Patentability and knowledge spillovers of basic RD

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Abstract

This study develops an R&D-based growth model with basic and applied research to analyze the growth and welfare effects of two patent instruments: (a) the patentability of basic R&D, and (b) the division of profit between basic and applied researchers. We find that for the purpose of stimulating basic R&D and economic growth simultaneously, increasing the share of profit assigned to basic researchers is more effective than raising the patentability of basic R&D, which has either a negative effect or an inverted-U effect on technological progress. However, a benevolent patent authority requires both patent instruments to achieve the socially optimal allocation in the decentralized economy.

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"Our ambition to build a knowledge-based society and a European Research Area requires a strong science base and high quality human capital. Basic research is the answer to both demands. Today's fundamental research will turn into tomorrow's growth, competitiveness and better quality of life. The US has understood this. The EU is still lagging behind. Ours is a wake-up call: we need to act now to reverse this situation and fill the gap." European Research Commissioner Philippe Busquin.[1]

1 Introduction

Basic (or fundamental) research is an important part of the innovation process by expanding the frontier of human knowledge. However, unlike applied research, it may not lead to immediate marketable applications; therefore, basic research is often underprovided and has to be funded by the public sector.[2] Given the importance of basic R&D, the European Research Council was officially launched in 2007 as the first European funding body to support and promote fundamental research. An important policy lever for incentivizing basic research is the patentability of basic R&D. For example, the European Union directive on biotechnological patents (passed in 1998 and implemented by all of the 27 EU member states by the end of 2006) has increased the patentability of biotechnological inventions in Europe. This directive provides that "inventions ... shall be patentable even if they concern a product consisting of or containing biological material or a process by means of which biological material is produced, processed or used" and that "biological material which is isolated from its natural environment or produced by means of a technical process may be the subject of an invention even if it previously occurred in nature." In other words, biological material could be patentable in Europe as a result of this directive. As for the US, in the court case of Diamond vs. Chakrabarty of 1980, the Supreme Court ruled that genetically modified organisms could be patentable.

Another important example of patentability of basic R&D is the Bayh-Dole Act (also known as the University and Small Business Patent Procedures Act) of 1980. As a result of this Act, universities in the US are granted the right to patent and license the results of federal government funded research. In a comprehensive review of the Bayh-Dole Act, Mowery *et al.* (2004) argue that although this Act is one of the several key factors that

[1] European Research Commission's press release "Commission calls for a boost in basic research." January 15, 2004.

[2] See for example Akcigit *et al.* (2011) for an interesting quantitative analysis.

contributed to the significant increase in patenting and licensing of university inventions, many of the historical contributions from universities to industrial innovation took place without patenting. Furthermore, when universities patent their inventions, other researchers are restricted from using these basic research outputs until the patents expire. Therefore, Mowery *et al.* (2004) conclude that it is important for universities to recommit to the free flow of knowledge that has historically enhanced industrial innovation. However, most OECD countries currently give universities the right to patent their government-funded inventions (OECD 2002).

In this study, we develop an R&D-based growth model with basic and applied research to analyze how the patentability of basic R&D affects economic growth and social welfare. On the one hand, we find that raising the patentability of basic R&D increases patented basic inventions that contribute to economic growth as the conventional argument suggests. On the other hand, it reduces knowledge spillovers because more basic research outputs are patented and become less accessible by other researchers. Given these opposing effects, we find that patentability of basic R&D may have an inverted-U effect on technological progress.[3] The intuition behind this non-monotonic effect is based on the tradeoff between patent protection and knowledge spillovers. Putting it simply, raising the patentability of basic R&D increases patented basic inventions that contribute to economic growth; however, this policy change also decreases the accumulation of what we call "pure knowledge," which is freely available to all researchers. Our analysis reveals that these two opposite forces interact with each other to potentially generate a non-monotonic relationship between patent protection for basic inventions and technological progress. Furthermore, we find that patentability of basic R&D has a monotonically negative effect on economic growth when knowledge spillovers depend only on pure knowledge (but not on patented basic inventions). Intuitively, an increase in the patentability of basic R&D generates a negative effect on the accumulation of pure knowledge as expected and an additional surprising negative crowding-out effect on basic R&D. This crowding-out effect on basic R&D occurs because raising the patentability of basic R&D increases the stock of industrially applicable basic inventions, which in turn improves incentives for applied R&D by so much that basic R&D falls. Therefore, patentability of basic R&D may not be an ideal policy instrument for stimulating basic research.

In addition to the patentability of basic R&D, we also consider a second related patent

[3] Recently, Qian (2007) and Lerner (2009) report empirical evidence that enhancing intellectual property rights protection reduces innovation activities when the protection is already strong. This suggests that the relationship between patent protection and innovation follows an inverted-U shape. See Furukawa (2010) for a review on theoretical models. The present study differs from previous theoretical studies by analyzing a novel mechanism through knowledge spillovers of basic R&D that generate an inverted-U effect of patent rights on innovation.

instrument that is the division of profit between basic and applied researchers. When an applied researcher develops an invention based on a patented basic invention, she has to pay a licensing fee to the patentholder of the basic invention. The profit-division rule in the model captures this licensing arrangement in which the relative bargaining power of the basic and applied researchers is influenced by the relative strength of patent protection on basic and applied inventions. Therefore, it is not unrealistic to treat the profit-division rule as a patent policy lever. We show that unlike the patentability of basic R&D, the share of profit assigned to basic researchers has a monotonically positive effect on the equilibrium growth rate. Intuitively, strengthening the bargaining power of basic researchers stimulates basic R&D without stifling the spillover effects of pure knowledge while increasing the patentability of basic R&D reduces spillovers from pure knowledge. Therefore, strengthening the bargaining power of basic researchers relative to applied researchers may be a superior policy instrument for achieving the dual objectives of stimulating basic R&D and economic growth. However, characterizing the optimal coordination of the two patent instruments, we find that a benevolent patent authority requires both patent instruments to achieve the socially optimal allocation in the decentralized economy.

Our study relates to the R&D-based growth literature; see Romer (1990), Segerstrom *et al.* (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) for seminal studies.[4] This literature emphasizes two fundamental factors for endogenous technological progress. First, the patent institution matters. Without sufficient patent protection, investors would have insufficient incentives to invest resources in R&D activities due to the public nature of knowledge. Another essence is the wide-spread spillover of knowledge, which plays the critical role as a source of long-run economic growth. An inevitable tradeoff emerges between patent protection and knowledge spillovers. Patent protection encourages private incentives for innovation but also limits the wide-spread use of patented inventions. This latter effect weakens knowledge spillovers from pure knowledge. Although patent protection and knowledge spillovers are fundamental for long-run technological progress, there hasn't been much analysis on their tradeoff in the context of an R&D-based growth model. We fill this gap in the literature by explicitly analyzing this tradeoff in an endogenous growth model with basic and applied R&D.

