Bundling revisited: substitute products and inter-firm discounts

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Abstract

This paper extends the standard model of bundling to allow products to be substitutes and for products to be supplied by separate sellers. Whether integrated or separate, firms have an incentive to introduce bundling discounts when demand for the bundle is elastic relative to demand for stand-alone products. When products are partial substitutes, this typically gives an integrated firm a greater incentive to offer a bundle discount (relative to the standard model with additive preferences), while product substitutability is often the sole reason why separate sellers wish to offer inter-firm discounts. When separate sellers negotiate their inter-firm discount, they can use the discount to relax subsequent competition.

1 Introduction

Bundling—the practice whereby consumers are offered a discount if they buy several distinct products—is used widely by firms, and is the focus of a rich economic literature. However, most of the existing literature discusses the phenomenon under relatively restrictive assumptions, namely: (i) a consumer's valuation for a bundle of several products is the sum of her valuations for consuming the items in isolation, and (ii) bundle discounts are only offered for products sold by the same firm. The two assumptions are related, in that when valuations are additive it is less often the case that a firm would wish to reduce its price to a customer who also buys a product from another seller. This paper analyzes the incentive to engage in bundling, and the consequent impact on prices and profits, when one or both of these assumptions is relaxed.

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There are very many situations in which modelling products as substitutes is relevant. For instance, when visiting a city a tourist may gain some extra utility from visiting art gallery A if she has already visited art gallery B, but the incremental utility is likely to be smaller than if she were only to visit A. Joint purchase discounts (or premia) on products offered by separate sellers are rarer, though several examples can be found, including:

- A tourist may be able to buy a "city pass", so that she can visit all participating tourist attractions at a discount on the sum of individual entry fees. Most major tourist cities have such schemes, either organized as a joint venture by the attractions themselves, or implemented by one or more third parties who put together their own bundles given the wholesale fees they negotiate with attractions.[1]

- Bundling is prevalent in markets for transport services. Sometimes customers can obtain inter-firm bundling discounts, as is the case with alliances between airlines, when different firms coordinate to offer a "travel pass" in a city, or when neighboring ski-lifts agree to offer a combined ticket.[2]

- Online music stores retail music by many different publishers to final consumers, often using bundling discounts.[3] Separately-owned television channels may be retailed separately as well as being offered as a bundle to viewers. Separately owned academic journals are marketed individually, and as part of a collection, to libraries.[4]

- Pharmaceuticals are sometimes used in isolation and sometimes as part of a "cocktail" with one or more drugs supplied by other firms. Drugs companies can set different prices depending on whether the drug is used on a stand-alone basis or in a cocktail. (One way to do this is for a firm to use a different name for the same chemical in

[1] An interesting example at the time of writing is the website *www.smartdestinations.com* (visited on June 30, 2011), which allows tourists to put together their own bundle of attractions for a number of American cities. Each bundle is sold at discount on the sum of individual entry fees, with the proportional discount typically increasing with the number of attractions chosen.

[2] In fact, in a famous antitrust case concerning ski-lifts in the Aspen resort, one ski-lift operator successfully sued another for not permitting it to participate in an inter-firm bundling scheme. See Easterbrook (1986) for further details.

[3] For some background to this market, see Shiller and Waldfogel (2009). While many distributors do use bundling schemes, including flat-rate schemes with unlimited downloads, for many years the market leader, Apple's iTunes music store, retailed every song at $0.99, regardless of the song or how many other songs were purchased.

[4] For background to this market, as well as a discussion of the merits and problems involved in bundling schemes involving separately-owned journals, see Armstrong (2010).

two different uses, and to obtain regulatory approval for one name to be used in the cocktail and the other name to be used for stand-alone treatment.)

- Products supplied by separately-owned firms are often marketed together, with discounts for joint purchase. Thus, supermarkets and gasoline stations may cooperate to offer a discount when both services are consumed. Airlines and car rental firms may link up for marketing purposes, and sometimes credit cards offer discounts proportional to spend towards designated flights or hotels. Currently, *Amazon.co.uk* offers its customers discounts (in the form of vouchers enclosed when its books are delivered) for seemingly unrelated products (such as wine) offered by independent sellers.

- Marketing data obtained by a firm may reveal useful information about a potential customer's purchase history which affects the firm's price offer to the customer. For instance, information that the customer has chosen to buy another firm's product 1 may induce the supplier of product 2 to discount its price, and an inter-firm discount for the joint purchase of the two products is implemented. In this situation the bundle discount is not announced *ex ante* and, depending on the sophistication of consumers, not anticipated either.[5]

- At a wholesale level, one manufacturer may offer a retailer a discount on its product if the retailer does *not* also purchase a rival manufacturer's product. (Such contracts are termed "loyalty contracts" or "market share discounts".) This is a situation in which there is joint purchase premium instead of a discount.

Although several of these examples involve a degree of coordination between sellers to achieve bundle discounts, in this paper (until section 6) I consider the extent to which firms can implement an inter-firm discount without recourse to coordination. That is, a firm unilaterally chooses two prices: a price for buying its product on its own, and a (lower) price if the consumer also buys the other firm's product. As I discuss later in the

[5] Taylor (2004) analyzes a model where two sellers interact sequentially with consumers, and where the second firm can base its price on whether the consumer previously purchased the other product. In his framework, valuations for the two products are additive but positively correlated. This implies that the second firm sets a higher price to those customers who also bought the other firm's product, so there is a joint-purchase premium. However, his model could easily be adapted to allow for negative correlation in valuations or a degree of substitution between the two products, which could induce the second firm to offer a discount to those customers who also buy the other product.

paper, historically it has often been hard for two separate sellers to implement an inter-firm discount without coordination, but technological advances in selling procedures mean that it is now easier for firms to introduce a discount non-cooperatively.

In more detail, the plan of the paper is as follows. In section 2 I briefly recapitulate the approach to bundling presented in Long (1984), which is used as a major ingredient for my analysis in the rest of the paper. In section 3, I present a fairly general analysis of the incentive to introduce bundling discounts, both with integrated and separate supply. In broad terms, there is a motive to offer a bundle discount when consumer demand for the bundle is elastic compared to demand for stand-alone items.

In section 4, I consider in more detail the case where an integrated firm supplies both products, and specialise to the case where products are symmetric. An integrated firm has an incentive to bundle whenever the proportion of those consumers who buy a product at price p and who go on to buy the other product at the same price decreases with p. Relative to the situation with additive preferences, the integrated firm typically has a greater incentive to offer a bundle discount when products are substitutable. Because the purchase of one product can decrease a consumer's incremental utility from the second item, the firm has a direct incentive to reduce the price for a second item, in addition to the rent-extraction motive for bundling familiar from the existing literature. In examples we see that the size of the discount can be above or below the corresponding discount with additive preferences.

In section 5 I turn to the situation where products are supplied by separate sellers. When valuations are additive, a firm has a unilateral incentive to offer a bundle discount when valuations for products are negatively correlated. When products are substitutes, whether a firm has a unilateral incentive to introduce a discount depends on the way that preferences are modelled. When there is a constant disutility of joint consumption, separate sellers typically wish to offer a joint-purchase discount: the fact that a customer has purchased the rival product implies that her incremental valuation for the firm's own item has fallen, and this usually implies that the firm would like to reduce its price to this customer. Alternatively, if a proportion of buyers only want a single item (for instance, a tourist in a city might only have time to visit a single museum) while other consumers have additive preferences, separate sellers would like, if feasible, to charge a *premium* when a customer also buys the rival product. In the examples solved, when this form of price discrimination is permitted, one price increases and the other decreases relative to the situation with uniform pricing, and price discrimination results in higher equilibrium profit.

Finally, in section 6 I investigate partial coordination between separate sellers. (Earlier parts of the paper consider the two polar cases where separate sellers did not coordinate their tariffs at all and—in the integrated firm analysis—where the two suppliers fully coordinated their tariffs.) Specifically, I suppose that symmetric firms first agree on an inter-firm discount (which they fund equally), and subsequently choose prices without coordination. When valuations are additive, it is shown that such a scheme will usually raise each firm's profit, and, at least when valuations are independent, its operation will also boost total welfare. However, when sellers offer substitute products, the negotiated bundle discount acts to reduce the effective substitutability between products, inducing firms to raise prices. Thus, the scheme can induce collusion and harm consumers.

This paper is not the first to investigate these issues. The incentive for an integrated seller to offer a discount for the purchase of multiple items is discussed by Stigler (1963), Adams and Yellen (1976), Long (1984) and McAfee, McMillan, and Whinston (1989), among many others. The latter two papers showed that it is optimal to introduce a bundle discount when the distribution of valuations is statistically independent and valuations are additive, suggesting that a degree of joint pricing is optimal even for entirely unrelated products. Except for Long, these papers assume that valuations are additive.[6]

Schmalensee (1982) and Lewbel (1985) study the incentive for a single-product monopolist unilaterally to offer a discount if its customers also purchased a competitively-supplied product. Since the two products can be independent or substitutes in their analysis, their argument is distinct from the idea that tying a monopoly product with a competitively-supplied *complementary* product can be used as a metering device. Consider Schmalensee's argument in more detail. There are two items for sale to a population of consumers, and item A is available at marginal cost due to competitive pressure, while item B is supplied

[6] Venkatesh and Kamakura (2003) analyze an integrated firm's incentive to engage in bundling when products are either complements or substitutes. The analysis is carried out using a specific uniform example, and a consumer's valuation for the bundle is some constant proportion (greater or less than one, depending on whether complements or substitutes are present) of the sum of her stand-alone valuations. The focus of their analysis is mostly on whether pure bundling is superior to linear pricing. Dansby and Conrad (1984) analyze a variant of the usual bundling model in which the act of bundling the two items can affect utility from the bundle (either upwards or downwards relative to the sum of stand-alone valuations). In their model, a consumer can buy both items separately from the seller (in which case their valuation, and the price they pay, is additive), or they can buy the bundle at the stipulated bundle price.

by a monopolist. Valuations are additive, but are not independent in the statistical sense, and the fact that a consumer is willing to buy item A is useful information for the monopolist. If there is negative correlation in the values for the two items, the fact that a consumer buys item A is "bad news" for the monopolist, who then has an incentive to set a lower price to its customers who also buy A. Lewbel performs a similar exercise but allows the two items to be partial substitutes. In this case, the fact that a consumer buys item A is also bad news for the monopolist, and provides a reason to offer a discount for joint consumption.

Bundling arrangements between separate firms are analyzed by Gans and King (2006), who investigate a model with two kinds of products (gasoline and food, say), and each product is supplied by two differentiated firms. When all four products are supplied by separate firms which set their prices independently, there is no interaction between the two kinds of product. However, two firms (one offering each of the two kinds of product) can enter into an alliance and agree to offer consumers a discount if they buy both products from the alliance. (In their model, the joint pricing mechanism is similar to that used in section 6 below: the firms decide on their bundle discount, which they agree to fund equally, and then they set prices non-cooperatively.) Gans and King observe that when a bundle discount is offered for joint purchase of otherwise independent products, those products are then converted into "complements". In their model, in which consumer tastes are uniformly distributed, a pair of firms does have an incentive to enter into such an alliance, but when both pairs of firms do this, their equilibrium profits are unchanged from the situation when all four firms set independent prices, although welfare and consumer surplus fall.[7]

A number of papers have investigated whether or not "code sharing"—i.e., coordinated pricing by separately-owned airlines for multi-flight itineraries—is an efficient practice. Multi-flight itineraries are products made up of complementary components, and so the inefficiency of uncoordinated pricing by separate airlines is due to double marginalization. An early theoretical contribution to this literature is Brueckner (2001), who provides a model in which two airlines need to cooperate to prevent double-marginalisation on some city-pair routes, but compete on other routes. In his model, if the two firms are permitted to coordinate prices on all routes, the benefits of price reductions on the non-competitive

[7]Brito and Vasconcelos (2010) modify this model so that rival suppliers of the same products are vertically rather than horizontally differentiated. They find that when two pairs of firms form an alliance all prices rise relative to the situation when all four products are marketed independently. This result resembles the analysis in section 6 below, where an agreed bundle discount induces collusion in the market.

routes tend to outweigh the harm done by allowing collusion on the competitive routes.

