of a trader depend only on the offers made when the trader is chosen to propose. Therefore, $V^B(v) = Q^P\left(v - P^P\right)$. This implies that, for any two types v and v', $V^B(v) \geq V^B(v') + Q^P(v')(v - v')$. Together with the observation that $Q_k^P(v)$ converges to one along any sequence of equilibria for which $Q_k(v)$ converges to one, Monotonicity and No Rent Extraction follow.

impossibility theorem suggests that asymmetric information is detrimental to efficiency. However, this is not the case here. Asymmetric information directly implies that two of the four conditions hold. Moreover, consider the example in which sellers have all the bargaining power and face no direct competition, $\beta = 0$ and $\zeta = 0$. Then, the limit outcome is efficient if information is asymmetric ($\eta = 1$), but the limit is inefficient if information is symmetric ($\eta = 0$), as shown in Lauermann (2011) and as discussed in the previous paragraphs. The current framework allows interpreting the counterintuitive findings about private information as ensuring the No Rent Extraction condition.

4.6 Weak Pairwise Efficiency

Bargaining protocols with symmetric information that specify a surplus sharing rule—such as Nash bargaining—satisfy Weak Pairwise Efficiency since the total expected surplus is always realized. In general, it is critical that the bargaining protocol is not "too inefficient." This is a new characterization of bargaining protocols. Specifically, a bargaining protocol is said to be not too inefficient, if, whenever the expected surplus between two traders is positive, at least one of the traders can realize a positive, non-vanishing fraction of this surplus (a formal definition follows).[16] Conversely, if the sum of the expected payoffs for two types of traders is zero despite the existence of a positive expected surplus for each of them, the condition does not hold. Two reasons for the existence of unrealized surplus are discussed at the end of this section. First, in the bargaining phase, traders might be stuck in a "bad" Nash equilibrium when actions are chosen simultaneously. Second, traders might not try to realize existing surplus for fear of "punishment" in the future.

Consider an exit rate δ and a constellation σ. Given a pair of types (v, c), I define

$$\begin{aligned} \Delta(v,c) &= \max\left\{v - c - (1-\delta)\left(V^S(c) + V^B(v)\right), 0\right\}, \\ x(v,c) &= \min\left\{X^B(v), X^S(c)\right\}, \end{aligned}$$

where Δ is the surplus available between the types, and where x is the minimum of the probabilities that the seller is matched with a buyer of type at least v, and the probability that the buyer is matched with a seller with cost at most c. When V^S and V^B are monotone with absolute slopes bounded by one, Δ is increasing in v and decreasing in c. If $x(v,c)\Delta(v,c)$ is positive, then, for each type, the expected surplus that is available in any given match is positive.

[16] This is again a condition on outcomes (of bargaining) to ensure that the analysis is general.

I rewrite payoffs recursively. Let π^S be the expected net gain conditional on trading. Payoffs are $V^S = q^S \left(\pi^S + (1-\delta) V^S\right) + \left(1 - q^S\right)(1-\delta) V^S$. Reordering terms yields $\delta V^S = q^S \pi^S$. Given some specification of the game, let γ^S be a uniform lower bound such that $\delta V^S \geq \gamma^S x \Delta$ for all types c, all exit rates δ, and all equilibria. The parameter γ^S measures how much of the expected surplus is realized by the seller. Let γ^B be an analogous uniform bound for buyers, so that $\delta V^B \geq \gamma^B x \Delta$. Such bounds exists trivially because $\gamma^S = \gamma^B = 0$ suffice. A bargaining protocol for which the sum of the bounds $\gamma^S + \gamma^B$ is not trivial is said to be "not too inefficient." As shown below, most standard bargaining protocols are not too inefficient.

If a positive share of the surplus can be realized, Weak Pairwise efficiency holds. Take a sequence of market constellations σ_k, and suppose that types v_x and c_x become available. By definition of γ^S and γ^B, the sum of their payoffs is bounded from below by

$$V_k^S (c_x) + V_k^B (v_x) \geq \left(\gamma^S + \gamma^B\right) \frac{x_k}{\delta_k} \Delta_k.$$

Suppose there are non-trivial lower bounds such that $\left(\gamma^S + \gamma^B\right) > 0$. Since the types v_x and c_x are available by hypothesis, $x_k / \delta_k \to \infty$, from (4). Therefore, it must be the case that the surplus between c_x and v_x becomes zero, $\Delta_k \to 0$, for otherwise the right-hand side of the displayed equation would become infinite. If $\Delta_k \to 0$, then, by definition of Δ_k, payoffs become pairwise efficient, $\bar{V}^S + \bar{V}^B \geq v_x - c_x$. Thus, Weak Pairwise Efficiency holds whenever nontrivial uniform lower bounds exist, that is, whenever the bargaining protocol is not too inefficient.