Our study also relates to the strand of literature on R&D-based growth models that distinguish between basic and applied R&D; see Aghion and Howitt (1996) for a seminal study. Subsequent studies by Michelacci (2003), Akiyama (2009) and Acs and Sanders (2011) consider R&D and entrepreneurship as two types of innovative activities instead of basic and applied R&D. Our variety-expanding model is also related to the quality-ladder

[4] See Jones (2005) for a comprehensive review of this literature.

model in Cozzi and Galli (2009, 2011), who consider basic and applied R&D as two stages of innovation. A recent study by Akcigit *et al.* (2011) provides a quantitative analysis on the underinvestment in basic R&D and policy implications of various types of R&D subsidies. Our study complements these interesting studies by analyzing the growth and welfare effects of patentability of basic R&D and its optimal coordination with the profit-division rule.

Finally, our study relates to the literature on patent design for which Nordhaus (1969) provides the seminal analysis on patent length. Scotchmer (2004) provides a comprehensive review on the subsequent developments in this patent-design literature. While studies in this literature are mostly based on a partial-equilibrium framework, our study follows more closely a related macroeconomic literature by providing a dynamic general-equilibrium (DGE) analysis on patent policy. In the macroeconomic literature on patent policy, the seminal DGE analysis on patent length is Judd (1985), who shows that optimal patent length can be infinite.[5] While these studies focus on patent length, a related branch of studies analyzes the growth and welfare effects of other patent instruments in R&D-based growth models. See for example Li (2001) on patent breadth,[6] Cozzi (2001) and Cozzi and Spinesi (2006) on intellectual appropriability, O'Donoghue and Zweimuller (2004) on forward patent protection and patentability requirement, Kwan and Lai (2003), Horii and Iwaisako (2007) and Furukawa (2007, 2010) on patent protection against imitation, Chu (2009) and Chu and Pan (2011) on blocking patents, and Acs and Sanders (2011) on the division of profit between entrepreneurs and inventors. A recent study by Acemoglu and Akcigit (2011) provides an interesting analysis on optimal state-dependent patent protection based on the endogenous technological gap between the leader and followers in an industry. The present paper complements these studies by analyzing the optimal coordination of multiple patent instruments in an R&D-based growth model with basic and applied R&D.[7]

The rest of this study is organized as follows. Section 2 sets up the model. Section 3 analyzes the effects of patent policies on economic growth and social welfare. The final section concludes. All proofs are relegated to Appendix A.

[5] Subsequent studies by Horowitz and Lai (1996), Iwaisako and Futagami (2003), Futagami and Iwaisako (2007) and Chen and Iyigun (2011) show that optimal patent length is usually finite.

[6] See Chu (2010) for a quantitative analysis on the effects of patent breadth on income and consumption inequality in addition to economic growth and also Chu (2011) for a quantitative analysis on uniform versus sector-specific optimal patent breadth in a two-sector quality-ladder growth model.

[7] See also Iwaisako and Futagami (2003) and Palokangas (2011) on optimal patent length and breadth and Chu and Furukawa (2011) on optimal patent breadth and profit division in research joint ventures. However, these studies do not distinguish between basic and applied R&D.

2 A simple R&D-based growth model with basic and applied research

In this section, we extend the seminal R&D-based growth model in Romer (1990) and Rivera-Batiz and Romer (1991) by introducing two types of innovative activities, namely basic R&D and applied R&D. Our model builds on Akiyama (2009) by introducing patentability of basic R&D and spillover effects of pure knowledge. To model the patentability of basic R&D, we assume that some basic research outputs are patentable while others are *not*. The probability that a basic research output is patentable captures the degree of patentability of basic R&D. A basic invention that is not patented becomes pure knowledge, which is freely available to all researchers. An example would be the Black–Scholes (1973) option-pricing formula (see footnote 8 for a more detailed discussion). A patented basic invention might be matched with an applied invention subject to a stochastic process. When this match occurs, the matched inventions generate monopolistic profits. Applied researchers pay a licensing fee to basic researchers by sharing profits subject to a profit-division rule. For simplicity, we assume that patent length is infinite as in the seminal Romer model.

2.1 The basic setup

Consider a continuous-time model, in which there is an infinitely lived representative household. As in Aghion and Howitt (1996), the household inelastically supplies L units of unskilled labor and H units of skilled labor at each date t. Unskilled and skilled labors are used for manufacturing and R&D respectively. The household is endowed with a standard log utility function $U = \int_0^\infty e^{-\rho t} \ln C_t dt$, where $\rho > 0$ is the discount rate and C_t is the consumption of final goods at date t. Final goods are used for consumption only. Given C_t as the numeraire, standard dynamic optimization yields the familiar Euler equation given by $\dot{C}_t/C_t = r_t - \rho$, where r_t is the interest rate. Consumption goods are produced by a standard CES (constant elasticity of substitution) aggregator over a continuum of patented intermediate goods $X_t(i)$ distributed on $[0, N_t]$.

$$C_t = \left(\int_0^{N_t} X_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where N_t is the number of intermediate goods (or industries) and $\varepsilon > 1$ is the elasticity of substitution. Consumption goods firms are perfectly competitive.

Each variety of intermediate goods is monopolistically manufactured by a monopolistic producer who holds the patents on the manufacturing technology for intermediate goods

i. Each unit of intermediate goods is produced with one unit of unskilled labor; therefore, the marginal cost is equal to the wage rate of unskilled labor w_t^u. The monopolistic price for patented intermediate goods i is equal to $[\varepsilon/(\varepsilon-1)]w_t^u$. From profit maximization of consumption goods firms, the conditional demand and profit functions of intermediate goods are as follows.

$$X_t(i) = \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{C_t}{w_t^u N_t} \equiv X_t, \quad \Pi_t(i) = \frac{C_t}{\varepsilon N_t} \equiv \Pi_t. \tag{2}$$

An industrial monopolist i holds the patents on the manufacturing technology of product i and produces $X_t(i) = X_t$ units of intermediate goods earning profit $\Pi_t(i) = \Pi_t$ at each date t. The value of being the monopolist in industry i is $V_t(i) = \int_t^\infty e^{-(R_\tau - R_t)}\Pi_\tau d\tau \equiv V_t$, where $R_\tau = \int_0^\tau r_s ds$ is the cumulative interest rates up to date τ.

2.2 Patented basic inventions and pure knowledge

The economy grows endogenously due to two forms of technological progress. The first growth engine is the accumulation of patented inventions. As mentioned above, an industrial monopolist i holds a pair of well-matched basic and applied inventions. A basic invention is a preliminary idea, which cannot bear any profit *by itself* but may establish the basis for a future invention that generates profits by introducing a new variety of intermediate goods. We call such a profitable invention an applied invention. When an applied invention is developed on a patented basic invention that previously has not been matched, a new industry is introduced into the intermediate goods market by the industrial monopolist holding the pair of basic and applied inventions. This process of patented inventions continuously occurs and contributes towards the variety N_t of intermediate goods in (1). The productivity of consumption goods firms thus increases over time.

