Fair division of income distribution, development and growth:evidence from a panel of countries.

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Fair Division of Income Distribution, Development and Growth[†]: Evidence from a Panel of Countries

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Abstract

I use an unbalanced panel data to explore the correlation between aggregate income per capita and income inequality. A lot of studies document controversial results using the Gini index or other summary measurements of income inequality. I measure income inequality by the two dimensions of a point on the Lorenz Curve, where the Lorenz curve has unit slope. It is called the fair division point, which involves the fair population share and the fair income share. The difference between the fair population share and the fair income share approximates the Gini index of an income distribution.

My analysis shows that a country's low income population relatively decreases (the fair population share drops slightly) as the economy grows; and at the same time, those low income households are relatively worse off (the fair income share falls even though the GDP per capita increases). Inversely, as an economy becomes rich, there are more low income households (the fair population share increases), but those low income households are better off (the fair income share goes up and GDP per capita increases as well). Overall, both the Gini index and the difference between the fair population share and the fair income share have been increasing during the last half century in the panel of countries. Therefore, income inequality increases as an economy is getting richer.

The analysis presents significant evidence for optimum income inequality regarding both the aggregate productivity and the growth rate of GDP, where income inequality is measured by either the Gini index or the fair division shares. But no evidence has been found for the Kuznets' hypothesis. Both high and low inequality of income distribution could harm an economy as we compare with its potential optimum inequality. Also developed economies show different optimum inequality from that in developing economies, and there is the growth-worst fair population share that results in the lowest growth in developed economies. Measurement of income inequality matters on its economic effects for the subsamples of the panel data.

Keywords: Gini index, Fair population share, Fair income share, Development and growth

JEL Classification: J24, E25, J62, O12.

1 Introduction

I use an unbalanced panel data to explore the correlation between aggregate productivity and income inequality. A lot of studies document controversial results using the Gini index or other summary measurements of income inequality. I use a two-dimensional metric to describe income inequality, where the Lorenz curve has unit slope. It is called fair division point, which involves the fair population share and the fair income share, and where the Lorenz curve has unit slope. The difference between the fair population share and the fair income share approximates the Gini index of an income distribution.

Economists have done a lot of work to explore how an economy will be affected by income inequality, but it turns out to be a complex problem. Ranking income distribution by a summary measurement of income inequality was an attempt to address the problem, which proved quite unsuccessful, because when two Lorenz curves cross each other with the same Gini index (coefficient), we have no way to rank them.

Partha Dasgupta et al. (1973) prove that strict Schur-concavity[1] is a sufficient and necessary property for a summary measure to rank income distribution. The Gini index is Schur-concave, but not strictly Schur-concave, so that the Gini index cannot appropriately rank income distribution. Newbery (1970) takes a simple example to demonstrate that the Gini ordering over income distributions is not implied by any additive social welfare function when the individual utility function is strictly concave and differentiable everywhere. Economic development is the primary indicator of social welfare. Hence, we may conclude that there is no strict linear relationship between development and the Gini index. Any summary measure of income inequality will sacrifice some information about income distribution. Currently, there is no strictly S-concave summary index found to measure income inequality.

I define the "fair division point" on a Lorenz curve as the point that presents a particular slope to characterize the economy of the Lorenz curve; then, the pair of income share and population share at the fair division point describes the overall inequality of income distribution. At the fair division point, the income share is called fair income share and the population share is called fair population share. Households ordered within the fair population share earn relatively lower income than those households ordered above the fair population share. The fair income share defines more people as high income earners if the

[1] Strict Schur-concave function $f : R^n \to R, \forall x, y \in R^n$, if x strictly majorizes y, then, $f(x) < f(y)$.

slope of fair division point is smaller. For different Lorenz curves with the same Gini index, the fair division point will be different, so that it is possible for the fair division point to tell us more information about development than the Gini index. I use both Gini index and fair division point in this study to describe income inequality so that we should be able to see some new information from the fair division point.

Some literatures (e.g., Atkinson, Piketty and Saez, 2010) use the income share of the top or bottom percentile population to describe income inequality, which is a two dimensional metric too; but it is not justified how much the top (or bottom) percentile income share could describe the income inequality of the entire income distribution, and it is not satisfactory to describe the relationship between aggregate development and the income share of the top (or bottom) percentile.

The Kuznets' hypothesis suggests that income inequality rises as an agricultural economy develops toward an industrial economy and then it decreases as the economy becomes rich, which has not been universally supported in empirical studies regardless of the measurements of income inequality. Classic growth models show that income inequality is persistent and enhances development due to agent heterogeneity and the marginal propensity to save increasing with wealth. Neoclassical growth models show that income distribution plays an insignificant role on development assuming representative agents and decreasing marginal returns in investment. The contemporary view proposes that income inequality shows up differently depending on whether the engine of growth goes from physical capital to human capital (Galor and Moav, 2004).

My analysis shows that a country's low income population relatively goes down (the fair population share drops slightly) as the economy develops; and at the same time, those low income households are relatively worse off (the fair income share falls). Inversely, as an economy becomes rich, there are more low income households (the fair population share increases), but those low income households are better off (the fair income share goes up and GDP per capita increases as well). Overall, both the Gini index and the difference between the fair population share and the fair income share have been increasing over the last half century. Therefore, income inequality increases as an economy is getting richer.

The analysis presents strong evidence for optimum income inequality regarding both the aggregate income per capita and the growth rate of GDP, where income inequality is measured by either the Gini index or the fair division shares. But no evidence has been found for Kuznets' hypothesis. Thus, both high and low inequality of income distribution could harm an economy as we compare with its potential optimum inequality.

The paper is organized as follows. Section 2 is for literature review, section 3 explains functional forms of the economic effects of income inequality, section 4 describes the data, section 5 is for econometric issues, section 6 discusses the development effects of income inequality, section 7 is for the growth effects of inequality and section 8 concludes.

2 Literature Review

There are two opposite empirical results about the trend of income inequality within an economy and over the world, which correspondingly represent two approaches about the economic effects of income inequality. The first one is the neoclassical approach which applies representative agents and proposes that income inequality within an economy falls in the process of development so that there would be absolute convergence in steady state equilibrium.

Becker and Tomes (1986) show that regression to the mean in earnings in rich countries appears to be fast, and almost all the advantages and disadvantages of ancestors disappear in three generations. Becker et al. (2005) find stark reduction in world inequality after incorporating gains in longevity. Sala-I-Martin (2006): Eight indices of income inequality show reductions in global inequality during the 1980s and 1990s.

Bourguignon and Morrisson (2002) find that inequality of world distribution of income worsened from the beginning of the 19th century to World War II and after that seems to have stabilized or to have grown more slowly. In the early 19th century most inequality was due to differences within countries; later, it was due to differences between countries. Inequality in longevity, also increased during the 19th century, but then was reversed in the second half of the 20th century, perhaps mitigating the failure of income inequality to improve in the last decades.

Actually, the world income inequality is determined by both the income inequality within each economy and the income inequality across economies, and the population weights of each country. Even though both of the two inequalities have been increased, but the weighted world inequality can be decreased. World income inequality is not a very meaningful concept no matter how it has been changing since it is about the income distribution of global economy; the world economy is not a really complete economy, because many countries are pretty much independent and have distinct economic systems.

The second one is called new neoclassical approach because it discards the assumption of representative agents and shows persistent income inequality with negative effects on development, which is contrast with the classical and neoclassical growth theory that income inequality is beneficial or neutral for development. Recent empirical studies present that both within (sectors, economies) and across (sectors, economies) wage inequalities have been increasing in developed economies in the last decades, and it is due to the advances of so-called skill-biased technology (Krusell et al., 2000; Acemoglu, 2002) and the indivisibility in investment of human capital (Galor and Joseph, 1993; Mookherjee and Ray, 2002).

Krusell et al. (2000) find that the rise in the college premium could be largely attributed to an increase in the rate of (capital-embodied) skill-biased technical progress. Heathcote, et al. (2004, 2009) document that wage inequality in the USA significantly rises from 1960s to 1990s which includes both permanent and persistent components, and transitory shocks as well; Acemoglu (2002) finds that technological progress has skill-biased impacts on wage inequality in the past sixty years in the US because the rapid increase of skilled labor has induced the development of skill-complementary technologies. Rios-Rull, et al. (2002) find that the basic facts about economic inequality in the United States did not change (improve) much during the 1990s, earnings, income and wealth had been unequally distributed just as they were at the beginning of the decade. Saez (2005) finds that the increase in annual top income shares in North America since 1970s is due to a surge in top wages and salaries, the United States reduced marginal tax rates for high income earners in the last 40 years but Canada didn't, and the mobility in top wage earners has been very stable in Canada, so that he conjectures a permanent income concentration in the last decades. Krueger et al. (2007) find that wage and skill premium inequalities have substantially increased over last three decades in most of the nine countries (U.S., Canada, U.K., Germany, Italy, Spain, Sweden, Russia and Mexico).

Finally, Banerjee and Duflo (2003) find nonlinearity correlation between growth and income inequality, which resolves the previous confusion of opposite evidences. This paper further finds that the correlation between development (growth) and income inequality is inverted U- shape, and income inequality shows different impacts in developed economies from developing economies.

3 Functional Forms for the Effects of Inequality

3.1 Fair Division of Income Distribution and Growth

Since two different income distributions may have the same Gini index when they intersect, then, the Gini index cannot rank income distribution for these intersection cases. Let p denote cumulative population percentage (share) in an economy that people are ordered by the amount of income they have earned, w denote cumulative wealth percentage (share) holding by the people within the population share of p in the economy. Figure 3.1 below shows that triangle Lorenz curves M and L intersect and have the same Gini index, but each Lorenz curve denotes a different income distribution and corresponds to distinct economic development levels, say, the two economies may have very different aggregate productivities.

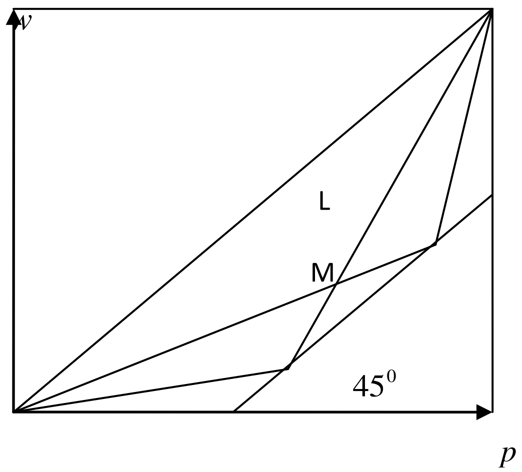


Figure 3.1.1 Lorenz Curves with Equal Gini Index

Refer to Figure 3.2 below; consider an income distribution expressed by a differentiable and strictly increasing Lorenz curve:

$$L(p) = \{ (p, w) : w = L(p), L'(p) > 0, L(0) = 0, L(1) = 1, x \in [0, 1] \} \tag{3.1.1}$$

We choose the point $F(p_l, w_l)$ on $L(p)$ so that

$$w_l = L(p_l), \quad L'(p)\Big|_{p_l} = s, \; s \in (0, 1] \tag{3.1.2}$$

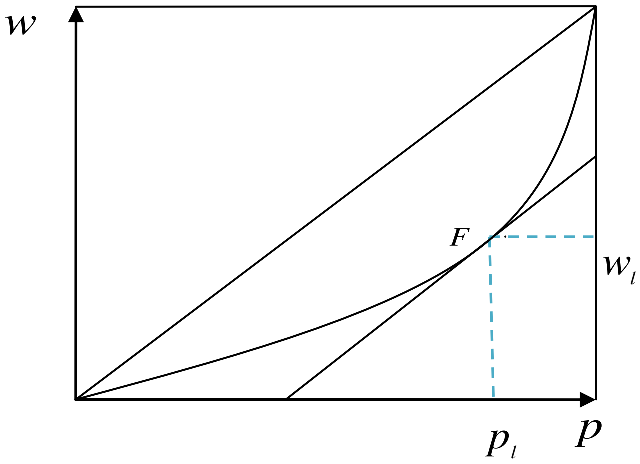


Figure 3.1.2 Lorenz curve and Fair Division Point

The slope s is determined by the characteristics of an economy at equilibrium, say, agents' preferences, markets' properties and aggregate production technology. The division point F is defined such that all people in the economy have the same growth rate of income; F is called **balanced division point** of income distribution, the corresponding growth of income is called balanced growth. Later (3.1.8) gives the explanation.

When $s = 1$, the marginal increase of income share is larger than the marginal increase of population share for all population shares larger than p_l, households located above p_l can be called high income households; and the marginal increase of income share is less than the marginal increase of population share for all population shares less than p_l , households located within p_l can be called low income households; so that $F(p_l, w_l)$ separates households in the economy into two parts on Lorenz curve. To ease exposition and computation, we assume the slope s to be unit in this study.

Let's call point $F(p_l, w_l)$ **Fair Division Point** of Lorenz curve $L(p, w)$ when $s = 1$, p_l is called the fair (division of) population share and w_l is called the fair (division of) income share.

For a given point $F(p_l, w_l)$, we ignore the redistribution within high income or low income groups, so that we assume the corresponding Lorenz curve is uniquely determined by the fair division point, for instance, when Lorenz curve is a triangle with the fair division point as one of its vertex. For given fair division point $F(p_l, w_l)$, any differences on Lorenz curve will be resource reallocation within the group of either low or high income households. We will only explore the economic effects of income redistribution between low income and high income groups, not within each group.

Actually, we can see that the Gini index can be approximated by the difference of fair division shares by approaching a Lorenz curve with a triangle. Figure 6.1.1b and Figure 6.1.4b Section 7 show that the Gini index and the difference of fair division shares empirically match very well by the estimation of fixed effects. For different Lorenz curves with the same Gini index, fair division point will be different, so that it is possible for fair division point to tell more information about development than Gini index. Following equation (3.1.3) and Figure 3.3 show this proposition.

$$\begin{aligned}\text{Gini Index} &\approx 2S_{\Delta FOB} = 2S_{\Delta AOB} = OA \\ &= p_l - w_l\end{aligned} \tag{3.1.3}$$

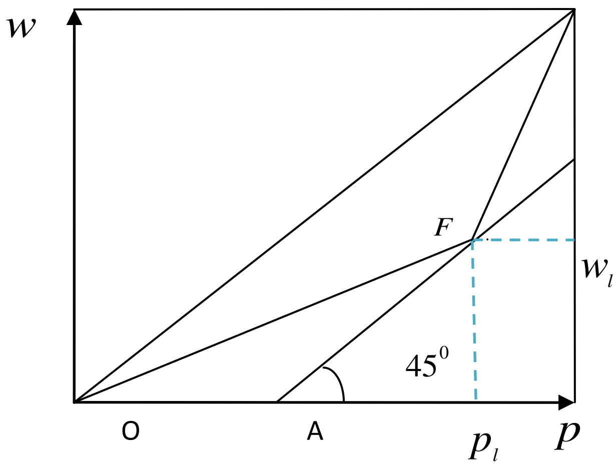


Figure 3.1.3 Gini Index and Fair Division Shares

Now, we discuss the condition of balanced growth that all people in the economy enjoy the same growth rate of income.

There are two groups of population regarding to income per capita in the economy. Low income households have a population share and income share of (p_l, w_l), and high income households have the population share and income share of (p_h, w_h). Let g_r denote the growth rate when the economy moves from income distribution (p_l, w_l) to $(p_l + \Delta p_l, w_l + \Delta w_l)$, W_i is the income per capita of type *i* household, *i*=*l, h*, and the change of income per capita for the people of group *i* is ΔW_i, and g_i is the growth rate of income for the type (group) *i* population, then:

$$w_i = \frac{W_i p_i}{W_l p_l + W_h p_h}, \quad g_i = \frac{\Delta W_i}{W_i}, i = l, h\,; \Delta p_l + \Delta p_h = 0\,, \Delta w_l + \Delta w_h = 0\,.$$

$$w_i + \Delta w_i = \frac{(W_i + \Delta W_i)(p_i + \Delta p_i)}{(W_l + \Delta W_l)(p_l + \Delta p_l) + (W_h + \Delta W_h)(p_h + \Delta p_h)},$$

$$g_r + 1 = \frac{(W_l + \Delta W_l)(p_l + \Delta p_l) + (W_h + \Delta W_h)(p_h + \Delta p_h)}{W_l p_l + W_h p_h}$$

$$= \frac{w_i}{w_i + \Delta w_i} \frac{(W_i + \Delta W_i)(p_i + \Delta p_i)}{W_i p_i},$$

$$\begin{cases} 1 + g_r = (1 + g_i)\dfrac{(1 + \Delta p_i / p_i)}{(1 + \Delta w_i / w_i)}, \quad i = l, h\,. \\ \Delta p_l + \Delta p_h = 0\,, \Delta w_l + \Delta w_h = 0 \end{cases} \tag{3.1.4}$$

Employing (3.1.4), we can find the interaction of income growth between the two groups of people, where the letter a denotes the ratio of growth rates of income for the two types of people, $a = \frac{1 + g_l}{1 + g_h} > 0$:

$$(1+g_l)\frac{(1+\Delta p_l / p_l)}{(1+\Delta w_l / w_l)} = (1+g_h)\frac{(1+\Delta p_h / p_h)}{(1+\Delta w_h / w_h)}, \quad \Rightarrow$$

$$a = \frac{1+g_l}{1+g_h} = \frac{(1+\Delta w_l / w_l)}{(1+\Delta w_h / w_h)} \frac{(1+\Delta p_h / p_h)}{(1+\Delta p_l / p_l)}, \qquad \Rightarrow$$

$$\frac{1+\Delta w_l / w_l}{1+\Delta w_h / w_h} = a\,\frac{1+\Delta p_l / p_l}{1+\Delta p_h / p_h}. \tag{3.1.5}$$

Let g_w be the growth rate of income share at w_l for the low income group, and g_p the growth rate of population share at p_l for the low income group:

$$g_w = \frac{\Delta w_l}{w_l}, \; g_p = \frac{\Delta p_l}{p_l}, \quad 1 + g_r = (1 + g_l)\frac{(1 + g_p)}{(1 + g_w)}; \tag{3.1.6}$$

For the balanced growth, we have the following results:

$$g_l = g_h \;\Rightarrow\; a = 1, \; (3.1.5) \;\Rightarrow\; \frac{1+\Delta w_l / w_l}{1+\Delta w_h / w_h} = \frac{1+\Delta p_l / p_l}{1+\Delta p_h / p_h}, \Rightarrow$$

$$w_l = \frac{c}{p_l}, \tag{3.1.7}$$

$$\left(p_l, w_l\right): \begin{cases} w = \dfrac{c}{p} \\ w = L(p) \end{cases} \qquad (3.1.8)$$

$$s = \left.\frac{dL(p)}{dp}\right|_{(p_l, w_l)} \qquad (3.1.9)$$

We define the unique point (p_l, c/p_l) of satisfying (3.1.8) on the Lorenz curve as the **balanced division point** of income distribution in the sense that all people have the same growth rate of income, constant c is determined by the initial condition. The slope s of the Lorenz curve at the balanced division point is determined by the characteristics of the economy, the constant c. The value of s is around unit. This is why we have the definition of (3.1.2) for the fair division point, which is an approximation of the balanced division point.