Let me consider a model similar to Gale (1987) with symmetric information, $\eta = 0$, no competition, $\zeta = 0$, and $\beta \in (0,1)$, where β measures the distribution of bargaining power. It is straightforward to verify that the buyer and the seller can expect a share $\gamma^B = \beta$ and $\gamma^S = (1-\beta)$, respectively. I verify this for the seller. The seller's payoffs are

$$\delta V^S (c_x) = (1-\beta) \int \Delta (c_x, v) \, dX^B (v) .$$

This equation holds because, whenever a seller is matched with a type v and chosen to be the proposer, the seller makes an acceptable offer that captures the entire surplus $\Delta (c_x, v)$. When chosen to be the responder, the seller captures nothing over and above the continuation value. Observing that in equilibrium the surplus $\Delta (c_x, v)$ is nondecreasing in v, the above formula implies in particular that $\delta V^S (c_x) \geq (1-\beta) \, x (v, c_x) \, \Delta (v, c_x)$ for all v; therefore, $\gamma^S = (1-\beta)$ is indeed a uniform lower bound on the seller's share of the expected surplus $x\Delta$. The bound is independent of the type, the exit rate, and the equilibrium as required.

Finally, let me consider a model that is similar to Butters (1979), Satterthwaite and Shneyerov (2008), and Lauermann (2008). In these models, offers are exclusively made by sellers, $\beta = 0$, and information is asymmetric, $\eta = 1$.[17] There may be competition $\zeta \in [0, 1)$. With asymmetric information, bargaining cannot be efficient. Still, traders can expect to realize a non-zero share of the expected surplus. In fact, for these parameters, $\gamma^S = (1 - \zeta)$. To see why, suppose the seller offers a price that is equal to the maximal willingness to pay of some type v_x; that is, suppose that $p_x = r(v_x)$, were $r(v_x) = v_x - (1 - \delta) V^B(v_x)$ by the equilibrium requirement. In equilibrium, the reservation price is increasing in the buyer's valuation so that all buyers with valuations $v \geq v_x$ accept the offer. The probability to be matched with such a buyer is $X^B(v_x)$. The probability to have no competitor is $(1 - \zeta)$. Therefore, the price offer is accepted with a probability of at least $(1 - \zeta) X^B(v_x)$. The payoff from offering p_x provides a lower bound on the equilibrium payoff,

$$\begin{aligned} \delta V^S(c_x) &\geq x(1-\zeta)\left(p_x - c_x - (1-\delta) V^S(c_x)\right) \\ &= x(1-\zeta)\left(v_x - (1-\delta) V^B(v_x) - c_x - (1-\delta) V^S(c_x)\right) = (1-\zeta) x\Delta. \end{aligned}$$

Therefore, $\gamma^S = (1 - \zeta)$ is a uniform lower bound of the seller's share of the expected surplus.[18] The bound is independent of the type, the exit rate, and the equilibrium.

As I have demonstrated, it is not necessary to fully calculate the equilibrium in order to check whether equilibrium payoffs admit a nontrivial lower bound. It is sufficient to show that individual agents have actions available that ensure a minimal payoff. I did use an equilibrium requirement on reservation prices that implies that the other agents accept an offer whenever accepting the offer is individually rational. The example below demonstrates the significance of this requirement.

Failure of Weak Pairwise Efficiency with Simultaneous Auctions. Serrano (2002) specifies the bargaining protocol as a simultaneous double auction.[19] He shows that equilibrium outcomes do not need to become competitive. I can replicate the main features of his bargaining protocol in the basic example by dropping the equilibrium requirement on reservation prices and analyzing the larger set of "Nash equilibria" instead. Suppose

[17] In Satterthwaite and Shneyerov (2008), the roles of buyers and sellers are reversed: A random number of buyers make price offers to a single seller. The seller cannot commit to an optimal ex-ante reservation price, but, ex post, he can either accept or reject the highest offered price.

[18] It can be verified that $\gamma^S + \gamma^B > 0$ if $\zeta = 1$ and sellers face competitors with certainty.

