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Abstract

Economic models of Fiscal Federalism, according to different settings, are generally linear and static, offering unique and deterministic solutions starting with simplifying assumptions. This paper rises from the idea to investigate how the decision-makers, abandoning their traditional economic models and focusing, instead, the attention on innovative components of evolutionary economics, can achieve better performance results, to organize and to optimize an economic system based on Fiscal Federalism. For this purpose, Fiscal Federalism must be understood as a dense network of economic relationships between different complex adaptive and co-evolving systems, the jurisdictions, linked by strong interdependencies. A better understanding of the links between interdependence will be provided by the Kauffman' NK-model. The relevance of the NK-model in the study of economic organizations has been detected several times in the literature. These studies, however, neglect the problem of co-evolution, which instead underpins this paper.

Key words: Evolutionary Economics, Fiscal Federalism, NK-model

1.Introduction

Federalism, both in theory and in reality, is a commonly used label to identify a wide range of political and institutional models characterized by the union of a functional and structural multiplicity of local authorities, variously named, but all have, more or less extensive powers of self government. Federalism, in this sense, means many things, among them often different and sometimes seemingly antithetical and, indeed, no model of Federalism, actually exists, equal to another.

Also from the theoretical point of view there are significant differences in the approach to this issue. Buchanan (1960) for example, prefers a reading near to political philosophy and. Using the analogy between clubs and local government, he proposed to explain the behavior of local governments in order to determine the optimal level both of size and

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activity; Musgrave (1959), however, considers Federalism primarily in terms of the theory of public finance, suggesting that the three are the functions assigned to the public sector: macroeconomic stabilization, income redistribution and resource allocation. The first two have to be the exclusive prerogative of the central government, while the allocative efficiency of the decentralized governments. It follows an "easy" translation of these assumptions in the theory of an appropriate system of Fiscal Federalism, which is to maximize the satisfaction of individual preferences over public goods and services through the decentralization of public expenditure and revenue decisions.

However, the pursuit of policies of public intervention, as any decision to maximize an objective function, in this case, the welfare of the community, through efficient public spending, cannot ignore the constraints of available resources. It would derive, otherwise, a set of distortions in the evaluation of policies that can affect in the long time the goodness of public intervention itself.

In fact, it is not possible to consider optimal a choice that identifies the benefits of the intervention, but not the costs related to it and especially without taking into account the effects that the marked differences that characterize the reality of the individual territoriality, could have on the sustainability of federal structure in the active pursuit of fiscal policies. We must therefore make suitable choices for a complex and complicated reality and discard others that, although theoretically valid, result inconvenient when put in a heterogeneous environment.

These observations do not seem to be taken into account by traditional economic modeling of Fiscal Federalism that aims to simplify the described reality. If, however, it is true that the cognitive process is at the same time a simplification process, (because it do not perceive the reality of things but its phenomenology), this does not mean it have to dismantle too the layer of complexity that surrounds the nature of things.

This is the basis of analysis of Complexity Theory. Complex is each phenomenon not completely framed in a linear, deterministic and predictable context, which is different than from what was represented until now by the science that have blindly followed the principles of separation, reduction and abstraction. These principles, imposed by the Cartesian paradigm of simplification have created a separation between reality and its formal representation. The Complexity Theory aims to study the phenomena not more by simplifying, linearizing and dividing them, but observing the relevance of inter-relationships among the components of systems - as well as their relationships with the environment and vice versa - in determining collective behaviors.

In this sense economics is a complex system but also a co-evolutionary system. The economic co-evolution describes the evolution of two or more agents that interact closely with another one and with the environment, reciprocally affecting each other's evolution.

Further because these agents are part of their environment, when they change, they change also their environment, and as it has changed they need to change again, and so it goes on as a continuous process. Each agent continually has to reorganize itself in order to seek a sufficient level of performance (fitness) to survive. In other words within this changing landscape, agents have to continually seek optimal positions and each strategic choice of a system leads to position changes of the others in unpredictable and unplanned ways. But from this mass interaction regularities emerge and start to form a pattern which feeds back on the system and informs the interactions of the agents.

From a mechanistic and linear vision, inspired by the Newtonian principles where the whole is always equal to the sum of its parts, they are now moving towards a complex approach where the whole is more than the sum of parts. The variables that measure the macroscopic state of a system, influenced by microscopic forces, can manifest linear or alternatively non linear dynamics, in this last case, coherent or purely chaotic dynamics.

The paper is structured as follows: section 1 starts with a critique of traditional models of Fiscal Federalism highlighting the limits of the capacity to adequately capture the behavioral dynamics of economic systems. We stress the innovative aspects of complexity theory, and the premises on which to base the analysis of Fiscal decentralization in that perspective. To this end we focus the attention to the centrality that has concepts such as interaction between agents, non-linearity and co-evolution. In section 2 we briefly describe the fitness landscape and the NK model of Kauffman (1993) as tools used to analyze the evolutionary dynamics of complex systems and we stress the use of such tools in economics. Then we proceed to model a landscape in which jurisdictions, complex systems of small size, that must find the optimal path to organize the local tax planning and to optimize their local economy. Finally we compare the properties of Kauffman's random exploration with a dynamic that reduces the randomness by introducing small constraints to be respected in the choice of fitness contributors. The work concludes with some considerations.