Finally, we explore the correlation between growth and fair division of income distribution. It is not guaranteed that the growth of income would be equal between people grouped by the fair division point, then, we have following unbalanced growth. Employing (3.1.5) and $g_l \neq g_h$, we have:

$$\frac{(1+g_w)w_h}{w_h - w_l g_w} = a\,\frac{(1+g_p)p_h}{p_h - p_l g_p} \qquad \Rightarrow$$

$$\frac{g_p + 1}{g_w + 1} = \frac{1}{a}\left[1 - \frac{p_l g_p}{p_h} + \left(\frac{g_p p_l}{p_h} - 1 + (1+g_p)a\right)w_l\right] \qquad (3.1.10)$$

Then, (3.1.6) and (3.1.10) yield

$$\begin{aligned} g_r &= (1+g_l)\frac{(1+g_p)}{(1+g_w)} - 1 \\ &= (1+g_h)\left(1 - \frac{p_l g_p}{p_h}\right) + \left[\frac{g_p}{p_h} + \left(\frac{g_p p_l}{p_h} - 1\right)g_h + (1+g_p)g_l\right]w_l - 1 \\ &= w_l g_l + w_h g_h + \frac{(1+g_h)[a g_w + (1-a)w_h(g_w+1)][a w_l - (w_h + a w_l)p_l]}{a(w_h - w_l g_w) + [(1-a)w_h + (w_h + a w_l)g_w]p_l} \end{aligned}$$

Above equation gives the following results about the correlation between aggregate growth and the fair income division (p_l, w_l):

$$Corr(g_r, w_l) = \frac{1}{p_h}\left[g_p + (g_p p_l - p_h)g_h + (1+g_p)p_h g_l\right]; \qquad (3.1.11)$$

$$Corr(g_r, p_l) = \frac{-w_h(1+g_l)[a g_w + (1-a)w_h(g_w+1)]}{\{a(w_h - w_l g_w) + [(1-a)w_h + (w_h + a w_l)g_w]p_l\}^2}; \qquad (3.1.12)$$

Conclusion:

The correlation between aggregate growth and fair division of income distribution is nonlinear, and can be affected by the fair division point (p_l, w_l), growth of income for the two groups (g_l, g_h), and the growth of fair division shares (g_p, g_w).

The growth effects of fair division point of income distribution can be both positive and negative.

Especially the negative correlation between growth and the fair income share necessarily rely on either a large population share of high income group $p_h > g_p p_l$, a large growth of income for high income group (g_h>0) or a decrease of the fair population share (g_p<0). Or say, a lower aggregate growth happens if the fair income share increases and the fair population share decreases.

The negative correlation between growth and the fair population share can be satisfied with a positive growth of fair income share and faster income growth of high income group, $ag_w+(1-a)w_h(1+g_w)>0$. Or say, even though the fair income share increases (g_w>0), if the growth of income for low income group is less than that of high income group (a<1), then, an increase of fair population share will result in a lower overall growth.

In the next section, we discuss the estimation of the economic (growth) effects of income inequality. Typically, we would like to use the quadratic functional form for the estimation.

3.2 Functional Form of the Economic Effects of the Gini Index

Income inequality is a median situation of income distribution; the two extremities are perfectly equal distribution and perfectly unequal distribution. The two extremities won't be a stable equilibrium of distribution due to the heterogeneity of agents in initial endowments, skills and preferences, etc., thus, we expect that an efficient market would develop an optimal income inequality over time so that all heterogeneous members are rewarded by their characteristic endowments, which especially include their physical and human capitals.

When an income inequality stays at competitive equilibrium, we would believe all agents are well off and social welfare stays at optimal state; otherwise, redistribution of resources between group-members will automatically start until social welfare is improved; for instance, it can be completed by a growth of GDP per capita. Put it in another way, efficient reallocations of resource will lead to growth and raise social welfare. During a growth, the reallocation improves social welfare in the aggregate level; during a recession, resources are reallocating between group members and some members are losing resources that were inefficiently allocated.

We will choose polynomial functional forms for either the fair division point or the Gini index, because it is always easy and possible to find a polynomial expression of independent variables to express any local and global fluctuations of a dependent variable. Another advantage of polynomial function is that it is easy to calculate the fair division point on the Matlab.

Denote the economic effects of income inequality as $f(p_l, w_l)$ for the fair division point, and $f(g)$ for the Gini index, $f(g, w_l)$ for the joint effects of the fair income share and the Gini index, and $f(g, p_l)$ for the joint effects of the fair population share and the Gini index, respectively. The economic effects *f*(...) of an inequality can be either on growth or development.

Due to the heterogeneity in initial endowments of wealth and innate characteristics, social welfare would be optimal when income distribution presents a median inequality that makes all agents well off up to their individual productivity and preference. Assume there is an optimal Gini index $g^{\bullet}$, then, a transitory Gini index *g* that deviates from $g^{\bullet}$ will lead to social welfare loss, and it is also assumed that a larger or a smaller non-optimal Gini index would have the same social welfare loss; thus, we write the economic effects of income inequality by Gini coefficient as follows:

$$f(g) = c - \alpha(g - g^{\bullet})^2 \qquad (3.2)$$

Where parameters c and α are positive constants, and α is the coefficient of marginal effects of the Gini index that deviates from the optimal Gini g^*. Function (3.2) means that any Gini index other than the optimum would lead to welfare loss and a redistribution of income toward the optimal Gini g^* improves the economic effects $f(g)$.

Any social welfare index can be the dependent variable of function (3.2). Since both GDP per capita and growth rate of GDP per capita can be an indicator of social welfare, in the following sections of econometric analysis we will use the two variables as dependent variable individually to explore growth and development effects of income inequality.

3.3 Functional Form of the Economic Effects of the Fair Division Point

Assume an optimal fair division point denoted by $F(p_l^{\bullet}, w_l^{\bullet})$ on the Lorenz curve, a transitory fair division point is $F(p_l, w_l)$. The distance between the two points would be a measure of social welfare deviation (inefficiency) due to non-optimal income inequality. A reallocation of resources that reduces the distance between the two points should be efficient and lead to growth so that social welfare is improved. Similar to function (3.2), we also assume that both a larger and smaller value of either fair division shares have the similar welfare loss, then, we have following (3.3.1):

$$f(p_l, w_l) = c - \alpha_1 (p_l - p_l^{\bullet})^2 - \alpha_2 (w_l - w_l^{\bullet})^2 \tag{3.3.1}$$

Where, $c > 0, \alpha_i > 0, i \in \{1, 2\}$, α_1 is the coefficient of marginal welfare effect of population share that deviates from the optimal population share, α_2 is the coefficient of marginal welfare effect of income share that deviates from the optimal income share. Function (3.3.1) gives a unique optimal fair division point.

We expect that regression of growth by the function (3.3.1) should present significance and the corresponding signs for each item.

It is possible that the Gini index and one of the fair division shares could jointly describe inequality better than any one of them, so that we would like to consider a hybrid metric as well. Assume there is an optimal income inequality expressed by the fair income share and the Gini index noted by $(g^{\bullet}, w_l^{\bullet})$, similar to functional form (3.3.1), we have the functional form of the hybrid effects of the fair income share and the Gini index as follows:

$$f(g, w_l):\ c - \alpha_1 (w_l - w_l^{\bullet})^2 - \alpha_2 (g - g^{\bullet})^2 \tag{3.3.2}$$

$$f(g, p_l):\ c - \alpha_1 (p_l - p_l^{\bullet})^2 - \alpha_2 (g - g^{\bullet})^2 \tag{3.3.3}$$

Where, $c > 0, \alpha_i > 0, i \in \{1, 2\}$, and α_1 is the coefficient of marginal welfare effect of income share that deviates from optimal income (or population) share, α_2 is the coefficient of marginal welfare effect of the Gini index that deviates from the optimal Gini index. We expect that (3.3.2) ~ (3.3.3) should be able to tell more information than (3.2.1) ~ (3.3.1) because it has one more dimension than (3.2.1) and include the effects of within and between redistribution among the low and high income household groups.

4. The Data

4.1 Data Description

We only consider a reduced form of the economic effects of income inequality, so that we will use variables including the Gini index, the fair division point of Lorenz curve, GDP per capita, growth rate of GDP per capita and population.

We use the unbalanced panel data “WIID2C”of income distribution from World Institute for Development Economic Research at United Nations University. The panel is unbalanced and ranks quality of all observations; high quality data satisfy following criteria: the underlying concepts of income or consumption are known, the quality of income concept and the survey can be judged as sufficient. The data was collected or adjusted by household. I choose 547 observations of 52 countries from 1956 to 2006 in the data, in which 390 observations are ranked as high quality, 157 observations are ranked as low quality because the income concepts or survey quality are not verified. And there are time gaps in the data of some countries.

The definition of income in the data is disposable income or monetary income per household and the data collection samples over entire population in each country. All those observations of consumption are dropped, because distribution of consumption gives very different results of inequality from income, so that the data of income inequality are calculated only by disposable income or monetary income.

The real GDP per capita and its growth rate are adopted from Penn World Table 6.3, which have been converted by PPP in 2005 constant price using chain method, denoted by rgdpch in the data. The growth rate is the growth rate of Real GDP Chain per capita (RGDPCH). Population is also adopted from PWT6.3.

Considering the economic effects of labor supply within a country, we assume the base population size of each country as 1 so that the population index for each observation represents the differences of labor supply within a country; I divide each observation of population by the biggest one of all observations in each country so that it becomes a population index, which is denoted by **popw** in the data, it approximately shows the trend of labor supply within a country over the period of observations. This population index controls for the effects of labor supply on the dependent variable within a country. We divided each observation of GDP by the maximum of all observations in each country to obtain **gdpw,** which assumes no difference of GDP at optimal state between countries. Considering the effects of population size across countries, I create another population index, denoted by **popb** in the data, which is the observation of population divided by the biggest one in the entire panel data, which is 2004 USA population. I also create between-GDP per capita, denoted by **gdpb**, which is the observation of GDP per capita divided by the biggest one in the entire panel which is 2000 Luxembourg GDP per capita.

To obtain fair division point (p_l, w_l) of each observation of income distribution, I approximate Lorenz curve for each observation by 4^{th} degree polynomial for the data of quintiles and 6 degree polynomial for the data of percentiles including two endpoints of (0, 0) and (1, 1), which is completed in excel; then the fair division points are calculated by the first derivative of the polynomial approximation which can be completed on MatLab. There is an example of calculating the fair division point (Figure 4.1) in the Appendix.

The panel data of income distribution were collected by many different public or private agents and verified by the project of “WIID2C”. Table 4.1.1 below shows the data summary.

Table 4.1.1 Variable Summary

Variable	Obs	Mean	Std. Dev.	Min	Max
p_l	547	0.643	0.067	0.530	0.820
w_l	547	0.379	0.040	0.288	0.691
g	547	0.369	0.107	0.196	0.64
gdpb	547	0.253	0.149	0.0199	1
gdpw	547	0.773	0.1999	0.090	1
popw	547	0.919	0.093	0.469	1
popb	547	0.169	0.217	0.0012	1
growth	547	0.030	0.036	-0.141	0.146

Table 4.1.2 in Appendix shows bilateral correlation between variables. We can find that there is very strong correlation between the Gini index and the fair division shares (0.898 with the fair population share, 0.572 with the fair income share), and strong correlation (0.5327) between GDPb and fair population share, which is larger than the correlation (0.4045) between GDPb and the Gini; GDPw and Popw are also strongly correlated (0.636).

4.2 The Data of Income Inequality

The Figures 4.2.1 below shows the observations of the fair division point against the Gini index. It is very clear that higher Gini index corresponds to a higher fair division of population share and lower division of income share; or say, more households become even lower income earners when the Gini index becomes larger.

Figure 4.2.2b below shows the observations of the fair division shares against GDPb, where P_l and W_l denote the fair population share and the fair income share, respectively. It seems that the fair income share is stationary in the interval of [0.3, 0.5] and the fair population share decreases as the sample countries become richer.

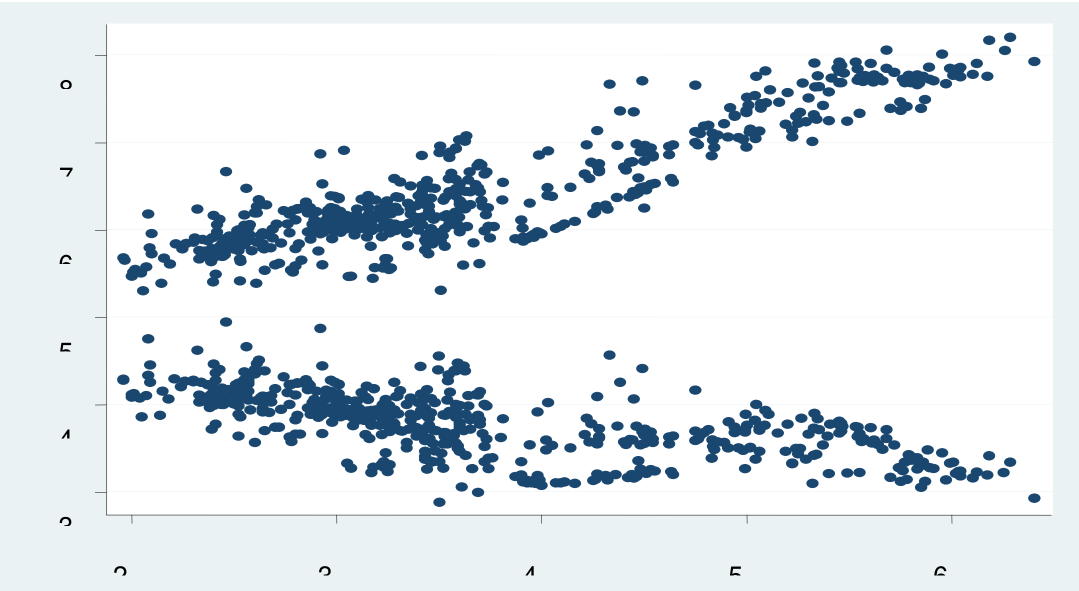


Figure 4.2.1 Observations of Fair Division Shares against Gini

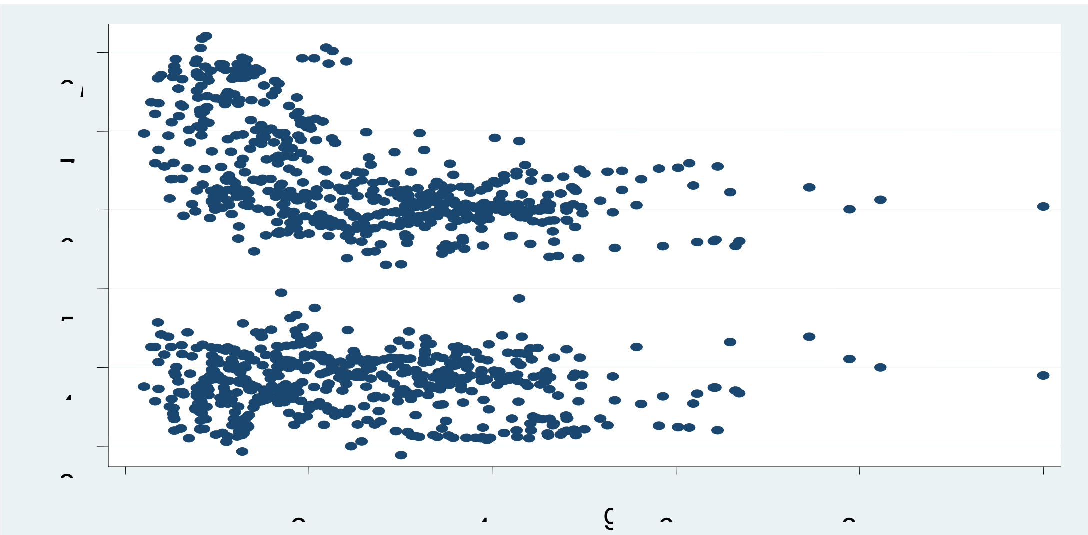


Figure 4.2.2b Observations of Fair Division Points against GDPb per Capita

Regression results on Table 4.2.1 and Table 4.2.2 in the Appendix show that the fair division point can explain more variations of the Gini index than the Gini index does on the fair division point, thus, we would expect that the fair division point would be able to tell more information about the economic effects of income inequality than the Gini index does.

