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Discrimination in the Equilibrium Search Model with Wage-Tenure Contracts[1]

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Abstract: This paper develops a discrimination search model with wage-tenure contracts to study race/gender differences in labor market outcomes. We find that based on the model: first, minority workers have a higher unemployment rate and a longer duration of unemployment; second, non-discriminating firms make higher profits than discriminating firms; third, the lowest acceptable wage for a minority worker is greater than that for a majority worker while the highest expected wage of a minority worker is lower; fourth, generally, on average minority workers earn less than majority workers and their wage increases more slowly than their counterparts'. In addition, our estimates show that productivity differences between blacks and whites (men and women) are 3% of whites (men's) productivity, while 91% of firms are prejudiced towards black workers and 93% towards female workers. The distaste they hold toward blacks is about 70% of the productivity of whites and towards women it is 95% of male productivity.

Key words: discrimination, random search, wage-tenure contract, wage gap

JEL: J31, J41, J71

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1. Introduction

Race and gender differentials in the labor market are persistent and widespread. The black-white pay gap has remained around 20% since the mid-1970s (Altonji and Blank, 1999). Even after controlling for human capital and other factors, blacks still earn about 10% less than whites on average (Lang and Lehmann, 2010).[1] In addition to wage differentials, blacks have historically higher unemployment rates and longer unemployment duration (Fairlie and Sundstrom, 1999). Similar stylized facts are also found in the gender literature. A series of papers by Blau and Kahn (2000, 2003, 2006) find that the gender pay gap in the US has stayed roughly constant at 25% since the mid-1990s; they also find that on average, there is a 0.3 log-point differential for 22 countries examined over the 1985-94 period. Gender differences in unemployment are also widely observed. For example, Azmat, Guell and Manning (2006) document a large gender gap in unemployment rates in many OECD countries.[2] Du and Dong (2009) find longer unemployment durations for women in post-restructuring urban China while Ollikainen (2003) observes longer duration for men in Finland.

One possible explanation for these race and gender differences in labor market outcomes is the presence of differences in endowments of characteristics related to productivity and preferences. The unexplained part, on the other hand, is either due to unobserved productivity skills or discrimination in the labor market. However, distinguishing between the two effects is far from straightforward. Recently, Bowlus and Eckstein (2002) developed an equilibrium search model and separately identified discrimination from unobserved productivity differences because each factor affects the earnings distribution differently. Using a sample of male high school graduates, they find that blacks produce 3.3% lower than whites, and 56% of firms in the labor market have a prejudice against blacks that the distaste is as high as 31% of the productivity of whites. In a study of gender discrimination, Flabbi (2010) uses

[1] Neal and Johnson (1996) find the unexplained wage gap between blacks and whites is significantly narrowed, or even disappeared in some subgroups after controlling for AFQT. However, other researchers find wage differentials re-emerge when years of schooling are further controlled with AFQT (Rodgers and Spriggs, 1996).

[2] Altonji and Blank (1999), however, find the unemployment among women has been as low or lower than among men since early 1980s. Participation rates, on the other hand, are historically lower among women. Therefore, when it comes to the overall rate of non-employment, it is always higher among women.

maximum likelihood estimation in a search framework with matching and bargaining and concludes that female workers are 6.5% less productive than male workers and that half of employers discriminate against women. This paper, built on the framework of search model with wage-tenure contracts (see for example, Burdett and Coles 2003), is able to empirically distinguish discrimination from unobserved productivity differences, and at the same time touches on wage-tenure profiles. Few papers on discrimination theory have attempted to generate predictions in this regard.[3]

In what follows, we will outline a discrimination search model with wage-tenure contracts and describe equilibrium results. To discuss the effect of discrimination on labor market outcomes, we introduce two types of workers and firms: (1) majority workers A and minority workers B; (2) discriminating firms D and non-discriminating firms N.[4] Workers are assumed to be identical except for their appearance. Firms who experience a disutility from hiring minority workers recruit them at a slower rate. So, for type A workers firms are homogenous while for type B workers they are heterogeneous. In this paper, discrimination is associated with 3 parameters: the fraction of D-firms, the degree of recruiting discrimination and the disutility taste D-firms have when hiring B-workers, all of which are assumed to be exogenously determined. Our model belongs to a class of random search models. Firms post tenure-based contracts for both types of workers, recruit workers and pay wages specified in the contracts. Workers, both unemployed and employed search for jobs randomly, accept the offers which arrive at an exogenous rate if and only if the expected lifetime value from the new offer is higher than the current one. Firms cannot fire workers or counter-offer workers' outside offers.

In equilibrium, the optimal contract for B workers provided by N-firms is uniformly better than that provided by D-firms. Though by offering a higher tenure-wages, the N-firm extracts a lower profit

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[3] The positive effect of tenure on wages has been identified in many studies (Altonji and Shakotko 1987; Topel 1991; Altonji and Williams 2005). However, there is competing empirical evidence on gender disparity in wage returns to tenure/experience. Some find that the overall wage return to tenure/experience is lower for women than men (Light and Ureta 1995; Munasinghe et al. 2008), while others find steeper wage-tenure profiles for women than men (Becker and Lindsay 1994; Hersch and Reagan 1997). The difference in returns to tenure between races is found to be insignificant in Bratsberg and Terrell (1998) and the returns to actual experience lower for blacks.

[4] One can think majority and minority workers are male and female in the context of gender, or white and black in the context of race.

from each B worker, it can hire more B workers who are willing to stay for a longer period so that the total profit B workers have created in the N firm exceeds that in a D firm. In addition, since both firms make the same profit from type A workers, the total profit is also higher for N firms than D firms.

The second finding of the discrimination search model with wage-tenure contracts concerns the relationship between the discrimination associated parameters and wage ranges for minority workers. It proves that, the fewer D firms are in the labor market, the higher the minimum wage and the lower the maximum wage B workers can expect in D firms. Similarly, the more severe the recruiting discrimination or distaste D firms hold, the higher the lower bound and the lower the upper bound for wages in D firms. The maximum wage in N-firms, is negatively related to all three parameters.

We also find that the lowest wage A workers are willing to accept is smaller than a B worker's lowest acceptable wage and both lowest wages are smaller than the unemployment insurance. This is because A workers can expect a faster wage increase and a larger probability of receiving a new offer than B workers and at the same time, both types of employed workers get a wage promotion that the unemployed do not get. The sign of the mean wage gap between type A and B workers, however, is uncertain. If D firms don't hire any B workers, it is shown that the average A worker earns more than the average B worker while in a general case,the fraction of discriminating firms and their distaste towards minority workers have to be large enough to generate the stylized average wage gap.

Subsequently, we show that in a special case of a CRRA utility function with the coefficient approaching zero, the model degenerates to a simplified version of Bowlus and Eckstein (2002) and has certain similar implications. How the average wage is affected by the discrimination-related parameters is next illustrated in the numerical example, where we also simulate the profile of wage dynamics for both types of workers. It is found that, the wage-tenure effect is positive and it is steeper for A workers than B worker in most cases.

Applying the search discrimination model with wage-tenure contracts using a sample of male high school graduates in 1985-88, we find that the productivity of blacks is 3% lower than that of whites and that 91% of firms in the labor market possess a distaste against blacks which is as high as 71% of the productivity of whites. In a second application using white high school graduates, we find that the gender difference in productivity is 3% and there are 93% of firms with a strong distaste against female workers which is about 95% of men's productivity. We compare the empirical hourly wage increase over a year with the predicted profiles and observe a certain correspondence between the two.

The contribution of this paper is the development of a discrimination search model with wage-tenure contracts that, among other things, generates race/gender differences in unemployment rates, durations of unemployment, and wage dynamics. In the theoretical literature on labor market discrimination, the taste-based theory of discrimination (Becker, 1971) and statistical discrimination (Aigner and Cain, 1977) are often subject to criticism on the grounds that discrimination cannot be sustained in the long run.[5] Taste discrimination models within a search framework, on the other hand, are very promising in explaining persistent wage differentials (Altonji and Blank, 1999). An early example is Black (1995) who studied discrimination in an equilibrium search model. In that paper, cost is introduced in job search processes and discriminating firms are assumed to hire only majority workers. He shows in the model that the wage minority workers receive is lower than the wage of their majority counterparts and the wage differential increases with the proportion of minority workers in the labor market. In a similar line of research, Bowlus and Eckstein (2002), by allowing for on-the-job search, construct a discrimination search model that generates wage dispersion among equally productive workers (see also Burdett and Mortensen, 1998). Moreover, they are able to distinguish the skill differences and discrimination in explaining the residual wage differentials between races. This paper follows the assumption of on-the-job search, but replaces the constant wage assumption with wage-tenure contracts which was first introduced in Burdett and Coles (2003). It allows for the possibility to predict differences in wage-tenure profiles.

[5]See Cain (1986) for a good review on the classic theories, Lang and Lehmann (2010) and Charles and Guryan (2011) for a recent review on progresses in both theories and empirics on race discrimination.

The next section sets up the model and discusses workers' and firms' optimal decisions. Section 3 characterizes the equilibrium solutions and section 4 shows the equilibrium properties. In section 5, we show in a special case, that the optimal wage-tenure contracts degenerate to a constant wage and our discrimination search model with wage-tenure contracts degenerates to a variant of Bowlus and Eckstein (2002). Further, to facilitate comparisons of average wages and their dynamics, we carry out a numerical exercise in section 6. Sections 7 and 8 put the model to data and estimate the race/gender differences in productivity and race/gender discrimination in the labor market. Finally, section 9 concludes and points out promising future research. All proofs are given in the appendix.

2. The Model

2.1 The Environment

Consider an economy consisting of two types of workers and firms. The total work force is n, of which the majority workers (type A) are $(1-\theta)n$ and the minority workers (type B) are θn. Among all the firms in the labor market, a fraction σ has a distaste for minority workers, denoted by D; and *(1-σ)* are non-discriminating firms denoted by N. Workers are assumed to be equally productive (productivity level P), and have utility function $u(w)$, where $u^{'} > 0, u^{''} < 0$. They are finitely lived, with a death rate δ. To balance the population, it's assumed that birth rate equals death rate and the newly born people enter the labor force immediately as unemployed. Unemployed workers can obtain an insurance compensation b per instant. Workers--both employed and unemployed--search for better opportunities to maximize their expected lifetime utility.

On the other hand, a firm posts a wage-tenure contract and hires workers to maximize its profit. The wage-tenure contract is denoted by $w(t)$, where t denotes tenure—the duration a worker stays in the firm. Suppose the offer arrival rate is λ for A-workers, both employed and unemployed; while for B-workers, it depends on the type of firm the offer originates from. If it is from N firms, the arrival rate is still λ; if it is from D firms, the offer arrival rate is $(1-k)\lambda$, where $k \in (0,1)$ reflects the degree of recruiting discrimination.[6] The larger k is, the more severe the discrimination. D firms experience a disutility d from hiring B workers, which enters the profit function directly. Therefore, the instantaneous profit from a B worker who has stayed in the D firm for a duration t is: $P - w_B^D(t) - d$. In addition, assume firms cannot fire workers but workers can quit for a better job without suffering any punishment from the previous employer. Time preferences of workers and firms are zero and there are no recalls in the process.

2.2 Workers' Optimal Decision

Let $V_i\left(t\middle|\widetilde{w_i^J}\right)$ be the expected lifetime utility of a type i ($i = A, B$) worker who has tenure t under the wage-tenure contract $\widetilde{w_i^J}$ and uses an optimal quit strategy in the future. The term $\widetilde{w_i^J}$ denotes the wage-tenure contract a type i worker has signed with firm j ($j = D, N$). $F_A(V_0)$, $F_B^N(V_0)$ and $F_B^D(V_0)$ are the offer distributions for A and B where superscripts N, D denote non-discriminating and discriminating firms and V_0 is the starting expected lifetime value of the offer. Thus, the offer distribution measures the proportion of firms who provide workers a starting offer value no greater than V_0. Since all firms treat A the same, there is no difference in the offer distributions for A provided by N or D firms. Let $\underline{V_A}$ ($\overline{V_A}$) denote the infimum (supremum) of the support of F_A and $\underline{V_B^j}$ ($\overline{V_B^J}$) the infimum (supremum) of the support of F_B^j where $j = N, D$.

First consider the situation of employed workers. The standard Bellman equations for employed type A and type B workers are:

$$0 = u\big(w_A(t)\big) - \delta V_A(t|\widetilde{w_A}) + \lambda \int_{V_A(t|\widetilde{w_A})}^{\overline{V_A}} [V_0 - V_A(t|\widetilde{w_A})]dF_A(V_0) + \frac{dV_A(t|\widetilde{w_A})}{dt} \tag{1a}$$

$$0 = u\big(w_B(t)\big) - \delta V_B\left(t\middle|\widetilde{w_B^J}\right) + (1-\sigma)\lambda \int_{V_B\left(t\middle|\widetilde{w_B^J}\right)}^{\overline{V_B^N}} \max\left\{0, \left[V_0 - V_B\left(t\middle|\widetilde{w_B^J}\right)\right]\right\} dF_B^N(V_0)$$

[6]Parameter k can also be interpreted as indicating the difference in search intensity. Therefore, it only reflects the degree of recruiting discrimination when we assume both types of workers exert the same level of effort in looking for jobs. Indeed, the existence of recruiting discrimination against minority workers such as blacks and women are widely documented (see, for example, Goldin and Rouse 2000; Bertrand and Mullainathan 2004; and Pager et al. 2009).

$$+\sigma(1-k)\lambda \int_{V_B\left(t|\widetilde{w_B^j}\right)}^{\overline{V_B^D}} \max\{0, \left[V_0 - V_B\left(t\middle|\widetilde{w_B^j}\right)\right]\} dF_B^D(V_0) + \frac{dV_B\left(t\middle|\widetilde{w_B^j}\right)}{dt} \tag{1b}$$

Note that, an A worker receives an offer at rate λ, whereas a B worker has a probability of $(1-\sigma)\lambda$ receiving an offer from N firms and a probability of $\sigma(1-k)\lambda$ receiving an offer from D firms. The optimal quit strategy implies that they will quit and accept the new offer if and only if its starting value is greater than the current value.[7] The last term in both equations calculates the instantaneous change in the expected lifetime value.

Similarly, we can get the Bellman equations for unemployed workers of both types:

$$0 = u(b) - \delta V_{AU} + \lambda \int_{V_{AU}}^{\overline{V}_A} [V_0 - V_{AU}] dF_A(V_0) \tag{2a}$$

$$0 = u(b) - \delta V_{BU} + (1-\sigma)\lambda \int_{V_{BU}}^{\overline{V_B^N}} [V_0 - V_{BU}] dF_B^N(V_0) + \sigma(1-k)\lambda \int_{V_{BU}}^{\overline{V_B^D}} [V_0 - V_{BU}] dF_B^D(V_0) \tag{2b}$$

The expected lifetime value of an offer from firms should be no less than the unemployed lifetime value V_U; otherwise, no worker would be hired. Therefore, $\underline{V_A} \geq V_{AU}$ and $\underline{V_B^j} \geq V_{BU}$ $(j = D, N)$.

