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Abstract

Using a simple two-group model of the private provision of public goods, this paper investigates how endogenous formation of within-group cooperation is affected by different types and degrees of between-group interactions. We show that when between-group interactions are of the same directions and weak (strong), within-group cooperation for providing public goods will (will not) occur in each group for strategic reasons. On the other hand, when between-group interactions are of the opposite directions or unidirectional, within-group cooperation will necessarily occur. In addition, endogenous formation of cooperation is independent of absolute (individual) levels of income as well as income distribution between agents, which corresponds to an extended version of Warr's neutrality theorem. We also show whether endogenous formation of within-group cooperation is beneficial or harmful to each group crucially depends on the degree of between-group interactions. The variation in the interaction degree leads to three different types of games concerning welfare consequences: the Prisoners' Dilemma, Coordination Game, and Invisible Hand.

Keywords: Private provision, Public goods, Cooperation.

JEL classification: H41, C72.

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1 Introduction

It is well known that when public goods are privately provided by individuals in a certain group, they can jointly increase their individual payoffs by cooperating to provide the public goods. However, such cooperation may not be as beneficial to them as non-cooperation if there are other outside (third-party) individuals or groups that strategically interact with them through public goods consumption. This is because cooperation in one group may induce negative reactions from other individuals or groups. In other words, the related individuals' or groups' reactions must be considered when calculating the profitability of cooperative provision of public goods in a certain group.

Economists have investigated the profitability of cooperation in providing public goods considering third-party behavior in a wide variety of topics. For example, in the context of military alliances or arm races, Bruce (1990) and Ihori (2001) demonstrate that all allied countries may be worse off when the allies cooperate on defense spending rather than when they do not if there are certain adversary alliances. This is because cooperative increase in defense spending in one alliance induces the adversary alliances to increase their defense spending. In the context of foreign aid, Torsvik (2005) considers a case where several altruistic donors provide aid to alleviate poverty in another country. He shows that donor cooperation may adversely change the domestic policy in the receiving country by aggravating the crowding out problem; thus, it may not be beneficial for the donors.[1] In the context of horizontal mergers, Salant et al. (1983) indicate that in a Cournot model with symmetric firms, a merger (or a cartel) will not be profitable because of the outside firms' business stealing reactions. In addition, a considerable number of studies on a public-goods experiment investigate the effectiveness of intergroup competition in promoting cooperative behavior.[2] The common element among these studies is that cooperative provision of public goods within one group depends on the nature of their interaction with other (outside) agents or groups.[3]

Using a simple model of the private provision of public goods with two distinct groups, this study investigates the effects of variations in between-group interactions on endogenous formation of within-group cooperative strategies. The model has two stages: first, the *cooperation stage* where agents decide whether to cooperate with other agents in the same group when providing a group-specific public good; second, the *contribution stage* where agents voluntarily contribute to the public good. The amount of group-specific public good in one group has positive or negative effects on that in another group, which we call *between-group interactions*. Within this framework, we examine the endogenous determination of within-group cooperation and its welfare implications.

This study contributes to the literature in three ways. First, we show that endogenous formation of within-group cooperation crucially depends on the characteristics of between-

[1] For an empirical study on foreign aid including the public-good nature of the aid and strategic interaction among donor countries, see Mascarenhas and Sandler (2006).

[2] See Bornstein and Ben-Yossef (1994), Bornstein et al. (2002), Tan and Bolle (2007), and Reuben and Tyran (2010) among many others for this point.

[3] In the field of anthropology, Kitchen and Beehner (2007) review the relationship between inter-group interaction and intra-group cooperation among non-human primates. For various types and properties of strategic interactions and cooperative behaviors in providing global public goods, see Cornes and Sandler (1996), Sandler (1997), and Barrett (2007).

group interactions, and not on any other variable. In the case of asymmetric direction of between-group interactions (i.e., the case where one group's contributions to public goods have positive external effects on agents in another group, but the reverse has negative external effects) and unidirectional interaction (i.e., the case where one group's contributions have positive or negative external effects on another group, but the reverse of either has no effect on the group), within-group cooperation necessarily emerges in each group. In the case of symmetric direction of between-group interactions, cooperation (non-cooperation) emerges in each group if the between-group interactions are weak (strong). If they are intermediate, a coordination situation emerges.

Second, we show that within-group income redistribution has no effect on the equilibrium utility of each agent as well as endogenous formation of cooperation, corresponding to the extended version of Warr's neutrality theorem (1983). Furthermore, interestingly, we obtain the result that between-group income redistribution and any kind of income changes have no effect on the result of endogenous formation of cooperation. These results correspond to the extended version of Warr's neutrality theorem concerning formation of cooperation.

