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Abstract

In the present work we propose an original analytical model of coopetitive games. We try to apply this analytical model of coopetition - based on game theory and conceived at a macro level - to the Greek crisis, suggesting feasible solutions in a cooperative perspective for the divergent interests which drive the economic policies in the euro area.

Keywords. Games and Economics, competition, cooperation, coopetition, normal form games

1 Introduction

In this contribution we focus on the Greek crisis, since we know that Greece is still in a very difficult economic situation, due to its lack of competitiveness and is at risk of insolvency, because of its public finance mismanagement. Although the EU Governments and IMF have recently approved more substantial financial aids to cover the refinancing needs of Greece until 2014, in exchange of a serious and tough austerity program. Germany, on the other hand, is the most competitive economy of the Euro area and has a large trade surplus with Greece and other Euro partners; hence significant trade imbalances occur within the euro area.

The main purpose of our paper is to explore win-win solutions for Greece and Germany, involving a German increasing demand of Greek exports. We do not analyze the causes of the financial crisis in Greece and its relevant political and institutional effects on the European Monetary Union. Rather we concentrate on stability and growth, which should drive the economic policy of Greece and the other Euro countries.

Organization of the paper. The work is organized as follows:

- section 2 examines the Greek crisis, suggesting a possible way out to reduce the intra-eurozone imbalances through coopetitive solutions within a growth path;
- sections from 3 to 17 provides an original model of coopetitive game applied to the Eurozone context, showing the possible coopetitive solutions;
- conclusions end up the paper.

Introduction and Section 2 of this paper are written by D. Schilirò, sections from 3 to 17 are written by D. Carfì; conclusions are written by the two authors.

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2 The Greek Crisis and the coopetitive solution

The deep financial crisis of Greece, which was almost causing the default of its sovereign debt, has revealed the weaknesses of Greek economy, particularly its lack of competitiveness, but also the mismanagement of the public finance and the difficulties of the banking sector.

2.1 The crisis and the Greek economy

With the outbreak of the global crisis of 2008-2009, Greece relied on state spending to drive growth. Moreover, the country has accumulated a huge public debt (over 320 billion euros in 2010). This has created deep concerns about its fiscal sustainability and its financial exposition has prevented the Greek government to find capitals in the financial markets. In addition, Greece has lost competitiveness since joining the European Monetary Union and, because of that, Greek's unit labor cost rose 34 percent from 2000 to 2009. The austerity measures implemented by the Greek government, although insufficient, are hitting hard the Greek economy, since its growth is expected to be negative also this year (2011), making the financial recovery very problematic [Mussa, 2010]. Furthermore, Greece exports are much less than imports, so the trade balance shows a deficit around 10%. Therefore, the focus of economic policy of Greece should become its productive system and growth must be the major goal for the Greek economy in a medium term perspective. This surely would help its re-equilibrium process.

2.2 The soundest European economy: Germany

Germany, on the other hand, is considered the soundest European economy. It is the world's second-biggest exporter, but its wide commercial surplus is originated mainly by the exports in the Euro area, that accounts for about two thirds. Furthermore, since 2000 its export share has gradually increased vis-à-vis industrial countries. Thus Germany's growth path has been driven by exports. We do not discuss in this work the factors explaining Germany's increase in export share, but we observe that its international competitiveness has been improving, with the unit labor cost which has been kept fairly constant, since wages have essentially kept pace with productivity. Therefore the prices of the German products have been relatively cheap, favoring the export of German goods towards the euro countries and towards the markets around the world, especially those of the emerging economies (China, India, Brasil, Russia). Finally, since 2010 Germany has recovered very well from the 2008-2009 global crisis and it is growing at a higher rate than the others Euro partners.

Therefore we share the view that Germany (and the other surplus countries of the Euro area) should contribute to overcome the crisis of Greek economy stimulating its demand of goods from Greece and relying less on exports towards the Euro area in general. Germany, as some economists as Posen [2010] and Abadi [2010] underlined, has benefited from being the anchor economy for the Eurozone over the last 11 years. For instance, in 2009, during a time of global contraction, Germany has been a beneficiary, being able to run a sustained trade surplus with its European neighbors. Germany exported, in particular, 6.7 billions euro worth of goods to Greece, but imported only 1.8 billion euro worth in return.