The current paper investigates when a seller wishes unilaterally to makes its price contingent on whether a customer also purchases from another seller. Focussing on the wholesale level, there is a growing literature on market share, or loyalty, contracts where one manufacturer conditions its wholesale price on whether a retailer also purchases a rival manufacturer's product. For example, Calzolari and Denicolo (2009) propose a model where consumers buy two products and each product is supplied by a single firm. Each firm potentially offers a nonlinear tariff which is a function of a consumer's demand for its own product and the consumer's demand for the other firm's product. They find that the use of these tariffs can harm consumers compared to the situation in which firms base their tariff only on their own supply. Their model differs in two ways from the one presented in this paper. First, in their model consumers have elastic (linear) demands, rather than unit demands, for the two products. Thus, they must consider general nonlinear tariffs, while the firms in my model merely choose a pair of prices which makes the analysis more tractable. Second, in my model consumers differ in richer way, and a consumer might like product 1 but not product 2, and can vary in the degree of substitutability between products. In Calzolari and Denicolo (2009), consumers differ by only a scalar parameter (the demand intercept for both products), and so all consumers view the two products when consumed alone as perfect substitutes.

Finally, Lucarelli, Nicholson, and Song (2010) discuss the case of pharmaceutical cocktails. Although the focus of their analysis is on situations in which firms set the same price for a drug, regardless of whether it is used in isolation or as part of a cocktail, they also consider situations where firms can set two different prices for the two kinds of uses. They document how a firm selling treatments for HIV/AIDS set different prices for similar chemicals depending on whether the drug was part of a cocktail or not. They estimate a demand system for colorectal cancer drugs, where there are at least 12 major drug treatments, 6 of which were cocktails which combine drugs from different firms. Although in this particular market firms do not price drugs differently depending whether the drug is used in a cocktail, they estimate the impact when one firm engages in this form of price discrimination. They find that a firm will typically (but not always) reduce the price for stand-alone use and raise the price for bundled use.

2 Long's Analysis of Bundling

In a clever note, Long (1984) presents what could be termed an "economic" model of bundling. Rather than following a diagrammatic exposition concentrating on the details of joint distributions of two-dimensional consumer valuations, he uses standard tools from demand theory to derive conditions under which offering a bundling discount is optimal. In this section, I recapitulate his analysis in its simplest, symmetric form. (Long also analyzes the situation where products are asymmetric.)

Suppose there are two symmetric products supplied by an integrated monopolist, labelled 1 and 2, each of which has constant marginal cost c. A consumer wishes to buy either zero or one unit of each product (and may wish to buy a unit of both products). Due to the assumed symmetry of demand and cost, suppose the firm sets the same price p for buying either product. Potentially, the firm offers a discount $\delta \geq 0$ if the consumer buys both products, so that the total price for buying both products is $2p - \delta$. Write the proportion of all potential consumers who buy a single item as X_s and the proportion who buy both items as X_b. The firm's profit is therefore $\pi = (p - c)X_s + (2p - \delta - 2c)X_b$, which can be re-written as

$$\pi = \delta N + (P - c)X \ , \tag{1}$$

where $N \equiv X_s + X_b$ is the proportion of consumers who buy something from the firm, $X \equiv X_s + 2X_b$ is the total number of units supplied, and $P = p - \delta$ is the incremental price of a product given the consumer buys the other product. Expression (1) shows how the bundling tariff can be viewed as a two-part tariff comprising a fixed charge δ and marginal price P. Viewing the two demands N and X as functions of (δ, P), given that there are no income effects standard demand theory indicates that cross-price effects are symmetric, so that $N_P \equiv X_\delta$ (where subscripts denote partial derivatives).

The question whether it is profitable to introduce a bundling discount is therefore equivalent to whether it is profitable to have a positive fixed charge in this two-part tariff. Let P^* be the monopolist's most profitable price when no bundle discount is offered, i.e., P^* maximizes $(P - c)X(0, P)$. Starting from this situation with linear pricing, consider the impact on profit of introducing a small discount $\delta > 0$, keeping the marginal price fixed at P^*. From (1), the impact on profit is

$$\left.\frac{\partial \pi}{\partial \delta}\right|_{\delta=0} = N + (P^* - c)X_\delta = N + (P^* - c)N_P = N - \frac{X}{X_P} N_P \overset{sign}{=} -\frac{\partial}{\partial P}\frac{X}{N}$$

(where every term on the right-hand side of the above is evaluated at $\delta = 0$). Here, the third equality follows from the first-order condition for the optimality of P^*. Thus, introducing a bundle discount raises profits if average demand per consumer, X/N, falls with price when $\delta = 0$. More exactly, if the firm offers linear price p for either item (and no bundle discount), write $x_s(p)$ and $x_b(p)$ respectively for the proportion of consumers who buy a single item and who buy both items. Since when $\delta = 0$ we have

$$\frac{X}{N} = \frac{x_s + 2x_b}{x_s + x_b} = 1 + \frac{x_b/x_s}{1 + x_b/x_s} ,$$

the condition requires that the ratio x_b/x_s decreases with price, so that demand for a single item is less elastic than demand for the bundle. We summarize this discussion as:

Result (Long, 1984): *Suppose an integrated monopolist supplies two symmetric products. The firm has an incentive to introduce a discount for buying the bundle whenever the elasticity of demand for buying a single item is lower than the elasticity of demand for buying both items, so that*

$$-\frac{x_s'(p)}{x_s(p)} < -\frac{x_b'(p)}{x_b(p)} . \tag{2}$$

In economic terms, condition (2) is intuitive: if the firm initially charges the same price for buying a single item as for buying a second item, and if demand for the latter is more elastic than demand for the former, then the firm would like to reduce its price for buying a second item (and to increase its price for the first item).

Consider the knife-edge case where a consumer's valuation for the bundle is simply the sum of her individual stand-alone valuations. That is, if the stand-alone valuation for product $i = 1, 2$ is v_i her valuation for the bundle is $v_1 + v_2$. With additive valuations, if the firm offers the linear price p for buying either item the consumer's buying decision is simple: she should buy product i whenever $v_i \geq p$, as shown on Figure 1. Suppose that the marginal c.d.f. for either valuation v_i is $F(v_i)$. A useful way to capture the extent of correlation in valuations is the function

$$\Psi(p) \equiv \Pr\{v_2 \geq p \mid v_1 \geq p\} . \tag{3}$$

Then, as shown on the figure, we have

$$x_s(p) = 2(1 - F(p))(1 - \Psi(p)) \ ; \ x_b(p) = (1 - F(p))\Psi(p) .$$

It follows that (2) holds whenever

$$\Psi(p) \text{ is strictly decreasing in } p\ . \tag{4}$$

Clearly, condition (4) holds if v_1 and v_2 are independently distributed, but it applies much more widely.[8] Indeed, the beauty of Long's approach is that condition (2) applies just as well to situations in which valuations are not additive, as I discuss in more detail in the following analysis.

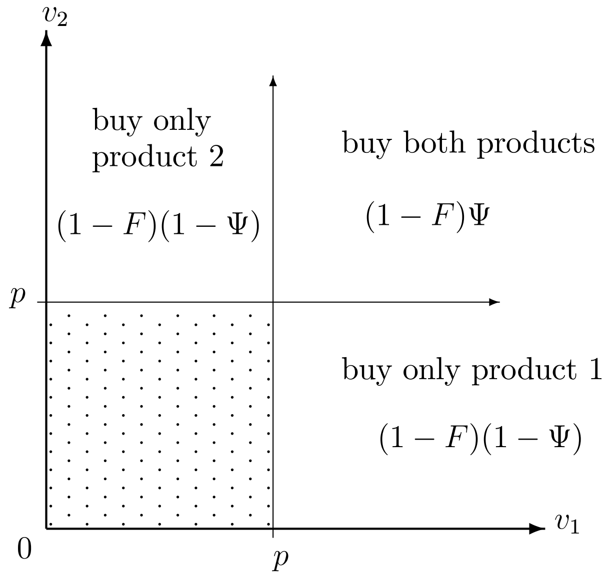

Figure 1: Pattern of demand with additive valuations

3 Bundling Revisited

Consider a market with two products, labeled 1 and 2, where there is a constant marginal cost of supplying product i equal to c_i. I consider situations where a monopolist supplies both products and where the two products are supplied by separate firms. Each consumer wishes to buy either zero or one unit of each product. A consumer is willing to pay v_i for product $i = 1, 2$ on its own, and to pay v_b for the bundle of both products. Thus a consumer's preferences are entirely described by the vector (v_1, v_2, v_b), and this vector is distributed across the population of consumers according to some known distribution. Unlike most of the bundling literature, I allow for non-additive preferences so that $v_b \neq$

[8] An example which violates condition (4) is when v_1 and v_2 are perfectly positively correlated, so that $\Psi \equiv 1$.

$v_1 + v_2$, and say that a consumer views the two products as partial substitutes whenever $v_b \leq v_1 + v_2$. Whenever there is free disposal (so that a consumer can discard one item without cost), we require that $v_b \geq \max\{v_1, v_2\}$ for all consumers.

Consumers face three prices: p_1 is the price for consuming product 1 on its own; p_2 is the price for product 2 on its own, and $p_1 + p_2 - \delta$ is the price for consuming the bundle of both products. Thus, δ is the discount for buying both products (which is zero if there is a linear price for each product, or negative if consumers are charged a premium for joint consumption). A consumer chooses the option which leaves her with the highest net surplus, i.e., she will buy both items whenever

$$v_b - (p_1 + p_2 - \delta) \geq \max\{v_1 - p_1, v_2 - p_2, 0\} \ ,$$

she will buy product $i = 1, 2$ on its own whenever

$$v_i - p_i \geq \max\{v_b - (p_1 + p_2 - \delta), v_j - p_j, 0\} \ ,$$

and otherwise she will buy nothing.

As functions of the three tariff parameters (p_1, p_2, δ), denote by Q_1 the proportion of potential consumers who buy only product 1, Q_2 the proportion of consumers who buy only product 2, and Q_b the proportion of consumers who buy both products. It will also be useful to discuss demand when no discount is offered, so let $q_i(p_1, p_2) \equiv Q_i(p_1, p_2, 0)$ and $q_b(p_1, p_2) \equiv Q_b(p_1, p_2, 0)$ be the corresponding demand functions when $\delta = 0$. Indeed, we will see that a firm's incentive to introduce a bundle discount is determined entirely by the properties of the "no-discount" demand functions q_i and q_b. This is important insofar as these demand functions are more likely to be identified from market data than the more hypothetical demands Q_i and Q_b.[9]

Finally, to avoid tedious caveats involving corner solutions in the following analysis, suppose that over the relevant range of linear prices there is some two-item demand, so that $q_b > 0$.

[9]The model of consumer preferences presented here is related to the small empirical literature which estimates discrete consumer choice when multiple goods are chosen simultaneously. For instance, see Gentzkow (2007) who estimates the degree of complementarity between print and online newspapers. In his illustrative model in section 1.A, he supposes that the value of the bundle is the sum of the values of the two individual products plus a constant term (which could be positive or negative), which is similar to Example 1 discussed later in this paper.