2.3 Firms' Optimal Decision

The optimization problem faced by a firm is to choose two wage-tenure contracts, one for A workers and the other for B workers, to maximize the total expected profit at the steady state. To begin with, we need to derive the expressions of total expected profit for each firm.

Since the quit rate of a type A worker who has stayed t periods under the wage-tenure contract $w_A(t)$ is $\lambda(1 - F_A(V_A(t|\widetilde{w_A}))$, the survival probability of such a worker is:

$$\psi_A(t|\widetilde{w_A}) \triangleq \exp\left\{-\int_0^t \left[\delta + \lambda\left(1 - F_A\left(V_A(s|\widetilde{w_A})\right)\right)\right] ds\right\} \tag{3a}$$

Similarly, the survival probability of worker B is:

$$\psi_B\left(t\middle|\widetilde{w_B^j}\right) \triangleq \exp\left\{-\int_0^t \left\{\delta + (1-\sigma)\lambda\left[1 - F_B^N\left(V_B\left(s\middle|\widetilde{w_B^j}\right)\right)\right] + \sigma(1-k)\lambda\left[1 - F_B^D\left(V_B\left(s\middle|\widetilde{w_B^j}\right)\right)\right]\right\} ds\right\}$$

[7] Since the relationship between the current expected lifetime value and the supremum of offers from $N(D)$-firm is not clear yet, the maximum of zero and instantaneous change that occurs when the worker accepts the offer ensures the non-negativity and economic meaning. Intuitively, the current value should always be smaller than $\overline{V_B^N}$, which means the first max is trivial; however, it may or may not be smaller than $\overline{V_B^D}$ which makes the second max indispensable.

(3b)

Let $G_A(V)$ denote the steady state proportion of A workers who have an expected lifetime utility less than or equal to V (including the unemployed); and correspondingly, $G_B(V)$ for worker B. Thus, at the steady state, a firm posting an offer V can recruit $[\lambda G_A(V)(1-\theta)n]$ A workers and $\lambda G_B(V)\theta n$ (if N-firm) or $\lambda(1-k)G_B(V)\theta n$ (if D firm) B workers. The steady state profits of N and D firms are then functions of the wage-tenure contracts:

$$\Omega^{\mathrm{N}}\left(V_0^A, V_0^B\right) =$$

$$\lambda G_A\left(V_0^A\right)(1-\theta)n\int_0^\infty \psi_A(t|\widetilde{w_A})[P-w_A(t)]\,\mathrm{dt} + \lambda G_B(V_0^B)\theta n\int_0^\infty \psi_B\left(t\middle|\widetilde{w_B^N}\right)[P-w_B^N(t)]\,\mathrm{dt} \tag{4a}$$

$$\Omega^{\mathrm{D}}\left(V_0^A, V_0^B\right) =$$

$$\lambda G_A\left(V_0^A\right)(1-\theta)n\int_0^\infty \psi_A(t|\widetilde{w_A})[P-w_A(t)]\,\mathrm{dt} + \lambda(1-k)G_B\left(V_0^B\right)\theta n\int_0^\infty \psi_B(t|\widetilde{w_B^D})[P-w_B^D(t)-d]\,\mathrm{dt} \tag{4b}$$

In each equation, the first part is the profit from A and the second part is the profit from B. The integration calculates the expected profit that each worker brings to the firm; the part before the integration measures the steady state number of workers hired at given offers. So, the multiplication reflects the firms' expected profit from each type of worker. As both firms treat A equally, profit earned from A is the same between firms in equilibrium.

To derive the optimal decisions of firms, we need to solve the profit maximization problems. Due to additivity, we can solve separately for A; B in N firms and B in D firms. Each sub-problem can be solved in two steps:

(i) Conditional on the offer chosen, the optimal wage-tenure contract solves:

$$\underset{w_i^j(t)}{\text{Max}} \int_0^\infty \psi_i\left(t\middle|\widetilde{w_i^j}\right)\left[P-w_i^j(t)\right]\mathrm{dt}$$

$$\text{s.t } \psi_i\left(t\middle|\widetilde{w_i^j}\right) \text{ satisfies (3)}$$

$$V_i\left(t\middle|\widetilde{w_i^j}\right) \text{ satisfies (1)}$$

$$\text{and, } \psi_i\left(0\middle|\widetilde{w_i^j}\right) = 1; V_i\left(0\middle|\widetilde{w_i^j}\right) = V_0^{ij}\ .$$

(ii) The optimal offer solves:

$$\mathrm{Max}_{V_0{}^{ij}}\, G_i\left(V_0{}^{ij}\right) \int_0^{\infty} \psi_i\left(t\middle|\widetilde{w_i^j}\right)\left[P - w_i^j(t)\right] \mathrm{dt}$$

$$\text{s.t } w_i^j(t) \text{ solves (i)}$$

where $i = A, B$; $j = N, D$.

When it comes to type B workers in D firms, the disutility taste d should be further subtracted from $P - w_i(t)$.

3 Equilibrium

Since worker A faces homogenous firms in the labor market, the market equilibrium outcomes for this sub-problem are exactly the same as specified in Burdett and Coles (2003). To solve for the steady state equilibrium for worker B, we first show in proposition 3.2 that the optimal offer for B provided by D firms is uniformly smaller than that provided by N-firms.

Proposition 1: Let V_{0N}^B denote the optimal offer for B given by N-firms and V_{0D}^B the optimal offer provided by D firms; then we have $V_{0N}^B \geq V_{0D}^B$.

Proposition 1 simplifies the subsequent analysis substantially.[8] As $V_{0N}^B \geq V_{0D}^B$, equations (1b) and (3b) can be rewritten for B in N and D firms separately. Specifically, the Bellman equation for B workers working in N firms is reduced to:

$$0 = u\left(w_B^N(t)\right) - \delta V_B^N\left(t\middle|\widetilde{w_B^N}\right) + (1-\sigma)\lambda \int_{V_B^N\left(t\middle|\widetilde{w_B^N}\right)}^{\overline{V_B^N}} \left[V_0 - V_B^N\left(t\middle|\widetilde{w_B^N}\right)\right] dF_B^N(V_0) + \frac{dV_B^N\left(t\middle|\widetilde{w_B^N}\right)}{dt} \tag{5}$$

For those working in D firms the Bellman equation becomes:

$$0 = u\left(w_B^D(t)\right) - \delta V_B^D\left(t\middle|\widetilde{w_B^D}\right) + (1-\sigma)\lambda\left[EV_B^N - V_B^D\left(t\middle|\widetilde{w_B^D}\right)\right]$$

$$+\sigma(1-k)\lambda \int_{V_B^D\left(t\middle|\widetilde{w_B^D}\right)}^{\overline{V_B^D}} \left[V_0 - V_B^D\left(t\middle|\widetilde{w_B^D}\right)\right] dF_B^D(V_0) + \frac{dV_B^D\left(t\middle|\widetilde{w_B^D}\right)}{dt} \tag{6}$$

Similarly, survival probabilities of B workers who are employed by N firms and D firms change from 3(b) to:

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[8] Burdett and Coles (2010) prove that offer values can be ranked according to the productivity level of firms. Consider the market of B workers only, if we think the marginal productivity of D firms as $P - d$ and N firms as P, proposition 3.2 here is implied by their result.

$$\psi_B\left(t\middle|\widetilde{w_B^N}\right) = exp\left\{-\int_0^t [\delta + (1-\sigma)\lambda(1 - F_B^N\left(V_B\left(s\middle|\widetilde{w_B^N}\right)\right))]ds\right\} \tag{7}$$

$$\psi_B\left(t\middle|\widetilde{w_B^D}\right) = exp\left\{-\int_0^t \left[\delta + (1-\sigma)\lambda + \sigma(1-k)\lambda(1 - F_B^D\left(V_B\left(s\middle|\widetilde{w_B^D}\right)\right))\right] ds\right\} \tag{8}$$

This makes disentanglement of the sub-problems for B workers in N and D firms possible. The following proposition describes the equilibrium outcomes in the labor market. The crucial step in the proof is to define $G_B^D(V_0)$ and $G_B^N(V_0)$ to replace $G_B(V_0)$. Let $G_B^D(V_0), V_0 \in [\underline{V_B^D},\ \overline{V_B^D})$ be the proportion of B workers who have an expected lifetime value no greater than V_0 in all B workers excluding those working in N firms and $G_B^N(V_0), V_0 \in [\underline{V_B^N},\ \overline{V_B^N})$ be the proportion of B workers with expected lifetime value no greater than V_0 in all B-workers. Then, the proof of the equilibrium outcomes could fit nicely in that of Burdett and Coles (2003). Moreover, through constructing the overall $G_B(V_0)$ from $G_B^D(V_0)$ and $G_B^N(V_0)$, we show that the lower bound of the starting wage in N-firms is the upper limit of starting wages offered by D firms. The assumption of differentiable $\mathrm{F}_\mathrm{B}^\mathrm{j}(\cdot)$ is necessary to derive the equilibrium. Otherwise, a mass point exists in $\mathrm{F}_\mathrm{B}^\mathrm{j}(\cdot)$ at the extreme offer value and wages in N-firms can be smaller than wages in D-firms when the rank of offer values remains (Burdett and Coles 2010). Detailed proof refers to the appendix.

Proposition 2:

(1) Given $\underline{\mathrm{w}_\mathrm{A}}, \underline{w_B} > 0$ and $\mathrm{F}_\mathrm{A}(\mathrm{V})$, $F_B^N(V)$, $F_B^D(V)$ are increasing and continuously differentiable, there exists a unique market equilibrium. At the steady state equilibrium, the baseline salary scale for worker A satisfies:

$$\frac{P-\overline{w_A}}{P-\underline{w_A}} = \left(\frac{\delta}{\delta+\lambda}\right)^2 \tag{9}$$

$$u\left(\underline{w_A}\right) = u(b) - \frac{\sqrt{P-\underline{w_A}}}{2}\int_{\underline{w_A}}^{\overline{w_A}} \frac{u'(x)dx}{\sqrt{P-x}} \tag{10}$$

The optimal wage-tenure contract for worker A follows the dynamic path**:**

$$\frac{dw_A}{dt} = \frac{\delta(P-w_A)}{u'(w_A)}\int_{w_A}^{\overline{w_A}} \frac{u'(x)dx}{\sqrt{(P-\overline{w_A})(P-x)}} \tag{11}$$

For worker B, the baseline salary scale satisfies:

$$\frac{P-\overline{w_B^D}-d}{P-\underline{w_B^D}-d}=\left(\frac{\delta+(1-\sigma)\lambda}{\delta+(1-\sigma k)\lambda}\right)^2 \tag{12}$$

$$u\left(\underline{w_B^D}\right)=u(b)-\frac{\sqrt{P-\underline{w_B^D}-d}}{2}\int_{\underline{w_B^D}}^{\overline{w_B^D}}\frac{u'(x)dx}{\sqrt{P-x-d}} \tag{13}$$

$$\underline{w_B^N}=\overline{w_B^D} \tag{14}$$

$$\frac{P-\overline{w_B^N}}{P-\underline{w_B^N}}=\left(\frac{\delta}{\delta+(1-\sigma)\lambda}\right)^2 \tag{15}$$

And the dynamics of baseline salaries are:

$$\frac{dw_B^D}{dt}=\frac{(\delta+(1-\sigma)\lambda)(P-w_B^D-d)}{u'(w_B^D)}\int_{w_B^D}^{\overline{w_B^D}}\frac{u'(x)dx}{\sqrt{\left(P-\overline{w_B^D}-d\right)(P-x-d)}} \tag{16}$$

$$\frac{dw_B^N}{dt}=\frac{\delta(P-w_B^N)}{u'(w_B^N)}\int_{w_B^N}^{\overline{w_B^N}}\frac{u'(x)dx}{\sqrt{(P-\overline{w_B^N})(P-x)}} \tag{17}$$

(2) At equilibrium, the earnings distributions are given by:

$$K_w^A(w)=\frac{\delta}{\lambda}\left[\sqrt{\frac{P-\underline{w_A}}{P-w}}-1\right] \tag{18}$$

$$K_w^B(w)=\begin{cases}\frac{\delta}{(1-\sigma k)\lambda}\left[\sqrt{\frac{P-\underline{w_B^D}-d}{P-w-d}}-1\right], & if\ w\in[\underline{w_B^D},\overline{w_B^D}]\\ \frac{\delta+(1-\sigma k)\lambda}{(1-\sigma k)\lambda}\sqrt{\frac{P-\overline{w_B^N}}{P-w}}-\frac{\delta}{(1-\sigma k)\lambda}, & if\ w\in[\underline{w_B^N},\overline{w_B^N}]\end{cases} \tag{19}$$

And the unemployment rates of each type of workers are:

$$u_A=\frac{\delta}{\lambda+\delta} \tag{20}$$

$$u_B=\frac{\delta}{\delta+(1-\sigma k)\lambda} \tag{21}$$

The maximized total profits earned by a D firm and a N firm are:

$$\Omega^{\mathrm{D}}=\lambda(1-\theta)n\frac{P-\overline{w_A}}{\delta}+\lambda(1-k)\theta n\delta\frac{P-\overline{w_B^D}-d}{[\delta+(1-\sigma)\lambda]^2} \tag{22}$$

$$\Omega^{\mathrm{N}}=\lambda(1-\theta)n\frac{P-\overline{w_A}}{\delta}+\lambda\theta n\frac{P-\overline{w_B^N}}{\delta} \tag{23}$$

Baseline salary scale is a succinct way to describe all the equilibrium solutions. For any starting value V_0 from the support of offer distribution F_A, there exists a point t_0 such that $V_0 = V_s(t_0)$ where the subscript s denotes baseline. So the wage-tenure contract with a starting value V_0 can be expressed as $w(t|V_0) = w_s(t + t_0)$; that is, any equilibrium wage-tenure contract can be found on the baseline salary scale starting with a specific point t_0. In this paper, we suppress the s-subscript for simplicity of presentation. The optimal decision implied in the proposition 2(1) is: for worker A, a firm can set any wage between $[\underline{w_A}, \overline{w_A})$ as the starting wage offer and backload it as described in the optimal wage-tenure dynamic (11); the total profit from A will be the same across firms no matter which wage-tenure contract they choose. Since $\frac{dw_A}{dt}$ is positive, the optimal wage increases with tenure and the upper limit of the increment is $\overline{w_A}$. Obviously, the wage support for type A workers can be solved by combining (9) and (10), from which the earnings distribution (18) can be derived.

Similarly, for worker B, D firms can set any starting wage between $[\underline{w_B^D}, \overline{w_B^D})$ and then backload the wage using the rule described in (16). Profit from type B workers is the same across the discriminating firms. N-firms can determine any starting wage between $[\underline{w_B^N}, \overline{w_B^N})$, increase the wage with tenure as described in (17) and make the same profit as any other N firms. One point to note is that although $\underline{w_B^N} = \overline{w_B^D}$, $\underline{V_B^N} \neq \overline{V_B^D}$. Rather, employees hired in N firms with a payment $\underline{w_B^N}$ have a higher expected lifetime value than the high-earners in D firms, i.e., $\underline{V_B^N} > \overline{V_B^D}$; because workers with $\underline{w_B^N}$ can expect an immediate increase in the payment while those approaching $\overline{w_B^D}$ cannot.

