Squaring the investment cycle

Kakarot-Handtke, Egmont

18 August 2011

Squaring the Investment Cycle

Egmont Kakarot-Handtke*

Abstract

The present paper replaces the standard behavioral axioms by structural axioms and applies these to the analysis of the accumulation and decumulation of capital. This yields a coherent view of the interrelations of real and nominal saving–investment, of profit–loss, of money–credit, and of internal–external financing. The main result is that asymmetric growth is indispensable for the viability of the market system.

JEL E10, E21, E22, E23, E40

Keywords New framework of concepts, Structure-centric, Axiom set, Symmetric investment cycle, Asymmetric investment cycle, Flux-reflux, Real rate of interest, Nominal rate of profit, Financial profit, Nonfinancial profit, Roundaboutness

*Affiliation: University of Stuttgart, Institute of Economics and Law, Keplerstrasse 17, D-70174 Stuttgart. Correspondence address: AXEC, Egmont Kakarot-Handtke, Hohenzollernstraße 11, D-80801 München, Germany, e-mail: handtke@axec.de

> The golden rule marked a climax in the development of growth theory. This was not because it was an intellectual achievement of an exceptionally high order but because it was the last theorem on economic growth that made a contribution to mainstream economics. ... After the golden rule, growth theory became the preserve of mathematical specialists with little apparent output for those interested in substantive economic insights. (Niehans, 1994, p. 465)

This phenomenon is not confined to growth theory. A lot of explanations have been advanced in the heterodox literature as to why standard economics in the most diverse avenues of theoretical analysis arrives with apparent necessity at the same formalistic cul-de-sac. The present paper circumvents this discussion in a constructive manner. Keynes pointed the way, alas, without going it himself:

> For if orthodox economics is at fault, the error is to be found not in the superstructure, which has been erected with great care for logical consistency, but in a lack of clearness and of generality in the premises. (Keynes, 1973, p. xxi)

Now in cannot in earnest be maintained that standard economics lacks clearness of premises. To the contrary. Its formal architecture is of exemplary transparency and height compared to the dusky and flat theoretical dwellings that surround it. This, though, is not of overriding importance. The question is, who stands on firmer ground.

It is not clearness, it is the content of the formal foundations that is at issue. Standard economics rests on specific behavioral assumptions that are formally expressed as axioms (Arrow and Hahn, 1991, p. v), (Kaldor, 1985, p. 13). The standard set of behavioral axioms is in the present paper at first replaced by structural axioms. These are applied to the intertwined real and nominal processes of capital accumulation and decumulation. The general thesis of the present paper is that human behavior does not yield to the axiomatic method, yet the axiomatization of the money economy's fundamental structure is feasible.

By choosing objective structural relationships as axioms behavioral hypotheses are not ruled out. On the contrary, the structural axiom set is open to *any* behavioral assumption and not restricted to the standard optimization calculus.

The case for structural axiomatization has been made at length elsewhere (2011a, 2011b, 2011c), thus we can leave the pros and cons of standard economics were they stand and throw a glance at the new avenue ahead. In the following the minimalistic formal frame that constitutes the pure consumption economy is set up in section 1. Then, in section 2, profit and the distribution of profits among two firms is derived from the axiom set. Profit is pivotal in the growing money economy. In sections 3 and 4 the interrelations between saving–dissaving, investment–disinvestment, money, credit and finance are consistently developed for the symmetric investment cycle. The same is done in section 5 for the complementary limiting case of the perfectly asymmetric cycle. Between these two limiting cases of capital accumulation

and decumulation reality is to be found. In section 5 the real rate of interest of the household sector and the nominal rate of profit of the business sector is defined. It turns out that they are unrelated which has some bearing on the coordination of real and nominal saving and investment. Section 7 concludes.

1 Axioms

The first three structural axioms relate to income, production, and expenditures in a period of arbitrary length. For the remainder of this inquiry the period length is conveniently assumed to be the calendar year. Simplicity demands that we have at first one world economy, one firm, and one product.

Total income of the household sector Y is the sum of wage income, i.e. the product of wage rate W and working hours L, and distributed profit, i.e. the product of dividend D and the number of shares N.

$$Y = WL + DN \quad |\mathrm{t} \tag{1}$$

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad |\mathrm{t} \tag{2}$$

Consumption expenditures C of the household sector is the product of price P and quantity bought X.

$$C = PX \quad |\mathrm{t} \tag{3}$$

The axioms represent the pure consumption economy, that is, no investment expenditures, no foreign trade, and no taxes or any other government activity.

2 Overall profit and firms's profits

The business sector's financial profit ΔQ_{fi} in period t is defined with (4) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditures C – and costs – here identical with wage income Yw:

$$\Delta Q_{fi} \equiv C - Y_W \equiv PX - WL \quad \Leftarrow \quad Y_W \equiv WL \quad |\mathrm{t} \tag{4}$$

For the business sector as a whole to make a profit consumption expenditures C have to be greater than wage income Yw in the simplest case. So that profit comes into existence in the *pure consumption economy* the *household* sector must run a deficit at least in one period. This in turn makes the inclusion of the financial sector mandatory. An economic theory that does not include at least one bank that supports the concomitant credit expansion cannot capture the essential features of the market economy (for details see 2011d, 2011e).

From (4) and (1) follows for the relation of financial profit and distributed profit:

$$\Delta Q_{fi} \equiv C - Y + Y_D \quad \Leftarrow \quad Y_D \equiv DN \quad |\mathrm{t} \tag{5}$$

The business sector consists of two firms. The household sector apportions its consumption expenditures between the purchase of two different consumption goods. Axiom (3) is accordingly differentiated to:

$$C = P_1 X_1 + P_2 X_2 = C_1 + C_2 \quad |\mathrm{t} \tag{6}$$

Total consumption expenditures are equal to total income at first. The overall expenditure ratio is defined as:

$$\rho_E \equiv \frac{C}{Y} \quad |\mathrm{t} \tag{7}$$

The household sector as a whole does neither save nor dissave, i.e. $\rho_E=1$ (for details see 2011d, 2011e).

The profit definition (4) is now also differentiated for the two firms:

$$\begin{aligned} \Delta Q_{fi1} &\equiv P_1 X_1 - W_1 L_1 \\ \Delta Q_{fi2} &\equiv P_2 X_2 - W_2 L_2 \quad |\mathrm{t} \end{aligned} \tag{8}$$

To take the simplest case first, profits are set to zero. Then, by implication, it must hold for each firm that sales revenues and wage costs are equal:

$$C_1 = W_1 L_1 \qquad C_2 = W_2 L_2 \quad |\mathrm{t} \tag{9}$$

The quantities produced by the firms O and bought by the households X are set equal for the time being in order not be distracted by inventory changes. The respective sales ratios are defined as:

$$\rho_{X1} \equiv \frac{X_1}{O_1} \qquad \rho_{X2} \equiv \frac{X_2}{O_2} \quad |\mathrm{t} \tag{10}$$

Under the condition that both markets are cleared, that is, both sales ratios are unity, (9) can be rewritten as:

$$\frac{W_1}{P_1 R_1} = 1 \quad \text{if} \quad \rho_{X1} = 1 \qquad \frac{W_2}{P_2 R_2} = 1 \quad \text{if} \quad \rho_{X2} = 1 \quad |\mathrm{t} \tag{11}$$

Overall profits are zero because of $C=Y$ and $Y_D=0$ according to (5). The zero profit condition for each firm reads $W/PR=1$ according to (11). In sum: both markets are cleared, the household sector's budget is balanced and profits are zero for both firms. With the zero profit condition the market clearing prices P_1 and P_2 for both firms are determined by the respective wage rates and productivities.