2.The Economic Models of Fiscal Federalism and their limits

The modeling that has dominated economic theory on Federalism until the 1980 (Buchanan 1960; Musgrave 1959, Oates 1972, Tiebout 1956) shares a common approach: the simplification and abstraction of the assumptions of generally linear and static models, able to offer unique and deterministic solution. The Oates'model suggests, for example, the absence of "spillover effects" and economies of scale, constant production costs, but also uniformity of preferences within local government jurisdictions and the heterogeneity of preferences among local jurisdictions. Oates achieves, in fact, "not ambiguous results," just because he departs from these assumptions. Moreover the respect of the "correspondence principle"[1] is made difficult not only by the difficult determination of the territorial scale of a single good, but also by the fact that, generally, different public goods will have different optimum dimensional areas.

The homogeneity of preferences also characterizes the model by Buchanan (1960), to which assumptions such as the existence of a revelation mechanism of preferences and a population with the same income are added.

Starting from the heterogeneity of preferences within the jurisdiction Tiebout assumed that the individuals can move freely among the different jurisdictions offering different baskets of goods (government services) at a variety of prices (tax rates). Given that individuals have different personal valuations on these services and different ability to pay the attendant taxes, they will move from one local community to another until they find the best mix of services and taxes which maximizes their utility. With enough variety among the jurisdictional offerings, each community will end up with people having identical preferences. Through this choice process, an equilibrium provision of local public goods in accord with the tastes of individuals will be determined. While the model has the advantage of solving two major problems with government provision of public goods: preference revelation and preference aggregation, however it relies on a very restricted set of assumptions. Perfect mobility, perfect knowledge of the differences among the various local governments in terms of taxes to be paid and services to be used; large number of jurisdictions, limited relevance of spill-over effects, constant-cost of services production allow to obtain an efficient provision of public goods.

It is clear that the traditional economic theory on Fiscal Federalism provides very general information from which is not always easy to draw practical guidance.

Mainly, theoretical structure and, consequently, the application of those models depend heavily on the basic assumptions, which represent their cornerstone but at the same time their Achilles' heel.

In fact these models are derived, in large part, by the translation in a simplified form of the insights of researchers on the reality, paying a hefty price due to the limits of this procedure, which is common to all sciences. It concerns the way in which the simplification is made and, above all, the level of simplification up to which we must or can be pushed without causing the loss of important and explicative information. Therefore it is not questioned whether the simplification must be made or not. The formal models that meet general approval are those which, although with some degree of abstraction, maintain a strong relationship with the represented phenomenon. More controversial, however, is the validity of those models that substantially deviate from

[1] We remember that the principle identifies the places where it is beneficial to provide public goods and services, which is determined using the geographic range of the positive and negative externalities that these goods and services cause.

what they are trying to approximate, in spite of their apparent ability to synthesize better than less formalized models.

There are many conditions in which opposite effects are algebraically added, neutralizing each other. Generally, to be able to catch them, it used an assumption that has the idea of an average that considerably summarizes the description. If, however, it disclaims to investigate the underlying interactions it loses much of the informational value of the result and it accepts the risk that, increasing the level of generality, the model will prove totally unfounded.

To overcome these limits and the growing interest in the dynamics of evolutionary systems, researchers from different disciplines (physics, biology, economics) have started on the one hand to test the goodness of traditional theories and models, proved, in fact, often unable to adequately capture the behavioral dynamics of systems, and on the other hand to explain the new principles that would provide a justification for such inadequacy, forming the foundation for the construction of a new interdisciplinary approach: the Complexity Theory (Bertuglia and Vaio 2005, Colander et alt. 2004, Arthur et al. 1997).

From a mechanistic and linear view of where the entire is always equal to the sum of the parts we are moving to a non-linear, complex view where "the entire is more than the sum of its parts". The linear view represents only one of many states in which a system can passes through: chaos and order coexist and the key to understanding all is the degree of interaction between the various elements that compose the system.

What is interesting is the analysis of the behavior of "system-model" located in "environment-model" in order to understand how, through co-evolution the system adapts to the environment and vice versa from time to time resulting different configurations (Oliver-Roos 1999, Merry 1999, Stacey 1995, 2003).

Systems and the environment have been studied often in the unique perspective, which had as its main, while not only, knowledge objective to determine the effects ex post generated from operating in the contexts of the subjects, without taking into consideration the "reciprocal" nature of the phenomena and, therefore, never resorting to the identification of a working scheme of their interaction in time and space by adopting i.e. a co-evolutionary approach.

Speaking of co-evolution, then, implies the need to have a dual and contextual perspective of investigation, the perspective of systems and of the environment, in which the economic and also the anthropological variable are strongly represented and interdependent.

It is a contextualized system in time and space the features of which are the fundamental variability of the environment (landscape) and the ability to use the environment as a source of competitive advantage (survival skills, levels fitness[2]). Therefore the study of the characteristics of the system- environment relationship must take into account that, because of the interaction, any evolutionary change of a system can lead to evolutionary changes in another, and that, under the co-evolution, the improvements for a system will provide competitive advantages for another allowing finding much of the available resources. In this context, the fitness increase of a system is due to decreased fitness of another system. The only possible solution for a system involved in this competition, is to adapt continuously as fast as in order to maintain its fitness level compared to that of other economic systems and alternately change its configuration.

Since the environment in which systems operate continuously changing as a result of co-evolution between them and the environment, the purpose of each system is to optimize their level of fitness as an expression of the attitude and the ability to survive via typical adaptation mechanisms of natural selection[3].

