Managers Compensation and Collusive Behaviour under Cournot Oligopoly

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Abstract

The aim of the present paper is to show that the existence of a concrete outside option for firms' executives can induce, under specific circumstances, every firm to adopt restrictive output practises. In particular, the paper characterizes the conditions for which, under Cournot oligopoly, existing firms behave more collusively than in a standard Cournot model. It is also shown that room exists for perfect and stable collusive agreements amongst firms. Other interesting findings are also twofold. Firstly, that the equilibrium executives' pay will usually be dependant upon the number of companies initially disposing of the technology and/or of the organizational knowledge required to set up the business. Secondly, that companies' procedures difficult to duplicate can constitute a beneficial form of competition policy in that they induce the firms to behave less collusively in the product market.

Keywords: *Managers' Compensation, Oligopoly.*

1. Introduction

Usually the outside option of companies executives officers (CEOs) and, in general, of highly trained workers, takes the form of alternative offers made by other (often competitor) firms. This form of at-will employment does not necessarily mean that each company is completely vulnerable to the disclosure of its strategic informations after the CEOs' departure. Trade Secret Acts (like the Uniform TSA in U.S.), corporate policies on trade secrets as well as postemployment restrictive covenants, such as non disclosure and nonsolicitation agreements, are all tools that large companies adopt to avoid a too great temptation for CEOs and other important employees to walk off stealing company's informations.[1]

However, there are well known cases in which CEOs decide to leave their company to set up independent business, mainly as a result of a solid organizational and managerial experience acquired in the field. In fact, in industries in which a relatively small number of cutthroat competitors control most of the market, a noncompete clause cannot realistically be imposed on executives. It might be either too expensive for the guaranteed contracts that senior officers would demand if asked to accept it, or it might simply be an uncommon practise in the industry. Moreover, when CEOs set up new ventures based on their organizational and market experience, companies do not really have grounds for a good lawsuit. It can be difficult to achieve evidence from which a court can infer that either customer lists, pricing and marketing plans or simply the company organizational style have been stolen.

The relevance of the outside option or "going rate" in affecting executives' pay is empirically recognized [see, for instance, Smith and Szymanski (1995)].[2] CEOs' defection to set up independent businesses can be considered more likely in industries in which fixed costs are not particularly high and the company's experience is easily duplicable.Whether the answer of existing firms' owners should be, on the one hand, that of increasing the existing market competition - to reduce the value of potential entrants - on the other it may simply be that of adopting a collusive output choice as a result. In any case, whenever the company's environment naturally discloses strategic informations to a few firm's insiders, executives' pay should be responsive of the existing outside options and hence, of the features of the market in which the firm operates.

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[1]See, for instance, the recent case study "When an Executive Defects", *Harvard Business Review,* January-February 1997, pp. 18-34.

[2]Related empirical works on managers' pay and firms' performance are, amongst others, Murphy (1985), Jensen and Murphy (1990) and Gregg et al. (1993).

The purpose of this paper is to show that the existence of a concrete outside option for firms' executives can induce, under specific circumstances, every firm to adopt restrictive output practises. In particular, the paper characterizes the conditions under which, in a Cournot oligopoly, existing firms behave more collusively than in a standard Cournot model. It is also shown that room exists for perfect and stable collusive agreements amongst firms. Other interesting findings are also twofold. Firstly, that the equilibrium executives' pay will usually be dependant upon the number of companies that initially dispose of the technology and/or of the organizational knowledge required to set up the business. Secondly, that companies' procedures difficult to duplicate can constitute a beneficial form of competition policy, by inducing the firms to behave less collusively in the product market. This is because firms are less worried to lose their informational advantages in favour of potential defecting firm's insiders.

Different setups related to the paper topic are contained, among the others, in Fenstein and Stein (1988), Mailath and Postlewaite (1990) and Stole and Zwiebel (1997). The results presented in this paper can also be compared to the well known (and opposite) result [Vickers (1985), Fershtman and Judd (1987), Sklivas (1987)] that under Cournot oligopoly the presence of managers' incentives related to sales can induce each company to behave less collusively than simple entrepreneurial firms (i.e., which managed by just the owner).

The paper is organized as follows. The next section briefly presents the structure of the model. Section 3 introduces a simple model specification to show the main paper findings. Section 4 is devoted to extend some of the model results to a more general framework. Section 5 concludes the paper.