3 The symmetric investment cycle

The investment cycle begins with the assembly of capital goods and ends with their elimination from the production of consumption goods. We consider two limiting cases in turn: the symmetric and the asymmetric cycle.

The structure of the economy is quite frugal in the initial period. From assigning convenient values to selected variables in the following no loss of formal generality arises. The full scope of the analysis is invariably defined by the unrestricted structural axiom set. The generalizations of the simplified exposition suggest themselve.

3.1 The initial period

The business sector consists of two consumption goods producing firms. Hence total income (1) is given by:

$$Y = \underbrace{W_1}_{W} L_1 + \underbrace{W_2}_{W} L_2 + \underbrace{(D_1 N_1 + D_2 N_2)}_{Y_D=0} \quad |0 \tag{12}$$

The wage rates are set equal for both firms and distributed profits are set to zero.

To take gratuitous complexity out of the analysis total employment L is henceforth taken as constant:

$$L \equiv L_1 + L_2 \quad |0 \tag{13}$$

Total consumption expenditures are initially equal to income, i.e. $\rho_E=1$, and spent on the output of both firms according to (6). In the initial period profits of both firms are zero according to (11). From this follow the market clearing prices in the initial period as:

$$P_{10} = \frac{W}{R_{10}} \quad \text{if} \quad \rho_{X10} = 1 \qquad P_{20} = \frac{W}{R_{20}} \quad \text{if} \quad \rho_{X20} = 1 \tag{14}$$

The prices are equal to the respective unit wage costs.

3.2 Investment and disinvestment

Period$_1$

There are no changes for firm$_1$, yet firm$_2$ now switches production from consumption good output to investment good output. Therefore X_{21} denotes a qualitatively different quantity. Productivity R_{21} and price P_{21} assume new dimensions and values. The previous consumption expenditures of the households C_{20} vanish and are replaced by investment expenditures of firm$_1$ in period$_1$ I_{11}. Labor input and wage costs remain unchanged in the two industries. The profit of the new investment good industry is set to zero. This implies that the investment expenditures cover exactly the wage costs:

$$\Delta Q_{fi2} \equiv \underbrace{P_2 X_2}_{I_1} - W_2 L_2 \quad \Rightarrow \quad I_1 = W_2 L_2 \quad \text{if} \quad \Delta Q_{fi2} = 0; \rho_{X2} = 1 \quad |1 \qquad (15)$$

In contrast to the initial period the households save in period$_1$. Financial saving is defined as[1]:

$$\Delta S_{fi} \equiv Y - C \quad |\text{t} \qquad (16)$$

It is assumed that consumption expenditures to firm$_1$ stay the same, i.e. C_{11}=C_{10}, and that households save exactly the former consumption expenditures to firm$_2$ C_{20}, i.e. C_{21}=0. Thereby the households adapt perfectly to the available consumption good output:

$$\Delta S_{fi} \equiv Y - C_1 - C_2 \equiv \underbrace{WL_1 + \overbrace{WL_2}^{I_1}}_{Y} - \underbrace{WL_1}_{C_1} - C_2 \quad |1 \qquad (17)$$

From this follows then as a corollary that household sector saving is equal to the investment expenditures of firm$_2$ in period$_1$:

$$\Delta S_{fi1} = I_1 \quad \text{if} \quad C_2 = 0 \qquad (18)$$

This is the familiar result – with all implicit assumptions made explicit. It would be patently misleading, though, to resume that saving and investment are equal by definition; they are equal by assumption – a host of assumptions, to be precise.

Money and credit

If income is higher than consumption expenditures the household sector's stock of money increases. The change in period t is defined as:

$$\Delta \bar{M}_H \equiv_m Y - C \quad |\text{t} \qquad (19)$$

Financial saving ΔS_{fi} as given by (16) is the residual Y-C as it appears at the household sector; the same residual appears now as a change of the household sector's stock of money $\Delta \bar{M}_H$. Saving and the change of the household sector's stock of money are *two aspects* of the *same* flow residual[2]. The monetary aspect is formally kept apart by the notation $\equiv_m$.

[1] The 6th axiom states that saving, like profit, has a financial and nonfinancial component. The nonfinancial component is neglected here.

[2] It is no accident that money gives rise to double-entry bookkeeping. This entails two definitions for the same thing which is not allowed in a purely formal notational system. In double-entry bookkeeping, though, it is just the opposite. One and the same transaction is artificially but consistently split into two appearances that carry different labellings. In the strictly formal sense double-entry bookkeeping is a closed system of meaningful semantic tautologies that is designed according to the archetype buying≡selling. The fact that numeric equality = and semantic tautology ≡ fall into one in double-entry bookkeeping is sometimes confusing. There are two types of error: miscalculation (numeric)

The stock of money $\bar{M}_H$ at the end $\bar{t}$ of an arbitrary number of periods is defined as the numerical integral of the previous changes of the stock plus the initial endowment:

$$\bar{M}_H \equiv \sum_{t=1}^{t} \Delta\bar{M}_{Ht} + \bar{M}_{H0} \quad |\bar{t} \tag{20}$$

The changes in the stock of money as seen from the business sector are symmetrical to those of the household sector:

$$\Delta\bar{M}_B \equiv_m C - Y \quad |t \tag{21}$$

The business sector's stock of money at the end of an arbitrary number of periods is accordingly given by:

$$\bar{M}_B \equiv \sum_{t=1}^{t} \Delta\bar{M}_{Bt} + \bar{M}_{B0} \quad |\bar{t} \tag{22}$$

In order to reduce the monetary phenomena to the essentials it is supposed that all financial transactions are carried out by the central bank[3]. The stock of money then takes the form of current deposits or current overdrafts. Initial endowments are set to zero. Then, if the household sector owns current deposits according to (20) the current overdrafts of the business sector are of equal amount according to (22). Money and credit are symmetrical.

In the initial period income and consumption expenditures are equal, i.e. $\rho_E=1$. The monthly income $Y/12$ is paid out at mid-month as shown in Figure 1. Expenditures are evenly distributed over the month.

With the beginning of period$_1$ households start to save and thereby their current deposits increase until period end according to (20). Business, taken as a whole, cannot recoup total wage income and by consequence its current overdrafts increase according to (22) (see also Schmitt, 1988). At the end of period$_1$ current deposits are numerically equal to financial saving ΔS_{fi}. Investment expenditures, which are equal to saving according to (18), are – at the moment – completely financed by overdrafts (alternative forms of financing are considered in section 4).