Second, from the expression of unemployment rate (21), we can see that disutility d has no effect on u_B; it is always higher than A 's unemployment rate given in (20) as long as there is discrimination in the labor market ($\sigma k \neq 0$). If any of the two indicators equals zero, there would be no discriminating firms in the labor market.

Third, from (22) and (23), it is easy to get the difference in profits in N and D firms:

$$\Omega^{\mathrm{N}} - \Omega^{\mathrm{D}} = \lambda\theta n \frac{\delta[k(P-\overline{w_B^D})+(1-k)d]}{[\delta+(1-\sigma)\lambda]^2} > 0 \tag{24}$$

This is a general finding in the discrimination literature. Though D firms earn higher profit from a single B worker by paying a lower wage, the total profit is less than that in N firms; because the negative effect of lower employment and higher quit rate in a D firm outweighs the positive effect of a lower wage. Besides, the disutility taste D firms have towards B workers widens the profit gap further. The larger θ, n, k and d is, the larger the gap.[9] This indicates that having more minority workers in the labor market places the discriminating firms in a worse situation; and, the more prejudiced the discriminating firms are, the higher loss they will bear.

4 Equilibrium Properties

To facilitate the comparisons of average wages between two types of workers, we calculate the mean wages from (9), (12), (14), (15), (18) and (19), which gives

$$Ew_A = \int_{\underline{w_A}}^{\overline{w_A}} w dK_w^A(w)$$

$$= \overline{w_A} + \frac{\delta}{\lambda}(\overline{w_A} - \underline{w_A}) - \frac{2\delta}{\delta+\lambda}(P - \underline{w_A}) \tag{25}$$

$$Ew_B = \int_{\underline{w_B^D}}^{\overline{w_B^N}} w dK_w^B(w)$$

$$= \overline{w_B^N} + \frac{\delta}{(1-\sigma k)\lambda}(\overline{w_B^N} - \underline{w_B^D}) - \frac{2}{1-\sigma k}\left[\frac{\delta\sigma(1-k)}{\delta+(1-\sigma k)\lambda}\left(P - \underline{w_B^D} - d\right) + \frac{(1-\sigma)[\delta+(1-\sigma k)\lambda]}{\delta}(P - \overline{w_B^N})\right] \tag{26}$$

Note that the unemployed workers are not included in the calculation.

Under some general conditions, we discuss the equilibrium properties in the following proposition:

Proposition 3 If $b \le \frac{3}{4}(P-d)$, $\frac{u'\left(\overline{w_B^D}\right)}{u'\left(\underline{w_B^D}\right)} > \frac{\eta}{\eta+2}$ and $\frac{\partial \overline{w_B^D}}{\partial\sigma} < \frac{2\lambda\left(P-\overline{w_B^D}\right)}{\delta+(1-\sigma)\lambda}$ where $\eta = \frac{\delta+(1-\sigma k)\lambda}{\delta+(1-\sigma)\lambda}$ is the relative hazard rate, then the equilibrium has the following properties:

(1) $\frac{\partial \underline{w_B^D}}{\partial\sigma} < 0 \quad \frac{\partial \overline{w_B^D}}{\partial\sigma} > 0 \quad \frac{\partial \overline{w_B^N}}{\partial\sigma} < 0$

---

[9]Though values of k and d also influence $\overline{w_B^D}$ in the expression of profit difference, the negative correlation between k, d and $\overline{w_B^D}$ (which to be shown in section 4) will enhance the positive relationship between k, d and the profit gap.

(2) $\frac{\partial \underline{w_B^D}}{\partial k} > 0 \quad \frac{\partial \overline{w_B^D}}{\partial k} < 0 \quad \frac{\partial \overline{w_B^N}}{\partial k} < 0$

(3) $\frac{\partial \underline{w_B^D}}{\partial d} > 0 \quad \frac{\partial \overline{w_B^D}}{\partial d} < 0 \quad \frac{\partial \overline{w_B^N}}{\partial d} < 0$

(4) $\underline{w_A} < \underline{w_B^D} \leq b < \overline{w_B^D} < \overline{w_A}$

(5) $Ew_A > Ew_B$ when discriminating firms only hire A workers (i.e., $k = 1$ and $\sigma \neq 0$)

The discriminating wage bounds solved from equations (12) and (13) and non-discriminating wage bounds solved from equations (14) and (15) are functions of productivity P, unemployment insurance b, birth-death rate δ, normal offer arrival rate λ and three discrimination indicators (σ, k, d). Under conditions specified in proposition 3, the comparative statics of wage bounds with respect to the three discrimination associated parameters, described in properties (1)-(3), can be easily obtained.

Property (1) shows that the higher the proportion of D-firms in the market, the wider the range of discriminating wages will be; and the range extends in both directions. On the contrary, the degree of recruiting discrimination has an opposite effect: severe discrimination in the hiring process will lead to a narrowing of the discriminating wage range which converges to the unemployment insurance (which is implied by property (4)). Disutility has the same effect on discriminating wage bounds. Finally, the highest non-discriminating wage decreases as any of the three parameters increases.

The next two properties compare the equilibrium wages between two types of workers. Several points are noteworthy. First, the lowest acceptable wage is lower than the unemployment insurance, which is a unique result within the search model with wage-tenure contracts. In Burdett and Mortensen (1998), firms set a constant wage rather than a wage-tenure contract, hence the lowest acceptable wage is the unemployment insurance b (when the offer arrival rate is the same for both the employed and the unemployed). Under the wage-tenure framework, however, workers are willing to work at a wage lower than the unemployment insurance only because they can expect an immediate increase in the payment. In fact, the expected lifetime value at the lowest wage is virtually equal to that at the status of unemployment.

Second, A's lowest acceptable starting wage is less than the lowest starting wage for B. This is because on the one hand, worker A's wage increases with tenure more quickly than B's; on the other hand, compared to B, A is more likely to get a new and better job offer in the labor market.

Third, that the upper bound of A's wages being higher than their counterpart's is within expectation, since discriminating firms are unlikely to set too high a wage due to their disutility tastes.

In a special case where discriminating firms hire only type A, property (5) shows that "minority workers receive lower wages than workers not facing discrimination" (Black, 1995). However, this finding cannot be generalized. In the numerical example, we will show that if D-firms can hire B ($0 \leq k < 1$), the average worker B might be able to earn a slightly higher wage than worker A.

5. A special case

In this section, a special case of the CRRA utility function: $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$ $(\gamma \to \infty)$ is considered. Tractable equilibrium solutions that are derived from proposition 2 can shed more light on the labor market with discrimination. Proposition 4 below summarizes the equilibrium results in this special case.

Proposition 4: Given that both types of workers have the same CRRA utility function: $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$ with $\gamma \to \infty$, the following statements hold:

(1) The optimal strategy of a firm is to set fixed wages instead of the wage-tenure contracts, i.e., $\frac{dw_i^j}{dt} = 0$.

(2) The wage bounds are:

$$\underline{w_A} = b \qquad \overline{w_A} = P - \left(\frac{\delta}{\lambda+\delta}\right)^2 (P-b)$$

$$\underline{w_B^D} = b \qquad \overline{w_B^D} = \underline{w_B^N} = P - d - \left(\frac{\delta + (1-\sigma)\lambda}{\delta + (1-\sigma k)\lambda}\right)^2 (P-b-d)$$

$$\overline{w_B^N} = P - \left(\frac{\delta}{\delta + (1-\sigma)\lambda}\right)^2 d - \left(\frac{\delta}{\delta + (1-\sigma k)\lambda}\right)^2 (P-b-d)$$

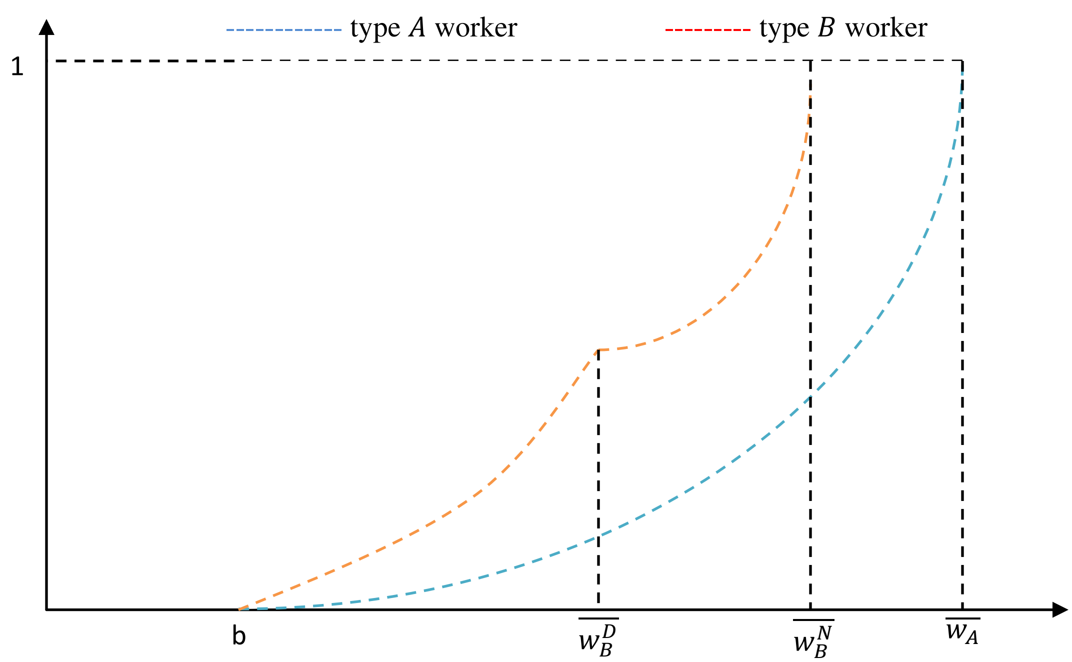
And, $\underline{w_A} = \underline{w_B^D} = b < \overline{w_B^D} < \overline{w_B^N} < \overline{w_A}$.

(3) A's earnings distribution first order dominates B's earnings distribution, i.e., $K_w^A < K_w^B$ for all w.

(4) $Ew_A > Ew_B$ and the mean wage gap increases with (σ, k, d).

As $\gamma \to \infty$, workers are infinitely risk averse; thus the optimal wage contract is constant wages. The equilibrium search model with wage-tenure contracts then degenerates to Burdett and Mortensen (1998) and the discriminating wage-tenure equilibrium search model degenerates to a simplified version of Bowlus and Eckstein (2002).[10] Figure 1 describes the earnings distributions for both types of workers and apparently A's cumulative earnings distribution first order dominates B's distribution. From first order dominance, property (4) is directly obtained. In addition, the same reservation wages between A and B is resulted from the assumption that the offer arrival rate is invariant between the employed and unemployed workers. The upper wage limit of B is less than that of A because of the existence of the three non-zero discrimination parameters (σ, k, d).

Figure 1: Earnings distributions



Moreover, the larger (σ, k, d) is, the smaller B's average wage is. Since (σ, k, d) does not enter type A worker's wage, the average wage gap increases as (σ, k, d) increases. This conclusion is in line with the empirical findings. For example, Charles and Guryan (2008) plot the black-white wage gap

[10] Bowlus and Eckstein (2002) extend Burdett and Mortensen (1998)'s model to discuss the contributions of discrimination and skill differences to the wage gaps. In their paper, the offer arrival rate is assumed to be different between the employed and the unemployed and therefore unlike what we get in this special case, the reservation wage is larger than the unemployment compensation

against prejudicial attitude and find a wider gap at regions where many people will not vote for the black candidate for presidency or are against interracial marriages.

6 A Numerical Example

As mentioned in section 4, it is interesting to examine the effect of the three discrimination-relevant parameters on the difference in the mean wages between type A and B workers. We assume in the section that all workers have the same CRRA utility function. Let $P = 300, b = 100, \lambda = 0.03$ and $\delta = 0.003$. If the coefficients of relative risk aversion are $0.9, 1.4$ and 1.9, equation (25) gives that A's average wages are 273.3307, 275.3025 and 276.8115 respectively. It seems that the more risk averse workers are, the higher the average wage they would earn.

For worker B, we vary the values of (σ, k, d) to see how the mean wage changes accordingly. Results are presented in table 1 in which the first panel fixes d and k, and changes the measure of discriminating firms σ; the second panel changes the recruiting discrimination k and keeps the other two measures unchanged; and the third one modifies disutility taste d given certain values of σ and k. The findings are as follows: First, the mean wage of type B worker decreases in σ and d, but increases in γ while the relationship with k is uncertain. Second, the fraction of D-firms plays a key role in the average wage; the other three parameters, though matter to some extent, have only limited influence on the wage outcomes. Third, if only D-firms exist in the labor market (see the case $\sigma = 1$ in Panel 1), the wage gap is very large; however, the gap will drop dramatically when N-firms begin to appear. In addition, Panel (2) indicates that the wage gap does not change much even when D-firms are forbidden to discriminate in hiring (see $k = 0$); on the other hand, what appears to be against expectation is that severe discrimination in recruitment leads to higher average wage for B and hence smaller wage gap (see $k = 0.9$). However, one should realize that this does not mean type B workers are better off because only a few will be hired in this situation and the overall welfare of type B workers is in fact jeopardized. Finally, compared to A's average wage, the numbers in Table 1 are almost consistently smaller, which accords with the common sense that discriminated workers have lower average wage.[11]

Table 1: The mean wage of type B workers

(1) $d = 80$ $k = 0.2$

	$\gamma = 0.9$	$\gamma = 1.4$	$\gamma = 1.9$
$\sigma = 0.2$	276.6618	276.7661	276.8648
$\sigma = 0.4$	269.3880	269.9290	270.3657
$\sigma = 0.6$	258.4842	259.7430	260.6239
$\sigma = 0.8$	238.7852	240.6891	241.9693
$\sigma = 1.0$	196.9306	199.2000	200.7968

(2) $d = 80$ $\sigma = 0.5$

	$\gamma = 0.9$	$\gamma = 1.4$	$\gamma = 1.9$
$k = 0.0$	264.9373	265.9374	266.6517
$k = 0.3$	264.4216	265.2213	265.8328
$k = 0.6$	264.4312	264.8563	265.2231
$k = 0.9$	265.7274	265.7662	265.8044

(3) $\sigma = 0.5$ $k = 0.5$

	$\gamma = 0.9$	$\gamma = 1.4$	$\gamma = 1.9$
d =10	269.3757	270.4330	271.2300
$d = 80$	264.3334	264.9009	265.3683
$d = 150$	259.9760	260.1040	260.2230

