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Investible Benchmarks & Hedge Fund Liquidity*

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Abstract

A lack of commonly accepted benchmarks for hedge fund performance has permitted hedge fund managers to attribute to skill returns that may actually accrue from market risk factors and illiquidity. Recent innovations in hedge fund replication permits us to estimate the extent of this misattribution. Using an option-based model, we find evidence that hedge fund returns actually reflect the value of liquidity options that investors grant managers at below market values. Coupled with the competition from hedge fund replication vehicles, this analysis may motivate hedge fund managers to relax their redemption terms.

Introduction

Good investments outperform relevant benchmarks. Thanks to Treynor (1965), Sharpe (1966), and Jensen (1969), who pioneered that field of study decades ago, investors in conventional assets have well established means of testing the hypothesis that truism implies. Largely because of their work, investors in equity mutual funds compare their results to investible performance indices of the markets in which their funds invest. Research by Markowitz (1953), Tobin (1958), and Sharpe (1964) established comparable paradigms for fixed income investments. As a result, bond investors measure nominal returns against historic term and risk premia, and real returns against inflation.

Alternative assets lack well-defined and widely accepted performance benchmarks. Consider hedge funds. The many indices that track hedge fund performance suffer a common set of problems that limits their utility as standards of performance. These include survivor and backfill biases in databases, a lack of consensus on classification, and the inability to invest in an index proxy. This paper argues that the recent growth of investible factor-based hedge fund replication vehicles begins to solve this problem. It also claims that the liquidity of such vehicles creates a framework for quantifying the price of hedge fund illiquidity.

Decomposing Hedge Fund Returns

In the absence of a consensus on accurate benchmarks for hedge funds, most consultants and professional allocators have limited their correlation analysis to broad long-only benchmarks such as equity indices. This has allowed hedge fund managers to claim that returns not correlated with these broad indices constitute evidence of their skill. They define these idiosyncratic returns as *alpha*.

Recent developments may alter standards for analyzing hedge fund returns, at least for some hedge fund strategies. For years, some analysts have reported and some investors have recognized the magnitude of hedge fund returns attributable to measurable risk factor exposures. Brown and Goetzmann (2001), Fung & Hseih (1997, 2000, 2004), Al-Sharkas (2005) and others have documented this extensively. Often called exotic *beta*, hedge fund *beta* or something similar, these statistical artifacts have served to date primarily as measures of correlation that true believers shun in a quixotic quest for the elusive *alpha* particle that allegedly shares the same subatomic return space.

At the most general level then, hedge fund returns comprise some idiosyncratic returns, some known and measurable returns, and some other "stuff" that in a linear regression of hedge fund returns and risk factors appears as statistical noise.

For a single hedge fund, we may describe this more formally as

$$R^f = \alpha^f + B^f X_T + \epsilon^f \tag{1}$$

where

$$B^f = [\beta_1^f \quad \beta_2^f \quad \cdots \quad \beta_n^f] \tag{2}$$

and

$$X_T = [X_T^1 \quad X_T^2 \quad \cdots \quad X_T^n] \tag{3}$$

In words, the returns of a hedge fund comprise its uncorrelated non-random returns, the correlation-weighted non-random returns of *n* known risk factors computed over time period *T*, and some random returns. The correlations B^f apply only to the specific fund *f* while the regressors X_T represent the returns of a single set of risk factors to which all funds may exhibit sensitivity. In any period, correlation coefficients or regressors may hold positive or negative values. In terms of expectations, however, the standard linear regression model requires that the expected value of the random returns equal zero.

The regression described above produces estimates of a fund's α^f and B^f from time $t = 0$ to time $t = T$. Conventionally, one would interpret B^f as a measure of the correlation of returns attributable to known variables; $\alpha^f > 0$ would indicate that the manager has demonstrated some skill, and $\alpha^f < 0$ a lack thereof. A skeptic may question the assertion that positive *alpha* equals skill because it may simply correspond to risk factors excluded from the regression. Such a claim may overstate a manager's insight, and undervalue its portfolio management skills which determined its risk factor exposure.