Properties of consumer demand: One property which is almost immediate is that total demand for a product is an increasing function of the bundle discount. To see this, observe that a consumer buys product 1, say, if and only if

$$\max\{v_b - (p_1 + p_2 - \delta), v_1 - p_1\} \geq \max\{v_2 - p_2, 0\} \ .$$

(The left-hand side above is the consumer's maximum surplus if she buys product 1—either in the bundle or on its own—while the right-hand side is the consumer's maximum surplus if she does not buy product 1.) Clearly, the set of such consumers is increasing (in the set-theoretic sense) in δ, and hence the measure of such consumers is as well. For convenience, we state this formally:

Lemma 1: *Total demand for product i, $Q_i + Q_b$, increases with the bundle discount δ.*

In the case of separate supply, this result implies that when one firm introduces an inter-firm bundle discount, the profits of its rival will rise. Similar arguments show that each of the demands Q_i, Q_b and $Q_i + Q_b$ decrease with the price p_i.

When linear prices are used, products are said to be gross substitutes if total demand for product i, $q_i + q_b$, increases with the other product's linear price p_j. As one would expect, if all consumers view products as partial substitutes, products are then gross substitutes:

Lemma 2: *Suppose that $v_b \leq v_1 + v_2$ for all consumers. Then when linear prices are used, demand for product i, $q_i + q_b$, weakly increases with p_j.*

(All omitted proofs are contained in the appendix.)

Importantly, when a bundle discount is offered, this result can be reversed. That is to say, if products are partial substitutes then when $\delta > 0$ the demand for a product can *decrease* with the stand-alone price of the other product. The observation that a bundle discount can mitigate or overturn the innate substitutability of products is a recurring theme in the following analysis.

Regardless of whether the underlying products are complements or substitutes, the three discrete purchasing *options* (buy product 1 only, buy product 2 only, or buy both products) are necessarily substitutes, in the sense that cross-price effects are non-negative:

$$\frac{\partial Q_i}{\partial \delta} \leq 0 \; ; \frac{\partial Q_j}{\partial p_i} + \frac{\partial Q_j}{\partial \delta} \geq 0 \; ; \frac{\partial Q_b}{\partial p_i} + \frac{\partial Q_b}{\partial \delta} \geq 0 \; . \tag{5}$$

(Concerning the second and third inequalities here, note that if price p_i and discount δ rise by the same amount, the prices for the bundle or product j on its own are unchanged, but the stand-alone price for item i rises.) We also necessarily have Slutsky symmetry of cross-price effects, so that

$$\frac{\partial Q_2}{\partial p_1} + \frac{\partial Q_2}{\partial \delta} \equiv \frac{\partial Q_1}{\partial p_2} + \frac{\partial Q_1}{\partial \delta} \; ; \; \frac{\partial Q_b}{\partial p_i} + \frac{\partial Q_b}{\partial \delta} \equiv -\frac{\partial Q_i}{\partial \delta} \; . \tag{6}$$

Note that setting $\delta = 0$ in the right-hand expression in (6) implies the following result:

Lemma 3: *The impact of a small bundle discount on the total demand for product i is equal to the impact of a corresponding price cut on the demand for the bundle, i.e.,*

$$\left. \frac{\partial (Q_i + Q_b)}{\partial \delta} \right|_{\delta=0} = -\frac{\partial q_b}{\partial p_i} \; . \tag{7}$$

This result will play a key role in the following analysis of the profitability of introducing a bundle discount.

Integrated supply: Suppose first that an integrated monopolist supplies both products. The firm's profit with the bundling tariff (p_1, p_2, δ) is

$$\pi = (p_1 - c_1)(Q_1 + Q_b) + (p_2 - c_2)(Q_2 + Q_b) - \delta Q_b \; . \tag{8}$$

Given that the three purchase options are substitutes, the most profitable bundling tariff will involve above-cost pricing for each option, so that

$$p_i \geq c_i \; ; \; p_1 + p_2 - \delta \geq c_1 + c_2 \; .$$

Consider the firm's incentive to offer a bundling discount. Starting from any pair of linear prices (p_1, p_2), by differentiating (8) we see that the impact on profit of introducing a small discount $\delta > 0$ is

$$\begin{aligned} \left. \frac{\partial \pi}{\partial \delta} \right|_{\delta=0} &= \left. \left\{ (p_1 - c_1) \frac{\partial}{\partial \delta}(Q_1 + Q_b) + (p_2 - c_2) \frac{\partial}{\partial \delta}(Q_2 + Q_b) - Q_b \right\} \right|_{\delta=0} \\ &= -(p_1 - c_1) \frac{\partial q_b}{\partial p_1} - (p_2 - c_2) \frac{\partial q_b}{\partial p_2} - q_b \; , \end{aligned} \tag{9}$$

where the second equality follows from (7). Let (p_1^*, p_2^*) be the most profitable linear prices. Therefore,

$$(p_1^*, p_2^*) \text{ maximizes } (p_1 - c_1)(q_1 + q_b) + (p_1 - c_2)(q_2 + q_b) \ ,$$

which has first-order condition for p_i^* given by

$$q_i + q_b + (p_1^* - c_1)\frac{\partial}{\partial p_i}(q_1 + q_b) + (p_2^* - c_2)\frac{\partial}{\partial p_i}(q_2 + q_b) = 0 \ . \tag{10}$$

If the products are gross substitutes, both price-cost margins are positive, and in particular $(p_2^* - c_2)\frac{\partial}{\partial p_1}(q_2 + q_b) \geq 0$ and $(p_1^* - c_1)\frac{\partial}{\partial p_2}(q_1 + q_b) \geq 0$. The first-order condition (10) therefore implies that

$$p_i^* - c_i \geq \frac{q_i + q_b}{-\partial(q_i + q_b)/\partial p_i} \quad \text{for } i = 1, 2 \ . \tag{11}$$

Substituting this pair of inequalities into (9) shows that offering a bundle discount is profitable whenever condition (12) holds, as summarized in this result:

Proposition 1: *Suppose that products are gross substitutes and that*

$$\frac{q_1 + q_b}{q_b}\frac{\partial q_b/\partial p_1}{\partial(q_1 + q_b)/\partial p_1} + \frac{q_2 + q_b}{q_b}\frac{\partial q_b/\partial p_2}{\partial(q_2 + q_b)/\partial p_2} > 1 \ . \tag{12}$$

Then the integrated monopolist has an incentive to offer a discount when a customer buys both products.

Condition (12) is satisfied when demand for the bundle is not "too much" less elastic than the overall demand for each product. A simple sufficient condition for (12) to hold is that each term on the left-hand side is greater than a half, so that a price rise which causes total demand for a particular product to fall by 10% causes demand for the bundle to fall by more than 5%.

Separate sellers: Next, suppose that each product is supplied by a separate seller, and the sellers set their tariffs simultaneously. When firms offer linear prices—i.e., prices which do not depend on whether the consumer also purchases the other product—firm i chooses its price p_i^*, given its rival's price, to maximize $(p_i - c_i)(q_i + q_b)$. In some circumstances, a firm can condition its price on whether a consumer also buys the rival firm's product. For instance, a museum could ask a visitor to show her entry ticket to the other museum to claim a discount. The next result describes when a firm has a unilateral incentive to offer a discount when a customer buys the other firm's product.

Proposition 2: *Suppose that demand for the bundle is more elastic than demand for firm i's stand-alone product, i.e., that*

$$-\frac{1}{q_b}\frac{\partial q_b}{\partial p_i} > -\frac{1}{q_i}\frac{\partial q_i}{\partial p_i} \ . \tag{13}$$

Starting from the situation where both firms set the equilibrium linear prices p_1^* and p_2^*, firm i has an incentive to offer a discount to those consumers who buy product j. If expression (13) is reversed, firm i would like, if feasible, to charge its customers a premium if they also buy product j.

Thus, discounts for joint purchase can arise even when products are supplied by separate firms and when a firm chooses, and funds, the discount unilaterally. The reason for this is straightforward: since demand for the bundle is more elastic than demand for its stand-alone product, a firm wants to offer a lower price to those consumers who also buy the other product. As Lemma 1 shows, the introduction of this discount will also benefit the rival firm.

At least for given stand-alone prices (p_1, p_2), condition (13) is stronger than condition (12).[10] Therefore, whenever a separate seller has an incentive to bundle, we expect that an integrated firm does also (but not necessarily *vice versa*). Intuitively, if it is profitable for a separate seller to introduce its own bundle discount even without taking into account the positive externality this discount brings to the other seller, it will also be profitable for an integrated firm to introduce a discount.

If firm $i = 1, 2$ offers the price p_i when a consumer only buys its product and the price $p_i - \delta_i$ when she also buys the rival's product, a consumer who buys both products pays the price $p_1 + p_2 - \delta_1 - \delta_2$. The issue then arises as to how the combined discount $\delta = \delta_1 + \delta_2$ is implemented. For instance, in some cases a consumer must buy the two items in order,

[10] Formally, if condition (13) holds for firm i, then demand for the bundle is more elastic than total demand for that firm's product, and so

$$\frac{q_i + q_b}{q_b}\frac{\partial q_b/\partial p_i}{\partial(q_i + q_b)/\partial p_i} > 1 \ .$$

Therefore, condition (12) is satisfied.

and both firms cannot simultaneously require proof of purchase from the other seller when they offer their discount.[11]

However, there are at least two natural ways to implement such an inter-firm bundling scheme. First, the bundle discount could be implemented via an electronic sales platform which allows consumers to buy products from several sellers simultaneously. The sellers choose their prices contingent on which other products (if any) a consumers buys, a website displays the total prices for the various combinations, and firms receive directly their stipulated revenue from the chosen combination. With such a mechanism there is no need for firms to coordinate their tariffs. Second, there may be "product aggregators" present in the market who put together their own packages from separate firms and retail them to final consumers. (See footnote 1 for an example of this practice.) In the two-product case discussed in this paper, aggregators bundle the two products and each firm chooses a wholesale price for its product contingent on being part of this package. If the aggregator market is competitive, the price of the bundle will simply be the sum of the two wholesale prices. Again, there is no need for firms to coordinate their prices.

A major difference between these inter-firm bundling discounts and the discount offered by an integrated supplier is that with separate sellers the bundle discount is chosen non-cooperatively. A bundle is, by definition, made up of two "complementary" components, namely, firm 1's product and firm 2's product, and the total price for the bundle, $p_1 + p_2 - \delta_1 - \delta_2$, is the sum of each firm's component price $p_i - \delta_i$. When one firm considers the size of its own discount δ_i, it ignores the benefit this discount confers on its rival. Thus, as usual with separate supply of complementary components, double marginalization will result and the overall discount $\delta = \delta_1 + \delta_2$ will be too small (for given stand-alone prices).

Without specifying consumer tastes in more detail, it is hard to derive further results. In

[11] However, even if proof of purchase could be provided, this is not necessarily an ideal way to implement the inter-firm bundle discount. In the Aspen case mentioned in the introduction, a small ski-lift operator (with only one mountain) wished to participate in the bundled ticket offered by the larger firm (which operated on three mountains). The large operator refused to do this on terms acceptable to the small operator, and so the latter attempted to put together its own four-mountain ticket. It tried to purchase the rival's three-mountain ticket (at full price) and add its own mountain to that. However, the larger operator refused to sell its ticket for that purpose. Presumably, the small operator could have announced to potential customers that if they show it their three-mountain ticket, the small firm will give them its own mountain ticket, thus making up a full four-mountain ticket. But such a scheme has disadvantages for the small firm: customers are instructed to visit the large rival first, and there is a danger that, due to transactions costs, many such customers would not bother coming back to the small firm. For whatever reason, though, the small firm did not choose to follow this strategy. See Easterbrook (1986, page 972) for more details.

the next sections—which cover respectively the cases of integrated and separate supply—we specialise the framework in various ways to obtain further insight.