The second engine of technological progress is the accumulation of "pure knowledge." We introduce pure knowledge into the model as a by-product of basic research activity in the following stylized manner. With our consideration on the patentability of basic R&D, some basic inventions are randomly chosen to be patentable by the patent institution while others are not patentable. Formally, each newly developed basic invention is patentable with probability ϕ. An unpatented basic invention contributes to pure knowledge that is freely available to all researchers, consequently having a knowledge spillover effect on R&D activity as in the knowledge-driven growth model of Romer (1990) and Rivera-Batiz and Romer (1991). Here we assume that applied inventions must be derived from patented basic inventions because any applied invention that is derived from public pure knowledge is not patentable.

Current research productivity depends on both patented basic inventions and pure knowledge. Denote by K_t the cumulative number of unpatented basic inventions that have been accumulated as of date t. This K_t represents the stock of pure knowledge in the economy at date t. In this setting, we intend to differentiate between pure knowledge and patented inventions in their external spillover effects. To do so, we assume that the knowledge spillover effect is a function of the stocks of pure knowledge and patented inventions, but these two inputs are not perfectly substitutable.[8] Furthermore, we consider two special cases in which the spillover effect depends only on either pure knowledge or patented basic inventions.

2.3 Basic and applied R&D

As mentioned above, the economy grows as the two factors of technological progress, N_t and K_t, accumulate. The two factors increase as basic inventions are developed and then industrially applied. In this section, we explain how inventions are made and applied to industries and how inventions increase pure knowledge of the economy. There are infinitely many potential researchers. Potential researchers can make both basic and applied inventions. Denote by B_t the cumulative number (stock) of total basic inventions. The stock of basic inventions B_t increases due to basic research activity by researchers. The accumulation of B_t gradually increases the two growth factors, N_t and K_t, through the three stages of research to be described below.

2.3.1 Stage I (Patented basic inventions and pure knowledge)

When a potential researcher invests $b/S\left(K_t, B_t - K_t\right)$ units of skilled labor in basic research, she can develop a basic invention j without any risk of failure. In this setting, the spillover function $S(.)$ captures the extent to which the stocks of pure knowledge K_t and patented basic inventions $B_t - K_t$ affect research productivity through knowledge spillovers. We

[8] It is useful to consider an alternative but related interpretation on ϕ. If we think of ϕ as a technological parameter instead of a policy instrument, ϕ could be interpreted as the share of basic inventions that are industrially applicable. These basic inventions are naturally granted a patent. The remaining share $1 - \phi$ of basic inventions is pure research output, such as a mathematical theorem (e.g., the Black-Scholes option-pricing formula) disseminated to other researchers through scientific journals. Under this interpretation, it is natural that applied inventions are developed only on patented (i.e., industrially applicable) basic inventions and that the stocks of pure knowledge and patented inventions are not perfectly substitutable. Also, we can relate this interpretation to the patentability of basic R&D. Increasing the patentability of basic R&D significantly encourages patenting of university inventions (Mowery *et al.* 2004), resulting into an increase in the share of industrially applicable basic inventions but potentially reducing research on pure knowledge. Therefore, increasing the patentability of basic R&D can also be captured by an increase in ϕ under this alternative interpretation.

adopt a simple Cobb-Douglas functional form for the spillover function $S(K_t, B_t - K_t) = K_t^{\psi}(B_t - K_t)^{1-\psi}$,[9] where $\psi \in [0,1]$ is a factor share parameter that controls how pure knowledge K_t affects the magnitude of knowledge spillovers. $\psi \in \{0,1\}$ captures the special cases in which the spillover effect depends only on either patented basic inventions $B_t - K_t$ or pure knowledge K_t.

Each newly developed basic invention is judged as patentable with probability $\phi \in (0,1)$, and basic inventions that are not patented become pure knowledge. Thus, at date t, $\phi \dot{B}_t$ units of patentable basic inventions and $(1-\phi)\dot{B}_t$ units of pure knowledge are introduced. We can describe the evolution of pure knowledge as

$$\dot{K}_t = (1-\phi)\dot{B}_t. \tag{3}$$

Denote Z_t as the market value of a basic invention that is patentable. Free entry guarantees the following no-arbitrage condition in equilibrium.

$$\phi Z_t = \frac{b w_t^s}{S(K_t, B_t - K_t)}, \tag{4}$$

where w_t^s denotes the wage rate of skilled labor at date t.

2.3.2 Stage II (Basic inventions waiting for applied inventions)

Basic inventions that are patentable at date t, with size $\phi \dot{B}_t$, immediately go to a waiting pool of patented inventions, where patented basic inventions (that have not been industrially applied) await for applied inventions. Denote W_t as the pool of patented basic inventions waiting for industrial applications. The inflow to the waiting pool is $\phi \dot{B}_t$, and the outflow is the number of applied inventions $\dot{N}_t$ that are recently matched with basic inventions in the waiting pool. Then, we have

$$\dot{W}_t = \phi \dot{B}_t - \dot{N}_t. \tag{5}$$

2.3.3 Stage III (Basic inventions becoming industrially applied)

We now turn to applied research activities. Applied researchers make applied inventions that can be matched with basic inventions in the waiting pool. As a result of a successful match, a new industry is introduced into the intermediate goods sector increasing N_t. When

[9] It is useful to note that our results are robust to generalizing the spillover function to a CES form. For simplicity, we focus on the Cobb-Douglas spillover function in this study.

an applied researcher invests $\iota_t a / S(K_t, B_t - K_t)$ units of skilled labor in matching with basic invention j, she can develop an applied invention that is well matched with the basic invention with probability ι_t. At the aggregate level, it holds that

$$\dot{N}_t = \iota_t W_t. \tag{6}$$

In other words, a fraction ι_t of the waiting basic inventions W_t becomes industrially applied at date t. See Figure 1 for a graphical illustration of the three stages of innovation.

[*Insert Figure 1 about here*]

Once basic and applied inventions are matched, the patentholders of these inventions can earn and share the final market value V_t. This value V_t is shared between the patentholders of basic and applied inventions according to a profit-division rule $s \in (0, 1)$. The applied researcher takes $(1 - s) V_t$ and pays a licensing fee of sV_t to the basic researcher. Then, we can describe the free-entry condition to applied R&D as

$$(1 - s) V_t = \frac{a w_t^s}{S(K_t, B_t - K_t)}. \tag{7}$$

Recall that the patentholder of a basic invention takes sV_t, earning a net value $sV_t - Z_t$. Therefore, the market value Z_t of a patented basic invention satisfies the following no-arbitrage condition.

$$r_t Z_t = \dot{Z}_t + \iota_t (sV_t - Z_t). \tag{8}$$

The final value of an invention V_t (i.e., the value of a pair of matched basic and applied inventions) follows the familiar Bellman equation.

$$r_t V_t = \Pi_t + \dot{V}_t. \tag{9}$$

Through the above three stages, basic and applied R&D governs technological progress and economic growth. There are two roles of basic inventions in technological progress. One is the role to increase the stock of pure knowledge K_t in the first stage. The other is the role to increase the number of industrially applicable basic inventions W_t and eventually the variety of intermediate goods N_t. These two roles interact with each other to drive technological progress and economic growth.

