# Endogenous categorization and group inequality

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# ENDOGENOUS CATEGORIZATION AND GROUP INEQUALITY

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**Abstract:** The objective of this paper is to integrate economic and sociological elements in a model of human capital accumulation by phenotypically distinct individuals. Both kinds of elements are influenced by the degree of categorization endogeneity (CE), meant as the influence of endogenous elements (e.g., behavioral traits) in group categorization. If CE is high, members of dominated groups can pass as members of dominant groups by adopting the behavioral norm associated with that group. CE facilitates group equality by decreasing the ability to discriminate between members of dominant and dominated groups, but it weakens intra-group neighborhood effects. It is argued that, under sufficiently low levels of discrimination, CE widens the range of values of the neighborhood effects parameter for which group inequality is stable.

**Key-words:** human capital, neighborhood effects, categorization endogeneity.

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## 1. Introduction

Group inequality is still a commonly observed phenomenon worldwide. It occurs in a broad range of contexts: countries with both large populations (India, United States, Brazil, and Japan) and small populations (Belize, Trinidad and Tobago, Israel, and New Zealand); higher-income countries (Australia, Canada, Japan, and the United States), as well as in lower income countries (India, Belize, and South Africa); countries with high growth rates (Malaysia, Japan, Belize, and India), as well as in countries with low or negative growth rates (South Africa and New Zealand); and countries with high levels of general inequality (Brazil, South Africa, and Malaysia), as well as in countries with low levels of general inequality (Canada, India, Australia, and Israel) (Darity Jr. and Nembhard 2000).

Traditional economic analysis has focused on two main explanations for the existence of group inequality. The first one is that members of discriminated groups enjoy smaller returns on human capital investment, due to taste-based (Becker 1957) or occupational discrimination (e.g., Bergmann 1974). In this case, two equally qualified individuals coming from different groups will receive different wages due to pure taste discrimination or because individuals coming from discriminated groups are confined to low-wage sectors.

The second explanation for group inequality is that members of discriminated groups accumulate less human capital than those coming from dominant groups. It can be explained, for instance, by statistical discrimination (Phelps 1972, Arrow 1973), discrimination in the access to and quality of schooling[1] or disadvantages accumulated over generations[2].

Economic elements alone are not able to fully explain group inequality. For instance, group discrimination is unprofitable to firms. Thus, competition would make discrimination in the labor market disappear in the long run (Arrow 1973). However, pure taste discrimination is still observed. The usual explanation for this, statistical discrimination, is not satisfactory, as it relies on unrealistic postulates as the observation

[1] It has as consequence not only smaller levels of education, but also smaller returns to education and earnings (Chiswick 1988).

[2] Becker and Tomes (1986) discuss how families concerned about the welfare of their children can transmit them assets and earnings, stimulating their human capital accumulation.

of individual's marginal productivity (Arrow 1998). This quotation from Arrow (1998) summarizes this view:

> I am going to suggest in this paper that market-based explanations will tend to predict that racial discrimination will be eliminated. Since they are not, we must seek elsewhere for non-market factors influencing economic behavior. The concepts of direct *social interaction* and networks seem to be good places to start (Arrow 1998: 2, italics added).

In this vein, economists are more and more convinced that social elements should be taken into account for the sake of a better understanding of group inequality. It is widely recognized that one's conditions and incentives to adopt a pro-mobility behavior depends on the characteristics of one's reference group, or *neighborhood*. For instance, the individual cost of acquiring education may be decreasing in the average effort in education in one's residential area (Brock and Durlauf 2001). Groups defined along social rather than physical lines can also contribute to the persistence of group inequality. Exclusion may lead discriminated social categories, as Blacks in most Western countries, to adopt a detrimental behavior as behavioral prescriptions (Akerlof and Kranton 2000).

Nonetheless, there is not a consensus among these sociological theories of group inequality regarding the role played by segregation on the persistence of group-based differences. The seminal study from Loury (1977) argues that racial inequality can persist due to racial segregation. This view is shared by other studies as Bowles et al. (2007), Bowles and Sethi (2006), Chaudhuri and Sethi (2008) and Sethi and Somanathan (2004). The concern here is with *inter-group* neighborhood effects: segregation prevents members of dominated groups from enjoying positive externalities stemming from dominant groups.

Other sociological-based explanations for group inequality, however, argue that integration rather than segregation can contribute to the persistence of group inequality. The constitution of "oppositional identities" (Bisin et al. 2006), where the behavioral norms associated with the dominant group are rejected, can be stronger in mixed neighborhoods. For instance, Fryer and Torelli (2005) find that the punishment to

"acting white" behaviors (e.g., put more effort on studies) among black students is stronger in racially mixed schools. Thus, segregation enforces group inequality through *intra-group* neighborhood effects, favoring the development of detrimental social norms among dominated groups.

The objective of this paper is to integrate economic and sociological elements in a model of human capital accumulation by phenotypically distinct individuals. Both kinds of elements are influenced by a common element: the degree of categorization endogeneity. By categorization endogeneity I mean the influence of endogenous elements, as behavioral traits, in group categorization. If categorization endogeneity is high, members of dominated groups can pass as members of dominant groups by, for example, adopting the behavioral norm associated with that group.

In the model, categorization endogeneity is set in the following way: self-categorization is exogenous, rooted in phenotypic characteristics, but heterocategorization (categorization by others) is endogenous, in such a way that members of the subaltern category may be seen by others as members of the dominant category if they choose to acquire education. On the one hand, categorization endogeneity facilitates the acquisition of qualification by members of subaltern groups on a market point of view. If higher wages, access to education, better jobs etc are reserved to members of dominant groups, individuals coming from dominated groups would be better-off if they can pass as members of the dominant group by adopting some behavioral traits, for example. In other words, categorization endogeneity decreases the "ability to discriminate": distinguishing between members of the dominant group and idiosyncratic members of dominated groups is difficult if categorization is essentially endogenous.

