On mental transformations

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Abstract

This paper presents an alternative interpretation of the experimental data published by Kahneman and Tversky in their 1992 paper describing Cumulative Prospect Theory. It was assumed that mental transformations such as mental adaptation, prospect scaling, and logarithmic perception should be considered when analyzing the experimental data. This led to the design of a solution that did not require the probability weighting function. The double S-type function obtained (the decision utility) resembles the utility curve specified by the Markowitz hypothesis (1952) and substitutes the fourfold pattern of risk attitudes introduced by Cumulative Prospect Theory.

Keywords: *Prospect/Cumulative Prospect Theory, Markowitz Utility Hypothesis, Mental Processes, Adaptation & Attention Focus, Aspiration Level, Decision & Perception Utility*

JEL classification: *D03, D81, C91*

1 Introduction

The first approach to describe economic behavior in terms of utility was proposed by Daniel Bernoulli as early as 1738. However, it was von Neumann and Morgenstern (1944) who proved that rational decision making can be described using a utility function. A concave utility function reflecting the general risk aversion for different wealth levels was widely accepted among economists. However, as early as in 1948 Friedman and Savage argued that the utility function needs to be partially convex and concave to explain why people buy lottery tickets and insurances. A similar approach was proposed by Markowitz (1952), but he was the first to consider the shape of the utility function around a "customary" level of wealth.

A growing body of experimental data indicated, however, that no utility function could

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satisfactorily explain human behavior. The best known counterexample was the Allais paradox (1953). This led to the creation of several theories collectively referred to as Non-Expected Utility Theories. Cumulative Prospect Theory (Kahneman, Tversky, 1979, 1992) presupposes a value function to account for risk aversion for gain prospects and risk seeking for loss prospects, as well as a general aversion to loss. Additionally, the theory gave rise to the concept of the probability weighting function, which is meant to show the non-linear transformation of probabilities when making decisions. This, in turn, is meant to explain people's willingness to participate in lotteries as well as their tendency towards less risky investments in the case of average probabilities (Camerer, Ho, 1994; Wu, Gonzalez, 1996, 1999; Prelec, 1998; Tversky, Wakker, 1995).

The theory was successfully used to explain several phenomena but has also met with criticism. Nwogugu (2006) has compiled a large collection of objections and draws on a bibliography of no fewer than 131 titles to support his claims. The author asserts that Prospect Theory was derived using improper methods and calculations and that it is not consonant with natural mental processes. Shu (1995) shows that it is wrong to assume the existence of probability weights. Neilson and Stowe (2002) demonstrate that Cumulative Prospect Theory cannot simultaneously explain participation in lotteries and the original Allais paradox. Blavatskyy (2005) shows that the theory does not explain the St. Petersburg paradox. Levy and Levy (2002) state that their experimental results negate Prospect Theory and confirm the Markowitz hypothesis[2].

The present paper, too, is critical of Prospect Theory. However, it is not criticizing individual components or individual methodological assumptions, but is rather focused on analyzing the entire process of experimental data interpretation. It has been stated that any analysis of the experimental data should include mental transformations such as mental adaptation, prospect scaling, and logarithmic perception (Point 2). This assumption finds its explanation in psychology, in particular cognitive psychology, and in research at the sensory and neuronal levels. On the other hand, probability weighting should be excluded from this list as it is a mathematical, rather than psychological, concept.

On the basis of the assumptions stated above and using exactly the same experimental data that were used to derive Cumulative Prospect Theory, a solution was obtained without the use of the probability weighting function (Point 3). This solution describes a direct relationship between probability and the relative certainty equivalent. The resulting curve (named the decision

[2] This result was severely criticized by Wakker (2003).

utility function) has a double S-shape (Point 4) consistent with the Markowitz hypothesis (1952) (Point 5). More importantly, the decision utility function explains how people's attitude towards risk depends on their aspiration levels. The description of risk attitudes given by the convex-concave-convex-concave shape of the decision utility function substitutes the fourfold pattern introduced by CPT.

The essential feature of the model presented is that lotteries under consideration are always framed before making decisions and decision utility is expressed in terms of probability (Point 6). This results in a multi-step valuation process, which reflects mental processes during decision making. A logarithmic perception of financial stimuli should be implemented for wider outcome ranges (Point 7). As the model is based on well documented mental transformations, and is described in simple mathematical terms, it may be regarded as superior to other theories of choice under condition of risk.

2 Review of Mental Transformations

2.1 Transformation of Probabilities

That perception of probabilities is distorted is simultaneously one of the key assumptions and key results of Prospect Theory. The concept of decision weights was introduced into the first version of Prospect Theory in 1979. Even at that early stage, Kahneman and Tversky were stating that decision weights were not probabilities and did not comply with the axioms of probability. This led to serious mathematical objections (failure to comply with the First-Order Stochastic Dominance). As a result, Rank-Dependent Expected Utility Theory (Quiggin, 1982) was developed to remedy the shortcomings of its predecessor. The key concepts of that theory were later adopted by Cumulative Prospect Theory (Tversky, Kahneman, 1992). The axiomatization is based on pretty complex topological models (Schmeidler, 1989, Wakker 1989, 1990).

It is important to note that Kahneman and Tversky (1979) distinguish *overestimation* (often encountered when assessing the probability of rare events) and *overweighting* (as a feature of decision weights). The latter phenomenon lacks psychological justification to the extent that the former has it (for instance by dint of insufficient knowledge). It is difficult to explain in psychological terms how a decision regarding an event whose probability is known seems to assume a different probability value. Furthermore no mechanism was posited to explain why this effect of probability transformation only manifests itself at the moment a decision is made. A failure to distinguish between *overestimation* (which can be referred to as a kind of *subjective* view of events whose probabilities are not known) and *overweighting* (a mathematical concept to explain the results of experiments regarding events whose probabilities are known) leads to the commonly accepted view that the probability weighting function has a profound psychological justification.

Despite the extensive research that has been conducted since Prospect Theory was introduced, the basic question, however, remains unanswered: *Why and how do people overweight small probabilities and underweight high probabilities when the probabilities are known?* Until an answer to this question is given, and a psychological explanation of this phenomenon presented, probability weighting cannot be regarded as the psychological explanation of the decision making behaviors observed in lottery experiments.

An important evidence against the "psychology" of the cumulative probability weighting concept was given by Birnbaum (2004). In several studies, subjects were assigned to receive different prospect formats: text displays, pie charts, tickets, list and semi-split lists, marbles in urns, and finally the cumulative probabilities. The last case is the way probabilities are used in Cumulative Prospect Theory. If the theory were psychologically true, this format should facilitate calculations and result in a lower percentage of violations. It turned out, however, that stochastic dominance violations were greatest when cumulative probabilities were supplied. This shows that cumulative representation is not a natural way of using probabilities when making decisions under conditions of risk. Birnbaum concludes his research: "*The weighting functions of RDU/RSDU/CPT are merely artifacts of wrong theory*".