2.4 Market equilibrium

The stock of total basic inventions B_t can be divided into three parts as follows.

$$B_t = K_t + W_t + N_t. \tag{10}$$

This equation states that the stock of basic inventions B_t is divided into pure knowledge K_t, patented basic inventions W_t that have not been industrially applied, and patented basic inventions N_t that have been industrially applied. We now close the model by considering the labor market equilibrium conditions.

$$L = N_t X_t \tag{11}$$

for unskilled labor, and

$$1 = H_{A,t} + H_{B,t} = \frac{a\dot{N}_t}{S\left(K_t, B_t - K_t\right)} + \frac{b\dot{B}_t}{S\left(K_t, B_t - K_t\right)} \tag{12}$$

for skilled labor. Here we normalize the total supply of skilled labor to unity. The left-hand side of (12) is the total supply of skilled labor and the right-hand side is the total demand for skilled labor from applied research and basic research, respectively.

From (2)–(12), we can completely characterize the equilibrium dynamics of our model. Before proceeding, we define a balanced growth path in our model. On the balanced growth path, variables for cumulative inventions B_t, K_t, N_t, and W_t grow at a constant rate g^*, which we call the equilibrium growth rate of technology, and the balanced growth rate of consumption is equal to $\dot{C}_t/C_t = g_c^* = g^*/(\varepsilon - 1)$. Taking into account the laws of motion (3), (5) and (6), we have the following steady-state ratios $K/B = 1-\phi$, $N/B = \phi\iota^*/(\iota^* + g^*)$ and $W/B = \phi g^*/(\iota^* + g^*)$ on the balanced growth path. Using these ratios and (12), the equilibrium growth rate of technology is

$$g^* = \frac{\dot{B}_t}{B_t} = (1-\phi)^{\psi}(\phi)^{1-\psi}\left(\frac{H_B^*}{b}\right), \tag{13}$$

where $H_B^* \in (0,1)$ is the equilibrium amount of high-skill labor allocated to basic R&D. Similarly, the equilibrium arrival rate ι^* of applied inventions is

$$\iota^* = \frac{\dot{N}_t}{W_t} = \frac{S\left(K_t, B_t - K_t\right)/B_t}{W_t/B_t}\left(\frac{H_A^*}{a}\right) = \left(\frac{1-\phi}{\phi}\right)^{\psi}\left(\frac{\iota^* + g^*}{g^*}\right)\left(\frac{H_A^*}{a}\right), \tag{14}$$

where H_A^* is the equilibrium amount of high-skill labor allocated to applied R&D.

Equating (4) and (7) yields a no-arbitrage condition between basic and applied R&D given by $(1-s)\,V_t/a = \phi Z_t/b$. Imposing the balanced growth condition on (8) yields $Z_t = sV_t\iota^*/(\iota^* + r^* - g_z^*)$, where $r^* = \rho + g_c^*$ from the Euler equation and $g_z^* = g_c^* - g^*$ from (2). Combining these two conditions yields

$$\frac{b}{a}\left(\frac{1-s}{s}\right)\frac{1}{\phi} = \frac{\iota^*}{\iota^* + \rho + g^*}. \tag{15}$$

This condition along with $1 = H_A^* + H_B^*$, (13) and (14) solves the model. In the following analysis, we restrict attention to the feasible region of ϕ given by $\phi \in (\phi_L, 1)$, where $\phi_L \equiv (b/a)\,(1-s)\,/s$. Given $\iota^* > 0$ and $g^* > 0$, the right hand side of (15) is less than one. Therefore, $\phi > \phi_L$ must hold in order for the left-hand side of (15) to be also less than one. Finally, on the balanced growth path, the equilibrium growth rate g^* of technology is unique, positive and determined by the following condition.[10]

$$\frac{(1-\phi)^{\psi}\,(\phi)^{1-\psi}/b - g^*}{\rho + g^*} = \frac{\phi\,(1-s)\,g^*}{\phi s g^* + \rho\,(1-s)\,b/a}. \tag{16}$$

3 Effects of patent instruments

In the previous section, we have developed an R&D-based growth model with basic and applied research. In this section, we investigate how the patentability of basic R&D and the division of profit between basic and applied researchers affect the growth rate g^* of technology. Equation (16) is a quadratic equation for which the solution is quite complicated. Here it is useful to first consider a limiting case of g^* given by ρ approaching zero.

$$\lim_{\rho\to 0} g^* = (1-\phi)^{\psi}\,(\phi)^{1-\psi}\left(\frac{s}{b}\right). \tag{17}$$

This special case previews our results that the growth rate g^* is a strictly increasing function in the share s of profit assigned to basic R&D and a potentially inverted-U function in patentability ϕ. To be more precise, $\lim_{\rho\to 0} g^*$ is an inverted-U function within the feasible range of ϕ if and only if $0 < \psi < 1 - \phi_L$. If $\psi \geq 1 - \phi_L$, then $\lim_{\rho\to 0} g^*$ would be a monotonically decreasing function in $\phi \in (\phi_L, 1)$. If $\psi = 0$, then $\lim_{\rho\to 0} g^*$ would be a monotonically increasing function in ϕ.

[10] We rule out the negative solution to (16) because $g^* < 0$ implies $H_B^* < 0$ from (13). In an unpublished appendix (see Appendix B), we provide an explicit solution for $g^* > 0$ and sufficient conditions under which the economy converges to a locally stable balanced growth path.

From the above expression, it may seem that the non-monotonic effect of ϕ, which is captured by $(1-\phi)^{\psi}(\phi)^{1-\psi}/b$ in (16), is entirely built-in through the spillover function because $\lim_{\rho\to 0} H_B^* = s$ is independent of ϕ. However, this is not true for the general case of $\rho > 0$. In the case of $\rho > 0$, patentability ϕ has two additional effects on the growth rate through the equilibrium allocation $H_B^*(s,\phi)$. There is a positive effect of ϕ on H_B^*, which arises from the direct effect of raising the patentability of basic inventions that increases the incentives for basic R&D. This positive effect of ϕ on the growth rate g^* is captured by $\phi s g^*$ in (16). There is a negative effect of ϕ on H_B^*, which arises from an indirect effect of patentability ϕ that increases applied R&D by so much to crowd out basic R&D through the resource constraint on skilled labor. Intuitively, raising ϕ may increase applied R&D H_A^* because a larger ϕ increases the waiting pool of industrially applicable basic inventions. For a given arrival rate of applied inventions, a larger waiting pool requires more applied R&D, which in turn crowds out basic R&D, and this negative effect of ϕ on the growth rate g^* is captured by $\phi(1-s)g^*$ in (16). When the discount rate ρ approaches zero, this negative crowding-out effect and the positive incentive effect cancel each other. When ρ is strictly positive, the positive incentive effect of ϕ on the growth rate g^* is dominated by the negative crowding-out effect. Therefore, in the case of $\rho > 0$, it is the interaction between these general-equilibrium effects and the spillover function that drives our results.