2. The structure of the model

The model describes an oligopolistic industry in which, at the beginning, only n owners can establish a business based on their knowledge. This exclusive knowledge represents the only barrier to entry for other potential competitors (for instance the firm's executives, henceforth labelled as *managers*) assumed to need a specific on-the-job training to start a new business. Thus, in the industry, the n owners are assumed to set up n (identical) firms behaving *à la* Cournot and producing a homogenous commodity y. The assumed sequence of strategies is quite simple. Firstly, every owner decides how much commodity to produce (and thus, how many identical managers to hire), according to the usual profit maximization procedure. Secondly, the owner has to fix the level of every manager's remuneration, indicated as v. Hence, every recruited managers can either decide to stay, accepting v, or leave, to set up a competing company in the industry, thus earning a profit of $\pi(n+1)$. Every manager that has become owner continues the game exactly as before (i.e., first deciding $y(n+1)$ and then $v(n+1)$

for her or his recruited managers) and the game goes on in this way, with an infinite horizon. The solution concept used to solve the game is a standard subgame Nash equilibrium.[3] An equilibrium of the game is thus a vector of Nash equilibrium quantities $\left(y_1^*\left(v^*\left(n+k\right)\right), y_2^*\left(v^*\left(n+k\right)\right), ..., y_{n+k}^*\left(v^*\left(n+k\right)\right)\right)$, where v^* is every manager's equilibrium remuneration and k is the number of new entrants that have entered the market in equilibrium.

3. A simple example

Let us assume that in an a certain industry n owners, initially disposing of the knowledge on how to produce a homogenous commodity y, set up n (identical) firms behaving à la Cournot. Let also the available technology be described by the following constant returns to scale production function:

$$y_i = m^\theta \cdot \ell^{(1-\theta)} \tag{3.1}$$

where respectively m is the number of managers recruited by every i-th firm ($i = 1, ...n$) and ℓ is the number of unskilled workers. Let us assume, for simplicity, that $\theta = 1/2$. Let also every firm's fixed cost be equal to zero. To reduce the notation, the wage paid to unskilled workers can be normalized to one, while v will denote each manager's compensation. Moreover, let the market demand be linear and equal to:

$$p\left(Y\right) = a - Y \tag{3.2}$$

where $Y = \sum_{i=1}^{n} y_i$ is the total quantity of commodity delivered to the market.

Deriving by (2.1) every firm's cost function as:

$$C_i\left(y_i\right) = 2\sqrt{v} \cdot y_i \tag{3.3}$$

it is straightforward to get every n-th firm's Cournot equilibrium profit as:

$$\pi_i^*\left(n\right) = \frac{\left(a - 2\sqrt{v}\right)^2}{\left(n + 1\right)^2} \tag{3.4}$$

The basic feature of the model is that, when managers are hired by a firm's owner, they immediately acquire the specific knowledge (e.g., something about the firm's organization) that enables them to become potential competitors of the existing firms. Hence, managers' compensation must be optimally decided by every owner knowing each manager's potential threat of leaving the firm to setup, through the use of unskilled workers (and possibly other managers), a new production unit.

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[3]Extensions of this game to coalitional deviations are contained in Marini (1997).

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3.1. Case I: every leaving manager can just set up an entrepreneurial firm

For introductory purposes, a first application of the model deals with the simplest case in which every leaving manager is able to set up just an entrepreneurial firm. This means that the stock of knowledge (e.g., organizational) stored within every firm is complex and cannot be acquired in full by each manager working for it. Since the knowledge that every hired manager can achieve is incomplete, it is assumed that her or his actual outside option amounts to the possibility of setting up an inefficient type of organization, i.e., an entrepreneurial firm.

It is not difficult to obtain from (2.1)-(2.3) the equilibrium quantity of every $(n+1)$-th entrepreneurial firm (with $m=1$) created by a leaving manager,

$$y_m^* (n+1) = \frac{a + 2n\sqrt{v}}{3n+4} \tag{3.5}$$

and the corresponding equilibrium profit as:

$$\pi_m^* (n+1) = \frac{2 (a + 2n\sqrt{v})^2}{(3n+4)^2} \tag{3.6}$$

For simplicity, expressions (2.5)-(2.6) assume that only managers that voluntarily leave a firm can decide to set up new companies, whereas managers fired as effect of every firm's market share reduction are assumed to become unemployed.[4] Another assumption, that will be shown later to be endogenous, is that every existing firm's owner whose manager(s) decides to leave can replace him (them) using unemployed managers. Furthermore, for every owner behaving in such a manner must be the best action to take.[5]

Every owner that wants to avoid her or his managers leaving, knowing (2.6) will fix their compensation such that:[6]

$$v = \pi_m^* (n+1) = \frac{2 (a + 2n\sqrt{v})^2}{(3n+4)^2} \tag{3.7}$$

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[4] Including in each firm's best-reply function the option for fired managers to create new firms complicates the analysis without really changing the type of results obtained. However, this assumption will be removed for the more general case treated in the next subsection.