[19] His interest stems from the prior use of simultaneous auctions in dynamic matching and bargaining games in the context of common values.

matching is pairwise and sellers are always chosen to propose, $\zeta = 0$ and $\beta = 0$. Consider the following action profile with trading at an arbitrary price $\bar{p}$: Sellers offer the price $\bar{p}$ if $c \leq \bar{p}$ and they offer $p = 1$ otherwise. Buyers choose a reservation price $r = \bar{p}$ if $v \geq \bar{p}$ and $r = 0$ otherwise. This action profile constitutes a mutual best response for all δ_k. Thus, the action profile together with the induced steady state stock implies an equilibrium outcome for every exit rate. Fixing an action profile with a price $\bar{p}$ as described before for a sequence of exit rates defines a sequence of equilibrium outcomes. The sequence of outcomes satisfies Monotonicity, No Rent Extraction, and Availability for every $\bar{p}$. Availability holds for every $\bar{p}$ because, by the observations from Section 4.4, the matching technology is such that Availability holds for *every* bargaining profile. Monotonicity and No Rent Extraction are immediate as well. Moreover, if the price $\bar{p}$ is equal to the Walrasian price, then the limit outcome will be Walrasian. However, if the price $\bar{p}$ is not equal to the Walrasian price, then no limit of the sequence is Walrasian and, as argued in the next paragraph, the Weak Pairwise Efficiency condition fails. Thus, the non-convergence result can be attributed to the failure of Weak Pairwise Efficiency.

To see that Weak Pairwise Efficiency fails, suppose that $\bar{p} > p^w$. By the specification of the bargaining profile, buyers with a valuation below $\bar{p} - \varepsilon$ never trade and, therefore, $\bar{Q}^B(\bar{p} - \varepsilon) = \bar{V}^B(\bar{p} - \varepsilon) = 0$. Since their trading probability is below one, these buyers are available. For sellers, note that all types below p^w make the same offer and, therefore, these types trade with the same probability. This trading probability must be bounded away from one by the feasibility constraints.[20] The fact that the sellers' trading probability is below one implies that, (i), sellers' payoffs are bounded away from $(\bar{p} - c)$, $\bar{V}(c) < (\bar{p} - c)$ and, (ii), sellers with costs below $\bar{p}$ are available. Take some pair of types with $c < \bar{p}$ and $v = \bar{p} - \varepsilon$. Since these types are available, Weak Pairwise Efficiency requires that $\bar{V}^S(c) + \bar{V}^B(\bar{p} - \varepsilon) \geq \bar{p} - c - \varepsilon$ for all ε. However, the earlier observations imply that, for small enough ε, $(\bar{p} - c - \varepsilon) > \bar{V}^S(c) + \bar{V}^B(\bar{p} - \varepsilon)$, since $\bar{V}^B(\bar{p} - \varepsilon) = 0$ and $\bar{V}^S(c) < \bar{p} - c$.

Failure of Weak Pairwise Efficiency with Observable Actions. I use the fact that the continuation payoffs are fixed when proving Weak Pairwise Efficiency. The assumption that continuation payoffs are independent of a trader's current action is motivated by the assumptions that histories are private information and that there is a continuum of traders. Therefore, deviations cannot trigger a change in continuation payoffs, since, given the random matching technology, a trader will almost never meet the same partner again (or the partner's partner or the partner's partner's partner, etc.).

[20]The mass of buyers with valuations above $\bar{p}$ exceeds the mass of sellers below $\bar{p}$, since, by definition of p^w and the assumption that $\bar{p} > p^w$, $G^S(\bar{p}) > 1 - G^B(\bar{p})$. Thus, feasibility implies that the trading probability of the sellers cannot be one; see Section 4.2.

If actions are publicly observable, however, Weak Pairwise Efficiency might fail. Traders do not necessarily have an incentive to deviate from the equilibrium in order to realize sure gains from trade because of the potentially negative impact of their deviation on their future trading opportunities. (Or, alternatively, because rejecting an offer is rewarded by a subsequent increase of the continuation payoffs.) Rubinstein and Wolinsky (1990) construct non-competitive equilibria in a model with a finite number of agents and observable actions. They also assume that traders are not forced to break up the match after no agreement is reached. The non-competitive equilibrium can be interpreted as a community enforcement of "fair" prices. In a recent paper, Gale and Sabourian (2005) introduce "complexity costs" and show that, if traders prefer "simple" strategies, such non-competitive equilibria cannot be supported.