We find that patentability ϕ has both positive and negative effects on economic growth, and this finding is consistent with the seminal result in O'Donoghue and Zweimuller (2004) but the underlying mechanism is drastically different. O'Donoghue and Zweimuller (2004) analyze the patentability requirement in a quality-ladder model. They show that increasing the patentability requirement makes it harder to develop an innovation and reduces its arrival rate, but the resulting larger step size of innovation also contributes to growth. In our variety-expanding model with basic and applied research, patentability ϕ of basic R&D has four effects on technological progress. First, it increases patented basic inventions by raising the probability that a basic invention is patentable. Second, increasing patentability ϕ reduces knowledge spillovers from pure knowledge. Finally, increasing patentability ϕ has the positive incentive effect and the negative crowding-out effect on basic R&D.

The following proposition summarizes our first key result, which demonstrates that the effect of raising the patentability ϕ of basic R&D on the equilibrium growth rate g^* is generally non-monotonic except when ψ is either sufficiently large or equal to zero. When ψ is sufficiently large, g^* is monotonically decreasing in ϕ. In the unlikely case that the spillover function is independent of pure knowledge (i.e., $\psi = 0$), g^* is monotonically increasing in ϕ.

Proposition 1 *If $\psi \geq 1 - \phi_L$, then g^* is monotonically decreasing in ϕ for $\phi \in (\phi_L, 1)$. If $0 < \psi \leq s(1 - \phi_L)$, then g^* is firstly increasing and eventually decreasing in ϕ; therefore, the relationship between g^* and ϕ is non-monotonic. If $s(1 - \phi_L) < \psi < 1 - \phi_L$, then g^* is either non-monotonic or strictly decreasing in ϕ depending on the value of ρ. If $\psi = 0$, then g^* is monotonically increasing in ϕ.*

Next we consider how the profit-division rule s affects economic growth. We may interpret an increase in s as a strengthening of patent protection for basic R&D relative to that of applied R&D, so that applied researchers have to pay a larger amount of licensing fee to basic researchers.[11] The profit-division rule s controls the capital gain that is received by basic researchers, $sV_t - Z_t$, whereas the patentability parameter ϕ affects the initial expected return on basic research, ϕZ_t. In summary, we find that g^* continues to be monotonically increasing in s when $\rho > 0$.

Proposition 2 *The relationship between the profit share s of basic R&D and the technology growth rate g^* is monotonically positive.*

Proposition 2 shows that the growth rate is an increasing function in the profit share s of basic research. Intuitively, the growth rate of B_t is determined by basic R&D H_B^*, which in turn is strictly increasing in s. Furthermore, on the balanced growth path, the growth rate of N_t is equal to the growth rate of B_t. Although increasing s reduces the equilibrium arrival rate ι^* of applied inventions, this reduction in ι^* does not affect the growth rate but only the steady-state ratio of N/B, which has a level effect on social welfare as shown in the next section. Therefore, strengthening patent protection for basic R&D relative to applied R&D results in a faster growth rate. Because $s < 1$, there does not exist an interior growth-maximizing profit-division rule. This result differs from Cozzi and Galli (2011), who analyze the division of profit between basic and applied researchers in a quality-ladder model. In their model, the development of a quality improvement is based on the combination of a basic invention and an applied invention. Therefore, in their quality-ladder model with basic and applied R&D, Cozzi and Galli (2011) show an interesting result that an intermediate value of s maximizes the arrival rate of innovation as well as the equilibrium growth rate. In our variety-expanding model, basic R&D has a growth effect while applied R&D has a welfare effect; therefore, the equilibrium growth rate is monotonically increasing in s.

[11] See also Cozzi and Galli (2011) for an interesting analysis of profit division between basic and applied R&D on wage inequality and human-capital accumulation in a quality-ladder model.

To summarize, the drastically different growth effects of the two patent instruments $\{\phi, s\}$ arise for the following reasons. Strengthening patent protection for basic R&D via the profit-division rule stimulates basic R&D without stifling the spillover effects of pure knowledge. However, increasing the patentability of basic R&D eventually decreases basic research due to a crowding-out effect on skilled labor and also reduces the spillover effect from pure knowledge.

3.1 Optimal coordination of patent instruments

Propositions 1 and 2 reveal that the two patent instruments, the patentability ϕ of basic R&D and the profit-division rule s, are useful policy levers for controlling the equilibrium growth rate in the decentralized economy. In this section, we demonstrate that the steady-state welfare-maximizing allocation can be achieved in the decentralized economy only if both patent instruments are present. Therefore, although the profit-division rule may be more effective than the patentability of basic R&D in stimulating basic research and technological progress simultaneously, a benevolent patent authority requires both patent instruments to achieve the socially optimal allocation.

Consider a social planner's problem as follows.[12]

$$\max_{\phi, H_A, H_B} U = \int_0^\infty e^{-\rho t} \ln C_t dt$$

subject to the resource constraint $H_A + H_B = 1$, the laws of motion (3), (5), and (6) along with $\dot{N} = aH_A/S(K_t, B_t - K_t)$ and $\dot{B} = bH_B/S(K_t, B_t - K_t)$. Imposing symmetry of $X_t(i)$ on (1) yields

$$C_t = \left(\int_0^{N_t} X_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} = (N_t)^{\frac{\varepsilon}{\varepsilon-1}} X_t. \tag{18}$$

The resource constraint for unskilled labor implies $X_t = L/N_t$, where L is the total supply of unskilled labor. Therefore, (18) becomes $C_t = (N_t)^{\frac{1}{\varepsilon-1}} L$. Taking log yields $\ln C_t = (\varepsilon - 1)^{-1} \ln N_t + \ln L$, and the utility of households on the balanced growth path becomes

$$U = \frac{1}{\rho(\varepsilon - 1)} \left(\ln N_0 + \frac{g}{\rho} \right), \tag{19}$$

where the exogenous L has been dropped. The equilibrium growth rate of technology in (13) and the equilibrium arrival rate of applied inventions in (14) hold along the balanced

[12] In the case of a social planner, it is more appropriate to view $1 - \phi$ as the fraction of basic inventions B that the planner chooses as pure knowledge K.

growth path for both the decentralized and centralized economies. Using (13) and (14), we can re-express N/B as[13]

$$\frac{N}{B} = \frac{(\phi)^{1-\psi}(1-\phi)^{\psi}}{g}\left(\frac{H_A}{a}\right) = \frac{b}{a}\left(\frac{H_A}{H_B}\right) \leq \phi.$$

Substituting N/B and g into the utility of households (19) yields

$$\rho(\varepsilon-1)U = \underbrace{\ln\left(\frac{b}{a}B_0\right) + \ln\left(\frac{H_A}{H_B}\right)}_{=\ln N_0} + \underbrace{\frac{(1-\phi)^{\psi}(\phi)^{1-\psi}}{\rho}\left(\frac{H_B}{b}\right)}_{=g/\rho}, \tag{20}$$

where B_0 is the initial number of basic inventions. The resource constraint for high-skill labor implies $H_A = 1 - H_B$, and $N/B \leq \phi$ implies $H_B \geq b/(b + a\phi)$.[14] Then, we have the following proposition characterizing the socially optimal allocation.

