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A note on the choice of Malmquist Productivity Index and Malmquist Total Factor Productivity Index

by

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Abstract

This paper by analyzing the two popular methodologies of productivity measurement provides an example that illustrates the differences when adopting the two methodologies. Furthermore, under the restriction of constant returns to scale raises some methodological issues regarding the theory of productivity measurement using the Malmquist Productivity Index and Malmquist Total Factor Productivity Index. Furthermore by using an illustrative example under the restriction of constant returns to scale the study indicates that the two indexes produce similar results. However, the differences observed are determining the choice of the methodology adopted when measuring productivity.

Keywords: Productivity measurement, Malmquist Productivity Index, Malmquist Total Factor Productivity Index.

JEL Codes: C69, D24, I10,

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I. Introduction

According to Yu [1] productivity measurement has many dimensions which can be distinguished between partial measures and total factor productivity. However, according to Lovell [2] there are two indexes that can be adopted in order to measure productivity change and are associated with the name Malmquist. Firstly, the Malmquist Productivity Index (MPI) pioneered by Caves et al. [3] based on ratios of output distance functions or on ratios of input distance functions. Secondly there is the Malmquist Total Factor Productivity Index (MTFPI) proposed by Bjurek [4] which is simultaneously oriented and it is based on a ratio of output distance functions contained in the output quantity index and a ratio of input distance functions contained in the input quantity index [2].

According to Lovell [2] there are a few studies providing empirical evidence comparing productivity measurements of the two indexes. This paper tests productivity changes using both indexes under the restriction of constant returns to scale by using data from the Greek health sector. The structure of the paper is as follows. Section two describes the computation of input and output distance functions whereas section three provides the construction of the two indexes. Finally, section four provides the empirical evidence whereas section five concludes our study.

II. Computing input and output distance functions

Consider a firm employing a vector of inputs $\boldsymbol{x}^t \in \Re^N_+$ to produce a vector of outputs $\mathbf{y}^t \in \Re^M_+$ where $\Re^N_+$, $\Re^M_+$, are non-negative *N*- and *M*-dimensional Euclidean spaces, respectively in period t (t =1,…,T).

$$F^t = \{(y^t,x^t): x^t \text{ can produce } y^t\} \qquad (1)$$

The production set is the set of all feasible output-input vectors in period t. The output sets associated with F^t are:

$$K^t(x^t) = \{y^t: (y^t,x^t) \in F^t\},\ \boldsymbol{x^t} \in \Re^N{}_{+,} \qquad (2)$$

the input sets associated with F^t are:

$$B^t(y^t) = \{x^t: (x^t,y^t) \in F^t\},\ \mathbf{y}^t \in \Re^M{}_{+,} \qquad (3).$$

Both sets are assumed to be closed, bounded, convex and they satisfy strong disposability of outputs and inputs. We assume that all functions refer to constant returns to scale.

Following Shephard's [5] calculations for output within period and adjacent period distance functions:

$$D_{oc}^t(x^t, y^t) = \min\{\theta : (y^t/\theta) \in K^t(x^t)\} \qquad (4)$$

$$D_{oc}^t(x^{t+1}, y^{t+1}) = \min\{\theta : (y^{t+1}/\theta) \in K^t(x^{t+1})\} \qquad (5)$$

$$D_{oc}^{t+1}(x^t, y^t) = \min\{\theta : (y^t/\theta) \in K^{t+1}(x^t)\} \qquad (6)$$

Furthermore calculations for input within period and adjacent period distance functions can be defined as:

$$D_{ic}^t(y^t, x^t) = \max\{\vartheta : (x^t/\vartheta) \in B^t(y^t)\} \qquad (7)$$

$$D_{ic}^t(y^{t+1}, x^{t+1}) = \max\{\vartheta : (x^{t+1}/\vartheta) \in B^t(y^{t+1})\} \qquad (8)$$

$$D_{ic}^{t+1}(y^t, x^t) = \max\{\vartheta : (x^t/\vartheta) \in B^{t+1}(y^t)\} \qquad (9).$$

Finally, we assume that F^t satisfies other axioms as specified by Shephard [5] in order to be produced meaningful output and input distance functions.