This is important since the majority of economic activity involves the integration and coordination of interdependent resources. Some of these interdependencies are that an element of the system needs other element to perform its function, or at least it can perform well its function if the other element is also present. It is therefore helpful to think of an economic system (enterprise, firm, production system, a jurisdiction) as a network of connected elements by a dense and complex links of interdependencies.

We can say the same to frame the Fiscal Decentralization in this framework of analysis. We consider the public sector as a big complex adaptive system in which different forces, hardly compatible, act with a multitude of human beings, with variables moods and continuous changes in political and economic scenarios. The fiscal decentralization as a prerequisite for organizing the entire fiscal structure led to the creation of local jurisdictions with fiscal autonomy.

The jurisdictions are economic systems at many dimensions characterized by complexity at different hierarchical levels. In this sense they are complex systems characterized by the connections between different levels and sizes through communications network. Economic agents are the nodes of the network, which produce knowledge by processing the information. (Barabasi 2002).

The jurisdictions play a very important role in the development of a country's competitiveness for economic development and that is why it stresses the need to develop an integrated and coordinated strategy that in a necessity bottom up logic, shifting the

[2] The theory of fitness has been proposed in evolutionary biology to represent the relationship between the number of genotypes of a certain class found in the present generation and the number of the same class of genes identified in the previous generation (Wright, 1932).

[3] Economic science has translated the concept of fitness in an evolutionary theory according to which heterogeneous organizations are selected on the base of their ability to develop different levels of fitness with the territory wherein they operate. (Nelson & Winter, 1982).

emphasis from static to dynamic optimization, is based on the use and development of new research tools such as genetic algorithms, exploration models, and simulations to analyze the potential long-term consequences of fiscal choices, their adaptability and robustness through the change of scenery.

3.Patching Theory and Jurisdictions

A system moves around its *fitness landscape* through various mechanisms: the *adaptive walk* that estimates the effects of individual changes on the entire system and the *patching*, (according to Kauffman more efficient), which estimates the effects on sub-system levels.

Patching theory proposes to divide a complex adaptive system, and then the problems, in several not overlapping parts, the *patches.* The *patches*, however, are not independent of each other, or each agent of each patch pays attention only to what happens in its borders, losing sight of the unity of the system and of the problems to solve. It is important to remember that the aim is always the efficiency and the survival of the global system, and then the originated sub-systems from its division constantly have to exchange information and co-evolve together.

Therefore, the *patching algorithm* searches improvements in the local fitness, inside the patch, rather than global improvements. Instead of adopting changes in the state that have a positive impact on the entire system, it shall state changes that have positive impact on subsets of the system.

This process seems to be particularly suitable to study social systems, those in which "[...] *Today people work in separate groups by creating and not resolving conflicts of various kinds ... ... because the individual solutions do not converge towards a single compromise that can properly address all the needs of departure ".* In particular, Kauffman (1993) argues that: *"For systems with various types of local autonomy, the analogy with the patches can be a key mechanism for understanding the evolution of economic systems, cultural ....".*

It is, therefore, that the theory of fiscal federalism and the patching theory propose to analyze complex economic-financial issues of a complex economic system/ State in the same way by identifying jurisdictions with patches.

Using the patching theory it also addresses the question of possibility, during the adaptive walk, to get into areas of fitness landscape with low efficiency and low fitness value. To avoid such mishaps it should leave the patches individually and freely evolve and auto-organize themselves.

Regarding fiscal federalism are the local jurisdictions that independently develop themselves and organize their own structure for the collection and spending of financial resources as they see fit. All this, however, within the limits set by national legislation, which must coordinate the process of adaptation of individual geographical areas in order to reach the highest peak of the *fitness landscape* for the entire State. In this situation the increase in efficiency can be spread with a proper management of externalities, trying to delete the negative ones and encourage positive ones.

To allow a complex system to move in the landscape by dividing it into several pieces can clarify the problems within the system that, oversized, has difficult to explore the entire territory, to design and to test new evolutionary paths. To divide the State in more local units of government cannot just give the entire tax system the needed degree of flexibility to adapt to the socio-economic changing, but also to find and to exploit all facets of the local microcosm. For happening this it is essential that the size of local jurisdictions is right. We saw earlier how Buchanan has resolved the matter.

From an economic point of view it is important to take into account also the size of the externality effects, the preferences of citizens, administrative costs and economies of scale. These constraints are also added to those brought as dowry by patching: the patches should be neither too large, otherwise the complex system is likely to crystallize in a single configuration and hang in an area of the landscape, nor too small, if it doesn't want that the pure chaos reigns supreme.

These new restrictions are necessary to ensure to the financial structure of a country an appropriate process for future development, aimed at achieving the goals of economic theory of fiscal federalism.

For example, to check what the right size is, we can use *the fitness landscape* in the following way. In the contemporary States the levels of government unlikely exceed the number of three: the central one, the middle (regions, Länder, cantons, ...) and local (municipalities, provinces Districts ...). Each level corresponds to a different dimension, taking into consideration economic and political considerations. On the intermediate, often in conflict with the central level, most often it is the focus of the system of territorial government. Consequently, the local units have very few skills. Considering these circumstances and leaving the central government, whose dimensions are not subject of the theory of fiscal federalism, we can construct two graphs of two landscapes. The first shows the fitness value of the various regions, the second of the municipalities.