A nice example to illustrate this point comes from the story of the Brazilian indigenous leader Marcos Terena[3]. He managed to study until become a pilot of the Brazilian Air Force saying he was a "Japanese", a term which is extended also to Japanese descendents in Brazil. Japanese is an ethnicity associated in Brazil with pro-mobility behavior, as diligence in school. Thus, by adopting the Japanese behavioral prescription and taking advantage of the phenotypic similarity, Terena was able to adopt a Japanese identity and had access to the elements which allowed his upward mobility.

[3] Terena published a letter telling his story in the Brazilian newspaper *Jornal do Brasil*.

On the other hand, categorization endogeneity can be harmful to the human capital accumulation among members of dominated groups from the sociological point of view, as it weakens the within-group neighborhood effects. Throughout this paper, I am especially concerned with intra-group neighborhood effects. As emphasized by the literature, mutual influences are stronger if individuals belong to the same category, that is, if they share the same sense of identity. If group identity is rooted in endogenous elements, as behavioral prescriptions, members of dominated groups acquiring qualification may be seen as deserters of the group. Thus, they may not be followed by the other members of the group. Continuing with Terena's story, he was victim of hostility by many indigenous and seen by them as "White" due to his behavior. If categorization was less endogenous, he could be imitated by more individuals from his group.

Beside this introduction, this paper is divided in four more parts. Section 2 sets the theoretical framework, arguing how an endogenous categorization may weaken within-group neighborhood effects. The model is presented in Section 3. Section 4 makes an analysis of the dynamics of the model. Some concluding remarks take the last section.

The insights provided in this paper shed some light on the debate about the role played by segregation on the persistence of group inequality. Integration tends to increase categorization endogeneity. Segregated societies, thus, can present higher levels of group inequality if anti-discrimination laws are weakly enforced. On the other hand, if anti-discrimination laws are strongly enforced, segregated societies can present lower levels of group inequality.

## 2. Theoretical framework

### *Neighborhood effects*

Neighborhood effects arise when individual decisions are influenced by the actions taken by other members of some reference group. In general they are justified

along psychological and sociological lines (Durlauf 2004)[4]. Using criminal behavior as reference, Glaeser and Scheinkman (2001) cite some channels through which neighborhood effects can take place, "ranging from pure physical externalities (while one person is being arrested, the police find it harder to arrest someone else), to learning from one's neighbors, to stigma (the more people who are committing a particular crime--the less likely is that crime to be a negative signal) to pure taste externalities (individuals just enjoy imitating others)" (Glaeser and Scheinkman 2001: 1).

Groups can be defined along various lines. The element which appears more frequently as a group glue is physical proximity: in this setting, individuals influence each other because they are physically near. In Evans et al. (1992), for example, neighbors are the students of some school.

Nevertheless, a widespread literature, bringing insights from other social sciences, notably social psychology and sociology, suggests that social influence goes beyond geographic proximity. Rather, it has to do with how individuals are located in what Akerlof (1997) calls "social space". This perception of neighborhood is supported by a sociological view of social interactions, based on "concepts that play little or no role in modern economics: class, community, culture, influence, status, gender roles, and so on" (Manski 2000: 12).

I will term the first type of neighborhood as *physical neighborhood* and the second one, *social neighborhood*. For example, one's physical neighborhood is formed by her residential neighbors, her classmates, and all others who share with her some physical space. On the other hand, her social neighborhood is composed by those she identifies with: people from the same racial/ethnic group, from the same religion, and so on.

The channels through which physical and social groups influence their members bear some similarities. With the exception of physical externalities, other channels as conformism and stigma are present in both kinds of groups. The norms enforced by social groups are called *social norms* (Elster 1989) or *behavioral prescriptions* (Akerlof and Kranton 2000). A universally familiar example of social categorization concerns gender. Within this kind of categorization, there are two social groups or categories,

[4] An important exception concerns local public finance of education. In this case, children of poor neighborhoods are affected by the low level of local per pupil expenditure. To more on this, see Hussar and Sonnenberg (2001).

man and woman. To each social category, it is associated a set of behavioral prescriptions, which can be interpreted as the ideal behavior for each category. Following this example, there are behavioral prescriptions for gender categories regarding dressing (e.g., only women are supposed to wear dresses), jobs (e.g., firefighters are men, while nurses are women), family roles (the father is responsible for providing the financial resources, while the mother should stay at home taking care of the children), among other aspects.

Some studies present empirical evidence on the influence of physical neighborhood on individual behavior. Case and Katz (1991), analyzing young men in poor Boston neighborhoods, found significant peer effects on criminal behavior, drug and alcohol use, church attendance and labor market activity. Crane (1991) concludes that the percentage of workers with professional or managerial job in PUMS neighborhoods affects 16-19 years old women behavior regarding dropping out of high school and fertility. However, the importance of neighborhood effects is not consensual among empirical studies. Evans et al. (1992), for instance, studying teenager behavior regarding dropping out of high school and pregnancy, found no evidence of neighborhood effects once endogeneity of neighborhood is controlled for by instrumental variables.

Empirical studies also show the importance of social neighborhoods. In general, such effects are analyzed together with the impact of physical neighborhoods. For instance, Borjas (1992) stress the importance of the so-called "ethnic capital", defined as the average skills of the ethnic group in the parents' generation, on children skills. It was shown that ethnic capital matters even among individuals who grow up in the same neighborhood (Borjas 1995). Another example is the study of Aizer and Currie (2004), which found evidence that the use of prenatal care public programs is highly correlated within groups defined using race/ethnicity and neighborhoods. Bertrand et al. (2000) show that individuals surrounded by others who speak the same language increase their use of welfare programs if their language group also does so.

*Neighborhood effects and group inequality*

Group-led behavior can contribute to the persistence of group inequality. Unfavorable initial conditions can persist in locally segregated groups. This link can emerge due to a lot of reasons, as exemplified by Brock and Durlauf (2001): the individual cost in acquiring education can be negatively associated to one's neighbors educational effort, children's educational decision is influenced by observed education/occupation outcomes among adults in their community and the chance of making a successful job application depends on information possessed by members of one's social network.