Proposition 3 *The socially optimal ratio of pure knowledge to basic inventions is*

$$1 - \phi^{**} = \psi, \tag{21}$$

and the socially optimal amount of basic R&D is

$$H_B^{**} = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4b\rho}{(\psi)^{\psi}(1-\psi)^{1-\psi}}}\right). \tag{22}$$

Finally, by (13), the market equilibrium H_B^* in the decentralized economy can be expressed as

$$H_B^*(s,\phi) = \left(\frac{b}{(1-\phi)^{\psi}(\phi)^{1-\psi}}\right) g^*(s,\phi), \tag{23}$$

where the equilibrium growth rate g^* satisfies (16). To achieve the optimal allocation in the decentralized equilibrium, a benevolent patent authority sets patentability ϕ to its optimal value $\phi^{**} = 1-\psi$ and the profit-division rule s to an intermediate value s^{**} that equates the equilibrium H_B^* to the optimal H_B^{**}. Intuitively, the patent instrument ϕ achieves the optimal ratio of pure knowledge to patented basic inventions whereas the patent instrument s achieves the optimal ratio of basic R&D to applied R&D. Although the equilibrium growth rate is

[13] It is useful to recall the following steady state ratios (a) $K/B = 1-\phi$, (b) $N/B = \phi\iota^*/(\iota^* + g^*)$, and (c) $W/B = \phi g^*/(\iota^* + g^*)$.

[14] When this constraint is violated, the arrival rate ι of applied inventions becomes negative.

monotonically increasing in s, the optimal profit-division rule s^{**} takes on an intermediate value because applied R&D also contributes to social welfare through the initial level of varieties N_0. Proposition 4 summarizes this result.

Proposition 4 *If $\psi < 1-\phi_L$, then the patent authority can achieve the steady-state welfare-maximizing allocation in the decentralized economy by using the two patent instruments $\{\phi, s\}$. To do so, the patentability parameter ϕ is set to its optimal level $\phi^{**} = 1 - \psi$, and the profit-division rule is set to an intermediate value s^{**} within the feasible range of s.*

4 Conclusion

In this study, we have developed a simple R&D-based growth model with basic and applied research to analyze the growth and welfare effects of two patent instruments: (a) the patentability of basic R&D, and (b) the division of profit between basic and applied researchers. We find that the patentability of basic R&D has either a monotonically negative effect or an inverted-U effect on technological progress. Therefore, although increasing the patentability of basic R&D may have contributed to economic growth since the 1980's with continual technological progress on biotechnology and information technology, the inverted-U relationship suggests that further increasing the patentability of basic R&D might eventually depress economic growth because increasing the patentability of basic R&D makes basic research discoveries less available to researchers resulting into a reduction in knowledge spillovers. Furthermore, we find that the equilibrium growth rate is monotonically increasing in the share of profit assigned to basic researchers. Therefore, strengthening the bargaining power of basic researchers relative to applied researchers may be a superior policy lever for achieving the dual objectives of stimulating basic R&D and economic growth. However, for a benevolent patent authority, both patent instruments are needed to achieve the socially optimal allocation in the decentralized economy.

To keep our analysis tractable, we have considered a stylized model that may be extended in the following ways. First, the patentability of basic R&D may be partially endogenized by allowing basic researchers to influence the industrial applicability of their inventions. Second, the matching process may be generalized to allow basic inventors to also devote resources towards successful matches. Finally, our model potentially features scale effects, which are removed by normalizing the supply of skilled labor to unity; see Jones (1999) and Li (2000, 2002) for a fruitful discussion on scale effects in R&D-based growth models. It would be interesting to analyze the patent instruments in other vintages of scale-invariant R&D-based growth models with basic and applied R&D. However, any of these extensions would complicate our analysis significantly, and hence, we leave them for future research.

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Appendix A

Proof of Proposition 1: In (16), the left-hand side (LHS) is decreasing in g while the right-hand side (RHS) is increasing in g. Furthermore, RHS is increasing in ϕ. Therefore, if LHS is weakly decreasing in ϕ, then the equilibrium growth rate g^* must be decreasing in ϕ. The condition for RHS to be weakly decreasing in ϕ is $\psi \geq 1 - \phi$. In other words, $\psi \geq 1 - \phi_L$ is a sufficient condition for g^* to be monotonically decreasing in ϕ over the parameter space of $\phi \in (\phi_L, 1)$. Taking the total differentials with respect to g^* and ϕ in (16) yields

$$\frac{dg^*}{d\phi} = \frac{\left[g^* - \psi[\phi/(1-\phi)]^{1-\psi}/b\right]/[\phi^2(g^*+\rho)] + s(1-s)(g^*)^2/[s\phi g^* + \rho(1-s)b/a]^2}{\rho(1-s)^2 b/[a(s\phi g^* + \rho(1-s)b/a)^2] + \left[\rho/\phi + (1/\phi - 1)^{\psi}/b\right]/(g^*+\rho)^2} .$$

As $\phi \to \phi_L = (b/a)(1-s)/s$, $g^* \to (1-s)\left[(a/b)s/(1-s) - 1\right]^{\psi}/a$. Therefore,

$$\lim_{\phi \to \phi_L} \frac{dg^*}{d\phi} = \frac{[s-(1-s)b/a-\psi][\phi/(1-\phi)]^{1-\psi}/[b\phi^2(g^*+\rho)] + s(1-s)(g^*)^2/[s\phi g^* + \rho(1-s)b/a]^2}{\rho(1-s)^2 b/[a(s\phi g^* + \rho(1-s)b/a)^2] + \left[\rho/\phi + (1/\phi - 1)^{\psi}/b\right]/(g^*+\rho)^2} > 0,$$

in which the inequality holds if $\psi \leq s - (1-s)b/a$, which is equivalent to $\psi \leq s(1 - \phi_L)$. Furthermore, we know that so long as $\psi > 0$, $\lim_{\phi \to 1} dg^*/d\phi < 0$. Therefore, g^* must be a non-monotonic function in ϕ if $0 < \psi \leq s(1-\phi_L)$. As for $s(1-\phi_L) < \psi < 1 - \phi_L$, we know from (17) that $\lim_{\rho \to 0} g^*$ is also non-monotonic in ϕ. However, when $\rho > 0$, g^* may be strictly decreasing in ϕ for $s(1-\phi_L) < \psi < 1 - \phi_L$. Finally, if $\psi = 0$, then $dg^*/d\phi > 0$.

Proof of Proposition 2: Recall that the LHS of (16) is decreasing in g while the RHS is increasing in g. Furthermore, RHS is decreasing in s and LHS is independent of s. Therefore, the equilibrium growth rate g^* must be increasing in s.

Proof of Proposition 3: Applying simple optimization on (20) yields the socially optimal $\phi^{**} = 1 - \psi$ and the socially optimal H_B^{**} characterized by the following equation.

$$H_B(1 - H_B) = \frac{b\rho}{(\psi)^{\psi}(1-\psi)^{1-\psi}}.$$

This quadratic equation has two solutions. Deriving the second-order condition, one can easily show that the larger solution is the locally optimal H_B^{**}, which is given in (22). To ensure that this interior optimum is achievable, we naturally assume that $H_B^{**} > b/(b + a\phi^{**})$, which must hold given a sufficiently small ρ. This parameter restriction simply implies that at $H_B = H_B^{**}$, $N/B < \phi^{**}$. To ensure that the locally optimal H_B^{**} is also the global optimum, we further impose $U|_{H_B = H_B^{**}} > U|_{H_B = b/(b+a\phi^{**})}$.[15]

[15] It can be shown that this equality must hold given a sufficiently small a.