Finally, we answer the question of whether endogenous formation of cooperation yields a superior outcome. We show that the variation of between-group interactions yields three different types of games in the cooperation stage: the Prisoners' Dilemma (PD), Coordination Game (CG), and Invisible Hand (IH). PD situations arise if between-group interactions are strongly positive or moderately negative. In the former case of strongly positive interactions, endogenous formation of cooperation leads to mutual non-cooperation, which is Pareto dominated by mutual cooperation. In the latter case of moderately negative interactions, it leads to mutual cooperation, which is Pareto dominated by mutual non-cooperation. CG situations arise if between-group interactions are rather strong either positively or negatively. In both cases, coordination situations emerge when deciding whether to cooperate. IH situations arise if between-group interactions are strongly negative and relatively weak, either positively or negatively. In these cases, self-interest in both groups is sufficient to guarantee a Pareto superior outcome.

Our simple framework of the model is applicable to a wide variety of socio-economic problems. Investigation of the case with negative between-group interactions can apply to problems such as arms races between two military alliances and advertising competitions between two tourist sites. The case with positive interactions corresponds to CO_2 reductions between two countries and team productions with two groups or departments in one company. The case of asymmetric direction of between-group interactions and the case of unidirectional interaction can apply to certain situations. One such situation is a unidirectional transboundary pollution problem: members of an upstream country make efforts to reduce pollutants, which is beneficial to members of a downstream country. However, the efforts made by members of the downstream country to prevent the pollutants from entering their country do not benefit the members of the upstream country. Another example is the relationship between domestic and foreign militaries against terrorists: domestic militaries contribute toward preemptively eliminating terrorists hiding in their own countries, which is also beneficial to foreign militaries. However, foreign militaries contributing toward defensive prevention of the terrorists infiltrating their national borders is not beneficial to domestic militaries in the above context.[4]

The paper is organized as follows. Section 2 presents the basic model. Section 3 characterizes the possible Nash equilibria in the second stage (contribution stage). Section 4 investigates endogenous formation of intra-group cooperation in the first stage (cooperation stage). Then, we characterize the subgame-perfect Nash equilibrium of the game. Section 5 conducts welfare analysis in the symmetric group case, and Section 6 concludes the paper.

2 The Model

Consider two groups A and B. Each group $i \in \{A, B\}$ consists of two agents 1 and 2: agent $A1$ and $A2$ are in group A and $B1$ and $B2$ are in group B.[5] In each group, there is one group-specific public good that is voluntarily provided by group members $k \in \{1, 2\}$. Let agent ik's (agent k in group i) contributions to the group-specific public good be g_{ik} and let

$$G_A = g_{A1} + g_{A2}, \text{ and } G_B = g_{B1} + g_{B2}$$

be the total amount of group-specific public goods in groups A and B, respectively (i.e., the group-specific public goods are pure public goods for its members). Agent k in group i is assumed to have initial endowment (income) y_{ik}, and allocates it between private consumption x_{ik} and contribution g_{ik} so as to maximize utilities. To make analyses simple and clear, we postulate a simple utility function:

$$U_{ik} = U^{ik}(x_{ik}, \Psi_i) = x_{ik} \times \Psi_i, \tag{1}$$

where Ψ_i is the total amount of public goods enjoyed (consumed) by members in group i.[6] We specify group i's consumption of public goods Ψ_i as:

$$\Psi_i = F_i + G_i + \gamma_i G_j, \tag{2}$$

where $F_i > 0$ is an initial endowment of group-specific public goods,[7] G_j ($j \neq i \in \{A, B\}$) is the level of public goods provided by the other group, and $\gamma_i \in [-1, 1]$ represents the direction and degree of externalities of public goods provided by group j to group i.[8]

Parameter γ_i represents between-group interaction, and the combination of γ_i and γ_j enables us to describe diverse socio-economic situations: If γ_i is positive (negative), then the public good of group j (G_j) has positive (negative) externality on group i's public good. If $\gamma_i = 1$ for all $i = \{A, B\}$, then the public good is a standard pure public good where each group's

[4] For the two counterterrorism policies, pre-emption and deterrence, see Sandler and Siqueira (2006).

[5] In this paper, we assume that the two groups and their members are exogenously determined by geographical, institutional, or historical reasons. Therefore, we do not consider the reconstruction of the group members and the movement of individuals between the groups.

[6] The utility function implies that the marginal propensity to consume public goods is 1/2, which seems to be quite high. Although the utility specification appears to be rather ad hoc, it enables us to derive certain interesting theoretical results concerning the relationship between within-group cooperation and between-group interaction. In addition, our corollary is qualitatively unchanged if we assume a more general utility function.

[7] The initial endowment of group-specific public goods F_i is incorporated into Ψ_i in order to assure that the total amount of public goods in group i is positive; $\Psi_i > 0$.