The norms engendered by social groups can also lead to group-based inequality. Indeed, as social norms are not outcome-oriented, there is no guarantee that will make all group members better-off (Elster 1989). Dominated groups can develop behavioral prescriptions which are incompatible with socio-economic mobility, perpetuating their inferior position. In Akerlof and Kranton (2000) framework, for example, this is explained by exclusion *per se*. Discriminated groups would feel a strong anxiety if they adopted pro-mobility behavioral prescriptions, as exclusion would prevent them to follow such prescriptions. For example, some studies point that African American youth, recognizing societal iniquity in confront with their group, may come to feel education as of little usefulness for their economic and social mobility. Thus, adolescents identified with the Black culture may present targets of low academic achievement (Chavous et al. 2003).

In other approaches, anti-mobility behavioral prescriptions are more implicit. According to the social dominance theory (Pratto et al. 2006), for example, group inequality is seen as something natural by both dominant and, in a less extent, dominated groups. Nothing should be done to change this situation. Group-based inequality is justified by the dissemination and acceptance of the so-called *hierarchy-enhancing legitimizing myths* (HE-LM):

> *Hierarchy-enhancing legitimizing myths (HE-LMs)* provide moral and intellectual justification for group-based oppression and inequality. Examples include myriad forms of racism, sexism, heterosexism, stereotypes, notions of "fate," just world beliefs, nationalism, Confucianism, the doctrine of meritorious karma, classism, the Divine Rights of Kings, Manifest Destiny, and internal attributions for poverty. Such disparate myths have been used to argue that

> inequality is fair, legitimate, natural, or moral. Hierarchy-enhancing legitimizing myths not only organize individual, group, and institutional behavior in ways that sustain dominance, they often lead subordinates to collaborate with dominants in the maintenance of oppression (Pratto et al. 2006: 7, italics in the original).

The same way neighborhood effects can lead to detrimental behavior, they can contribute to the erosion of group-based differences through the so-called social multiplier (Glaeser et al. 2002). Deviations from the group enforced behavior by some members can lead other group members to change their behavior too, eventually guiding the whole group towards a new set of norms, in such a way that group inequality is eliminated or at least diminished. Deviant behavior can have a lot of sources: individuals being affected by public policies; noises or mutations, as in stochastic evolutionary game theory models (e.g., Kandori et al. 1993); intentional idiosyncratic play (Bowles 2004); or the presence of individuals not influenced by others' actions, the fixed agents (Glaeser and Scheinkman 2001)[5].

Not just physical neighborhoods, but also social neighborhoods can have their norms affected by deviant behavior. One example is the feminist revolution. Popularly, the feminist movement was launched in U. S. by the book *The Feminine Mystique*, written by Betty Friedan in 1963. After that, there was a great incursion of females in predominantly male occupations. In U. S., between 1970 and 1990, there was an increase of the percent female in virtually all 45 Census occupations, except in some traditionally female jobs (as dietitians and speech therapists). Moreover, some former predominantly male occupations turned to be predominantly female, as underwriters and psychologists (Blau et al. 1998). It represented a change in female behavioral prescriptions concerning family roles (weakening of the housewife stereotype), as well as those related to occupations.

*Endogenous and exogenous categorizations*

[5] Fixed agents are not just a theoretical curiosity. Experiments run by Falk et al. (2003) show that about 10% of experiment' participants were not influenced by their peers' decisions regarding group contributions.

It was argued that groups – of both physical and social nature – can enforce behaviors that can be detrimental to their members. A crucial point to understand what kind of neighborhood effects will emerge in a given society is the way individuals place themselves and others in social groups – the process known as *categorization*. To what categories the individual identifies with? Which individuals will be considered as members of these categories? On the answers to these questions depends the formation of both physical and social groups. These questions refer to the two dimensions of categorization: the way individuals group themselves and how individuals place others – self-categorization and heterocategorization, respectively.

Some categorizations are essentially exogenous. An example of this first type of categorization is gender categorization. In this case, individuals generally group themselves and others in one of the two categories – males and females – in an unambiguous way driven by easily observable exogenous characteristics.

Notwithstanding, other types of categorization are at least partially endogenous. Ethnic/racial categorizations are good examples of this second kind of grouping. In this case, categorization is driven not just by exogenous individual characteristics – as phenotype – but also by endogenous elements regarding the individual (e.g. social status) and the environment (e.g. social and historical context).

The fluidity of racial categorization is partially explained by the fact that an individual has at least three racial identities: an internal one (what she tells himself about his race), an external one (what others believe his race to be) and an expressed one (what she wants his actions and words to indicate to others about his race). These three types of racial identity need not be either identical (although they are not independent) or consistent across social contexts, being influenced by racial composition and ideology of contexts, as well as by the extent to which an individual is known in a particular setting (Harris and Sim 2000).

The literature brings abundant evidence that ethnic/racial categorization may be essentially endogenous. For example, Miguel and Posner (2006), analyzing data from twelve African countries, concluded that ethnic identification is positively related to employment in non-traditional economic sectors and to the proximity of the survey to a competitive national election. Working with U.K. data, Bisin et al. (2006) found that the main determinants of ethnic identity include past racial harassment experiences, language spoken at home and with friends, quality of housing, structure of the family

and degree of neighborhood segregation. Even a country as United States, in which there already was a legal definition of race – and, thus, supposed to have a more exogenous racial categorization – presents a significant degree of racial endogeneity[6]. In the U. S. prior to the Civil War, marked by racist rules, African descendents, especially mulattoes, could be "white" by behavior and reputation (Bodenhorn and Ruebeck 2003).

If categorization is significantly endogenous, the boundaries between dominant and dominated groups are much more fluid. Membership in the dominant (dominated) group is associated with the adoption of a pro-(anti-) mobility behavior. For instance, in Rwanda, the Tutsi is the dominant group and the Hutu, the dominated one. Thus, those with lots of cattle were classified as a Tutsi, and the poorer ones, as Hutu (Bowen 1996).

