The theorem of consumer surplus and demand elasticity at equilibrium price in a monopolist competition case

Grebennikov, Petr I

The Higher School of Economy. Russia

6 February 2010

Petr I. Grebennikov,
DSc, Professor, Saint-Petersburg University of Economics and Finance.

The theorem of consumer surplus and demand elasticity at equilibrium price in a monopolist competition case

Theorem: Let the monopolist competitor production demand has a linear function type, and its total production cost is given by formula $TC = F + vQ$, where F and v stand for fixed and average variable costs, and Q is a production quantity. Then, at the price of a long-run equilibrium, the consumers' surplus is equal to a half of fixed cost value, and the price elasticity is equal to the ratio of total to fixed costs.

The proof:
1) In the case of monopolist competition long-run equilibrium, the average cost curve $AC = F/Q + v$ is tangent to the demand curve $P = g - hQ$ (see the figure below). Total consumers' surplus value is presented, then, by the gEP_0 triangle area that is equal to $0{,}5Q_0$ times the length of the leg gP_0; the latter being equal to $h \cdot Q_0$ product, where $h = |dAC/dQ| = F/Q_0^2$. Finally, one gets that the surplus equals to $0{,}5Q_0 \cdot F/Q_0^2 \cdot Q_0 = 0{,}5F$.

2) In the long-run equilibrium, $P = AC = F/Q + v$, hence giving $Q = F/(P - v)$, and $dQ/dP = -F/(P - v)^2$. Therefore:

$$e^D = \frac{dQ}{dP} \cdot \frac{P}{Q} = -\frac{F}{(P-v)^2} \cdot \frac{P(P-v)}{F} = \frac{P}{P-v} = \frac{AC}{AFC} = \frac{TC}{TFC}.$$