The 3D graph of Figures 4.2.3 below shows the fixed effect regression the Gini index on the fair division shares, which is the Table 4.2.1 in Appendix. Figure 4.2.3 shows that there is strong positive linear relationship between the Gini index and the fair population share, and strong non-linear relationship between the Gini index and the fair income share.

Figure 4.2.3 Estimation of Gini on Fair Division Point

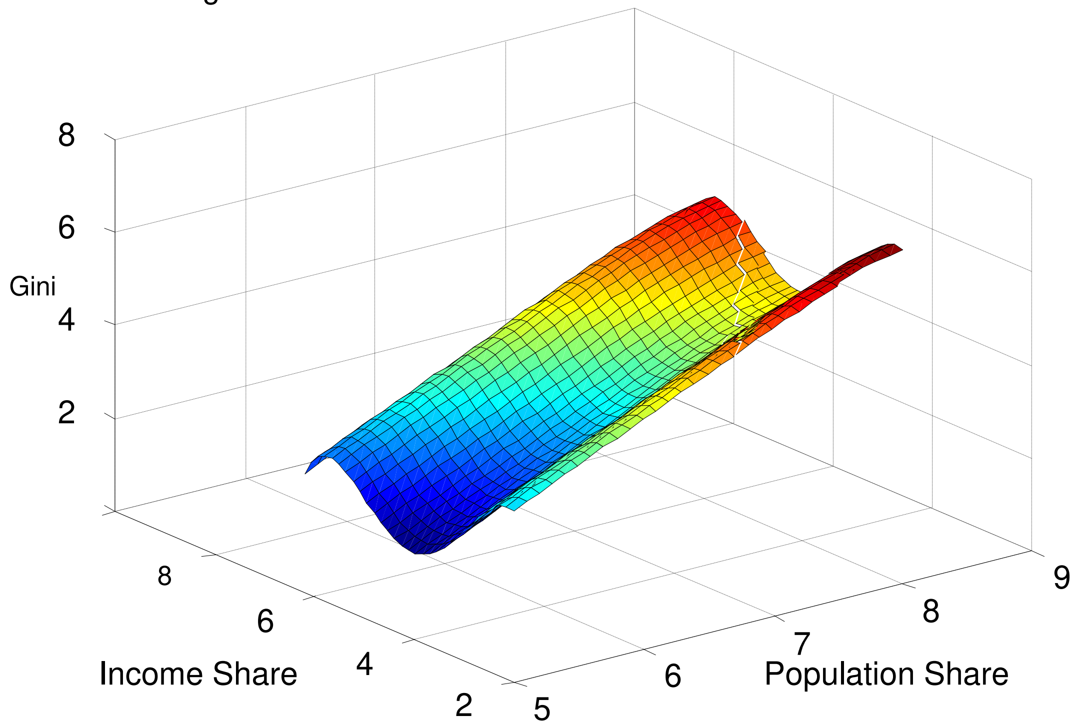


Table 4.2.3 in the Appendix shows that the fair population share is significantly correlated to the fair income share within a country in the data; the regression presents small within and between R^2, and very little overall R^2. The regressions present some correlations, FE corr(u_i, Xb), between the error term and explanatory terms, thus there are some kind of endogeneity problem on the regression; possibly there are other omitted factors to determine the fair division shares even though the two factors were significantly correlated.

Figure 4.2.4b shows the observations of the Gini index and growth rate against GDPb. Using GDPb, Figure 4.2.2b and Figure 4.2.4b show that both the Gini index and the fair population share seem to fall slightly at the early stages of development, but stopped falling further once development has reached some level and they rise slowly in developed economies; fair income share does not present very clear trend respect to development. Growth rate has much larger variance in developing countries than developed countries.

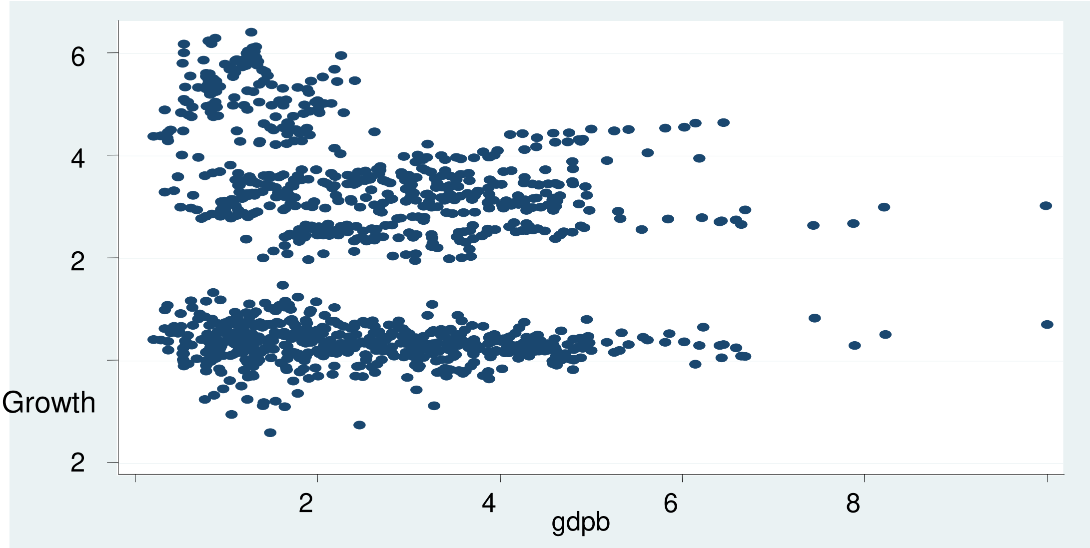


Figure 4.2.4b Observations of Gini and Growth of GDP against GDPb

Finally Figure 4.2.5 below shows the observations of fair division shares.

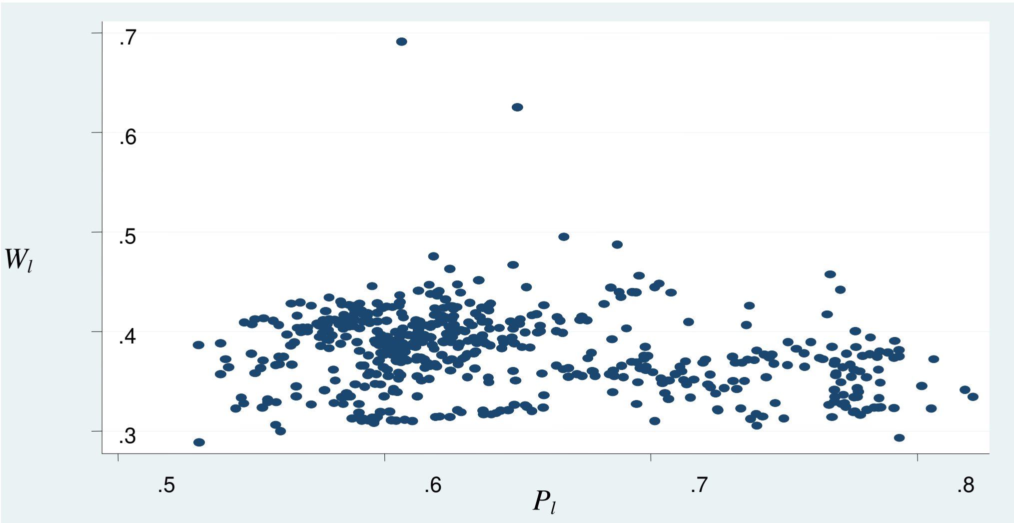


Figure 4.2.5 Observations of Fair Income Share Against Fair Population

5. Econometric Analysis

5.1 Functional Forms

We are going to employ following three models for econometric analysis on the panel data:

$$X_{it} = \alpha_0 + \alpha_1 Pop_{it} + \alpha_2 GDP_{it}^2 + \alpha_3 GDP_{it} + v_i + u_{it} \quad (5.1)$$

$$Growth_{it} = \alpha_0 + \alpha_1 GDP_{it} + \alpha_2 Pop_{it} + \alpha_3 X_{it}{}^2 + \alpha_4 X_{it} + v_i + u_{it} \quad (5.2)$$

$$GDP_{it} = \alpha_0 + \alpha_1 GDP_{it-1} + \alpha_2 Pop_{it} + \alpha_3 X_{it}{}^2 + \alpha_4 X_{it} + v_i + u_{it} \quad (5.3)$$

$$u_{it} \sim IID\left(0, \sigma_u^2\right), Eu_{it} = E(u_{is}u_{it}) = 0, \ s \neq t \quad (5.4)$$

Where, X_{it} is a measure of income inequality, the quadratic form of X_{it} will take the functional forms, (3.2) ~ (3.3.3), in section 3. We assume the unobserved error term over time and countries of (5.1) ~ (5.2) satisfy the condition of IID and no serially correlation.

For the models (5.1) ~ (5.2), we know pooled OLS is biased due to unobserved heterogeneity[2], thus, we will apply either fixed effect or random effect regressions which depend on the correlation between explanatory variables and country specific error term. If there is no within heterogeneity, we apply fixed effects model; if there is within heterogeneity, we employ the Hausman test to see if the random effect model is consistent.

Equation (5.1) is used to verify Kuznets hypothesis that income inequality would show an inverted U-shape in the process of development. If this hypothesis exists in the data, then we should be able to get significant estimation for the coefficients and a negative sign for the estimation of quadratic item. If we do not find significant quadratic effects of inverted U-shape, we will also explore other polynomial forms of GDP per capita that show significant estimations.

Model (5.2) explores the growth effects of income inequality and the existence of optimal income inequality for growth. It applies to fixed and random effects regressions. When there is no endogeneity problem, say the correlation between covariates and panel-level unobserved effects is trivial, we will employ the Hausman test to choose if random effect regression presents the same consistent but more efficient estimation.

[2] Meghir and Pistaferri (2004) find strong evidence of sizeable ARCH effects as well as evidence of unobserved heterogeneity in the variances.

Equation (5.3) is a linear dynamic panel model. It explores the development effects of income inequality and it also shows the existence of an optimal income inequality on development from the data. This dynamic model is originated by the idea that development effects of income inequality works as a shock to transitory GDP per capita. Actually, (5.2) and (5.3) use similar covariates to explain two different measures of the economic effects of income inequality, (5.3) measures the level effects of GDP, (5.2) measures the growth of level effects GDP.

The model (5.3) includes one lag of the dependent variable as covariate and contains unobserved panel-level effects, fixed or random. The lagged dependent variable is correlated to u_{it}, the unobserved panel-level effect which makes OLS estimator inconsistent. We employ the consistent and efficient GMM estimator created by Arellano and Bond (1991), which also deals with panels with short time periods and large number of panels. The unbalanced panel data I use here give 278 observations of 32 countries for the dynamic panel model, in which the minimum observation is 1 and maximum observation is 40. At time T, the valid instrument set to apply GMM in model (5.3) includes all earlier lags:

$$\left(GDP_{i1}, GDP_{i2}, \ldots\ldots, GDP_{iT-2}, Pop_{i1}, \ldots, Pop_{iT-2}, X_{i1}^2, \ldots, X_{iT-2}^2, X_{i1}, \ldots, X_{iT-2}\right)$$

Where, we assume population and income inequality measure are predetermined; and the moment condition is

$$E\left(\Delta u_{it} \cdot GDP_{it-j}\right) = 0,\ j = 2, \cdots, t-1, t = 3, \cdots, T$$

Let W denote the matrix of instruments, then, we can perform generalized least squares (GLS) on following estimator:

$$W^{'}\Delta GDP_t = W^{'}\Delta GDP_{t-1}\alpha_{t1} + W^{'}\Delta Pop\_Index\alpha_{t2} + W^{'}\Delta X^2\alpha_{t3} + W^{'}\Delta X\alpha_{t4} + W^{'}\Delta u_t\ ,$$

Which gives us the preliminary one-step consistent estimation; and using the first step estimated error, we perform the second-step GLS to get the consistent and efficient estimation.

For the causality interpretation between development and income inequality in the models (5.1) ~ (5.3), if we assume that the income inequality is exogenous due to agents' heterogeneity in innate endowments and initial resource endowments, then, based on the assumption (5.4), then, the significant estimations in (5.1) ~ (5.3) tell the causality effects of income inequality on growth and development. Of course, this is only an assumed causality interpretation. We do not justify the causality interpretation in this exercise. We would prefer to interpret the estimation as a correlation of the reduce forms.

In the following Section 7, we will see that the growth model (5.2) presents no endogeneity since the fixed effect regression shows very trivial correlation between the error term and covariates.

5.2 Assumptions about Population and GDP per capita Between Countries

Assumptions about the differences of population size and development need to be specified for the above models. Since we are not sure if population size and its growth could affect income inequality and economic growth, we would like to explore the economic effect of income inequality in two cases: one is that the base population size differs across countries, the other one is that the base population sizes are the same across countries.

If we assume that the bases of population size and GDP are the same (a very strong assumption!) across countries, it means the base population and base GDP are standardized to unit in each country and the only difference of population (GDP) between countries is the growth rate of population (GDP). We complete this by dividing each observation of population (GDP) with the maximum population (GDP) over time of the entire observations within each country. We use popw to denote the variable of population index and gdpw for GDP for this assumption. The assumption makes sense in the long run when the welfare effects of income inequality are not dominated by the size of population but by their human capital. This assumption means that any two countries with different population sizes but possessing the same technology would present the same welfare effects of income inequality in the long run.

If we assume the base population and base development are different across countries, it means that population and GDP per capita are standardized in the panel level by dividing each observation by the maximum of entire panel data, respectively; and we use popb and gdpb as the two variables of population and development index, respectively. In this case, all countries can be ordered by either population size or per capita GDP. This assumption can be more practical since both physical and human capital are very different across countries in the panel data.

None of the two assumptions is a perfect approximation of real economy; we take each of the two assumptions for each issue to see how outcome will vary by the assumptions. Since the assumption of same base population is very strong, later we will mainly report the work using the assumption of different base population and development (popb and gdpb) in the contexture, and the corresponding figures and tables of using popw and gdpw are not reported here to reduce the volume of the paper but they are available upon request.

5.3 The Choices of Regression Method

We will apply robust OLS, fixed effects and random effects regressions in the study. Robust OLS assumes the error term has zero mean which can be a strong assumption for panel data; so if robust OLS has very different results from fixed effect regression, we will choose fixed effects regression as our tool. We use the Hausman test to choose between fixed effect model and random effect model; if the Hausman test gives a probability larger than 0.05 with positive chi2 value, then, we will choose the consistent and efficient random effect regression as our tool, otherwise, we will report both random and fixed effect regressions.

Distinct results between OLS and fixed effect model come from their different assumptions about the error term. It is a strong assumption on the panel data for OLS model to assume independent and identical distribution of the error term because each country is subject to different natural endowments and economic institutions; and fixed effect model controls for unobserved fixed heterogeneity in the error term. Thus we take the fixed effect model as our basic regression tool.

6. Development and Income Inequality

Section 6.1 explores if Kuznets hypothesis would fit in the data by both the Gini index and the fair division point, which is the model (5.1). Section 6.2 discusses the development effects of income inequality using model (5.3).

6.1 Test the Kuznets' Hypothesis

Kuznets' hypothesis proposes that income inequality increases in GDP per capita for poor countries and then decreases in GDP per capita for rich countries. It is an inverted U-shape between income inequality and development. Employing functional form (5.1), we do not see any significant estimation to support the hypothesis, but either the linear form or the third degree of polynomial form for GDP per capita can give us different and significant estimations. We apply the Gini index and the fair division shares individually to following analysis.

6.1.1 Regression of the Gini Index on GDP per Capita

We run regressions of gdpb on the Gini index with explanatory popb, and choose polynomial form of gdpb up to the degree of significance. Table 6.1.1b in the Appendix reports the result. The robust OLS regression presents significance with third degree of polynomial, fixed effect regression presents significance with only the linear form and random effect regression does not show any significance at all with any degree of polynomials. The robust OLS regression shows very significant development effects on the Gini index, but the estimation is very different from fixed effect model. Random effects models do not present any significant development effects on the Gini index, but population size shows significant and positive effects on the Gini index.

The fixed effect linear model gives the following results:

- There is significant and positive linear relationship between the Gini index and development over time and across countries;
- Population size (popb) plays positive significance on the Gini index. There is very significant and positive constant effect for the Gini index, which is at least 0.335;
- The regression shows very little within and between R^2 with explanatory popb and gdpb.

But with the explanatory variables gdpw and popw, robust OLS, fixed effect and random effect models all present more significant and positive correlation between the Gini index and development, popw is not significant. Figure 6.1.1b below shows the fixed effect estimation of the Gini index by explanatory gdpb and popb.