Using this information to establish a benchmark holds more promise for fund evaluation. It requires only a small but subtly different interpretation of the regression results. From this perspective, we consider the quantity $B^f X_T$ a return available from known risk factors and, as such, a benchmark for the period from 0 to *T*. To observe the over or under-performance of the fund, we rearrange (1) to obtain

$$\alpha_T^f + \epsilon_T^f = R_T^f - B^f X_T \tag{4}$$

We measure the difference between the return of the fund and the benchmark only in terms of its random and non-random components. We do not define any of it as skill, or a lack thereof, because true skill encompasses the ability to manage a portfolio's risk factor allocation as well as its idiosyncratic security selection. Since $E[\epsilon_T^f] = 0$, a fund outperforms its benchmark only when $\alpha_T^f > 0$ or

$$R_T^f > B^f X_T \tag{5}$$

Using such a proxy as a benchmark for the correlated portion of a fund's expected return establishes it as a floor on the minimum return an investor should expect from a fund over a time period of length *T*. In practice, this approach still has only limited value because most individual funds exhibit low correlations with risk factors. Moreover fund-specific benchmarks do not permit us to compare funds fairly with each other.

Constructing Strategy Benchmarks with Risk Factors

In their efforts to avoid correlation with known *betas*, hedge fund analysts and investors have used regression primarily to estimate correlation while overlooking the possibility of using the identifiable components of fund returns as benchmarks. Until recently, they may have done so as well because one could not invest directly in many of the risk factors identified in regression analyses of hedge fund returns. Absent the ability to invest in risk factors correlated with hedge fund returns, analysts reasonably declined to cite them as benchmarks for hedge fund performance.

By themselves, even investible risk factor proxies function poorly as benchmarks. Comparing hedge fund returns to those of multiple risk factors creates a one-to-many relationship between a hedge fund and a set of risk factor proxies. Knowing a fund's correlations with a range of risk factors tells us something about its historic exposures but little, in aggregate, about its performance relative to its peers. Unfortunately, such analysis adds more confusion than insight to the process of performance assessment.

Bundling risk factor allocations according to their degree of correlation with large universes of hedge funds mitigates this problem to a significant extent. The recent proliferation of listed ETFs, ETNs and futures contracts has expanded the universe of investible risk factor proxies and accordingly reduced the constraint on using them to construct legitimate benchmarks for hedge fund returns. Following Hasanhodzic & Lo (2007), we may construct such a benchmark from investible risk factor proxies by weighting them by their correlation with the fund's returns over a specified period.

Risk Factor Correlation at the Strategy Level

We characterize funds with similar correlations to common sets of risk factors as hedge fund strategies. Applying the same regression methodology described above to such strategy universes produces robust correlations for most of the major categories into which we tend to classify hedge funds. For clarity, we rewrite the equations above with slightly different notation. The regression becomes

$$R^s = \alpha^s + B^s X_T + \epsilon^s \tag{6}$$

where

$$B^s = [\beta_1^s \quad \beta_2^s \quad \cdots \quad \beta_n^s] \tag{7}$$

While identical to (1) and (2) but for a change in notation, the results of this regression require a different interpretation from that given for (1). The regression results

$$\hat{B}^s = [\hat{\beta}_1^s \quad \hat{\beta}_2^s \quad \cdots \quad \hat{\beta}_n^s] \tag{8}$$

quantify the average exposure of all funds in the universe to a common set of risk factors. As such, they identify the factors that dominate the aggregate behavior of the funds in the universe of similar funds. At the same time, they highlight the insignificance to the universe as a whole of factors that may explain a lot of the behavior of only a few of the funds in the universe.

We measure the goodness-of-fit of a regression, roughly the percentage of the behavior of strategy returns R^s explained by the risk factors $\hat{B}^s X_T$, by its so-called R-square coefficient. The following table shows the R-square values of regressions of returns of some major hedge fund strategies against baskets of investible risk factor proxies.

Hedge Fund Strategy[1]	**R-square**[2]
Long/Short Equity North America	95%
Event Equity	89%
Long/Short Equity Emerging Markets	84%
Distressed Investment	79%
Credit Strategies	78%
Convertible Bond Arbitrage	62%
Market Neutral Equity	58%
Directional Macro	55%
Statistical Arbitrage	53%
CTAs	34%
Volatility Arbitrage	13%

The data suggests clearly that portfolios of risk factor proxies explain significant percentages of the returns of some strategies, and insignificant portions of others. Funds in strategies with high correlations to investible risk factors tend to have two characteristics in common: an concentration on corporate securities and a low rate of monthly portfolio turnover. In general, we observe that long/short equity and corporate credit strategies exhibit high correlations with investible risk factors. In contrast, relative value and arbitrage strategies, especially those focused on interest rates and foreign exchange, and those with rapid turnover such as Directional Macro and Statistical Arbitrage, exhibit much less correlation with such instruments.