2.3 A win-win solution for Greece and Germany

Thus we believe that an economic policy that aims at adjusting government budget and trade imbalances and looks at improving the growth path of the real economy in the medium and long term in Greece is the only possible one to assure a stable re-balancing of the Greek economy and also to contribute to the stability of the whole euro area [Schilirò, 2011]. As we have already argued, German modest wage increases and weak domestic demand favored the export of German goods towards the euro countries. We suggest, in accordance with Posen [2010], a win-win solution (a win-win solution is the outcome of a game which is designed in a way that all participants can profit from it in one way or the other), which entails that Germany, which still represents the leading economy, should re-balance its trade surplus and thus ease the pressure on the southern countries of the euro area, particularly Greece. Obviously, we are aware that this is a mere hypothesis and that our framework of coopetition is a normative model. However, we believe that a cooperative attitude must be taken within the members of the European monetary union. Thus we pursue our hypothesis and suggest a model of coopetitive game as an innovative instrument to analyze possible solutions to obtain a win-win outcome for Greece and Germany, which would also help the whole economy of the euro area.

2.4 Our coopetitive model

The two strategic variables of our model are investments and exports for Greece, since this country must concentrate on them to improve the structure of production and its competitiveness, but also shift its aggregate demand towards a higher growth path in the medium term. Thus Greece should focus on innovative investments, specially investments in knowledge [Schilirò, 2010], to change and improve its production structure and to increase its production capacity and its productivity. As a result of that its competitiveness will improve. An economic policy that focuses on investments and exports, instead of consumptions, will address Greece towards a sustainable growth and, consequently, its financial reputation and economic stability will also increase. On the other hand, the strategic variable of our model for Germany private consumption and imports.

The idea which is driving our model to solve the Greek crisis is based on a notion of coopetition where the cooperative aspect will prevail. Thus we are not talking about a situation in which Germany and Greece are competing in the same European market for the same products, rather we are assuming a situation in which Germany stimulates its domestic demand and, in doing so, will create a larger market for products from abroad. We are also envisaging the case where Germany purchases a greater quantity of Greek products, in this case Greece increases its exports, selling more products to Germany. The final results will be that Greece will find itself in a better position, but also Germany will get an economic advantage determined by the higher growth in the two countries. In addition, there is the important advantage of a greater stability within the European Monetary system. Finally our model will provide a new set of tools based on the notion of coopetition, that could be fruitful for the setting of the euro area economic policy issues.

2.5 The coopetition in our model

The concept of coopetition was essentially devised at micro-economic level for strategic management solutions by Branderburger and Nalebuff [1995], who suggest, given the competitive paradigm [Porter, 1985], to consider also a cooperative behavior to achieve a win-win outcome for both players. Therefore, in our model, coopetition represents the synthesis between the competitive paradigm [Porter, 1985] and the cooperative paradigm [Gulati, Nohria, Zaheer, 2000; Stiles, 2001]. Coopetition is, in our approach, a complex theoretical construct and it is the result of the interplay between competition and cooperation. Thus, we suggest a model of coopetitive games, applied at a macroeconomic level, which intends to offer possible solutions to the partially divergent interests of Germany and Greece in a perspective of a cooperative attitude that should drive their policies.

3 Coopetitive games

3.1 Introduction

In this paper we develop and apply the mathematical model of *coopetitive game* introduced by D. Carfì in [] and []. The idea of coopetitive game is already used, in a mostly intuitive and non-formalized way, in Strategic Management Studies (see for example [] and []).

The idea. A coopetitive game is a game in which two or more players can interact *cooperatively and non-cooperatively at the same time.* But even Brandenburgher and Nalebuff, creators of coopetition, did not defined precisely a *quantitative way to implement coopetition* in Game theory context.

The problem in Game Theory to implement the notion of coopetition is:

- *how do, in normal form games, cooperative and non-cooperative interactions can live together simultaneously, in a Brandeburgher-Nalebuff sense?*

Indeed, consider a classic two-player normal-form gain game $G = (f, >)$ - such a game is a pair in which f is a vector valued function defined on a Cartesian product $E \times F$ with values in the Euclidean plane $\mathbb{R}^2$ and $>$ is the natural sup-order of the Euclidean plane. Let E and F be the strategy sets of the two players in the game G. The two players can choose the respective strategies $x \in E$ and $y \in F$ cooperatively (exchanging information) or not-cooperatively (not exchanging informations), but these two ways are mutually exclusive in normal-form games: the two ways can't happen simultaneously (without using probability, but this is not the way suggested by Brandenburgher and Nalebuff) in the model of normal-form game. There is no room, in the classic game model, for a simultaneous (non-probabilistic) employment of the two extremes *cooperation* and *non-cooperation*.