Period$_2$

In period$_2$ firm$_2$ switches back to consumption good production. Labor input and consumption good output are exactly as they were in the initial period. Firm$_1$ now employs the investment good output of the previous period in the current production of its consumption good. With the same labor inputs L_1 and L_2 total income remains unvaried at the level of the foregoing periods.

---

and addressing the wrong account (semantic). Numerical equality is no guarantee that no semantic error has occurred (see for example $\Sigma\equiv I\equiv S$, 2011a, p. 21, see also the reference to the Dijon-Fribourg school in Rochon and Rossi, 2003, p. xxxvi)

[3] For a more detailed account of the central bank's role see (2011d, 2011e).

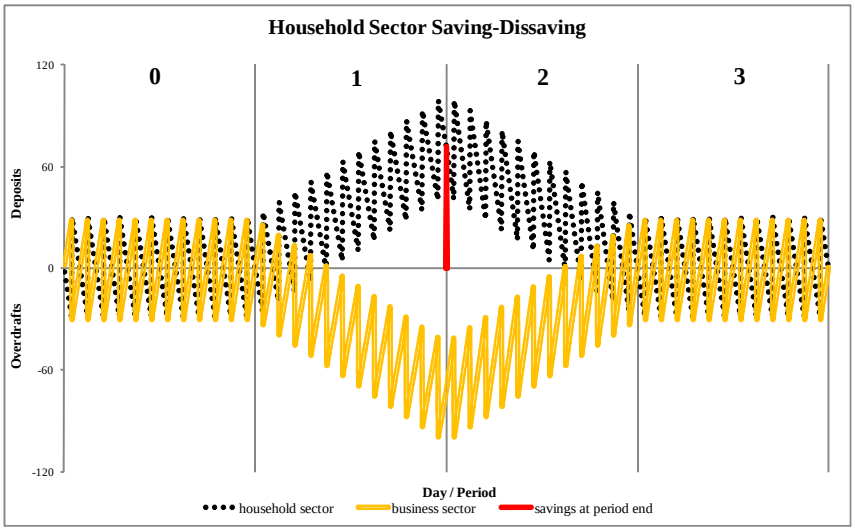

Figure 1: Saving leads in period$_1$ to an increase of the household sector's current deposits and the business sector's current overdrafts

With unaltered productivity the output of firm$_1$ would stay at the level of the foregoing periods. It is assumed now that the service-input of the investment goods leads to a productivity increase in firm$_1$ from R_{11} to R_{12}.

The market clearing price P_1 should according to (11) fall compared to the previous period, i.e. $P_{12}<P_{11}$.

We assume, in addition, that the households dissave in period$_2$ exactly the same amount that they saved in period$_1$. Consumption expenditures increase from C_{11} to $C_{12}=C_{11}+\Delta S_{fi1}$. The increased nominal demand requires a higher market clearing price. The increased productivity on the other hand requires a lower price. The net result depends on the relative magnitudes of the countervailing impacts. The new price is determined by:

$$P_{12}=\frac{C_{12}}{R_{12}L_1}=\frac{C_{11}+\Delta S_{fi1}}{R_{12}L_1}=\frac{W}{R_{12}}\left(1+\frac{L_2}{L_1}\right)\quad\text{if}\quad\rho_{X12}=1 \tag{23}$$

Given the conditions enumerated above we obtain the same result as Minsky (2008, p. 164). His theory can therefore be taken as a special case of the structural axiomatic approach with zero distributed profits.

Financial profits, which were zero in the previous period, increase with the additional nominal demand:

$$\Delta Q_{fi12}\equiv(C_{11}+\Delta S_{fi1})-WL_1 \tag{24}$$

This boils down to:

$$\Delta Q_{fi12}=\Delta S_{fi1}\quad\text{if}\quad C_{11}=WL_1 \tag{25}$$

Profit in period$_2$ is equal to dissaving which, because of (18), is in turn equal to investment expenditures of period$_1$. Hence:

$$\Delta Q_{fi12} = I_1 \tag{26}$$

This result coincides with Minsky's 'powerful truth' (2008, p. 163).

Now it is time to broaden the conceptual basis. The 5th axiom[4] states that *total profit* is the sum of financial and nonfinancial profit:

$$\Delta Q = \Delta Q_{fi} + \Delta Q_{nf} \quad |\mathrm{t} \tag{27}$$

Nonfinancial profit ΔQ_{nf} spans all positive and negative changes of value of the firms's real assets, here investment goods, in a given period and consists in this elementary case only of depreciation. Thus we have for firm$_1$:

$$\Delta Q_{nf1} \equiv G_1^+ - G_1^- \quad \text{with} \quad G_1^+ = 0 \quad |2 \tag{28}$$

The firm has some discretion in valuing its real assets, at least until they are brought to market and fetch a market price. It is assumed that the investment goods are *fully* depreciated in period$_2$, i.e. G^-=I. In combination with (26) and (28) then follows for total profit (27):

$$\Delta Q = I_1 - G^- = 0 \quad |2 \tag{29}$$

Total profit in period$_2$ is zero. Financial profit in period$_2$ is equal to dissaving. Since we have stipulated that the households spend, in addition to their current income, what they have saved in period$_1$, and since saving ΔS_{fi1} was equal to investment expenditures I_1 in period$_1$, financial profit is in turn equal to I_1 in period$_2$. If, on the other hand, the investment goods are *fully* depreciated then by consequence profit is zero in the disinvestment phase. Hence profits of the business sector as a whole are zero over *all* phases of the symmetric cycle. The physical input of investment goods does neither generate additional factor incomes nor profits.

At first sight this appears to contradict the microeconomic evidence because the very purpose of investment is to earn higher profits and at most times the firms succeed. This effect indeed shows up when the business sector is *differentiated*. For the business sector *as a whole*, though, things look different. The productivity effect of the investment goods does not affect overall profits as given by (5) but only the distribution of profits among firms. Since we consider one single investing firm this distributional effect does not materialize. To extrapolate the firm's evidence onto the whole economy is a fallacy of composition. The experience of business men is therefore often misleading in theoretical economics. Marshall's definition: 'Economics is a study of men as they live and move and think in the ordinary business of life' (2009, p. 12) is ill-conceived. In theoretical economics we are not primarily interested in what business men think but how the economic system works (see also Rochon and Rossi, 2003, p. xxxviii).

[4] The 4th axiom, see (2011b, p. 6), is not required in the present context.

The application of investment goods in the production of consumption goods affects productivity but not total profit. The distribution of the consumption good output is not affected. The wage earning households absorb the whole output in each period. They only swap, more precisely: are made to swap, a part of consumption good output in period_1 for a higher output in period_2. Whether this temporal shift is worthwhile depends on the productivity effect of the investment goods (see section 6.1).

The financial profit of period_2 makes it possible to pay off the business sector's current overdrafts as shown in Figure 1. At the end of period_2 the stock of money is – after dissaving – again zero (cf. the flux-reflux principle; Seccareccia 2003, p. 173) and the investment goods are valueless in bookkeeping terms. The investment cycle is completed.