[5] Actually, as long as there exists a pool of unemployed managers, owners are completely indifferent whether to hire an unemployed manager or just recruit one of those currently working for other identical firms.

[6] Here the assumption is that an owner bargains with each manager, taking other managers' behaviour as *given*. Including a collective bargaining process or a different owners' conjecture would not drastically change the nature of ensuing results.

from which an equilibrium managers' compensation as a function of the number of firms active in the market can be obtained as:[7]

$$v^* (n) = \frac{a^2 \left[\sqrt{2} \left(3n + 4 \right) - 4n \right]^2}{\left(n^2 + 24n + 16 \right)^2} \tag{3.8}$$

Moreover, as long as there is positive profitability, every owner will always prefer to keep managers inside the firm rather than let them leaving to reduce their remuneration. This is proved by the following lemma.

Lemma 1. *As long as,* $p \left(Y^* \left(n \right) \right) > AC_i \left(y_i^* \left(n \right) \right),$ *the following inequality holds for every* $i = 1, ..n$:

$$\pi_i^* \left(v^* \left(n \right), n \right) > \pi_i^* \left(v^* \left(n + 1 \right), n + 1 \right) \tag{3.9}$$

Proof. (See Appendix).

Now, knowing that each owner hires a certain number of managers (otherwise, due to production function (2.1), firm's output and profit would be equal to zero) and, under positive profitability, she or he pays them the equilibrium compensation $v^* \left(n \right)$ (by lemma 1), a first result of the analysis is presented below.

Proposition 1. *Under Cournot equilibrium and under the assumptions of the model, when the initial number of firms is no greater than* $\underline{n}$, *the equilibrium number of managers* m^* *selected by every owner is less than it would be under standard neoclassical assumptions.*

Proof. Since $m^* = \frac{y^{i*}}{\sqrt{v^*}}$, and y_i^* is monotonically decreasing in v, it ensues that $m^* \left(v^* \left(n \right) \right) < m^* \left(\overline{v} \right)$ as long as $v^* \left(n \right) > \overline{v}$, where $\overline{v}$ indicates the neoclassical market clearing wage. From expression (2.8) condition $v^* \left(n \right) > \overline{v}$ exactly happens for $n < \underline{n} = \frac{a\sqrt{2}\sqrt{17 + 12\sqrt{2}} - \sqrt{\overline{v}}\left(12 + 8\sqrt{2} \right)}{\sqrt{\overline{v}}\left(17 + 12\sqrt{2} \right)}$. ∎

In the figure below $v^* \left(n \right)$ is plotted against different number of firms initially active in the market and compared to a given market clearing wage $\overline{v}$. Managers' compensation decreases with the number of firms existing in the market. Notice also that, since for certain levels of n the equilibrium managers' compensation is higher than the given market level, there will usually be a number of unemployed managers. This endogenous availability of managers allows for their substitution when they decide to leave the firm.

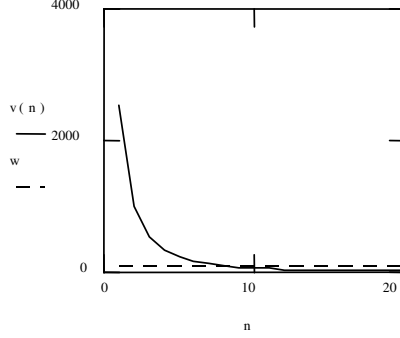[7]Equation (2.7) presents two solutions, among which, expression (2.8) (the lowest one) will be selected by every firm's owner.

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Fig 2.1-Values of $v^*(n)$ and $\dot{w}=\overline{v}$ for $a=350$ and $n=1,..20$.
In this example $v^*(n)$ is equal to $\overline{v}=10$ for $\underline{n}=8$.