Towards a possible solution. David Carfì (in [2010]...) has proposed a manner to pass this *impasse*, according to the idea of coopetition:

- in a Carfì's coopetitive game, the players of the game have their respective strategy-sets (in which they can choose cooperatively or not cooperatively) and a common strategy set C containing other strategies (possibly of different type with respect to the previous one) that *must be chosen cooperatively.* This strategy set C can also be structured as a Cartesian product, but in any case the strategies belonging to this new set C must be chosen cooperatively.

Remark. A particular aspect of the question of Carfì's coopetitive games is that: when we consider a coopetitive game, we necessarily build up a family of classic normal-form games; so that such coopetitive games can be defined as a family of normal-form games.

3.2 The model for n players

We give in the following the definition of coopetitive game proposed by Carfì (in []).

Definition (of n-player coopetitive game). *Let E be a finite n-family of non-empty sets and let C be another non-empty set. We define n-**player coopetitive gain game over the strategy support** (E, C) any pair $G = (f, >)$, where f is a vector function from the Cartesian product ${}^{\times}E \times C$ (here ${}^{\times}E$ is the classic strategy-profile space of n-player normal form games, i.e. the Cartesian product of the family E) into the n-dimensional Euclidean* *space $\mathbb{R}^n$ and $>$ is the natural sup-order of this last Euclidean space. The element of the set C will be called* ***cooperative strategies of the game****.*

Definition (the family of normal-form games associated with a coopetitive game). *Let $G = (f, >)$ be a coopetitive game over a support (E, C). And let g be the family of classic normal-form games $(G_z)_{z\in C}$ whose member g_z is defined, for any cooperative strategy z in C, by the normal-form game*

$$G_z := (f(., z), >),$$

where the payoff function $f(., z)$ is the section

$$f(., z) : {}^{\times}E \to \mathbb{R}^n$$

defined (as usual) by

$$f(., z)(x) = f(x, z),$$

for every point x in the strategy profile space ${}^{\times}E$. We call the family g (so defined) ***family of normal-form games associated with the game*** *G.*

We can prove this (obvious) theorem.

Theorem. *The family g of normal-form games associated with a coopetitive game G uniquely determines the game. In more rigorous and complete terms, the correspondence $G \mapsto g$ is a bijection of the space of all coopetitive games - over the strategy support (E, C) - onto the space of all families of normal form games - over the strategy support E - indexed by the set C.*

Proof. This depends totally from the fact that we have the following natural bijection between function spaces:

$$\mathcal{F}({}^{\times}E \times C, \mathbb{R}^n) \to \mathcal{F}(C, \mathcal{F}({}^{\times}E, \mathbb{R}^n)) : f \mapsto (f(., z))_{z\in C},$$

which is a classic result of theory of sets. ∎

Thus, the exam of a coopetitive game should be equivalent to the exam of a whole family of normal-form games (in some sense we shall specify).

In this paper we suggest how this latter examination can be conducted and what are the solutions corresponding to the main concepts of solution which are known in literature for the classic normal-form games, in the case of two-player coopetitive games.

3.3 Two players coopetitive games

In this section we specify the definition and related concepts of two-player coopetitive games, sometimes we repeat some definitions of the preceding section.

Definition (of coopetitive game). *Let E, F and C be three nonempty sets. We define **two player coopetitive gain game carried by the strategic triple** (E, F, C) any pair of the form $G = (f, >)$, where f is a function from the Cartesian product $E \times F \times C$ into the real Euclidean plane $\mathbb{R}^2$ and the binary relation $>$ is the usual sup-order of the Cartesian plane, defined component-wise, for every couple of points p and q, by $p > q$ iff $p_i > q_i$, for each index i.*

Remark. The difference among a two-player normal-form gain game and a two player coopetitive game is the fundamental presence of the third strategy Cartesian-factor C. The presence of this third set C determines a total change of perspective with respect to the usual two-player normal form games, since we now have a normal form game $G(z)$, for every element z of the set C; we have, then, to study an entire ordered family of normal form games in its own totality, and we have to define a new manner to study these kind of families.

3.4 Terminology and notation

Definitions. *Let $G = (f, >)$ be a two person coopetitive gain game carried by the strategic triple (E, F, C). We will use the following terminologies:*

- *the function f is called the **payoff function of the game** G;*
- *the first component f_1 of the payoff function f is called **payoff function of the first player** and analogously the second component f_2 is called **payoff function of the second player**;*
- *the set E is said **strategy set of the first player** and the set F the **strategy set of the second player**;*
- *the set C is said the **cooperative strategy set of the two players**;*

- *the Cartesian product $E \times F \times C$ is called the* ***coopetitive strategy space of the game*** *G.*

Memento. The first component f_1 of the payoff function f of a coopetitive game G is the function of the strategy space of the game G into the real line defined by the first projection

$$f_1(x, y, z) = \mathrm{pr}_1(f(x, y, z)),$$

for every triple (x, y, z) in $E \times F \times C$, analogously we proceed for the second component f_2.