*Endogenous categorization and neighborhood effects*

As argued before, deviations from group enforced behavior can lead to the displacement of group norms. Nonetheless, the endogeneity of categorization poses a challenge to such a process. If grouping is rooted in exogenous, easily observable elements, identity is not affected by behavior. In this case, idiosyncratic individuals will be still seen as members of some reference group. On the other hand, if categorization is endogenous, membership in the reference group is evaluated on the basis of the commitment with group's behavioral prescriptions. If one plays idiosyncratically, departing from group's behavioral prescriptions, she can be expelled from the reference group.

The transition to a new set of group enforced norms is easier in the first case than in the second one. An example will illustrate better this point. Suppose some group is characterized by a low level of human capital accumulation. In order to combat group inequality, the government implements some public policy with the objective of increasing human capital accumulation by the members of this group. Due to variation

[6] In this country, "the records of governmental specification of rules of racial identity associated with peoples of African descent were established both through legislation and court decisions (Darity Jr. et al. 2006: 288)".

in the personal willingness of accumulate human capital, just some members of this group, in a first moment, decide to augment their human capital stock.

What will happen with the remaining, more conformist members of the group? If categorization is exogenous, it is not affected by individual choices. Thus, the deviant members will be still seen as members of the group. Consequently, the more conformist members of the group will perceive an increase in the average human capital accumulated by group members and they will be stimulated to doing the same. With the majority of group members changing their choices, the group prescribed behavior would transit from a low-level human capital accumulation to a high-level human capital accumulation.

On the other hand, if categorization is endogenous, deviant members would no more be seen as members of the group, as they are breaking with the group' prescribed behavior. So, conformist members would not perceive an increase in the average accumulated human capital of the group. They would tend not to change their levels of human capital accumulation and the transition to a new group rule would not occur.

It clearly has important implications regarding the overriding of group inequality. If categorization is endogenous, members of dominated groups adopting a pro-mobility behavior can lose identity links with the more conformist members of the group. The group stimulus to these members to choose a pro-mobility behavior is weaker, as the idiosyncratic members would be seen as "deserters" of the group.

## 3. The Model

The following model will formalize the ideas presented above. There is population composed by genetically distinct individuals. There are two social categories in this society: the Blues and the Greens. A fraction $\beta$ of this population owns a gene G, which generates a physical, easily observable characteristic associated with the Green social group. For instance, this gene can be associated with skin darkness and the Green group can be the Black social category. This society is also characterized by group hierarchy, in such a way that the Blues are the dominant group and the Greens, the dominated one.

Individuals live for two periods. In the first one, or childhood, they choose to acquire education or not. In the second period, adulthood, they will work as qualified or unqualified employees. When she arrives at the second period, the individual generates, by cloning, a genetically identical child. At the end of the second period, the individual dies and her child becomes an adult. Thus, the population is composed by two generations of equal and constant size. Let us normalize this size to one.

Self-categorization is exogenously – that is, genetically – determined. Thus, individuals with (without) the gene G self-classify as Green (Blue). On the other hand, heterocategorization follows both exogenous and endogenous lines. Individuals are classified by others as Green or Blue not just according to their physical appearance, but also according to their educational choice. As the Blues are the social dominant group, they have as behavioral prescription the pro-mobility attitude, that is, the acquisition of education. Similarly, the Greens are supposed not to acquire education. The probability of an individual i be classified as member of group j is equal to

$$P_{i,j} = (1-\mu_j)G_{i,j} + \mu_j E_{i,j}$$

In the equation above, $G_{i,j}$ is equal to one if the phenotype of individual i corresponds to the phenotype associated with category j, and zero otherwise. Similarly, $E_{i,j}$ is equal to one if the behavior of individual i matches the prescribed behavior of category j, being zero otherwise. The parameter $\mu_j$ is equal to $0.5\alpha_j$, where $\alpha_j$ varies between zero and one and measures the degree of endogeneity of category j, being $\alpha_G$ greater than $\alpha_B$. For sake of simplicity, let us set $\alpha_B$ equal to zero[7] and call $\alpha_G$ simply $\alpha$.

Therefore, individuals without gene G are classified by others as Blues. Individuals with the gene are classified as Green with probability 1 if not acquiring education, but they will be classified as Blue with probability $\mu$ if acquiring education.

[7] This simplifying assumption has a strong empirical support. In their study using Brazilian data, Telles and Lim (1998) shows that, while 88% of Whites were consistently classified (that is, self-classification is consistent with other classification), this value was 58.5% for Blacks. Moreover, more educated self-classified Blacks were more likely to be classified as Whites. Analyzing data from 15-59-year-old women from Recife (Brazil), Miranda-Ribeiro and Caetano (2005) concluded that consistence between self- and other-categorization is greater among White women It suggests that categorization endogeneity works much more as an *upward* mechanism rather than a *downward* device: the entrance in the dominant group through the adoption of its behavioral prescriptions is easier than the downward grading to the dominated group of misbehaving members of the dominant group.

Note that if α is equal to one, an educated Green is randomly classified by others as Blue or Green.

In case of acquiring education, an individual self-classified as member of group j enjoys the following pay-off:

$$\pi_{i,j,t} = a_i r_j - (1 - a_i)\gamma(1 - \hat{s}_{j,t})$$

In the first term of the equation above, $r_j$ is the expected pecuniary pay-off for educated members of group j. It encompasses the expected pecuniary return to education and the expected pecuniary costs of educational effort. In the second term, $\gamma(1 - \hat{s}_{j,t})$ is the social cost of acquiring education, where γ is the social interactions parameter which measures the strength of neighborhood effects and $\hat{s}_{j,t}$ is the observed fraction of educated members of group j at time t. It captures the idea that the individual is influenced by the educational choices made by the members of the group she belongs to. The higher the observed fraction of educated contemporaneous members of her reference group, the easier will it be for her to become educated too. These two terms are weighted by $a_i$, the individual level of non-conformism, which ranges between zero and one. Less conformist individuals have a pay-off which is more dependent on the pecuniary pay-off; on the other hand, more conformist individuals face greater costs when departing from their peers' choices. In case of not acquiring education, the individual will have a pay-off equal to zero.