III. Measurement of Malmquist productivity index and Malmquist Total Factor Producivity Index

The Malmquist Productivity Index is defined by Caves, Christensen and Diewert [3] (hereafter CCD), with reference to the technology of the initial period (t) as:

$$m_{CCD}^{t} = \frac{D_{oc}^{t}(x^{t+1}, y^{t+1})}{D_{oc}^{t}(x^{t}, y^{t})} \qquad (10)$$

However, we can also choose the technology in period (t+1) as the reference in defining a productivity index. The Malmquist Productivity Index in relation to the technology of the final period (t+1) can be defined as:

$$m_{CCD}^{t+1} = \frac{D_{oc}^{t+1}(x^{t+1}, y^{t+1})}{D_{oc}^{t+1}(x^{t}, y^{t})} \qquad (11).$$

The two indexes appear to be identical however, they may or may not be the same in the cases of multiple inputs and varying returns to scale (VRS) technology. However this study uses the assumption of constant returns to scale. To avoid the arbitrariness in choosing the benchmark, Färe et al. [6] [7] specify the Malmquist Productivity Index as the geometric mean of the above two indexes:

$$MPI(x^{t+1}, y^{t+1}, x^{t}, y^{t}) = \left[\frac{D_{oc}^{t}(x^{t+1}, y^{t+1})}{D_{oc}^{t}(x^{t}, y^{t})} \times \frac{D_{oc}^{t+1}(x^{t+1}, y^{t+1})}{D_{oc}^{t+1}(x^{t}, y^{t})} \right]^{1/2} \qquad (12).$$

If the value of Malmquist Productivity Index (hereafter MPI) is greater than unity then the firm is regarded to be efficient improving it's productivity over time however when it is less than unity deterioration in firm's productivity is recorded over time and thus the firm is regarded to be inneficient.

The Hicks-Moorsteen index was introduced as a "Malmquist total factor productivity index" (hereafter MTFPI) by Bjurek [4]. In accordance with the above stated, the rationale offered by Bjurek [4] for computing this index was that the classic Malmquist productivity index does not measure properly the changes in productivity at the time of changes in returns to scale. What is important is the fact that instead of defining an output or input oriented index, this specification measures the change in output quantities in output direction, and the change in input quantities in input direction, a detail of considerable meaning when employing variable returns to scale [4]. However, as indicated previously this study tests empirically the differences of the two indexes under the assumption of conctant returns to scale.

$$MTFPI(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{D^t_{oc}(x^t, y^{t+1}) / D^t_{oc}(x^t, y^t)}{D^t_{ic}(y^t, x^{t+1}) / D^t_{ic}(y^t, x^t)} \qquad (13).$$

In MTFPI is comprised of both output and input distance functions. Again if the value of Malmquist Total Factor Productivity Index (MTFPI) is greater than unity then the firm is regarded to be efficient improving it's productivity over time however when it is less than unity deterioration in firm's productivity is recorded over time and thus the firm is rgarded to be inneficient.

IV. An illustrative example

The data used for our analysis are the second revised edition of 'Greece in Figures' presenting a panorama of the Greek corporate sector based on the balance sheets and income statements of 2004 and 2005. The data were collected and processed by ICAP's Business Information Division and include all financial statements and other companies' data which were published within the time limits set by the Greek law [8]. In our analysis we use the top-20 companies in health sector ranked by total assets. For the construction of the two productivity indexes we use two inputs: total assets (measured in thousands of euros), number of employees and one output pre-tax profits (measured in thousands of euros).

Table 1: Description of company names, Malmquist productivity index scores, Malmquist total factor productivity index scores and productivity differences.