For highly correlated strategies then, we claim that investible portfolios of correlation-weighted risk factor proxies constitute *ex post* benchmarks for significant percentages of the relevant strategies' expected returns. Specifically, to the extent that a strategy's aggregate *beta* $\hat{B}^s X_T$ accounts for a relatively stable percentage of its returns over time, a time series of returns of a portfolio designed to mimic that *beta* constitutes a lower bound on the performance one might have reasonably expected to earn from investments in that strategy over that period.

Using Replication Benchmarks to Price Hedge Fund Liquidity

Accounting for the explanatory variables of a regression of strategy level returns still leaves us without an explanation for the α^s, i.e. the non-random portion of the strategy returns not correlated with known risk factors. Clearly it does not describe *skill* since we cannot apply such a concept in the aggregate, a point that should raise questions about its use for that purpose at the fund level. Excluding skill as a possible meaning for α^s requires us to develop alternative explanations for it. By definition, we know that it may simply refer to unidentified risk factors not included among the X_T. Until we can identify them, we cannot incorporate them into the benchmark. No investible risk factor proxy exists for α^s. An insight into the features of an investible benchmark suggests a more tangible and significant alternative.

By their construction, factor-based replication funds lack the ability to generate *alpha*. Unlike hedge

1 The authors have classified funds in the hedgefund.net™ database into the listed strategies based on the commonality of their risk factor correlations.

2 Based on regressions of monthly performance for all funds in the universes for any consecutive twelve month period from January 2, 1999 through March 31, 2011.

funds, they can explain 100% of their returns at all times in terms of the performance of the risk factor proxies in which they invest. They differ from the hedge funds whose strategies they attempt to mimic in other ways as well. Specifically, they can offer investors daily mark-to-market transparency and liquidity. A large investor in a replication fund that invests only in listed ETFs, ETNs, and futures contracts, for example, can close out such an investment in no more than a few hours. Smaller ones can convert their investments to cash in a matter of minutes.

In contrast, few hedge funds offer their investors even monthly liquidity. Most offer quarterly liquidity with significant notice periods. Some place even stricter limits on withdrawals. Many impose lockup periods of one or more years on new investments. Some permit "early" redemptions only with the payment of an exit fee. At the same time managers retain the right to return capital to investors at any time.

Chacko (2005) finds evidence that liquidity premia account for statistically significant portions of returns of corporate bonds. In his analysis, he introduces a concept of *latent liquidity* or *accessibility* that depends essentially on the willingness of bond holders to sell, what one might also call *behavioral liquidity*. In contrast, this paper examines *contractual liquidity* between a hedge fund investor and a hedge fund manager. To reconcile these two phenomena, we recognize behavior as a contract with oneself. For example, an insurance company with an investment policy that limits reviews of holdings to quarterly periods has essentially imposed on itself a policy of quarterly liquidity.

The obvious differences in liquidity between factor-based replication funds and hedge funds suggest that a liquidity premium accounts for at least some portion of α^s, the excess returns observed at the strategy level. Surely a hedge fund investor subject to limited liquidity should earn more than an investor in a fund with more frequent liquidity that replicates only that hedge fund's known risk factors. Excess returns α^s observed at the strategy level then should include both idiosyncratic performance and compensation for the liquidity that hedge fund investors forego relative to investors in replication funds.

Chacko, Das & Fan (2011) model *ex post* behavioral illiquidity as a sum of two American options, one call and one put. To answer the *ex ante* question of how much of a liquidity premium an investor should expect to earn in exchange for entering into a contract that limits liquidity for the convenience of a hedge fund manager, we present a similar option-based model similar to those of Longstaff (1995) and Koziol & Sauerbier (2007) that contrasts the quasi-continuous liquidity of equity markets with scheduled liquidity of hedge funds. Like Longstaff and Koziol & Sauerbier, our model posits a perfectly liquid proxy against which we compare an identical asset with intermittent liquidity.

Continuous liquidity in a replication fund corresponds to the right to sell the fund at any time. In option terms we may describe this as the right to put the replication fund back to the market continuously. Mathematically we may express this value as a sum of an infinite series of at-the-money put options of infinitesimal duration[3]. Let λ^s represent the value of continuous liquidity available to a replication fund investor.

$$\lambda^s = \int_0^\infty P^s(t) \tag{9}$$

3 Appendix A contains a detailed explanation of the theory that allows us to estimate liquidity premia with option prices.