4 Integrated Supply

For maximum transparency of the analysis, suppose now that the two products are symmetric, so that $c_1 = c_2 = c$ and the same density of consumers have taste vector (v_1, v_2, v_b) as have the permuted taste vector (v_2, v_1, v_b). As in section 2, let $x_s(p)$ denote the proportion of consumers who buy any single item when the linear price for either item is p and let $x_b(p)$ denote the proportion of consumers who buy both items when the linear price is p. Then section 2 shows that an integrated monopolist wishes to introduce a bundle discount whenever (2) holds. As before, let $F(\cdot)$ denote the marginal c.d.f. for either stand-alone valuation v_i, and in the following assume that F has an increasing hazard rate, so that

$$\frac{f(v)}{1 - F(v)} \text{ strictly increases with } v \ . \tag{14}$$

Suppose the products are partial substitutes, so that $v_b \leq v_1 + v_2$ for all consumers. For a type-(v_1, v_2, v_b) consumer, define

$$V_1 \equiv \max\{v_1, v_2\} \ ; \ V_2 \equiv v_b - V_1 \ , \tag{15}$$

so that V_1 is her maximum utility if she buys only one item and V_2 is her incremental utility from buying the second item. The assumption that products are substitutes implies

$$V_2 \leq \min\{v_1, v_2\} \leq V_1 \ ,$$

so that the support of (V_1, V_2) lies under the 45^0 line, as shown on Figure 2. Note that $v_b = V_1 + V_2$, so that valuations are additive after the change of variables (15). With a linear price p for either item, a type-(V_1, V_2) consumer will buy both items whenever $V_2 \geq p$, and will buy only one item whenever $V_2 < p \leq V_1$, as depicted on the figure. As in expression (3), define

$$\Phi(p) \equiv \Pr\{V_2 \geq p \mid V_1 \geq p\} \ . \tag{16}$$

If we write $G(P) \equiv \Pr\{V_1 \leq P\}$ for the marginal c.d.f. for V_1, by examining Figure 2, we see that[12]

$$x_s(p) = (1 - G(p))(1 - \Phi(p)) \ ; \ x_b(p) = (1 - G(p))\Phi(p) \ . \tag{17}$$

[12] In terms of the c.d.f. $F(\cdot)$ and function $\Psi(\cdot)$ in (3), we have $1 - G(p) = (1 - F(p))(2 - \Psi(p))$.

It follows immediately that when Φ is decreasing condition (2) holds, and we deduce the following generalization of Long's original condition (4) to the case where products are partial substitutes:

Proposition 3: *Suppose products are substitutes and Φ in (16) is strictly decreasing. Then an integrated monopolist has an incentive to offer a bundle discount.*

Note that Proposition 3 applies equally to an alternative framework where the monopolist supplies a *single* product, and where consumers wish to buy zero, one or two units of this product. Here, the parameter V_1 represents a consumer's value for consuming one unit of the good, and V_2 is her incremental value for a second unit (so her total value for two units is $V_1 + V_2$). Then Proposition 3 applies in this framework, so that when (16) holds in the population of consumers, the single-product firm will wish to offer a tariff which involves a quantity discount.[13] (However, this alternative interpretation of the model is not as natural in the separate sellers context, since we would have to assume that for some reason each supplier could only sell a single unit of the product to any consumer.)

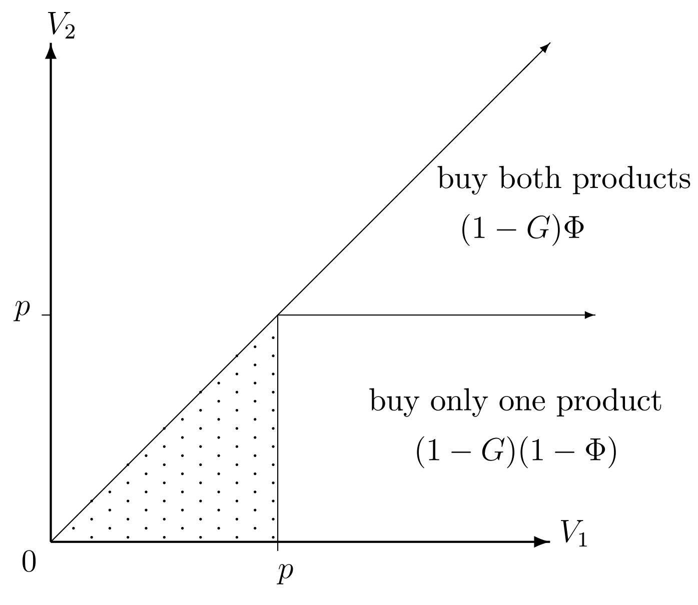

Figure 2: Pattern of demand with substitutes

A natural question is whether products being partial substitutes makes it more or less likely that the integrated firm wishes to introduce a bundle discount, relative to the

[13] See Maskin and Riley (1984) for an early contribution to the theory of quantity discounts, where—in contrast to the current paper—consumers differ by only a scalar parameter.

corresponding situation with additive valuations. To gain insight into this issue, consider a market where the stand-alone valuations, v_1 and v_2, have a given (symmetric) distribution. In scenario (i), each consumer's valuation for the bundle is additive, so that $v_b \equiv v_1 + v_2$. Then we know from section 2 that the firm has an incentive to offer a bundle discount whenever x_b/x_s is decreasing in the linear price p, which is equivalent to the condition that x_b/N decreases with p. (Recall that $N = x_s + x_b$ is the fraction of consumers who buy something from the firm.) Now consider an alternative situation (ii) which is the same as (i) except that $v_b \leq v_1 + v_2$. Write the fraction of consumers who buy both items at linear price p with these preferences as $\hat{x}_b(p)$. Then, as in (i), it is profitable to offer a bundle discount whenever $\hat{x}_b/N$ decreases with p. (Note that N is exactly the same function in the two scenarios, and given by $(1 - F)(2 - \Psi)$ as shown on Figure 1.) Thus, if $\hat{x}_b/x_b$ (weakly) decreases with price, then if bundling is profitable under scenario (i) it is sure to be profitable under scenario (ii) as well. That is to say, if bundle demand in the case of substitutes is no less elastic than it would be with additive valuations, then (3) decreasing implies that (16) is also decreasing. We summarize this formally as:

Corollary 1: *All else equal, if bundle demand when products are partial substitutes is more elastic than bundle demand with additive valuations, then if it is profitable to offer a bundle discount with additive valuations it is also profitable to offer a bundle discount with substitute products.*

It is plausible, though not inevitable, that demand $\hat{x}_b$ is no less elastic than demand x_b. Since $V_2 \leq \min\{v_1, v_2\}$, it follows that $\hat{x}_b \leq x_b$. Thus, for $\hat{x}_b$ to be no less elastic we require that the slope $-\hat{x}'_b$ not be "too much" smaller than $-x'_b$.[14]

In this rest of this section, I describe three special cases of this analysis. (I revisit the same examples when presenting the analysis for separate supply.)

[14] An example where the substitutability of products makes the firm *less* likely to engage in bundling is as follows. Suppose that $v_b = v_1 + v_2$ if $\min\{v_1, v_2\} \geq k$ and $v_b = \max\{v_1, v_2\}$ otherwise, where k is some constant. Thus, preferences are additive when both stand-alone valuations are high, while if one valuation does not meet the threshold the incremental value for the second item is zero. With these preferences, whenever the linear price satisfies $p < k$ those consumers with $\min\{v_1, v_2\} \geq k$ will buy both items, and this set does not depend on the price. Therefore, bundle demand x_b is completely inelastic for $p < k$, while in the corresponding example without substitution (i.e., setting $k = 0$), bundle demand is elastic. Whenever k is large enough (so that the equilibrium linear price is below k), one can check that starting from the most profitable linear price p, the firm makes strictly less profit if it offers a small bundle discount.

Example 1: *Constant disutility of joint consumption.*

Consider the situation in which for all consumers

$$v_b = v_1 + v_2 - z \tag{18}$$

for some constant $z \geq 0$. Here, to ensure free disposal we need to assume that the minimum possible realization of v_i is greater than z. Then with a linear price p_i for buying product i, the pattern of demand is as shown on Figure 3.[15] The next result provides a sufficient condition for bundling to be profitable in this setting.

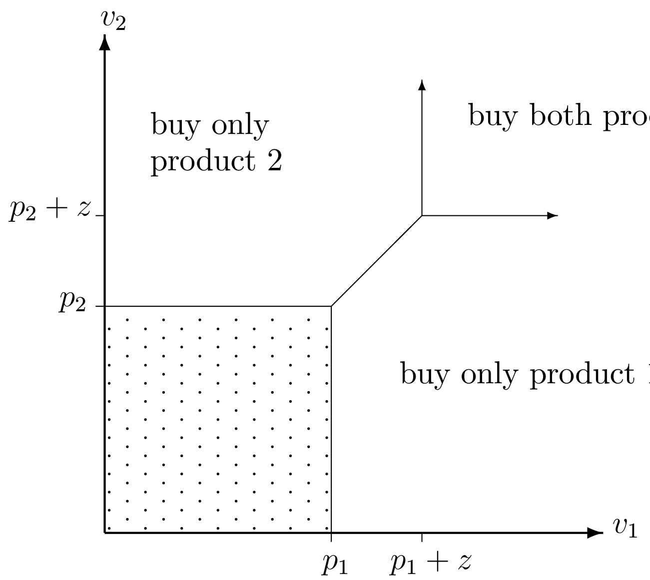

Figure 3: Pattern of demand with constant disutility of joint purchase

Proposition 4: *Suppose that bundle valuations are given by (18). Then an integrated monopolist has an incentive to offer a bundle discount when condition (4) holds.*

To illustrate, suppose that (v_1, v_2) is uniformly distributed on the unit square $[1, 2]^2$, that $z = \frac{1}{4}$ and that $c = 1$.[16] Then an integrated monopolist which uses linear prices

[15]Note that the pattern of demand with linear pricing and a disutility of joint consumption $z > 0$ is the same as that corresponding to additive valuations and a tariff *premium* for buying both items. (The latter is illustrated in Long, 1984, Figure 8.) Thus, just as a bundle discount can convert independent products into complements, a bundle premium converts independent products into substitutes.