Moreover, it can be noticed that $y_i^*(v^*(n),n)$ is, for every firm, dependent upon the equilibrium managers' compensation. Thus, since $v^*(n)$ is, for $n <\underline{n}$, higher than $\overline{v}$, it turns out that, whenever $n < \underline{n}$, $y_i^*(v^*(n),n) < y_i^*(\overline{v},n)$. This means that each firm is, within a given range of n, more collusive in terms of output than under the usual market clearing conditions. There also exists an initial number of firms for which $y_i^*(v^*(n),n)$ exactly coincides with the perfectly collusive output choice, i.e., that obtained when all firms cooperatively maximize their joint profit. The next proposition describes these results.

Proposition 2. *The output selected by every firm under Cournot equilibrium and managers' threat to leave is more collusive than under Cournot equilibrium and managers' competitive market for $n <\underline{n}$, that is, $y_i^*(v^*(n),n) < y_i^*(\overline{v},n)$ for $n <\underline{n}$. Moreover, there exists a level of $n = n^*$ such that, $y_i^*(v^*(n^*),n^*) = y_i^{c*}(\overline{v},n^*)$, where y_i^{c*} is the output resulting by cooperative agreement among firms.*

Proof. By proposition 1, for $n <\underline{n}$, $v^*(n)$ is greater than $\overline{v}$. Since firm's equilibrium output is monotonically decreasing in v, it follows that, for $n <\underline{n}$, $y_i^*(v^*(n),n) < y_i^*(\overline{v},n)$.

Moreover, substituting expression (2.8) for $v^*(n)$ into y_i^* yields:

$$y_i^*(v^*(n),n) = \frac{a\left[n^2 + 32n + 16 - \sqrt{2}(6n-8)\right]}{(n^2+24n+16)(n+1)} \tag{3.10}$$

while, the collusive quantity under market clearing managers' compensation $\overline{v}$ is:

$$y_i^{c*}(\overline{v},n) = \frac{a - 2\sqrt{\overline{v}}}{2n} \tag{3.11}$$

Thus, expression (2.10) is equal to (2.11), for $n = n^*$, where n^* is the only positive solution of an equation that, for ease of brevity, is not presented here. It is obvious that

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the higher the managers' market clearing wage is and the lower will be the number of firms for which $y^{i*}\left(v^{*}\left(n\right),n\right)=y_{i}^{c*}\left(\overline{v},n\right).$ ∎

A particular example of the result above is presented in figure 2.2. Obviously, for $n=1$ and given $\overline{v}$, the perfect collusive equilibrium quantity $y_{i}^{c*}\left(\overline{v},n\right)$ coincides with the Nash equilibrium quantity $y_{i}^{n*}\left(\overline{v},n\right)$. Interestingly, for $n=2$, the collusive quantity $y_{c}^{i*}\left(\overline{v},n\right)$ exactly coincides with $y^{i*}\left(v^{*}\left(n\right),n\right).$ Moreover, since every firm's quantity $y^{i*}\left(v^{*}\left(n\right),n\right)$ is also a Nash equilibrium quantity, it will be stable against each firm's temptation to deviate from the equilibrium choice of output, differently to what usually happens under collusive agreement. Finally, it can be noticed that such a particular example of *non-cooperative collusive* solution can take place either through mergers among firms (when the initial number of firms n is higher than n^{*}) or through a controlled departure of managers induced by a firm (when n is lower than n^{*} and the profit is negative).
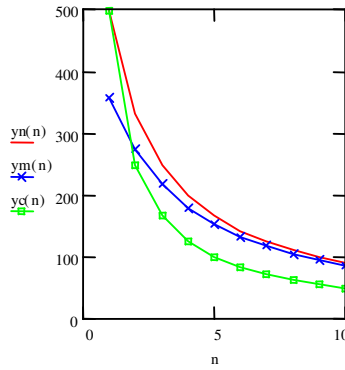

Fig.2.2 - Equilibrium quantity respectively for a usual Cournot firm with
a given wage $\overline{v}$ y$n(n)$, for a Cournot firm firm under managers' threat $ym(n)$
and for a perfectly collusive firm (y$c(n)$) (a=1000, $\overline{v}$=10,n=1,..10).

Thus, in this first example whether the behaviour of existing firms is less or more collusive than in a usual Cournot model depends upon the initial number of firms that know how to organize the firm's activity.