5 Extensions and Conclusion

5.1 Entry

Many steady-state models include an entry stage. With an entry stage, traders who choose not to enter are unavailable even though they do not trade with probability one. An entry stage typically implies multiplicity of equilibrium because it is an equilibrium for all traders to not enter. If no other trader enters, not entering is a best response. A sequence of no-trade equilibrium outcomes violates the last two conditions of the main result. The trading probability is zero for every pair of types but the sum of their payoffs is not pairwise efficient.

An example of a steady-state model with entry is considered in Gale (1987). Abstracting from sequences of equilibria in which the number of actual trades vanishes, it is shown that every sequence of equilibrium outcomes becomes competitive when frictions become small. Since the stated conditions of Proposition 1 are necessary for convergence, sequences of equilibrium outcomes with non-vanishing trade satisfy the conditions. However, for a model with entry it is not possible to directly verify the Availability condition, with availability defined as in the parameterized model: With an entry stage, there are some strategy profiles for which the life-time matching probabilities for some types are zero even though the trading probability is below one (this is trivial if the types do not enter). Thus, to prove that sequences of outcomes with non-vanishing trades satisfy the Availability condition, one would need to calculate equilibrium outcomes first.

It is possible to modify the Availability and Weak Pairwise Efficiency condition so that they are directly verifiable. Specifically, one may require the following: If, for any pair of types c_x and v_x for which it is true that there is either a non-empty interval (c', c_x) such that $0 < \bar{Q}^S(c) < 1$ for all $c \in (c', c_x)$ or there is a non-empty interval (v_x, v') such that $0 < \bar{Q}^B(v) < 1$ for all $v \in (v_x, v')$, then payoffs are $\bar{V}^S(c) + \bar{V}^B(v) \geq v - c$ for all $c \in (c', c_x)$ and $v \in (v_x, v')$. The requirement of a strictly positive trading probability ensures that types from the respective intervals must have entered. This modified condition can be directly verified in models with entry without deriving equilibrium. Intuitively, if there is a set of types (c', c_x) who enter the stock and who trade with probability less than one, then these types must make up a large share of the stock and buyers must be matched frequently with such types. One can prove the following analogous result to Proposition 1: Let A_k be a sequence of feasible outcomes with uniformly bounded variation and non-vanishing trading volume.[21] Then, a limit outcome exists and is equal to the Walrasian outcome if and only if the sequence satisfies Monotonicity, No Rent Extraction, and the modified condition discussed above.

The conditions can be verified directly[22] in the models by Gale (1987) and by Satterthwaite and Shneyerov (2008), using the methods introduced in Section 4. One finding is that, with entry, convergence requires a more efficient bargaining protocol. This is reflected by the modified condition which requires pairwise efficiency already if only one market side is "available" (trades with probability strictly between zero and one).[23]

5.2 Exogenous Stocks and the Failure of Feasibility

In their seminal paper on dynamic matching and bargaining games, Rubinstein and Wolinsky (1985) consider a model where the stock of agents is exogenous. The composition and the size of the stock are kept constant over time by replacing exiting agents. A model with a similar feature has also been used by De Fraja and Sakovics (2001). Since the stock is exogenous, it is natural to interpret the stock of traders as the relevant economy. In both models it is shown that when discounting is removed, outcomes generically do not become Walrasian with respect to the economy defined by the stock of traders. Rubinstein and Wolinsky's finding of a non-competitive yet "frictionless" limit outcome has sparked

[21] Formally, the trading volume must satisfy $\liminf \int Q_k^S dG^S + \int Q_k^B dG^B > 0$.

[22] However, outcomes fail the feasibility condition because of the presence of discounting and entry costs in these models. The feasibility condition requires that the expected payoff equals the trading surplus *exactly* while discounting and entry costs may lead to "waste". This failure is not consequential; one needs only feasibility of the limit outcome which holds in both models.

[23] For example, Satterthwaite and Shneyerov (2008) cannot allow the seller to run an optimal auction with an ex-ante reservation price, for otherwise the limit fails to be competitive. In constrast to the model with entry, one can use the current results to show that in an analogous model without entry the limit is also competitive when sellers use optimal auctions (set ex-ante reservation prices).

much of the interest in dynamic matching and bargaining games. In the following, I will relate their result to the failure of the feasibility condition.