[16]This example gives rise to a linear demand system when linear prices are used, and when prices are such that there is some two-item demand and some consumers who buy nothing, Figure 3 shows that the total demand for product i is equal to $k - p_i + \frac{1}{4}p_j$ for a constant k.

will choose price $p \approx 1.521$, generating profit of around 0.407. At this price, around 73% of potential consumers buy something, although only 5% buy both products. The most profitable bundling tariff can be shown to be

$$p \approx 1.594 \; ; \; \delta \approx 0.380 \; , \tag{19}$$

which generates profit of about 0.449, and about 66% of potential consumers buy something but now 28% buy both items. In particular, note that the bundle discount is large enough to outweigh the innate substitutability of the products (i.e., $\delta > z$). Faced with this bundling tariff consumers now view the two products as complements rather than substitutes, and the pattern of demand looks like Figure 5 below rather than Figure 3. Nevertheless, the discount in (19) is smaller than in the corresponding example with additive valuations (i.e., when $z = 0$).[17]

Example 2: *Time-constrained consumers.*

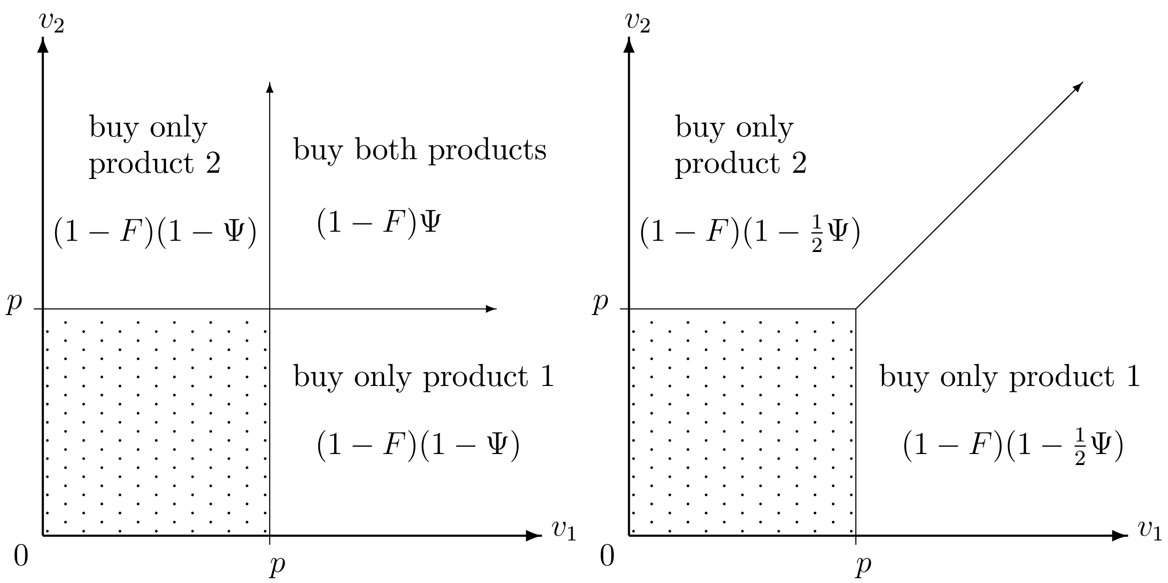

Figure 4A: Additive valuations   Figure 4B: Time-constrained consumers

A natural reason why products might be substitutes is that some buyers are only able to consume a restricted set of products, e.g., due to time constraints.[18] For instance, a

[17]When $c = 1$, (v_1, v_2) is uniformly distributed on $[1, 2]^2$ and $v_b \equiv v_1 + v_2$, one can check that $p = \frac{5}{3}$ and $\delta = \frac{\sqrt{2}}{3} \approx 0.47$.

[18]In the context of competitive *intra*-firm bundling, Thanassoulis (2007) also analyzes the situation where an exogenous fraction of consumers wish to buy a single product.

tourist may have the time only to visit a single museum in a city. To that end, suppose that an exogenous fraction λ of consumers have valuation v_i for stand-alone product $i = 1, 2$ and valuation $v_b = v_1 + v_2$ for the bundle, while the remaining consumers can only buy a single item (and have valuation v_i if they buy item i). For simplicity, suppose that the distribution for (v_1, v_2) is the same for the two groups of consumers. Let the marginal c.d.f. for each v_i be $F(v)$, and let $\Psi(\cdot)$ be as defined in (3). (See Figure 4 for an illustration.)

The central feature of this scenario is that the time-constrained consumers have zero incremental value for the second item (i.e., $V_2 = 0$). It is then straightforward to show that

$$\Phi(p) = \lambda \frac{\Psi(p)}{2 - \Psi(p)} ,$$

so that Φ is decreasing if and only if Ψ is decreasing. Proposition 3 therefore has the corollary:

Proposition 5: *When some consumers are time-constrained, an integrated firm has an incentive to offer a bundle discount if and only if (4) holds, i.e., under the same condition as when consumers have additive preferences.*

Example 3: *Stand-alone values* (v_1, v_2) *are uniformly distributed on the unit square* $[0, 1]^2$*, and given* (v_1, v_2) *the bundle value* v_b *is uniformly distributed on the interval* $[\max\{v_1, v_2\}, v_1 + v_2]$*. Production is costless.*

(Recall that with free disposal we require that v_b be at least $\max\{v_1, v_2\}$, and $v_b \leq v_1 + v_2$ if products are substitutes.) In contrast to the previous examples, this example has a full three-dimensional distribution for consumer valuations. Moreover, it is worthwhile studying this example since the method used here can be adapted to solve any specific (two–product) bundling problem.

The detailed calculations for this example are presented in the appendix. One can show that the optimal linear price is approximately $p \approx 0.540$, which yields industry profit of 0.406. Note that about 70% of potential consumers buy something given this price, although only 4% of consumers buy both items. One can show that Φ is strictly decreasing in this example, and so Proposition 3 implies that the firm will wish to offer a bundle discount. Indeed, the optimal bundling tariff is

$$p \approx 0.648 \ ; \ \delta \approx 0.588 \tag{20}$$

which yields profit 0.463. Notice that, compared to the corresponding example with additive values, the bundle discount is deeper.[19] With this bundling tariff, where the incremental price for the second item is rather small, about 51% of potential consumers now buy both items and only 15% of consumers buy a single item.

Summary: This section focussed on the case when an integrated firm supplies products which are partial substitutes. A general condition was derived (Proposition 3) which governs when the firm wishes to offer a bundle discount, and a number of special cases were solved. We saw in examples that the bundle discount could be higher or lower than the corresponding case with additive utility. We saw that in most cases the presence of substitutability made it more likely that the firm will wish to offer a discount, relative to the corresponding situation with additive preferences. For instance, in the case of time-constrained consumers, the condition governing when bundling is used was exactly the same as when values were additive, and when there was a fixed disutility of joint consumption, bundling was profitable in more cases than the additive case.

In broad terms, when products are substitutes there is an extra motive to offer a bundle discount, relative to the additive case, which is to try to serve customers with a second item even though the incremental utility of the second item is lowered by the purchase of the first item. Intuitively, once a customer has purchased one item, this is bad news for her willingness-to-pay for the other item, and this often gives the firm a motive to reduce price for the second item. With additive preferences, the only motive in this model to use a bundle discount is to extract information rent from consumers, and this motive vanishes if the firm knows consumer preferences. With sub-additive preferences, the firm may wish to offer a bundling tariff even when it knows the customer's tastes.[20] While with integrated supply sub-additive preferences merely give an additional reason to bundle, with separate sellers such preferences will often be the *sole* reason to offer a bundle discount, as I discuss in more detail in the next section.

[19]When $c = 0$, (v_1, v_2) is uniformly distributed on $[0, 1]^2$ and $v_b \equiv v_1 + v_2$, one can check that $p = \frac{2}{3}$ and $\delta = \frac{\sqrt{2}}{3} \approx 0.47$.

[20]For instance, suppose the consumer has known sub-additive valuations $v_1 = v_2 = 3$ and $v_b = 4$. If production is costless, then with linear pricing the most profitable strategy for the firm is to sell just one item for price $p = 3$. Clearly, with a bundling tariff the firm can extract the first-best profit of 4.

5 Separate Sellers

In this section I turn to the situation where the two products are supplied by separate sellers. I first consider the situation where the sellers do not compete, in the sense that consumer valuations are additive, and then go on to consider the three special cases with substitute products presented earlier in the context of integrated supply in section 4.

Additive valuations: Suppose that $v_b = v_1 + v_2$ for all consumers. With separate sellers, there is no particular benefit in assuming that the products are symmetric. Let $F_i(v_i)$ and $f_i(v_i)$ be respectively the marginal c.d.f. and the marginal density for v_i, and define

$$H_i(p_i \mid v_j) = \Pr\{v_i \le p_i \mid v_j\}$$

to be the conditional c.d.f. for value v_i when the other value is v_j. The next result provides a sufficient condition for a firm to offer a discount when its customers buy the other firm's product:

Proposition 6: *Suppose that valuations are additive. Starting from the situation where firms set equilibrium linear prices, firm i has an incentive to offer a discount to those consumers who buy the other product whenever $H_j(p_j \mid v_i)$ strictly increases with v_i (where p_j is firm j's equilibrium linear price).*

Thus, whenever the valuations are negatively correlated, in the strong sense that $H_j(p_j \mid v_i)$ strictly decreases with v_i, a firm has an incentive to offer a discounted price for joint purchase. It is intuitive that negative correlation is associated with the incentive to engage in inter-firm bundling when valuations are additive. If firm i knows that a potential consumer has purchased firm j's product, i.e., the consumer has a relatively high value for item j, then negative correlation implies that this is bad news for the consumer's likely value for i's product, which will usually induce the firm to lower its price to this consumer.

We next consider the three examples with non-additive valuations discussed in the previous section.

Example 1. Here, the pattern of consumer demand is as illustrated in Figure 3. For simplicity, I focus on the situation where v_1 and v_2 are identically and independently distributed. (From Proposition 6, we already know that negative correlation will tend to give an incentive to offer a unilateral bundle discount.) The next result shows that a firm typically does have a unilateral incentive to offer a bundle discount.

Proposition 7: *Suppose that v_1 and v_2 are identically and independently distributed with c.d.f. satisfying (14) and that the bundle valuations satisfy (18). When the two products are supplied by separate sellers, each seller has an incentive to offer a discount to those consumers who buy the rival product.*

It is economically intuitive that products being substitutes of the form (18) will give an incentive to a firm to offer a discount when its customers purchase the rival product. If the potential customer purchases the other product, this is bad news for the firm as the customer's incremental value for its product has been shifted downwards by z, and this will give an incentive to offer the customer a lower price.

Consider the same specific example as presented in section 4 (that is, (v_1, v_2) uniform on $[1, 2]^2$, $z = \frac{1}{4}$ and $c = 1$) applied to the case with separate sellers. The equilibrium linear price is $p \approx 1.446$ and industry profit is about 0.399. Around 9% of consumers buy both items with this linear price, and 80% of all consumers buy something. The equilibrium inter-firm bundling tariff is

$$p_1 = p_2 = 1.476 \ ; \ \delta_1 = \delta_2 = 0.05 \ .$$

Thus, the discount $\delta = \delta_1 + \delta_2$ when a consumer buys the second product is about 7% of the stand-alone price. This bundle discount is approximately one quarter the size of the discount with integrated supply (see expression (19) above), reflecting the discussion in section 3 that separate firms will unilaterally choose too small a discount. Now, around 14% of consumers buy both items, and industry profit rises to 0.421. Intuitively, when firms offer a bundle discount, this reduces the effective degree of substitution between products, which in turn relaxes competition between firms. Note that the equilibrium linear price lies between the two discriminatory prices when firms engage in this form of price discrimination.[21]

[21] The same feature is seen in Examples 2 and 3 which follow. This is to be expected in the light of the analysis in Corts (1998), who shows that when the two firms wish to set their lower price to the same group of customers (the "weak" market, which in this example is the set of customer who buy both products),

Example 2. Consider next the situation with time-constrained consumers when separate sellers supply the products:

Proposition 8: *Suppose that v_1 and v_2 are identically and independently distributed with c.d.f. satisfying (14) and that some consumers are time-constrained. When the two products are supplied by separate sellers, a seller has no incentive to offer a discount to those consumers who buy the rival product. (They would, if feasible, like to charge their customers a higher price when a customer buys the rival product.)*

In this setting, the observation that a consumer wishes to buy both items implies she belongs to the "non-competitive" group of consumers, and a firm would like to exploit its monopoly position over those consumers if feasible.[22] Of course, in many situations, a consumer can hide her purchase from a rival firm, in which case a firm cannot feasibly levy a premium when a customer buys another supplier's product.

Example 3. Following the approach discussed in the appendix, one can show that when separate firms supply the two products the equilibrium linear price in this example is $p \approx 0.426$ which yields industry profit of 0.388. Here, about 82% of potential consumers buy something at this price, and 9% of consumers buy both products. When firm i unilaterally offers a discount δ_i to its customers when they also purchase the rival product, so that the total bundle discount is $\delta = \delta_1 + \delta_2$, then the symmetric equilibrium bundling tariff is

$$p \approx 0.440 \; ; \; \delta = 0.081 \tag{21}$$

and a firm offers about a 10% discount when a customer also buys the rival product. Here, equilibrium profit rises to 0.404, about 81% of consumers buy something and 14% now buy the bundle. Note that the price for the bundle with separate sellers in (21) is greater than the cost of the bundle with integrated supply in (20), which reflects the earlier observation

then the equilibrium non-discriminatory price lies between the two discriminatory prices. However, we cannot apply Corts' result directly, since his argument relies on there being no cross-price effects across the two consumer groups, which is not the case in the current setting.