Qualified Blue employees enjoy an economic pay-off equal to r. Thus, $r_B$ is simply r. Additionally, qualified Green workers may suffer a decrease in this pay-off in an amount equal to d. This parameter reflects the degree of discrimination in this society and is smaller than r. This can be due to any form of direct discrimination, as taste discrimination in the labor market, denied access to good schools or education borrowing constraints. However, in μ percent of the time a qualified Green worker will be classified as Blue, not suffering this discount d. Thus, the expected economic return for qualified Green workers will be

$$r_G = r - (1 - \mu)d$$

Let $s_{B,t}$ and $s_{G,t}$ be, respectively, the *real* fractions of Blues and Greens who opted to acquire education at time t. The observed fractions will be

$$\hat{s}_{B,t} = \frac{(1-\beta)s_{B,t} + \mu s_{G,t}}{(1-\beta) + \mu s_{G,t}}$$

$$\hat{s}_{G,t} = \frac{\beta(1-\mu)s_{G,t}}{\beta[(1-s_{G,t}) + (1-\mu)s_{G,t}]} = \frac{(1-\mu)s_{G,t}}{1-\mu s_{G,t}}.$$

In the equation that represents the observed fraction of qualified Blue individuals, the denominator shows the fraction of individuals who are viewed as Blues in the society, that is, all individuals who are genetically Blue and the fraction μ of idiosyncratic Greens. The numerator is the share of such individuals that acquired education. The observed fraction of educated Greens can be explained in a similar fashion. It is important to stress that, if CE is null, observed and real fractions of skilled workers are equal.

Individuals will decide to acquire education if their respective pay-offs are greater than zero. The less conformist individuals are more prone to become educated, so it can be said that an individual will acquire education if her non-conformism level is above some threshold level ã. Ignoring time subscripts, these levels are, for each social group, equal to

$$\tilde{a}_B = \frac{\gamma(1-\hat{s}_B)}{r + \gamma(1-\hat{s}_B)}$$

$$\tilde{a}_G = \frac{\gamma(1-\hat{s}_G)}{r_G + \gamma(1-\hat{s}_G)}$$

Therefore, at time t + 1, the fraction of educated individuals will be equal to the fraction of individuals whose non-conformism level was greater than ã at time t. To simplify, let us suppose that a is uniformly distributed between zero and 1 for both genetic groups. In this case, we have

$$s_{B,t+1} = 1 - \tilde{a}_{B,t} \tag{1.A}$$

$$s_{G,t+1} = 1 - \tilde{a}_{G,t} \tag{1.B}$$

## 4. Dynamics of the model

A point $\{s_B^*, s_G^*\}$ is stationary if it is a solution to the system of difference equations represented by 1.A and 1.B:

$$s_B^* = 1 - \tilde{a}_B(s_B^*, s_G^*)$$

$$s_G^* = 1 - \tilde{a}_G(s_B^*, s_G^*)$$

Furthermore, the stationary point is stable if the Jacobean matrix of the dynamical system at this point has all eigenvalues strictly less than one. The system analyzed here has three possible stable stationary points[8]:

$$\{s_B^*, s_G^*\} = \left\{ [1,1], \left[1, \frac{2r_G}{r_G - (1-\alpha)r + 2\gamma}\right], \left[\frac{r}{1-\beta}\left(\frac{1}{\gamma} - \frac{2\beta}{r_G - (1-\alpha)r + 2\gamma}\right), \frac{2r_G}{r_G - (1-\alpha)r + 2\gamma}\right] \right\}$$

I will call these states, respectively, the high level state (HS), the unequal state (US) and the low level state (LS). It is important to stress that LS is also an unequal

[8] There is a fourth fixed point, but it is not stable. See Appendix for details.

state, since in this situation the fraction of qualified Blue individuals will never be inferior to the share of educated Green individuals. The existence and stability of these points depend on the neighborhood effects parameter. I will set two crucial values of γ, $\gamma^G = (1 - \mu)r_G$ and $\gamma^B = \gamma^G + k$, where

$$k = \frac{1}{8}\left[[2r - d(4-\alpha)](\alpha - 2) + \sqrt{(\alpha-2)^2(2r+\alpha d)^2 + \frac{32\alpha r r_G}{1-\beta}}\right]$$

is a non-negative constant. It can be shown that[9]:

i) If $0 < \gamma < \gamma^G$, HS is the unique stationary stable state (Situation 1).

ii) If $\gamma^G < \gamma < \gamma^B$, US is the unique stationary stable state (Situation 2).

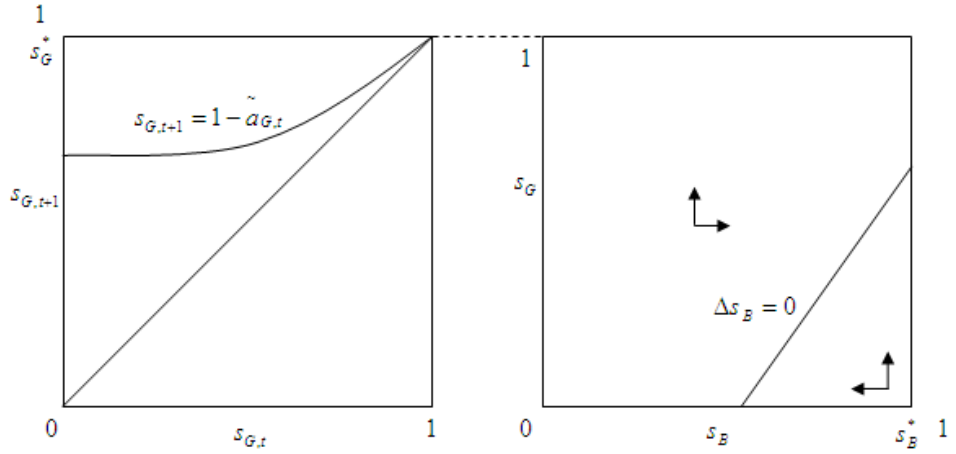
iii) If $\gamma > \gamma^B$, LS is the unique stationary stable state (Situation 3).