Evaluating this integral lies beyond the scope of this paper. Fortunately, we know from elementary calculus that it equals or exceeds the value of a sum of discrete at-the-money options of measurable duration, i.e.

$$\int_0^\infty P^s(t) \geq \sum_{t=0}^{T} P^s(t) \tag{10}$$

We will use the right hand term of this equation to estimate the value of the liquidity of a replication fund offering daily liquidity.

In contrast to the replication fund, we may value the periodic liquidity of a hedge fund as a single European put option expiring on the fund's redemption date. We let λ^f represent the value of this option *ex ante,* i.e. at the time of investment.

$$\lambda^f = P^f(T) \tag{11}$$

We cannot know *ex ante* the true at-the-money strike price of this option because we cannot know the value of the fund at time *T*. If we believe, however, that funds are profitable more often than not, then we may assume that an at-the-money put struck at the inception of an investment period will be at least slightly out of the money on most redemption dates. Thus λ^f will underestimate slightly the *ex post* value $\hat{\lambda}^f$ if we value it as an at-the-money put option struck at the beginning of an investment period that expires on the redemption date allowed by the fund.

$$\lambda^f \leq \hat{\lambda}^f = P^f(T) \tag{12}$$

Thus the actual put option an investor holds at the end of an investment period has more value than that implied by an at-the-money put struck at the beginning on an investment period. Since the same logic applies to the values of the replication fund's continuous puts of infinitesimal duration, i.e. their true strike price actually changes with an upward bias over time, this has no meaningful effect on the analysis.

To estimate the value of the liquidity denied by a hedge fund with liquidity available only at time *T* in comparison to that of a replication fund offering continuous liquidity, we simply take the *ex ante* difference between the two option values.

$$\Lambda^T = \lambda^s - \lambda^f = \sum_{t=0}^{T} P^s(t) - P^f(T) \tag{13}$$

This expression captures the effect of an investor exchanging daily liquidity for a single fixed redemption date.

Estimated Values of Hedge Fund Illiquidity

We use the standard discrete Black-Scholes option pricing model to price European puts of different durations. To value the sum of one day at the money puts, we discount each one by the appropriate forward rate so as not to overstate the combined value of the puts. Of course the term put $P^f(T)$ requires only a single straight-forward calculation. The model produces some startling results when one compares the value of daily liquidity to that of almost any other term. They appear considerably more reasonable when one compares longer redemption periods to each other. Significantly, our results resemble those of Longstaff (1995) who commented in his paper that "discounts for lack of marketability can be large even when the length of the marketability restriction is very short."[4]

To compare different liquidity regimes, we compare different liquidity preferences with different liquidity profiles. For example, we compute the value to an investor seeking monthly liquidity of investments with quarterly and annual liquidity. This approach allows us to map the liquidity premia that investors should demand for investments that exceed their preferred liquidity.

We assume annualized volatility of 8% for both funds, a number similar to the long-term level observed for funds that replicate the performance of US-focused long/short equity hedge funds, and an annual risk-free rate of 2%, a more realistic estimate than that imposed by current Fed policy. Because liquidity premia vary directly with volatility and inversely with the risk-free rates, investors should demand higher liquidity premia for more volatile assets with equal volatility and accept lower ones from all assets in higher interest rate environments.

Yield Compensation for Less-than-preferred Liquidity							
% per deferral period		**Preferred Liquidity**					
		Daily	Weekly	Monthly	Quarterly	Semi-Annual	Annual
Actual Liquidity	Daily	0					
	Weekly	0.73%	0				
	Monthly	4.10%	0.98%	0			
	Quarterly	13.60%	4.15%	1.18%	0		
	Semi-Annual	28.12%	9.22%	3.28%	0.93%	0	
	Annual	57.71%	19.79%	7.89%	3.16%	1.30%	0

As a caveat, we recognize that the extreme sensitivity of option prices to volatility means that hedge funds could justify their restrictive redemption terms easily if they needed to compare their performance only to more liquid investments with much lower volatility. For example, an investor would be indifferent between an investment with continuous liquidity and an annualized volatility of just over 0.8% and a hedge fund with quarterly liquidity and an annualized volatility of 8%. That does not apply, however, to investible hedge fund replication strategies with volatilities similar to those of hedge funds themselves. In general, the volatility differentials required to equate typical hedge fund redemption terms with far more liquid alternatives exceed by far those available to investors today. In other words, investors have available to them many investment options with risk-return profiles similar to those of hedge funds that offer far more attractive liquidity than hedge funds.