[22] Consider the specific example where $c = 0$, (v_1, v_2) is uniformly distributed on $[0,1]^2$ and half of consumers can only buy a single product ($\lambda = \frac{1}{2}$). Then one can check that when firms use linear pricing, the equilibrium price is $p \approx 0.464$, whereas when they engage in price discrimination the stand-alone price falls to $p \approx 0.454$ and a firm's price when its customer buys the other product rises to about 0.477.

that an integrated firm has an incentive to offer a deeper discount than separate sellers.

Summary: This section considered a firm's incentive to offer a discount when a customer also buys the rival product. Two broad forces may provide such an incentive. First, if a consumer's value for one product is negatively correlated with the other, the information that a consumer has purchased the rival product (i.e., its value for the rival product is relative high) is bad news for a firm, and typically induces it to lower its price to that customer. Second, if purchasing the rival product causes a consumer's incremental value for the firm's product to fall, due to substitution, then the firm may wish to reduce its price to these customers (Example 1). However, Example 2 showed that an alternative form of substitution makes a firm wish to set a higher price when its customers buy the rival product. Thus, the precise form in which products are substitutes is important for a firm's incentive to offer inter-firm bundling discounts.

It is plausible that the framework studied here, where customers are final consumers, could sometimes be extended to situations where rival manufacturers supply products to a retailer, which then supplies one or both products to final consumers. (Indeed, traditional retailers are the most prevalent of the "product aggregators" discussed in the introduction.) If the manufacturers supply products which are partial substitutes, this analysis suggests that one manufacturer could have an incentive to charge a lower price if the retailer also chooses to supply the rival product. This is the opposite pricing pattern to the "loyalty pricing" schemes which often worry antitrust authorities. On the other hand, if the situation is more like the time-constrained consumer case—i.e., some retailers can only stock one of the two products, perhaps because of shelf or refrigeration constraints—then a supplier has an incentive to charge the retailer less if the retailer does not stock the rival product, which is the more conventional prediction.

6 Partial Coordination Between Sellers

The analysis to this point has considered the two extreme cases where (a) there is no tariff coordination between separate sellers, and (b) where there is complete tariff coordination between sellers. (The integrated-firm analysis in section 4 describes the outcome when two sellers coordinate their pricing to maximize industry profit.). The problem with complete coordination is that any competition between rivals is eliminated. As discussed in section

3, though, the welfare problem with a policy of permitting no coordination between sellers is that the resulting bundle discount may be inefficiently small (or non-existent). It would be desirable, if feasible, to obtain the efficiency gains which may accrue to bundling without permitting the firms to collude over their regular prices.[23] One way this might be achieved is if firms first negotiate an inter-firm bundle discount and then compete in the usual way by choosing their stand-alone prices independently.

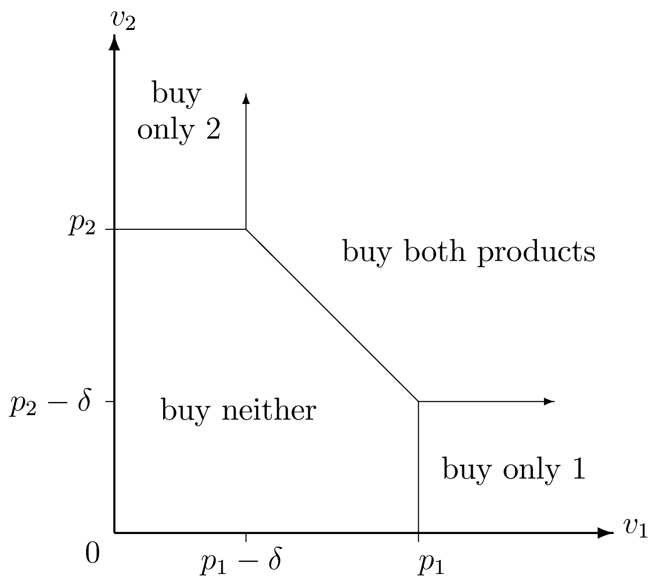

Figure 5: Pattern of demand with additive values and bundling discount $\delta > 0$

To consider this situation in more detail, suppose that two symmetric firms supply two products. The firms interact in two stages in a similar manner to the procedure in the four-firm analysis of Gans and King (2006). First, the two firms agree on a bundle discount, δ say, which they agree to fund equally. That is to say, if firm $i = 1, 2$ chooses stand-alone price p_i, the consumer pays this price if she buys only that firm's product (and the firm receives that revenue), but if she buys both products she pays $p_1 + p_2 - \delta$ and firm i receives revenue $p_i - \frac{1}{2}\delta$. After δ is chosen, firms choose their stand-alone prices unilaterally. Far-sighted firms will choose δ after taking into account how this discount will affect their interaction in the second stage. Since separate firms tend to set lower prices when products are more substitutable, and since a bundle discount mitigates or overturns a consumer's view of the products as substitutes, it will usually be the case that an agreed

[23] In the context of code-sharing by airlines, ideally one would like to allow airlines to coordinate their pricing when they jointly offer multi-flight itineraries so as to avoid double marginalization, but not when they compete along similar routes.

bundle discount δ will induce firms to set higher stand-alone prices. To the extent this is so, a joint-pricing scheme of this form could act as an instrument of collusion.[24]

Consider first the case in which valuations are additive. Then for an agreed inter-firm discount δ, the pattern of demand for the two firms is as illustrated in Figure 5. The following result shows that this joint pricing scheme leads to higher industry profit, and describes when the scheme also increases total welfare.

Proposition 9: *Suppose that products are symmetric and valuations are additive. The marginal c.d.f. for either value v_i satisfies (14). For given $\delta > 0$ consider the joint pricing scheme in which if firm $i = 1, 2$ sets the stand-alone price p_i then the price for buying both products is $p_1 + p_2 - \delta$ and firm i receives revenue $p_i - \frac{1}{2}\delta$ when a bundle is sold. If condition (4) holds, for sufficiently small $\delta > 0$ this inter-firm bundling scheme increases each firm's profit, relative to the situation where the products are marketed independently. In addition, if the function $H(p, v) \equiv \Pr\{v_2 \leq p \mid v_1 = v\}$ weakly increases with v, the scheme increases total welfare for small δ.*

This result suggests that joint bundling schemes should, in theory, be both profitable and welfare-enhancing for many groups of suppliers, even if they supply seemingly unrelated products. Proposition 9 could be seen as the "separate seller" analogue of the result for integrated monopoly derived by Long (1984) and McAfee, McMillan, and Whinston (1989), who showed that when valuations were additive and condition (4) was satisfied it was profitable for a monopolist to introduce a bundle discount.

The reason that a small agreed inter-firm discount will boost profit is intuitive. A small $\delta > 0$ will have some effect on each firm's choice of stand-alone price, but this has no first-order impact on a firm's profit. (A small change in the firm's own price does not significantly affect its profit, since the original price was at the optimal level. And with additive valuations a small change in the other firm's price does not affect the firm's profit

[24] The mechanism discussed in this section, whereby firms initially choose a bundle discount and then independently choose their regular prices, is just one of many possible joint pricing schemes firms could organize. Another kind of mechanism involves firms jointly choosing the *price* for the bundle, and then setting their regular prices independently. In any such scheme, firms need to agree on a rule for how to allocate the revenue from bundle sales. Possibilities include sharing the revenue based on "usage" of the various products (as was done by the two Aspen ski-lift operators before their joint pricing scheme was abandoned), according to a fixed share regardless of firms' prices or consumer demands, or in proportion to the firms' regular prices. See Ginsburgh and Zang (2004) and Armstrong (2010) for further discussions of the merits and drawbacks of these various options.

when the bundle discount is zero.) The first-order impact of δ on industry profit is that, for a *fixed* stand-alone price p, the introduction of a bundle discount boosts profit whenever expression (4) is satisfied. The impact on total welfare is more complex, as the impact of the discount on equilibrium prices needs to be considered. A bundle discount tends to induce firms to raise their stand-alone prices. A bundle discount converts independent products into complements, and this typically induces separate firms to set higher prices. However, when values are independently distributed or negatively correlated (in the sense that $H(p,v)$ increases with v), the impact of the price rise is not large enough to outweigh the efficiency benefits of the bundle discount, and total welfare rises when the scheme is used.

To illustrate, consider the example where (v_1, v_2) is uniformly distributed on the unit square $[0,1]^2$ and $c = 0$. Using Figure 5, one can show that each firm's equilibrium stand-alone price as a function of the agreed discount is

$$p(\delta) = \frac{3\delta + 2\delta^2 + 2}{3\delta + 4} ,$$

which is indeed increasing in δ. For small δ, Proposition 9 shows that this scheme benefits the firms and efficiency.[25] However, in this example the scheme reduces aggregate consumer surplus.[26]

While the operation of this joint pricing scheme appears to be relatively benign when values are additive, this can easily be reversed when firms offer substitutable products. Consumers benefit, and total welfare rises, when firms are forced to set low prices due to products being substitutes. However, an agreed inter-firm discount can reduce the effective substitutability of products, and thus relax competition between suppliers. While this effect can be demonstrated more generally, for maximum clarity consider the following simple example:

Example 4: There are two profit-maximizing museums in a city, and the marginal cost of a museum visit is zero. All tourists have identical tastes, and the two museums are

[25] One can check that the most profitable choice of δ for the firms is $\delta \approx 0.38$, with corresponding stand-alone price $p \approx 0.67$. Thus, compared to the tariff chosen by an integrated monopolist (where $p = \frac{2}{3}$ and $\delta \approx 0.47$), the stand-alone price is essentially unchanged but the chosen discount is reduced. In particular, even when firms coordinate on their bundle discount—let alone when this is chosen unilaterally as emphasized earlier in the paper—this discount is smaller than that which would be chosen by a monopolist.

[26] When $\delta = 0$, we have $p'(0) = \frac{3}{8}$. Therefore, when δ is small that half of the consumer population who only buy one item experience a price rise of $\frac{3}{8}\delta$, while that quarter of consumers who buy both items experience a net price fall of $\frac{1}{4}\delta$. Thus, the net impact on consumers is a loss of $\frac{1}{8}\delta$.

homogenous in the sense that if a tourist visits just one museum, she does not mind which one it is. A tourist values visiting any single museum at V_1 and gains incremental utility $V_2 < V_1$ from visiting the second museum. Because of the declining marginal value of visits, the two museums compete to some extent. If each museum sets an independent entry charge, one can check that the equilibrium entry charge is the incremental value of a second visit, V_2. The result is that tourists visit both museums and obtain strictly positive surplus $V_1 - V_2$. Suppose next that the two museums are free to choose their own entry charge but agree in advance to offer a discount δ on the sum of stand-alone prices if a tourist visits both museums, and they fund this discount equally. (That is to say, if museum i chooses the entry fee p_i, the charge for visiting both museums is $p_1 + p_2 - \delta$ and museum i receives revenue $p_i - \frac{1}{2}\delta$ when a tourist visits both museums.) Since with a bundle discount δ a tourist's incremental utility from a second visit is now $V_2 + \delta$, the equilibrium entry fee with discount $\delta \leq V_1 - V_2$ is $p = V_2 + \delta$, with the result that tourists visit both museums and pay the joint price $2V_2 + \delta$. In particular, by choosing $\delta = V_1 - V_2$ firms can induce the fully collusive outcome.

