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Abstract. The paper aims at providing a non-linear Game Theory model of coopetition which addresses the problem of the ***global Green Economy***. The *Green Economy* is a theoretical model of economic development that suggests economic, technical and legislative solutions to ***reduce the consumption of energy***, ***of natural resources*** and ***environmental damage*** while promoting a ***sustainable development model*** for the economy. Our coopetitive model is non-linear with respect to each strategy of the game.

Keywords. *Sustainability, coopetition, conservation and improvement of natural resources, Differentiable Pareto Analysis, macroeconomic and global interactions*

Jel classification. C7, C71, C72, C78, D21.

AMS classification. 91Bxx, 91B38, 91Axx. 91A10, 91A12, 91A20 91A40, 91A44, 91A80.

1 Introduction

The *Green Economy* includes those activities and sectors that focus on enhancement of “traditional goods" such as: landscape, nature, culture, traditions, food and wine. In this paper we apply the notion of coopetition devised by Branderburger and Nalebuff (1995). These authors argue that firms operate within a competitive environment, but in some cases they realize that the outcome of competition will not be a win-lose solution for the players, but it will be a lose-lose result. Thus it is convenient for the firm playing the game to change the game and find a win-win solution, that indicates a situation in which the firm thinks about both cooperative and competitive ways to change the game (ibid., p. 59). The win-win solution is therefore a situation in which the firm must cooperate and compete at the same time.

Thus in the present work we apply the notion of coopetition ***at country level***, instead of microeconomic firm level. The country has to decide whether it wants to collaborate with the rest of the world in getting an ***efficient Green Economy***, even if the country is competing in the global scenario.

Our model will provide different win-win solutions which are going to show the convenience for each country to participate actively to a ***program of sustainability and efficient resource allocation*** within a non-linear coopetitive framework.

The three main variable of our coopetitive model are:
x representing the strategy of any country c;
y representing the strategy of the rest of the world w;
z representing the ***coopetitive sustainability strategy***.

In this paper we suggest an original analytical framework of coopetitive games applied at the global environment, with the aim to enrich the set of tools for environmental policies.

The paper will show the strategies that could bring to feasible solutions in a coopetitive perspective between each country and the rest of the world, by offering win-win outcomes and to establish a true efficient resource Green Economy at a global level.

2 A model of coopetitive games

The coopetitive model we propose hereunder must be interpreted as normative models, in the sense that it will show win-win strategies within a cooperative perspective.

The main variables of the two models are:

strategies *x* of a certain country *c* (*the investment in agricultural and food production)*, which directly influence both pay-off function;

strategies *y* of the rest of the word (*the investment in agricultural and food production)* which directly influence both pay-off function;

a shared strategy *z* which is determined together by c and the rest of the world w: *z* is the global level of investment for environmental and natural resources. Therefore, in the model we assume that c and w define the set C of coopetitive strategies.

Main strategic assumptions.

We assume that any real number x, in the unit interval U = [*0,1*]*, can be an investment of c in agricultural and food production and any real number y, in the same unit interval U, can be an analogous investment of w, moreover any real number z, in C =* [*0,6*]*, can be the total investment of c and w for sustainability of natural resources and* *for the environmental protection. Let assume that the country c and the rest of the world w contribute for z with percentages* (q, r), in such a way that $z = qz + rz$.

We also consider as payoff functions of c and w two Cournot type payoff functions.

***Payoff function of** c*

We assume that the payoff function of c is the function f_1 of the set $S := U^2 \times C$ into the real line, defined by

$$f_1(x, y, z) = x\,(1 - x - y) + z,$$

for every triple (x, y, z) in the set $U^2 \times C$.

***Payoff function of** w*

We assume that the payoff function of w is the function f_2 of the set $S := U^2 \times C$ into the real line, defined by

$$f_2(x, y, z) = y\,(1 - x - y) + (-1/6)(z - 3)^2 + 3/2,$$

for every triple (x, y, z) in the set $U^2 \times C$.

Payoff function of the game

We so have build up a coopetitive gain game G = (f, >) with payoff function given by

$$\begin{aligned} f(x, y, z) &= (x\,(1 - x - y) + z,\ y\,(1 - x - y) + (-1/6)(z-3)^2+3/2) = \\ &= (x\,(1 - x - y),\ y\,(1 - x - y)) + (z, (-1/6)(z-3)^2+3/2\ ), \end{aligned}$$

for every triple (x, y, z) in the set $U^2 \times C$.