Interpretation. We have:

- two players;
- anyone of the two players has a strategy set in which to choose his own strategy;
- the two players can cooperatively choose strategies z in a third set C;
- the two players will choose (after the exam of the game) their cooperative strategy z in order to maximize (in some sense we shall define) the gain function f.

3.5 Normal form games of a coopetitive game

Let G be a coopetitive game in the sense of above definitions. For any cooperative strategy z selected in the cooperative strategy space C, there is a corresponding normal form game

$$G_z = (p(z), >),$$

upon the strategy pair (E, F), where the payoff function $p(z)$ is the section

$$f(., z) : E \times F \to \mathbb{R}^2,$$

of the payoff function f of the coopetitive game - the section is defined, as usual, on the competitive strategy space $E \times F$, by

$$f(., z)(x) = f(x, z),$$

for every bi-strategy x in the bi-strategy space $E \times F$.

Let us formalize the concept of game-family associated with a coopetitive game.

Definition (the family associated with a coopetitive game). *Let $G = (f, >)$ be a two player coopetitive gain game carried by the strategic triple (E, F, C). We naturally can associate with the game G a family $g = (G_z)_{z \in C}$ of normal-form games defined by*

$$G_z = (f(., z), >),$$

for every z in C, which we call ***the family of normal-form games associated with the coopetitive game*** *G.*

Remark. It is clear that with any above family of normal form games

$$g = (G_z)_{z \in C}$$

we can associate

- a family of payoff spaces

$$(\mathrm{im} f(., z))_{z \in C};$$

- a family of Pareto maximal boundary

$$(\partial^* G_z)_{z \in C};$$

- a family of suprema

$$(\sup G_z)_{z \in C};$$

- a family of Nash zone

$$(N(G_z))_{z \in C};$$

- a family of conservative bi-values

$$v^{\#} = (v_z^{\#})_{z \in C};$$

and so on, for every meaningful feature of normal form games.

And we can interpret any of the above families as set-valued paths in the strategy space $E \times F$ or in the payoff universe $\mathbb{R}^2$.

It is just the study of these induced families which becomes of great interest in the study of a coopetitive game G and which will enable us to define the various possible solutions of a coopetitive game.

4 Solutions of a coopetitive game

4.1 Introduction

The two players should choose the cooperative strategy z in C in order that:

- the Nash equilibria of G_z are "better" than the Nash equilibria in each other game $G_{z'}$;
- the supremum of G_z is greater than the supremum of any other game $G_{z'}$;
- the Pareto maximal boundary of G_z is "higher" than that of any other game $G_{z'}$;
- the Nash bargaining solution is better in G_z than that in $G_{z'}$;
- and so on, fixed a common kind of solution for any game G_z, say $S(z)$ the set of these kind of solutions, we can consider the problem to find the optimal solutions in set valued path S, defined on the cooperative strategy set C;

The payoff function of a two person coopetitive game is (as in the case of normal-form game) a vector valued function with values belonging to the Cartesian plane $\mathbb{R}^2$. We shall consider the maximal Pareto boundary of the payoff space $\mathrm{im}(f)$ as an appropriate zone for the bargaining solutions.

We note the fundamental circumstance that in general the above criteria are multi-criteria and so they generate multi-criteria optimization problems.

Let us define rigorously some kind of solution for two player coopetitive games based on a bargaining method. But first of all we have to precise what kind of bargaining method we are going to use.

4.2 Bargaining problems

In this paper we shall use the following definition of bargaining problem.

Definition (of bargaining problem). *Let S be a subset of the Cartesian plane $\mathbb{R}^2$ and let a and b be two points of the plane with the following properties:*

- *they belong to the small interval containing S;*
- *they are such that $a < b$;*
- *the intersection*
$$^{\leq}[a, b] \cap \partial^* S,$$
*among the interval $^{\leq}[a, b]$ with end points a and b (it is the set of points greater than a and less than b, **it is not** the segment $[a, b]$) and the maximal boundary of S is non-empty.*

*In this conditions, we call **bargaining problem on** S, **corresponding to the pair of extreme points** (a, b), the pair*

$$P = (S, (a, b)).$$

*Every point in the intersection among the interval $^{\leq}[a, b]$ and the Pareto maximal boundary of S is called **possible solution of the problem** P. Some time the first extreme point of a bargaining problem is called **initial***

point of the problem *(or* ***disagreement point*** *or* ***threat point****) and the second extreme point of a bargaining problem is called utopia point of the problem.*

In the above conditions, when S in convex, the problem P is said to be convex and for this case we can find in the literature many existence results for solutions of P enjoying prescribed properties (Kalai-Smorodinsky solutions, Nash bargaining solutions and so on ...).