3.2. Case II: every leaving manager can set up a managerial firm

This section removes the assumption made previously that every leaving manager can set up only entrepreneurial firms. Here a manager can leave the firm and set up a company that is virtually identical to the one she or he is working for. Widening every manager's outside option is a simple device to describe the case in which existing firms' (organizational) knowledge is not very complex and can thus be fully acquired by every single manager.

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In this situation, expression (2.6) simply becomes:

$$\pi_i^* (n+1) = \frac{\left(a - 2\sqrt{v(n+1)}\right)^2}{(n+2)^2} \tag{3.12}$$

and the equilibrium managerial compensation is thus equal to:

$$v^* (n) = \frac{\left(a - 2\sqrt{v^*(n+1)}\right)^2}{(n+2)^2} \tag{3.13}$$

Expression (2.13) is a non linear difference equation that can be solved by iteration on the number k of new entrants. The fact that $v^* (n) = \pi_i^* (n+1)$, implies, when profits are decreasing in n, that there exists an arbitrary number of entrants k for which $v^* (n+k) = \pi_i^* (n+k+1) = 0$. This fact can be used to achieve the equilibrium manager's compensation. The appendix proves that lemma 1 also applies in this case.

When lemma 1 holds, expression (2.13) can be solved for v, obtaining:

$$v^* (n) = \frac{\left\{ \sum\limits_{i=0}^{k} \left[(-1)^i \left(2^i a\right)\right] \prod\limits_{j=k}^{i} (n+3+j) + (-1)^{k+1} \left(2^{k+1} a\right) \right\}^2}{\prod\limits_{i=0}^{k+1} (n+2+i)^2} \tag{3.14}$$

An interesting feature of expression (2.15) is that it converges to a finite value for a low number of k new entrant firms. Moreover, as in the other example, the value of v decreases monotonically with the number of existing firms. These features allow to achieve the results presented in the next proposition.

Proposition 3. *Under managers' threat to set up new managerial firms, the equilibrium number of managers hired by every existing firm is equal to one, i.e., $m^* (v^* (n)) = 1$. Moreover, there exists a number of firms $\underline{n}$ for which the choice of managers coincides with that of a neoclassical setting. The output selected by every firm will thus be more collusive than in a standard Cournot model for $n < \underline{n}$, that is, $y_i^* (v^* (n), n) < y_i^* (\overline{v}, n)$ for $n < \underline{n}$. Moreover, there exists a level of $n = n^{**}$ for which, $y_i^* (v^* (n), n^{**}) = y_i^{c*} (\overline{v}, n^{**})$, where y_i^{c*} is the output resulting by a perfect cooperative agreement among firms.*

Proof. Straightforward manipulations of expression (2.15) show that it converges to a finite value equal to:

$$v^* (n) = \frac{a^2 \left(n^5 + 23n^4 + 205n^3 + 881n^2 + 1818n + 1412\right)^2}{\left[(n+2)^2 \left[(n+3)^2 \left[(n+4)^2 \left[(n+5)^2 \left[(n+6)^2 (n+7)^2\right]\right]\right]\right]\right]} \tag{3.15}$$

From (2.1), (2.3) and (2.16) it follows that:

$$m^{*}\left(v^{*}\left(n\right)\right) = \frac{y_i^{i*}(v^{*}(n))}{\sqrt{v^{*}(n)}} = \frac{\left(n^6 + 25n^5 + 249n^4 + 1255n^3 + 3342n^2 4392n + 2192\right)}{\left(n^5 + 23n^4 + 205n^3 + 881n^2 + 1818n + 1412\right)^2} \in [1, 1.5)$$

whatever is the number of existing firms. The closest integer to $m^{*}\left(v^{**}\left(n\right)\right)$ is thus equal to one. Hence, it turns out that $m^{*}\left(v^{*}\left(n\right)\right) = m^{*}\left(\overline{v}\right)$ for $\underline{n} = \frac{a-3\sqrt{\overline{v}}}{\sqrt{\overline{v}}}$, and $y_i^{*}\left(v^{*}\left(n\right),n\right) < y_i^{*}\left(\overline{v},n\right)$ for $n < \underline{n}$. As in proposition 2, also in this case there exists a n^{*} such that $y_i^{*}\left(v^{*}\left(n\right),n^{*}\right) = y_i^{c*}\left(\overline{v},n^{*}\right)$. The figure shows a simple example of this result for given values of parameters and a range of n. It can be noticed that, for $n = 220$, $y_i^{*}\left(v^{*}\left(n\right),n\right) = y_i^{c*}\left(\overline{v},n\right)$. ∎
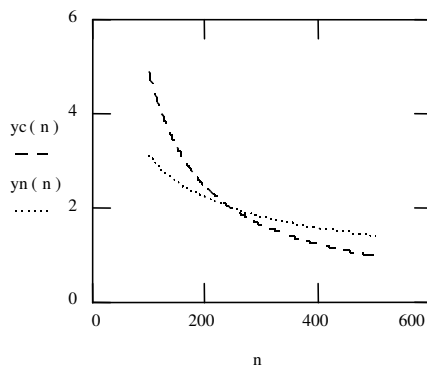