3 Study of the game $G = (p, >)$

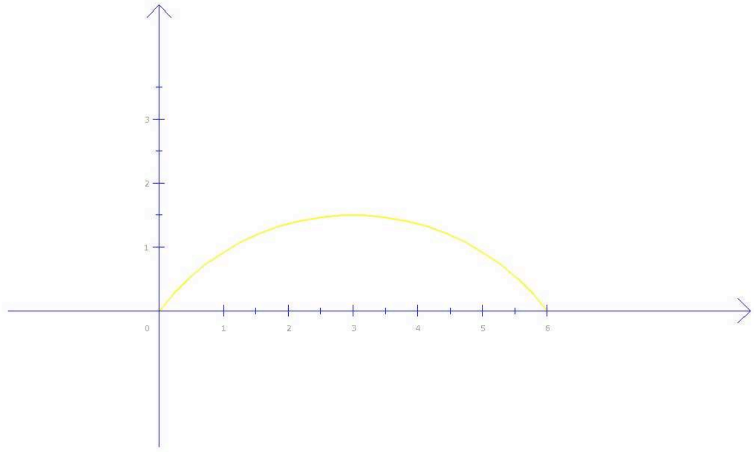
Note that, fixed a cooperative strategy z in C, the game $G(z) = (p(z), >)$ with payoff function $p(z)$, defined on the square U^2 by

$$p(z)(x, y) = f(x, y, z),$$

is the translation of the game $G(0)$ by the vector

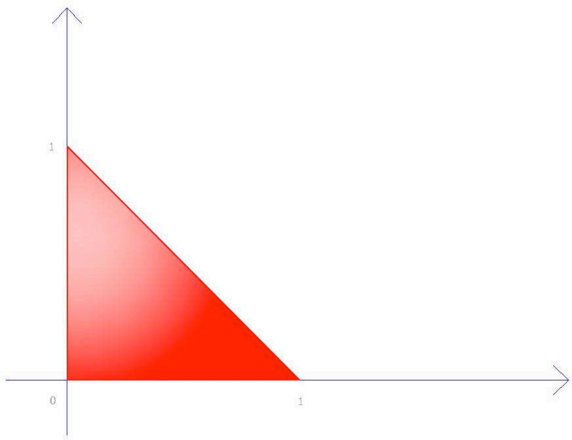
$$v(z)= (z, -(1/6)(z - 3)^2 + 3/2\ ),$$

represented pointwise in the following figure, for every z.



so that we can study the game $G(0)$ and then we can translate the information on the game $G(0)$ by the vector v(z).

So let us consider the game $G(0)$. The last classic Cournòt game G_0 has been studied completely by D. Carfì in *Topics in Game Theory*, Gabbiano 2011. The conservative part (the part of interest in the sense of J.P. Aubin) of the payoff space is the canonical 2-simplex T, convex envelope of the origin and of the canonical basis e of the Euclidean plane $\mathbf{R}^2$.



3.1 Payoff space and Pareto Boundary of the payoff space of G(z).

The Pareto boundary of the payoff space of $G(z)$ is the segment $[e_1, e_2]$, with end points the two canonical vectors of the plane $\mathbf{R}^2$, translated by the vector v(z).

The payoff space of the coopetitive game G, the image of the payoff function f, is the union of the family of payoff spaces

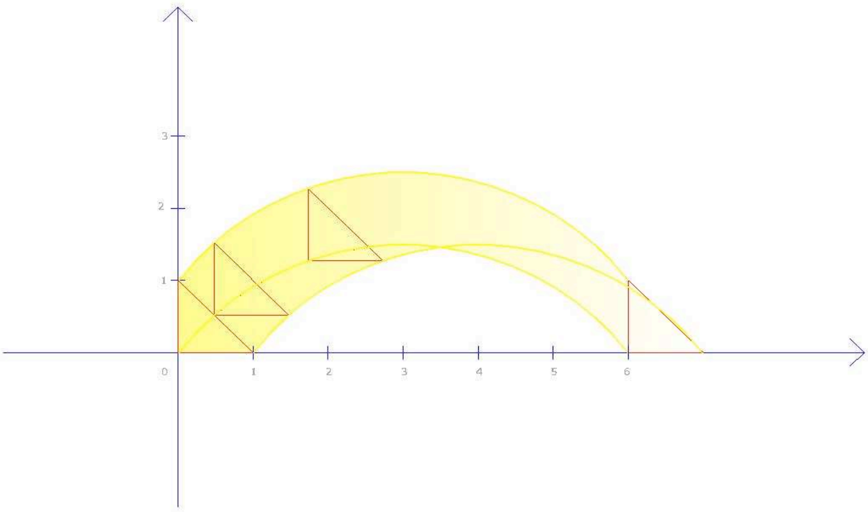
$(\mathrm{im}\, p(z))_z$,

that is the convex envelope of the of points 0, e_1, e_2, and of their translations by the vector

$v(z) = (z,(-1/6)(z-3)^2+3/2)$,

for every z in C.

The payoff space of the game G is represented below.