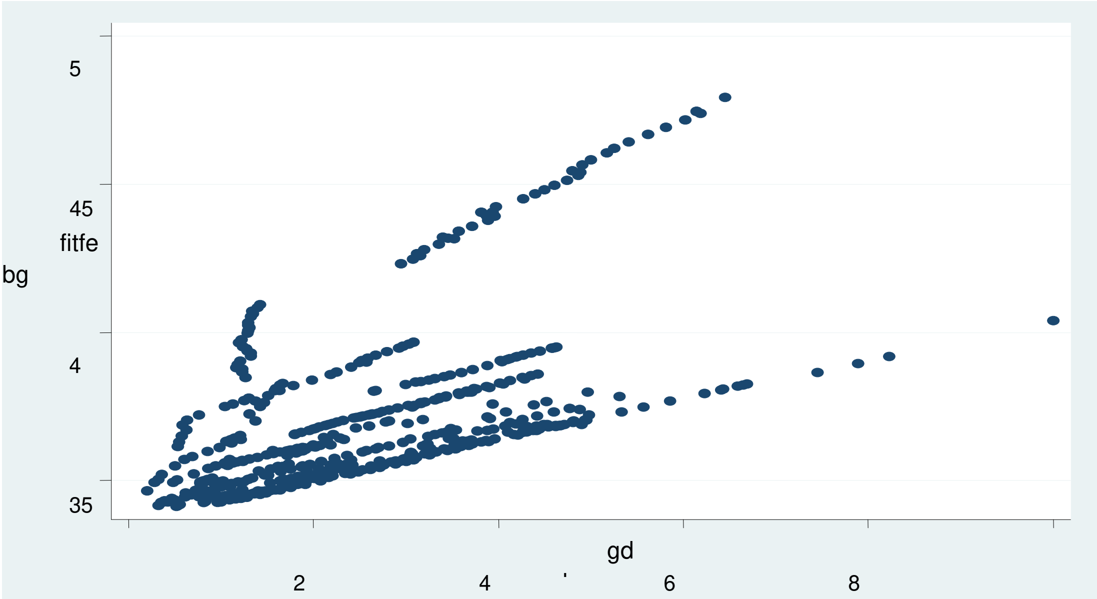


Figure 6.1.1b Fitted Gini with GDPb by Fixed Effect Model

Conclusion: Fixed effect model shows development has significant and positive constant effects, and significant and positive effects on the Gini index. Population size (gdpb) plays positive significance on the Gini index between countries in both fixed and random effect models, but population growth (popw) is insignificant over time within a country. Hence, we find no evidence for the Kuznets hypothesis by the measure of Gini index.

6.1.2 Regression of the Fair Population Share on GDP per Capita

Using the third degree polynomial form for GDP per capita, we run the robust OLS, fixed effects and random effects regressions; Table 6.1.2b~w in the Appendix present the results for the two assumptions about the base of population and GDP indices. Table 6.1.2b shows the following results:

- The fair population share falls in poor countries as they grow but rises dramatically later in rich countries. Population size (popb) presents positive significance between countries. There is very significant constant effects on fair population share, which is at least 0.651;
- The largest within R^2 is in the fixed effects model, which is 0.1168; the largest between R^2 is in the random effect model, which is 0.0931.

Let $fitfeb\_p_l$ denote the fitted fair population share by fixed effect regression and explanatory popb and gdpb. Figure 6.1.2b below graphs the estimations, graphs for random effects and regressions with gdpw and popw can be found as Figure 6.1.2w in the Appendix.

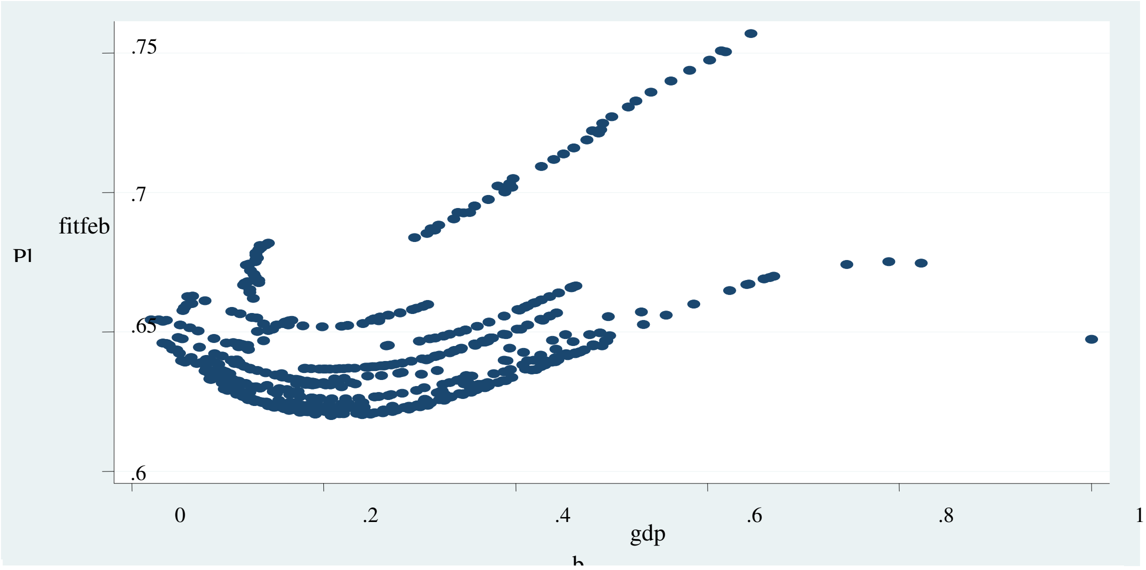


Figure 6.1.2b Fixed Effect Fitted Fair Population Share Against gdpb

Conclusion: There are very significant and positive constant effects on fair population share as an economy grows; the fair population share decreases as a country gets rid of poverty and then it increases as it becomes rich. Population size contributes significant and positive effects on the fair population share between countries in both fixed and random effect models; but population growth plays insignificance within a country.

A country's low income population relatively decreases (the fair population share drops slightly) as the economy grows; inversely, as an economy becomes rich, there are more low income households (the fair population share increases).

6.1.3 Regression of the Fair Income Share on GDP per Capita

Table 6.1.3b in the Appendix gives the following results for the regression of fair income share on GDP per capita:

- The fair income share falls in poor countries as they grow and rises dramatically later in developed countries as they become richer;
- Population size makes no significance on the fair income share; There is very significant constant income effects, which is at least 0.404;
- The within R^2 is 4.9% and between R^2 is 16.33% in the fixed effect regression, and the R^2 in random effect regression are pretty small.

Let $fitfeb\_W_l$ be the fitted fair income share in random effect model with explanatory gdpb and popb. Figure 6.1.3b below graphs the fixed effect estimation with explanatory popb and gdpb. Figure 6.1.3w in the Appendix graphs the fixed effect estimation with explanatory popw and gdpw, and the graphs for random and robust OLS results are Figure 6.1.3w in the Appendix too. Table 6.1.3w in the Appendix shows that the fair income share decreases in the early stages of development in an economy and keeps stable in the middle stages and then turns down slightly in developed stages in both fixed effect and random effect models; there is significant constant effects at least 0.403 and popw is significant.

Conclusion: There are significant and positive constant fair income share as an economy grows; the fair income share decreases in poor countries and it increases later in rich countries in the process of development. Population size between countries does not contribute any significant effects on the fair income share in both fixed and random effect models, but population growth does show significant and positive effects on fair income share. Thus, fair income share does not support Kuznets hypothesis too.

As an economy grows, low income households are relatively worse off (the fair income share falls even though the GDP per capita increases); Inversely, as an economy becomes rich, those low income households are better off (the fair income share goes up and GDP per capita increases as well).

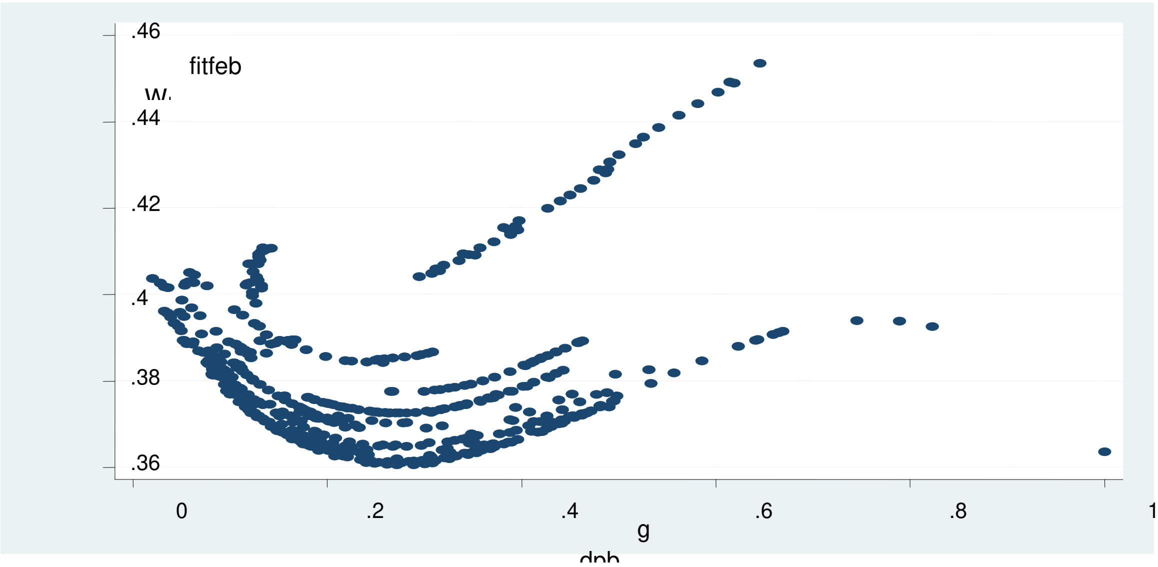


Figure 6.1.3b Fitted Fair Income Share Against GDPb

6.1.4 Regression of the Difference of Fair Division Shares on GDP per Capita

The difference between the fair population share and the fair income share is another measure of income inequality. This difference tells if the low income population is better off in the sense of comparing their income share with their population share as an economy becomes richer. The results are reported in Table 6.1.4b in the Appendix.

Figure 6.1.4b below graphs the results of fixed effect regression on gdpb and popb. The difference goes up as an economy grows in fixed effect model, but it does not show any significant increase in random effect model, robust OLS shows very different estimations that the difference drops a lot in developing countries and rises slightly in developed economies, and there is very significant constant effect over time and across countries which is at least 0.247.

Comparing Figure 6.1.4b with Figure 6.1.1b, we can find the two graphs look like very similar. This is the evidence in section 2.2 that we suggest to interpret the Gini index as the difference of the fair division shares. The difference of fair division shares has been increasing over time and across countries in most of the economic history in this panel data, and the robust OLS, fixed effect and random effect model all give similar results; the significant constant effect is at least 0.261. Thus, even though above section 6.1.2 and 6.1.3 tell that both the fair population share and the fair income share fall first and rise later as an economy becomes richer, but the fair population share goes up faster than fair income share, thus, income inequality does not improve over time and across countries in the sense of the difference of the fair division shares, which agrees with the performance of the Gini index in fixed effect models in section 6.1.1.

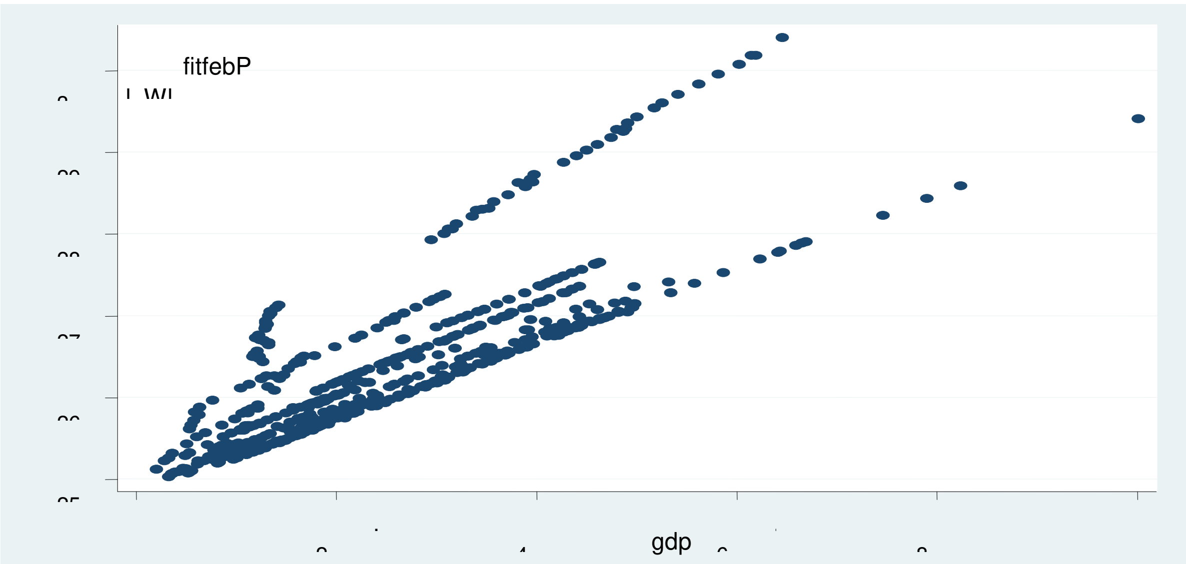


Figure 6.1.4b Fixed Effect Fitted Difference of Fair Division Shares Against

Conclusion: Both the fair population share and the fair income share fall slightly in poor countries, but they rise dramatically in rich countries, and the difference between the two dimensions has been rising! The Gini index also falls a little bit in poor countries and has been rising dramatically over most of the process of development.

A country's low income population relatively decreases (the fair population share drops slightly) as the economy grows; and at the same time, those low income households are relatively worse off (the fair income share falls even though the GDP per capita increases). Inversely, as an economy becomes rich, there are more low income households (the fair population share increases), but those low income households are better off (the fair income share goes up and GDP per capita increases as well).

Population size and its growth play some roles in the evolution of income inequality. Table 6.1 below summarizes all the estimations of population effects in above regressions. Population size presents positive significance on the fair population share and the Gini index, but no significant effects on the fair income share; population growth shows positive significance on the fair income share, but no significance on the fair population share and the Gini index. It shows that population size tends to significantly increase the fair population share, the Gini index and the difference of the fair division shares by fixed effect models; but population growth tends to be insignificant on the fair population share and the Gini index, and it shows significant effects to increase the fair income share and insignificance to decrease the Gini index and the difference of fair division shares.

Table 6.1 Population Effects on Inequality

Dependent / Independent	Fair population share		Fair income share		Difference of fair division shares		Gini index	
	FE	RE	FE	RE	FE	RE	FE	RE
popb	**0.091 (2.52)**	**.0975 (3.31)**	0.0647 (1.45)	**-.0374 (-1.7)**	.0264 (0.55)	**.084 (2.13)**	**0.0999 (1.99)**	**.1424 (3.25)**
popw	.03514 (1.62)	.0186 (0.84)	**.0501 (1.89)**	**.0899 (3.62)**	-.0222 (-0.81)	**-.0492 (-1.77)**	-.0137 (-0.48)	-.0373 (-1.27)

Both the Gini index and the fair division point show the same fact that income inequality increases in GDP within and between countries. Above empirical results abide by the empirical results of the increasing wage inequality since 1970s in developed countries

(Acemoglu, 2002; Heathcote, et al., 2009). Atkinson, et al. (2010) have got the similar results that employ the top percentile income to measure the income inequality. But these results differ from some other empirical results, (Braulke (1983), Galor and Tsiddon (1996), Nielsen and Aiderson (1997), Sala-I-Martin (2006)).

6.2 Development Effects of Income Inequality

If the above fact that income inequality increases in GDP per capita has been an event of efficient allocation of resources, then we should be able to find the development effects of income inequality in the panel. Considering there might be many transitory noises on economic development, we would expect that an efficient income inequality regarding development is optimal in the long run to maximize GDP per capita over time and across countries.

Employing model (5.3) in section 5, we explore the empirical evidence of income inequality on development. (5.3) is a dynamic panel model, Arellano and Bond(1991) give a consistent and efficient GMM method to estimate it. We use two types of metric for income inequality: one employs only one measurement of income inequality as shown in the Table 6.2.1 ~ Table 6.2.3 in the Appendix; the other type, called hybrid metric, takes both the Gini index and one of the fair division shares as shown in Table 6.2.4 in the Appendix. We also discuss the different effects between rich economies and poor economies for each model.

6.2.1 Development Effects of the Gini Index

Table 6.2.1b below presents the empirical results for the GMM regressions using the Gini index to describe inequality. Regressions with either explanatory gdpb and popb or gdpw and popw give very similar results except of the estimation sizes.

Table 6.2.1b GMM Regression of GDPb on the Gini Index[3]

Dependent / Independent	GDPb Per Capita		
	laggedgdpb<0.26	laggedgdpb>0.26	Entire Sample
gdpb L1.	**1.0032(44.04)**	**.9098(36.99)**	**.9758(70.89)**
popb	-.0347(-0.50)	**.1306(3.99)**	**.0472(2.37)**
g	-.0023 (-0.00)	**.5029(3.06)**	**.1934(3.1)**
g^2	-.0234 (-0.25)	**-.6587(-2.63)**	**-.2236(-2.83)**
Intercept	.0132 (0.63)	**-.0834(-2.92)**	**-.0349(-2.88)**
$g^{\bullet}$	\	**0.3818(11.96)**	**0.4326(10.41)**

The value in parenthesis is for the statistics of the regression. The panel covers total 32 countries and 278 dynamic observations, in which he subsample of rich economies (gdpb>0.26) includes 14 countries and 132 observations; the subsample of poor economies (gdpb<0.26) includes 22 countries and 146 observations.

The regression results in Table 6.2.1b and Table 6.2.1w in the Appendix are summarized as follows:

- Given the history of GDP per capita, current GDP per capita is an inverted U-shape function of income inequality either for the entire sample or rich countries, so that there is a development-optimal income inequality over time and across (rich) countries;
- But poor countries do not show optimal Gini index for development;
- For the entire sample or rich countries, both population and its growth play positive significance; but population size shows insignificant effects and population growth shows positive significance in poor countries;
- There is significant and negative constant effect on development in the entire sample and rich countries.