To conclude, the incremental return required to make an investor indifferent between two investments that differ significantly only in their liquidity terms seems reasonable when one compares, for example, quarterly to annual liquidity. The extreme differences between the value of daily liquidity and any other term raises, however, serious questions about the practice of funds offering less liquidity than those of

4 Longstaff (1995), pg. 1768.

the assets in which the invest. Specifically, with respect to the primary topic of this article, funds that replicate hedge fund returns, these results raise serious questions about the restrictions managers place on redemptions relative to the α^s they appear to generate at the strategy level, and by implication the α^f they produce individually.

Conclusions

The contrasting results of risk factor regressions on individual funds and on collections of funds classified as "strategies" force hedge fund investors to assess their commitment to the paradigm of strategy classification of hedge funds. An investor who views hedge funds as unique investment vehicles for which strategy classifications serve merely an accounting function may discount the value of bundles of risk factor proxies. In contrast, an investor who acknowledges that strategy classifications properly reflect similarities in the behavior of many hedge funds may embrace the notion of factor-based replication vehicles as benchmarks for their allocations to the corresponding hedge fund strategies.

Recognizing factor-based replication funds as legitimate benchmarks permits us to view the uncorrelated non-random portion of strategy level returns as a proxy for differences in liquidity between replication funds, and the funds whose correlated non-random returns they emulate. Using an option model, we observe that daily liquidity has a value far in excess of any excess returns one might expect to earn from most hedge funds with limited liquidity.

These results suggest that hedge fund managers in strategies that investors can emulate to a significant extent with more liquid alternatives may need to relax their redemption terms as replication funds grow in popularity. More significantly, it implies that most hedge fund returns come not from manager skill but from the value of options that investors, when they invest in a hedge fund, "sell" implicitly to fund managers at prices far below their market values.

Appendix A

Deriving Liquidity Premia from Option Prices

Since the arrival of the Black-Scholes option pricing model (1975), practitioners of academic and applied finance have focused their research on volatility and on the effects of irregular cash flows. Invariably they have viewed *time*, one of the five variables required to price options, as given, or, in comparing option prices for different assets, as equal for each asset.

In this paper we argue that an investment manager may only claim to have produced excess returns over a benchmark after compensating investors for any differences in liquidity between the managed asset and the benchmark. The main body of the paper contains the argument for treating an investible replication fund as a benchmark but presents only an abridged justification for using the option pricing model to evaluate the differences in liquidity between a benchmark fund and a hedge fund to for which it serves as a reference. This appendix contains a more complete explanation of this approach.

To begin, we invert the normal representation of options as derivatives of some financial asset. We argue, instead, that financial assets themselves lack inherent existence. Instead they derive their value from continuous series of at-the-money options on the real assets whose values they embody. From this perspective, we observe that an exchange-traded equity, for example, comprises an infinite series of at-the-money calls and puts on some real assets. *Stock* is merely a name that we impute to this collection of options. We define this phenomenon as follows:

Definition 1: *A financial asset comprises infinite series of at-the-money calls and puts of infinitesimal duration. All such series have lower bounds of t=0. Series for assets with final maturity dates have upper bounds of t=T; series for assets without a final maturity date have no upper bound.*

Formally we may describe the unbounded scenario with the equation

$$S=\int_0^{\infty} C(t)+\int_0^{\infty} P(t) \tag{A1}$$

Most readers will see instantly how the right of an owner of a share of stock to sell it at any time corresponds to a continuous series of at-the-money put options; but where, one may wonder, are the call options? To see this we introduce two rules: the long rule and the short rule.

Long Rule: *A long position in a financial asset remains a long position until and unless the owner exercises one of the infinite series of put options available to terminate the long exposure. Such put options may be available continuously or intermittently. The asset owner implicitly exercises each of the infinite series of calls at every moment that it does not choose to exercise a put to negate the long exposure.*

Short Rule: *A short position in a financial asset remains a short position until and unless the owner exercises one of the infinite series of call options available to terminate the short exposure. Such call options may be available continuously or intermittently. The holder of the short position implicitly exercises each of the infinite series of puts at every moment that it does not choose to exercise a call to negate the short exposure.*

We may incorporate these rules into Definition 1 to define long and short positions in terms of the characteristics of the options they comprise.