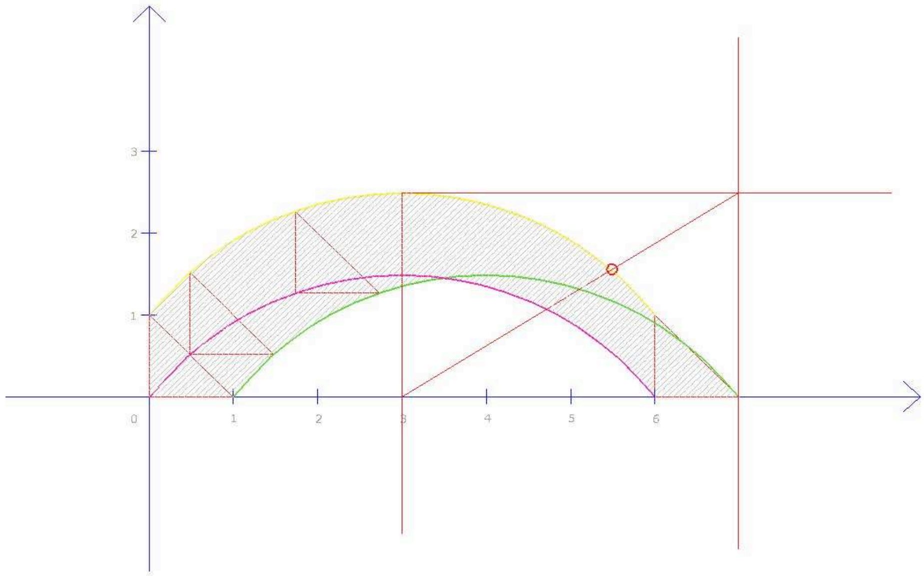
3.2 Pareto maximal boundary and the best compromise

The Pareto maximal boundary M of the payoff space $f(S)$ of the game G is the union of two curves, specifically of the parabolic segment

$(0,1) + v([3,6])$

and of the segment $[P', Q']$, where the point P' is the translation $e_1 + v(6)$ and Q' is the point $e_2 + v(6)$. In the below figure we see the Pareto boundary M and the bargaining (Kalai Smorodinsky) solution of the classic bargaining problem

(M,(inf M,sup M).



3.3 *Properly coopetitive solution*

The Nash equilibrium payoff path N is represented below in blue, it is nothing but the curve

$$N = (1/9,1/9) + v([0,6]),$$

that is the orbit of the Cournot equilibrium of the game G(0) determined by the curve

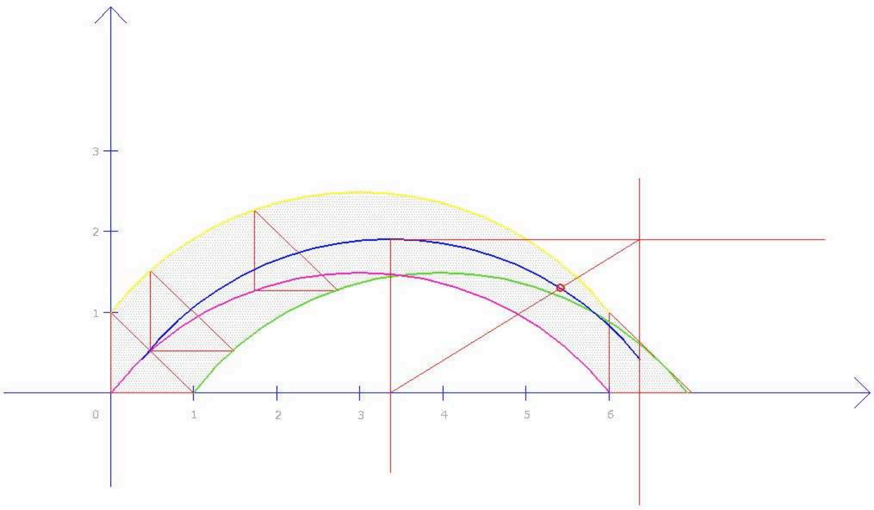
$$v([0,6]).$$

We see also the properly coopetitive solution: the Kalai Smorodinsky solution of the classic bargaining problem

$$(\partial^*N,(\inf \partial^*N,\sup \partial^*N)),$$

it is the intersection of the segment $[\inf \partial^*N,\sup \partial^*N]$ with the curve ∂^*N (Pareto maximal boundary of the Nash path), by the way we note that

$$\partial^*N = (1/9,1/9) + v([3,6]).$$

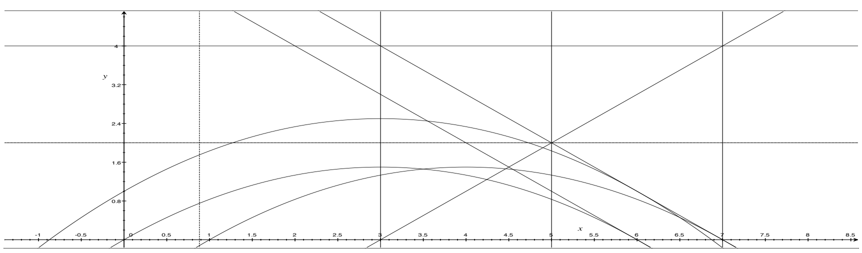


3.4 Transferable utility coopetitive solution

Below, we show in the figure the transferable utility coopetitive solution (5,2), obtained as Kalai Smorodinsky solution of the bargaining problem

(T, (inf T, sup T)),

where T is the portion [(3,4),(7,0)] of the transferable utility Pareto boundary of the game G. Note that a maximum point of the collective gain function g(X,Y) = X + Y upon the Pareto boundary M of G is the point (0,7) (or (6,1) or any point in the segment [P',Q'] = [(6,1),(7,0)]).



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