Follow-up productivity

It may well be the case that the investment goods do the job much longer than one period. What we observe then is a higher productivity of firm_1 in subsequent periods compared to the initial period. Under the condition of market clearing the price of the firm's product must be lower according to (14). In all other respects there is no difference to the initial period.

When we assume, as a parable, that we have only agricultural production and a change of climate leads to a higher output for the same labor input, there would be *no economic* difference between the two cases.

Investment goods that operate longer than they are valued in the books are like manna from heaven. The manna effect depends on the physical lifespan of the investment goods. Seen from beginning to end the two-period process of investment–disinvestment lifts the pure consumption economy onto a higher productivity level. The investment economy can therefore be seen as a temporary deviation from the pure consumption economy. In the historical context temporary means the last 250 years.

4 Money and Finance

Figure 1 gave a rough impression of the interrelation of saving–dissaving and the accumulation–decumulation of current deposits and overdrafts. Figure 2 offers more details about what happens in the business sector.

There is a noticeable difference between the two firms in period_1. While firm_1 produces and sells its consumption good as before, firm_2 switches production and does no longer sell to the households. The firm pays the monthly wages but has, at first, no revenues. Therefore current overdrafts increase stepwise until period end. The zigzag pattern of the business sector as a whole is composed of the horizontal movement of deposits and overdrafts of firm_1 and the downward movement of the overdrafts of firm_2.

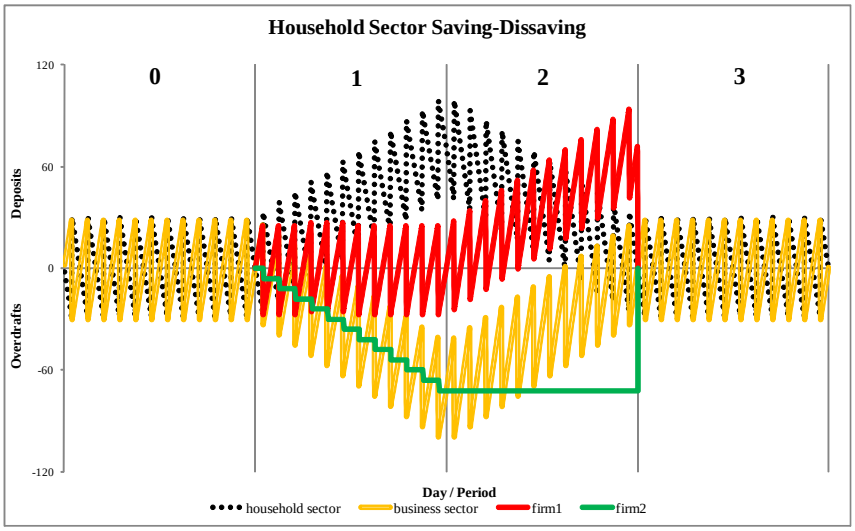

Figure 2: Development of deposits and overdrafts of the consumption good producing firm_1 and the investment good producing firm_2 with final redemption of the business-to-business loan

In the second period the households dissave and accordingly the current deposits of firm_1 increase until period end. Firm_2 switches back to consumption good production and to the prior pattern of payments and receipts which is not explicitly shown in Figure 2. It is assumed here that firm_2 sells the investment goods to firm_1 at the end of period_1 and at the same time grants a credit until the end of period_2. The current overdrafts of firm_2 remain therefore unchanged for the time being.

At the end of period_2 firm_1 has accumulated enough deposits in order to settle its liabilities. Current deposits of firm_1 reduce simultaneously with current overdrafts of firm_2 to zero. A certain amount of money and credit disappears from the economy. Thereafter the payment pattern is the same as in the initial period.

Interest payments between the firms or between the firms and the central bank have been completely left out of the picture. The rate of interest has been dealt with in more detail in (2011d; 2011e).

The financing of investment by the seller of the investment goods is, of course, one alternative among many others. It puts a lot of strain on the seller who in fact finances the process of capital accumulation–decumulation from the very beginning to the very end. This simplifies the exposition of the main point considerably but does normally not happen. Normally capital investments are financed in advance by the buyer of the investment goods.

Figure 3 shows that firm_1 takes up a two-period loan from the central bank at the beginning of period_1 and that it hands over the corresponding current deposits to firm_2. Firm_1 advances exactly the wage payments of firm_2 during the production process of the investment goods (see Keynes, 1937, p. 246; Cencini, 2003, pp. 306-307; Seccareccia, 2003, p. 176).

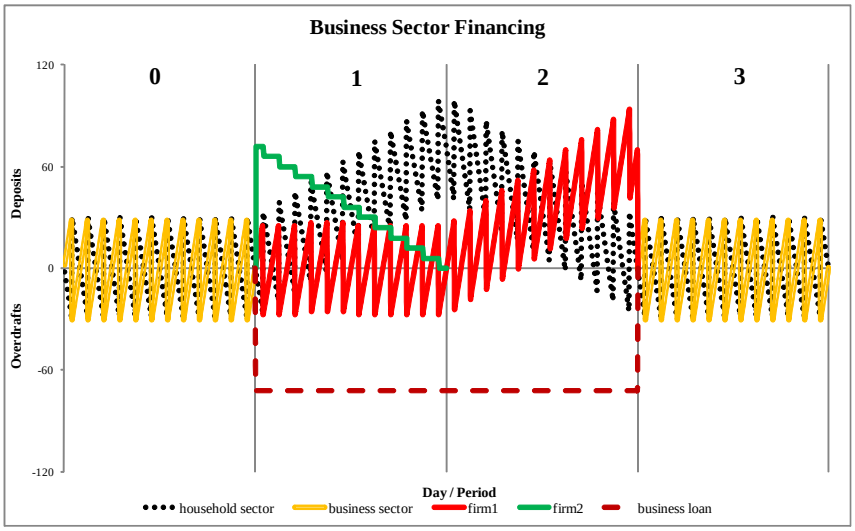

Figure 3: Investment expenditures are financed in advance by firm_1 with a two-period loan from the central bank

In period_2 firm_2 switches to – zero-profit – consumption good production while firm_1 makes a profit and accumulates current deposits. At the end of period_2 the firm is in the position to redeem the two-period loan. Subsequently there is again business as usual.

It is obvious from the two cases above that there is virtually an unlimited number of financing alternatives with regard to timing and the maturity of loans that lead to the same outcome at the end of period_2. They are of considerable practical interest. For the moment, however, the basic cases are sufficient to give a fairly complete picture of the real, monetary and financial interrelations of the symmetric investment cycle.

For the sake of completeness it has to be added that financing real investment is not the sole occupation of the banking industry. This may cause specific problems that, however, are only indirectly related to the financing of real investments in the consumption good industry:

> Our economy is unstable because of capitalist finance. If a particular mix of hedge and speculative financing of positions and of internal and external financing of investment rules for a while, then there are, internal to the economy, incentives to change the mix. (Minsky, 2008, p. 244)

This problems cannot occur for the time being due to the simplifying assumption that the banking industry consists of the central bank. To provide the financing of the symmetric investment cycle is an easy task for a central bank that necessitates neither hedging nor speculation. Nothing but the application of time-tested banking rules is required.