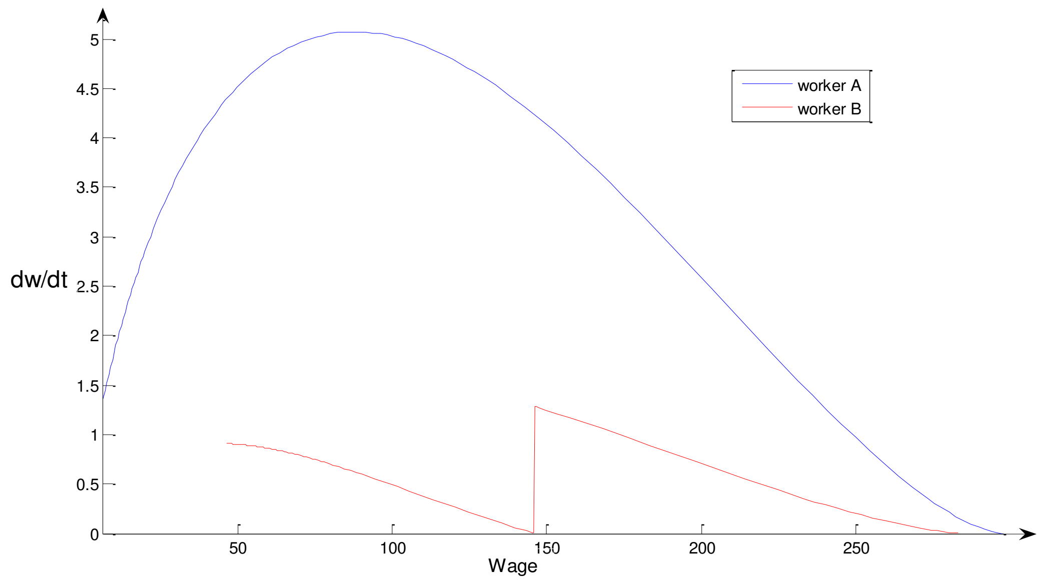
Next, we discuss the difference in wage dynamics between the two types of workers. To be representative, we choose a most realistic case where $\gamma = 0.9$, $k = 0.8$, $\sigma = 0.8$ and $d = 80$ and an extreme case in which B's mean wage exceeds that of type A worker (See Figure 2).[12]

Figure 2: Wage Dynamics

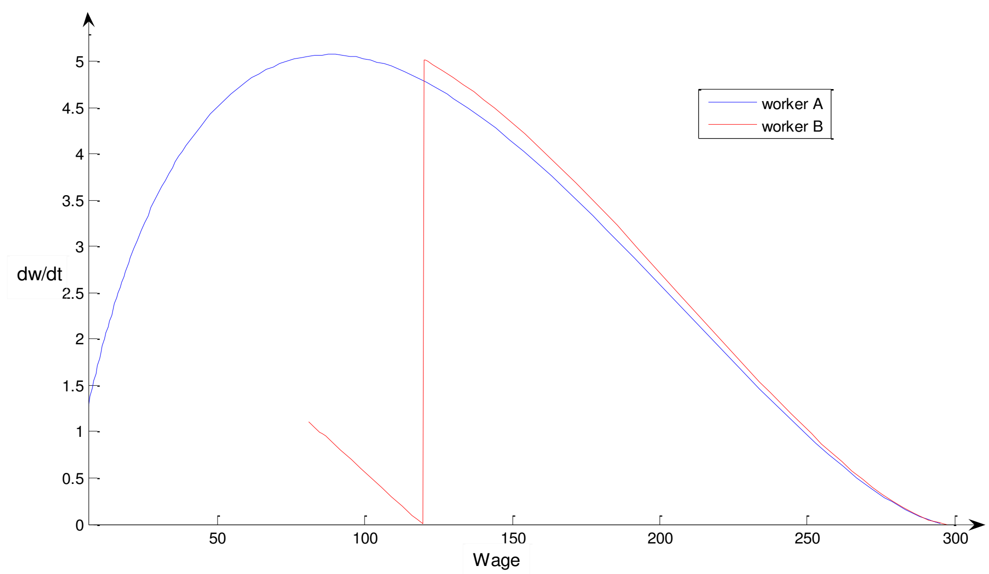
(a) Realistic case

[11] One exception is when $\sigma = 0.2$ in Panel 1, B's average wage is slightly larger than A's. These rare cases seem to imply that the fraction of discriminating firms has to be large enough to generate the result of minority workers earning less than majority workers on average. Becker (1971) gives the exact condition σ should satisfy to derive the wage differential in the framework of competitive labor market. Aigner and Cain (1977) find a similar result in a group of low skilled workers, that discriminated-against workers have a higher average wage than their counterparts under the assumption of same mean productivity and different variances.

[12] Given those values, the simulated average wages for A and B are 273.3307 and 229.4995 respectively, very close to 273.9 and 230.96 derived from real data (Bowlus and Eckstein, 2002).



(b) Extreme case



There are several points worth noting. First, the slope of the wage-tenure contract is positive, meaning that the wage always increases with tenure. Second, for type A workers, the increase accelerates at the beginning, and slows down gradually; on the other hand, for type B workers the increasing rate drops from the very beginning. Besides, the slope of A's wage-tenure contract is, in general, larger than B's, especially in D-firms. N-firms, though owning no prejudice towards worker B, have less incentives to backload their wages as quickly as they do to worker A because there are fewer outside opportunities to worker B. If, however, only a small number of firms discriminate against worker B so that they can still seek many job offers from non-discriminating firms, then the slope of wage-tenure contracts designed for B workers by N-firms can be very close to, or even exceed the wage increase rate of worker A (figure b).

7. Application One: Racial wage discrimination

One empirical difficulty in the discrimination literature is how to distinguish the unobserved productivity differences and discrimination in the residual wage gaps. Bowlus and Eckstein (2002) build a structural model they are able to identify due to the different impacts productivity differences and discrimination have on the earnings distribution. In this exercise, using the same data as Bowlus and Eckstein (2002) and similar identification strategy, we estimate the structural parameters of the wage-tenure discrimination model, and compare how the inferences on the extent of productivity differences versus discrimination in explaining racial wage gaps differ between the two models. Table 2 is a summary of the data that is useful in the estimation; detailed data description refers to Bowlus and Eckstein (2002).

Table 2 Summary of NLSY Data for Male High School Graduates, 1985-1988

	Whites	Blacks	Pooled
Unemployment rate	0.077	0.157	0.089
Unemployment duration in weeks	22.15	29.05	23.65
Minimum weekly wage	118.18	120.39	118.18
Maximum weekly wage	605.97	428.16	605.97
Mean weekly wage	273.90	230.96	268.03

7.1 Identification

In the model section, we assume no differences between the two types of workers except the observable characteristic which in this exercise refers to race. Now, to match the empirical observations in data and distinguish productivity difference from discrimination that attribute to the wage gap, we allow for racial differences in productivity P and the death rate δ. The wage and unemployment data is used for identification. The following illustrates the identification in the most general case where both productivity difference and discrimination exist. Identification of the structural parameters in other cases is just straightforward.

First, using the unemployment duration, rate of unemployment rate and wage bound of whites, λ, δ_A and P_A are identified, as $\text{duration of unemployment}_{\text{A}} = \frac{1}{\lambda}$, $u_A = \frac{\delta_A}{\delta_A + \lambda}$ and the equilibrium condition $\frac{P_A - \overline{w_A}}{P_A - \underline{w_A}} = {u_A}^2$.

Next, σk and δ_B are identifiable using blacks' unemployment rate and duration of unemployment data as $\text{duration of unemployment}_{\text{A}} = \frac{1}{(1-\sigma k)\,\lambda}$ and $u_B = \frac{\delta_B}{\delta_B + (1-\sigma k)\,\lambda}$. Note that hiring discrimination is key to matching the racial difference in unemployment duration and varying death rate is crucial in determining the different unemployment rates.

Third, parameters associated with discrimination, d, σ, k and $\overline{w_B^D}$, and blacks productivity P_B are simultaneously identified from the system of equations: the estimated σk, two equilibrium conditions $\frac{P_B - \overline{w_B^D} - d}{P_B - \underline{w_B} - d} = \left(\frac{\delta_B + (1-\sigma)\lambda}{\delta_B + (1-\sigma k)\lambda}\right)^2$, $\frac{P_B - \overline{w_B}}{P_B - \overline{w_B^D}} = (\frac{\delta_B}{\delta_B + (1-\sigma)\lambda})^2$, the mean wage $Ew_B = \overline{w_B} + \frac{\delta_B}{(1-\sigma k)\lambda}(\overline{w_B} - \underline{w_B}) - \frac{2}{1-\sigma k}\left[\frac{\delta_B \sigma (1-k)}{\delta_B + (1-\sigma k)\lambda}\left(P_B - \underline{w_B} - d\right) + \frac{(1-\sigma)[\delta_B + (1-\sigma k)\lambda]}{\delta_B}(P_B - \overline{w_B})\right]$ and the median wage $w_B^{.5} = \begin{cases} P_B - d - \left(P_B - \underline{w_B} - d\right)\left(\frac{\delta_B}{\delta_B + (1-\sigma k)\lambda q}\right)^2 & if\ \frac{\delta_B (1-k)\sigma}{(1-\sigma k)[\delta_B + (1-\sigma)\lambda]} > 0.5 \\ P_B - (P_B - \overline{w_B})\left(\frac{\delta_B + (1-\sigma k)\lambda}{\delta_B + (1-\sigma k)\lambda q}\right)^2 & if\ \frac{\delta_B (1-k)\sigma}{(1-\sigma k)[\delta_B + (1-\sigma)\lambda]} \le 0.5 \end{cases}$. The estimation procedure is as follows: first, try the value of P_A as the estimate of P_B and get all other parameters through the system of equations but the median one; then, predict the median wage according to the median wage equation, if it approximates the empirical median wage, keep all the parameter estimates, if not, modify the estimate of P_B accordingly and redo all the above until the predicted median wage tallies with the empirical one. One can also compare the predicted and empirical value at any other quantile to determine the appropriate estimates of parameters.

7.2 Estimation

We estimate the parameters in six versions from a simple case where there are no differences between whites and blacks towards a complete model with both productivity difference and discrimination. In the simplest scenario where $P_A = P_B = P$; $\delta_A = \delta_B = \delta$ and no discrimination present $(d = k = 0, \sigma = 1)$, parameters λ and δ are solved from the pooled unemployment duration and rate of unemployment. Productivity level P is then identified from the equilibrium condition $\frac{P-\overline{w}}{P-\underline{w}} = \left(\frac{\delta}{\delta+\lambda}\right)^2$ where $\underline{w}$ and $\overline{w}$ are replaced with 118.18 and 605.97 respectively. The assumptions and estimation results are presented in column (1) of Table 3. In scenarios (2) and (3), we calculate δ_i and P_i using the separate unemployment rate and equilibrium condition by race instead. The productivity levels differ significantly between whites and blacks and P_B/P_A ratios are indeed smaller than mean wage ratio as predicted in Bowlus and Eckstein (2002). Varying δ is important in explaining the unemployment rate differential in data.

Scenario (4) begins to incorporate the assumption of discrimination by allowing for the disutility experienced by prejudiced firms from hiring blacks. P_B, d, σ and $\overline{w_B^D}$ are calculated simultaneously using two equilibrium conditions and the equations of mean and median wages for blacks. The disutility level is found to be 72.4% of the white productivity, and 81.2% of the firms are prejudiced against blacks. When the restriction on equal hiring rate is relaxed in scenario (5), 91.1% of firms in the labor market have a distaste of 70.6% of the white productivity and offer to hire blacks at a rate 26.1% lower than the offer rate to whites.[13] The productivity of blacks is only 3% lower than their counterpart. In this estimation, the model can not only match the racial differences in unemployment rates but unemployment durations as well. Finally, we get the parameter estimates in the scenario of pure discrimination in column (6).The last row of Table 3 presents the estimate of $\overline{w_B^D}$ when there is discrimination present in the labor market.

[13] These estimates are obtained to match the wage of blacks at 10% percentile. When the median is matched in the estimation, the result implies blacks are more productive than whites and 96.5% of firms are prejudiced with a distaste as high as 97.5% of the white productivity and an offer rate 24.6% lower to blacks ($P_B = 812.41; d = 593.59; k = 0.2462; \sigma = 0.9649; \overline{w_B^D} = 215.04$).

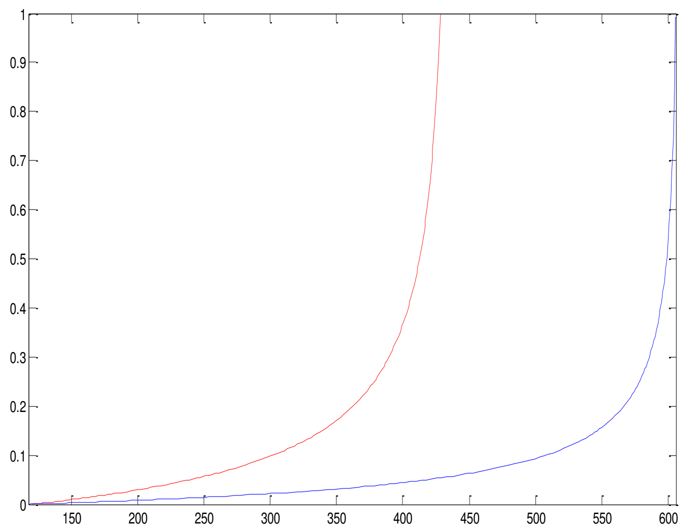
Table 3 Parameter Estimates

Parameter	(1)	(2)	(3)	(4)	(5)	(6)
	$P_A = P_B$; no discrimination; $\delta_A = \delta_B$	$P_A \neq P_B$; no discrimination; $\delta_A = \delta_B$	$P_A \neq P_B$; no discrimination; $\delta_A \neq \delta_B$	$P_A \neq P_B$; discrimination present ($k = 0$); $\delta_A \neq \delta_B$	$P_A \neq P_B$; discrimination present; $\delta_A \neq \delta_B$	$P_A = P_B$; discrimination present; $\delta_A \neq \delta_B$
$\boldsymbol{\lambda}$	0.0423	0.0423	0.0423	0.0423	0.0451	0.0451
$\boldsymbol{\delta_A}$	0.0041	0.0041	0.0035	0.0035	0.0038	0.0038
$\boldsymbol{\delta_B}$	0.0041	0.0041	0.0079	0.0079	0.0064	0.0064
$\boldsymbol{P_A}$	609.86	609.86	608.88	608.88	608.88	608.88
$\boldsymbol{P_B}$	609.86	430.62	435.94	537.21	591.02	608.88
$\boldsymbol{d}$	0	0	0	441	429.67	436.34
$\boldsymbol{k}$	0	0	0	0	0.2608	0.2580
$\boldsymbol{\sigma}$	1	1	1	0.8122	0.9106	0.9206
$\overline{\boldsymbol{w_B^D}}$	-	-	-	99	158.65	169.41

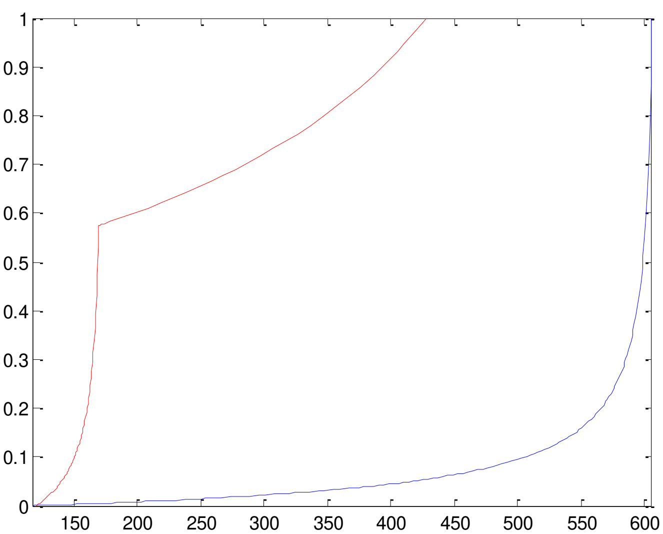
The earnings distributions predicted in the pure productivity difference (scenario (3)), pure discrimination (scenario (6)) and mixed cases (scenario (5)) are shown in Figure 3, which clearly demonstrates the distinguishing effects productivity difference versus discrimination have on the earnings distribution. Besides, the wage-tenure discrimination model also allows for the depiction of wage dynamics. Figure 4 shows how the wage increase varies between blacks and whites at each wage level. In the case of pure productivity difference, the two lines have a similar shape and the blue line lies above the red one, suggesting a higher wage increasing rate in whites compared to blacks at any wage level. In the case of pure discrimination, there is a striking gap between the blue line and the red line; moreover, the red line is discontinuous at the jump point. It indicates that discrimination is an influential factor in wage-tenure contracts, and the presence of discrimination leads to a sharp decrease in the wage increasing rate for blacks in both discriminating and non-discriminating firms. Intuitively, although non-discriminating firms do not discriminate against blacks, they have incentives to offer a less attractive contract to blacks than otherwise as there is now less competition among firms to hire blacks and black workers will be willing to stay and accept the less attractive contract because of no better options outside. When there are both productivity difference and discrimination in the labor market, the line indicating the wage dynamic of blacks is a combination of the two effects.