Each region and each municipality, in each case providing a degree of autonomy, takes its own internal organization, a configuration somewhat different from all others. So, on the graphs it observes the offered solutions by the intermediate and local levels of government for the problems to be solved with the Federal tax. In this way you can understand what the current level of efficiency, such as the possible future development and what is the process of adaptation at all levels. Obviously, the efficiency should be measured on subjects in which all levels of Government have responsibilities, shared or not. Therefore the fitness landscape of regions and municipalities are compared taking into account, from time to time, the efficiency of the bureaucracy, the ability to contribute to local development, the efficiency of collection and spending of resources, etc.. In this way it can get guidance on what is a size that ensures a better solution of different issues and, therefore, at what level of government should entrust of responsibility of a certain field of public administration.

The entire system must be flexible so as to monitor the behavior of all patches of all sizes and changes its organizational and space structure, by facilitating and encouraging the more efficient jurisdictions. It may also be that the optimal size is a cross between the regional and local level. Patching and fitness landscape can also be used to find the best size for every possible configuration. As mentioned, each patch can organize its own management structures and obtain different results. Both between regions and between the municipalities will be preferred to the more efficient level which is on the highest peak of the landscape.

This level, by taking into account the socio-economic differences, can be a model for all others which conform, to the final step of the adaptive walk.

4.The use of the NK model in Economics

Although there are several equivalent models to analyze the effects of interdependences on the complexity of a system, for the purposes of the paper we will deal with the theoretical core of fitness landscapes associated with Stuart Kauffman (1995) and the NK model where the fitness landscape is its basis. This model consists in the search of optimization for problems characterized by a large number of variables in conflict with each other.

We consider a system composed of N elements that can have different states (0 and 1). These elements may have also different degrees of inter-dependence. Not to get into the details of these interdependencies, we'll just treat them as if they were determined randomly. The only thing that we want to check in detail is the "degree of interdependence" in the system, i.e., the average number of other elements with which each element is interdependent.

Denote by K the measure of interdependence whose values are between 0 and N-1. We define as system configuration each possible combination of states of individual components and as fitness the measure of the system performance. Each possible configuration of the elements of the system will have its own degree of fitness, more or less dependent on the exploitation of complementarity[4] and the greater or lesser effects of conflicts between systems. The set of fitness values associated with different configurations of the system draws a kind of "surface" of the fitness of the system called fitness landscape.

NK model can be considered as composed of two distinct components: a specific problem and a searching algorithm in the space of possible solutions. As we have said, the problem is a set of possible solutions represented as binary strings, each associated with a value of fitness, which is the pay-off of that solution. The NK model analyzes the evolution of a single string, which represents the state (or configuration) of a system, and it is important, although preliminary, for the construction of more elaborate models, in order to suggest possible avenues for self-organization in situations characterized by co-evolution.

The relevance of the NK model in the study of economic organizations has been detected several times in the literature. These studies (Westhoff, Yarbrough e Yarbrough 1996, Pagano (998), Levinthal 1997), however, neglect the problem of co-evolution, which instead underpins this work.

To clarify our intent, we consider a process of co-evolution between jurisdictions induced, for example, by the need to reorganize their economic system as a result of tax reform. The reorientation of the possibilities for tax planning and opportunities for economic growth, starting as a direct consequence, given the scarce resources, gives rise to a competition between jurisdictions. This is a phase of substantial uncertainty, caused by the fact that new opportunities are still ill-defined and can evolve rapidly. This situation gives rise to new dominant solutions in the tax planning of jurisdictions as a result of an extensive process of co-evolution influenced by interdependence.

The choice to represent the co-evolution tends to emphasize that a change in the tax planning of a jurisdiction creates new and different opportunities or disadvantages for other jurisdictions. In other words, a movement of a jurisdiction along the fitness *landscape* can deform the fitness landscape of other jurisdictions. Here the fitness landscape of a jurisdiction is interpreted as the graph of a map that associates each possible variant of the state (configuration) of a jurisdiction with his fitness level, interpreted as a measure of its efficiency in a given environment and in a given time.

If the effects of interdependence between jurisdictions are strong enough, the results of co-evolution in each jurisdiction are disturbed by the systematic deformation induced by simultaneous evolution of the fitness landscape in the other jurisdictions.

[4] Complementarity in this case means that the elements must be used and to act together in order to maximize the degree of fitness of the system to which they belong.

In this scenario, the constraints of interdependence plays a selective role because they affect the likelihood that the systems are well adjusted. This occurs because the interdependence constraints, by limiting the set of advantageous movements in the space of representation of the possible solutions, they increase the probability of evolution towards a stable configuration (despite the fact that the configuration may not be optimal ex post). In this way, the interdependence constraints help to reduce uncertainty and disorder in a system, considered as a set of evolving complex systems.

Let us we assume that the possible levels of public spending of jurisdictions are uniformly distributed in space *K*, where *K* is obviously the measure of interdependence. To take a systematic relationship between interaction and contributions means that every fitness landscape is drawn from a distribution such that the degree of interaction of an element is correlated with its contribution fitness. A stronger interaction leads to stronger constraint of complementarity.

On this premise, more integrated fitness landscape is even more rugged in the average. Then, as evidenced by Kauffman's results in rugged landscape local optima are more numerous, although their average fitness value may be lower. In addition, routes to the local optima involve fewer steps. These properties can be used to prove that, at every stage of a co-evolutionary process evolving systems on a rugged landscape are more likely to be simultaneously on a peak of landscape, and then to move towards a local optimum.