Remark. Let S be a subset of the Cartesian plane $\mathbb{R}^2$ and let a and b two points of the plane belonging to the smallest interval containing S and such that $a \leq b$. Assume the Pareto maximal boundary of S be non-empty. If a and b are a lower bound and an upper bound of the maximal Pareto boundary, respectively, then the intersection

$$^{\leq}[a, b] \cap \partial^* S$$

is obviously not empty. In particular, if a and b are the extrema of S (or the extrema of the Pareto boundary $\partial^* S$) we can consider the following bargaining problem

$$P = (S, (a, b)),$$

and we call this particular problem a standard bargaining problem (or standard bargaining problem with respect to the Pareto maximal boundary).

4.3 Kalai solution for bargaining problems

Note the following property.

Property. *If $(S, (a, b))$ is a bargaining problem with $a < b$, then there is at most one point in the intersection*

$$[a, b] \cap \partial^* S,$$

where $[a, b]$ is the segment joining the two points a and b.

Proof. Since if a point p of the segment $[a, b]$ belongs to the Pareto boundary $\partial^* S$, no other point of the segment itself can belong to Pareto boundary, since the segment is a totally ordered subset of the plane (remember that $a < b$). ■

Definition (Kalai-Smorodinsky). *We call* ***Kalai-Smorodinsky solution*** *(or* ***best compromise solution****)* ***of the bargaining problem*** $(S, (a, b))$ *the unique point of the intersection*

$$[a, b] \cap \partial^* S,$$

if this intersection is non empty.

So, in the above conditions, the Kalai-Smorodinsky solution k (if it exists) enjoys the following property: there is a real r in $[0, 1]$ such that

$$k = a + r(b - a),$$

or

$$k - a = r(b - a),$$

hence

$$\frac{k_2 - a_2}{k_1 - a_1} = \frac{b_2 - a_2}{b_1 - a_1},$$

if the above ratios are defined; these last equality is the *characteristic property of Kalai solutions.*

Definition (of Pareto boundary). *We call* ***Pareto boundary*** *every subset M of an ordered space which has only pairwise incomparable elements.*

4.4 Nash solution of a coopetitive game

Let $N(G)$ be the union of the Nash zone families of the coopetitive game G, that is the union of the family $(N(G_z))_{z \in C}$ of Nash zones of the family $g = (g_z)_{z \in C}$ associated to the coopetitive game G. We consider the bargaining problem

$$P_N = (N(G), \inf(N(G)), \sup(N(G))).$$

Definition. *If the above problem P_N has a Kalai Smorodinsky solution k, we say that k is* ***the properly coopetitive solution of the coopetitive game*** *G.*

The term properly coopetitive is clear:

- *this solution is determined by cooperation on the common strategy set C and to be selfish (competitive in the Nash sense) on the bi-strategy space $E \times F$.*

4.5 Bargaining solutions of a coopetitive game

Obviously, it is possible for coopetitive games, to define other kind of solutions, which are not properly coopetitive, but realistic and sometime affordable. These kind of solutions are, we can say, *doubly cooperative.*

Let us show some of these kind of solutions.

Consider a coopetitive game G and

- its Pareto maximal boundary M and its extrema (a_M, b_M);
- the Nash zone $N(G)$ of the game in the payoff space and its extrema (a_N, b_N);
- the conservative set-value $G^\#$ (the set of all conservative values of the family associated with G) and its extrema $(a^\#, b^\#)$.

We call:

- ***Pareto compromise solution of the game*** *G the best compromise of the problem $(M, (a_M, b_M))$, if this solution exists;*
- ***Nash-Pareto compromise solution of the game*** *G the best compromise of the problem $(M, (b_N, b_M))$, if this solution exists;*
- ***conservative-Pareto compromise solution of the game*** *G the best compromise of the problem $(M, (b^\#, b_M))$, if this solution exists.*

4.6 Transferable utility solutions

Other possible compromises we suggest are the following one:

Let consider the Pareto boundary of transferable utilities M of the game G, that is the set of all non-negative points p in the Euclidean plane (universe of payoffs) such that their 1-norm $p_1 + p_2$ is equal to the maximum value of the 1-norm over the payoff space of the game G. This Pareto boundary M is a compact segment if the game has an upper bounded (with respect to the usual order of the plane) payoff space.