Figure 3 Predicted earnings distributions

(a) Pure productivity difference



(b) Pure discrimination



(c) Mixed: both productivity difference and discrimination

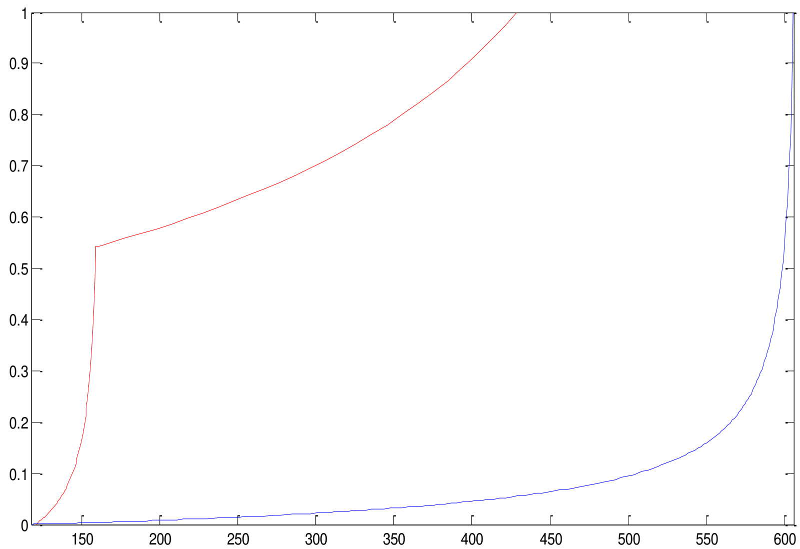
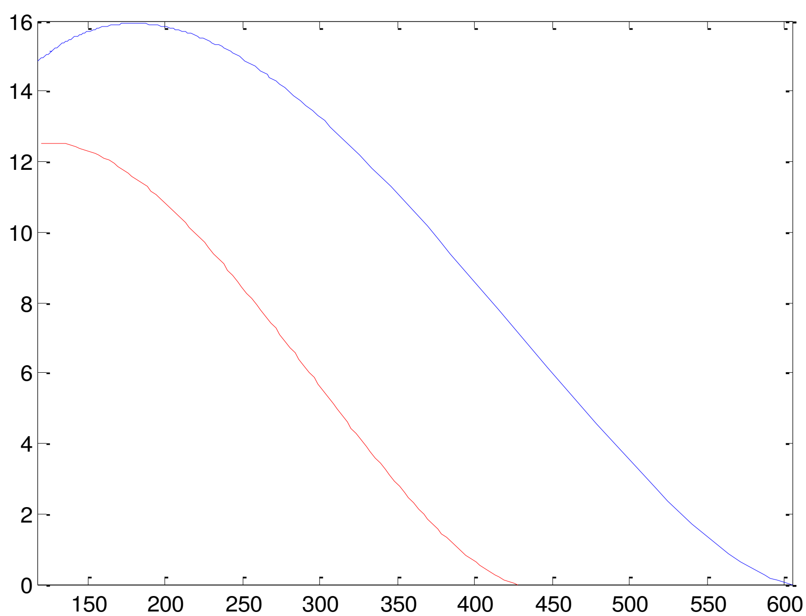
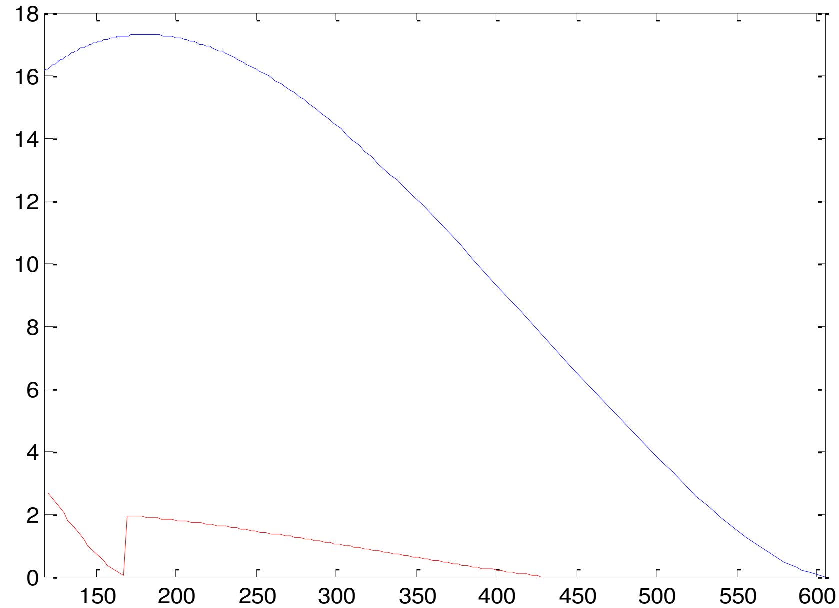


Figure 4 Predicted wage dynamics

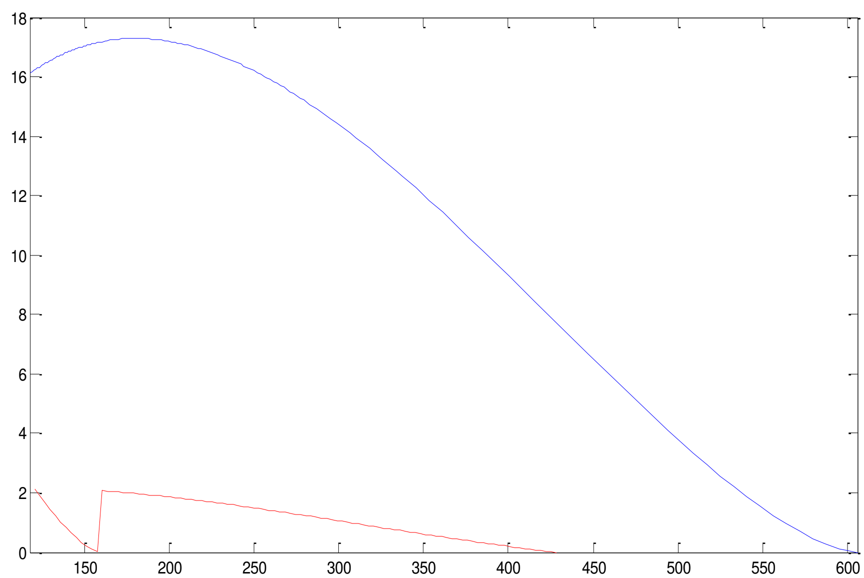
(a) Pure productivity difference



(b) Pure discrimination



(c) Mixed: both productivity difference and discrimination

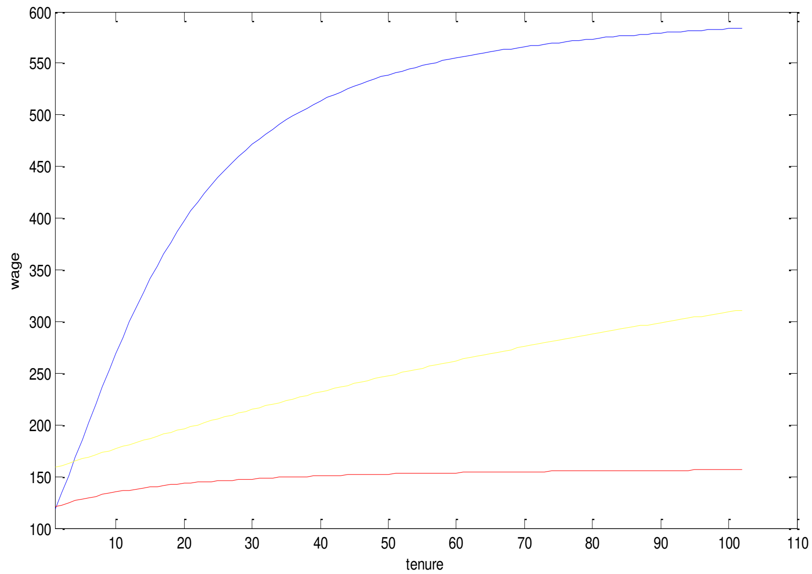


7.3 Comparison

The estimation results of our model imply that for the high school male graduates in 1985-1988, the productivity of black workers is 3% less than that of whites; and 91.1% of firms in the labor market have a distaste against blacks that is as high as 70.6% of whites productivity. Besides, these firms offer to hire blacks at a rate 26.1% lower than the offer rate they send to whites. Compared with the results in Bowlus and Eckstein (2002), we come to the same conclusion regarding to the productivity differential between race. However, in our model, the fraction of discriminating firms in the labor market is higher, the disutility discriminating firms feel in hiring black workers is stronger, while the recruiting discrimination is not as severe as that predicted in Bowlus and Eckstein (2002). We speculate, the reason may be that there are so many firms that have a strong prejudice towards blacks that it turns out to be costly to have such a distaste and profitable by relaxing the discrimination a little during recruitment.

One should notice a few differences between the wage-tenure discrimination model and Bowlus and Eckstein (2002) (BE henceforth). First and foremost, a constant wage is assumed in BE while our model assumes wage changes with tenure. Therefore, except for using mean wages in estimation, we also utilize wage ranges. Second, productivity is different in interpretation. BE interpret P as the average productivity level in a market with firm heterogeneity while P in our model is the marginal productivity a worker brings to the firm that not varying across firms. It implies that P must be greater than the maximum wage observed in the data. The estimated P in our model is thus much greater than those in BE. Third, offer arrival rates are different among employed and unemployed workers in BE but they are the same in our model and δ is the destruction rate in BE but birth/death rate in our model. These simplifications help us focus on different effects productivity differential and discrimination have on the wage gap and their respective wage dynamics. Figure 5 depicts how wage increases with tenure when there are both productivity differential and discrimination in the labor market. Blue line represents the wage dynamic for whites, red line for blacks hired in discriminating firms and yellow line for blacks hired in non-discriminating firms. Obviously, the slope of blue line is greater than the slope of yellow line, which is greater than that of red line. It implies that white workers will experience a steeper wage increase over tenure, followed by the black workers in non-discriminating firms. Black workers employed in discriminating firms have to search for opportunities in non-discriminating firms after around 50 weeks otherwise the wage will stagger and remain almost unchanged.

Figure 5 Wage dynamics



8. Application Two: Gender wage discrimination

The second exercise is to estimate gender wage discrimination in the labor market. Besides, we will see how the predicted pattern of wage dynamics matches the empirical one.

8.1 data

The sample used is extracted from the NLSY79 for the period 1985-1987. To be included in our sample, an individual must be a white, either employed or unemployed in week 390, graduated from high school and not enrolled in further education in the period 1985-1987. For the unemployed worker, we calculate the unemployment duration. There are two versions of unemployment duration. $DurU1$ is the period that dated back from the week the unemployed worker began unemployed (no earlier than week 314 in year

1984) till the week he/she was either employed or out of labor force (no later than week 522 in year 1987). $DurU2$ is the period that dated back from the week the unemployed worker began not employed (either unemployed or out of labor force, but no earlier than week 314 in year 1984) till the week he/she was employed (no later than week 522 in year 1987). Since $DurU2$ takes out-of-labor-force into the calculation of unemployment duration, it is greater than $DurU1$ which only counts the period unemployed.[14] For the employed worker, we keep workers who have been employed all the time in 1985 (week 367) -1986 (week 470), and calculate the increase in hourly wages. Table 4 summarizes the statistics that are useful in the estimation. Figure 6 plots the cdf and pdf of hourly wage for both genders.

Table 4 Summary of NLSY Data for High School White Graduates, 1985-1987

	Male	Female	Pooled
Unemployment rate	4.314%	5.257%	4.749%
$DurU1$ (weeks)	29.237	19.668	24.237
$DurU2$ (weeks)	34.661	36.771	35.664
Wage range (hourly pay)	[41, 1923]	[45, 1511]	[41,1923]
Average wage (hourly pay)	823.3616	634.6458	743.4436
Median wage (hourly pay)	769	591	682

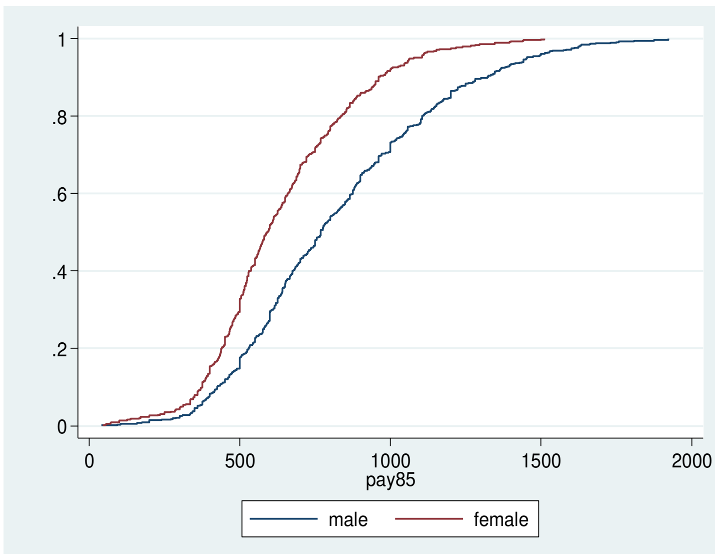
Next, we explore the difference in patterns of wage increase between men and women. Figure 7 draws two scatter plots to show a rough relationship between wage increase and 1985 hourly wage, one is not weighted and the other weighted. It is found that points cluster in the lower middle part where hourly wage is between 250 and 1200 and wage increase is below 500; and, substantially more "male" points lie at higher wages. The predicted lines describe the trends of wage increase over levels. To compare the wage profiles of men and women more clearly, Figure 8 plots the magnitude of average wage increase over twenty or forty wage categories for male and female workers. Numbers on X-axis represent the middle point of each wage category. It is observed that women's wage increase is almost consistently smaller than men's wage increase. In addition, for both men and women, the magnitude of wage increase

[14] Bowlus (1997) built a three stage model (employment, unemployment and nonparticipation) to study the role of gender differences in behavior patterns on wage differentials. Our model, however, only allows for two stages, i.e., employment and unemployment. Therefore, when applying it to gender labor market, "unemployment" refers to the status of unemployment or nonparticipation. $Dur1$(real unemployment duration) and $Dur2$(nonemployment duration) generate two groups of estimates that help us to compare the implications.

is much higher at low wages than at high wages. At very high wages (above 1600), only male workers are observed. In the next subsection, we will see whether our model is able to explain the regularities found in data, i.e., gender wage gap, differences in unemployment rate and unemployment duration and the patterns of wage increase. Since identification strategy is the same as specified in last exercise, we present the estimation results directly.