Fig. 2.3 - Equilibrium quantity both for the managerial firm
$ym(n)$ and for the perfectly collusive firm ($yc(n)$) (a=1000, $\overline{v}$=10,n=0,..600).

It can be noticed that, by including among leaving managers those fired as effect of every company's market share reduction would not change the final result that just one manager is hired in equilibrium by every firm. Furthermore, the set of results presented in proposition 1, 2 and 3 allows for a unifying conclusive proposition. This expresses the fact that, under the model simplifying assumptions, the easier the acquisition of the firm's knowledge by the managers is, the higher will be the degree of collusion put in place by every firm.

Proposition 4. *Under the model assumptions, the firms' degree of output collusion is increasing in the level of managers' outside option.*

Proof. It follows from proposition 1, 2 and 3. In fact, it is easy to see that the number of managers recruited and the output level of every firm in equilibrium is higher in the case I than in the case II. A continous set of outside options between these two extreme cases can easily be modeled by imposing that an intermediate number of managers (more than 1 but less than that of a managerial firm) can be hired by every new entrant firm. This would imply the result. ∎

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4. Some generalizations of the model

This section is devoted to give some generality to the results obtained above as well as to discuss which basic assumptions are strictly required to achieve the main findings of the model.

Under the following standard assumptions that:

A.3.1 The payoff of each firm is a function of its own strategy and the sum of strategies of all existing firms (usually defined as *aggregation axiom*; see, for instance, Dubey, Mas-Colell and Shubik (1980));

A.3.2 The strategy sets Y_i are, for each firm, compact and convex;

A.3.3 The payoff function of every firm, $\pi_i : Y_i \times \Re_+ \to \Re_+$, is twice continuously differentiable;

A.3.4 $\frac{\partial^2 p(Y)}{\partial Y^2} y_i + \frac{\partial p(Y)}{\partial Y} < 0$;

A.3.5 $\frac{\partial p(Y)}{\partial Y} - \frac{\partial^2 C_i(y_i)}{\partial y_i^2} < 0$, (A.3.4 and A.3.5 are standard assumptions for second order conditions to hold, see, for instance, Friedman (1977));

A.3.6 The output of every firm is strictly decreasing in the manager's pay v;

It can be proved that:

i) A Cournot-Nash equilibrium $(y_1^*, y_2^*, ..., y_n^*)$ always exists and is unique (see, for instance, Corchon (1996), p.15);

ii) Condition $\pi_i^*(n) > \pi_i^*(n + k)$ for $(k = 1, 2, ..., \infty)$ always hold under firm's positive profitability (see lemma 2, in the appendix).

From i) and ii), the following proposition can be derived.

Proposition 5. *Under Cournot oligopoly and managers' threat to leave, when assumptions A.3.1-A.3.6 hold, there is always a number of firms below which equilibrium manager compensation is higher than market clearing neoclassical wage. Moreover, within this range of n, the output is more collusive than in a standard Cournot model.*

Proof. Result ii) implies that, for $(k = 0, 1, ..., \infty)$, $v^*(n)$ is monotonically decreasing in n. Hence, it is always possible to find a $n = \underline{n}$ such that $v^*(\underline{n}) = \overline{v}$ and then for $n < \underline{n}$, $v^*(\underline{n}) > \overline{v}$. Furthermore, from A.3.5 it ensues that, for $n < \underline{n}$, $y_i^*(v^*(n)) < y_i^*(\overline{v})$. ∎

It can be interesting to spend a few words in discussing the meaning of A.3.5. Coupled with A.3.4, it ensures that every firm's payoff is concave, from which, second orders conditions for a maximum are satisfied. This in turn requires either a "not too