[8] Throughout the paper, we mean that $i \neq j$ (and $k \neq l$) when we use i and j (and k and l) at the same time.

contribution to the public good is perfectly substitutable. An archetypical example of this is voluntary reduction of greenhouse gas emissions by two countries, each of which have two districts.[9] If $\gamma_i = -1$ for all $i = \{A, B\}$, then the amount of public goods is reduced exactly by the public goods provided by the other group. An archetypical example is arms race between two military blocks, each of which has two countries (Ihori 2001). National defense is a pure public good for countries in the same block, and an equal increase in national defense by two blocks leaves the national security of both blocks unchanged. If $\gamma_i > 0$ and $\gamma_j < 0$, then the amount of public goods in group i is augmented by increases in G_j, but the amount of public goods in group j is reduced by increases in G_i. An example of this asymmetric direction of externalities is the relationship between domestic and foreign militaries against terrorists: country j's efforts to eliminate terrorists hiding in country j reduce country i's risk of being attacked; thus, they are beneficial to country i. However, country i's efforts to prevent the terrorists from infiltrating its national border increase country j's risk of domestic terrorism and are not beneficial to country j.

The budget constraint of each agent is given by $x_{ik} + g_{ik} = y_{ik}$, where y_{ik} is the exogenously given endowed resources (incomes) of agent ik. Notationally, let

$$Y_A = y_{A1} + y_{A2}, \quad Y_B = y_{B1} + y_{B2}$$

be the total endowed resources of group A and B, respectively.

The timing of the game is as follows; in the first stage of the game (*cooperation stage*), each agent simultaneously decides whether to cooperate in providing group-specific public goods with other agents in the same group, taking the other group's decision on cooperation as given. In the second stage of the game (*contribution stage*), each agent determines his/her voluntary contributions to group-specific public goods either cooperatively or non-cooperatively according to the commitment in the cooperation stage.

3 Equilibrium in the Contribution Stage

We solve the model backwards. In the second stage, we have four cases to consider: (1) Case NN refers to the situation where each agent non-cooperatively determines his/her own contribution to group-specific public goods in both groups, taking the behavior of every other agent as given. (2) Case CN refers to the situation where agents in group A cooperatively decide their contribution and each agent in group B non-cooperatively decides his/her contribution. (3) Case NC refers to the opposite of case CN, i.e., the situation where group A does not cooperate but group B does. (4) Case CC refers to the situation where in both group, agents cooperatively determines their contributions to group-specific public goods.[10]

[9] The case with $\gamma_i > 0$ and $\gamma_j = 0$ can illustrate a situation of unidirectional transboundary pollutions where country j's pollution harms country j and partly country i, and members of country i try to prevent the pollutants from entering their country.

[10] In the appendix, we briefly investigate a case where an agent in one group chooses to cooperate with an agent in another group.

3.1 Case NN: Cooperation does not occur in both groups

In the case where within-group cooperation does not occur in both groups, the reaction function of agent k in group i can be derived by maximizing (1) with respect to x_{ik} and g_{ik}, subject to his/her budget constraint, while taking G_j as given. For $i,\ j = \{A, B\}, k,\ l = \{1, 2\}$, we have

$$g_{ik} = \frac{1}{2}\left(y_{ik} - F_i\right) - \frac{1}{2}\left(g_{il} + \gamma_i G_j\right). \tag{3}$$

Clearly, agent ik's contribution to public goods is a strategic substitute for agent il's contribution, and is also a strategic substitute (complement) for the other group's contribution when γ_i is positive (negative). We obtain the second-stage equilibrium g_{ik} and G_i:

$$g_{ik}^{NN} = \frac{3F_i + 2y_{ik}(3 - \gamma_i\gamma_j) - y_{il}(3 - 2\gamma_i\gamma_j) - \gamma_i(Y_j - 2F_j)}{9 - 4\gamma_i\gamma_j}, \tag{4a}$$

$$G_i^{NN} = \frac{3(Y_i - 2F_i) - 2\gamma_i(Y_j - 2F_j)}{9 - 4\gamma_i\gamma_j}. \tag{4b}$$

Similarly, we have agent ik's utility in the second-stage Nash equilibrium:

$$U_{ik}^{NN} = \frac{\left[3F_i + (3 - 2\gamma_i\gamma_j)Y_i + \gamma_i(Y_j - 2F_j)\right]^2}{(9 - 4\gamma_i\gamma_j)^2}. \tag{5}$$

We find that U_{ik}^{NN} and G_i^{NN} depend on total income Y_i and Y_j and not on its individual income (y_{ik} and y_{jk}). This implies that ex-ante within-group income redistribution does not affect the equilibrium utility of all agents in both groups, which corresponds to the famous *neutrality result* demonstrated by Warr (1983) and Bergstrom et al. (1986). However, ex-ante income redistribution among *different* groups affects the utility of agents as well as the total amount of public goods unless $\gamma_A = \gamma_B = 1$. Since $(3 - 2\gamma_i\gamma_j) > \gamma_i$ holds for all $\gamma_i, \gamma_j \in [-1, 1]$, we find that income redistribution among different groups benefits the recipient and harms the donor agents. Note that when $\gamma_A = \gamma_B = 1$, both intra-group and inter-group income redistributions do not affect the equilibrium unless all agents continue to make positive contributions after the redistribution.