Thus, under small values of γ, all members of both groups would acquire education. If γ is sufficiently high, just a fraction of both groups will acquire education. Finally, under intermediate values of γ, all members of the dominant group and a fraction of the dominate group would acquire education. It is important to stress that $\gamma^B$ represents the neighborhood effects parameter below which all Blue individuals will choose to be qualified and $\gamma^G$ is the parameter below which all Greens will choose to acquire education. The better condition for the qualification of Blue individuals can be seen by the fact that $\gamma^B$ is greater than $\gamma^G$. Note also that $\gamma^G$ is the expected return for qualified Greens multiplied by the "ability to discriminate", which reaches its maximum value (one) when endogeneity is zero.

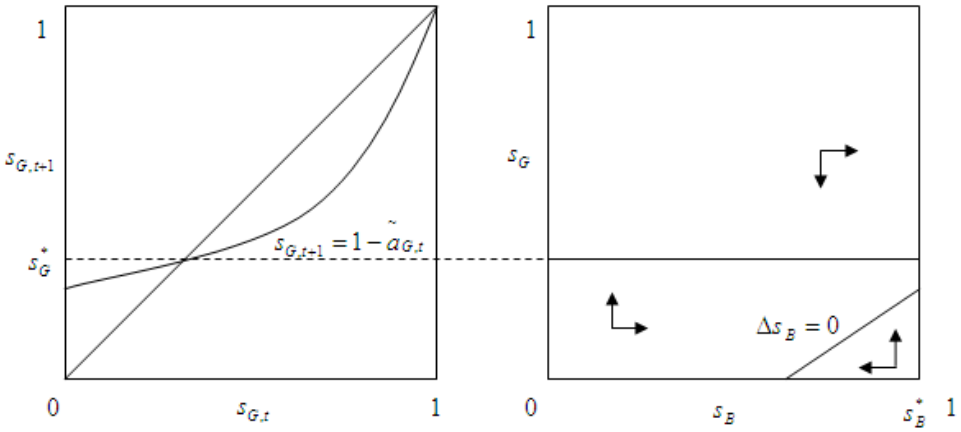
The following phase diagrams will illustrate these points. Note that, while $s_{G,t+1}$ depends solely on $s_{G,t}$, $s_{B,t+1}$ is a function of $s_{B,t}$ and $s_{G,t}$. Thus, $s_G^*$ is determined first and, then, $s_B^*$. If γ is sufficiently low, $s_G^*$ will be equal to 1. In this case, $s_B^*$ will necessarily be equal to 1 too. This situation corresponds to the HS and is depicted in

[9] The demonstration is in the Appendix.

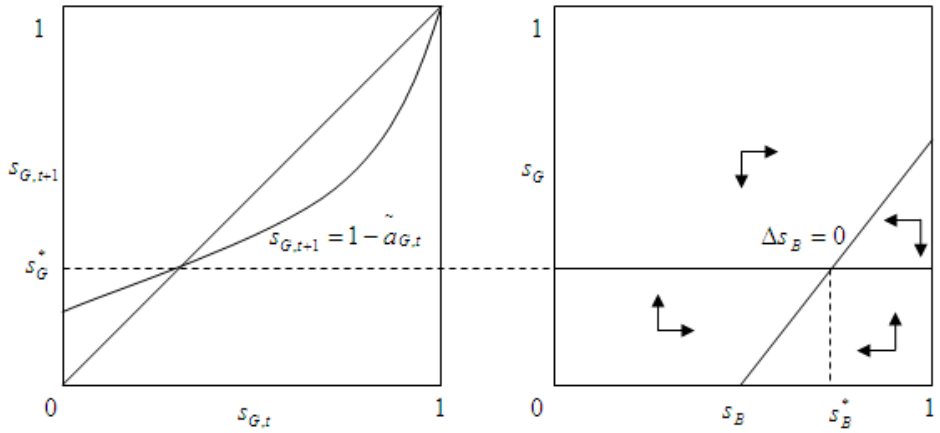
Figure 1(a). For higher levels of γ, $s_G^*$ is smaller than 1. In this case, $s_B^*$ can be equal to [Figure 1(b)] or – if γ is sufficiently high – smaller than 1 [Figure 1(c)], engendering US or LS, respectively.




(a)




(b)




(c)

Figure 1: Situations 1 (a), 2 (b) and 3 (c).

It is important to stress that the equilibria are Pareto-rankable. LS is the worst one. US is Pareto-superior to LS, as the fraction of qualified Blue individuals increases without a decrease in the fraction of qualified Green individuals. Finally, HS is Pareto-superior to US, given that the fraction of qualified Green individuals goes to one.

It can be shown that

$$\frac{\partial \gamma^B}{\partial \alpha} = \frac{1}{4}\left[ d(1-\alpha) - r - \alpha d - \frac{r_G[r - d(1-2\alpha)]\ (2-\alpha)[\alpha d(1-\beta) - 2r(1+\beta)]}{(1-\beta)\sqrt{(\alpha-2)^2(2r+\alpha d)^2 + \frac{32\alpha r r_G}{1-\beta}}} \right],$$

which is non-negative. In fact, the higher the level of categorization endogeneity, the greater is the probability of incorporation of qualified Green workers in the Blue reference group. Moreover,

$$\frac{\partial \gamma^G}{\partial \alpha} = \frac{1}{2}[d(2-\alpha) - r],$$

which is negative if $d < r/(2 - \alpha)$. The effect of higher levels of CE in $\gamma^G$ is twofold. First, it increases the expected wage for qualified Green workers, as they will be more likely misclassified as Blue. Second, it decreases the "ability to discriminate", weakening the positive externalities. It occurs because, in this situation, qualified Green workers are "expelled" from the Green reference group with a greater probability.