2.2 Mental Adaptation

Evolutionary adaptation was first described by British natural theologians John Ray (1627–1705) and William Paley (1743–1805). The theory was later refined by Charles Darwin (1809–82). Peter Medawar, winner of the Nobel Prize for Medicine and Physiology in 1960, describes the term as "*a process allowing organisms to change to become better suited for survival and reproduction in their given habitat"*. On the other hand, the Oxford Dictionary of Science defines adaptation as "*any change in the structure or functioning of an organism that makes it better suited to its environment"*. More definitions can be found in Rappaport (1971) and Williams (1966). Summarizing *"adaptation can refer to a trait that confers some fitness on an ani-*

mal, but it also represents the process by which that trait has come about" (Greenberg, 2010).

"*Neural or sensory adaptation is a change over time in the responsiveness of the sensory system to a constant stimulus. More generally, the term refers to a temporary change of the neural response to a stimulus as the result of preceding stimulation*". This Wikipedia definition is close to those met in academic texts: "*Adaptation in the context of sensation refers to the fact that a prolonged and uniform sensory stimulus eventually ceases to give rise to a sensory message*" (Medawar, 1983, more in Laughlin, 1989, and Hildebrandt, 2010). The best example of neural adaptation is eye adaptation. Similar mechanisms are well attested for smell, temperature, taste, pain and touch (Gregory, Colman, 1995, Medawar, 1983).

The definitions presented so far all assume that it is the living organism which adapts to changing environmental conditions. However, from the standpoint of a human being, adaptation may be seen as a process of changing the external world to suit its requirements. This was best expressed by Leakey (1981) as follows: *"Animals adapt themselves to environment, hominids adapt environment to themselves using tools, language and complex cooperative social structures*". Mutual human - environment interaction was described by the famous Swiss psychologist Jean Piaget, who *"considers in fact intelligence rising from mental adaptation, where the adaptation is the equilibration of the action of an organism on the environment (assimilation) and of the action of the environment on the organism (accommodation)"* (Maniezzo, Roffilli, 2005).

In the author's opinion, the term "mental adaptation" is best expressed as *"the state of not thinking about certain phenomena"*. This definition follows the Sulavik (1997) paper on mental adaptation to death in the case of professional rescuers, although it can easily be extended to cover many other situations like stress, major illness, bereavement, financial loss, immigration (Jasinskaja-Lahti, 2006), disasters (Leon, 2004), and even space travel (NASA). It should be borne in mind that mental adaptation occurs in positive situations as well – financial windfalls, professional achievements, falling in love etc. "Hedonic treadmill" is another term for mental adaptation coined by Brickman and Campbell (1971) *"to describe the now widely accepted notion that though people continue to accrue experiences and objects that make them happy – or unhappy – their overall level of well-being tends to remain fairly static."* (Mochon et al., 2008, also Kahneman, 1999). There are several other meanings of adaptation encountered in the literature (e.g. social adaptation). A wide coverage of hedonic adaptation examples is given by Frederick and Loewenstein (1999). Nevertheless, most of them have a common feature, viz. they signify a shift of either the organism's structure or its perception system to a new level. As a result, people (and animals) become better suited to external conditions, do not sense any more external stimuli, and cease to think about certain phenomena. Therefore, the mathematical representation of adaptation is shift (translation).

2.3 Prospect Scaling

Prospect scaling is of key significance for deriving the solution presented in this study. The springboard for discussion is the Weber law[3], one of the fundamental laws of psychophysics. The law states that the Just Noticeable Difference (JND) is a constant proportion of the initial stimulus magnitude. It follows from the Weber law that the same change in stimulus (for instance 0.2 kg) can be strongly felt, slightly noticed or not perceived at all depending on the magnitude of the initial stimulus. It further follows that an unambiguous and absolute perception level of a specific stimulus change cannot be determined, as this depends on the situational context.

This applies equally to financial stimuli. The human sensory system adapts itself to financial quantities, just as it does to physical ones. This means that when looking at a financial prospect (project, investment, lottery etc.), its size becomes a reference value in the entire mental process, causing an absolute amount of money (say 10 USD) to be relevant (for instance when shopping) or irrelevant (when buying a house), i.e. depending on the context. This conclusion constitutes a fundamental difference to Prospect Theory, which regards profits and losses in absolute terms, and tries to draw a value function as a function of absolute amounts of money.

Although not implemented by Prospect Theory, the phenomenon is well known to behavioral researchers. Thaler (1999) considers the example that "*most people will travel to save the $5 when the item costs $15 but not when it costs $125*". Thaler (1980) proposes that "*search for any purchase will continue until the expected amount saved as a proportion of the total price equals some critical value. This hypothesis is a simple application of the Weber-Fechner law of psychophysics*". Kahneman and Tversky (1985) define minimal, topical, and comprehensive accounts, where: *"a topical account relates the consequences of possible choices to a reference level that is determined by the context within which the decision arises*". They suggest *"that people spontaneously frame decisions in terms of topical account"* and that *"the topical organization of mental accounts leads people to evaluate gains and losses in <u>relative</u> rather than in absolute terms"*

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[3] Not to be confused with the Weber-Fechner Law discussed in 2.4.

(emphasis added). Kahneman and Tversky stated this, however, only a few years *after* the introduction of Prospect Theory. Baltussen, Post and Van den Assem (2008) used an extensive sample of choices from ten different editions of the high stakes TV game show "*Deal or No Deal*": *"In each sample, contestants respond in a similar way to the stakes relative to their initial level, even though the initial level differs widely across the various editions. Amounts therefore appear to be primarily evaluated relative to a subjective frame of reference rather than in terms of their absolute monetary value"*.

The mechanism responsible for this mental transformation is attention – one of the most thoroughly examined concepts in cognitive psychology. According to one definition, attention is the process of selectively concentrating on a single perceived object, source of stimulation, or topic from among the many available options (Nęcka, 2007). The existence of attention is indispensable on account of a living organism's need to adapt to the demands of the environment (Broadbent, 1958) and on account of the finite ability of the brain to process information (Duncan, Humphreys, 1989). Several models of attention division are discussed, especially in relation to focused attention. The entire mechanism can be explained by such aspects of attention as selection and gain (amplification) control, the existence of which is evidenced by attention research at the neuronal level. Hillyard et al. (1998) state that attention has a gain (amplification) control character which aims to increase the signal to noise ratio of the stimuli on which attention is focused. The signal of most interest to the brain is maintained at a stable and optimal level as a result. It may well be assumed that the amplification control mechanism operates at a higher mental level as well. This leads to problems differing in scale being perceived as equally significant when attention is focused. Very clearly, the mathematical equivalent of amplification is homothety (scaling).