Company Names	MPI	MTFPI	Differences	MPI	MTFPI
ATHENS MEDICAL CENTER S.A.	1,92	0,83	1,10	*Productive*	Not Productive
ATHINAIKI GENERAL CLINIC S.A.	0,19	0,56	-0,37	Not Productive	Not Productive
BIOMED S.A.	0,76	0,27	0,49	Not Productive	Not Productive
BLUE CROSS S.A.	0,48	0,63	-0,15	Not Productive	Not Productive
CENTRAL CLINIC OF ATHENS S.A.	1,70	0,93	0,77	*Productive*	Not Productive
EUROCLINIC OF ATHENS S.A.	0,95	0,73	0,22	Not Productive	Not Productive
EUROMEDICA KRITIS S.A.	0,61	0,99	-0,38	Not Productive	Not Productive
EUROMEDICA S.A.	0,44	0,94	-0,50	Not Productive	Not Productive
FILOKTITIS S.A.	1,52	2,16	-0,64	*Productive*	*Productive*
GENESSIS S.A.	0,07	0,00	0,06	Not Productive	Not Productive
GENIKI KLINIKI DODEKANISSOU S.A.	0,23	0,00	0,23	Not Productive	Not Productive
GENIKI KLINIKI OF THESSALONIKI S.A.	0,80	0,63	0,17	Not Productive	Not Productive
HYGEIA DIAGNOSTIC S.A.	2,34	1,00	1,33	*Productive*	*Productive*
IASO GENERAL S.A.	0,46	0,28	0,18	Not Productive	Not Productive
IASO S.A.	0,93	1,09	-0,16	Not Productive	*Productive*
IATROPOLIS MAGNETIC TOMOGRAPHY S.A.	1,73	1,12	0,61	*Productive*	*Productive*
MITERA S.A	1,05	0,81	0,24	*Productive*	Not Productive
NOSSILEFTIKI S.A.	1,20	0,62	0,57	*Productive*	Not Productive
OLYMPION THERAPEFTIRIO S.A.	0,96	0,23	0,72	Not Productive	Not Productive
PERSEFS S.A.	2,13	0,70	1,43	*Productive*	Not Productive

Table 1 illustrates the results of our analysis in such a way that we can compare the two indexes. The first column illustrates the names of the twenty companies used in our analysis whereas the second and third describe the results derived from our calculations of the two indexes (MPI and MTFPI). The differences of the two indexes are presented in the fourth column whereas the results of whether a company is efficient or not are presented in the last two columns. Looking at the results we realise that the two indexes have some notisable differences under the restrictions of constant returns to scale. Looking at the fifth and sixth column we realise that in six cases (ATHENS MEDICAL CENTER S.A., CENTRAL CLINIC OF ATHENS S.A., IASO S.A., MITERA S.A, NOSSILEFTIKI S.A. and PERSEFS S.A.) the indexes provide completely different results (in terms of decision making i.e. productive/not productive). Furthermore, when using the MPI index eight firms are appearing to be productive (score >1) whereas in the case of MTFPI only four firms are recorded to be overall productive.

Figure 1: Diagrammatical comparison of the two productivity indexes under constant returns to scale.