6.2.2 Development Effects of the Fair Division Shares

Table 6.2.2b below shows the GMM regression results for using the fair division shares. Table 6.2.2b is for the models that both fair population share and fair income share take quadratic form, where fair population share does not show strong significance for entire sample. The significance of fair division shares present if we just apply one of them to quadratic form, which shows in Table 6.2.3b in the Appendix.

[3] Highlighted values denote the significance at 5% confidence level, and the same for all tables hereafter.

Table 6.2.2b GMM Regression GDPb on the Fair Division Shares

Dependent / Independent	GDPb Per Capita		
	laggedgdpb<0.26	laggedgdpb>0.26	Entire Sample
gdpb L1.	**1.0017(42.3)**	**.9307(42.00)**	**.9936(82.22)**
popb	.00007(0.00)	**.0690(2.5)**	.0139(0.73)
P_l	.2207(0.62)	-.4390 (-0.54)	**.4602(1.69)**
P_l^2	-.1462(-0.56)	.4610(0.69)	-.3235(-1.53)
W_l	**.8599(1.97)**	.0789(0.54)	**.1951(1.82)**
W_l^2	**-1.0628(-1.82)**	-.1058(-0.72)	**-.2148(-1.94)**
Intercept	**-.2490(-1.75)**	.09888(0.4)	**-.1979(-2.41)**
$P_l^\bullet$	/	/	**0.7113(11.43)**
$W_l^\bullet$	**0.4045(15.04)**	/	**0.4541(13.0)**

We summarize the results in Table 6.2.2b and Table 6.2.2w in the Appendix as follows:

- There is development optimal fair division shares for entire sample, but no optimal fair division shares for rich countries;
- There is development optimal fair income share, but no optimal fair population share for poor countries;
- Population size and its growth play positive and significant effects on development for rich countries, but population size plays insignificance in entire sample and poor subsample;
- Population growth (popw) presents positive and significant effects on development for entire sample and any subsamples.

The question in the above models is that developing economies do not present development-optimal Gini index, but developed economies do; and fair division shares show no development-optimal values for developed countries but developing economies do show optimum income share regarding aggregate per capita income.

We may doubt either the data or the approximation of the fair division point has some problems. All developing economies didn't have long periods of observations, and only a few developed economies had long periods of observations in the sample, which is possibly one of the reasons of losing the development effects of inequality. Actually following section 7 shows the growth effects of inequality performs perfectly about the growth-optimum inequality since it is a short run effect. Another problem could be the slope of defining the fair division point, which could be different between developing and developed economies.

6.2.3 Hybrid Models for the Development Effects of Inequality

The GMM regression results of hybrid models are reported in Table 6.2.4w~b in the Appendix. It shows the following results:

- There are development-optimal inequalities measured by the hybrid metric of the Gini index and the fair division shares for entire sample and the subsample of rich countries;
- Population size plays positive significance for rich countries, but insignificance in entire sample and poor countries;
- Population growth (regarding popw) shows positive significance for entire sample and any subsamples.

It seems that the hybrid metric performs better than the fair division shares because we can find develop-optimal inequality in both the entire and the subsample of rich countries.

There is another problem among these empirical results of the different models, that is, the optimal income inequality differs across models within a sample. Since both the fair division point and the Gini index each does not uniquely describe Lorenz curve, so that they may give different optimal inequalities in the hybrid models from the models of either the Gini index or the fair division point, this is the one reason; the other one could be the problem of data quality. Because this is a very unbalanced panel with gaps and short periods, the panel data has only 32 countries and 278 observations, which may be not big enough to expose significant effects of the long run. Table 6.2.5 below summarizes the development-optimal inequality by the GMM regressions with explanatory gdpb and popb.

Table 6.2.5 Development-Optimum Inequality[4]

Measure of Inequality	$Gini^*$	p_l^*, w_l^*	$Gini^*$, p_l^*	$Gini^*$, w_l^*
Entire Sample	0.433	0.711, 0.454	0.374, /	0.522, 0.483
Rich Countries	0.382	/, /	0.352, /	0.411, 0.492
Poor Countries	/	/, 0.404	/, 0.704	/, 0.419

[4] The result of development-optimum income inequality conforms to the Corollary 3.3 in Shao (2010) in a framework of new neoclassical model economy.

6.2.4 Summary

Income inequality presents significant effects on economic development. There is development-optimal income inequality over time and across countries, and so that both high and low inequality can be a bad event for an economy comparing its potential development-optimal inequality. Specifically, the entire sample and rich countries show optimal development at an appropriate inequality of income distribution; developing economies may not show development-optimal income inequality, which depends on the measurement of inequality; specifically, developing countries do not show development-optimal Gini index, but there is development-optimal inequality jointly measured by the Gini index and the fair division shares.

The fair division point of income distribution gives clear policy implications that a change of either the fair population share or the fair income share could lead to an improvement of efficiency toward the development-optimal income inequality; developing economies do not share with developed economies the same properties of inequality on development and thus neither their polices about income distribution.

Population size plays positive significance on development in rich countries, but it is insignificant for poor countries. The difference of development effects from population size could come from the differences of human capital among countries.

Population growth presents significant and positive effect on development in the countries of entire sample and any subsamples. But it is not clear within the reduce form of models why population growth significantly enhances development in developing economies and population size plays the opposite. It is probably because that development induces population growth within a developing economy and a larger population size shows negative effects on development between developing economies.

7. Growth Effects of Income Inequality

Section 6 shows that income inequality has significant effects on economic development, but how does this happen? There should be many channels in microeconomic level to bridge development and income inequality; on macroeconomic level, growth rate of GDP per capita can be a direct bridge leading a short run event to long run impact. If we take the optimal inequality for development as a long-run equilibrium of income distribution, then, the optimal inequality for growth is the event of income distribution in short run.

To regress growth rate of GDP on income inequality, GDP per capita will be controlled for since we know total production function has diminishing returns to inputs, thus we let GDP per capita take quadratic form which allows growth rate varies according to development.

This section will study this topic by three types of model corresponding to the measurement of inequality, which is the Gini index, the fair division point, and the hybrid measurement of the Gini index and the fair division shares.

7.1 Growth Effects of Inequality by the Gini Index

Employing the functional form (5.2), we can test the growth effects of the Gini index by fixed effect and random effect models, the Hausman test shows that random effect models are preferred to fixed effect models assuming no endogeneity problem; actually we can see in the regressions that the correlation between the error terms and covariates are pretty small in the fixed effects regression. Assuming the differences of base population and aggregate productivity, Table 7.1w~b in the Appendix presents the regression results, which are summarized as follows:

- There is growth-optimal Gini index for both the entire panel and the subsample of poor countries, but no growth-optimal Gini index for developed countries. The optimal Gini index is between 0.365 for entire sample and 0.359 for poor countries;
- Population size and its growth present insignificant roles for growth in entire sample and any subsamples; thus either population size or growth would not be an issue for growth;
- Growth rate of aggregate productivity is significantly a U-shape in development for the entire sample; both poor countries and developed countries could have high growth, but those median developed countries may have lower growth rate;
- There is no significant constant effects on growth in the model;
- The consistent and efficient random effect model explains about 28.7% between variations of growth for the entire panel of countries, about 26.6% for poor countries. The within R^2 is pretty small for the both samples.

Figure 7.1b below shows the fitted growth of random effect regression with explanatory gdpb and popb.

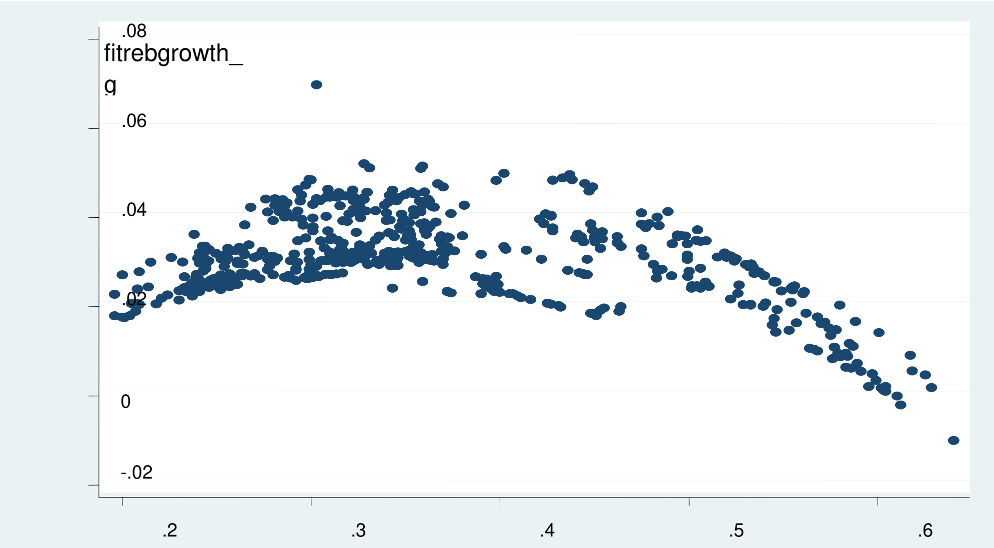


Figure 7.1b Fitted Growth by Random Effects, Gini, explanatory GDPb

Section 7.2 shows that the fair division point does a much better job than the Gini index in explaining growth, and the regression models are very convincing by the Hausman test.

7.2 Growth Effects of Inequality by the Fair Division Point

We employ the model (5.2) as well in this section and use the fair division point to describe income inequality. The Hausman test shows that random effect models are always consistent and efficient in all these growth models. Assuming the differences of base population and aggregate productivity, Table 7.2b in Appendix presents the regression results, which are summarized as follows.

- There is growth-optimal fair division point for entire panel of countries, but there is only growth-optimal fair population share and no optimal fair income share for poor countries; the optimal fair population share is 0.666, and the optimal fair division of income share is between 0.489 for the entire panel;
- There is the **"worst"** fair population share in developed economies, which is 0.575;
- <u>There are significant and negative constant effects on growth for entire sample and poor economies, but significant and positive constant effects for developed economies;</u>
- GDP per capita has no significant effect on growth, even if there is some effect, it is negative for developing economies and positive for developed countries;
- Both popb and popw show no significance on growth, thus either population size or growth would not be an issue for growth;

- The model accounts for 26.34% between variation, and 2.61% within variation of growth in the entire sample; 25.1% between variation, and 3.2% within variation of growth in the sample of poor economies when assuming gdpb and popb. The between R^2 41.9% and within R^2 is 3.15% for entire sample, and between R^2 is 38.43% and within R^2 is 3.84% for poor countries when assuming gdpw and popw.

Above results do not change even if we use level form for one of the fair division shares and quadratic form for the other fair division share.

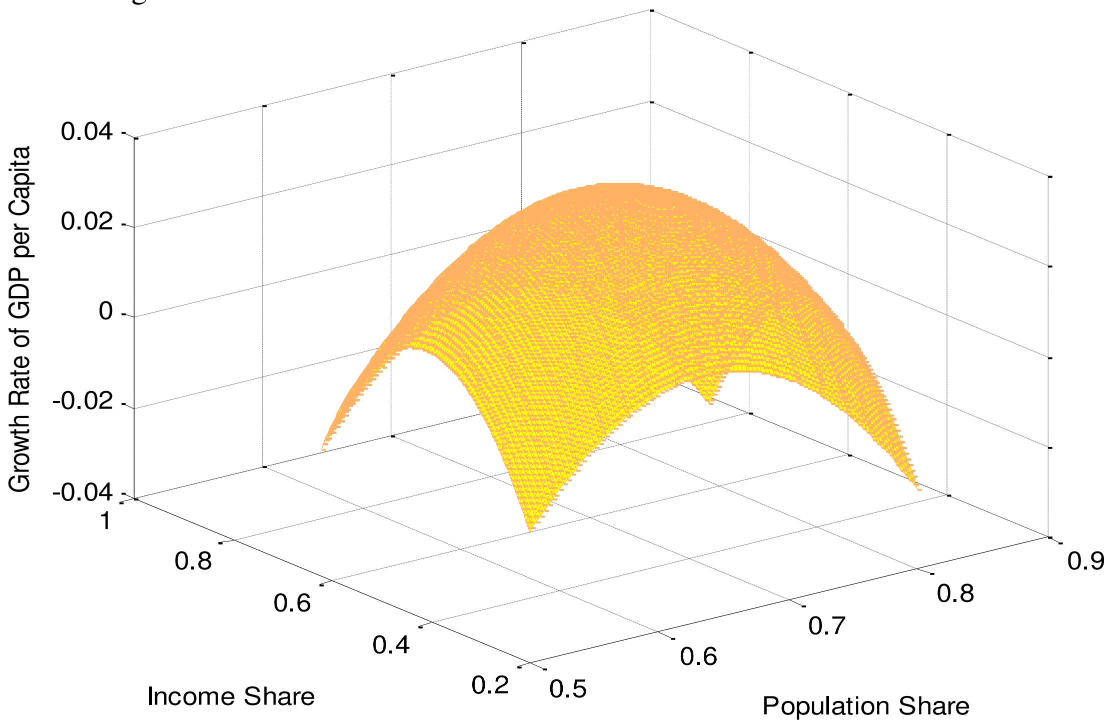
Both the fair division point and the Gini index present significant effects on growth, but there are also some differences between the two measurements in explaining growth. Table 7.1b and Table 7.2b present the following differences:

- In developed economies, there is no growth-optimal Gini index, but there is a significant and the **"worst"** fair population share, which is between 0.575. The model with the fair division point accounts for about 53.1% between variation, but the model with Gini accounts for only 11.2% between variation for developed economies;
- The significance of GDP per capita is reduced in the model of using the fair division point instead of using the Gini index;
- The constant effects are insignificant in the model with the Gini index, but it is significant and negative in developing economies, and significant and positive in developed economies in the model with the fair division point;

Thus, the fair division shares give more information than the Gini index.

Figure 7.2.1 below graphs the random effect estimation of growth rate using explanatory gdpb and popb for entire sample, where GDP per capita was 0.254 and population index was 0.9192.

Figure 7.2.1 Random Effects Estimation of Growth on Fair Division Point



Actually, the fair division point ignores redistribution within group and the Gini index may fail to tell redistribution between groups, but a hybrid metric is possible to do a better job than a single one of them in explaining growth. Following section 7.3 does this exercise.

7.3 Growth Effects of Inequality by the Hybrid Metric

Now we discuss the final functional form of (5.2), which assumes that simultaneously there are growth-optimal Gini index and one of the fair division shares, letting the other dimension of fair division point be undetermined. We employ the Hausman test to see if random effects regression is consistent and efficient; otherwise both fixed effect and random effect models will be reported. Table 7.3w~b in the Appendix reports the regression results, which are summarized as follows:

- There is growth-optimal inequality jointly described by the Gini index and the fair income share in the entire sample, the optimal Gini is 0.3991, and optimal fair income share is 0.5092;
- In those developed countries, there is optimal fair income share, which is between 0.4857, and no optimal Gini index with the hybrid metric;
- In those poor economies, there is only the optimal Gini coefficient which is 0.4044, and no optimal fair income share with the hybrid metric;

- Population size and population growth do not show significance on growth;
- Development level does not show much significance on growth in the subsamples of either developing or developed economies; but it shows significant negative effects for developing countries (gdpb<0.4457) and significant positive effects on growth for rich countries (gdpb>0.4457) in the entire sample;
- The between R^2 is 40.4% for the entire sample, 41.3% for developing economies, 29.9% for developed economies, and all of within R^2 are around 2%.

The regression results for the hybrid measurement of the Gini index and the fair population share in Table 7.4w~b in the Appendix are summarized as follows:

- There is only growth-optimal Gini index and no growth-optimal fair population share for the entire sample when the Gini index is controlled for;
- For those developed economies, there are jointly the growth-optimal Gini index and the growth-worst fair population share;
- We don't see growth-optimal hybrid metric of the Gini index and the fair population share for developing economies, but for given one of the two dimensions, there is growth-optimal inequality by the other dimension (refer to Table 7.4bb in the Appendix).

Now we report the R^2 for above growth models in Table 7.5 below, which shows that the Gini index and the fair division point jointly explain much higher between variation of growth than either one of the two metrics, but the within R^2 is pretty small for entire sample and the two subsamples, which could be caused by the short periods of observations in each country.

Table 7.5 R^2 by Sample and Inequality Metric in Growth Models[5]

Measure of Inequality		g	p_l, w_l	g, p_l	g, w_l
For Entire Sample	Within R^2	0.01	0.026	0.026	0.022
	Between R^2	**0.287**	**0.263**	**0.417**	**0.404**
	Total R^2	0.098	0.095	0.126	0.119
For Rich Countries	Within R^2	0.025	0.009	0.016	0.018
	Between R^2	**0.112**	**0.531**	**0.568**	**0.299**
	Total R^2	0.049	0.092	0.103	0.064

Table 7.6 below summarizes all growth-optimal inequalities for entire panel and subsamples assuming the differences of base population and base productivity.

[5] R^2 values in Table 7.5 are from the random effect regressions with explanatory gdpb and popb.

Table 7.6 Growth-Optimum Inequality

Measure of Inequality	$Gini^*$	p_l^*, w_l^*	$Gini^*$, p_l^*	$Gini^*$, w_l^*
Entire Sample	0.365	0.666, 0.489	0.278, /	0.399, 0.509
Rich Countries	/	0.575[6], 0.449	0.314, 0.581[7]	/, 0.486
Poor Countries	0.359	0.683, /	/, /	0.404, /

If we compare Table 6.2.5 and Table 7.6, we find that the optimum of income inequality in a subsample depends on the measurement, but the entire sample does present optimal inequality regarding either growth or development.