If the systems have a sufficiently large number of N elements, there is a trade-off between the probability that a process coevolves towards a stable local peak and the average fitness of a peak. So, it turns out that systems with an intermediate degree of interaction have a selective advantage against competitors, characterized by very high or low complementarity constraints. These properties are always true, no matter if evolution proceeds by random exploration of such trial and error (as assumed by Kauffman, 1993, 1995), or by imposing constraints that help to identify optimal choices within a set local choices. Once the systems are simultaneously at peak fitness, co-evolution tends to decrease.

In what follows, we try to show how the Kauffman's model can be used to construct a formal model of the phenomena mentioned above.

4.1A Landscape for Jurisdictions

We define a level of public expenditure *(t)* in the jurisdiction *i* at time *t*. The information on the level of public spending are coded in a number of binary elements, each of which may have the value 0 or 1. We can think of the string as a way to encode a specific combination of supply of public goods and services. In each stage of research the number of potentially available combinations tends to grow over time and the length of the string. To emphasize that the suggested approach has little to do with determining size of the problem, it is assumed that the length of all strings is finite and fixed. The efficiency of the chosen level of public spending, represented by the fitness value, defines the competitive strength of the jurisdiction.

There are G jurisdictions in the country. The level of public spending in the jurisdiction i ($i = 1, ..., G$) is a string of N binary elements ($x_{i1}, x_{i2}, ..., x_{iN}$), where each x_{ij}, $j = 1, ..., N$ can take value 0 or 1. Then there are 2^N possible levels of public spending for the Jurisdiction, corresponding to the number of different states in the space $\{0, 1\}^N$ that define the set A_i. We assume for simplicity that at the initial moment there is the same level of public spending in each jurisdiction. The configuration (planning) of the tax jurisdiction is defined by the level of public spending in i. Let x_i and x'_i N-strings in A_i. The distance between x_i and x'_i is defined by the number of different elements that arise.

More formally:

$$d(x_i, x'_i) = \sum_{j=1}^{N} (x_{ij} - x'_{ij})^2 \tag{1}$$

We therefore define two or more neighboring combinations that differ for a single element: $d = 1$. The neighborhood of x_i is the set of strings in A_i with distance from $x_i \leq 1$ and it is composed of x_i and its N neighbors. The fitness function of the jurisdiction is the map $F_i : A_i \to \Re$ that associates each configuration of the jurisdiction i with its fitness value (real number).

The fitness value of a string is the sum of fitness contributions of its N elements. More formally we define the map F_i as:

$$F_i(x_{i1}, x_{i2}, ...x_{iN}) = \sum_{j=1}^{N} F_{ij}(x_{i1}, x_{i2}, ...x_{iN}) \tag{2}$$

where F_{ij} $(x_{i1}, x_{i2}, ..., x_{iN})$ is the fitness contribution of the string element xij, given its configuration ($x_{ij} = 0$ or 1).

F_{ij} is treated as a random real number in a unit interval. The above notation is used to formalize the concept of interdependence, since the fitness contribution of x_{ij} may depend not only on the configuration of this element, but also the configuration of the other elements of the string.

$K_{ij} \leq N - 1$ is the number of string elements that are interdependent with respect to x_{ij}, so $K_{ij} + 1$ is the number of the non-redundant argument of F_{ij} $(x_{i1}, x_{i2}, ..., x_{iN})$. For simplicity we assume that K_{ij} is constant in all jurisdictions:

$K_{ij} = K =, j = 1, ..., N \quad i= 1, ..., G$ 3

In the absence of interdependence ($K = 0$), F_{ij} $(x_{i1}, x_{i2}, ..., x_{iN})$ can be written as:

$$F_{ij}(x_{ij}) \quad e \quad F_{ij}\left(x_{i1}, x_{i2}, \ldots\ldots x_{iN}\right) = \sum_{j=1}^{N} F_{ij}\left(x_{ij}\right) \qquad 4$$

The level of public expenditure with the highest fitness in A_i is then identified by the string such that the configuration of each element x_{ij} maximizes the fitness contribution F_{ij} (x_{ij}) of that element. The fitness landscape of the Jurisdiction i is the graph of F_i on A_i. In a walk that combines x_i to x'_i is a sequence of strings such that x_i and x'_i are respectively the first and last element of the sequence, and the distance between each pair of adjacent elements of the sequence is $d = 1$.

A walk that joining x_i to x'_i and is minimal if the distance to x'_i is strictly decreasing on this "walk".

x'_i is a local maximum of F_i $(x_{i1}, x_{i2}, ..., x_{iN})$ on A_i if and only if on every walk that joining x'_i to a string y_i such that F_i (y_i)> F_i (x'_i) there is a y'_i such that F_i (y'_i) <F_i (x'_i) *e* d (x'_i, y'_i) <d (x'_i, yi)[5].

Suppose that $K = 0$. If x_i is a global maximum of F_i on A_i, and y_i is an arbitrary string in this set, then F_i does not diminish in any shortest walk joining x_i to y_i.

The proposition is self-evident. Because the walk is minimal, there must be many steps along the path as there are elements of y_i, which differ in their configuration, from the corresponding element of x_i. At every step along the path, the distance from x_i decreases, since there is another element of y_i that has the same value of the corresponding element of x_i. This value maximizes the fitness contribution of the element because x_i is a global maximum, without reducing the fitness contribution of the other elements (Because K = 0)[6]. If $K = 0$, the fitness landscape of the jurisdiction i has at most one local optimum of F_i on A_i which corresponds to global optimum[7].