Thus, the apparently pro-consumer policy of offering a discount for joint purchase can act as a device to sustain collusion. This suggests that inter-firm discounting schemes operated by firms supplying substitutable products should be viewed with some suspicion by antitrust authorities.[27]

7 Conclusions and Managerial Implications

This paper has extended the standard model of bundling to allow products to be partial substitutes and for products to be supplied by separate sellers. Building on the somewhat neglected paper by Long (1984), simple formulas were derived which governed when a firm wishes to introduce a bundle discount. With monopoly supply, we typically found that the firm has an incentive to offer a bundle discount in at least as many cases as with the traditional model with additive valuations. Sub-additive preferences give the firm an

[27] The UK competition authority, the Office of Fair Trading, has recently developed policy towards multi-operator travel cards, whereby several travel companies coordinate to provide combined travel tickets. The authority argued that such multi-operator tickets can provide efficiency gains, but they should only be permitted under competition law if they do not significantly eliminate competition. See their press release on 16 December 2010, at www.oft.gov.uk/news-and-updates/press/2010/141-10.

additional reason to offer a bundle discount, which is to better target a low price for a second item at those customers who are inclined (with linear prices) to buy a single item. We observed that the impact of substitutability could amplify or diminish the size of the most profitable bundle discount.

When products were supplied by separate firms, we found that a firm often has a unilateral incentive to offer a joint-purchase discount when their customers buy rival products. In such cases, inter-firm bundle discounts are achieved without any need for coordination between suppliers. The two principal situations in which a firm might wish to do this are (i) when product valuations are negatively correlated in the population of consumers, and (ii) when products are partial substitutes so that consumption of a rival product reduces the incremental utility derived from a firm's own product. In either case, when a customer buys another supplier's product, this is bad news about a customer's willingness to pay for a firm's product and gives the firm an incentive to cut its price.[28] When firms price discriminate in this manner, we saw that, relative to the uniform pricing regime, a firm typically raises its price for stand-alone purchase and lowers its price for joint purchase. In addition, equilibrium profits typically are higher with price discrimination. One reason why profits rise is that when firms offer an inter-firm bundle discount, this mitigates the innate substitutability of their products, and thus competition is relaxed. In sum, when conditions (i) or (ii) hold, a firm should consider conditioning its price on whether customers also buy products from rival sellers; its profit increases not only if it follows this strategy in isolation, but also if its rivals follow suit.

Historically, this form of price discrimination was not often observed. In many cases, in order to condition price on a purchase from a rival supplier, a firm would need a "paper trail" such as receipt from the rival. One problem with this system is that customers are then encouraged to visit the rival firm first, and because of transaction and travel costs, this might mean that fewer customers would actually come to the firm. A second problem is that it is hard for two firms to offer such discounts, since a customer might have to visit the firms sequentially. However, these two (related) problems can nowadays often be overcome when products can be purchased simultaneously, which can be facilitated either by online buying platforms or by other kinds of product aggregators. For instance, a website could

[28] We also discussed the situation where some consumers could only buy a single product (e.g., because of time constraints), and in this case a firm actually has an incentive to raise its price when a customer buys the rival product.

be set up which allows tourist attractions in a city to post retail prices which could be conditioned on which other attractions are chosen. A consumer then constructs her own bundle in the light of the menu of prices, and pays each attraction its stipulated price. Alternatively, intermediaries could provide ready-made packages for consumers, with retail prices based on the bundle-specific wholesale prices offered by sellers. Arrangements of this kind require no price coordination between rival suppliers. Thus, modern methods of shopping and paying make it easier for firms to pursue this kind of pricing strategy, and we may see greater use of it in future.

A more traditional way to implement inter-firm bundling is for firms to coordinate aspects of their pricing strategy. In this paper I focussed on one particular kind of coordination, which is where firms agree on a joint purchase discount, and subsequently choose their prices non-cooperatively. Because a bundle discount mitigates the innate substitutability of rival products, separate sellers can use this mechanism to lessen rivalry in the market. Thus, firms often have an incentive to explore joint pricing schemes of this form, and regulators have a corresponding incentive to be wary of such schemes.

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APPENDIX

Proof of Lemma 2: A type-(v_1, v_2, v_b) consumer buys product 1 if and only if

$$\max\{v_b - p_1 - p_2, v_1 - p_1\} \geq \max\{v_2 - p_2, 0\} \ . \tag{22}$$

I claim that the difference between the two sides in (22), that is

$$\max\{v_b - p_1 - p_2, v_1 - p_1\} - \max\{v_2 - p_2, 0\} \ , \tag{23}$$

is weakly increasing in p_2 for all (v_1, v_2, v_b). (This then implies that the set of consumer types who buy product 1 is increasing, in the set-theoretic sense, in p_2, and so the measure of such consumers is increasing in p_2.) The only way in which expression (23) could strictly decrease with p_2 is if

$$v_b - p_1 - p_2 > v_1 - p_1 \text{ and } v_2 - p_2 < 0 \ .$$

However, since products are substitutes we have $v_b \leq v_1 + v_2$, which implies that the above pair of inequalities are contradictory. This establishes the result. ■

Proof of Proposition 2: Firm i's equilibrium linear price p_i^* maximizes $(p_i - c_i)(q_i + q_b)$, so that

$$0 = q_i \left[1 - (p_i^* - c_i)\frac{-\partial q_i/\partial p_i}{q_i}\right] + q_b \left[1 - (p_i^* - c_i)\frac{-\partial q_b/\partial p_i}{q_b}\right] \ . \tag{24}$$

Suppose now that firm i offers a discount $\delta_i > 0$ from its price p_i^* to those consumers who purchase product j as well. (Those consumers who only buy product i continue to pay p_i^*.) Then firm i's profit is

$$\pi_i = (p_i^* - c_i)(Q_i + Q_b) - \delta_i Q_b \ , \tag{25}$$

and the impact of a small joint purchase discount is governed by the sign of $\left.\frac{d\pi_i}{d\delta_i}\right|_{\delta_i=0}$, which from (7) is equal to

$$-q_b - (p_i^* - c_i)\frac{\partial q_b}{\partial p_i} \ . \tag{26}$$

When (13) holds, the second term $[\cdot]$ in (24) must be strictly negative, i.e., expression (26) is strictly positive. Therefore, offering a small discount for joint purchase will raise the firm's profit. ■

Proof of Proposition 4: From Figure 3 we see that with linear price p for either product we have

$$x_b(p) = (1 - F(p+z))\Psi(p+z) \ ; \ x_s(p) = (1 - F(p))(2 - \Psi(p)) - x_b(p) \ ,$$

and so (16) is given by

$$\Phi(p) = \frac{x_b(p)}{x_s(p) + x_b(p)} = \frac{(1 - F(p+z))\Psi(p+z)}{(1 - F(p))(2 - \Psi(p))} \ .$$

Differentiating shows that Φ is strictly decreasing with p if and only if

$$\frac{\Psi'(p)}{2 - \Psi(p)} + \frac{\Psi'(p+z)}{\Psi(p+z)} < \frac{f(p+z)}{1 - F(p+z)} - \frac{f(p)}{1 - F(p)} \ .$$

Since F is assumed to have an increasing hazard rate, the right-hand side of the above is non-negative, while if condition (4) holds then the left-hand side is strictly negative. Therefore, Φ is strictly decreasing and Proposition 3 implies the result. ■

Calculations for Example 3: Suppose that the price for either product on its own is p and the discount for buying both products is δ (so the total charge for the bundle is $2p - \delta$). Assume that $0 \le \delta \le p$. Then the pattern of demand can be understood with the help of the Figure 6. Here, in region A_i consumers buy product $i = 1, 2$ on its own for sure, in region B_i consumers either buy the bundle or buy product i on its own, and in region C_i consumers either buy the bundle or nothing (and the superior stand-alone product is product i).

Consider a point (v_1, v_2) in region B_1. What fraction of these consumers with stand-alone valuations (v_1, v_2) buy only product 1? Since $v_1 - p \ge 0$ and $v_1 - p \ge v_2 - p$, it is clear that the consumer will either buy the bundle or product 1 alone. The consumer prefers to buy product 1 alone if $v_1 - p \ge v_b - (2p - \delta)$. i.e., if $v_b \le v_1 + p - \delta$. Since for these consumers v_b is uniformly distributed on the interval $[v_1, v_1 + v_2]$, and $v_2 \geq p - \delta$ in this region, the fraction of these consumers who buy only product 1 is $\frac{p-\delta}{v_2}$, and the rest buy the bundle. It follows that the total fraction of consumers (including those in region A_1) who buy only product 1 with these prices is

$$Q_1(p, p, \delta) = (p - \delta)(1 - p) + \iint_{B_1} \frac{p - \delta}{v_2} dv_1 dv_2 \ .$$

(The same expression holds for Q_2.)

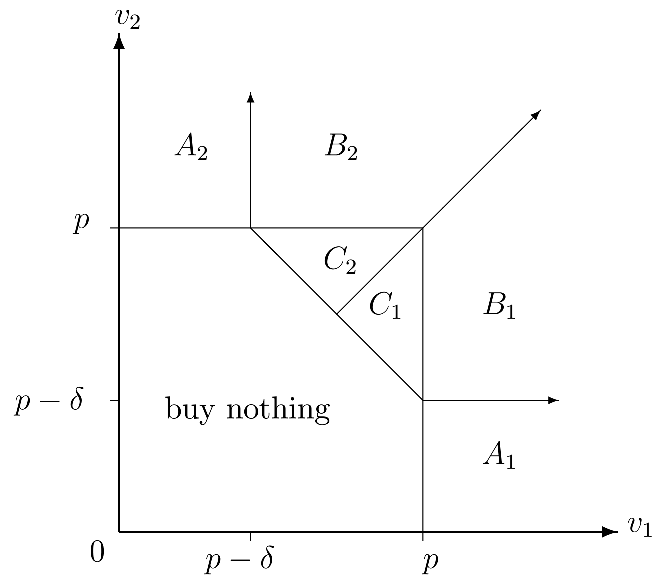

Figure 6: Pattern of demand in Example 3

Consider next the consumers with stand-alone valuations (v_1, v_2) which lie in region C_1. Since both $v_1 < p$ and $v_2 < p$, the only relevant choice is whether the consumer buys the bundle or nothing at all. The former is the better option whenever $v_b \geq 2p - \delta$. Since v_b is uniformly distributed on $[v_1, v_1 + v_2]$, and $v_1 \leq 2p - \delta \leq v_1 + v_2$, it follows that a fraction $\frac{v_1 + v_2 - (2p - \delta)}{v_2}$ of such consumers will choose the bundle. Therefore, the total fraction of consumers who buy the bundle (including those in region B_1, B_2 and C_2) is

$$Q_b(p, p, \delta) = 2 \times \left(\iint_{B_1} \frac{v_2 - (p - \delta)}{v_2} dv_1 dv_2 + \iint_{C_1} \frac{v_1 + v_2 - (2p - \delta)}{v_2} dv_1 dv_2 \right) \ .$$

(The factor 2 is introduced in the above expression include the regions B_2 and C_2, which by symmetry are equal to B_1 and C_1.) These integral expressions for Q_1 and Q_b can be written as explicit, if tedious, functions of p and δ. The linear price which maximizes industry profit therefore maximizes $2p[Q_1(p, p, 0) + Q_b(p, p, 0)]$, while the bundling tariff which maximizes profit maximizes the expression $2pQ_1(p,p,\delta)+(2p-\delta)Q_b(p,p,\delta)$, and these tariffs are reported in the main text in section 4.