Definition. *We call* ***transferable utility compromise solution of the coopetitive game*** G *the solution of any bargaining problem* $(M, (a, b))$*, where* a *is a point in the payoff space of* G *and* b *is a point strongly greater than* a *and smaller or equal to the supremum of* M*.*

Remark. In the applications, if the game G has a member G_0 of its family which can be considered as an "initial game" - in the sense that the pre-coopetitive situation is represented by this normal form game G_0 - the aim of our study (following the standard ideas on coopetitive interactions) is to enlarge the pie and to obtain a win-win solution with respect to the initial situation; so that we will choose as threat point a in our problem $(M, (a, b))$ the supremum of the initial game G_0. A good choice for the utopia point b is the supremum of the portion of M which is upon (greater than) this point a. Another **rebalancing solution** can be to choose b as the supremum of the portion of M that is bounded between the minimum and maximum value of that player i that gains more in the coopetitive interaction, in the sense that

$$\max(\mathrm{pr}_i(\mathrm{im}f)) - \max(\mathrm{pr}_i(\mathrm{im}f_0)) > \max(\mathrm{pr}_{3-i}(\mathrm{im}f)) - \max(\mathrm{pr}_{3-i}(\mathrm{im}f_0)).$$

5 Coopetitive games for Greek crisis

Our first hypothesis is that Germany must stimulate the domestic demand and to re-balance its trade surplus in favor of Greece. The second hypothesis is that Greece, a country with a declining competitiveness of its products and a small export share, aims at growth by undertaking innovative investments and by increasing its exports primarily towards Germany and also towards the other euro countries.

The coopetitive model that we propose hereunder must be interpreted as a normative model, in the sense that it shows the more appropriate solutions of a win-win strategy chosen by considering both competitive and cooperative behaviors.

The strategy spaces of the two models are:

- the strategy set of Germany E, set of all possible consumptions of Germany (in our model); we shall assume that the strategies of Germany directly influence only Germany pay-off;
- the strategy set of Greece F, set of all possible investments of Greece (in our model); we shall assume that the strategies of Greece directly influence only Greece pay-off;
- a shared strategy set C, whose elements are determined together by the two countries, when they choose their own respective strategies x and y, Germany and Greece. Every z in C is a given amount of Greek exports imported by Germany.

Therefore, in the two models we assume that Germany and Greece define the set of coopetitive strategies.

6 The mathematical model

Main Strategic assumptions. We assume that:

- any real number x, belonging to the unit interval $U = [0, 1]$, can represent a consumption of Germany;
- any real number y, in the same unit interval U, can represent an investment of Greece;
- any real number z, again in U, can be the amount of Greek exports which is imported by Germany.

6.1 Payoff function of Germany

We assume that the payoff function of Germany f_1 is its *gross domestic demand*:

- f_1 is the private consumption function c_1 plus the gross investment function i_1 plus government spending (that we shall assume equal 0) plus export function E_1 minus import function I_1, that is

$$f_1 = c_1 + i_1 + E_1 - I_1.$$

  We assume that:

- the private consumption function c_1 is the first projection of the product U^3,

$$c_1(x, y, z) = x,$$

  since we assume the private consumption of Germany the first strategic component of strategy profiles in U^3;

- we assume the gross investment function i_1 constant on the cube U^3 and by translation we can suppose i_1 equal zero;

- the export function E_1 is defined by

$$E_1(x, y, z) = (1 + x)^{-1},$$

  for every consumption x of Germany; so we assume that the export function E_1 is a strictly decreasing function with respect to the first argument;

- the import function I_1 is the third projection of the product,

$$I_1(x, y, z) = z,$$

  since we assume the import function depending only upon the cooperative strategy z of the coopetitive game G, our third strategic component of the strategy profiles in U^3.

Recap. We then assume as payoff function of Germany its gross domestic demand f_1, which in our model is defined, for every triple (x, y, z) in the cube U^3, as the sum of the strategies x with $-z$ and also with the export function E_1, viewed as a reaction function with respect to its domestic consumption (so that f_1 is the difference of the first and third projection of the Cartesian product U^3 plus the function E_1).

Concluding, the payoff function of Germany is the function f_1 of the cube U^3 into the real line $\mathbb{R}$, defined by

$$f_1(x, y, z) = x + 1/(x + 1) - z,$$

for every triple (x, y, z) in the cube U^3; where the reaction function E_1, defined from the unit interval U into the real line $\mathbb{R}$ by

$$E_1(x) = 1/(x + 1),$$

for every consumption x of Germany in the interval U, is the export function of Germany mapping the level x of consumption into the level $E_1(x)$ of export.