Figure 6 Hourly wage distributions

(a) Cumulative distribution function of hourly pay



(b) Kernel density of hourly pay

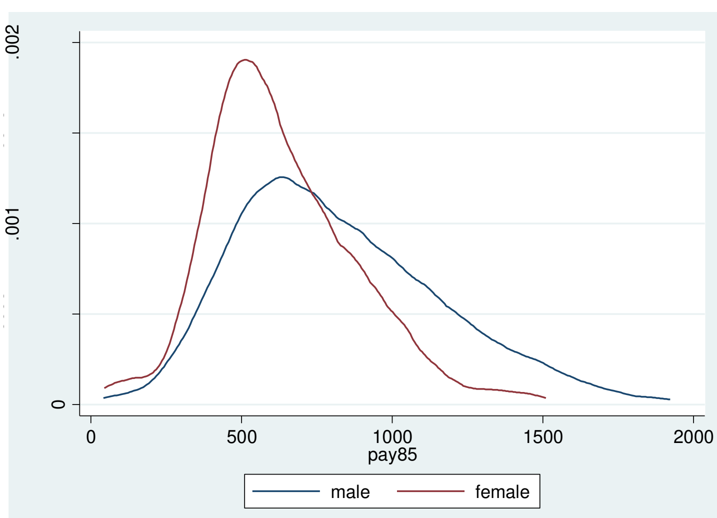
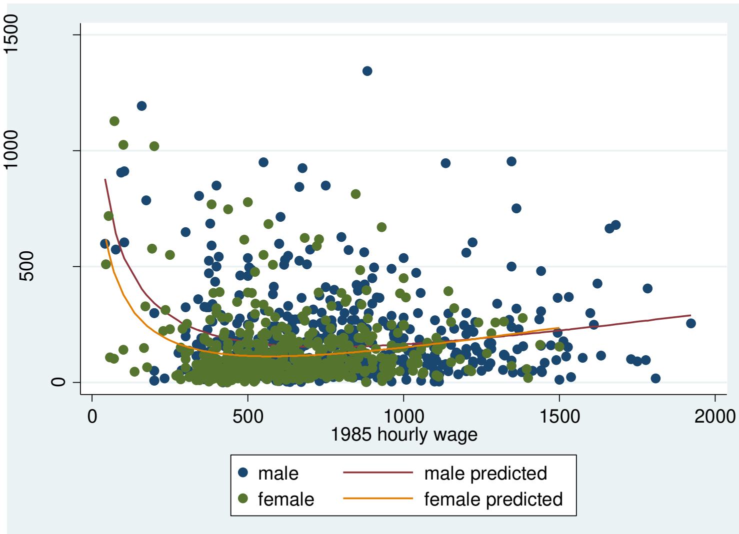


Figure 7 Scatter plots of wage increase and hourly pay

(a) Unweighted scatter plot



(b) Weighted scatter plot

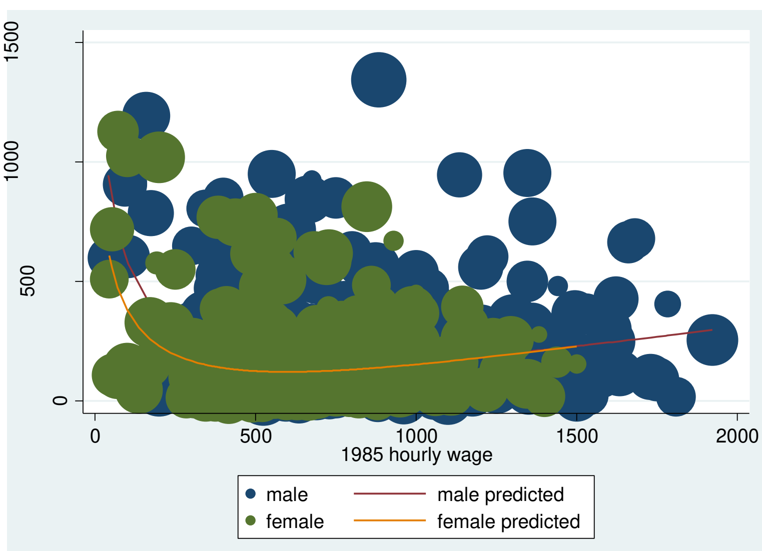
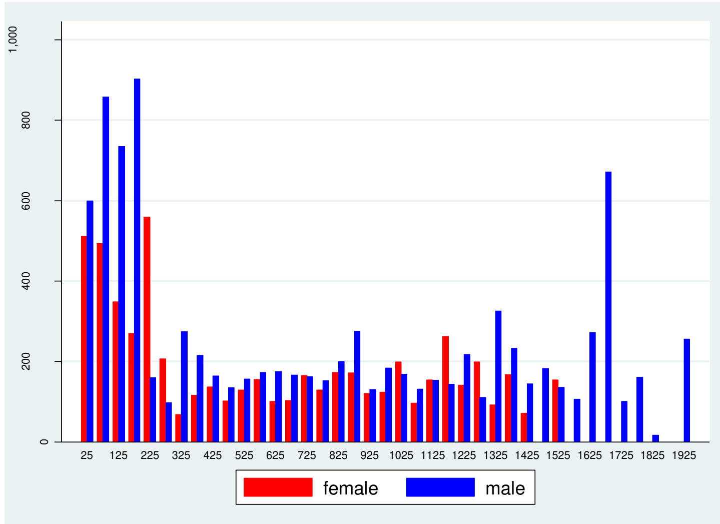
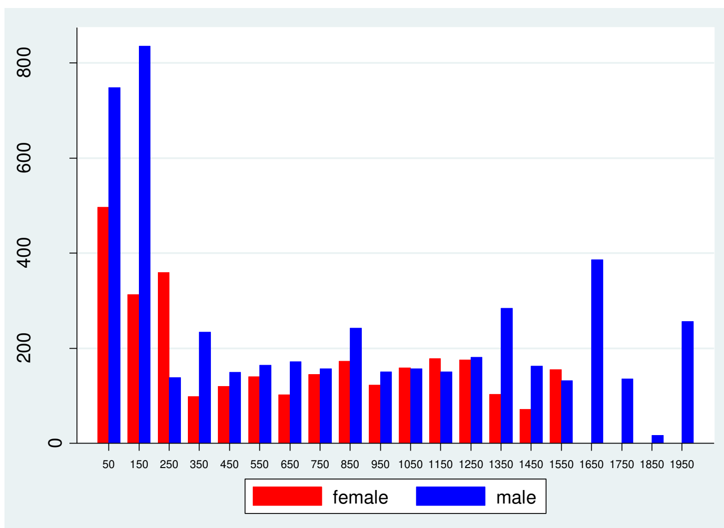


Figure 8 Hourly Wage Increase 1986-1985

(a) Over 40 categories



(b) Over 20 categories



8.2 Estimation

Table 5 reports the estimation results in three scenarios under two measures of unemployment duration. Assume no discrimination present in the labor market, the productivity of women is estimated 21% lower than men's.[15] If discrimination is taken into account, the productivity gap shrinks to 3% of men's productivity, smaller than 6.5% reported in Flabbi (2010) who uses the sample of white, college graduates

[15] Using a search model that does not allow for discrimination, Bowlus (1997) finds the average productivity differential between male and female college graduates is 17.1% and 25.3% for high school graduates.

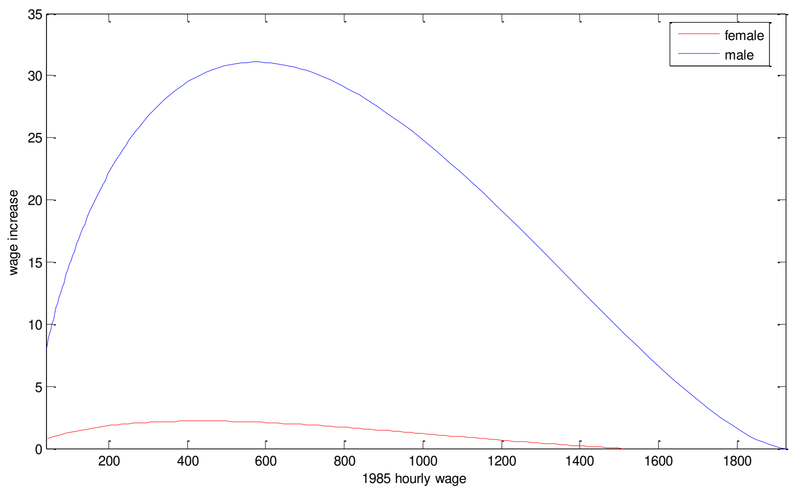
from CPS 1995. Looking at the estimates of discrimination parameters, one can find k is negative under $DurU1$, suggesting offer goes to women more frequently. This is completely opposite to our expectation and problematic. As a matter of fact, compared to men, women are more often ending the status of unemployment by not participating in the labor market rather than finding a job. Therefore, it results in a seemingly lower unemployment duration among female unemployed than male unemployed, when it is measured by $DurU1$. $DurU2$, on the other hand, avoids this problem and is more appropriate in the situation of male-female discrimination. Indicated in column (5), when there is no productivity difference between men and women and all gender wage gap is attributable to discrimination, about 94.3% of the firms are prejudiced against women, with a distaste as high as 94% of the productivity, and search for female workers 6% less intensively than for male. If there are both productivity differential and discrimination, it is estimated that fewer firms (93.2%) have a slightly stronger distaste (96%) and stronger recruiting discrimination (6.1%) against women. Our estimate of fraction of discriminating firms is higher than 52% in Flabbi (2010) and 56% in Bowlus and Eckstein (2002).

Table 5 Parameter Estimates

Parameter	$DurU1$			$DurU2$		
	(1)	(2)	(3)	(4)	(5)	(6)
	$P_A \neq P_B$; no discrimination; $\delta_A \neq \delta_B$	$P_A = P_B$; discrimination present; $\delta_A \neq \delta_B$	$P_A \neq P_B$; discrimination present; $\delta_A \neq \delta_B$	$P_A \neq P_B$; no discrimination; $\delta_A \neq \delta_B$	$P_A = P_B$; discrimination present; $\delta_A \neq \delta_B$	$P_A \neq P_B$; discrimination present; $\delta_A \neq \delta_B$
$\boldsymbol{\lambda}$	0.0413	0.0342	0.0342	0.0280	0.0289	0.0289
$\boldsymbol{\delta_A}$	0.0019	0.0015	0.0015	0.0013	0.0013	0.0013
$\boldsymbol{\delta_B}$	0.0023	0.0028	0.0028	0.0016	0.0015	0.0015
$\boldsymbol{P_A}$	1926.5	1926.50	1926.5	1926.5	1926.5	1926.5
$\boldsymbol{P_B}$	1515.1	1926.50	1863.4	1515.1	1926.5	1863.4
$\boldsymbol{d}$	0	1801.6	1851.2	0	1801.6	1851.2
$\boldsymbol{k}$	0	-0.5342	-0.5445	0	0.0608	0.0615
$\boldsymbol{\sigma}$	1	0.9107	0.8935	1	0.9434	0.9324
$\overline{\boldsymbol{w_B^D}}$	-	123.91	12.638	-	123.91	12.638

How well does our model match the empirical wage dynamics? Using the estimates in column (6) of Table 5, we plot the theoretical pattern of wage increases for both men and women in Figure 9, which also shows a greater wage increase among male workers than female workers and a steeper increase in the lower wages as in Figure 7 and Figure 8, although the magnitude of wage increase differs.

Figure 9 Predicted wage increase



9. **Conclusions**

This paper develops a discrimination search model with wage-tenure contracts and predicts: 1) minority workers have a higher unemployment rate and a longer duration of unemployment; 2) non-discriminating firms make higher profits than discriminating firms; 3) the lowest acceptable wage for a minority worker is greater than that for a majority worker while the highest expectable wage of a minority worker is lower; 4) generally, minority workers earn less than majority workers on average, and their wage increases more slowly than their counterpart. Moreover, we also show how the fraction of discriminating firms, distaste and recruiting discrimination affect the wage ranges and mean wages for both types of workers.

Applying the model to data in 1985-1988 from NLSY79, we investigate race/gender discrimination in the labor market. Productivity differences between blacks and whites are estimated to be 3% of whites productivity; productivity differences between men and women are estimated to be 3% of male productivity. 91% of firms possess prejudice towards black workers and 93% towards female workers. The distaste they hold toward blacks is about 70% of whites productivity and that towards women is 95% of male productivity. Compared to estimates in Bowlus and Eckstein (2002) and Flabbi

(2010), we got similar results on productivity differences, but much higher estimation on discrimination. In addition, the predicted patterns of wage increase and that from data seem to exhibit some common characteristics. First, wage increases faster for men than women; second, wage increases faster at low wages than high wages.

There are some limitations in the discrimination search model with wage-tenure contracts. First, it does not consider the status of nonparticipation and other characteristics of jobs but wages in the labor market. This is crucial in comparing gender differences in labor market outcomes. Bowlus (1997) shows women have a greater tendency to exit jobs to nonparticipation due to family, pregnancy or health issues. Flabbi and Moro (2010) measure women's preference for work flexibility and find an impact on wage distributions. The second limitation of the model exists in the empirical application. We follow the identification strategy in Bowlus and Eckstein (2002) but it would be better if we can generate an econometric approach from the model and do some robustness check. Finally, we suggest some future researches on this line. One can study taste discrimination in the directed search model with wage-tenure contract (Shi, 2009) and see what different predictions can be obtained. Or, it may be modified to some extent to explain glass ceiling/sticky floor effects found in empirical work.