Proof of Proposition 4: First, set ϕ to the optimal level $\phi^{**} = 1-\psi$ as implied by (21). Then, we show that there exists a feasible value of $s = s^{**}$ that equates the equilibrium H_B^* to the optimal H_B^{**}. By (22), the optimal $H_B^{**} \in (0,1)$ is independent of s while the equilibrium H_B^* is strictly increasing in s by Proposition 2. As $s \to 1$, it can be shown by using (16) and (23) that $H_B^* \to 1$. Therefore, it suffices to show that as s approaches its lower bound given by $b/\left(b + a\phi^{**}\right)$ (from $\phi^{**} > \phi_L$), H_B^* approaches a value that is below the optimal H_B^{**}. As $s \to b/(b + a\phi^{**})$, it can be shown by using (16) and (23) that $H_B^* \to b/(b + a\phi^{**})$, which is less than H_B^{**}.

Appendix B (not for publication)

In this appendix, we provide sufficient conditions under which the economy converges to a locally stable balanced growth path (BGP). On the BGP, the equilibrium growth rate g^* of technology is positive and determined by (16). By using the profit function in (2) and the free-entry and non-arbitrage conditions in (4), (7), (8), and (9), and by defining $c_t \equiv C_t/(\varepsilon N_t V_t)$, $k_t \equiv K_t/B_t$, $n_t \equiv N_t/B_t$ and $u_t \equiv W_t/B_t$, we obtain

$$\iota_t \left(\frac{as\phi}{b\,(1-s)} - 1 \right) = c_t. \tag{24}$$

Claim 1 *Note that $\phi > (b/a)(1-s)/s$ must hold in order for $\iota_t > 0$, which gives rise to the lower bound of ϕ given by $\phi_L \equiv (b/a)(1-s)/s$.*

Since $\iota_t = \dot{N}_t/W_t$ by (5),

$$\frac{\dot{N}_t}{N_t} = \frac{b\,(1-s)}{as\phi - b\,(1-s)} \frac{u_t c_t}{n_t}. \tag{25}$$

By using the Euler equation,

$$\frac{\dot{C}_t}{C_t} - \frac{\dot{V}_t}{V_t} = c_t - \rho \tag{26}$$

is also derived. From (3),

$$\frac{\dot{K}_t}{K_t} = (1-\phi)\frac{1}{k_t}\frac{\dot{B}_t}{B_t}. \tag{27}$$

From (6),

$$\frac{\dot{W}_t}{W_t} = \frac{\phi}{u_t}\frac{\dot{B}_t}{B_t} - \frac{n_t}{u_t}\frac{\dot{N}_t}{N_t}. \tag{28}$$

From (2), (7), (12), and (25), noting the definition of the spillover function $S(.)$,

$$\frac{\dot{B}_t}{B_t} = \frac{k_t^{\psi}\,(1-k_t)^{1-\psi}}{b} - \frac{a\,(1-s)\,c_t u_t}{as\phi - b\,(1-s)}. \tag{29}$$

From (2), (7), and (11), the skill premium is given by

$$\frac{w_t^s}{w_t^u} = \frac{(1-s)\,L}{a\,(\varepsilon-1)} k_t^{\psi}\,(1-k_t)^{1-\psi}\,\frac{1}{c_t n_t}. \tag{30}$$

From (24)–(29), the equilibrium dynamical system of our model is given by

$$\frac{\dot{c}_t}{c_t} = c_t - \rho - \frac{\dot{N}_t}{N_t}, \tag{31}$$

$$\frac{\dot{k}_t}{k_t} = \left(\frac{1-\phi}{k_t} - 1\right)\frac{\dot{B}_t}{B_t}, \tag{32}$$

$$\frac{\dot{n}_t}{n_t} = \frac{b(1-s)}{as\phi - b(1-s)}\frac{u_t c_t}{n_t} - \frac{\dot{B}_t}{B_t}, \tag{33}$$

and

$$\frac{\dot{u}_t}{u_t} = \left(\frac{\phi}{u_t} - 1\right)\frac{\dot{B}_t}{B_t} - \frac{b(1-s)}{as\phi - b(1-s)}c_t, \tag{34}$$

in which $\dot{B}_t/B_t$ satisfies (29).

Firstly, steady states of the system are identified in what follows. In a BGP, all variables grow at constant rates; specifically, in our model, $\dot{K}_t/K_t = \dot{B}_t/B_t = \dot{W}_t/W_t = \dot{N}_t/N_t = \dot{C}_t/C_t - \dot{V}_t/V_t$ holds. Denote by g^* the steady-state growth rate of variables B_t, K_t, N_t, and W_t.[16] Denote the values of (c_t, k_t, n_t, u_t) along a BGP by (c^*, k^*, n^*, u^*).

The trivial BGP is excluded by assuming $g^* > 0$, which implies

$$k^* = 1 - \phi, \tag{35}$$

in which the use has been made of $\dot{K}_t/K_t = \dot{B}_t/B_t$ with (27). Noting $\dot{W}_t/W_t = \dot{B}_t/B_t$ with (28), $g^* > 0$ also implies

$$n^* = \phi - u^*. \tag{36}$$

By $\dot{C}_t/C_t - V_t/V_t = \dot{N}_t/N_t$, equations (25) and (26) imply

$$u^* = \frac{as\phi - b(1-s)}{b(1-s)}\left(1 - \frac{\rho}{c^*}\right)n^*. \tag{37}$$

Cancelling out n^* from (36) and (37),

$$u^* = \phi\left[\frac{as\phi - b(1-s)}{b(1-s)}\left(1 - \frac{\rho}{c^*}\right)\right]\left[1 + \frac{as\phi - b(1-s)}{b(1-s)}\left(1 - \frac{\rho}{c^*}\right)\right]^{-1}, \tag{38}$$

and

$$n^* = \phi\left[1 + \frac{as\phi - b(1-s)}{b(1-s)}\left(1 - \frac{\rho}{c^*}\right)\right]^{-1}. \tag{39}$$

By $\dot{N}_t/N_t = \dot{B}_t/B_t$ with (25) and (29),

$$\frac{(k^*)^{\psi}(1-k^*)^{1-\psi}}{b} = \frac{b(1-s)u^*}{as\phi - b(1-s)}\left(\frac{c^*}{n^*} + \frac{a}{b}c^*\right). \tag{40}$$

[16] From (1) and (11), $\frac{\dot{C}_t}{C_t} = \frac{1}{\varepsilon - 1}g^*$, which implies $\frac{\dot{V}_t}{V_t} = \left(\frac{1}{\varepsilon-1} - 1\right)g^*$.