3.2 Case CC: Cooperation occurs in both groups

We next consider the case CC where cooperation occurs in both groups. In each group, contribution to public goods are decided by maximizing the sum of group members' utility $\sum_k U_{ik} = (Y_i - G_i)(F_i + G_i + \gamma_i G_j)$ subject to budget constraint $x_{i1} + x_{i2} + G_i = Y_i$ given G_j. Thus, we have the following reaction function:

$$G_i = \frac{1}{2}\left(Y_i - F_i\right) - \frac{1}{2}\gamma_i G_j. \tag{6}$$

Because of the existence of within-group cooperation, there is no strategic relationship between agents in the same group. Thus, we have

$$G_i^{CC} = \frac{2\left(Y_i - F_i\right) - \gamma_i\left(Y_j - F_j\right)}{4 - \gamma_i\gamma_j}, \tag{7}$$

$$U_{ik}^{CC} = \frac{\left[2F_i + \left(2 - \gamma_i\gamma_j\right)Y_i + \gamma_j\left(Y_j - F_j\right)\right]^2}{2(4 - \gamma_i\gamma_j)^2}. \tag{8}$$

It follows from (7) and (8) that within-group income redistribution does not affect the amount of public goods and the utility in Nash equilibrium, while between-group income redistribution does affect them (except for the case of $\gamma_A = \gamma_B = 1$).[11]

3.3 Cases CN and NC: One group cooperates and the other does not

We next consider the cases where one group decides to cooperate, while another decides not to cooperate in the first stage. Here, we derive the equilibrium of case CN where members in group A cooperate while members in group B do not.

It can be easily confirmed that group A's reaction function is given as (6), while that of agents in group B is given as (3). In the equilibrium, the amount of group A's public goods, individual contributions made by members in group B, and the amount of group B's public goods are obtained by

$$G_A^{CN} = \frac{3\left(Y_A - F_A\right) - \gamma_A\left(Y_B - 2F_B\right)}{6 - 2\gamma_A\gamma_B}, \tag{9a}$$

$$g_{Bk}^{CN} = \frac{\left(4 - \gamma_A\gamma_B\right) y_{Bk} - \left(2 - \gamma_A\gamma_B\right) y_{Bl} - \gamma_B\left(Y_A - F_A\right) - 2F_B}{6 - 2\gamma_A\gamma_B}, \tag{9b}$$

$$G_B^{CN} = \frac{Y_B - 2F_B - \gamma_B\left(Y_A - F_A\right)}{3 - \gamma_A\gamma_B}. \tag{9c}$$

The equilibrium utility of each member is obtained by

$$U_{Ak}^{CN} = \frac{[3F_A + (3 - 2\gamma_A\gamma_B)Y_A + \gamma_A(Y_B - 2F_B)]^2}{8(3 - \gamma_A\gamma_B)^2}, \tag{10a}$$

$$U_{Bk}^{CN} = \frac{[2F_B + (2 - \gamma_A\gamma_B)Y_B + \gamma_B(Y_A - F_A)]^2}{4(3 - \gamma_A\gamma_B)^2}. \tag{10b}$$

Note that regardless of whether a group cooperates, intra-group income redistribution does not affect the total amount of public goods and the utility of all agents.

Finally, the equilibrium outcome g_{Ak}^{NC}, G_A^{NC}, G_B^{NC}, U_{Ak}^{NC}, and U_{Bk}^{NC} in the reverse case (Case NC) can easily be obtained by replacing A to B in Eqs. (9a), (9b), (9c), (10a), and (10b).

4 Equilibrium in the Cooperation Stage

We now characterize the cooperation stage where each agent simultaneously decides whether to cooperate with their group members. Figure 1 shows the payoff matrix for the cooperation stage. In the figure, N is the strategy '*Not Cooperate*' and C is the strategy '*Cooperate.*'

Now we define $\Gamma \equiv \gamma_A\gamma_B \in [-1, 1]$. We obtain the following lemmas:

Lemma 1 *$U_{Ak}^{CC} \gtreqless U_{Ak}^{NC}$ and $U_{Bk}^{CC} \gtreqless U_{Bk}^{CN}$ hold when $\Gamma \lesseqgtr \left(2 - \sqrt{2}\right) \approx 0.59$.*

[11] We assume that, irrespective of income heterogeneities among agents, x_{ik} and g_{ik} in cooperating group i are decided so that each member in the cooperating group obtains the same utility level. This assumption is reasonable because the equilibrium utility of agents in the non-cooperating group (5) is identical even when their incomes are different. For example, if we consider the case where gains from cooperation are allocated to cooperating members by Nash bargaining, then they should be allocated equally because the utility of members at the threat point is identical.