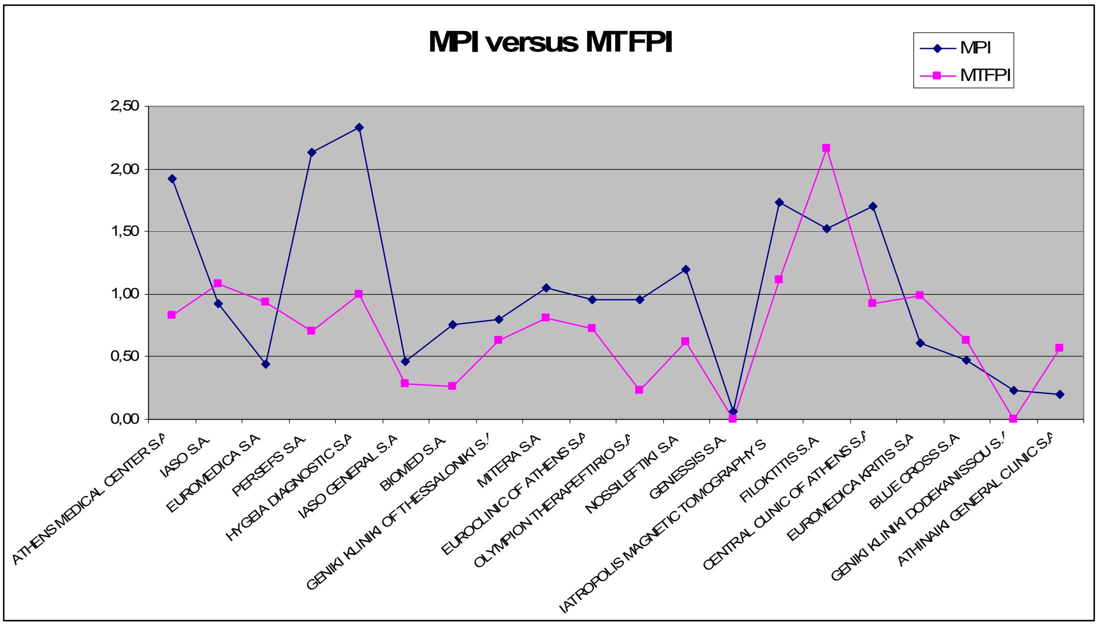

Table 2: Descriptive statistics and Mann-Whitney test of differences between Malmquist productivity index scores (MPI) and Malmquist total factor productivity index scores (MTFPI).

Variables	Mean	StDev	Minimum	Median	Maximum
MPI	1,024	0,669	0,070	0,940	2,340
MTFPI	0,726	0,479	0,000	0,715	2,160
Reference	**Mann-Whitney Test**	**Asymptotic significance (two tailed)**			
MPI vs MTFPI	454	0,239			

In addition, Figure 1 presents the two indexes where as can be realised the trends are similar in many cases except in the presence of outliers observed for MPI. Table 2 provides evidence of the similarity of the two indexes. When looking at the descriptive statistics we realise that the indexes have similar characteristics in terms of their standard deviation, median, minimum and maximum values. Due to the fact that the two indexes have been calculated from a nonparametric method (DEA), we have run the Mann-Whitney test in order to compare the two productivity indexes. The results provided in table 2 indicate that the null hypothesis of median equality cannot be rejected and therefore the two indexes are similar. This supports the argument by Lovell [2] and Färe *et al.* [7] under the restrictions of constant returns to scale.

However, in reality there are differences in the productivity scores. That is the MPI gives a lot more productive companies (eight) compared to MTFPI index (four). The properties of the two indexes are different and therefore, when using MTFPI index which takes simultaneously into consideration both input and output orientation provides 'strict' conditions of measurement and thus to our opinion provides different results which are subject to the methodology adopted by the practitioners.

V. Conclusions

This study using a sample of twenty companies operating in the Greek health sector, examines their productivity changes over a period of two years (2004-2005). By using assets and number of employees as inputs and pre-tax profits as output, we compute two productivity indexes. The study measures companies' productivity by using MPI pioneered by Caves et al. [3] and then compares it with results derived by calculating MTFPI introduced by Bjurek [4] under the restriction of constant returns to scale.

Our empirical results provide evidence that the two indexes give similar results. This supports the studies by Färe et al. [7] and Lovell [2] which illustrate the theoretical conditions that under constant returns to scales the two indexes are similar. If we don't follow strict statistical hypotheses and we adopt a non-parametric test like Mann-Whitney then the results provide no statistical justification for any differences between the medians. But when looking thoroughly at the results we notice considerable changes that may influence the decisions when measuring productivity using the two indexes. This is subject to the practitioners' choice upon the decision of the use of input/output orientation. It appears that if orientation in a study is not an issue then the results obtained by MTFPI are more rigid compared to the results obtained by MPI.

It has to be mentioned that according to Lambert [9] when allowing variable returns to scale there are problems using MPI and thus the results will be different relative to MTFPI. Finally Bjurek [4] suggests that MTFPI in that case will measure more accurately productivity changes.

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