5 Nominal and real asymmetry

The asymmetric cycle is different from the symmetric cycle in that the households keep consumption expenditures constant in period$_1$, that is, they do not save. The consumption expenditure ratio stays at unity throughout. The allocation of the constant labor input between the two firms is the same as in the symmetric case.

Period$_1$

Since the households can in period$_1$ only buy the consumption good output of firm$_1$ it holds: C=Y=C_{11}. The market clearing price has to rise analogous to (23):

$$P_{11} = \frac{C_{11}}{R_{10}L_1} = \frac{W(L_1+L_2)}{R_{10}L_1} = \frac{W}{R_{10}}\left(1+\frac{L_2}{L_1}\right) \quad \text{if} \quad \rho_{X11} = 1 \tag{30}$$

This price is higher than (14) and (23) because the productivity is the same as in the initial period.

With higher nominal demand profit is no longer zero. Financial profit of firm$_1$ in period$_1$ is given by:

$$\Delta Q_{fi11} \equiv C_{11} - WL_1 \equiv WL_1 + WL_2 - WL_1 \quad \text{if} \quad C = Y = C_{11} \tag{31}$$

Profit is therefore exactly equal to investment expenditures:

$$\Delta Q_{fi11} = I_1 \quad \text{if} \quad WL_2 = I_1 \tag{32}$$

Figure 4 shows that financing is now a much simpler affair compared to the symmetric case in Figure 2. Since profit at the end of period$_1$ is equal to investment expenditures I_1, firm$_1$ is in the position to settle its liabilities with firm$_2$ immediately. Current deposits of firm$_1$ and current overdrafts of firm$_2$ vanish simultaneously. Investment is now completely financed out of current profit. The stock of money is zero at the end of period$_1$. In firm$_2$ everything is exactly as it was in the symmetric cycle with the exception that current overdrafts are zero at period end.

From the business sector's viewpoint the asymmetric investment phase has a lot of advantages compared to the symmetric case. Instead of ΔS_{fi}=I; ΔQ_{fi}=0 we now have ΔQ_{fi}=I; ΔS_{fi}=0 and since positive profits is what we observe in the real world it is obvious that reality is always to be found *between* the two analytical limiting cases of perfect symmetry and perfect asymmetry. This means that ΔS_{fi}=I; ΔQ_{fi}=0 cannot be the general case. The commonplace story about saving and investment, viz.

> Saving is refraining from consumption today in order to have consumption in the future, for oneself or one's heirs; or, more precisely, to have the power to consume in the future. Overall, savings must equal investment, for current output set aside as an input to future production must necessarily be unavailable for current consumption. (Arrow, 1980, p. 142)

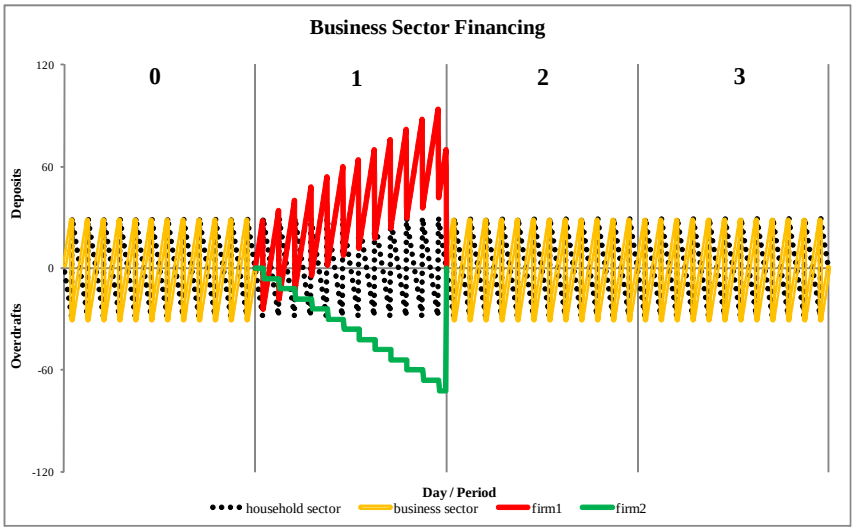

Figure 4: Investment financing out of current profit of firm_1

confounds the real and the nominal side of saving and investment, and therefore has to be rejected.

Period_2

The investment good industry disappears in period_2. Firm_2 switches back to consumption good production and gets its previous share of consumption expenditures. This entails a decline of nominal demand for the output of firm_1 compared to period_1. The overall consumption expenditure ratio is still unity.

The additional input of investment good services now boosts productivity and output of firm_1. In contrast to the symmetric disinvestment phase the households do not dissave because they have saved nothing in the investment phase. Therefore the market clearing price has to fall because there is now no additional nominal demand that coincides with additional output:

$$P_{12} = \frac{C_{12}}{R_{12}L_1} = \frac{WL_1}{R_{12}L_1} = \frac{W}{R_{12}} \quad \text{if} \quad \rho_{X2} = 1 \tag{33}$$

Compared to the initial period (14) the price is lower because the productivity is higher. Compared to period_1 (30) the price is lower because nominal demand is lower.

The financial profit of the consumption good industry is zero. When depreciation is taken into account then according to (29) there is a loss. This loss is exactly equal to the profit in period_1 since $G^{-}=I$. Summed up over the complete asymmetric investment cycle total profit is zero exactly as in the symmetric case.

The business sector's stock of money is not affected by the loss in period_2 because financial profit is zero and the loss is nonfinancial.

The whole consumption good output goes to the households in each period just as in the symmetric case. Investment and disinvestment provoke no real distributional effects.

In the asymmetric cycle financial profit is simply brought forward to period$_1$. The myopic agents cannot realize that their profit is the effect of a *time shift* and that it will be wiped out by an eventual loss of equal magnitude. This is due to the fact that any clue is lacking *when* the time shift will be reversed. For agents with perfect foresight and an infinite time horizon it would therefore not necessarily be rational to invest[5]. The general precondition of profit is that business investment and household saving are out of phase and this real and nominal asymmetry is what can be observed. Since the households neither save nor dissave in the perfectly asymmetric investment cycle there exists *no logical connection* between financial saving and investment expenditures. The familiar ex ante/ex post rationalization is off the mark (for details see 2011a, pp. 19-22).

It seems to be reasonably clear from the asymmetric investment cycle that one can observe *positive* real saving (in the sense of unavailable current output) and *zero* financial saving (in the sense of spending power for the future) *in the same period*. The lack of a rigorous conceptual differentiation in standard and Keynesian economics is, in the last instance, the source of the $I \equiv S$-flaw which in turn makes nonsense of employment theories that rely either on the wage mechanism or on effective demand – but in unison on $I=S$ as equilibrium condition.