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Appendix

A1. Proof of proposition 1

Since ${V_{0N}}^B$ and ${V_{0D}}^B$ are offers chosen by N-and D-firms to maximize their respective profit flow at the steady state, it implies

$$\lambda G_B({V_{0N}}^B)\theta n \int_0^\infty \psi_B\left(t\middle|\widetilde{w_B^N}\right)[P - w_B^N(t)]\,\mathrm{dt} \geq \lambda G_B({V_{0D}}^B)\theta n \int_0^\infty \psi_B\left(t\middle|\widetilde{w_B^D}\right)[P - w_B^D(t)]\,\mathrm{dt}$$

and

$$(1-k)\lambda G_B({V_{0D}}^B)\theta n \int_0^\infty \psi_B\left(t\middle|\widetilde{w_B^D}\right)[P - w_B^D(t) - d]\,\mathrm{dt}$$
$$\geq (1-k)\lambda G_B({V_{0N}}^B)\theta n \int_0^\infty \psi_B\left(t\middle|\widetilde{w_B^N}\right)[P - w_B^N(t) - d]\,\mathrm{dt}$$

Note that $\widetilde{w_B^j}$ $(j = N, D)$ is the wage-tenure contract designed to deliver the offer, so it's a function of ${V_{0j}}^B$. The two inequalities then imply:

$$G_B({V_{0N}}^B)\theta n \int_0^\infty \psi_B\left(t\middle|\widetilde{w_B^N}\right)[P - w_B^N(t)]\,\mathrm{dt} - G_B({V_{0N}}^B)\theta n \int_0^\infty \psi_B\left(t\middle|\widetilde{w_B^N}\right)[P - w_B^N(t) - d]\,\mathrm{dt}$$
$$\geq G_B({V_{0D}}^B)\theta n \int_0^\infty \psi_B\left(t\middle|\widetilde{w_B^D}\right)[P - w_B^D(t)]\,\mathrm{dt} - G_B({V_{0D}}^B)\theta n \int_0^\infty \psi_B\left(t\middle|\widetilde{w_B^D}\right)[P - w_B^D(t) - d]\,\mathrm{dt}$$

If we define:

$$\Psi({V_0}^B) \triangleq dG_B({V_0}^B)\int_0^\infty \psi_B(t|\widetilde{w_B})\,\mathrm{dt}$$

Then the above inequality is:

$$\Psi({V_{0N}}^B) \geq \Psi({V_{0D}}^B)$$

Because,

$$\Psi'({V_0}^B) = d\frac{\partial G_B({V_0}^B)}{\partial {V_0}^B}\int_0^\infty \psi_B(t|\widetilde{w_B})\,dt + dG_B({V_0}^B)\int_0^\infty \frac{\partial(\psi_B(t|\widetilde{w_B}))}{\partial {V_0}^B}\,dt > 0$$

due to the increasing property of $G_B({V_0}^B)$ and $\psi_B(t|\widetilde{w_B})$ with respect to ${V_0}^B$, we have ${V_{0N}}^B \geq {V_{0D}}^B$.

A2. Proof of proposition 2

For the derivation of equilibrium results for worker A, refer to Burdett and Coles (2003). Below is a similar derivation of equilibrium results for worker B.

(1) First consider the optimal wage-tenure contract designed for B-workers by discriminating firms. Given the starting offer V_0, the wage-tenure function solves:

$$\max_{w(t)>0} \int_0^\infty \psi_B\left(t\middle|\widetilde{w_B^D}\right)[P - w_B^D(t) - d]\,\mathrm{dt}$$

where
$$\dot{\psi_B} = -[\delta + (1-\sigma)\lambda + \sigma(1-k)\lambda(1 - F_B^D(V_B^D))]\psi_B \tag{A1}$$

$$\dot{V_B^D} = \delta V_B^D - u\left(w_B^D(t)\right) - (1-\sigma)\lambda[EV_B^N - V_B^D] - \sigma(1-k)\lambda\int_{V_B^D}^{\overline{V_B^D}}[x - V_B^D]\,dF_B^D(x) \tag{A2}$$

with starting values $\psi_B(0) = 1;\ V_B^D(0) = V_0$

To solve the dynamic optimization problem, define the Hamiltonian:

$$H = \psi_B[P - w_B^D(t) - d] - \Lambda_\psi \left[\delta + (1-\sigma)\lambda + \sigma(1-k)\lambda\left(1 - F_B^D(V_B^D)\right)\right]\psi_B$$
$$+\Lambda_V \left[\delta V_B^D - u\left(w_B^D(t)\right) - (1-\sigma)\lambda[EV_B^N - V_B^D] - \sigma(1-k)\lambda \int_{V_B^D}^{\overline{V_B^D}} [x - V_B^D] dF_B^D(x)\right]$$

Where Λ_ψ, Λ_V are costate variables with respect to ψ_B and V_B^D.

The necessary conditions are:

$$H_w = -\psi_B - \Lambda_V\, u'\left(w_B^D(t)\right) = 0 \tag{A3}$$

$$\dot{\Lambda_\psi} = -H_\psi = -[P - w_B^D(t) - d] + \Lambda_\psi[\delta + (1-\sigma)\lambda + \sigma(1-k)\lambda(1 - F_B^D(V_B^D))] \tag{A4}$$

$$\dot{\Lambda_V} = -H_V = -\Lambda_V \left[\delta + (1-\sigma)\lambda + \sigma(1-k)\lambda[1 - F(V_B^D)]\right] - \Lambda_\psi \sigma(1-k)\lambda F_B^{D\prime}(V_B^D)\psi_B \tag{A5}$$

And the two differential equations ψ_B and V_B^D should satisfy (A1), (A2).

Integrate (A4) with the integrating factor ψ_B yields:

$$\Lambda_\psi \psi_B = \int_t^\infty \psi_B\left(s\middle|\widetilde{w_B^D}\right)[P - w_B^D(s) - d]\,\mathrm{ds} + \mathrm{C}_1$$

Define the expected future profit flow from tenure period t onwards as:

$$\Pi_B^D\left(t\middle|\widetilde{w_B^D}\right) = \int_t^\infty \frac{\psi_B\left(s\middle|\widetilde{w_B^D}\right)}{\psi_B\left(t\middle|\widetilde{w_B^D}\right)}[P - w_B^D(s) - d]\,\mathrm{ds}$$

Then,

$$\Lambda_\psi = \Pi_B^D\left(t\middle|\widetilde{w_B^D}\right) + \frac{\mathrm{C}_1}{\psi_B\left(t\middle|\widetilde{w_B^D}\right)}$$

Since it's an autonomous control problem, the optimized Hamiltonian is zero, i.e., $H = 0$. Substituting Λ_ψ, Λ_V in H out yields:

$$0 = [P - w_B^D(t) - d] - \left\{\Pi_B^D\left(t\middle|\widetilde{w_B^D}\right) + \frac{\mathrm{C}_1}{\psi_B\left(t\middle|\widetilde{w_B^D}\right)}\right\}\left[\delta + (1-\sigma)\lambda + \sigma(1-k)\lambda\left(1 - F_B^D(V_B^D)\right)\right]$$
$$-\frac{1}{u'\left(w_B^D(t)\right)}\left[\delta V_B^D - u\left(w_B^D(t)\right) - (1-\sigma)\lambda[EV_B^N - V_B^D] - \sigma(1-k)\lambda\int_{V_B^D}^{\overline{V_B^D}}[x - V_B^D]dF_B^D(x)\right]$$

Therefore, C_1 has to be zero to make Π_B^D bounded. Thus $\Lambda_\psi = \Pi_B^D\left(t\middle|\widetilde{w_B^D}\right)$ and (A4) turns to be

$$\frac{d\Pi_B^D\left(t\middle|\widetilde{w_B^D}\right)}{dt} = -[P - w_B^D(t) - d] + \Pi_B^D\left(t\middle|\widetilde{w_B^D}\right)[\delta + (1-\sigma)\lambda + \sigma(1-k)\lambda(1 - F_B^D(V_B^D))] \tag{A6}$$

And (A2), (A6) and $H = 0$ give:

$$\frac{dV_B^D\left(t\middle|\widetilde{w_B^D}\right)}{dt} = -u'\left(w_B^D(t)\right)\frac{d\Pi_B^D\left(t\middle|\widetilde{w_B^D}\right)}{dt} \tag{A7}$$

Integrating (A5) with the integrating factor $\frac{1}{\psi_B}$ and substituting Λ_ψ with Π_B^D yields:

$$\frac{\Lambda_V}{\psi_B} = -\int_0^t \Pi_B^D \sigma(1-k)\lambda F_B^{D\prime}(V_B^D)ds + C_2$$

To Substitute Λ_V in (A3) using the above expression and differentiate with respect to t, we get:

$$-\frac{u''(w_B^D)}{u'(w_B^D)^2}\frac{dw_B^D(t)}{dt} = \sigma(1-k)\lambda F_B^{D\prime}(V_B^D)\Pi_B^D \tag{A8}$$

In addition, the transversality condition implies $\lim_{t\to\infty} V_B^D\left(t\middle|\widetilde{w_B^D}\right) = \overline{V_B^D}$.

(2) Next, we present the equilibrium results in terms of baseline wage.

If the solution to the above optimization problem with $V_0 = \underline{V_B^D}$ is taken as the baseline, then for any starting offer $V_0 \in [\underline{V_B^D}, \overline{V_B^D})$, there exists t_0 such that $V_s^{BD}(t_0) = V_0$. So, the optimal wage contract of any firm and all the equilibrium solutions could be expressed in terms of the baseline. For example, $w_B^D(t|V_0) = w_s^{BD}(t_0+t)$, $V_B^D\left(t\middle|\widetilde{w_B^D}\right) = V_s^{BD}(t_0+t)$ and $\Pi_B^D\left(t\middle|\widetilde{w_B^D}\right) = \Pi_s^{BD}(t_0+t)$. Then, it's easy to derive $w_s^{BD} \uparrow \overline{w_B^D}$ and $\Pi_s^{BD} \uparrow \overline{\Pi_B^D}$. Further, from (A2) we can obtain $\overline{V_B^D} = \frac{u\left(\overline{w_B^D}\right)+(1-\sigma)\lambda EV_B^N}{\delta+(1-\sigma)\lambda}$; and from (A6), we get $\overline{\Pi_B^D} = \frac{P-\overline{w_B^D}-d}{\delta+(1-\sigma)\lambda}$.

Let u_B denote the unemployment rate, d_B denote the share of B workers employed in D-firms and n_B the share employed in N-firms. The flow conditions imply

$$\delta = u_B(\delta + (1-\sigma)\lambda + \sigma(1-k)\lambda);$$
$$u_B\sigma(1-k)\lambda = d_B(\delta + (1-\sigma)\lambda);$$
$$(1-n_B)(1-\sigma)\lambda = n_B\delta$$

So, the unemployment rate is $u_B = \frac{\delta}{\delta+(1-\sigma k)\lambda}$.

And the employment rate of type B workers in D-firms and N-firms are:

$$d_B = \frac{\delta\lambda\sigma(1-k)}{[\delta+(1-\sigma k)\lambda][\delta+(1-\sigma)\lambda]}; \qquad n_B = \frac{(1-\sigma)\lambda}{\delta+(1-\sigma)\lambda}$$

Let $G_B^D(V_0), V_0 \in [\underline{V_B^D}, \overline{V_B^D})$ be the proportion of B workers who have an expected lifetime value no greater than V_0 in all the B workers excluding those working in N-firms. Then $G_s^{BD}(t)$ is the corresponding baseline expression which satisfies:

$$G_s^{BD}(0) = \frac{u_B}{u_B+d_B} = \frac{\delta+(1-\sigma)\lambda}{\delta+(1-\sigma k)\lambda} \tag{A9}$$

and the flow condition for B workers employed in D firms with salary point greater than t:

$$[\delta + (1-\sigma)\lambda]\left(1-G_s^{BD}(t)\right) = \frac{dG_s^{BD}(t)}{dt} + G_s^{BD}(t)\sigma(1-k)\lambda\left(1-F_s^{BD}(t)\right) \tag{A10}$$

As every D-firm makes the same profit from B-workers at the equilibrium, and $G_s^{BD} \to 1$, $\Pi_{\mathrm{s}}^{\mathrm{BD}} \to \overline{\Pi_B^D}$, from the profit function:

$$\Omega_B^D = \lambda(1-k)G_s^{BD}(t)\theta n(1-n_B)\Pi_{\mathrm{s}}^{\mathrm{BD}}(\mathrm{t})$$

we can get $G_s^{BD}(t)\Pi_{\mathrm{s}}^{\mathrm{BD}}(\mathrm{t}) = \frac{P-\overline{w_B^D}-d}{\delta+(1-\sigma)\lambda}$. So,

$$\frac{dG_s^{BD}}{dt}\Pi_{\mathrm{s}}^{\mathrm{BD}} + \frac{d\Pi_{\mathrm{s}}^{\mathrm{BD}}}{dt}G_s^{BD} = 0$$

Then substituting out $\frac{dG_s^{BD}}{dt}$and $\frac{d\Pi_{\mathrm{s}}^{\mathrm{BD}}}{dt}$ using (A6) and (A10) and combining it with (A10) yields:

$$G_w^{BD} = \sqrt{\frac{P-\overline{w_B^D}-d}{P-w_B^D-d}}$$

$$\Pi_w^{BD} = \frac{1}{\delta+(1-\sigma)\lambda}\sqrt{\left(P-\overline{w_B^D}-d\right)(P-w_B^D-d)}$$

Putting the expression of G_w^{BD} into (A9) thus gets,

$$\frac{P-\overline{w_B^D}-d}{P-\underline{w_B^D}-d} = \left(\frac{\delta+(1-\sigma)\lambda}{\delta+(1-\sigma k)\lambda}\right)^2.$$

The offer distribution could be derived from (A6), (A7), (A8) and the expression of Π_w^{BD}:

$$1-F_w^{BD} = \frac{\delta+(1-\sigma)\lambda}{\sigma(1-k)\lambda}\left[\sqrt{\frac{P-w_B^D-d}{P-\overline{w_B^D}-d}}-1-\frac{1}{2u'(w_B^D)}\int_{w_B^D}^{\overline{w_B^D}}\frac{u'(x)dx}{\sqrt{\left(P-\overline{w_B^D}-d\right)(P-x-d)}}\right] \tag{A11}$$

Further, $V_s^{BD}(0) = V_{BU}$ at the equilibrium.

Since,

$$\frac{dV_s^{BD}(0)}{dt} = u(b) - u(\underline{w_B^D})$$

which is derived from the baseline expression of (A2) at $V_B^D = V_s^{BD}(0)$ and the Bellman equation for unemployed B workers; and,

$$\frac{dV_s^{BD}(0)}{dt} = \frac{\sqrt{P-\underline{w_B^D}-d}}{2}\int_{\underline{w_B^D}}^{\overline{w_B^D}}\frac{u'(x)dx}{\sqrt{P-x-d}}$$

which could be derived from substitutions using (A6), (A7), (A11) and the expression of Π_w^{BD}; we can derive another relationship between the bounds of the support of discriminating wages, i.e., $u\left(\underline{w_B^D}\right) = u(b) - \frac{\sqrt{P-\underline{w_B^D}-d}}{2}$.

Besides, the dynamics of baseline tenure-wages (equation (16)) could be easily derived from (A8), (A11) and Π_w^{BD} expression.

(3) By the same token, we can get the equilibrium outcomes for B workers in the non-discriminating firms. Following the same procedures, we can prove that (17) holds. However, the support of the non-discriminating wages is somewhat different in the derivation.