Cancelling out c^*, k^*, and u^* from (40), with (35)–(39) and $g^* = c^* - \rho$ by (26),

$$\frac{(1-\phi)^{\psi}(\phi)^{1-\psi}/b - g^*}{\rho + g^*} = \frac{\phi(1-s)g^*}{\phi s g^* + \rho(1-s)b/a}.$$

Because the left-hand side is a strictly decreasing function in g^* and the right-hand side is a strictly increasing function in g^*, a unique positive BGP growth rate g^* exists. Because $g^* > 0$ holds, $c^* > \rho$ holds ($1 - \rho/c^* > 0$). This ensures the feasible values of the fractions n^* and u^*: $n^* \in (0,1)$ and $u^* \in (0,1)$ hold by (38) and (39). The fraction of pure knowledge k^* also satisfies $k^* \in (0,1)$ by (35).

In what follows, the saddle-path stability of the dynamical system for (c_t, k_t, n_t, u_t), (31)–(34), is proved. From (32) and (35), since $\frac{\dot{B}_t}{B_t} > 0$ by the assumption, $\dot{k}_t > 0$ holds for all $k_t < k^*$ and $\dot{k}_t < 0$ holds for all $k_t > k^*$. Thus, for any small $\delta > 0$, there exists a sufficiently large $T < \infty$ such that $k_t \in (k^* - \delta, k^* + \delta)$ holds for all $t \geq T$. Due to the continuity of the system, it would suffice to analyze the stability of the 3×3 dynamical system omitting $\dot{k}_t$ where $k_0 = k^*$. We consider this abbreviated system omitting $\dot{k}_t$. Using (35)–(40), the log-linearized version of this abbreviated system with $(\hat{c}_t, \hat{n}_t, \hat{u}_t) = \left(\ln \frac{c_t}{c^*}, \ln \frac{n_t}{n^*}, \ln \frac{u_t}{u^*}\right)$ has the following coefficient matrix, M:[17]

$$M = \begin{pmatrix} \rho & g^* & -g^* \\ g^* + \zeta & -g^* & g^* + \zeta \\ -\zeta \frac{b(1-s)}{as\phi - b(1-s)} \frac{g^*+\rho}{g^*} - \frac{b(1-s)(g^*+\rho)}{as\phi - b(1-s)} & 0 & -\frac{as\phi g^* + b(1-s)\rho}{as\phi - b(1-s)} - \zeta \frac{b(1-s)}{as\phi - b(1-s)} \frac{g^*+\rho}{g^*} \end{pmatrix}$$

where $\zeta = \frac{a\phi}{b} \frac{g^*(g^*+\rho)}{\frac{as\phi}{b(1-s)} g^* + \rho}$. We can show that the determinant of M is always positive.[18] The trace of M is negative if $\rho - \frac{as\phi g^* + b(1-s)\rho}{as\phi - b(1-s)} < 0$ (a sufficient condition). This inequality can be re-expressed as $g^* > \rho(1 - 2\phi_L/\phi)$, where $\phi_L \equiv (b/a)(1-s)/s$. Solving the quadratic equation in (16) yields

$$g^* = \frac{1}{2}\left(-\Phi + \sqrt{\Phi^2 + 4\left(\frac{1-\phi}{\phi}\right)^{\psi}(1-s)\frac{\rho}{a}}\right),$$

[17] We can formally prove that the coefficient matrx for the original 4×4 linearized system (including $\dot{k}_t$) has the three same eigenvalues as those of the coefficient matrix for the abbreviated 3×3 linearized system (omitting $\dot{k}_t$ with $k_t = k^*$). We can also show that the remaining one eigenvalue is a multiple root that is equal to $-g^*$, which is negative. Therefore, since k_t is a non-jumpable variable, it suffices to verify only the stability of the abbreviated 3×3 system in our model.

[18] Note that the determinant of M is equal to

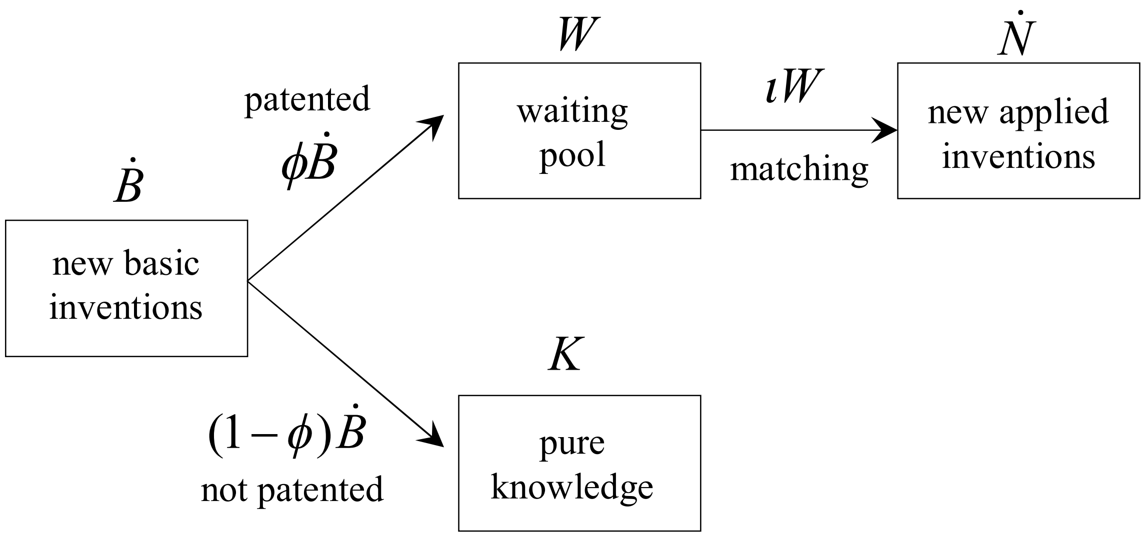
$$b(1-s)\rho\left(\rho(g^* + \zeta) + (g^*)^2 + 2\zeta g^*\right) + as\phi(g^*)^2(g^* + \rho + \zeta)\ .$$

where $\Phi \equiv \rho(1-s)[1+b/(a\phi)] - (1-\phi)^{\psi}(\phi)^{1-\psi}s/b$ is a composite parameter. Using this expression, we find that $g^* > \rho(1-2\phi_L/\phi)$ can be re-expressed as

$$\frac{s(1-\phi)^{\psi}(\phi)^{1-\psi}}{(2-s)b} > \rho(1-2\phi_L/\phi).$$

When this inequality holds, the eigenvalues of M, denoted by λ_1, λ_2, and λ_3, satisfy $\lambda_1\lambda_2\lambda_3 > 0$ and $\lambda_1+\lambda_2+\lambda_3 < 0$. This implies that M has two negative and one positive eigenvalues, which proves the saddle-path stability of the system.

Claim 2 *If ρ is sufficiently small (or $\phi < 2\phi_L$), then the dynamical system is locally saddle-path stable.*



W
$\dot{N}$
patented
$\phi\dot{B}$
waiting pool
ιW
matching
new applied inventions
$\dot{B}$
new basic inventions
K
$(1-\phi)\dot{B}$
not patented
pure knowledge

Figure 1