There is optimal income inequality measured by the Gini index or fair division point regarding either growth or development for the entire sample; Rich countries do not show growth-optimal Gini index, and no optimal fair population share, but they have the development-optimal Gini index and growth-optimal fair income share; poor countries seems to have either growth-optimal Gini index or fair population share, but no development-optimal Gini index and no growth-optimal fair income share.

7.4 Summary

We have following findings about the growth effects of income inequality when we apply both the Gini and the fair division point to measuring income inequality:

- There is growth-optimal inequality measured by the Gini index, the fair division shares, or the hybrid measurement of the two metrics for the entire panel of countries;
- For those rich countries, there is growth-optimal inequality described by the fair income share; and there is the "worst" fair population share that leads to lowest growth rate, but the Gini index does not give much implication on growth;
- For those poor countries, there is growth-optimal inequality measured by the Gini index, or fair population share;
- Population size and its growth are insignificant for growth in entire sample and any subsamples;
- The consistent and efficient random effect model explains the between variations of growth about 41.75% for the entire sample, and 56.8% for rich countries by the hybrid measurement

[6], [7], The red numbers (0.5747, 0.5813) are the "growth-worst" fair population share in the model.

of the Gini index and fair population share. A measurement of inequality with the fair population share can give a much higher between R^2.

It requires further work to explain why income inequality does not explain as much within variations as between variations of growth. This could be caused by two causes: one is that the unbalanced panel data does not include sufficient time periods in each country, the other one could be that income inequality has lag effect on growth. Once we have sufficient time periods for each country, we can do dynamic growth models to see if there is significant improvement in explaining the growth effect of income inequality within an eocnomy.

7.5 Policy Implication

There are very striking policy implications according to the fair division point. First, the theory of fair division point of income distribution supports the tax policy that tax rate varies according to the fair division point in order to respond to the growth effects of fair division shares.

Second, there is optimum income inequality regarding either aggregate productivity or growth rate of GDP, only a change (either increase or decrease) of inequality towards the optimum inequality will be an improvement of the economic effects of inequality.

Third, developing economies show different optimum inequality from that in developed economies, so that a developing country should not copy development policies from developed economies regarding income distribution.

Fourth, there is the growth-worst fair population share that results in the lowest growth in developed economies, so that a deviation from the "worst" fair population share would improve resource allocations and enhance growth for a developed economy.

8. Conclusion

We introduce the "fair division point" to describe inequality of income distribution. It shows unit slope on a Lorenz curve and involves the fair income share and the fair population share. The fair division point approximates the balanced income inequality that shows equal growth of income from both high and low income groups in the economy. The households which are income-ordered within the fair population share are called low income population; the others are called high income population. The Gini index can be practically interpreted as the difference of the fair population share and the fair income share.

Employing a panel data of countries, the analysis shows that a country's low income population will decrease (the fair population share drops slightly) as the country grows; and at the same time, those low income households are relatively worse off (the fair income share decreases). Inversely, as an economy grows rich, there are more low income households (the fair population share rises), but those low income households are relatively better off (the fair income share rises and GDP per capita increases). Overall, both the Gini index and the difference between the fair population share and the fair income share have been increasing, therefore, income inequality increases as an economy is getting richer. But there is no evidence for Kuznets hypothesis in this study.

Income inequality presents significant effects on economic development. For the entire panel of countries, there is development-optimum income inequality measured by either the Gini index and/or the fair division shares regarding aggregate income per capita, so that both high and low inequality could harm an economy as we compare with its potential development-optimum inequality. The development-optimum Gini index was about 0.433, the optimum fair population share was about 0.711, and the optimum fair income share was about 0.454. Subsamples of the panel show different development-optimum inequalities. Developed economies show development-optimum Gini index, but no optimal fair division point; developing economies show development-optimum fair income share but no optimal Gini index.

The analysis demonstrates growth-optimum income inequality for the entire panel of countries and any subsamples. Both high and low income inequalities might impede growth. But in developed economies there are the growth-optimum fair income share and the growth-worst fair population share that leads to the lowest growth; in developing economies there is growth-optimum Gini index or fair population share.

Growth-optimal inequality is different from development-optimal inequality that could be due to the differences of time horizon and regression method. Growth-optimum inequality responds to efficient allocation in the short run, but development-optimum inequality is an efficient allocation in the long run.

The fair division shares give more information than the Gini index in growth models. In developed economies, there is no growth-optimal Gini index, but there is a significant and the **"worst"** fair population share, which is between 0.575. The growth models of fair division point account for 53.1% between-variations, but only 11.2% between-variations are accounted for by the growth models of the Gini index for developed economies. The significance of GDP per capita is reduced in the growth models of using the fair division point instead of using the Gini index to express inequality. The constant effects are insignificant in the growth model with the Gini index, but significant and positive in developed economies and negative in developing economies and the entire sample in the growth model with the fair division point.

Population size presents positive significance on development for the entire sample and those rich countries, but negative insignificance for those poor countries. The difference of development effects from population size could come from the differences of aggregate human capital among countries. Population growth presents significant and positive effects on development for all countries. But population size and its growth do not show significant effects on growth rate of GDP per capita.

Appendix

Table 4.1.2 Correlation between Variables

	x	*y*	*g*	growth	gdpb	gdpw	popb	popw
x	1							
y	-0.2168	1						
g	**0.8985**	**-0.5716**	1					
growth	-0.0221	0.1811	-0.1077	1				
gdpb	**-0.5327**	-0.0046	**-0.4045**	-0.0792	1			
gdpw	0.2335	-0.0878	0.238	-0.1989	0.2412	1		
popb	0.1242	-0.388	0.3144	-0.1149	0.2048	-0.0235	1	
popw	-0.1802	0.2613	-0.2614	-0.1011	0.2546	**0.636**	-0.2843	1

Figure 4.1 below is an example of Lorenz curve for Latvia in 1998, following is the calculation on Matlab for its fair division point.

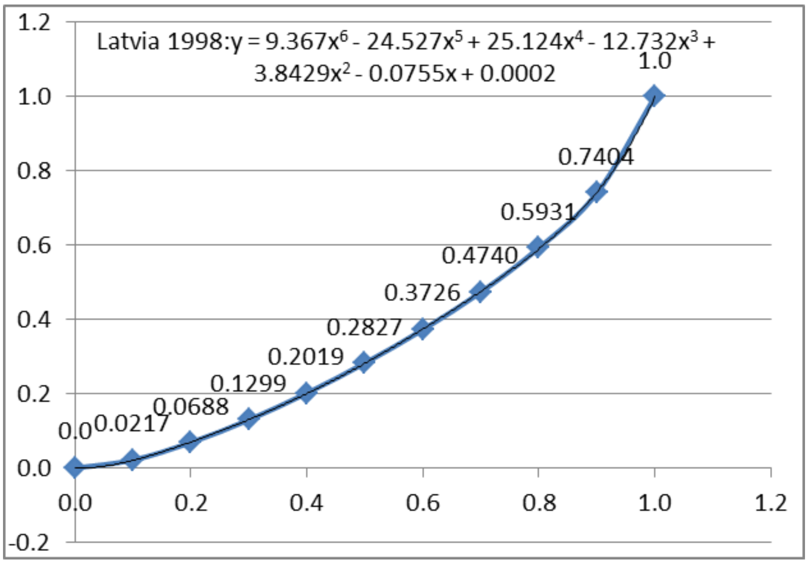


Figure 4.1

Then, here are the commands to calculate a fair division point (x, y) in Matlab for 1998 Latovia:

```
Y=[ 9.367   -24.567   25.124   -12.732   3.8429   -0.0755  0.0002   ]
Dc=polyder(Y)
One=[0  0  0  0  0  -1]
Ec=Dc+One
```

x=roots(Ec)
y=polyval(Y, x)

The output of above calculation is (0.6856, 0.4542), at which the slope of Lorenz curve is unit; it means that 68.56% of low income households hold 45.42% total disposable income in 1998 in Latvia.

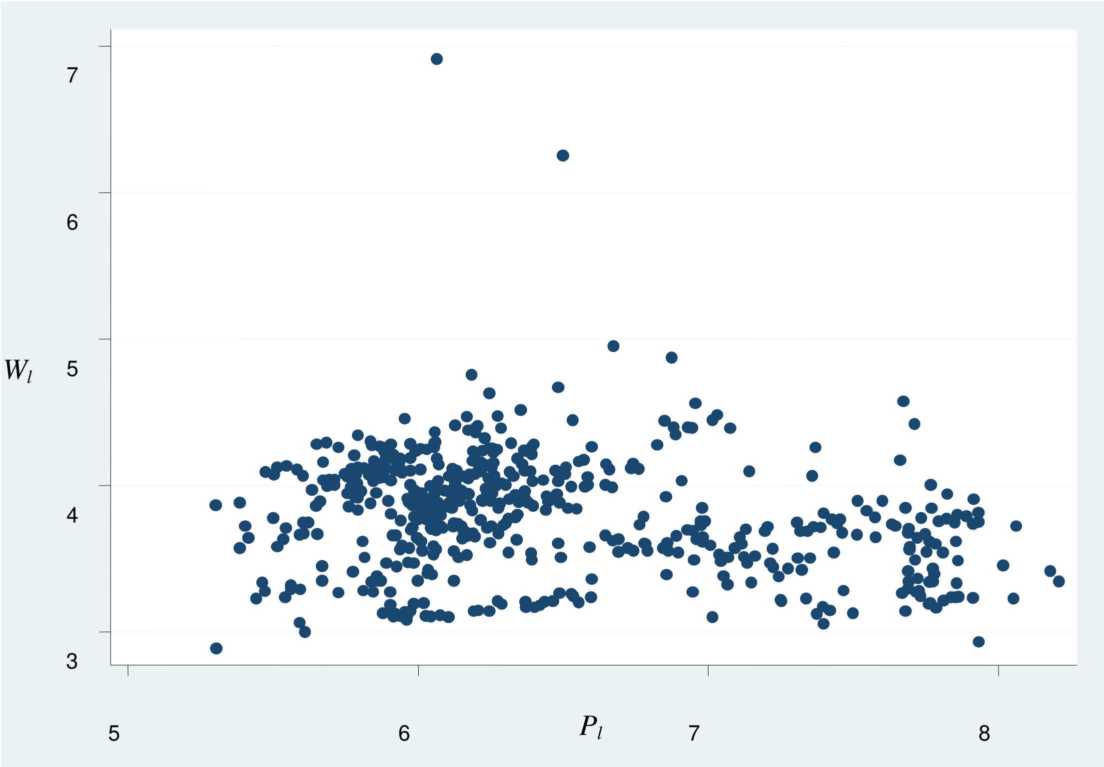


Figure 4.2.5 Observations of Fair Income Share Against Fair Population Share

Table 4.2.1 Regression of the Gini Index on the Fair Division Point

Independent Variables	**FE**	RE MLE	FE
P_l	**35.1911 (2.17)**	**1.2346 (40.60)**	**1.0894 (29.88)**
P_l^2	**-80.569 (-2.19)**		
P_l^3	**84.6778 (2.29)**		
P_l^4	**-33.224 (-2.4)**		
W_l	**55.6946 (22.31)**	**-.7724(-22.87)**	**-.6641 (-21.72)**
W_l^2	**-195.729(-23.09)**		
W_l^3	**288.2418 (23.02)**		
W_l^4	**-151.656(-22.42)**		
Intercept	**-11.3374 (-4.22)**	**-.1325(-6.96)**	**-.07958 (-3.54)**
corr(u_i, Xb)	0.086		0.7734
Within R^2	0.9645		0.6801
Between R^2	0.9975		**0.9810**
Overall R^2	0.9952		0.9490

Numbers in parenthesis are t-values. Cells left empty denote that the corresponding variables are not included in the model. The Hausman test shows that RE models are not consistent.

Table 4.2.2 RE Regression of the Fair Division Point on the Gini Index

Dependent / Independent	W_l		P_l	
	RE	MLE	RE	MLE
g	**30.835(3.46)**	**30.8523 (3.47)**	**29.9674(4.66)**	**29.9724 (4.69)**
g^2	**-161.01(-3.39)**	**-161.101(-3.41)**	**-154.547(-4.50)**	**-154.574(-4.53)**
g^3	**397.4945(3.24)**	**397.7388(3.25)**	**386.528(4.35)**	**386.6121(4.38)**
g^4	**-468.862(-3.05)**	**-469.178(-3.06)**	**-462.777(-4.15)**	**-462.912(-4.18)**
g^5	**212.3269 (2.83)**	**212.484(2.84)**	**213.426(3.93)**	**213.5093(3.95)**
Intercept	**-1.8143(-2.79)**	**-1.8154(-2.81)**	**-1.6790(-3.59)**	**-1.6793(-3.61)**
FE corr(u_i, Xb)	0.0082		0.282	
H-Test Prob(chi2)	0.6176 (3.54)		0.0634 (10.45)	
Within R^2	0.1352		0.4016	
Between R^2	0.5258		0.9056	
Overall R^2	0.3702		0.8296	

Table 4.2.3 Regression between the Fair Division Shares W_l and P_l

Independent Variable	P_l		W_l	
	RE	FE	RE	FE
W_l	**3.5985(6.86)**	**2.9059(4.99)**		
W_l^2	**-2.6179(-6.60)**	**-1.9506(-4.32)**		
P_l			**.8870(4.03)**	**.9725(4.46)**
P_l^2			**-.7157(-2.87)**	**-.7946(-3.22)**
Intercept	**-.8396(-4.87)**	**-.6736(-3.6)**	**.4225(8.63)**	**.3903(8.19)**
FE corr(u_i, Xb)		-0.6300		-0.4289
Within R^2	0.1225	0.1440	0.1298	0.1298
Between R^2	0.0029	0.1274	0.1585	0.1584
Overall R^2	0.0086	0.0132	0.0536	0.0537

Table 6.1.1b Regression of the Gini Index on Real GDPb and Popb

Dependent / Independent	Gini Index (g)		
	Robust OLS	Fixed Effect	Random Effect
gdpb	**-1.4312(-8.04)**	**0.0686(2.86)**	.03035(1.31)
$gdpb^2$	**2.4395(4.85)**		
$gdpb^3$	**-1.264(-3.19)**		
popb	**0.2018(11.42)**	**0.0999(1.99)**	**.1423(3.25)**
Intercept	**0.5314(30.71)**	**0.3351(42.61)**	**.3600(26.58)**
corr(u_i,Xb)		-0.1424	
H-Test Prob(Chi2)			0.000(246.52)
Within R^2		**0.044**	0.0393
Between R^2		**0.016**	0.0098
Overall R^2	0.4056	**0.0107**	0.0602

GDP is the real GDP in 2005 constant price by chain method and has been converted by PPP. Chi2 statistic of hausman test is not positive definite for the both models.

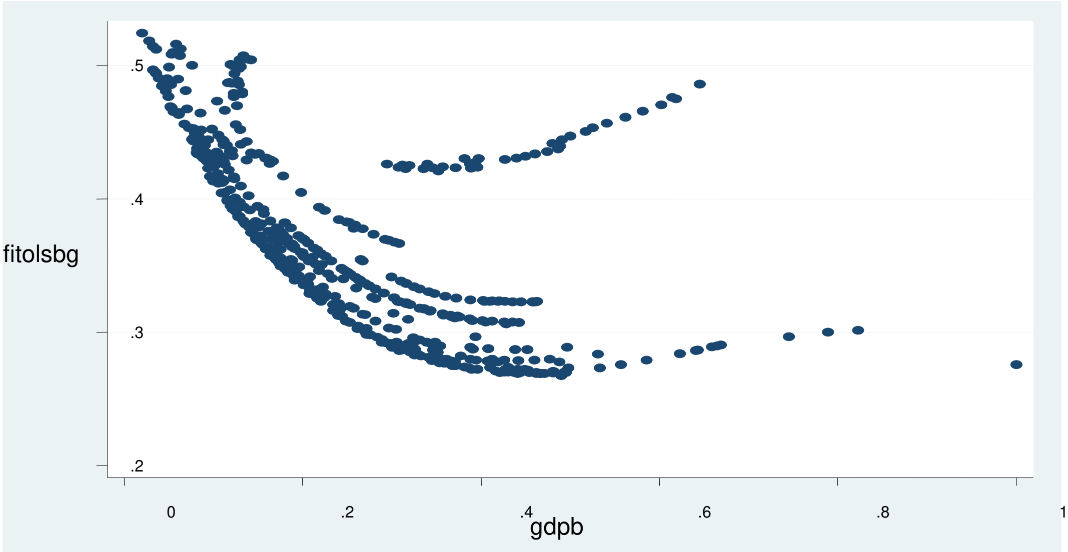


Figure 6.1.1b Robust OLS Fitted Gini Against GDPb

Table 6.1.2b Regression of the Fair Population Share on Real GDPb and Popb

Dependent / Independent	Fair Population Share (P_l)		
	Robust OLS	Fixed Effect	Random Effect
gdpb	**-0.9828(-9.29)**	**-0.3154(-4.57)**	**-.4106(-5.85)**
gdpb^2	**1.5844(5.3)**	**0.9121(5.15)**	**1.039(5.71)**
gdpb^3	**-0.7699(-3.27)**	**-0.6007(-4.46)**	**-.6627(-4.77)**
popb	**0.0747(7.12)**	**0.0908(2.52)**	**.0974(3.31)**
Intercept	**0.7700(74.9)**	**0.6512(71.53)**	**.6826(65.42)**
FE corr(u_i,Xb)		-0.2794	
H-Test Prob(Chi2)			0.000(212.79)
Within R^2		**0.1168**	**0.1065**
Between R^2		0.0031	**0.0931**
Overall R^2	0.4346	0.0054	**0.0524**