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convex" demand function or a "not too concave" cost function with respect to output. By assuming (without loss of generality) a linear technology, it is easy to see that:

$$p' - C_i'' < 0 \Rightarrow p' - \frac{\partial^2 v^*(n)}{\partial y_i^2} y_i^* - 2\frac{\partial v^*(n)}{\partial y_i} < 0 \qquad (4.1)$$

Note in general that the model assumptions always ensure that both $\frac{\partial \pi^*(n+1)}{\partial y_i} < 0$ and thus, $\frac{\partial v^*(n)}{\partial y_i} < 0$ and that $\frac{\partial^2 \pi^*(n+1)}{\partial y_i^2} < 0$ and then $\frac{\partial^2 v^*(n)}{\partial y_i^2} < 0$. Hence, every firm's cost function will be concave. Given a regular demand function, what is thus required for the collusive effect to take place is "a not too concave" cost function. Condition (3.1) shows that for every firm the choice of y_i yields, beside the usual negative effect on the demand function, an indirect effect on managers' compensation. This effect has two components: the first is which to reduce the pay of every manager through the fall of her or his outside option, i.e., the threat to set up a new firm. The second is which to reduce the other subsequent potential $(n + k)$ -th leaving managers' outside option and, hence, increasing which of the initial $(n + 1)$-th manager. By taking the second derivative of $v^*(n)$ with respect to y_i and applying the model specification used in section 2, it turns out that:

$$\frac{\partial^2 v^*(n)}{\partial y_i^2} = \lim_{k \to \infty} \sum_{j=1}^{k} (-1)^{(j+1)} \cdot p'' \cdot \prod_{j=1}^{k} y_{n+j}^* \qquad (4.2)$$

Condition (3.1) may certainly hold when expression (3.2) is not "too negative". This implies to impose that the chain effect on every $(n + k)$-th entrants has sufficient strenght to almost offset the direct effect of an output increase on every $(n + 1)$-th leaving manager's outside option. The same result would ensue by assuming that every manager's learning process requires a certain period of time to be completed. In this case, there would always be a discount factor $\delta \in (0, 1)$ sufficiently high for firms' payoff to fall with the number of entrants k and hence, for the collusive effect to take place.

5. Concluding Remarks

The paper has described, through an extremely simple model, that companies owners' need to fix a level of compensation high enough to keep managers within the firm can give rise to a collusive choice of output stable against individual firm's deviations. The result holds when the depressive effect of leaving managers on firms' profit prevails on the positive effect due to a reduction of their compensation. Furthermore, the model generates the empirically appealing property (see, for instance, Watson et al.(1994)) that managers'compensation is decreasing with the number of firms existing in the market and, consequently, with their size. The nature of every company's knowledge also plays

a role. When the firm's (organizational or technological) procedures are difficult to duplicate, the owner can hire a reasonably high number of managers without being afraid that they will become competitors. In this case, the equilibrium output can be close to which of a Cournot standard model, even if, a form of tacit collusion can also be reached when a small number of firms operate in the market. Conversely, when companies' procedures are easily duplicable (as in traditional businesses) the model predicts a very low number of recruited managers ($m^* \cong 1$) coupled with companies' output restrictions. In this respect, given the initial number of firms operating in an industry, complex and heterogenous company's procedures can be beneficial in terms of level of output and competition generated in the market.

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6. Appendix

Proof of Lemma 1. *As long as,* $p\left(Y^*\left(n\right)\right) > AC_i\left(y_i^*\left(n\right)\right)$, t*he following inequality holds for every* $i = 1, ..n$:

$$\pi_i^*\left(v^*\left(n\right), n\right) > \pi_i^*\left(v^*\left(n+1\right), n+1\right) \tag{.1}$$

Proof. The meaning of expression (.1) is that, under positive profitability of existing firms, every company's owner finds convenient to pay each manager the equilibrium wage $v^*\left(n\right)$ rather than let him go and start a new negotiation with another manager. Let us prove the lemma by contradiction.