Never stop growing

Reality is to be found somewhere in between perfect symmetry and perfect asymmetry. Moreover, one period with positive investment is followed by another one. The investment phase lasts longer than one single period and is not in the next period immediately followed by the full depreciation of the whole stock of investment goods. The two phases of investment and disinvestment *overlap continuously*. Total profit for the business sector as a whole is positive as long as

$$I - G^{-} > \Delta S_{fi} \quad |\mathrm{t} \tag{34}$$

and that means that net investment has to grow faster than financial saving. If financial saving is zero throughout then investment expenditures have only to grow faster than depreciation. Overall economic conditions are especially favorable when the households (private and public) dissave.

[5] "That agents seek to make their investment decisions rationally is taken as a fundamental premise of capital theoretic models. The rationality hypothesis is implemented by assuming that agents maximize a utility function over paths of future consumption and that producers maximize the present discounted value of their profits." (Becker, 2008, p. 1)
It is obvious that discounting future profits and perfect foresight is inconsistent if investment is immediately followed by disinvestment. Future profits are simply non-existent in this case. A theory that is based on behavioral axioms is *a priori* incapable to apprehend the structural reality. That there are discountable profits for an infinite time horizon needs explanation, otherwise it is wishful thinking.

It is obvious that the economy is inherently unstable. If investment expenditures are reduced then profits fall and this chain of events is self-reinforcing. This positive feedback is normally aggravated by financial problems due to a particular mix of internal and external financing. Whether this induces a self-reinforcing loop in the banking industry depends on the leverage and interconnectedness of the actual structure.

In the general case, with distributed profits included, the condition for positive total profits can be rewritten as:

$$I - G^{-} + Y_D > \Delta S_{fi} \quad |\mathrm{t} \tag{35}$$

All other things equal, profit distribution makes life easier for the business sector and decouples profit from the growth of the nominal capital stock. Profit distribution, though, establishes a positive feedback loop of its own.

In sum two points deserve emphasis. First, profit for the economy as a whole does not depend on the productivity of investment goods. It depends on *subsequent* investment expenditures, that is, it feeds on itself. With regard to expected rates of return on real (as opposed to financial) investment this means self-referentiality, i.e. there is no such thing as an objective marginal efficiency of capital. If the business sector as a whole wants a higher return on real investment in the future it has to increase investment expenditures in the future – with saving, profit distribution, et cetera unchanged, of course. For the economy as a whole investment expenditures validate their profitability with further investment expenditures.

Second, growth is indispensable for the viability of the market system. The preoccupation of standard economics with equilibrium and efficiency therefore misses the point. In marked contrast, the classics and Keynes were fully aware that capital accumulation is the mover of the market system. Strong growth overrides all market imperfections, inefficiencies and the tendency of the rate of profit to fall. To maintain that the efficient use of given resources is the ultimate secret of the market system's success is, at best, naïve.

It nearly goes without saying that the simplifying assumptions that define perfect symmetry and asymmetry are expository and not descriptive. In the general case, for example, profits are greater than zero in the investment good industry. But this leads only to a redistribution of profits within the business sector and does not affect our main conclusions. And, of course, it is well known that there occur lags of variable length between the production of investment goods and the eventual productivity effect. These practical peculiarities do not affect the end result. Lags of arbitrary length do not lengthen the logical distance between the beginning and the end of the investment cycle. Reality is composed of a multitude of overlapping asymmetric investment cycles of varying length.

The *first half* of the asymmetric investment cycle is, in a nutshell, the theoretical counterpart of the market system's development since the industrial revolution.

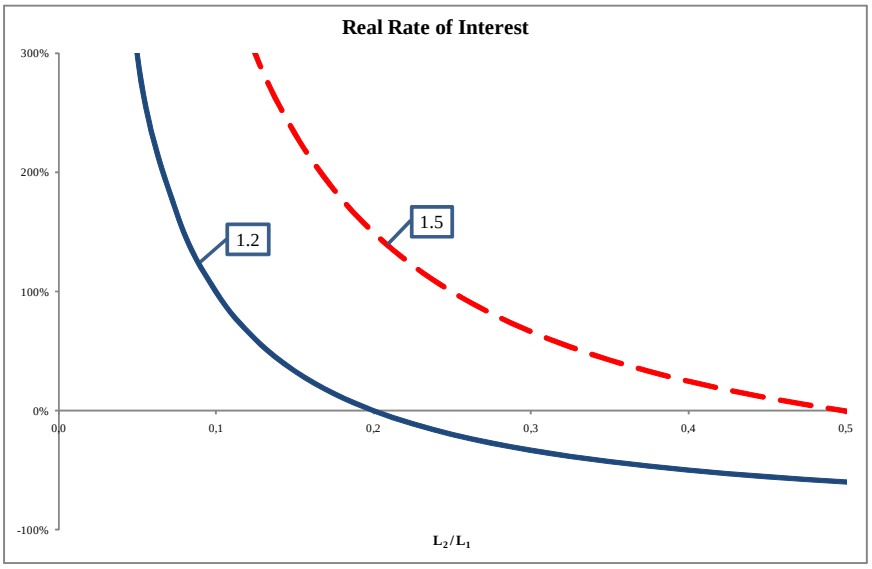

Figure 5: Relation between real rate of interest and allocation of labor inputs in the investment phase for a given productivity effect in the disinvestment phase

6 Properties of roundaboutness

6.1 Households's real rate of interest

By allocating a certain part of the constant total labor input to the production of the investment goods the output of the consumption goods is reduced in period$_1$. This reduction is compensated for by a higher output in period$_2$ due to the productivity effect of the investment goods. By setting the additional output in period$_2$ in relation to the foregone output in period$_1$ one gets – ex post – a real rate of interest:

$$J^{real} \equiv \frac{\Delta O_{12}}{\Delta O_{21}} - 1 \equiv \frac{\frac{R_{12}}{R_{10}} - 1}{\frac{L_2}{L_1}} - 1 \quad \text{if} \quad R_{10} = R_{20} \quad |2 \tag{36}$$

The relation has been simplified with the assumption that the productivities in both firms are initially equal, that is to say, outputs are qualitatively identical and the price is the same. Hence, if labor input is allocated between both industries in period$_1$ in the relation $L_1=0.75L$ respectively $L_2=0.25L$, and if the productivity effect in period$_2$ is $R_{12}=1.2R_{10}$ then the real rate of interest is zero. It is negative if the productivity effect falls short of $1.2R_{10}$ and it is positive for all productivity increases greater than 20 percent as depicted in Figure 5.

Conversely, given a definitive productivity effect in period$_2$ for the investment goods in question, the real rate of interest depends on the varying distribution of labor input between firm$_2$ (investment good production) and firm$_1$ (consumption good production) in period$_1$. The real rate of interest in period$_2$ turns out to be

the lower the higher the labor input in the investment good industry in period$_1$ was. Figure 5 shows this structural axiomatic version of a decreasing efficiency of capital. The inverse relationship follows, though, directly from definition (36) without reference to a conveniently postulated production function ('a powerful instrument of miseducation'; Robinson, 1953, p. 81) and without reference to the vague notion of capital.