The arguments cited indicate that the attention focused on specific payments in the conducted experiments seems to be a natural effect that has to be factored into any analysis of the results. This is especially the case under experimental conditions as those surveyed are remunerated for their participation; it means they are paid to focus their whole attention on the analyzed problems. The assumption that the size of a prospect becomes a reference value in the conducted experiments leads to a completely different solution than that which Prospect Theory proposes.

2.4 *Logarithmic Perception*

Logarithmic perception is the last mental transformation necessary to derive the results presented in the final part of the study. Here, the reference point for discussion is also a fundamental psychophysical law, viz. the Weber-Fechner law. "*Fechner assumed that JNDs correspond to equal increments in subjective intensity and that JNDs are proportional to the physical variable being studied (Weber's law). These assumptions led to the famous, logarithmic Weber-Fechner law of subjective intensity*" (Johnson, 2002). Hearing, as measured using the decibel scale, is an example of this sort of perception. The law applies to many other stimuli. Zauberman et al. (2008) argue that "*error in time estimation following the Weber-Fechner law can explain both sub-additive and hyperbolic discounting*".

Bernoulli stated that the St. Petersburg Paradox could be explained using logarithmic utility as early as 1738. Logarithmic utility expresses the diminishing marginal value of money. Interestingly, Sinn (2002) proves that "*expected utility maximization with logarithmic utility is a dominant preference in the biological selection process in the sense that a population following any other preference will disappear as time goes to infinity*".

The Weber-Fechner law, however, has been severely criticized by Stevens (1957), who claimed that the power function better describes human perception. The power function was included in Prospect Theory to describe the value function. Surprisingly, the difference in approach turns out to be insignificant as both functions (logarithmic and power) have an almost identical shape for low x-coordinate values.[4] This leads to the conclusion that the perception of monetary amounts used in Kahneman and Tversky's experiments could be equally well described using a logarithmic curve. Some arguments in favor of a logarithmic perception are provided by other results presented in CPT, which states that mixed prospects are accepted when gains are at least twice as high as losses. This effect can be easily explained by noticing that in logarithmic terms, a 100% profit corresponds to a 50% loss. More arguments in support of a logarithmic, rather than a power, perception of monetary amounts are given in Point 7 of this paper.

3 Solution Using Mental Adaptation & Prospect Scaling Transformations

This section contains an alternative analysis of the experimental data presented by Kah-

[4] Within the range [0, 0.6], $x^{0.88}/1.34$ is the best approximation of the $ln(1+x)$ function using a power function. The coefficient 0.88 is exactly the same as the power coefficient of the value function in Prospect Theory.

neman and Tversky in their 1992 paper (these data are replicated in the Appendix 1 of this paper). The analysis is based on the assumption that mental adaptation and prospect scaling transformations should be considered when analyzing the experimental data. During the experiment conducted by Kahneman and Tversky, certainty equivalents *CE* were collected for the prospects of payment P_{min} with probability 1 - p or payment P_{max} with probability p, where:

$$|P_{min}| < |P_{max}| \tag{3.1}$$

It is assumed that there is a function *F* such that:

$$CE = F(P_{min}, P_{max}, p) \tag{3.2}$$

The variables *CE'* and P_{max}' are now introduced to account for the mental adaptation process. These are a P_{min} translation of *CE* and *P*:

$$CE' = CE - P_{min} \tag{3.3}$$

$$P' = P_{max} - P_{min} \tag{3.4}$$

This step was implemented by Prospect Theory as well, but was abandoned by its Cumulative version. The payment P_{min} is interpreted as the riskless component. If $P_{min} = 0$ then $CE' = CE$ and $P' = P_{max}$. Introducing these new variables presupposes the existence of a function *G* such that:

$$CE' = G(P', p) \tag{3.5}$$

This formula shows that the certainty equivalent is a function of both the prospect payment and the probability of its winning. Prospect Theory thus assumes that both the value and the probability weighting functions are required to describe the experimental data. At this point, however, the approaches diverge, and (3.5) is transformed in such a way that probability p, and not *CE'*, becomes the value to be determined. Due to the fact that *CE'* is monotonic with respect to p, it may be assumed that there is an inverse function *H* such that:

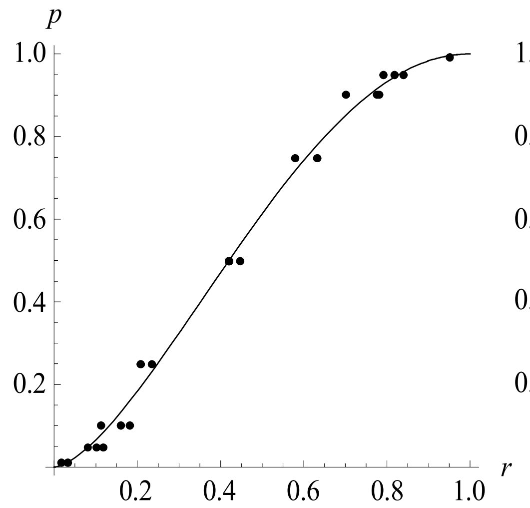
$$p = H(CE', P') \tag{3.6}$$

In order to take prospect scaling into account, it is assumed that the value of payment *P'* becomes the reference value for the certainty equivalent *CE'* and that the equivalent values are scaled by a coefficient $1/P'$. As a result, a variable $r = CE' / P'$ is introduced as the relative certainty equivalent with a value in the range [0, 1]. This also supports the existence of the following *D* function defined over the range [0, 1]:

$$p = D\left(CE' / P'\right) = D\left(r\right) \tag{3.7}$$

For example, for the specific values listed in Table 3.3 of Kahneman and Tversky's paper, the relationships 0.10 = D(9/50), 0.50 = D(21/50), and 0.90 = D(37/50) are obtained for the prospect (0, 50), and the relationships 0.05 = D(14/100), 0.25 = D(25/100) are obtained for the prospect (0, 100). For the prospect with the riskless component (50, 150), the relationships 0.05 = D((64-50)/100) = D(14/100), 0.25 = D((72.5-50)/100) = D(22.5/100), and 0.5 = D((86-50)/100) = D(36/100), are obtained after the mental adaptation transformation.

The obtained values are plotted on the graph $p = D(r)$ and approximated using the least squares method with the assistance of the Cumulative Beta Distribution $I_r(\alpha, \beta)$ (i.e. regularized incomplete beta function). This particular function was selected because it is defined in the domain [0, 1] and because of the extraordinary flexibility the two parameters α and β give its shape. Approximations were made separately for the loss (P < 0)[5] and gain (P > 0) prospects. The results are presented in Figure 3.1.