We suppose $K > 0$. The choice of configuration to maximize the fitness contribution of the element x_{ij}, given the configuration of the other N-1 elements of the string, cannot positively contribute to the general fitness level of public spending of jurisdiction i. The reason is that interdependence implies the possibility of a feed-back of uncertain sign stemming from the new configuration of the x_{ij} to the fitness contribution of the other elements. This is equivalent to the possibility that there may be more local optima. The situation is illustrated in Figure 1 with reference to the simple case $N = 2$ and $K = 1$.

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[5] With reference to the case K=0, it is important to remember how a change from 0 to 1, or vice versa, in the configuration of a single string element, does not affect the fitness contribution of the other components

[6] The probability that a randomly chosen string in a landscape K=0 is a local peak is $1/2^N$. Let $F^*(N,K)$ be the expected fitness of a local peak. $F^*(N,0)$ is independent of N and can be expressed as $E\left[\sum_{j=1}^{N} Max\left(a_j, b_j\right)\right] = 0.666$ where (a_j,b_j) are N couples of real random numbers uniformly distributed on the unit interval.

[7] This is easily demonstrable by supposing the contrary. If x_i is a maximum of F_i on A_i There may be in the same space an isolated maximum (local or global), $y_i \neq x_i$ of the fitness function F_i. By construction F_i has a non-monotonic behavior on every minimal path joining y_i and x_i.

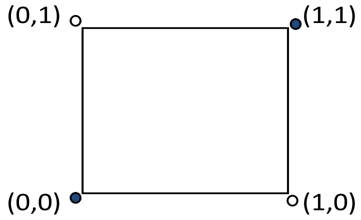

Fig.1

In this example, the set A_i of the possible levels of public spending in the jurisdiction is composed of 4 strings. Strings *(0.0)* and *(1.1)* are the local optima. The path that joints the strings is done on the sides of a square but not on its diagonal because the diagonal steps involve simultaneous changes of many elements, not just one. By construction we know that in each path joining *(0.0)* to *(1.1)* the fitness function does not have a monotonic behavior.

Finally, we consider the greatest interdependence *(K=N-1)*. The fitness landscape is random in the sense that the fitness values of the neighbors are totally uncorrelated. A change (from *0* to *1*), or vice versa in the configuration of a single element, say element j of the level of public spending of the jurisdiction, not only assigns a new random fitness contribution to F_{ij}, but also a new random contribution F_{hi} to each component of h $(h = 1, ..., N$[8] . The reason is that now x_{ji} is not a redundant argument of F_{hi} $(h = 1, N)$[9] . The statements are based on the following assumptions.

Since $K = N-1$ the fitness values are not correlated; each string in a landscape has a probability $1/(N+1)$ to be a local optimum and the expected value of local optima is $2^N/(N+1)$.

In each landscape the lower local optimum has a higher fitness value than the fitness value of the other N strings. The fitness value of the local optimum can be understood as the maximum in a set of 2^N fitness values[10].

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[8] Footnote 9 implies that $F^*(1,0)=0.666$. If $N>1$ $F^*(N,N-1)$ first grows above 0.666 and then decreases monotocally to 0.5. Moreover if $K=N-1$ then $F^*(mN,K)=F^*(N,K)$ for any $m \geq 1$. This suggests that $F^*(N,K)$ remains approximately constant as N grows to infinity , while K is fixed at $N-1$.

[9] The fitness value of each element on a landscape $K = N - 1$ is a random number, uniformly distributed between 0,1. The probability that a randomly chosen element of the landscape is a local peak (its fitness value is higher than its N neighbours) is $1/(N+1)$. Then there are on average $2^N/(N+1)$ local peak on a landscape $K = N-1$.

[10] This involves lower and upper bounds to F*(N,N-1):

$$E[Max(\alpha_1, \alpha_2, ..., \alpha_m)] < F^*(N, N-1) < E[Max(\beta_1, \beta_2, ..., \beta_m)]$$

Where each α_m e β_m i san average of N random numbers in the unit interval m=N+1, M=$2^{N.}$ Since the expected fitness value of intermediate local optima uniformly distributed between the lower and upper bounds above, we have:

$$F^*(N, N-1) = \{E[Max(a_1, a_2, ..., a_m)] + E[Max(b_1, b_2, ..., b_M)]\}/2$$ Order statistic shows that

$$\{E[Max(\alpha_1, \alpha_2, ..., \alpha_m)] + E[Max(\beta_1, \beta_2, ..., \beta_M)]\}/2 \approx 0.7$$ for $4 \leq N \leq 10$.

Consequently F*(N,N-1) decreases as N increases and converges to 0.5 as N grows to infinity because each single sample average α_m and β_m must behave accordingly. Moreover, we consider *K=N-1* and *F*(N,K)* where *N=mN* and *K=K*. Through a possible re-ordering of elements, every string of length N can be thought of as being composed of m segments of N elements each. Within each segment each element is connected to the K other elements. Thus, the fitness contribution of each component depends on its configuration (0 or 1) and on the configuration of every other component of the same segment. Hence the expected fitness value of

On average, the higher K implies that: the higher is the number of local optima, the shorter is the minimal path that connects a random string in A_i to the nearest local optimum; the lower the correlation between fitness values F_i of neighboring strings $x_{i,}\ y_i$.

4.2Fully vs. constrained randomness in a landscape exploration.