Thus, for sufficiently low levels of discrimination, the endogeneity of categorization turns the unequal state more likely. This is shown in Figure 2. If d is smaller than $r/(2 - \alpha)$ [Figure 2(a)], higher values of endogeneity of categorization increases the range of values of the social interactions parameter for which the unequal state is stable. Conversely [Figure 2(b)], if discrimination is high, this range is broader and is more constant regarding the level of categorization endogeneity.

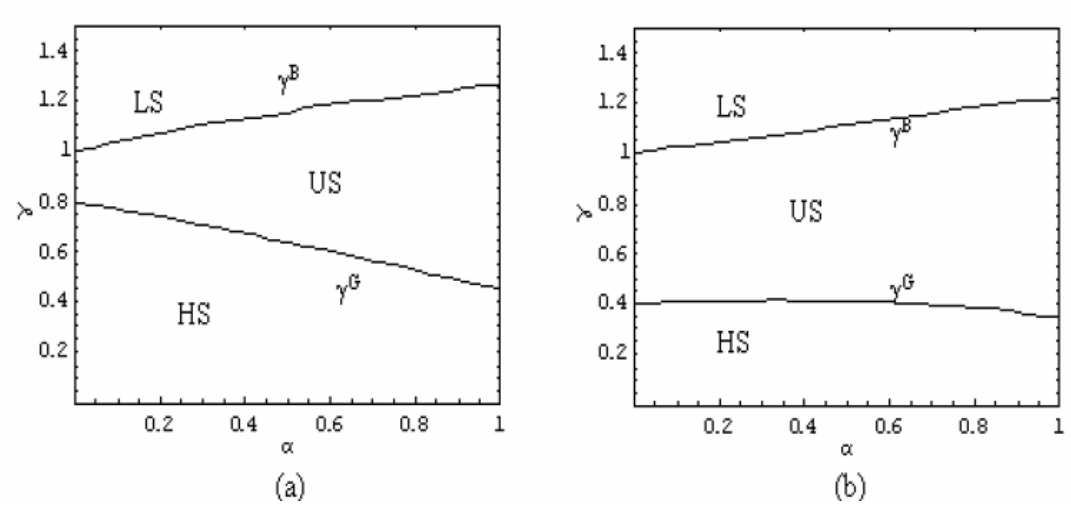



Figure 2: States which are stable for each combination of α and γ. The graphic on the left (a) corresponds to d = 0.2 and the one on the right (b), to d = 0.6. The other parameters assumed in the construction of the figure are r = 1 and β = 0.5.

Therefore, group inequality may be more likely in a less discriminatory society than in a more discriminatory one if they differ in their level of categorization endogeneity. Suppose there are two societies, A and B, which differ just in two elements: the level of discrimination and the degree of categorization endogeneity. Discrimination is higher in A ($d_A > d_B$), but categorization is more endogenous in B ($\mu_B > \mu_A$). Both societies are composed by two groups. It is possible that HS is less likely to take place in society B than in society A – that is, that $\gamma^G_A$ is higher than $\gamma^G_B$. Simple algebraic manipulation shows such will occurs only if

$$\Delta\mu > \frac{(1-\mu_A)^2}{r - d_B(2-\mu_A-\mu_B)}\Delta d\ ,$$

where $\Delta\mu = \mu_B - \mu_A$ and $\Delta d = d_A - d_B$, with the condition that $d_B < r/(2 - \mu_A - \mu_B)$. Thus, a less discriminatory society can experience a greater difficulty in reaching full educational achievement than a more discriminatory one, if the difference between their levels of categorization endogeneity were sufficiently higher than the difference between their degrees of discrimination.

4.1. The "melting pot solution"

An important shortcoming of the model presented here is that self-categorization is exogenous. This premise is unrealistic, as discussed in the previous section. In fact, one's identity choice is influenced by endogenous elements. By allowing individuals to choose their membership, it is opened a new room to group inequality erosion: it is possible that some members of the subaltern group may found reasonable to adopt the dominant group identity in order to avoid greater social losses when acquiring education. I will name it the "melting pot solution".

It poses the importance of the definition of *group* when discussing group inequality. In the traditional economic analysis, which considers group categorization as exogenous, the conception of group as a set of individuals who share the same identity sense coincides with the idea of group formed by individuals with the same exogenous characteristic (skin color, ancestry etc). Nevertheless, if CE is taken into consideration, there is a disjunction between these two concepts of group. By treating self-categorization as exogenous, I am going in hand with Telles and Lim' (1998) argument that self-classification is not the most appropriate method for determining group inequality.

Made this warning, the results of this model may be reinterpreted in the following way: if a given society finds itself out of the full educational achievement equilibrium, the only way to reach it is through the cultural assimilation of the dominated group by the dominant one. That is, the former should give up its identity and adopts the latter's identity. In other words, the results presented here hold just if the "melting pot solution", by some reason, is not available.

## 5. Concluding remarks

A model of qualification acquisition by phenotypically distinct individuals was presented above. It was shown that, under sufficiently small levels of discrimination, the endogeneity of the categorization turns the unequal state more likely: the higher the degree of endogeneity, the wider is the range of values of the social interactions parameter for which the unequal state is asymptotically stable.

The results presented here shed some light on the discrepancies regarding the impact of segregation on the persistence of group inequality. Segregation limits ethnic or racial admixture, making easier the categorization through individual phenotypic characteristics. Conversely, when categorization through exogenous, easily observable features is difficult or not possible, individuals are placed according to other elements, as behavioral traits (Humphreys et al. 2002). Thus, segregation reduces the endogeneity of categorization.