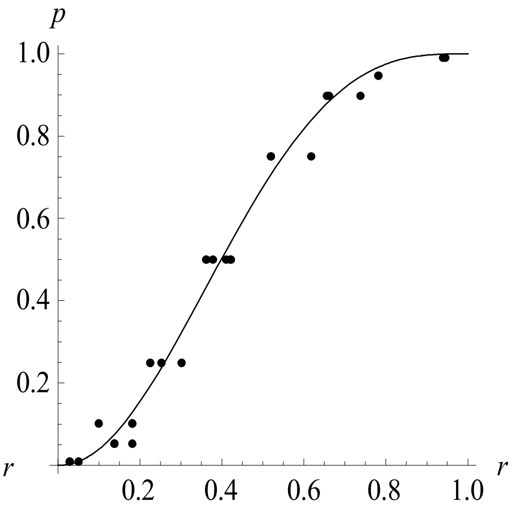


Figure 3.1 Transformed experimental points and approximation $p = D(r)$ using cumulative beta distribution function for loss prospects (left) and for gain prospects (right).

The approximations obtained for the function $p = D(r)$ for loss and gain prospects allow the following conclusions to be drawn:

1. The function $p = D(r)$ is S-shaped for both loss and gain prospects.
2. The respective values of the parameters α and β are 2.03 and 2.83 for gain prospects and 1.60 and 2.09 for loss prospects. The disparity between the parameters α and β in both cases confirms

[5] It should be noted that for the loss prospects, the value of relative certainty equivalent r is also positive, as CE' and P' are both negative in this case.

the asymmetry of the function $D(r)$ with respect to the center point $(p, r) = (1/2, 1/2)$.

3. The intersection of the approximation functions $p = D(r)$ with the straight line $p = r$ occurs when r has a value of approximately 0.27 for gains and 0.25 for losses. This value is called the aspiration level, as (in case of gains) the risk seeking attitude is present for lower values of the relative outcome r, and risk aversion is present for greater values of r. This implies a change of attitude to risk at the aspiration level, which is in accordance with generally accepted interpretations of this term. The pattern for losses is reversed.

Assuming mental adaptation and prospect scaling transformation thus led to a different solution than that presented by Prospect Theory. The entire description has been reduced to the relationship:

$$p = D(r) \tag{3.8}$$

where

$$r = \frac{CE - P_{min}}{P_{max} - P_{min}} \tag{3.9}$$

The function D is called here “decision utility” because it describes how decisions are made under conditions of risk. The value function and the probability weighting function have disappeared altogether as they are not needed to describe the experimental results. Decision utility is defined in the [0, 1] range: transformation (3.9) combines mental adaptation and prospect scaling, and normalizes the certainty equivalent within the [0, 1] range. Therefore, transformation (3.9) can simply be referred to as “framing”. This notion of framing is slightly different than the one normally discussed in the Prospect Theory context. In the latter case, framing is usually understood as considering the problem as a prospective gain or loss. In the present case, the problem is bounded from both sides, which is closer to the dictionary definition of frames. According to the present model, problems are always framed before making decisions.

Please note that the decision utility function was derived using linearly assessed outcomes. Despite this, the curve approximation is very good for lotteries from \$50 to \$400. However, a nonlinear perception utility function might be needed for wider outcome ranges. This is discussed in Point 7.

4 Combining Gains and Losses

The solutions obtained so far comprise two $p = D(r)$ functions with one describing losses, the other gains. The two functions need to be scaled before they can be used together. The simplest assumption has been adopted with the use of the loss aversion coefficient λ. This is similar to the Prospect Theory approach when defining the value function, and allows both parts of decision utility to be presented on a single graph (Figure 4.1). The derivation of the loss aversion coefficient λ is presented later in this paper (see Point 6.4).

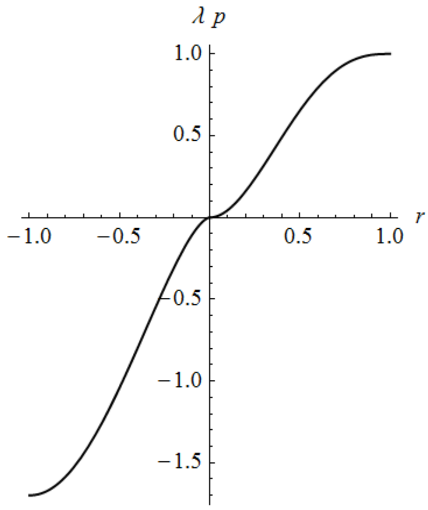


Figure 4.1 Functions $D_l(r)$ for losses and $D_g(r)$ for gains presented together on a single graph. Function $D_l(r)$ is multiplied by the negative loss aversion coefficient λ.

This decision utility curve presents the sum total of all the knowledge that has come out of Prospect Theory and its cumulative version.

1. The fourfold pattern of risk attitudes, which was presented by CPT, is evident:

    a). in case of gain prospects, the curve is convex for probabilities below 27% (corresponding to risk taking), and becomes concave for probabilities above 27% (corresponding to risk aversion);

    b). in case of loss prospects, the curve is concave for probabilities below 25% (corresponding to risk aversion), and becomes convex for probabilities above 25% (corresponding to risk seeking).

2. The convex-concave-convex-concave shape of the decision utility substitutes therefore the fourfold pattern of risk attitudes described by CPT.

3. Both parts of the curve are of different magnitude to reflect the loss aversion phenomenon.

4. The function's more linear shape for loss prospects confirms the results of other studies that people's attitude to risk for losses is rather neutral in nature[6].

[6] See Wakker (2003) for reference, who confirms that the pattern for losses is less clear than in the case of gains.

5 The Markowitz Utility Function Hypothesis

In 1952, Markowitz published an article "The Utility of Wealth" presenting his hypothesis on the shape of the utility function. While this article was known to Kahneman and Tversky, they believed that neither this nor any other utility function could explain certain psychological experiments. This led to the development of Prospect Theory as an alternative to classical economic theories based on utility functions. That the decision utility curve so closely resembles the curve presented in the Markowitz article (Figure 5.1) is highly surprising given the result was obtained using the same experimental data used to derive Cumulative Prospect Theory.

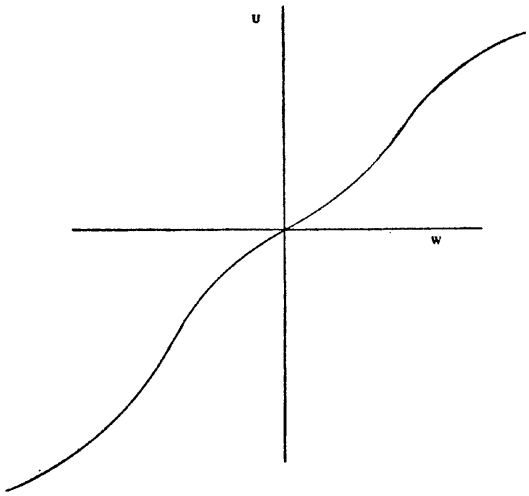


Figure 5.1 The shape of the utility function according to the Markowitz hypothesis of 1952.