Definition 1a: *A long position in a financial asset comprises an infinite series of at-the-money call options of infinitesimal duration and put options of infinitesimal or intermittent duration with the call options deemed exercised automatically unless the holder exercises one of the put options to liquidate the position.*

Definition 1b: *A short position in a financial asset comprises an infinite series of at-the-money put options of infinitesimal duration and call options of infinitesimal or intermittent duration with the put options deemed exercised automatically unless the holder exercises one of the call options to cover the position.*

From this perspective, both owners and short sellers of assets buy options continuously. Long holders pay their option premia implicitly with a combination of the foregone interest on the cash paid for the asset, and some portion of its future returns. *Ceteris paribus*, a long position has a gain when

$$S_{t+\eta} > X_t \rightarrow \pi > 0 \tag{A2}$$

where S equals the market price of the real assets underlying the options at time $t+\eta$, X the strike price of the at-the-money options at time t, π the profit on the position and η, an infinitesimal amount of time past t. Similarly

$$S_{t+\eta} < X_t \rightarrow \pi < 0 \tag{A3}$$

Clearly the reverse conditions apply to short positions. Thus in this paradigm, returns on financial assets equal the aggregate differences between the value of all the options they comprise from the time one acquires them (or establishes a short position in them), and the cost of those options that expire while one owns them (or maintains a short position in them).

To apply this analysis to the benchmarking example at issue in this paper, we let S_b represent the price of the benchmark replication fund and S_f the price of a hedge fund claiming to offer returns in excess of the benchmark. Because the volatility of the investible benchmark, and the funds it references should be similar, we assume that only redemption terms differentiate S_b from S_f with the former offering continuous liquidity and the latter liquidity only on a specific date T. Then following (A1) we may describe the price of the benchmark fund with the following equation.

$$S_b = \int_0^\infty C(t) + \int_0^\infty P(t) = \int_0^\infty C(t) + \int_0^T P(t) + \int_{T+\eta}^\infty P(t) \tag{A4}$$

We describe the hedge fund in equation (A5).

$$S_f = \int_0^\infty C(t) + P(T) + \int_{T+\eta}^\infty P(t) + \Lambda_L^T \tag{A5}$$

where Λ_L^T represents the value of the liquidity differential between a long position in a benchmark with continuous liquidity and the hedge fund that offers liquidity initially only at time T. For the hedge fund to perform at least as well as its benchmark, we must have

$$S_f \geq S_b \rightarrow S_f - S_b \geq 0 \tag{A6}$$

Substituting (A4) and (A5) into (A6) and simplifying the resulting equation produces

$$\int_0^T P(t) - P(T) - \Lambda_L^T \geq 0 \tag{A7}$$

$$\Lambda_L^T \geq \int_0^T P(t) - P(T) \tag{A8}$$

Thus the liquidity premium for a long asset with continuous liquidity over an identical asset with redemption available only at time T equals the value of the continuous infinitesimal at-the-money put options expiring from inception of the position until time T less the value of a single European put option expiring at time T.

A similar analysis comparing a short asset with continuous liquidity with another short asset offering only a single opportunity to cover the short produces a value derived from option prices of

$$\Lambda_S^T = \int_0^T C(t) - C(T) \tag{A9}$$

The liquidity valuation hypothesis assumes only that the hedge fund's volatility equals that of its benchmark. To analyze the hypothesis, we compare at-the-money European put option prices on the benchmark fund and the hedge fund that differ only by their expiration schedules, i.e. the variable of *time*.

We examined the benchmark asset under two scenarios: as a standalone entity with continuous liquidity; and as a benchmark embedded in an asset such as a hedge fund. In using the Black-Scholes option model to price the liquidity of each scenario, we differentiated between them only by using times until expiration. To make the option prices equivalent to percentage values, we set both the underlying price and the exercise price at 100. We used a constant volatility of 8% slightly more than that of the ten year historical level of the benchmark derived from a back test of hedge fund data. We set the risk-free rate at 2%, a more historically reasonable level than that of the post-2008 central bank regime.

Because these two calculations produce different results, and because differences in permissible times for execution correspond to different liquidity profiles, we interpret price differences between options resulting exclusively from differences in the time variable of the option pricing model as reasonable estimates of the relative value of the differences in liquidity schedules.

In analyzing the benchmarking example in this paper, we have assumed that the volatility of the benchmark equals that of the assets to which one would compare it. While this makes sense in the hedge fund example, it need not be true in general. This approach to pricing liquidity should apply to assets that differ in both the frequency with which they trade and in volatility. As we mentioned in the body of the paper, the values of liquidity differentials shrink dramatically when one compares liquid assets with low volatility to illiquid ones with high volatility. In the context of hedge funds and the emerging market for investible benchmarks, however, such differences in volatility exceed by far the levels required to justify the limited liquidity most hedge funds offer their investors.

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