		B-member N	B-member C
A-member	N	U_{Ak}^{NN}, U_{Bk}^{NN}	U_{Ak}^{NC}, U_{Bk}^{NC}
	C	U_{Ak}^{CN}, U_{Bk}^{CN}	U_{Ak}^{CC}, U_{Bk}^{CC}

Figure 1: Cooperation game in normal form (N: Not cooperate; C: Cooprate)

Proof. Using and arranging (8) and (10b), we have

$$U_{Ak}^{CC} - U_{Ak}^{NC} = \frac{\left[2F_A + (2-\Gamma)Y_A + \gamma_A(Y_B - F_B)\right]^2}{4(4-\Gamma)^2(3-\Gamma)^2}\left[2 - 4\Gamma + \Gamma^2\right],$$

$$U_{Bk}^{CC} - U_{Bk}^{CN} = \frac{\left[2F_B + (2-\Gamma)Y_B + \gamma_B(Y_A - F_A)\right]^2}{4(4-\Gamma)^2(3-\Gamma)^2}\left[2 - 4\Gamma + \Gamma^2\right].$$

Thus, we find that when another group chooses *Cooperate*, each group benefits from choosing *Cooperate* if and only if $[2 - 4\Gamma + \Gamma^2] > 0$ (i.e., $\Gamma < 2 - \sqrt{2} \approx 0.59$). $\square$

The lemma implies that the strategy *Cooperate* is the best response to another group's *Cooperate* if the interdependencies are not very strong in the same direction or are opposite directions ($\Gamma < 0.59$). In contrast, the strategy *Not Cooperate* is the best response to another group's *Cooperate* if the interdependency is strong in the same direction ($\Gamma > 0.59$). Interestingly, this condition is the same for all agents even when y_{ik}, Y_i, and F_i differ among agents or groups.

Lemma 2 *$U_{Ak}^{CN} \gtreqless U_{Ak}^{NN}$ and $U_{Bk}^{NC} \gtreqless U_{Bk}^{NN}$ hold when $\Gamma \lesseqgtr 3(2-\sqrt{2})/4 \approx 0.44$.*

Proof. Using and arranging Eqs. (5) and (10a), we have

$$U_{Ak}^{CN} - U_{Ak}^{NN} = \frac{\left[3F_A + (3-2\Gamma)Y_A + \gamma_A(Y_B - F_B)\right]^2}{8(3-\Gamma)^2(9-4\Gamma)^2}\left[9 - 24\Gamma + 8\Gamma^2\right],$$

$$U_{Bk}^{NC} - U_{Bk}^{NN} = \frac{\left[3F_B + (3-2\Gamma)Y_B + \gamma_B(Y_A - F_A)\right]^2}{8(3-\Gamma)^2(9-4\Gamma)^2}\left[9 - 24\Gamma + 8\Gamma^2\right].$$

Thus, we find that the strategy *Cooperate* is the best response to another group's *Cooperate* if and only if $[9 - 24\Gamma + 8\Gamma^2] > 0$ (i.e., $\Gamma < 3(2-\sqrt{2})/4 \approx 0.44$). $\square$

The lemma implies that the strategy *Cooperate* is the best response to another group's *Not Cooperate* if the interdependencies are not very strong in the same direction and are opposite directions ($\Gamma < 0.44$). The threshold value of Γ is smaller than that in Lemma 1. Also note that, as in Lemma 1, the condition is the same for all agents when y_{ik}, Y_i, and F_i differ among agents or groups.

From Lemmas 1 and 2, we have the following proposition.

Proposition 1 *In the cooperation stage,* *(i)* (C, C) *is a unique Nash equilibrium for* $\Gamma \in [-1, 0.44)$,

(ii) (C, C) *and* (N, N) *are two Nash equilibria for* $\Gamma \in [0.44, 0.59]$.

(iii) (N, N) *is a unique Nash equilibrium for* $\Gamma \in (0.59, 1]$.