The plain message of (36) is that the real rate of interest might be negative for the households despite the fact that the productivity effect is positive. Conversely, if a target rate of interest is given the curve can be used to determine the appropriate allocation of labor input between consumption good and investment good production. The consumers, to be sure, have not much to decide about investment. The firms decide. And for the firms the real rate of interest is not directly of interest.

6.2 Business's nominal rate of profit

The magnitude of the productivity effect is not predictable. This, though, does not matter much for total profit. As we have seen above, profit for the business sector as a whole depends on the symmetric or asymmetric time pattern of consumption expenditures and not on productivity. Hence total profit in period$_2$ may be positive while the consumers's real rate of interest might be negative.

The business sector's profit depends on the time pattern of the households's consumption expenditures and not on any right or wrong investment decision. These decisions are relevant for the *distribution* of profits among firms. Total profit follows from (27) and (8) and reads for the general case:

$$\Delta Q = C + I - W_1 L_1 - W_2 L_2 + \Delta Q_{nf} \quad |\mathrm{t} \tag{37}$$

Substituting (12) and (5) gives:

$$\Delta Q = C + I - (Y - Y_D) + \Delta Q_{nf} \quad |\mathrm{t} \tag{38}$$

The alternative formulation using the definition of financial saving (16) then reads:

$$\Delta Q = I - \Delta S_{fi} + Y_D + \Delta Q_{nf} \quad |\mathrm{t} \tag{39}$$

The final simplification is implemented with the assumption that saving and distributed profits are equal, i.e. ΔS_{fi}=Y_D. Now total profit is equal to net investment:

$$\Delta Q = I - G^{-} \quad |\mathrm{t} \tag{40}$$

The profit rate in period t is calculated here as the ratio of total profit (40) to the actual nominal value of the capital stock[6]:

[6] "In 1953 I tried to find out what [capital] meant. ... Does a quantity of capital mean a number of dollars or a list of machine tools, railway lines and other hard objects? And which is it that has a 'marginal product'? The only answer we got was: Let us pretend that it doesn't make any difference." J. Robinson, 1974; quoted in (Gram, 1991, p. 126)

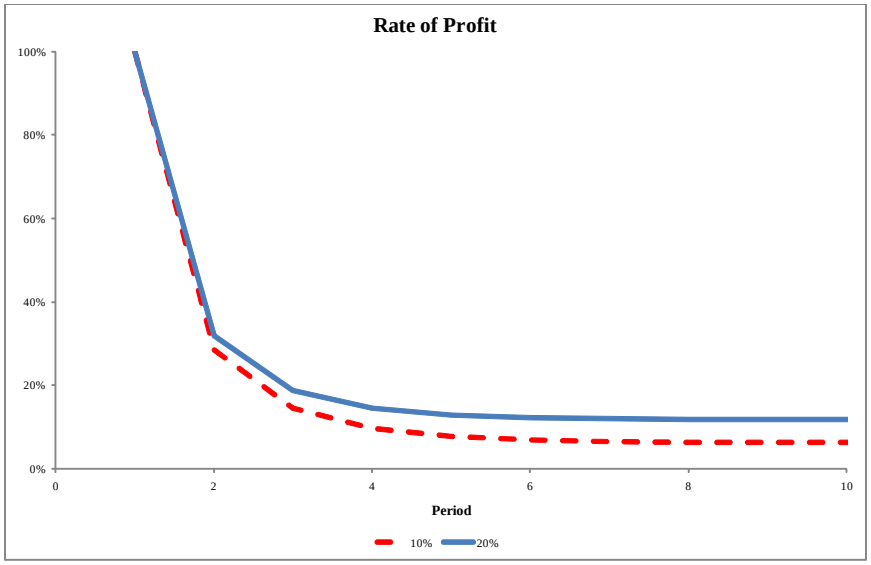

Figure 6: Development of the nominal rate of profit given a growth rate of investment expenditures of 10 respectively 20 percent and a depreciation rate of 50 percent

$$\pi \equiv \frac{\Delta Q}{\bar{K}} \quad |\mathrm{t} \tag{41}$$

At the end of the first period the nominal capital stock is I_1. At the end of the second period it is $(I_1\text{-}G_{-2})+I_2$. And so on. To make things concrete we proceed on the assumption that 50 percent of the actual value of the investment goods of each vintage are depreciated and that investment expenditures I grow with 10 respectively 20 percent in each consecutive period. The result is depicted in Figure 6.

Increasing investment expenditures and increasing depreciation eventually balance and the profit rate stabilizes after six periods around 6 respectively 12 percent. The development of the nominal rate of profit is determined by the growth rate of investment expenditures and the depreciation rate. When the growth rate of investment expenditures is set to zero, the rate of profit falls and eventually becomes negative. From the structural axiom set and the 'laws of algebra' (Shaik, 1980) therefore follows under elementary conditions that only a *growing* nominal capital stock yields a positive nominal rate of profit for the economy as a whole. As soon as the growth ends the 'tendency of the rate of profit to fall' gets the upper hand (cf. Sinn, 1975). The productivity effect, in contrast, is related to the real stock and persists for an indefinite time span.

It is not terribly important whether depreciation reflects the true obsolescence of the investment goods. Measurement errors lead only to a time shift of total profits. As Adam Smith already observed (2008, p. 89): 'Profit is so very fluctuating that the person who carries on a particular trade cannot always tell you himself what is the average of his annual profit.' In the end, though, the value of the investment goods is zero, that is, if the scrap value is zero, and all measurement errors finally

cancel out. The fact that the period to period measurements of the rate of profit are unreliable on the firm's level merely obliterates the underlying straightforward relationship between the rate of profit and the growth of the nominal capital stock for the economy as a whole. Financial profits are in any case measurable with a two-digit precision. Thus the concept of an overall nominal rate of profit is axiomatically well defined.

Mention should be made that the consumers's real rate of interest is different from the profit rate and both are in turn different from the central bank's rate of interest (for details see 2011d; 2011e; see also 2011f). There exists *no logical connection* between these rates (see Wicksell, 1936, p. xxv). Hence profits cannot in the classical way be interpreted as 'well-deserved remuneration for forgone consumption' (Graziani, 2003, p. 39).

A *real* rate of profit does not exist because total profit, as defined by (27), is a purely nominal magnitude. In a "real" economy it cannot occur. The surplus in a barter economy is conceptually entirely different from profit in a money economy. Hence there is no need to busy ourselves with the notion of a real capital stock (Harcourt, 1972, pp. 16-33). Because there is no real numerator available for the definition of the real profit rate, we do not need the real denominator. Productivity is a real phenomenon and profit is a monetary phenomenon. The former concerns the households, the latter the firms. They cannot be made to fit together in an analysis that restricts itself to purportedly real phenomena; yet they complement each other symmetrically or asymmetrically in the structural axiomatic analysis of the money economy.

7 Conclusions

Behavioral assumptions, rational or otherwise, are not solid enough to be eligible as first principles of theoretical economics. Hence all endeavors to lay the formal foundation on a new site and at a deeper level actually need no further vindication. The present paper suggests three non-behavioral axioms as groundwork for the formal reconstruction of capital accumulation and decumulation including the flux–reflux of money, credit and finance.