So far we have given a formal description of what could be a fitness landscape of a jurisdiction. Let’s see how the jurisdiction can precede this exploration. Here we compare the properties of Kauffman’s random exploration with a dynamic that reduces the randomness by introducing small constraints to be respected in the choice of fitness contributions. The choice rises by the assumption that the introduction in a totally random model, of some qualitatively and quantitatively important information, without falling into over-simplistic, increases the effectiveness of the use of complex tools. At each time t, the jurisdiction i doesn’t have a perfect knowledge of A_i, because the perception of a potentially profitable combination of elements $x_i \epsilon A_i$, and even more, the information on its fitness $F(x_i)$, is available only if x_i is in the neighborhood of the string that defines the tax configuration of jurisdiction i at time t. The information, even when it can be codified, does not immediately translate into knowledge that can be exploited for useful purposes. The transformation of information into knowledge requires understanding, learning and adaptation.

We can assume that this not encode information can be gained through experience. Unlike sectors where every change is always associated with a random mechanism, here we try to know how the research can proceed through the combination of random explorations and more targeted explorations aimed to achieve pre-selected goals. According to the dynamic of *NK* model induced by random exploration on a fitness landscape the neighbor element x'_i of the current state x_i is randomly selected at any time[11]. The fitness value $F(x'_i)$ was then examined, and a movement toward x'_i occurs if $F(x'_i)>F(x_i)$.

A greater focus on the intentional components of research generates the assumption that at any moment a system moves one step from pre-determined state to the state identified by the string with the highest fitness value in the given neighborhood.

This modeling strategy produces a slightly different dynamic on A_i. This comes out when the sequence of a neighbor x_i of x'_i uses combinations of intentional and random choices: in each time $n<N$ components of x_i with relatively low contributions to fitness are intentionally selected, one of which is randomly selected and its configuration modified.

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each segment is an average of N random numbers in the unit interval and is identical to the expected fitness contribution of every other segments. This holds independently of the size of m.

[11] This amounts to a random selection of one element of xi and a change of its configuration (from 0 to 1 or vice versa).

As before, a move towards x'_i occurs if $F(x'_i)>F(x_i)$. The figure shows the dynamics of a single jurisdiction on the fitness landscape $N=2$ and $K=1$ described in Fig. 1. Black points identify the local peaks of the fitness landscape. The string *(1.1)* is the global optimum F_i on A_i. The string *(0.0)* is a degenerate basin of attraction, which coincides with the string itself.

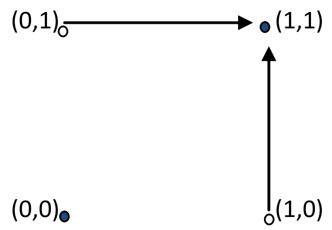

Fig.2

Fig.2 clarifies how the landscape in this example is completely robust in the sense that each neighbor of a string that is not a local optimum is a local isolated peak.

It may be instructive to compare the asymptotic average properties of NK model of Kauffman's random exploration and the dynamics of the model with bounded randomness on fitness landscape randomly generated for extreme cases of absence and complete interdependence. The main differences are: when $K=0$, the full randomly exploration and the bounded randomness exploration at the end reach both the global optimum of the landscape, the number of steps required is smaller in the bounded randomness, because every step is taken towards the pre-selected direction[12].

$0 < K < N - 1$ the dynamics of randomly exploration converges to the average of global optimum of the landscape. The average fitness value $F^*(N, K)$ of a local optimum changes with N and K.

For finite values N, the asymptotic deterministic dynamic on a landscape $0<K<N-1$ climbs a local optimum of fitness, which is surely above average. If $K=N - 1$, the fitness of highest optimum at average drops to 0.5 when N tend to infinity. The same is not true if N grows to infinity, but K remains constant.

4.3.The Effect of Co-evolution of Interdependence.

We consider pairs of levels of public spending in G jurisdictions of the state. We also consider the hypothesis that a single level is evolving in each jurisdiction. Therefore, the efficiency of public spending here refers to the level of public spending of jurisdiction i $(i = 1, ..., G)$. On any given landscape, the dynamics are assumed to be with reduced randomness, but, in line with the conclusions of the preceding paragraph, the same qualitative results are obtained when exploratory dynamics are considered totally random.

[12] The average fitness value $F^*(N,0)$ of a global optimum is 0,666.

The opportunity to optimize its tax system that is based on the interdependence between the elements, we remember, imply that, in general, the fitness of the 2^N states of *i*, depends on the current state of the other *G-1* jurisdictions. Following Kauffman (1993), we can predict these effects such as deformities of the fitness landscape of the jurisdiction *i*, triggered by changes in the other *G-1* jurisdictions. More precisely, we consider the changes in fitness levels of public spending in the jurisdiction *i (i = 1, ..., G).* The changes in the landscape can be global or local. If the relationships of interdependence between jurisdictions are limited to small segments of the string, the change of a single element does not induce a change in the global fitness landscape of another jurisdiction. However, because the State is composed of many jurisdictions, a multiplicity of individual change takes place simultaneously. Hence *G* is larger than *N*, the greater the probability of a global change of the landscape. If *G* is very small relative to *N*, the case of deformation bases on the assumption that global interdependence across jurisdictions is pervasive. Situations of complete interdependence are defined by the fact that each component of each string is connected to every other component of every other string. A single change in a state of an element is therefore sufficient to set up an entirely new landscape for any other jurisdiction. We use this rather extreme hypothesis, because it suggests an approach that strongly takes into account the co-evolution, whence the general qualitative effects of complementarity are more easily detected. So we define *C* the number of co-evolving systems.