This proposition holds independently of y_{ik}, Y_i, F_i *for all* $i \in \{A, B\}$ *and* $k \in \{1, 2\}$ *as long as all agents are positive contributors in equilibrium.*

Proof. Immediately from Lemmas 1 and 2. □

Proposition 1 implies when γ_A and γ_B have opposite signs (i.e., asymmetric direction of between-group interactions), Γ must be negative implying that cooperation necessarily occurs in each group.[12] When between-group interactions are in the same direction and are sufficiently weak (strong), cooperation occurs (does not occur) in each group. More interestingly, the threshold level of Γ is the same for each group irrespective of the difference of y_{ik}, Y_i, F_i. The striking properties of the results are summarized by the following corollary:

Corollary *As long as all agents are positive contributors in equilibrium,*

(a) income redistribution within a group has no effect on the equilibrium utility of each agent as well as the decision to cooperate,

(b) income redistribution between groups has no effect on the decision to cooperate.

(c) any kind of income growth has no effect on the decision to cooperate.

Corollary-(a) corresponds to the famous Warr's neutrality theorem (Warr 1983). Corollary-(b) and -(c) are novel and can be considered as new types of neutrality properties: the agents' choice between cooperating or not is independent of absolute income levels, income distributions, and initial levels of group-specific public goods.

5 Welfare Implications

We next investigate whether endogenous formation of within-group cooperation is beneficial for both groups. For the sake of simplicity, we hereafter assume that both groups are symmetric[13] ($F_A = F_B = F$ and $Y_A = Y_B = Y$) and the directions of between-group interactions are the same between groups ($\gamma_A = \gamma_B = \gamma$).[14] Thus, we have the following lemma:

Lemma 3 *In the symmetric case,* $U_{ik}^{CC} \gtreqless U_{ik}^{NN}$ *holds when* $\gamma \gtreqless (\sqrt{2} - 2)/2 \approx -0.29$.

[12] In addition, when between-group interactions are unidirectional (i.e., either γ_A or γ_B equals zero, then cooperation necessarily occurs in each group.

[13] Note that we still allow the income differences between agents as long as the sum of the members' income are same between the two groups.

[14] The welfare effect of endogenous formation of within-group cooperation in the case of asymmetric directions of between-group interactions seems to be obvious. If $\gamma_A > 0$ and $\gamma_B < 0$, then cooperation occurs in both groups A and B (Proposition 1), and the mutual cooperation is beneficial to the member in group A and is not beneficial to the member in group B as compared to the case of mutual non-cooperation.

Proof. From a symmetric assumption (i.e., $F_A = F_B = F$, $Y_A = Y_B = Y$, and $\gamma_A = \gamma_B = \gamma$) and Eqs. (5) and (8), we have

$$U_{ik}^{CC} - U_{ik}^{NN} = \frac{\big[F + Y(1+\gamma)\big]^2}{2(2+\gamma)^2(3+2\gamma)^2}\Big[1 + 4\gamma + 2\gamma^2\Big],$$

which has the positive sign if $(1 + 4\gamma + 2\gamma^2) > 0$. This condition leads to $\gamma < -(2+\sqrt{2})/2 \approx -1.70$ and $\gamma > -(2-\sqrt{2})/2 \approx -0.29$. Because $\gamma \in [-1, 1]$, we have proven that $\gamma > (<) - 0.29$ is sufficient for $U_{ik}^{CC} > (<)\ U_{ik}^{NN}$. $\square$

This lemma implies that when the externality is negative and sufficiently strong that $\gamma < -0.29$, then the situation where within-group cooperation occurs take in both groups is Pareto dominated by the situation where cooperation does not occur in both groups.

Proposition 2 *In a symmetric equilibrium,*

(i) For $\gamma \in [-1, -0.77)$, the Nash equilibrium (N, N) Pareto dominates the outcome (C, C).

(ii) For $\gamma \in [-0.77, -0.66)$, the Nash equilibria are (N, N) and (C, C), and the outcome (N, N) Pareto dominates the outcome (C, C).

(iii) For $\gamma \in [-0.66, -0.29)$, the Nash equilibrium (C, C) is Pareto dominated by the outcome (N, N).

(iv) For $\gamma \in [-0.29, 0.66)$, the Nash equilibrium (C, C) Pareto dominates the outcome (N, N).

(v) For $\gamma \in [0.66, 0.77)$, the Nash equilibria are (N, N) and (C, C), and the outcome (C, C) Pareto dominates the outcome (N, N).

(vi) For $\gamma \in [0.77, 1]$, the Nash equilibrium (N, N) is Pareto dominated by the outcome (C, C).