Markowitz specified the utility function as follows: "*The utility function has three inflection points. The middle inflection point is defined to be at the 'customary' level of wealth. The first inflection point is below customary wealth and the third inflection point is above it. The distance between the inflection points is a non-decreasing function of wealth. The curve is monotonically increasing but bounded from above and from below; it is first concave, then convex, then concave, and finally convex. We may also assume that $|U(-X)| > U(X)$, $X > 0$ (where $X = 0$ is customary wealth)*[7]".

It is clear that all but one of the requirements of the utility curve expressed by Markowitz in his hypothesis are met by the curve presented in Figure 4.1. The decision utility curve has three inflection points right where Markowitz predicted they would be. The function is monotonically

[7] Markowitz also assumed that in the case of recent windfall gains or losses, the second inflection point may, temporarily, deviate from present wealth. This requirement does not influence the shape of the curve.

increasing and is bounded from the top and from the bottom. Convexities and concavities occur in the order assumed by Markowitz. The condition related to the function value for X values having opposite signs is also met. The only condition not met is that the distances between the inflection points depend on people's wealth. Markowitz noted: "*If the chooser were rather rich, my guess is that he would act as if his first and third inflection points were farther from the origin. Conversely, if the chooser were rather poor, I should expect him to act as if his first and third inflection points were closer to the origin*". In the Markowitz hypothesis the position of inflection points changes because the w (wealth)-axis is expressed in absolute terms. This differs from the decision utility curve, where the r-axis is expressed relative to the size of the prospect.

There is no reason to regard this approach as being in any way anomalous. People commonly say "I have gained 15% on my stock investments" rather than "I have gained 5% *of my wealth* on my stock investments". It is clear enough that the former sentence assumes the value of the stock investment as the reference for gain/losses considerations. Moreover, according to Thaler (1985), people keep mentally separate accounts, so that investments and expenditures are considered as separate parts rather than as a whole. As a result people typically consider "I have lost 5% on my house but I have gained 15% on stocks" despite the fact that the absolute values of stock and house investments may differ substantially. It follows that the decision utility applies for each separate account, albeit with different reference values established by the attention focus. This may mean that people could be risk seeking and risk averse at the same time depending on the status and prospects of each account.

Markowitz's assumption that the shape of the utility curve corresponds with the value of wealth precluded his curve (however tempting its shape) from being able to explain experiments on financial payments which were not directly related to the wealth of the people being studied. This is what led Kahneman and Tversky to reject the Markowitz hypothesis and develop Prospect Theory. The result presented here, however, may signal a return to an approach based on the utility-like function and lead to a negation of Prospect Theory. Accepting that gains and losses need not be considered in relation to wealth, but to any other value depending on where a person's attention is focused, is all that it would take to come back to this earlier concept. The payoff is a simpler and more accurate description of people's behavior.

6 Lottery Valuation

6.1 Two-Outcome Lotteries

The essential feature of the model presented is that decision utility is expressed in terms of probability (see (3.8)). The greater the relative certainty equivalent r, the greater the probability p of winning the lottery and hence the greater the lottery decision utility. This would suggest using probabilities to compare lotteries. This, however, is only possible when lotteries have the same outcome range.

Decisions are always framed according to the model (see (3.9)). Some further steps are therefore required to compare lotteries of different ranges. As presented in Point 3, the relative certainty equivalent r is directly transformed into probability p (and vice versa). This means that in order to determine the certainty equivalent CE for probability p, the respective value of r need only be multiplied by the value of payment P'. For example, $r = 0.75$ for $p = 0.95$. Hence, $CE' = 75$ for $P' = 100$ (the value obtained experimentally was 78). In case of prospects with riskless components, e.g. (50, 150), the riskless component P_{min} (here 50) increases the value of the certainty equivalent: $CE = P_{min} + CE' = 50 + 75 = 125$ (the value 128 was obtained in the experiment).

This can be presented in a more general way. The certainty equivalent CE is derived from (3.8) and (3.9):

$$CE = P_{min} + D^{-1}(p)(P_{max} - P_{min}) \tag{6.1}$$

For $P_{min} = 0$, (6.1) simplifies to:

$$CE = P_{max} D^{-1}(p) \tag{6.2}$$

It may be noticed that (6.2) looks very similar to the certainty equivalent calculation postulated by Prospect Theory, with $D^{-1}(p)$ being the probability weighting function $w(p)$. This may lead to the conclusion that Decision Utility and Cumulative Prospect Theory are alternative models, the only difference being the inverse representation of the probability weighting function. This conclusion is only partially legitimate. One of the main differences between the decision utility $D(p)$ and the CPT probability weighting function $w(p)$ is that the former is expressed in terms of individual probabilities, whereas the latter concerns probabilities and probability weights expressed in a cumulative form. This form is required to avoid the stochastic dominance violations introduced by the original Prospect Theory. Prospect Theory, Cumulative Prospect Theory, and Decision Utility Theory fortuitously reduce to essentially the same model in the case of two-outcome lotteries. However, the last two models differ significantly for multi-outcome lotteries (Prospect Theory is restricted to a maximum of two non-null outcomes).

6.2 Multi-Outcome Lotteries

Decision utility of multi-outcome lotteries is likewise expressed in terms of probability. This time, however, it is the probability of winning the greater outcome in the equivalent two-outcome lottery. This probability is therefore termed "equivalent probability" for convenience.

From a psychological viewpoint, substituting a multi-outcome lottery by a two-outcome lottery should be regarded as a simplification of the problem. It results in a single value (equivalent probability) describing the lottery considered, whatever the number of its outcomes. This likewise holds for the continuous outcome distribution. The operation is easy once the expected value of the lottery is considered. For example, let us assume a lottery offering x_1 = \$100 with a probability of $p_1 = 0.5$, x_2 = \$60 with a probability of $p_2 = 0.3$ and x_3 = \$0 with a probability $p_3 = 0.2$. The expected value of the lottery can be obtained from:

$$EX = x_1 p_1 + x_2 p_2 + x_3 p_3 = 0.5\times 100 + 0.3\times 60 + 0.2\times 0 = 68 \tag{6.3}$$

Very obviously this lottery can be substituted by a two-outcome lottery offering x_1 = \$100 with a probability of $p_1 = 0.68$, and x_2 = \$0 with a probability of $p_2 = 0.32$. The equivalent probability p' of 0.68 can be calculated as:

$$p' = \sum_{i=1}^{n} r_i p_i \tag{6.4}$$

where r_i denotes a relative outcome[8]:

$$r_i = \frac{x_i - P_{min}}{P_{max} - P_{min}} \tag{6.5}$$

with P_{max} and P_{min} being the maximum and minimum lottery outcomes (here p_1 and p_3 respectively). As relative outcomes r_i are normalized they can be represented by respective probabilities $p_i^{'}$. Thus (6.4) takes the form of the joint probability calculation:

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[8] So far in this paper, r has been defined as the relative certainty equivalent (3.9). There is no conflict between this definition and the more general (6.5). The latter defines relative outcomes r_i for given values of x_i. In the same way, it defines the relative certainty equivalent r for a given value of certainty equivalent CE.