Rubinstein and Wolinsky (1985) prove that the limit outcome is characterized by a price $\hat{p}$ which depends on the parameters of the game (the bargaining power). Given the price, payoffs are $v - \hat{p}$ for the buyers and $\hat{p} - c$ for the sellers. Therefore, the outcome is pairwise efficient: $V^B(v) + V^S(c) \geq v - \hat{p} + \hat{p} - c = v - c$. Lemma 2 implies that a pairwise efficient outcome is competitive if and only if it is feasible. Therefore, limit outcomes from Rubinstein and Wolinsky (1985) are, in fact, competitive whenever they are feasible. However, as shown by Rubinstein and Wolinsky, for generic bargaining power, the price is not competitive relative to the stock of traders. Thus, generically, the outcome is also not feasible relative to the stock. De Fraja and Sakovics (2001) report a similar finding: The limit outcome is characterized by trade at a common price and the limit is pairwise efficient. Therefore, limit outcomes are competitive if and only if they are feasible. But, again, it is shown that the common trading price is generically not competitive and, by Lemma 2, the outcome is generically not feasible, too.[24] Thus, in these papers, non-convergence to the competitive outcome is implied by the fact that limit outcomes are not feasible. Outcomes that are not feasible cannot possibly be competitive.[25]

The fact that the stock is exogenous has two implications. First, all types are available. A trader has a positive chance to be matched with any set of types that has a positive share in the stock. Therefore, in the limit, all pairs are formed frequently, which helps in establishing pairwise efficiency directly. Second, since the stock is exogenous, it is possible that all traders from a large set of sellers trade with a small set of buyers, as measured by their shares in the stock. This allows for the failure of feasibility of the outcome relative to the stock. For example, in Rubinstein and Wolinsky's model, even if the mass of sellers in the stock exceeds the mass of buyers, all sellers can end up trading in equilibrium. This is not possible in a model in which the inflow is exogenous: If the mass of sellers exceeds the mass of buyers in the inflow, only a fraction of the sellers can trade, see Section 4.2.

Gale (1987) makes two well-known observations about the finding by Rubinstein and Wolinsky (1985). These observations are different from the observation that the feasibility constraints fail. First, he argues for making the stock an endogenous equilibrium object

[24] De Fraja and Sakovics show that there exist parameter combinations for their model for which the trading price is competitive, see their Proposition 5 and Proposition 7. This set of parameter combinations is a plane in the three-dimensional parameter space of their model. Generically, the price is not competitive.

[25] Note that the observations follow already from the original characterization result by Shapley and Shubik (1971). Thus, the observations from this section demonstrate the usefulness of cooperative characterization results in general rather than the usefulness of the Proposition 1 in particular.

instead of taking the stock as a primitive. His main argument is that, in many economic applications, the stock and, hence, the matching probabilities are endogenous (Gale, 1987, p. 21). Second, he argues that, in his model, the exogenous flow is the appropriate static economy relative to which one should interpret the limit price. The stock is not the appropriate benchmark because, in his model, the stock is endogenously determined by the equilibrium conditions and, therefore, the stock cannot be interpreted as a primitive of the model (Gale, 1987, p. 28).

5.3 Concluding Remarks

The paper introduces a modification of a well-known characterization result from cooperative game theory for quasilinear economies which states that a feasible outcome is competitive if and only if it is pairwise efficient. I argue that this existing characterization result cannot be used effectively to argue when and why outcomes of decentralized markets are competitive when frictions are small. In particular, I provide a simple example which demonstrates that in a dynamic search and bargaining game not all traders are available to be matched with. The reason is that it is inherently difficult to match with those types that trade fast. Thus, there might be unrealized gains from trade for pairs of types when one type is not available.

Motivated by this observation, I derive a new characterization result: An outcome is equivalent to the competitive outcome if and only if (i) it is pairwise efficient for a subset of types that trade with probability less than one and (ii) payoffs have a slope that is bounded in a particular way. I argue that these conditions are directly verifiable in many games—in contrast to the original characterization result. I discuss extensively what properties of the matching technology and of the bargaining protocol ensure that these conditions holds and I explain for what properties the conditions fail. A parameterized example demonstrates how the characterization result can be used to investigate what causes divergent results in the literature and how the characterization result generalizes insights from the analysis of specific dynamic search and bargaining games.

Decentralized markets typically lack clearly specified trading procedures.[26] Aumann (1987) argues that methods from cooperative game theory are particularly well suited to gain insights into such "amorphous" economic environments. This paper demonstrates that cooperative methods might indeed be useful for the analysis of decentralized markets.

[26] "The markets which are best organized from a competitive standpoint are those in which purchases and sales are made by auctions .... City streets with their stores and shops of all kinds —baker's, butcher's, grocer's, taylor's, shoemaker's, etc.— are markets where competition, though poorly organized, nevertheless operates quite adequately." (Walras, 1874); quoted in Daggan, Serrano, Volij (2000).

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