Proof. Applying the symmetric assumption to Lemmas 1 and 2, we have

$$U_{Ak}^{CC} \gtreqless U_{Ak}^{NC} \ \Leftrightarrow\ |\gamma| \lesseqgtr \pm\sqrt{2-\sqrt{2}} \approx \pm 0.77,$$

$$U_{Ak}^{CN} \gtreqless U_{Ak}^{NN} \ \Leftrightarrow\ |\gamma| \lesseqgtr \pm\frac{\sqrt{3(2-\sqrt{2})}}{2} \approx \pm 0.66.$$

Thus, the strategy *Cooperate* is the dominant strategy for $-0.66 \le \gamma < 0.66$, and the strategy *Not Cooperate* is the dominant strategy for $\gamma < -0.77$ and $\gamma \ge 0.77$. If $-0.77 \le \gamma < -0.66$ and $0.66 \le \gamma < 0.77$, (N, N) and (C, C) are both Nash equilibria in the cooperation stage. Combining them with Lemma 3 proves the proposition. $\square$.

Proposition 2 indicates the welfare implication of within-group cooperation. Figure 2 illustrates the results. The prisoners' dilemma (PD) situations emerge when $\gamma \in [-0.66, -0.29)$ and $\gamma \in [0.77, 1]$, which are represented by the dark shaded areas in the figure. In the former situation ($\gamma \in [-0.66, -0.29)$), within-group cooperation emerges in both groups, but the utility of each agent is smaller compared to that in mutual non-cooperation. In the latter situation ($\gamma \in [0.77, 1]$), no cooperation occurs even though mutual within-group cooperation is beneficial to all agents. In contrast, the invisible hand (IH) situations emerge when $\gamma \in [-1, -0.77)$

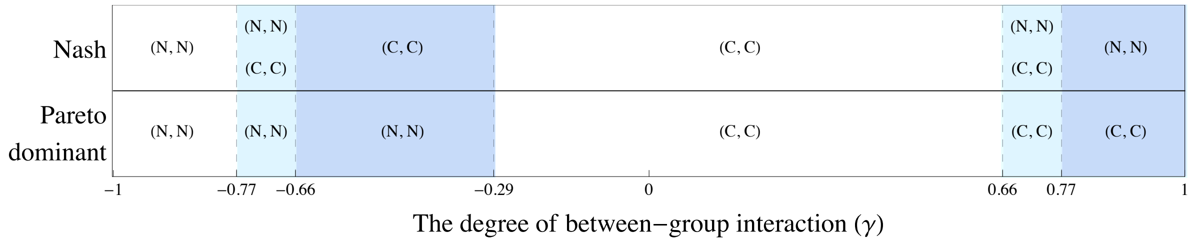

Figure 2: Nash and Pareto-dominant (symmetric) equilibria and the degree of between-group interactions

and $\gamma \in [-0.29, 0.66)$. IH situations mean that endogenous decisions to cooperate lead to Pareto superior outcomes. In the former situation ($\gamma \in [-1, -0.77)$), the between-group interactions are strongly negative; thus, each group cannot cooperate out of fear that cooperation will induce another group to contribute more. Thus, mutual non-cooperation is beneficial to all agents in this arms race situation. In the latter situation, between-group interactions are positively or negatively mild. Therefore, each group can decide whether to cooperate, without much regard for another group's reaction, stimulating beneficial mutual cooperation. Finally, the coordination game (CG) situations emerge when $\gamma \in [-0.77, -0.66)$ and $\gamma \in [0.66, 0.77)$, which are represented by the lighter shaded areas in Figure 2. In the former (latter) situation, mutual non-cooperation (cooperation) is better for all agents, but there are two Nash equilibria (N, N) and (C, C).[15] Figure 3 provides numerical examples in which the parameter values are $(Y = 20,\ F = 10)$ for negative γ and $(Y = 20,\ F = 0)$ for positive γ.[16] In each payoff matrix, players' best responses are underlined. The figure confirms the results shown in Proposition 2.

6 Concluding Remarks

When externalities exist both within and beyond the boundaries of a group, within-group cooperation may not occur for strategic reasons. This paper studies the effect of between-group interaction on endogenous determination of within-group cooperation in a simple two-group model of private provision of public goods. Our major results can be summarized as follows. First, between-group interactions play a dominant role in promoting within-group cooperation. In particular, when between-group interactions are in the same direction and weak (strong), within-group cooperation to provide public goods will (will not) occur in each group. On the other hand, when between-group interactions are in opposite directions or unidirectional, within-group cooperation will necessarily occur. Second, endogenous formation of cooperation is independent of the absolute (individual) levels of income as well as of income distributions

[15] We can solve the coordination issue regarding multiple Nash equilibria by using the concept of risk dominance as defined by Harsanyi and Selten (1988). If we apply this concept, tedious calculations indicate that $\gamma < -0.7002$ is sufficient to ensure the equilibrium (N, N) to be a risk-dominant equilibrium in the former situation. Likewise, $\gamma < 0.7144$ is sufficient to ensure the equilibrium (C, C) to be a risk-dominant equilibrium in the latter situation. In addition, if we consider the case where decisions to cooperate are made sequentially in the first stage, the unique Nash equilibrium coincides with Pareto dominant equilibrium.