$$p' = \sum_{i=1}^{n} p_i^{'} p_i \tag{6.6}$$

This form helps to derive a solution for a non-linear relationship between outcomes and probabilities (as stated in real lottery experiments). Here, each relative outcome r_i is equivalent to the probability $p_i^{'} = D(r_i)$, according to the decision utility model. Using (6.6), the equivalent probability is thus defined as:

$$p' = \sum_{i=1}^{n} D(r_i)\, p_i \tag{6.7}$$

Equivalent probability (6.7) represents the decision utility of a multi-outcome lottery, similarly as the single probability p represents decision utility in the two-outcome case. NB: for a two-outcome lottery, (6.7) simplifies to the probability of winning the greater outcome:

$$p' = D(0)(1-p) + D(1)\,p = 0(1-p) + 1p = p \tag{6.8}$$

Please note the strong resemblance of (6.7) to the Expected Utility Theory valuation. In fact, the decision utility model follows Expected Utility Theory with a transformed outcome domain. It is also worth noting that (6.7) can be easily extended to continuous outcome distributions:

$$p' = \int_{0}^{1} D(r)\, f(r)\, dr \tag{6.9}$$

where $f(r)$ denotes the probability density function defined over the [0, 1] range.

The derivation presented is straightforward as the decision utility model considers probabilities within the framework of the classical axioms. This is not the case with Cumulative Prospect Theory. To recapitulate, this theory requires that outcomes are first ranked, their respective probabilities are then transformed into cumulative form, these cumulative probabilities are then transformed into cumulative weights using the probability weighting function $w(p)$, and finally the cumulative weights are decumulated. Only then are the resulting individual probability weights applied together with their corresponding outcomes in the expected utility formula. As presented, none of these operations is required by the decision utility model, which thus offers a much more straightforward valuation of multi-outcome lotteries.

Cumulative Prospect Theory uses the presented cumulative approach because the original Prospect Theory suffers from stochastic dominance violations as a result of applying individual probabilities. Most importantly, the decision utility model, despite using individual probabilities, obviates this problem. A basic proof of this statement is presented in Appendix 2 of this paper.

6.3 Psychology of the Valuation Process

As already discussed in Point 6.1, only lotteries having the same outcome range can be compared using decision utility *p'* from a normative standpoint. In other cases, decision utility is applied in (6.1) or (6.2) instead of the single probability *p* to determine the lottery certainty equivalent; this finally serves to compare lotteries.

Let us now summarize the entire valuation process from a psychological point of view. The process consists of three steps. In the first step, mental adaptation and prospect scaling transformations result in problem framing (6.5) and in considering it in purely relative terms. In the second step, the multi-outcome lottery is simplified into a two-outcome one, and its equivalent probability (6.7) is determined. In the third step, the inverse transformation (6.1) to the absolute outcome scale is performed in order to compare lotteries using their certainty equivalents[9].

It needs to be pointed out that not all steps are required in all cases. The second step is obviously redundant for a two-outcome lottery; the (equivalent) probability is known beforehand. As each step requires time and brain resources, it may well be assumed that some steps will be omitted if this is likely to simplify the decision making process. For instance, the third step is unnecessary when the lotteries under consideration are of the same range; their respective equivalent probabilities are sufficient to compare them. This step may also be omitted when the outcome ranges do not differ significantly; (equivalent) probabilities may still decide in this case.

The last conclusion is in agreement with the choice heuristics proposed by Brandstätter et. al (2006). These are claimed to more satisfactorily explain lottery choices than “holistic” theories like EU or CPT. The authors state that people consider the probabilities (heuristic 2) first and only consider the lottery ranges when these are similar (heuristic 3).

For the same reason, i.e. economy of time and brain resources, it may well be assumed that none of the steps is performed in some cases. People would then base their decisions on, for instance, minimum lottery payments. This coincides (and this is most likely no accident) with the first heuristic postulated by Brandstätter, which is applied even before probabilities are taken into account. Please note that the order of heuristics corresponds with the steps of the process; later

[9] Please note that similar three-step processes are effectively applied in purely mathematical operations. The Laplace and Fourier transforms are the best known examples.

heuristics appear in later steps of the valuation process.

In this context, the three-step process presented here, while seemingly lengthy and convoluted, seems to reflect the mental processes that take place during decision making. It provides a fully “holistic” approach while at the same time allowing the inclusion of heuristics and stopping rules at each step of the decision process. As each step is additionally well described in psychological and simple mathematical terms, the process described here can be regarded as having an advantage over other choice theories.

6.4 Mixed Lotteries

The valuation of mixed prospects is a matter of great concern. Gain-loss separability is a basic premise underlying Prospect Theory. This axiom requires that preferences for gains be independent of preferences for losses and that the valuation of a mixed lottery be the sum of the valuations of the gain and loss portions of that lottery. Unfortunately, this is more a theoretical assumption than an experimentally confirmed fact. Kahneman and Tversky (1979, 1992) generally examined gain and loss prospects separately. The only case where mixed lotteries were examined was limited to a probability of 0.5, and concerned the determination of the loss aversion coefficient rather than the shape of descriptive functions.

Gain-loss separability has recently become a subject of criticism, as its violations have been evidenced (Wu, Markle, 2008; Birnbaum, Bahra, 2007). The authors have presented several explanations of the phenomenon based on Prospect Theory and the TAX model. Kontek (2011) also put forward a hypothesis, which explains the case by assuming a perception utility with a very different shape and properties than the Prospect Theory value function (see also Point 7). While this re-opens the subject of mixed prospect valuation, it is definitely too soon to give a definite answer. The decision utility function has been derived in this paper separately for gains and losses based on Tversky and Kahneman’s data, and no decision utility function can be presented for mixed prospects without any further data sets or new experiments.

At this stage, a solution *assuming* separability is presented and this works well in many cases. This is because people usually first evaluate prospective gains and losses separately to assess how much they can expect to win and lose when considering mixed prospects. Their decision will then be a trade-off between these two values. This method presupposes that positive outcomes are evaluated separately as a gain prospect and negative outcomes as a loss prospect.

These two valuations determine the respective certainty equivalents of $CE_g > 0$ for a gain portion and $CE_l < 0$ for a loss portion. The two certainty equivalent values are then compared and integrated into the certainty equivalent of the mixed prospect.