Turning next to the analysis when separate firms supply the two products, note that given the symmetric single-product price p, firm 1's profit when it chooses unilateral discount δ_1 and the rival chooses discount δ_2 is

$$pQ_1(p,p,\delta_1+\delta_2)+(p-\delta_1)Q_b(p,p,\delta_1+\delta_2) \ ,$$

and thus the first-order condition for the symmetric equilibrium discount, given p, can be derived from the above expressions for Q_1 and Q_b. However, the calculation of the equilibrium price p (given aggregate bundle discount δ) cannot be deduced from these expressions, as we need the impact on demand of a rise in the price p_1 for product 1 on its own, keeping $p_2=p$ fixed. However, careful examination of the regions in Figure 6 reveals that

$$-\frac{\partial Q_1}{\partial p_1}\bigg|_{p_1=p_2=p} = p-\delta+\int_{p-\delta}^{1}\frac{p-\delta}{v}dv \ ;$$

$$-\frac{\partial Q_b}{\partial p_1}\bigg|_{p_1=p_2=p} = \iint_{B_2}\frac{1}{v_2}dv_1dv_2+2\iint_{C_1}\frac{1}{v_2}dv_1dv_2 \ .$$

Again, these expressions have explicit form, and can be used to derive the equilibrium stand-alone price p given δ, which satisfies the first-order condition

$$Q_1+Q_b+p\frac{\partial Q_1}{\partial p_1}+(p-\frac{1}{2}\delta)\frac{\partial Q_b}{\partial p_1}=0 \ . \tag{27}$$

The equilibrium linear price is obtained from expression (27) by setting $\delta=0$. The equilibrium tariff with and without bundling are reported in the main text in section 5.

Proof of Proposition 6: From Figure 1, we see that

$$q_i(p_i,p_j)=\int_{p_i}^{\infty}H_j(p_j\mid v_i)f_i(v_i)dv_i \ ; \ q_b(p_i,p_j)=\int_{p_i}^{\infty}(1-H_j(p_j\mid v_i))f_i(v_i)dv_i \tag{28}$$

and

$$-\frac{\partial q_i}{\partial p_i}=H_j(p_j\mid p_i)f_i(p_i) \ ; \ -\frac{\partial q_b}{\partial p_i}=(1-H_j(p_j\mid p_i))f_i(p_i) \ .$$

Since H_j is assumed to be strictly increasing in v_i, it follows from (28) that

$$q_i(p_i,p_j)>H_j(p_j\mid p_i)(1-F_i(p_i)) \ ; \ q_b(p_i,p_j)<(1-H_j(p_j\mid p_i))(1-F_i(p_i))$$

and so
$$-\frac{1}{q_i}\frac{\partial q_i}{\partial p_i} < \frac{f_i(p_i)}{1-F_i(p_i)} < -\frac{1}{q_b}\frac{\partial q_b}{\partial p_i}$$
and Proposition 2 implies the result. ■

Proof of Proposition 7: If F and f are respectively the c.d.f. and density for each valuation v_i, by examining Figure 3 we see that
$$-\frac{\partial q_b}{\partial p_1} = f(p+z)(1-F(p+z))$$
and
$$-\frac{\partial q_1}{\partial p_1} = f(p)F(p) + \int_p^{p+z} (f(v))^2 dv$$
(where these derivatives are evaluated at symmetric prices $p_1 = p_2 = p$). At the symmetric price p we have
$$q_b = (1-F(p+z))^2 \ ; \ q_1 = \tfrac{1}{2}\left(1-(F(p))^2-(1-F(p+z))^2\right) \ .$$
We need to show that inequality (13) holds so that Proposition 2 can be applied.

Since F has an increasing hazard rate in (14), we have
$$\begin{aligned}\int_p^{p+z} (f(v))^2 dv &= \int_p^{p+z} \frac{f(v)}{1-F(v)} f(v)(1-F(v))dv \\ &\leq \frac{f(p+z)}{1-F(p+z)} \int_p^{p+z} f(v)(1-F(v))dv \\ &= \frac{1}{2}\frac{f(p+z)}{1-F(p+z)}\left((1-F(p))^2-(1-F(p+z))^2\right) \ .\end{aligned}$$
Therefore, a sufficient condition for (13) to hold is that
$$\frac{f(p+z)}{1-F(p+z)} > \frac{2f(p)F(p) + \frac{f(p+z)}{1-F(p+z)}\left((1-F(p))^2-(1-F(p+z))^2\right)}{1-(F(p))^2-(1-F(p+z))^2}$$
which can be rearranged to give
$$\frac{f(p+z)}{1-F(p+z)} > \frac{f(p)}{1-F(p)} \ .$$
Since F has a strictly increasing hazard rate, the claim is established. ■

Proof of Proposition 8: By examining Figure 4, we see that
$$-\frac{\partial q_b}{\partial p_1} = \lambda f(1-F) \ ; \ q_b = \lambda(1-F)^2$$

and

$$-\frac{\partial q_1}{\partial p_1} = fF + (1-\lambda)\int_p^\infty (f(v))^2 dv \ ; \ q_1 = \lambda F(1-F) + \tfrac{1}{2}(1-\lambda)(1-F^2)$$

(where these expressions are evaluated at symmetric prices $p_1 = p_2 = p$ and the dependence of f and F on p is suppressed). We need to show that inequality (13) is reversed.

Since F has an increasing hazard rate, we have

$$\begin{aligned}
\int_p^\infty (f(v))^2 dv &= \int_p^\infty \frac{f(v)}{1-F(v)} f(v)(1-F(v))dv \\
&> \frac{f}{1-F}\int_p^\infty f(v)(1-F(v))dv \\
&= \tfrac{1}{2}\frac{f}{1-F}(1-F)^2 \\
&= \tfrac{1}{2}f(1-F) \ .
\end{aligned}$$

Thus (13) is reversed whenever

$$\frac{f}{1-F} < \frac{2fF + (1-\lambda)f(1-F)}{2\lambda F(1-F) + (1-\lambda)(1-F^2)}$$

which some rearranging shows to be always the case provided $\lambda < 1$. ■

Proof of Proposition 9: Firm i's profit under the proposed joint-pricing scheme is

$$(p_i - c)(Q_i + Q_b) - \tfrac{1}{2}\delta Q_b \ . \tag{29}$$

The impact of introducing a small $\delta > 0$ on firm i's equilibrium profit is therefore governed by the sign of

$$\frac{d}{d\delta}\left\{(p_i - c)(Q_i + Q_b) - \tfrac{1}{2}\delta Q_b\right\}\Big|_{\delta=0}$$

$$= \frac{dp_i}{d\delta}\frac{\partial}{\partial p_i}[(p_i - c)(Q_i + Q_b)]\Big|_{\delta=0,p_i=p_i^*} + \frac{dp_j}{d\delta}\frac{\partial}{\partial p_j}[(p_i - c)(Q_i + Q_b)]\Big|_{\delta=0,p_i=p_i^*} \tag{30}$$

$$-\tfrac{1}{2}Q_b\big|_{\delta=0} + (p_i^* - c)\frac{\partial}{\partial\delta}(Q_i + Q_b)\Big|_{\delta=0}$$

$$= -\tfrac{1}{2}q_b - (p^* - c)\frac{\partial q_b}{\partial p_i} \tag{31}$$

(where this final expression is evaluated at optimal linear price p^*). Here, the terms in line (30) vanish, the first because p^* is the optimal price for firm i when firms choose linear prices (i.e., p_i maximizes $(p_i - c)(q_i + q_b)$), and the second because changing the other firm's price has no impact on a firm's demand when there is no bundling discount (i.e., $q_i + q_b$ does not depend on p_j when values are additive). The final expression follows from (7). Following by-now familiar arguments, the term (31) is strictly positive if and only if (4) holds.

Consider next the impact of the joint pricing scheme on total welfare. To calculate this we need to understand how the introduction of δ affects equilibrium prices p_i. Firm i's profit is given by (29) and so the first-order condition for p_i given δ (and p_j) is

$$Q_i + Q_b + (p-c)\frac{\partial(Q_i+Q_b)}{\partial p_i} - \frac{1}{2}\delta\frac{\partial Q_b}{\partial p_i} = 0 \ . \tag{32}$$

This expression then determines the symmetric stand-alone price $p(\delta)$ as a function of the discount δ. Totally differentiating (32) with respect to δ yields

$$\begin{aligned} 0 = &\frac{\partial(Q_i+Q_b)}{\partial\delta} + 2p'\frac{\partial(Q_i+Q_b)}{\partial p_i} + p'\frac{\partial(Q_i+Q_b)}{\partial p_j} \\ &+(p-c)\left[\frac{\partial^2(Q_i+Q_b)}{\partial p_i\partial\delta} + p'\frac{\partial^2(Q_i+Q_b)}{\partial p_i^2} + p'\frac{\partial^2(Q_i+Q_b)}{\partial p_i\partial p_j}\right] - \frac{1}{2}\frac{\partial Q_b}{\partial p_i} \ , \end{aligned}$$

where $p' = \frac{d}{d\delta}p(\delta)$. When $\delta = 0$ this simplifies to

$$0 = -\frac{3}{2}\frac{\partial q_b}{\partial p_i} - 2fp' + (p-c)\left[-\frac{\partial^2 q_b}{\partial p_i^2} - p'f'\right] \ . \tag{33}$$

Note that

$$-\frac{\partial q_b}{\partial p_1} = f(p_1)(1 - H(p_2 \mid p_1))$$

and so

$$\begin{aligned} -\left.\frac{\partial^2 q_b}{\partial p_1^2}\right|_{p_1=p_2=p} &= f'(p)(1-H(p\mid p)) - f(p)\frac{\partial}{\partial p_1}H(p_2 \mid p_1) \\ &\le f'(p)(1-H(p \mid p)) \\ &= -\frac{f'(p)}{f(p)}\frac{\partial q_b}{\partial p_1} \ , \end{aligned} \tag{34}$$

where the inequality follows when $H(p \mid v)$ weakly increases with v. Thus, expression (33) implies

$$\begin{aligned} [2f + (p-c)f']p' &= -\frac{3}{2}\frac{\partial q_b}{\partial p_i} - (p-c)\frac{\partial^2 q_b}{\partial p_i^2} \\ &\le -\frac{\partial q_b}{\partial p_i}\left[\frac{3}{2} + \frac{f'}{f}(p-c)\right] \\ &\le -\frac{\partial q_b}{\partial p_i}\left[2 + \frac{f'}{f}(p-c)\right] \\ &= -\frac{1}{f}\frac{\partial q_b}{\partial p_i}\left[2f + f'(p-c)\right] \ . \end{aligned} \tag{35}$$

Here, the first inequality follows from (34), and the second follows from the fact that $\frac{\partial q_b}{\partial p_i}$ is negative. Since the term $[2f + f'(p-c)]$ is strictly positive due to the second-order condition for p to be the equilibrium price when $\delta = 0$ (the second-order condition is sure to be satisfied given (14)), we deduce that

$$fp' \leq -\frac{\partial q_b}{\partial p_i} \ . \tag{36}$$

By inspecting Figure 5, one can see that the impact of a small discount δ on total welfare is equal to

$$W' = 2f(p)(p-c)\left\{(1 - H(p \mid p))(1-p') - H(p \mid p)p'\right\} \ .$$

(Here, the first term represents the welfare gain when more single-item consumers buy two items, as the incremental cost of the second item falls to $p(\delta) - \delta$, while the second term represents the welfare loss when some single-item consumers decide to buy nothing due to the price rising to $p(\delta)$.) This welfare change has the sign of

$$f\left\{1 - H - p'\right\} = -\frac{\partial q_b}{\partial p_i} - fp' \geq 0 \ ,$$

where the inequality follows from (36). Thus, when $H(p \mid v)$ weakly increases with v, the joint pricing scheme will increase total welfare when δ is small. ■