[16] The difference in the value of F serves to ensure the interior solution.

		B	
		N	C
A	N	$\underline{100}$, $\underline{100}$	$\underline{56}$, 78
	C	78, $\underline{56}$	50, 50

(a) IH: $\gamma = -1$

		B	
		N	C
A	N	$\underline{189}$, $\underline{189}$	140, 186
	C	186, 140	$\underline{143}$, $\underline{143}$

(b) CG: $\gamma = -0.7$

		B	
		N	C
A	N	225, 225	186, $\underline{238}$
	C	$\underline{238}$, 186	$\underline{200}$, $\underline{200}$

(c) PD: $\gamma = -0.5$

		B	
		N	C
A	N	204, 204	222, $\underline{226}$
	C	$\underline{226}$, 222	$\underline{247}$, $\underline{247}$

(d) IH: $\gamma = 0.25$

		B	
		N	C
A	N	$\underline{239}$, $\underline{239}$	310, 235
	C	235, 310	$\underline{317}$, $\underline{317}$

(e) CG: $\gamma = 0.7$

		B	
		N	C
A	N	$\underline{256}$, $\underline{256}$	$\underline{400}$, 200
	C	200, $\underline{400}$	356, 356

(f) PD: $\gamma = 1$

Figure 3: Numerical Examples: $Y = 20$, $F = 10$ (when $\gamma \leq 0$), $F = 0$ (when $\gamma > 0$).

between agents. This result corresponds to an extended version of Warr's neutrality theorem. Finally, we show whether endogenous formation of within-group cooperation is beneficial or harmful to each group depending on the degree of between-group interactions. The variation in the interaction degree yields three different types of games in the cooperation stage: the Prisoners' Dilemma, Coordination Game, and Invisible Hand.

Admittedly, our analysis is conducted in a highly simplified framework and may overlook several features. First, we assume both the number of members in each group and the number of groups to be two. The differences between group sizes will affect the profitability of forming within-group cooperation in each group. Second, we consider only the linear representation of between-group externalities (i.e., we assume that one group's contributions have linear effects on another group's public goods provisions). However, between-group interactions concerning public goods provision may be diverse, including non-linear relationships such as conflict between two groups (e.g., Niou and Tan 2005), contest between two groups (e.g., Baik 2008), and rent-seeking between two groups (e.g., Baik and Lee 2000; Cheikbossian 2010). Thus, our results obtained from the linear specification of between-group interactions should be interpreted as a benchmark. Finally, the validity of our theoretical results on endogenous formation of cooperation could be tested by laboratory experiments. For instance, Bornstein and Ben-Yossef (1994), Bornstein et al. (2002), and Reuben and Tyran (2010) experimentally investigate the effects of inter-group competition or conflict on intra-group cooperation. It will be interesting to experimentally investigate how within-group cooperation is affected by different types and degrees of between-group interactions. These issues will be examined in future research.

Appendix

In this appendix, we investigate whether agents have unilateral incentive to cooperate across group boundaries. We consider the case where agents $A1$ and $B1$ cooperate with each other and agents $A2$ and $B2$ do not. In this case, agents $A1$ and $B1$ choose g_{A1} and g_{B1} so as to maximize joint payoffs $U_{A1} + U_{B1}$, given g_{A2} and g_{B2}. Agent $A2$ ($B2$) independently chooses g_{A2} (g_{B2}) so as to maximize its own payoff U_{A2} (U_{B2}). We assume that all agents are symmetric. Then, after some manipulations, we obtain the equilibrium utility of agents $A1$ and $B1$ as

$$U_{A1}^{A1\cup B1} = U_{B1}^{A1\cup B1} = \frac{[F + (1+\gamma)Y]}{(3+\gamma)^2(3+2\gamma)^2}.$$

Thus, we have

$$U_{A1}^{A1\cup B1} - U_{A1}^{NN} = -\frac{\gamma\big[F + (1+\gamma)Y\big]^2\big[3+\gamma(3+\gamma)\big]}{(1+\gamma)(3+\gamma)^2(3+2\gamma)^2} \gtreqless 0 \Leftrightarrow \ \gamma \lesseqgtr 0.$$

This shows that agents have (do not have) incentives for unilaterally cooperating with an agent in another group for $\gamma < 0$ ($\gamma > 0$). The intuition is quite straightforward: if between-group interactions are positive, agents $A2$ and $B2$ can free ride on increases in the amount of public goods that are brought about by cooperation between agents $A1$ and $B1$. On the other hand, if between-group interactions are negative, agents $A2$ and $B2$ reluctantly increase their contribution because the amount of public goods is reduced by cooperation between agents $A1$ and $B2$.

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