This requires the value of the loss aversion coefficient λ. Tversky and Kahneman (1992) conducted additional experiments in order to determine this value. The results, presented in Table 3.6 of the original publication, indicate that the mixed prospects are accepted if the profit is at least twice as great as the loss. An exact ratio of 2.07, as the mean value of Θ resulting from problems 1-6, is assumed for further calculations[10]. The certainty equivalent of the loss outcome -x should thus correspond to the certainty equivalent of the gain outcome of 2.07x. This can be presented using (6.2):

$$-\lambda\, x\, D_l^{-1}(0.5) = 2.07\, x\, D_g^{-1}(0.5) \tag{6.10}$$

where D_l and D_g denote decision utility functions for losses and gains respectively. This leads to

$$\lambda = -1.99 \approx -2 \tag{6.11}$$

It follows that a mixed prospect has a value of 0 for $CE_g = -2CE_l$. The certainty equivalent CE of a mixed prospect can thus be derived as:

$$CE = CE_g + 2CE_l \quad \text{if } CE_g \geq -2CE_l \tag{6.12}$$

or:

$$CE = CE_g / 2 + CE_l \quad \text{if } CE_g < -2CE_l \tag{6.13}$$

A simple application of the approach to explain the WTA-WTP disparity in risky choices is presented. It has been stated in many experiments that both the ‘willingness-to-accept’ and ‘willingness-to-pay’ values differ both when riskless and risky options are considered. For example, subjects with a commodity to sell invariably demand a price substantially in excess of that which subjects in a position to purchase it are prepared to pay (Kahneman, Knetsch and Thaler (1990). The disparity between the two values has also been observed for risky options (Schmidt, Traub, 2009). The experimentally determined WTA/WTP ratio assumes a value in the neighborhood of 2 for a wide range of probabilities.

The notion of WTA coincides with that of the lottery certainty equivalent; both reflect the amount of money one is ready to accept in order to forgo the game. The risk that the lottery may

[10] The statistic Θ is the ratio of the “slopes” at a higher and lower region of the value function, according to Tversky and Kahneman (1992). For further details, please refer to Table 3.6 in their paper.

result in a zero outcome has to be taken into account. In this case, the investment will be negative and equal in value to the price paid (WTP). WTP can thus be regarded as the certainty equivalent of the loss portion of the lottery. Following (6.12), both values should satisfy the condition:

$$0 = \mathrm{WTA} + 2\,\mathrm{WTP} \tag{6.14}$$

Thus

$$\mathrm{WTA} / \mathrm{WTP} = -2 \tag{6.15}$$

which result is in accordance with the experimental data. The WTA-WTP disparity of risky options thus finds an interpretation in the phenomenon of loss aversion, according to the model presented.

The question may be raised as to whether the proposed method of mixed prospect valuation negates the mental adaptation process described in this paper. This is because people should adapt to the minimum or maximum lottery outcome, according to the process. The answer is inconclusive at this moment. The level of 0, which separates gains and losses, may prevail over all other levels in the adaptation process. This leads, however, to the gain-loss separability violations mentioned earlier in this Point. A solution that satisfies the new evidence while taking the adaptation to those extreme lottery outcomes into account is certainly a future possibility.

6.5 Illustrative Example

Let us consider the following two lotteries to illustrate the valuation method:

A: \$300 with probability 3/5	B: \$100 with probability 1/3
-\$200 with probability 2/5	\$50 with probability 1/3
	-\$100 with probability 1/3

Not only are the lotteries mixed, they are of different ranges, so their certainty equivalents have to be determined. The gain and loss parts of both lotteries are considered separately. In those cases where they contain only one outcome, (6.2) applies.

Lottery A is considered first. The certainty equivalent of its gain part is equal to $CEA_g = \$300 D_g^{-1}(3/5) = \$139.7$ and that of its loss part is equal to $CEA_l = -\$200 D_l^{-1}(2/5) =$ -\$70.6. The certainty equivalent of A is then determined to be \$139.7 /2 - \$70.6 = -\$0.75, according to (6.13). The value of A is thus close to \$0; the gain and loss parts of A have a similar impact on the valuation.

Lottery B is next considered. The certainty equivalent of its loss part equals

$CEB_l = -\$100 D_l^{-1}(1/3) = -\$30.8$. The gain part of B, however, has more than 1 outcome, so its equivalent probability has first to be calculated using (6.7):

$$p' = \frac{1}{3} D\left(\frac{100}{100}\right) + \frac{1}{3} D\left(\frac{50}{100}\right) + \frac{1}{3} D\left(\frac{0}{100}\right) = \frac{1}{3} + \frac{1}{3} D\left(\frac{1}{2}\right) = 0.551 \tag{6.16}$$

The gain part of lottery B is thus substituted by a lottery offering a single prize of \$100 with a probability of 0.551. It follows that its certainty equivalent equals CEB_g = $\$100 D_g^{-1}(0.551)$ = \$43.6. It further follows that the certainty equivalent of B equals \$43.6 / 2 – \$30.8 = -\$9.0. It turns out that A is better than B by \$8.25.

A different result is achieved using Cumulative Prospect Theory. The value of prospect A is -21.6, whereas the value of prospect B is -20.4. This means that both prospects are of similar *negative* values with a small preference for B[11]. A clear disadvantage of this approach is that the prospect values are expressed in meaningless units.

7 Including Logarithmic Perception of Financial Stimuli

The decision utility function was derived in Point 3 using linearly assessed outcomes. The function derived this way works well in most cases. An important objection raised by one reviewer was that the proposed model fails to interpret the following case. Let us suppose that P_{min} = 0 and p = 0.5. The model implies that CE/P is constant for a given probability according to (6.2). This seems unrealistic, because this ratio should decrease as P becomes very large. For instance, somebody may well be indifferent between a certain \$40 and a 50% chance of winning \$100, but will definitely prefer a certain \$40 million to a 50% chance of winning \$100 million.

This objection is justified. Therefore, a nonlinear perception utility function should be implemented, especially for wider outcome ranges, as in the case considered. Such a function leads to a different evaluation of relative outcomes, than presented by (6.5):

$$r_i = \frac{u(x_i) - u(P_{min})}{u(P_{max}) - u(P_{min})} \tag{7.1}$$

where u denotes the perception utility. “Perception utility” is a psychophysical function describing how stimuli, e.g. monetary outcomes, are perceived. Therefore, r_i would be more aptly

[11] The example considered was suggested by a reviewer. In order to compare the predictive power of the two models, a quick test was conducted among the staff of Acnet, a small telecommunications company in Warsaw, Poland. 19 people of varying sex, age, and educational level responded. Prospect A was chosen by 14 people (73.7%) and prospect B by 5 people (26.3%). The decision utility model therefore made a more accurate prediction in this case.

named the relatively perceived outcome in this case. Per contra, “decision utility” describes how these perceived and framed outcomes are factored into risky decisions. In principle, perception utility represents the same concept as the Prospect Theory value function. The two functions, however, differ in detail in almost every respect (Kontek, 2011). One of the differences is that the perception utility function is assumed to be logarithmic as the power function used by Cumulative Prospect Theory surprisingly fails to explain the case considered. This means that increasing prospect size does not alter preference, according to Cumulative Prospect Theory. This property of the theory is not accidental; Tversky and Kahneman (1992) refer to it as “preference homogeneity”.