Let $G_B^N(V_0), V_0 \in \left[\underline{V_B^N},\ \overline{V_B^N}\right)$ be the proportion of B workers (including the unemployed) who have an expected lifetime value no greater than V_0. Then, for the baseline expression, we have :

$$G_w^{BN} = \sqrt{\frac{P-\overline{w_B^N}}{P-w_B^N}}\,.$$

So, the overall proportion of type B workers (including the unemployed) who earn less than or equal to w at the steady state is:

$$G_w^B(w) = \begin{cases} \frac{\delta}{\delta+(1-\sigma)\lambda} G_w^{BD}, & if\ w \in [\underline{w_B^D}, \overline{w_B^D}] \\ G_w^{BN}, & if\ w \in [\underline{w_B^N}, \overline{w_B^N}] \end{cases}$$

Since $G_w^B\left(\overline{w_B^D}\right) = G_w^B\left(\underline{w_B^N}\right)$ and $G_w^B(w)$is monotonically increasing, $\overline{w_B^D}$=$\underline{w_B^N}$. Further, as $G_s^{BN}(0) = \frac{\delta}{\delta+(1-\sigma)\lambda}$, we can get:

$$\frac{P-\overline{w_B^N}}{P-\underline{w_B^N}} = \left(\frac{\delta}{\delta+(1-\sigma)\lambda}\right)^2$$

Thus, (14) (15) are proved.

(4) Finally, we derive the earnings distribution of type B workers.

Given G_w^{BD} and G_w^{BN}, the earning distributions of B workers in the D-and N- firms at the steady state are:

$$K_w^{BD} = \frac{u_B+d_B}{d_B}\left[G_w^{BD}(w) - \frac{u_B}{u_B+d_B}\right]$$

And:

$$K_w^{BN} = \frac{1}{n_B}\left[G_w^{BN}(w) - (u_B + d_B)\right]$$

So, the overall earning distribution is:

$$K_w^B(w) = \begin{cases} \frac{d_B}{d_B+n_B} K_w^{BD}, & if\ w \in [\underline{w_B^D}, \overline{w_B^D}] \\ \frac{d_B}{d_B+n_B} + \frac{n_B}{d_B+n_B} K_w^{BN} & if\ w \in [\underline{w_B^N}, \overline{w_B^N}] \end{cases}$$

Substituting the expressions of G_w^{BD}, G_w^{BN}, u_B, d_B and n_B inside, gives equation (19).

A3. Proof of Equations (22), (23)

As shown above:

$$\Omega_B^D = \lambda(1-k)G_s^{BD}(t)\theta n(1-n_B)\Pi_s^{\mathrm{BD}}(\mathrm{t}) = \frac{\lambda(1-k)\theta n\delta}{[\delta+(1-\sigma)\lambda]^2}\left(P-\overline{w_B^D}-d\right)$$

Similarly,

$$\Omega_B^N = \lambda G_s^{BN}(t)\theta n\Pi_s^{\mathrm{BN}}(\mathrm{t}) = \lambda\theta n\frac{P-\overline{w_B^N}}{\delta}$$

Profits from A are:

$$\Omega_{\mathrm{A}} = \lambda G_s^A(t)(1-\theta)n\Pi_s^{\mathrm{A}}(\mathrm{t}) = \lambda(1-\theta)n\frac{P-\overline{w_A}}{\delta}$$

So, $\Omega^{\mathrm{D}} = \Omega_{\mathrm{A}} + \Omega_B^D$ and $\Omega^{\mathrm{N}} = \Omega_{\mathrm{A}} + \Omega_B^N$.

A4. Proof of proposition 3

First, let's consider properties (1)-(3).

Taking partial derivatives of equation (13) with respect to (σ, k, d) yields:

$$A\frac{\partial \underline{w_B^D}}{\partial\sigma} + B\frac{\partial\overline{w_B^D}}{\partial\sigma} = 0; \qquad A\frac{\partial \underline{w_B^D}}{\partial k} + B\frac{\partial\overline{w_B^D}}{\partial k} = 0; \qquad A\frac{\partial \underline{w_B^D}}{\partial d} + B\frac{\partial\overline{w_B^D}}{\partial d} = \frac{1}{4}C;$$

where

$$A = \frac{u'\left(\underline{w_B^D}\right)}{2} - \frac{1}{4\sqrt{P-\underline{w_B^D}-d}}\int_{\underline{w_B^D}}^{\overline{w_B^D}}\frac{u'(x)dx}{\sqrt{P-x-d}} > 0\ ;$$

$$B = \frac{u'\left(\overline{w_B^D}\right)}{2}\sqrt{\frac{P-\underline{w_B^D}-d}{P-\overline{w_B^D}-d}} > 0\ ;$$

$$C = \frac{1}{\sqrt{P-\underline{w_B^D}-d}}\int_{\underline{w_B^D}}^{\overline{w_B^D}}\frac{u'(x)dx}{\sqrt{P-x-d}} - \sqrt{P-\underline{w_B^D}-d}\int_{\underline{w_B^D}}^{\overline{w_B^D}}\frac{u'(x)dx}{(P-x-d)^{3/2}} < 0.$$

Similarly, partial differentiation of equation (12) gives:

$$\frac{\partial \underline{w_B^D}}{\partial\sigma} = {\eta_1}^2\frac{\partial\overline{w_B^D}}{\partial\sigma} - \eta_2(P-\underline{w_B^D}-d)\ ;\ \frac{\partial \underline{w_B^D}}{\partial k} = {\eta_1}^2\frac{\partial\overline{w_B^D}}{\partial k} + \eta_3(P-\overline{w_B^D}-d);\ \frac{\partial \underline{w_B^D}}{\partial d} = {\eta_1}^2\frac{\partial\overline{w_B^D}}{\partial d} + {\eta_1}^2 - 1;$$

where:

$$\eta_1 = \frac{\delta+(1-\sigma k)\lambda}{\delta+(1-\sigma)\lambda}; \qquad \eta_2 = \frac{2\lambda(\delta+\lambda)(1-k)}{[\delta+(1-\sigma)\lambda][\delta+(1-\sigma k)\lambda]}; \qquad \eta_3 = \frac{2\sigma\lambda}{\delta+(1-\sigma k)\lambda}.$$

Substituting them into the first group of equations thus proves:

$$\frac{\partial \underline{w_B^D}}{\partial\sigma} < 0;\quad \frac{\partial\overline{w_B^D}}{\partial\sigma} > 0;\ \frac{\partial \underline{w_B^D}}{\partial k} > 0;\quad \frac{\partial\overline{w_B^D}}{\partial k} < 0;\ \frac{\partial\overline{w_B^D}}{\partial d} < 0.$$

In addition, as $(A{\eta_1}^2+B)\frac{\partial \underline{w_B^D}}{\partial d} = \frac{\eta_1(\eta_1-1)}{2}\left[u'\left(\overline{w_B^D}\right)(\eta_1+2) - u'\left(\underline{w_B^D}\right)\eta_1\right]$, when $\frac{u'\left(\overline{w_B^D}\right)}{u'\left(\underline{w_B^D}\right)} > \frac{\eta_1}{\eta_1+2}$,

we have $\frac{\partial \underline{w_B^D}}{\partial d} > 0$.

Since $\underline{w_B^N} = \overline{w_B^D}$, the partial derivative with respect to (σ, k, d) in (15) yields:

$$\frac{\partial \overline{w_B^N}}{\partial k} = (\frac{\delta}{\delta + (1-\sigma)\lambda})^2 \frac{\partial \overline{w_B^D}}{\partial k};$$

$$\frac{\partial \overline{w_B^N}}{\partial d} = \left(\frac{\delta}{\delta + (1-\sigma)\lambda}\right)^2 \frac{\partial \overline{w_B^D}}{\partial d};$$

$$\frac{\partial \overline{w_B^N}}{\partial \sigma} = \left(\frac{\delta}{\delta + (1-\sigma)\lambda}\right)^2 \frac{\partial \overline{w_B^D}}{\partial \sigma} - \frac{2\delta^2\lambda}{[\delta + (1-\sigma)\lambda]^3}\left(P - \overline{w_B^D}\right).$$

So, $\frac{\partial \overline{w_B^N}}{\partial k}$ and $\frac{\partial \overline{w_B^N}}{\partial d}$ have the same sign as $\frac{\partial \overline{w_B^D}}{\partial k}$ and $\frac{\partial \overline{w_B^D}}{\partial d}$; and, $\frac{\partial \overline{w_B^N}}{\partial \sigma} < 0$ if $\frac{\partial \overline{w_B^D}}{\partial \sigma} < \frac{2\lambda\left(P - \overline{w_B^D}\right)}{\delta + (1-\sigma)\lambda}$.

Next, prove property (4).

From (13) we get : $u\left(\underline{w_B^D}\right) \leq u(b)$. Thus, $\underline{w_B^D} \leq b$ because of the increasing property of $u(w)$.

To prove the other side, let's assume $\overline{w_B^D} \leq b$. The integrated variable hence satisfies $\underline{w_B^D} \leq x \leq \overline{w_B^D} \leq b$. So we have:

$$\frac{\sqrt{P - \underline{w_B^D} - d}}{2}\int_{\underline{w_B^D}}^{\overline{w_B^D}} \frac{u'(x)dx}{\sqrt{P-x-d}} \leq \frac{\sqrt{P - \underline{w_B^D} - d}}{2}\int_{\underline{w_B^D}}^{b} \frac{u'(x)dx}{\sqrt{P-b-d}} = \frac{\sqrt{P - \underline{w_B^D} - d}}{2\sqrt{P-b-d}}\left[u(b) - u(\underline{w_B^D})\right]$$

If $b \leq \frac{3}{4}(P-d)$, then $\underline{w_B^D} > 4b - 3(p-d)$. Thus, $\frac{\sqrt{P - \underline{w_B^D} - d}}{2\sqrt{P-b-d}} < 1$ and $\frac{\sqrt{P - \underline{w_B^D} - d}}{2}\int_{\underline{w_B^D}}^{\overline{w_B^D}} \frac{u'(x)dx}{\sqrt{P-x-d}} < u(b) - u(\underline{w_B^D})$ which violates equation (13). Therefore, the assumption is false and we have proved $\overline{w_B^D} > b$ if $b \leq \frac{3}{4}(P-d)$.

Besides, the wage bounds of worker A can be seen as a special case of worker B's where $d = 0, k = 0$ and $\sigma = 1$. From properties (1)-(3), property (4) is easily derived, i.e., $\underline{w_A} < \underline{w_B^D}$ and $\overline{w_A} > \overline{w_B^D}$.

As for property (5), if $k = 1$, equation (26) is reduced to

$$Ew_B = \overline{w_B^N} + \frac{\delta}{(1-\sigma)\lambda}(\overline{w_B^N} - \underline{w_B^N}) - \frac{2\delta}{\delta + (1-\sigma k)\lambda}(P - \underline{w_B^N})$$

where $\underline{w_B^N}$ and $\overline{w_B^N}$ satisfy:

$$u\left(\underline{w_B^N}\right) = u(b) - \frac{\sqrt{P - \underline{w_B^N}}}{2}\int_{\underline{w_B^N}}^{\overline{w_B^N}} \frac{u'(x)dx}{\sqrt{P-x}} \quad \text{and} \quad \frac{P - \overline{w_B^N}}{P - \underline{w_B^N}} = (\frac{\delta}{\delta + (1-\sigma)\lambda})^2.$$

The only difference in the system of equations compared with those for type A workers is the offer arrival rate, i.e., $\sigma = 0$ for type A while $\sigma > 0$ for type B.

Let $k \triangleq \frac{\delta}{\delta + (1-\sigma)\lambda}$, after some algebra the mean wage could be rewritten as:

$$Ew = P - (k^3 + k^2 + k)(P - \underline{w})$$

From the system of equations about $\underline{w}$ *and* $\overline{w}$, we can get:

$$\frac{\partial \underline{w}}{\partial k} = \frac{u'(\overline{w})(P - \underline{w})}{A + \frac{u'(\overline{w})k}{2}}$$

where $A = \frac{u\prime(\underline{w})}{2} - \frac{1}{4\sqrt{P-\underline{w}}}\int_{\underline{w}}^{\overline{w}} \frac{u'(x)dx}{\sqrt{P-x}}$>0.

So, $\frac{\partial Ew}{\partial k} = -(3k^2 + 2k + 1)\left(P - \underline{w}\right) + (k^3 + k^2 + k)\frac{\partial \underline{w}}{\partial k}$

$$= \frac{(P-\underline{w})}{A+\frac{u'(\overline{w})k}{2}}\left[\frac{u'(\overline{w})(k-k^3)}{2} - A(3k^2 + 2k + 1)\right] < 0$$

where the last inequality holds due to:

$$\frac{u'(\overline{w})(k-k^3)}{2} - A(3k^2 + 2k + 1) < \frac{u'(\overline{w})(k-k^3)}{2} - \frac{u'(\underline{w})(3k^3+2k^2+k)}{2} < 0$$

In addition, as k is increasing in σ, we get $\frac{\partial Ew}{\partial \sigma} < 0$. So the proposition is proved.

A5. Proof of proposition 4

(1) and (2) can be directly derived from proposition 1 and proposition 2. $\overline{w_B^N} < \overline{w_A}$ because

$$\overline{w_A} - \overline{w_B^N} = (P - b)\left[\left(\frac{\delta}{\delta+(1-\sigma k)\lambda}\right)^2 - \left(\frac{\delta}{\lambda+\delta}\right)^2\right] + d\left[1 - \left(\frac{\delta+(1-\sigma)\lambda}{\delta+(1-\sigma k)\lambda}\right)^2\right]\left(\frac{\delta}{\delta+(1-\sigma)\lambda}\right)^2 > 0$$

Next, consider the comparison of earning distributions.

Since

$$K_w^A = \frac{\delta}{\lambda}[\sqrt{\frac{P-b}{P-w}} - 1],$$

$$K_w^B = \begin{cases} \frac{\delta}{(1-\sigma k)\lambda}[\sqrt{\frac{P-b-d}{P-w-d}} - 1], & if\ w \in [\underline{w_B^D}, \overline{w_B^D}] \\ \frac{\delta}{(1-\sigma k)\lambda}[\frac{\delta+(1-\sigma k)\lambda}{\delta+(1-\sigma)\lambda}\sqrt{\frac{P-\underline{\overline{w_B^N}}}{P-w}} - 1], & if\ w \in [\underline{w_B^N}, \overline{w_B^N}] \end{cases}$$

and

$$\frac{\delta}{\lambda} < \frac{\delta}{(1-\sigma k)\lambda}, \quad \sqrt{\frac{P-b}{P-w}} < \sqrt{\frac{P-b-d}{P-w-d}}, \quad \sqrt{\frac{P-b}{P-w}} < \frac{\delta+(1-\sigma k)\lambda}{\delta+(1-\sigma)\lambda}\sqrt{\frac{P-\underline{\overline{w_B^N}}}{P-w}}$$

we can get $K_w^A < K_w^B$ for all w, i.e., A's earnings distribution first-order stochastically dominantes B's earnings distribution. Therefore, $Ew_A > Ew_B$ and $w_A^q > w_B^q$.

Through tedious calibration, we can get the comparative statics of $Ew_A - Ew_B$: $\frac{\partial[Ew_A - Ew_B]}{\partial(\sigma,k,d)} > \vec{0}$.