The function E_1 is a strictly decreasing function, and only this monotonicity is the relevant property of E_1 for our model.

6.2 Payoff function of Greece

We assume that the payoff function of Greece f_2 is again its gross domestic demand (private consumption plus gross investment plus government spending plus exports minus imports),

$$f_2 = c_2 + i_2 + E_2 - I_2.$$

We assume that:

- the function c_2 is irrelevant in our analysis, since we assume the private consumption independent from the choice of the triple (x, y, z) in U^3, in other terms we assume the function c_2 constant on the cube U^3 and by translation we can suppose c_2 equal zero;

- the function i_2 is defined by

$$i_2(x, y, z) = y + nz,$$

for every (x, y, z) in U^3 (see later for the justification);

- the function E_2 is defined by

$$E_2(x, y, z) = z + my,$$

for every (x, y, z) in U^3 (see later for the justification);

- the function I_2 is irrelevant in our analysis, since we assume the import independent from the choice of the triple (x, y, z) in U^3, in other terms we assume the function I_2 constant on the cube U^3 and by translation we can suppose I_2 equal zero.

So the payoff function of Greece is the linear function f_2 of the cube U^3 into the real line $\mathbb{R}$, defined by

$$f_2(x, y, z) = (y + z) + my + nz = (1 + m)y + (1 + n)z,$$

for every pair (x, y, z) in the Cartesian cube U^3.

We note that the function f_2 does not depend upon the strategies x in U chosen by Germany and that f_2 is a linear function.

The definition of the functions i_2 and E_2 must be studied deeply and carefully, and are fundamental to find the win-win solution.

- For every y in U, the term my represents the quantity effect of the Greek investment y on the Greek exports. In fact, the investments, specially innovative investments, contribute at improving the competitiveness of Greek goods, favoring the exports.

- For every z in U, the term nz is the cross-effect of the coopetitive variable z representing the additive level of investment required to support the production of z itself. We assume the factors m and n strictly positive.

6.3 Payoff function of the game

We so have build up a coopetitive gain game with payoff function given by

$$\begin{aligned} f(x,y,z) &= (x+1/(x+1)-z,(1+m)y+z) = \\ &= (x+1/(x+1),(1+m)y)+z(-1,1+n) \end{aligned}$$

for every x, y, z in $[0,1]$.

6.4 Study of the game $G = (f, >)$

Note that, fixed a cooperative strategy z in U, the section game $G(z) = (p(z), >)$ with payoff function $p(z)$, defined on the square $U \times U$ by

$$p(z)(x,y) = f(x,y,z),$$

is the translation of the game $G(0)$ by the "cooperative" vector

$$v(z) = z(-1, 1+n),$$

so that we can study the initial game $G(0)$ and then we can translate the various informations of the game $G(0)$ by the vector $v(z)$.

So, let us consider the initial game $G(0)$. The strategy square $S = U^2$ of $G(0)$ has vertices 0_2, e_1, 1_2 and e_2, where 0_2 is the origin, e_1 is the first canonical vector $(1,0)$, 1_2 is the sum of the two canonical vectors $(1,1)$ and e_2 is the second canonical vector $(0,1)$.

6.5 Topological Boundary of the payoff space of G_0

In order to determine the Pareto boundary of the payoff space, we shall use the technics introduced by carfi in []. We have

$$p_0(x,y) = (x+1/(x+1),(1+m)y),$$

for every x, y in $[0, 1]$. The transformation of the side $[0, e_1]$ is the trace of the (parametric) curve $c : U \to \mathbb{R}^2$ defined by

$$c(x) = f(x, 0, 0) = (x + 1/(x + 1), 0),$$

that is the segment

$$[f(0), f(e_1)] = [(1, 0), (3/2, 0)].$$

The transformation of the segment $[0, e_2]$ is the trace of the curve $c : U \to \mathbb{R}^2$ defined by

$$c(y) = f(0, y, 0) = (1, (1 + m)y),$$

that is the segment

$$[f(0), f(e_2)] = [(1, 0), (1, 1 + m)].$$

The transformation of the segment $[e_1, 1_2]$ is the trace of the curve $c : U \to \mathbb{R}^2$ defined by

$$c(y) = f(1, y, 0) = (1 + 1/2, (1 + m)y),$$

that is the segment

$$[f(e_1), f(1_2)] = [(3/2, 0), (3/2, 1 + m)].$$

Critical zone of $G(0)$**.** The Critical zone of the game $G(0)$ is empty. Indeed the jacobian matrix is

$$J_f(x, y) = \begin{pmatrix} 1 + (1 + x)^{-2} & 0 \\ 0 & 1 + m \end{pmatrix},$$

which is invertible for every x, y in U.