The inability of the power function to explain this particular case might account for the other known Cumulative Prospect Theory failures. Neilson and Stowe (2002) demonstrate that Cumulative Prospect Theory cannot simultaneously explain participation in lotteries and the original Allais paradox. Blavatskyy (2005) similarly demonstrated that Cumulative Prospect Theory, with its power value function, could not explain the St. Petersburg Paradox. This preference change can only be explained by assuming a value function of decreasing elasticity, whereas the power function is of constant elasticity (Scholten & Read, 2010). They propose using the logarithmic function $v(x) = 1/a \ln(1 + a\,x)$ in order to explain similar observations.

Using a logarithmic function to describe perception utility explains the case considered on the basis of the decision utility model. Please note that the relatively perceived outcome r is greater in the case of the lottery with greater outcomes:

$$\frac{\ln(1 + a\,40)}{\ln(1 + a\,100)} < \frac{\ln(1 + a\,40{,}000{,}000)}{\ln(1 + a\,100{,}000{,}000)} \tag{7.2}$$

This shifts considerations more to the right part of the decision utility function, which represents greater risk aversion, and explains the preference for a sure payment in the case of a million dollar lottery. Such an explanation would not be possible with linearly assessed outcomes, in which case the relative outcomes are the same:

$$\frac{40}{100} = \frac{40{,}000{,}000}{100{,}000{,}000} \tag{7.3}$$

or with perception utility described using a power function, in which case the ratio CE/P remains constant whatever the power coefficient value:

$$\frac{40^{\alpha}}{100^{\alpha}}=\frac{40,000,000^{\alpha}}{100,000,000^{\alpha}} \tag{7.4}$$

The latter case incidentally shows why Cumulative Prospect Theory with its power value function fails to interpret lotteries with increasing stakes. The ratio of outcome values does not change when these outcomes are multiplied by the same number; this property of the power function, known as “scale invariance”, leads to “preference homogeneity”.

8 Summary

This article presents an alternative interpretation of the experimental data published by Kahneman and Tversky in their 1992 paper "Advances in Prospect Theory". Mental transformations, crucial to deriving the results, were discussed in the introduction. Later, the solution was derived without using the probability weighting function. The obtained function has a double S-type shape that strongly resembles the utility curve specified by the Markowitz hypothesis (1952). The presented decision utility function shows that risk seeking appears for relative outcomes below the aspiration level. On the other hand, risk aversion is present for relative outcomes above the aspiration level. This pattern is reversed for losses. The description of risk attitudes given by the convex-concave-convex-concave shape of the decision utility function substitutes the fourfold pattern introduced by CPT.

The essential feature of the model presented is that lotteries under consideration are always framed before making decision and decision utility is expressed in terms of probability. This results in a multi-step valuation process reflecting mental processes during decision making. The lottery is first framed as the result of mental adaptation and prospect scaling transformation. This operation plays a crucial role in the model. Framing leads people to consider changes in wealth in relative, rather than absolute terms (see also Kontek, 2009). In the next step the lottery is simplified to a two-outcome lottery and its decision utility (equivalent probability) is determined. This utility is finally transformed back to the absolute outcome scale in order to calculate the lottery certainty equivalent. The psychophysical perception of outcomes should additionally be implemented in the process if the lotteries under consideration are of wider outcome ranges. As the whole, the model proposed provides a fully “holistic” approach to lottery valuation similarly to Expected Utility or Cumulative Prospect Theory. It allows, however, the inclusion of heuristics and stopping rules at each step of the decision process. This reflects the tendency to simplify decisions in order to save time and limited brain resources.

Each step of the process presented in this paper is based on well known and well documented mental transformations, and is described in simple mathematical terms. The decision utility model follows the Expected Utility Theory method of assessing multi-outcome lotteries. Despite using individual probabilities, it avoids stochastic dominance violations. Returning to the classical notion of probability makes the model easy to apply in real world conditions. The model may be regarded as superior to other choice theories by virtue of the arguments presented.

One issue requires further clarification. Recently evidenced gain-loss separability violations have called the approach of evaluating mixed prospects into question. Tversky and Kahneman's data, obtained separately for gain and loss prospects, were used in this paper to derive the decision utility function. Therefore no decision utility function can be presented for mixed prospects without any further data sets or new experiments. Although the approach proposed seems to perform better than that of Cumulative Prospect Theory, the valuation of mixed prospects seems to be one of the most important subjects for future research.

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Appendix 1
Original experimental data published by Tversky and Kahneman (1992).

x_1 (in \$)	x_2 (in \$)	Probability p of x_2	*CE* for Gains (\$)	*CE* for Losses (\$)
0	50	0.10	9	-8
0	50	0.50	21	-21
0	50	0.90	37	-39
0	100	0.05	14	-8
0	100	0.25	25	-23.5
0	100	0.50	36	-42
0	100	0.75	52	-63
0	100	0.95	78	-84
0	200	0.01	10	-3
0	200	0.10	20	-23
0	200	0.50	76	-89
0	200	0.90	131	-155

0	200	0.99	188	-190
0	400	0.01	12	-14
0	400	0.99	377	-380
50	100	0.10	59	-59
50	100	0.50	71	-71
50	100	0.90	83	-85
50	150	0.05	64	-60
50	150	0.25	72.5	-71
50	150	0.50	86	-92
50	150	0.75	102	-113
50	150	0.95	128	-132
100	200	0.05	118	-112
100	200	0.25	130	-121
100	200	0.50	141	-142
100	200	0.75	162	-158
100	200	0.95	178	-179

Appendix 2
First-Order Stochastic Dominance

Let us assume two lotteries, A and B, which have the same outcome range but different cumulative outcome distributions: F_A and F_B respectively. Lottery A first-order stochastically dominates lottery B if:

$$F_A(x) \le F_B(x) \tag{1.1}$$

for all x, with strict inequality at some x. According to the decision utility model, F_A and F_B are normalized to obtain the cumulative relative outcome distributions G_A and G_B defined in the [0, 1] range. A condition similar to (1.1), viz.

$$G_A(r) \le G_B(r) \tag{1.2}$$

likewise holds in this case for all r in the [0, 1] range, with strict inequality at some r. The decision utility formula (6.9) takes the following form:

$$p' = \int_0^1 D(r)\,dG(r) \tag{1.3}$$

where $dG(r)$ denotes the probability density function. Integrating (1.3) by parts results in the following decision utility for lottery A:

$$p_A' = D(r)G_A(r)\Big|_0^1 - \int_0^1 dD(r)G_A(r) = 1 - \int_0^1 dD(r)G_A(r) \tag{1.4}$$

Similarly, for lottery B one gets:

$$p_B' = 1 - \int_0^1 dD(r) G_B(r) \tag{1.5}$$

It is easy to state that $p_A' > p_B'$ if condition (1.2) holds. This means that if A first-order stochastically dominates B then the decision utility of A is greater than that of B. This holds for any decision utility function *D*(*r*). Considerations on A and B having different ranges are left for future writings.

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