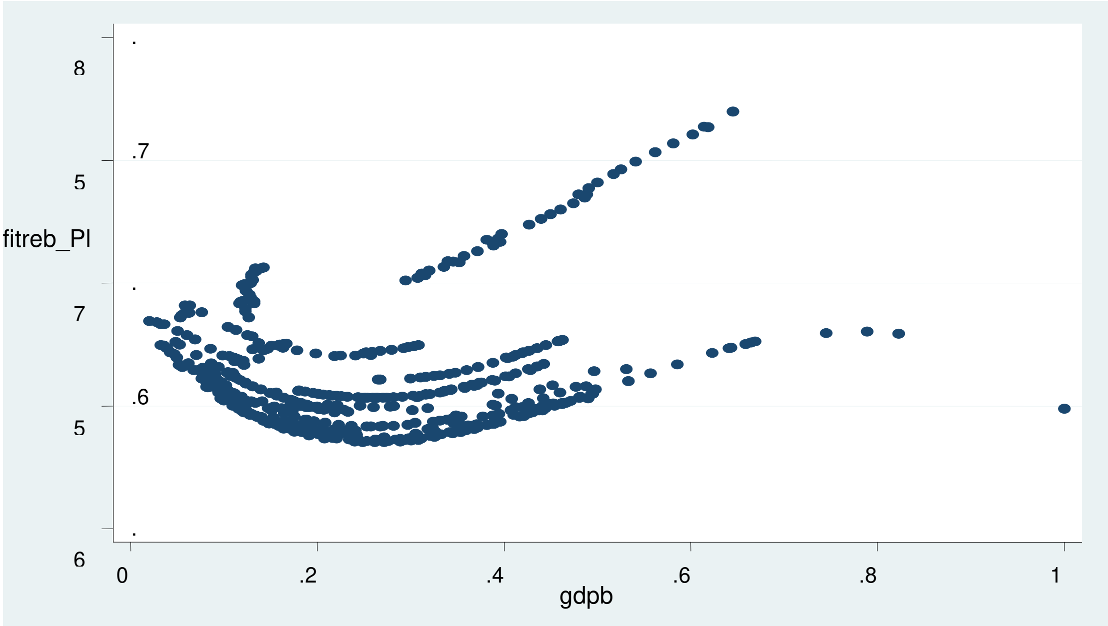


Figure 6.1.2b Random Effect Fitted Fair Population Share Against

Table 6.1.3b Regression of the Fair Income Share on GDPb and Popb

Dependent / Independent	Fair Income Share (W_l)		
	Robust OLS	Fixed Effect	Random Effect
gdpb	**0.1748(2.61)**	**-0.3628(-4.25)**	**-.2571(-3.3)**
$gdpb^2$	**-0.4000(-2.11)**	**0.8954(4.09)**	**.7189(3.45)**
$gdpb^3$	0.2598(1.74)	**-0.5733(-3.44)**	**-.4720(-2.94)**
popb	**-0.0849(-12.76)**	0.0647(1.45)	**-.0373(-1.7)**
Intercept	**0.3746(57.46)**	**0.4041(35.9)**	**.4051(44.87)**
FE corr(u_i,Xb)		-0.7611	
H-Test Prob(chi2)			0.000 (21.37)
Within R^2		**0.0494**	0.0367
Between R^2		**0.1633**	0.0145
Overall R^2	0.1600	**0.1386**	0.0670

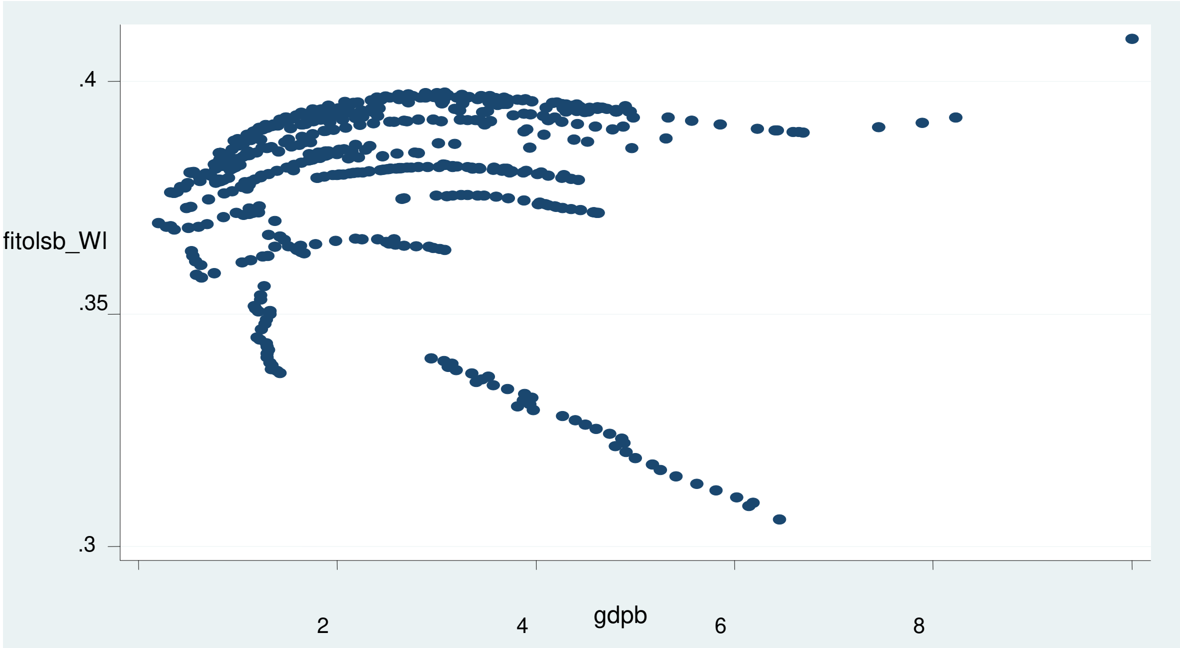


Figure 6.1.3b Robust OLS Fitted Fair Income Share Against GDPb

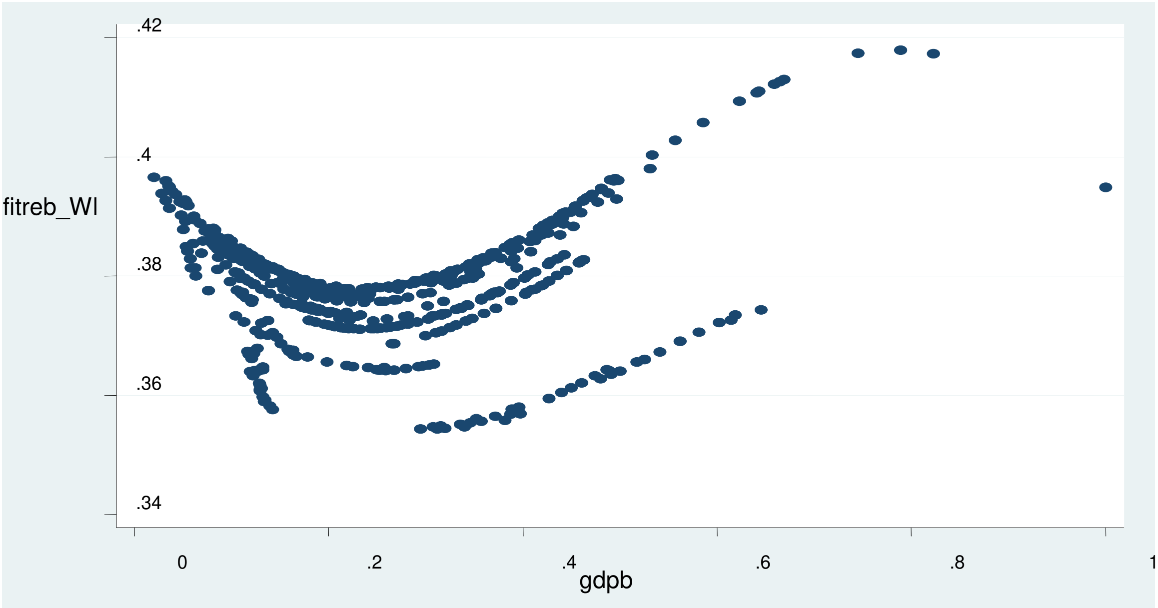


Figure 6.1.3b Random Effect Fitted Fair Income Share Against GDPb

Table 6.1.4b Regression of Difference Between the Fair Division Shares on GDPb and Popb

Dependent / Independent	Difference Between Fair Division Shares $P_l - W_l$		
	Robust OLS	Fixed Effect	Random Effect
gdpb	**-1.1218(-8.1)**	**.04634(2.01)**	.0022(0.1)
$gdpb^2$	**1.9217(4.91)**		
$gdpb^3$	**-1.0003(-3.25)**		
popb	**.1565(11.38)**	.02644(0.55)	**.0840(2.13)**
Intercept	**.3910(29.05)**	**.2477(32.82)**	**.2649(23.94)**
FE corr(u_i,Xb)		-0.2531	
H-Test Prob(Chi2)			0.000(60.24)
Within R^2		0.0139	0.0736
Between R^2		**0.1367**	0.0353
Overall R^2	0.3846	0.0168	0.0736

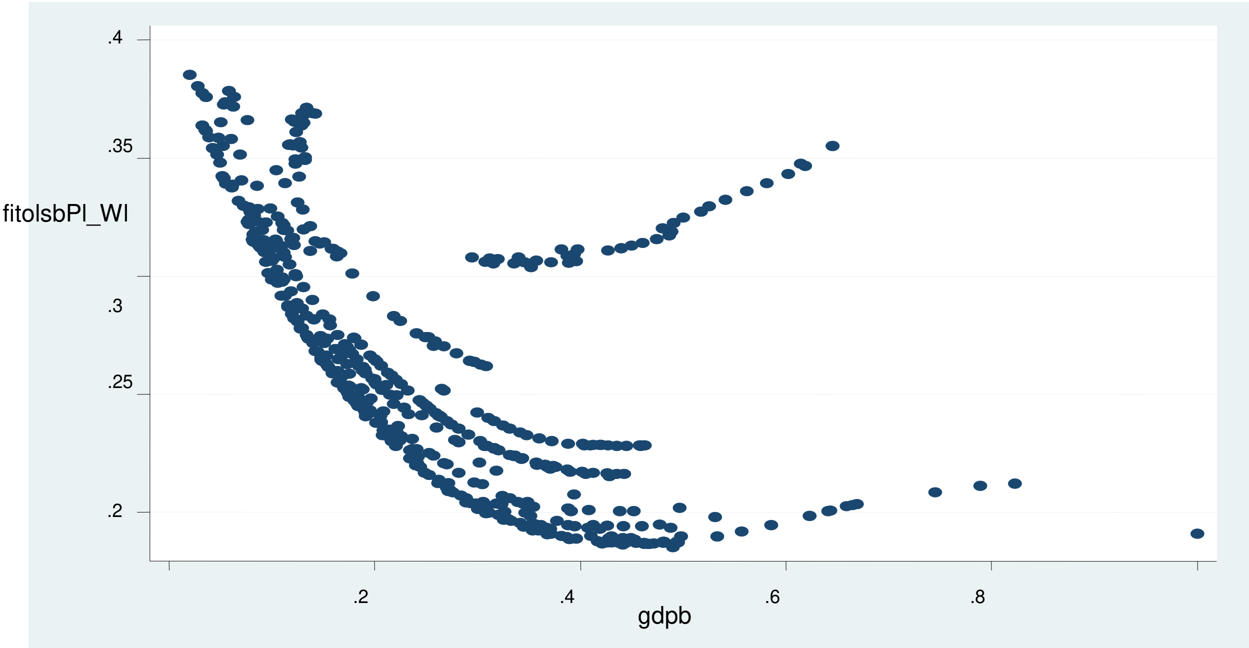


Figure 6.1.4b Robust OLS Fitted Difference of Fair Division Shares Against GDPb

Table 6.2.1w GMM Regression of GDPw on the Gini Index

Dependent / Independent	GDPw Per Capita		
	laggedgdpb<0.26	laggedgdpb>0.26	Entire Sample
gdpwL1.	**.943(28.53)**	**.909 (33.32)**	**.948(45.76)**
popw	**.191(2.16)**	**.219 (3.5)**	**.155 (3.03)**
g	.196(0.52)	**1.017 (3.18)**	**.590 (2.99)**
g^2	-.366(-0.88)	**-1.37(-2.81)**	**-.761(-3.01)**
Intercept	-.129(-1.23)	**-.294(-3.62)**	**-.191(-3.42)**
$g^{\bullet}$	/	**0.371(13.06)**	**0.389(11.14)**

The panel covers total 32 countries and 278 dynamic observations, in which the subsample of rich economies (gdpb>0.26) includes 14 countries and 132 observations; the subsample of poor economies (gdpb<0.26) includes 22 countries and 146 observations.

Delta method to estimate the variance of the optimal inequality $g(a,b)=\frac{-b}{2a}$:

Let $\sqrt{n}(\hat{a}-a)\xrightarrow{d}N(0,\delta_a^2)$, $\sqrt{n}(\hat{b}-b)\xrightarrow{d}N(0,\delta_b^2)$, δ_a^2 is the variance of estimation $\hat{a}$, δ_a^2 is the variance of estimation $\hat{b}$, covariance σ_{ab} of estimation $\hat{a}$ and $\hat{b}$, and $\hat{a}\neq 0$, then, delta method gives following result:

$$\sqrt{n}\left(g(\hat{a},\hat{b})-g(a,b)\right)\xrightarrow{d}N(0,JVJ'),$$

$$J=\left(\frac{\hat{b}}{2\hat{a}^2},-\frac{1}{2\hat{a}}\right),V=\begin{pmatrix}\delta_a^2 & \sigma_{ab}\\ \sigma_{ab} & \delta_b^2\end{pmatrix}.$$

The variance of the estimator $g(a,b)$ is as follows:

$$JVJ'=\left(\frac{\hat{b}}{2\hat{a}^2},-\frac{1}{2\hat{a}}\right)\begin{pmatrix}\delta_a^2 & \sigma_{ab}\\ \sigma_{ab} & \delta_b^2\end{pmatrix}\begin{pmatrix}\frac{\hat{b}}{2\hat{a}^2}\\ -\frac{1}{2\hat{a}}\end{pmatrix}$$

Table 6.2.2w GMM Regression of GDPw on the Fair Division Shares

Dependent / Independent	GDPw Per Capita		
	laggedgdpb<0.26	laggedgdpb>0.26	Entire Sample
gdpw L1.	**.9488(29.38)**	**.9412(38.57)**	**.9708(50.64)**
popw	**.2287(2.56)**	**.0923(1.66)**	**.1033 (2.07)**
P_l	1.235(0.75)	-1.526 (-0.94)	1.418 (1.56)
P_l^2	-.8662(-0.71)	1.4289(1.07)	-1.049(-1.48)
W_l	2.775(1.39)	.1670(0.56)	**.7075(1.97)**
W_l^2	-3.1424(-1.17)	-.2238(-0.75)	**-.7603(-2.04)**
Intercept	**-1.1861(-1.85)**	.3483(0.7)	**-.6853(-2.47)**
$P_l^\bullet$	/	/	**0.6758(14.76)**
$W_l^\bullet$	/	/	**0.4653(14.8)**

Table 6.2.3b GMM Regression of GDPb on the Fair Division Shares

Dependent / Independent	GDPb Per Capita					
	laggedgdpb<0.26		laggedgdpb>0.26		Entire Sample	
gdpb L1.	**1.0(42)**	**1.01(44.18)**	**.932(42.19)**	**.929(41.93)**	**.99(85)**	**.99(83.04)**
popb	-.004(-.05)	-.024(-0.35)	**.071(2.57)**	**.0718(2.62)**	.010(0.53)	.021(1.12)
P_l	.024(0.73)	.231(0.65)	**.121(2.91)**	-.392(-0.48)	**.046(2.03)**	**.603(2.29)**
P_l^2		-.17(-0.67)		.429(0.64)		**-.431(-2.1)**
W_l	**.895(2.06)**	.068(1.57)	.073(0.5)	-.025(-1.51)	**.241(2.34)**	-.01(-0.75)
W_l^2	**-1.1(-1.88)**		-.099(-0.67)		**-.259(-2.42)**	
Intercept	**-.19(-2.05)**	-.09(-0.83)	**-.070(-2.09)**	.106(0.43)	**-.078(-3.1)**	**-.2(-2.39)**
$P_l^\bullet$	/	/	/	/	/	0.6994
$W_l^\bullet$	0.408	/	/	/	0.465	/

The value in parenthesis is for the statistics of the regression. The panel covers total 32 countries and 278 dynamic observations, in which he subsample of rich economies (gdpb>0.26) includes 14 countries and 132 observations; the subsample of poor economies (gdpb<0.26) includes 22 countries and 146 observations.