The structural axiomatic analysis compares two limiting cases: a) nominal and real symmetry of saving–accumulation and dissaving–decumulation; and b) real and nominal asymmetry. The main results are:

- The productivity effect of the investment goods does not affect overall profits but only the distribution of profits among firms.
- From the business sector's viewpoint the asymmetric investment phase has a lot of advantages compared to the symmetric case. Instead of saving=investment/profit=0 one has profit=investment/saving=0.

- Since positive profits is what can be observed in the real world it is obvious that reality is always to be found *between* the two analytical limiting cases of perfect symmetry and perfect asymmetry.
- Profits of the business sector as a whole sum up to zero over all phases of the symmetric and asymmetric investment cycle. The physical input of investment goods does neither generate additional factor incomes nor profits.
- In the asymmetric cycle financial profit is simply brought forward. The myopic agents cannot realize that their profit is the effect of a time shift and that it will be wiped out by an eventual loss of equal magnitude. This is due to the fact that any clue is lacking *when* the time shift will be reversed.
- The whole consumption good output goes to the households in each period in the symmetric and asymmetric case. Investment and disinvestment provoke no real distributional effects.
- Since the households neither save nor dissave in the perfectly asymmetric investment cycle there exists no logical connection between financial saving and investment expenditures. The familiar ex ante/ex post rationalization is off the mark.
- One can observe positive real saving (in the sense of unavailable current output) and zero financial saving (in the sense of spending power for the future) in the same period.
- Profit for the economy as a whole depends on subsequent investment expenditures. With regard to expected rates of return on real investment this means self-referentiality, i.e. there is no such thing as an objective marginal efficiency of capital.
- The real rate of interest might be negative for the households despite the fact that the productivity effect is positive.
- Only a growing nominal capital stock yields a positive nominal rate of profit for the economy as a whole.
- The consumers's real rate of interest is different from the profit rate and both are in turn different from the central bank's rate of interest. There exists no logical connection between these rates.
- A real rate of profit does not exist because total profit is a purely nominal magnitude.
- Asymmetric growth is indispensable for the viability of the market system.

The first half of the asymmetric investment cycle is, in a nutshell, the theoretical counterpart of the market system's development since the industrial revolution.

References

Arrow, K. J. (1980). Real and Nominal Magnitudes in Economics. *Public Interest*, (Special Issue: The Crisis in Economic Theory): 139–150.

Arrow, K. J., and Hahn, F. H. (1991). *General Competive Analysis*. Amsterdam, New York, NY, etc.: North-Holland.

Becker, R. A. (2008). Capital Theory. In S. N. Durlauf, and L. E. Blume (Eds.), *The New Palgrave Dictionary of Economics Online*. Palgrave Macmillan, 2nd edition. URL http://www.dictionaryofeconomics.com/article?id=pde2008_C000575.

Cencini, A. (2003). IS-LM: A Final Rejection. In L.-P. Rochon, and S. Rossi (Eds.), *Modern Theories of Money*, pages 295–321. Cheltenham, Northampton, MA: Edward Elgar.

Gram, H. (1991). Value and Capital, Robinson Style. In I. H. Rima (Ed.), *The Joan Robinson Legacy*, pages 121–135. Armonk, NY, London: M. E. Sharpe.

Graziani, A. (2003). *The Monetary Theory of Production*. Cambridge, New York, NY, etc.: Cambridge University Press.

Harcourt, G. C. (1972). *Some Cambridge Controversies in the Theory of Capital*. Cambridge: Cambridge University Press.

Kakarot-Handtke, E. (2011a). Keynes's Missing Axioms. *SSRN Working Paper Series*, pages 1–29. URL http://ssrn.com/abstract=1841408.

Kakarot-Handtke, E. (2011b). Properties of an Economy Without Human Beings. *SSRN Working Paper Series*, pages 1–23. URL http://ssrn.com/abstract=1863788.

Kakarot-Handtke, E. (2011c). The Pure Logic of Value, Profit, Interest. *SSRN Working Paper Series*, pages 1–21. URL http://ssrn.com/abstract=1838203.

Kakarot-Handtke, E. (2011d). Reconstructing the Quantity Theory (I). *SSRN Working Paper Series*, pages 1–22. URL http://ssrn.com/abstract=1895268.

Kakarot-Handtke, E. (2011e). Reconstructing the Quantity Theory (II). *SSRN Working Paper Series*, pages 1–18. URL http://ssrn.com/abstract=1903663.

Kakarot-Handtke, E. (2011f). Uniform Profit Ratios. *SSRN Working Paper Series*, pages 1–13. URL http://ssrn.com/abstract=1906578.

Kaldor, N. (1985). *Economics Without Equilibrium*. Armonk, NY: M.E. Sharpe.

Keynes, J. M. (1937). Alternative Theories of the Rate of Interest. *Economic Journal*, 47(186): 241–252. URL http://www.jstor.org/stable/2225525.

Keynes, J. M. (1973). *The General Theory of Employment Interest and Money*. The Collected Writings of John Maynard Keynes Vol. VII. London, Basingstoke: Macmillan. (1936).

Marshall, A. (2009). *Principles of Economics*. New York, NY: Cosimo, 8th edition. (1890).

Minsky, H. P. (2008). *Stabilizing an Unstable Economy*. New York, NY, Chicago, IL, San Francisco, CA: McGraw Hill, 2nd edition.

Niehans, J. (1994). *A History of Economic Theory*. Baltimore, MD, London: Johns Hopkins University Press.

Robinson, J. (1953). The Production Function and the Theory of Capital. *Review of Economic Studies*, 21(2): pp. 81–106. ISSN 00028282. URL http://www.jstor.org/stable/2296002.

Rochon, L.-P., and Rossi, S. (2003). Introduction. In L.-P. Rochon, and S. Rossi (Eds.), *Modern Theories of Money*, pages xx–lvi. Cheltenham, Northampton, MA: Edward Elgar.

Schmitt, B. (1988). The Identity of Aggregate Supply and Demand in Time. In A. Barrère (Ed.), *The Foundations of Keynesian Analysis*, pages 169–193. Houndmills, London: Macmillan.

Seccareccia, M. (2003). Pricing, Investment and the Financing of Production within the Framework of the Money Circuit: Some Preliminary Evidence. In L.-P. Rochon, and S. Rossi (Eds.), *Modern Theories of Money*, pages 173–197. Cheltenham, Northampton, MA: Edward Elgar.

Shaik, A. (1980). Laws of Production and Laws of Algebra: Humbug II. In E. J. Nell (Ed.), *Growth, Profits, and Property*, pages 80–95. Cambridge, New York, NY, Melbourne: Cambridge University Press.

Sinn, H.-W. (1975). Das Marxsche Gesetz des tendenziellen Falls der Profitrate. *Zeitschrift für die gesamte Staatswissenschaft*, 131: 646–696.

Smith, A. (2008). *An Inquiry into the Nature and Causes of the Wealth of Nations*. Oxford: Oxford University Press. (1776).

Wicksell, K. (1936). *Interest and Prices*. London: Macmillan.