Payoff space of the game $G(0)$**.** So, the payoff space of the game $G(0)$ is the transformation of the topological boundary of the strategic square, that is the rectangle with vertices $f(0)$, $f(e_1)$, $f(1, 1)$ and $f(e_2)$.

Nash equilibria. The unique Nash equilibrium is the bistrategy $(1, 1)$. Indeed,

$$1 + (1 + x)^{-2} > 0$$

so the function f_1 is increasing with respect to the first argument and analogously

$$1 + m > 0$$

so that the Nash equilibrium is $(1, 1)$.

6.6 The payoff space of the coopetitive game G

The image of the payoff function f, is the union of the family of payoff spaces

$$(\mathrm{im} p_z)_{z \in C},$$

that is the convex envelope of the of the image $p_0(S)$ and of its translation by the vector $v(1)$.

6.7 The Pareto maximal boundary of the payoff space of G

The Pareto sup-boundary of $f(S)$ is the segment $[P', Q']$, where $P' = f(1, 1)$ and

$$Q' = P' + v(1).$$

Possibility of global growth. It is important to note that the absolute slope of the Pareto (coopetitive) boundary is $1+n$. Thus the collective payoff $f_1 + f_2$ of the game is not constant on the Pareto boundary and, therefore, the game implies the possibility of a global growth.

Trivial bargaining solutions. The Nash bargaining solution on the segment $[P', Q']$ with respect to the infimum of the Pareto boundary and the Kalai-Smorodinsky bargaining solution on the segment $[P', Q']$, with respect to the infimum and the supremum of the Pareto boundary, coincide with the medium point of the segment $[P', Q']$.

6.8 Transferable utility solution

In this coopetitive context it is more convenient to adopt a transferable utility solution: indeed:

- the point of maximum collective gain is the point

$$Q' = (1/2, 2 + m + n).$$

6.9 Rebalancing best compromise solution

Thus we propose a rebalancing kind of coopetitive solution, as it follows (in the case $m = 0$):

1. we consider the portion s of transferable utility Pareto boundary

$$M = (0, 5/2 + n) + \mathbb{R}(1, -1),$$

   obtained by intersecting (M itself) with the strip determined by the straight lines $e_2 + \mathbb{R}e_1$ and

$$(2 + n)e_2 + \mathbb{R}e_1,$$

   these are the straight lines of maximum gain for Greece in games $G(0)$ and G respectively.

2. we consider the Kalai-Smorodinsky segment s' of vertices $(3/2, 1)$ - supremum of the game $G(0)$ - and the supremum of the segment s.

3. our best payoff coopetitive compromise is the unique point K in the intersection of s and s', that is the best compromise solution of the bargaining problem

$$(s, (\sup G_0, \sup s)).$$

6.10 Win-win solution

This best payoff coopetitive compromise K represents a win-win solution with respect to the initial supremum $(3/2, 1)$. So that, as we repeatedly said, also Germany increases its initial profit from coopetition.

Win-win strategy procedure. The win-win payoff K can be obtained in a properly coopetitive fashion in the following way:

1) the two players agree on the cooperative strategy 1;

2) they implement their respective Nash strategies of determined (preceeding point) game $G(1)$; the unique Nash equilibrium of $G(1)$ is the bistrategy $(1, 1)$;

3) they share the "social pie"

$$(f_1 + f_2)(1, 1, 1)$$

cooperatively according to the decomposition K.

7 Conclusions

- The model of coopetitive game, provided in the present contribution, is essentially a normative model. It has showed some feasible solutions in a cooperative perspective to the Greek crisis. Our model of coopetition has pointed out the strategies that could bring to solutions in a cooperative perspective for Greece and Germany.

We have found:

- a Nash bargaining solution;
- which coincides with the Kalai-Smorodinsky bargaining solution on the coopetitive Nash path.
- Finally, a remarkable analytical result of our work consists in the determination of the win-win solution by a new selection method on the transferable utility Pareto boundary of the coopetitive game.

The solution offered by our analytical framework aims at "sharing the pie fairly", showing a win-win outcome for both countries, within a growth path represented by a non-zero (or better, non-costant) sum game.

Our analytical results allow us to find a "fair" amount of Greek exports which Germany must import, in order to re-balance the trade surplus of Germany, as well as the investments necessary to improve the Greek economy, thus contributing to growth and to the stability of the Greek economy.

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