Table 6.2.4w GMM Regression of GDPw on Income Inequality for Hybrid Models

Dependent / Independent	GDPw Per Capita					
	laggedgdpb<0.26		laggedgdpb>0.26		Entire Sample	
gdpw L1.	**.959** **(29.05)**	**.951** **(29.38)**	**.905** **(33.16)**	**.905** **(33.59)**	**.953** **(45.40)**	**.950** **(46.33)**
popw	**.223** **(2.49)**	**.233** **(2.6)**	**.207** **(3.19)**	**.1865** **(2.91)**	**.1515** **(2.82)**	**.138** **(2.67)**
g	-.60 (-1.04)	-.04 (-0.10)	**1.187** **(3.49)**	**1.027** **(3.26)**	**.624** **(2.22)**	**.586** **(2.99)**
g^2	.3796 (0.55)	.139 (0.30)	**-1.72** **(-3.33)**	**-1.26** **(-2.6)**	**-.944** **(-2.42)**	**-.663** **(-2.62)**
P_l	3.44 (1.37)		**-3.44** **(-1.95)**		-.665 (-0.49)	
P_l^2	-2.31 (-1.2)		**2.939** **(2.02)**		.666 (0.62)	
W_l		3.371 (1.54)		**.794** **(2.43)**		**1.026** **(2.70)**
W_l^2		-3.72 (-1.3)		**-.82** **(-2.49)**		**-1.06** **(2.69)**
Intercept	-1.24 (-1.6)	**-.908** **(-2.1)**	.7104 (1.36)	**-.455** **(-4.4)**	-.026 (-0.07)	**-.423** **(-4.14)**
g^*	\	\	0.3454	0.4062	0.3306	0.4423
P_l^*	\	\	0.5861	\	\	\
$W_l^{\bullet}$	\	\	\	0.4826	\	0.4834

The panel covers total 32 countries and 278 dynamic observations, in which there are 14 countries and 132 observations for the subsample of rich countries, 22 countries and 149 observations for the subsample of poor countries.

Table 6.2.4b GMM Regression of GDPb on Income Inequality for Hybrid Metrics

Dependent Independent	GDPb Per Capita					
	laggedgdpb<0.26		laggedgdpb>0.26		Entire Sample	
gdpb L1.	**1.0203** **(42.47)**	**1.007** **(41.89)**	**.9029** **(36.71)**	**.8868** **(34.28)**	**.9794** **(69.73)**	**.9771** **(72.49)**
popb	-.04932 (-0.71)	-.0177 (-0.25)	**.1247** **(3.76)**	**.1286** **(3.77)**	.0380 (1.75)	.0321 (1.56)
g	-.1844 (-1.45)	-.0540 (-0.64)	**.5470** **(3.15)**	**.5316** **(3.15)**	**.1695** **(1.89)**	**.1808** **(2.96)**
g^2	.1752 (1.15)	.0948 (0.94)	**-.7774** **(-2.95)**	**-.6465** **(-2.53)**	**-.2268** **(-1.87)**	**-.1732** **(-2.2)**
P_l	**.8860** **(1.64)**		-1.224 (-1.4)		.0047 (0.01)	
P_l^2	**-.6293** **(-1.53)**		1.072 (1.49)		.0359 (0.11)	
W_l		**1.072** **(2.28)**		**.6185** **(3.78)**		**.3665** **(3.20)**
W_l^2		**-1.2803** **(-2.11)**		**-.6279** **(-3.78)**		**-.3793** **(3.21)**
Intercept	-.2563 (-1.59)	-.2098 (-2.26)	.2671 (1.03)	**-.2275** **(-4.69)**	-.0427 (-0.36)	**-.1186** **(-4.08)**
$g^{\bullet}$	\	\	0.3518	0.4112	0.3739	0.5219
P_l^*	0.704		\	\	\	\
$W_l^{\bullet}$	\	0.4187	\	0.4925	\	0.4831

The panel covers total 32 countries and 278 dynamic observations, in which there are 14 countries and 132 observations for the subsample of rich countries, 22 countries and 149 observations for the subsample of poor countries.

Table 7.1b RE Regression of Growth on the Gini, Popb and GDPb

Independent Variable	gdpb<0.26	gdpb>0.29	Entire Sample
gdpb	.0367 (0.18)	.0305(0.32)	**-0.1139(-2.89)**
$gdpb^2$	-.4148(-0.62)	.0042(0.05)	**0.1278(2.44)**
popb	.0119 (0.65)	-.0246(-1.29)	-0.0094(-0.71)
g	**.6203(3.57)**	.4682(1.43)	**0.4496(3.33)**
g^2	**-.8639 (-4.14)**	-.5852(-1.19)	**-0.6261(-3.77)**
Intercept	-.0600(-1.57)	-.0699 (-1.16)	-0.024(0.9)
FE corr(u_i, Xb)	0.0842	-0.1159	0.2710
H-Test Prob(Chi2)	0.367(5.42)	0.7178(2.88)	0.7763(2.5)
Within R^2	0.0105	0.0247	**0.0104**
Between R^2	0.2658	0.1125	**0.2874**
Overall R^2	0.1095	0.0488	**0.0979**
Optimal g^*	**0.359(18.97)**	/	**0.3648(16.82)**

There are 212 observations from 24 countries for gdpb>0.29, 308 observations from 41 countries for gdpb<0.26.

Table 7.1w RE Regression of Growth on the Gini, Popw and GDPw

Independent Variable	gdpb<0.26	gdpb>0.29	Entire Sample
$gdpw$	-.0964(-1.56)	-.0047(-0.04)	**-.1133(-2.43)**
$gdpw^2$	.0619(1.36)	0.0043(0.06)	**.0694(2.13)**
$popw$	-.0345(-1.00)	0.0112(0.23)	-.0091(-0.34)
g	**.5699(3.38)**	0.3590(1.15)	**.3913(3.11)**
g^2	**-.7702(-3.81)**	-0.4836(-1.03)	**-.5225(-3.41)**
Intercept	.0058(0.13)	-0.0448(-0.55)	.0155(0.45)
FE corr(u_i, Xb)	-0.0108	-0.3262	-0.0285
H-Test Prob(Chi2)	0.1183(8.78)	0.0973(9.31)	0.0001(26.95)
Within R^2	0.0179	0.0109	0.0159
Between R^2	0.2359	0.0104	0.2627
Overall R^2	0.1284	0.0113	0.0865
Optimal g^*	**0.370(18.11)**	/	**0.3745(18.02)**

The fixed effect regression for entire sample and rich countries do not show significance on the quadratic form of Gini.

Table 7.2b RE Regression of Growth on the Fair Division Point, GDPb and Popb

Independent Variable	gdpb<0.26	gdpb>0.26	Whole Sample
gdpb	-.0773(-0.39)	-.0858(-1.09)	-.0674(-1.67)
$gdpb^2$	.1243(0.19)	.0979(1.26)	.0751(1.41)
popb	.0101(0.56)	-.0153 (-1.12)	-.0065(-0.49)
W_l	1.2097(1.11)	.2264(0.79)	**.694(2.29)**
W_l^2	-1.185(-0.84)	-.2778(-0.89)	**-.7095(-2.01)**
P_l	**2.942(3.87)**	**-2.8901(-1.75)**	**1.840(3.26)**
P_l^2	**-2.1784(-3.91)**	**2.5147(1.87)**	**-1.3809(-3.27)**
Intercept	**-1.2287(-3.72)**	**.8284(1.68)**	**-.7251(-3.87)**
FE corr(u_i,Xb)	-0.0148	-0.1140	-0.3054
H-Test Prob(chi2)	0.2670(8.8)	0.4164 (7.12)	0.5522(5.89)
Within R^2	0.0321	0.0092	**0.0261**
Between R^2	0.2508	**0.5308**	**0.2634**
Overall R^2	0.1361	0.0925	**0.0948**
P_l^*	**0.675(68.97)**	**0.5747(23.55)♣**	**0.666(51.84)**
W_l^*	/	/	**0.489(10.52)**

Dependent variable is growth rate of GDP in all above regressions. The linear form of GDP will lead to negative effects for entire sample and developing economies, positive effects for developed economies, but all are insignificant in 90% confidence interval. ♣This is the "worst" value of fair population share regarding to growth.

Table 7.2w RE Regression of Growth on the Fair Division Point, GDPw and Popw

Independent Variable	gdpb<0.26	gdpb>0.26	Entire Sample
$gdpw$	-.056(-0.92)	-.048(-0.39)	**-.093(-2.05)**
$gdpw^2$	.028(0.63)	.034(0.46)	**.050(1.58)**
$popw$	-.037(-1.08)	-.027(-0.6)	-.0212(-0.82)
W_l	1.334(1.23)	**.469(1.75)**	**.871(3.16)**
W_l^2	-1.369(-0.97)	**-.522(-1.77)**	**-.907(-2.75)**
P_l	**2.731(3.69)**	**-3.970271(-2.45)**	**1.907(3.95)**
P_l^2	**-2.001(-3.69)**	**3.389(2.56)**	**-1.406(-3.91)**
Intercept	**-1.137(-3.53)**	**1.125(2.29)**	**-.747(-4.47)**
FE corr(u_i,Xb)	0.0862	-0.2485	-0.084
H-Test Prob(chi2)	0.031(15.44)	/(-23.46)	0.00(33.42)
Within R^2	0.0355	0.0105	0.0315
Between R^2	0.3843	**0.4647**	0.4189
Overall R^2	0.1642	0.0777	0.1220
P_l^*	**0.682(63.76)**	**0.585♣(42.18)**	**0.678(71.35)**
W_l^*	/	**0.449(10.55)**	**0.480(14.14)**

The fixed effect regression for entire sample, rich countries and poor countries do not show significance on fair division shares. ♣This is the “worst” value of fair population share regarding to growth.

Table 7.3b RE Regression of Growth on the Fair Income Shares, the Gini and GDPb

Independent Variable	gdpb<0.26	gdpb>0.26	Entire Sample
gdpb	.05563(0.28)	-.0796(-0.97)	**-.0848(-2.18)**
$gdpb^2$	-.3644(-0.56)	.0883(1.09)	**.0951(1.85)**
popb	.01888(1.04)	-.0198(-1.15)	-0.002(-0.18)
g^2	**-.9502(-4.31)**	-.4482(-0.98)	**-.6935(-4.44)**
g	**.7685(4.17)**	.4258(1.4)	**.5536(4.28)**
W_l^2	1.089(0.73)	**-.7291(-2.04)**	**-.8476(-2.3)**
W_l	-.5034(-0.43)	**.7081(2.1)**	**.8632(2.68)**
Intercept	-.0774(-0.36)	**-.2035(-2.04)**	**-.2625(-3.25)**
H-Test Prob (chi2)	0.1645(10.45)	0.7474(4.28)	0.4749(6.57)
FE corr(u_i, Xb)	-0.1135	-0.2668	-0.3038
Within R^2	0.0226	0.0182	0.0218
Between R^2	**0.4128**	**0.2986**	**0.4041**
Overall R^2	0.1446	0.0644	0.1191
g^*	0.4044	/	0.3991,
W_l^*	/	0.4857	0.5092

GDP per capita is still insignificant for subsamples if it takes linear form in above models. Gini has significant and positive effects and other explanatory items do not show difference on significance and sign if it takes linear form in the subsample of rich economies. Fair income share has significant and positive effects and other items do not show difference of significance and sign if it takes linear form in the subsample of poor economies.

Table 7.3w RE Regression Growth on the Fair Income Shares, the Gini and GDPw

Independent Variable	gdpb<0.26	gdpb>0.26	Entire Sample
$gdpw$	-.0687317(-1.12)	-.0857564(-0.69)	**-.0948953(-2.12)**
$gdpw^2$	.0392584(0.87)	.0470697(0.61)	**.0535933(1.7)**
g^2	**-.8035143(-3.77)**	-.3656125(-0.85)	**-.6762245(-5.09)**
g	**.677315(3.79)**	.362981(1.27)	**.5776307(5.2)**
W_l^2	.6238193(0.41)	**-.9166467(-2.54)**	**-1.178806(-3.52)**
W_l	-.1348345(-0.12)	**.8878724(2.62)**	**1.188412(4.18)**
$popw$	-.0351753(-1.02)	.012146(0.27)	-.024338(-0.96)
Intercept	-.0779775(0.12)	**-.2271746(-2.24)**	**-.3007162(-4.2)**
H-Test Prob (chi2)	0.0258(15.93)	0.4053(7.23)	0.0002(28.54)
FE corr(u_i, Xb)	-0.0381	-0.2588	-0.1242
Within R^2	0.0268	0.0132	0.0283
Between R^2	**0.4972**	**0.2787**	**0.5226**
Overall R^2	0.1663	0.0429	0.1373
g^*	0.4215	/	0.4271
W_l^*	/	0.4843	0.5041

All fixed effect regressions do not show significance on inequality items.

Table 7.4w Regressions of Growth on the Fair Population Share, the Gini and GDPw

Independent Variable	gdpb<0.26 FE	gdpb>0.26 RE	Entire Sample RE
$gdpw$	-.085029(-1.15)	.0146069(0.12)	**-.0923854(-2.07)**
$gdpw^2$	.0841076(1.59)	-.004474(-0.06)	**.0516132(1.64)**
g^2	.5262248(1.04)	**-.9494806(-2.06)**	**-.7758644(-3.84**
g	-.6202943(-1.35)	**.5646259(1.85)**	**.4224562(2.85)**
P_l^2	**-2.256247(-1.87)**	**4.163199(2.91)**	-.1418632(-0.28)
P_l	**3.291752(2.04)**	**-4.843631(-2.79)**	.4985642(0.77)
$popw$	-.0538683(-1.23)	-.0255697(-0.55)	-.017651(-0.70)
Intercept	-.9297213(-1.87)	**1.365678(2.64)**	-.2175686(-1.12)
H-Test Prob(chi2)	-0.085	0.0020(22.57)	0.0001(31.31)
FE corr(u_i, Xb)		-0.0966	-0.0068
Within R^2	0.0558	0.0173	0.0301
Between R^2	**0.0451**	**0.4847**	**0.5207**
Overall R^2	0.0659	0.0890	0.1409
g^*		0.2973	0.2723
P_l^*	0.7295	0.5817♣	/

Fixed effect regression for entire sample, rich countries do not show significance on inequality items. ♣This is the "worst" value of fair population share regarding to growth.

Table 7.4b RE Regression of Growth on the Fair Population Share, the Gini and GDPb

Independent Variable	gdpb<0.26	gdpb>0.26	Entire Sample
$gdpb^2$	-.2334(-0.35)	-.0837(-1.08)	**.0928 (1.82)**
$gdpb$	.0250(0.13)	.1003(1.31)	**-.0814 (-2.11)**
$popb$	.0147(0.81)	-.0100(-0.75)	-.0027(-0.24)
g^2	**-.7190(-2.4)**	**-.9088(-1.94)**	**-.7494 (-3.37)**
g	.3334(1.40)	**.5700(1.90)**	**.4166 (2.53)**
P_l^2	-1.014(-1.28)	**3.612(2.51)**	-.2077 (-0.38)
P_l	**1.6941(1.59)**	**-4.2002(-2.41)**	.5217(0.74)
Intercept	-.6485(-1.99)	**1.1749(2.29)**	-.2488 (-1.17)
H-Test Prob (chi2)	0.0715(13.02)	0.3465(7.84)	0.3215 (8.13)
FE corr(u_i, Xb)	-0.0651	0.027	-0.1661
Within R^2	0.0265	0.0156	0.0257
Between R^2	**0.3805**	**0.5681**	**0.4175**
Overall R^2	0.1509	0.1034	0.126
g^*	/	0.3136	0.2779
P_l^*	/	**0.5813♣**	/

GDP per capita is still insignificant for subsamples if it takes linear form in above models and other explanatory items do not show difference on significance and sign. ♣This is a "worst" value of fair population share regarding to growth.

Table 7.4bb RE Regression of Growth on the Fair Population Share, the Gini and GDPb

Independent	gdpb<0.26		gdpb>0.26		Entire Sample	
$gdpb^2$	-.0704 (-0.36)	-.4259 (-0.65)	.1017 (1.31)	.0934 (1.16)	.07497 (1.39)	**.09406** **(1.85)**
$gdpb$	.10278 (0.16)	.07544 (0.38)	-.0893 (-1.14)	-.0829 (-1.02)	-.0670 (-1.63)	**-.0822** **(-2.14)**
g^2		**-.9965** **(-4.79)**		-.52279 (-1.15)		**-.81066** **(-5.16)**
g	**-.22277** **(-3.99)**	**.54628** **(3.18)**	-.00598 (-0.12)	.34715 (1.16)	**-.1205** **(-2.76)**	**.45827** **(3.68)**
P_l^2	**-2.3839** **(-4.32)**		**2.4467** **(1.82)**		**-1.434** **(-3.4)**	
P_l	**3.498** **(4.67)**	**.3370** **(3.6)**	**-2.8004** **(-1.72)**	**.1751** **(2.06)**	**2.0654** **(3.72)**	**.2545** **(3.94)**
popb	.00574 (0.32)	.0160 (0.89)	-.01673 (-1.22)	-.01904 (-1.13)	-.0078 (-0.56)	-.00215 (-0.19)
Intercept	**-1.131** **(-4.4)**	**-.2394** **(-3.84)**	**.84731** **(1.71)**	**-.11344** **(-1.66)**	**-.6425** **(-3.48)**	**-.1701** **(-3.87)**
H-Test Prob(chi2)	0.5003 (5.35)	0.0792 (11.31)	0.2778 (7.49)	0.6276 (4.36)	0.854 (2.63)	0.3861 (6.34)
FE corr(u_i, Xb)	0.0039	-0.1051	-0.2892	-0.310	-0.2452	-0.2046
Within R^2	0.0344	0.021	0.0066	0.0161	0.027	0.0247
Between R^2	**0.2405**	**0.427**	**0.5264**	**0.3177**	**0.2483**	**0.4267**
Overall R^2	0.1346	0.1462	0.0883	0.0653	0.0934	0.1261
g^*		**0.274**				0.2829
P_l^*	0.7337		0.5723♣		0.72	

GDP per capita is still insignificant for subsamples if it takes linear form in above models and other explanatory items do not show difference on significance and sign. ♣This is the "worst" value of fair population share regarding to growth.

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