Suppose inequality (.1) does not hold, that is:

$$\left[p\left(Y^*\left(n\right)\right) - 2\sqrt{v^*\left(n\right)}\right] y_i^*\left(n\right) \leq \left[p\left(Y^*\left(n+1\right)\right) - 2\sqrt{v^*\left(n+1\right)}\right] y_i^*\left(n+1\right)$$

This expression can be solved by iteration and, for each firm under the potential market entry of k entrepreneurial firms, the following result ensues:

$$v^*\left(n\right) \geq \frac{p\left(Y^*\left(n\right)\right)^2}{4} - \frac{\left[p\left(Y^*\left(n+k\right)\right) y_i^*\left(n+k\right)\right]^2 - 4\left[y_i^*\left(n+k\right)\right]^2 v^*\left(n+k\right)}{4\left(y_i^*\left(n\right)\right)^2}$$

Since in this model there are no entry costs, room potentially exists for an infinite number of entrants. Thus, taking the limit of expression above for k that tends to infinite, we get:

$$v^*\left(n\right) \geq \frac{p\left(Y^*\left(n\right)\right)^2}{4} - \lim_{k\to\infty} \frac{\left[p\left(Y^*\left(n+k\right)\right) y_i^*\left(n+k\right)\right]^2 - 4\left[y_i^*\left(n+k\right)\right]^2 v^*\left(n+k\right)}{4\left(y_i^*\left(n\right)\right)^2}$$

from which:

$$2\sqrt{v^*\left(n\right)} \cdot y_i^*\left(n\right) \geq p\left(Y^*\left(n\right)\right) \cdot y_i^*\left(n\right) - \lim_{k\to\infty} \pi_i\left(v^*\left(n+k\right), n+k\right)$$

Now, deriving from expressions (2.4) and (2.8) every $n-$th managerial equilibrium profit as:

$$\pi_i\left(v^*\left(n+k\right), n+k\right) = \frac{a^2\left[\left(n+k\right)^2 + 32\left(n+k\right) + 16 - 6\sqrt{2\left(n+k\right)} - 8\sqrt{2}\right]}{\left[\left(n+k\right)^2 + 24\left(n+k\right) + 16\right]^2 \left(n+k+1\right)^2}$$

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and applying the Hopital rule to solve the limit of the above expression, it ensues that:

$$2\sqrt{v^*\left(n\right)} \cdot y_i^*\left(n\right) \geq p\left(Y^*\left(n\right)\right) \cdot y_i^*\left(n\right) - 0$$

from which:

$$2\sqrt{v^*\left(n\right)} = AC\left(y_i^*\left(n\right)\right) \geq p\left(Y^*\left(n\right)\right)$$

that contradicts the assumption of every managerial firm's positive profitability. Note that the proof of the lemma can similarly be applied to case II, by substituting the expression for every firm's profit $\pi_i\left(v^*\left(n+k\right),n+k\right)$ with the following expression:

$$\pi_i\left(v^*\left(n+k\right),n+k\right) =$$

$$= \frac{a^2\left((n+k)^6+25(n+k)^5+249(n+k)^4+1255(n+k)^3+3342(n+k)^2+4392(n+k)+2192\right)^2}{\left[(n+k+2)^2\left[(n+k+3)^2\left[(n+k+4)^2\left[(n+k+5)^2\left[(n+k+6)^2(n+k+7)^2(n+k+1)^2\right]\right]\right]\right]\right]}$$

that also converges to zero for k that tends to infinite. ∎

Proof of Lemma 2. *Under the following standard assumptions:*

$$\frac{\partial^2 p\left(Y\right)}{\partial Y^2}y_i + \frac{\partial p\left(Y\right)}{\partial Y} < 0 \tag{A.2.1}$$

and,

$$\frac{\partial p\left(Y\right)}{\partial Y} - \frac{\partial^2 C_i\left(y_i\right)}{\partial y_i^2} < 0 \tag{A.2.2}$$

the following inequality holds:

$$\pi_i^*\left(v^*\left(n\right),n\right) > \pi_i^*\left(v^*\left(n+1\right),n+1\right) \tag{.2}$$

Proof. (Standard) Assumptions (A.2.1) and (A.2.2) always imply that $Y^*\left(n\right) < Y^*\left(n+1\right)$ and $y_i^*\left(n\right) > y_i^*\left(n+1\right)$. Given this, and using first order conditions, it ensues that:

$$\frac{d\pi_i^*(n)}{dn} = \frac{dy_i^*}{dn}p + y_i^*\frac{dp^*}{dY^*}\frac{dY^*}{dn} - \frac{dC}{dy_i^*}\frac{dy_i^*}{dn} =$$

$$= \left(p-C'\right)\frac{dy_i^*}{dn} + y_i^*p'\frac{dY^*}{dn} = y_i^*p'\left(\frac{dY^*}{dn} - \frac{dy_i^*}{dn}\right) < 0. \; \blacksquare$$