Therefore, segregation would be positively related with group inequality when anti-discriminatory laws are not effectively enforced. In this situation, although intra-group neighborhoods effects are strong, members of subaltern groups are heavily penalized by discrimination. It seems to be the case, for instance, of the scheduled castes in India[10].

On the other hand, segregation would contribute to the erosion of group differences when discrimination was bellowing some degree. In this case, the losses coming from the weakening of neighborhood effects would overcome the gains stemming from the smaller "ability to discriminate". It helps to understand why group inequality is decreasing faster in a more segregated country, as South Africa, than in a more integrated country, as Brazil, even if discrimination seems to be higher in the former than in the latter[11].

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[10] The Indian Constitution, existing since 1950, prohibits any kind of discrimination against inferior castes. Nonetheless, the effective application of this rule is not observed. Crimes against *Dalits*, the lower caste, in general while they are attempting to exercise legal rights, are still very common in the country (Meerman 2001). As a result, caste differences, including regarding educational attainment, persists in the country (Kijima 2006).

[11] Racial segregation in Brazil is only moderate when compared with that of South Africa (Telles 1992). Moreover, discrimination in the labor market is apparently lower in the former country than in the latter. A study conducted by Lam (2002), for example, shows that, after controlling for education and other variables, the White/Black wage gap in South Africa and Brazil were, respectively, 3.4 and 1.3.However, educational mobility among Blacks is higher in South Africa than in Brazil (Lam 1999, 2002).

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## Appendix: Fixed points and stability

Making $s_{G,t} = s_{G,t+1}$ and $s_{B,t} = s_{B,t+1}$ and simplifying, we have the following solutions to the system formed by equations (1.A) and (1.B):

[1,1],

$$\left[\frac{2r_G}{r_G-(1-\alpha)r+2\gamma},1\right]$$

$$\left[1,\frac{r[1-\beta(1-\mu)]}{\gamma(1-\beta)}\right],$$

$$\left[\frac{2r_G}{r_G-(1-\alpha)r+2\gamma},\frac{r}{1-\beta}\left(\frac{1}{\gamma}-\frac{2\beta}{\alpha r-d(1-\mu)+2\gamma}\right)\right]$$

These points will be termed FP (fixed point) 1, 2, 3 and 4, respectively. The Jacobean matrix of the form

$$J(s_G,s_B)=\begin{bmatrix} \dfrac{\partial s_{B,t+1}}{\partial s_{B,t}} & \dfrac{\partial s_{B,t+1}}{\partial s_{G,t}} \\ \dfrac{\partial s_{G,t+1}}{\partial s_{B,t}} & \dfrac{\partial s_{G,t+1}}{\partial s_{B,t}} \end{bmatrix}$$

has the following eigenvalues (ignoring time subscripts):

$$\left\{\frac{4\gamma(2-\alpha)[2r-d(2-\alpha)]}{\{[2r-d(2-\alpha)](\alpha s_G-2)-4(1-s_G)\gamma\}^2},\ \frac{2r(1-\beta)[2-\beta(2-\alpha s_B)]\gamma}{\{r[2-\beta(2-\alpha s_B)]+2\gamma(1-s_B)(1-\beta)\}^2}\right\} \quad \text{(A.1)}$$

A FP $\{s^*_G, s^*_B\}$ will be stable if the eigenvalues of $J(s^*_G, s^*_B)$ are strictly smaller than 1. Substituting FP1 in (A.1) and simplifying, we have

$$\left\{\frac{\gamma}{r_G(1-\mu)}, \frac{\gamma(1-\beta)}{r[1-\beta(1-\mu)]}\right\}$$

The first eigenvalue is greater than the second one. To see this, note that $r \geq r_G$ and $(1 - \beta)/[1 - \beta(1 - \mu)] \leq 1$. Thus, if the first one is smaller than one, so are both. It will occur if

$$\gamma < (1-\mu)r_G$$

Substituting FP2 in (A.1) and simplifying, we have

$$\left\{\frac{(1-\mu)r_G}{\gamma}, \frac{\gamma(1-\beta)(\alpha r_G + 2\gamma)}{\alpha r r_G + 2r\gamma(1-\beta)}\right\}.$$

The first eigenvalue is smaller than one if $\gamma$ is greater than $(1 - \mu)r_G$. This is also the condition which guarantees that $s^*_G$ exists, that is, is between zero and one. The second one is smaller than one if $\gamma$ is smaller than

$$\frac{1}{8}\left((2-\alpha)(2r+\alpha d) + \sqrt{(2-\alpha)^2(2r+\alpha d)^2 + \frac{32\alpha r r_G}{1-\beta}}\right)$$
$$= (1-\mu)r_G + \frac{1}{8}\left[[2r - d(4-\alpha)](\alpha-2) + \sqrt{(\alpha-2)^2(2r+\alpha d)^2 + \frac{32\alpha r r_G}{1-\beta}}\right].$$

Substituting FP3 in (A.1) and simplifying, we have

$$\left\{\frac{\gamma}{(1-\mu)[r-d(1-\alpha)]}, \frac{r[1-\beta(1-\mu)]}{\gamma(1-\beta)}\right\}.$$

The first eingenvalue is smaller than one if γ is smaller than (1 – μ)[r – d(1 – α)]. However, under this condition, the second eigenvalue is greater than one. Thus, FP3 cannot be stable.

Finally, substituting FP4 in (A.1) and simplifying, we have

$$\left\{\frac{(1-\mu)r_G}{\gamma}, \frac{r}{1-\beta}\left(\frac{1}{\gamma}-\frac{2\beta}{\alpha r-d(1-\mu)+2\gamma}\right)\right\}.$$

The second eigenvalue is greater than the first one. Thus, both will be smaller than one if γ was greater than

$$(1-\mu)r_G+\frac{1}{8}\left[[2r-d(4-\alpha)](\alpha-2)+\sqrt{(\alpha-2)^2(2r+\alpha d)^2+\frac{32\alpha r r_G}{1-\beta}}\right].$$

It also guarantees that both $s^*_G$ and $s^*_B$ are between zero and one.