The economic dynamics of *G* jurisdictions are determined by their interdependences, and the following tables describe the list of fitness values of each element corresponding to each state of level of public spending in the remaining jurisdictions.

The first element of the list is the one with the highest fitness value. The possibility that adjacent elements in the list have the same fitness value is excluded, because the event could be an irrelevant fluke.

Two examples are shown below for the case: *N=2*, *C=2*. The two jurisdictions are called α and β, and, by way of example, α00 is the state of public expenditure (*0,0*) of jurisdiction α. Table 1 refers to the case *K = 0*, Table 2 to case *K = 1*.

Se β_{00} : α_{10}	α_{11}	α_{01}	α_{00}
Se β_{01} : α_{11}	α_{10}	α_{01}	α_{00}
Se β_{10} : α_{01}	α_{00}	α_{11}	α_{10}
Se β_{11} : α_{00}	α_{10}	α_{01}	α_{11}

Se α_{00} : β_{11}	β_{10}	β_{01}	β_{00}
Se α_{01} : β_{10}	β_{00}	β_{01}	β_{11}

Se α_{10} : β_{00}	β_{10}	β_{11}	β_{01}
Se α_{11} : β_{01}	β_{11}	β_{00}	β_{10}

Table 1

Se α_{00} : β_{00}	β_{11}	β_{01}	β_{10}
Se α_{01} : β_{00}	β_{11}	β_{10}	β_{01}
Se α_{10} : β_{11}	β_{00}	β_{10}	β_{01}
Se α_{11} : β_{11}	β_{00}	β_{01}	β_{10}
Se β_{00} : α_{00}	α_{11}	α_{01}	α_{10}
Se β_{01} : α_{00}	α_{11}	α_{01}	α_{10}
Se β_{10} : α_{11}	α_{00}	α_{10}	α_{01}
Se β_{11} : α_{11}	α_{00}	α_{01}	α_{10}

Table 2

The time is discrete, and at each t time each configuration moves from the present state to fittest neighbor through simultaneous changes. The representation space of the dynamics induced by a given interdependence pattern between G jurisdictions, given the co-evolution, is the hypercube $\{0.1\}$ NG. Each hyper-row, or hyper-column of this representation space consists of an ordered series of 2^N elements, where each element, or a point $(x_1, ..., x_G)$ is an ordered list of tax configurations[13] one for each jurisdiction.

The neighbor of a point in state space is an ordered list $(y_1, ..., y_g)$ such that each y_i is a string of N binary codes N and $d(x_i, y_i) < 1$.

A point in the state space has NG neighbors. Each element of a hyper-row (or hyper-column) is therefore a configuration of a jurisdiction, and moves on the same hyper-line (or hyper-column) on which we meet all the possible states of the jurisdiction i, while the state of other G-1 jurisdiction is unchanged.

Recall that for $K=0$ each fitness landscape has one peak and that, by construction, each hyper-row (or hyper-column) refers to the fitness landscape of a given jurisdiction. Suppose that the level of interdependence is given.

A rest point in the state space corresponding to this model is that all jurisdictions are simultaneously on a peak of fitness. If and only if $K=0$, on every hyper-row (or hyper-column) in the state space there is at most one rest point in which the co-evolution slows down as shown in Figures 3 and 4. Figure 3 shows, in fact, associated dynamics in the representation space of possible solutions $\{0,1\}$ 4 determined by the model of interdependence indicated in Table 2.

[13] Strings of N binary codes

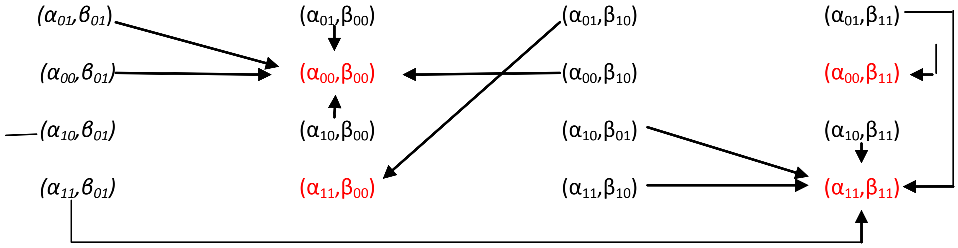

Fig. 3

When $K>0$, when the co-evolution begins to decrease, not all jurisdictions are necessarily on a global optimum of their landscape (see, for example, the state (α_{11}, β_{00}) of Fig 3. Some may be at the global peak while some others at a strictly local peak, or they may be simultaneously at a strictly local peak.

The number of admissible patterns of interdependence depends on the parameters N and K and the co-evolution. Since there are 2^N different states of a given jurisdiction, there are $2^N!$ re-ordering of these different strings based on their fitness value.

When $K=N-1$, each of these re-ordering is admissible. However, if $K=0$, two adjacent strings differ on every admissible re-ordering in one and only one element.

Since every configuration to the jurisdiction i $(i = 1, ..., G)$ can be coupled with $2^{N(G-1)}$ different states of the remaining jurisdictions, we obtain $[(2^N!)2^{N\,(G-1)}]^S$ possible patterns of interdependence for the case K = N - 1, where the parameter S identifies the degree of co-evolution, and a considerably lower number of possibilities for $K=0$.

Any admissible model gives rise to an evolutionary dynamics in phase space, which is a set of 2^{NS} trajectories, each starting from a different initial condition in phase space (see Fig 3 and 4).

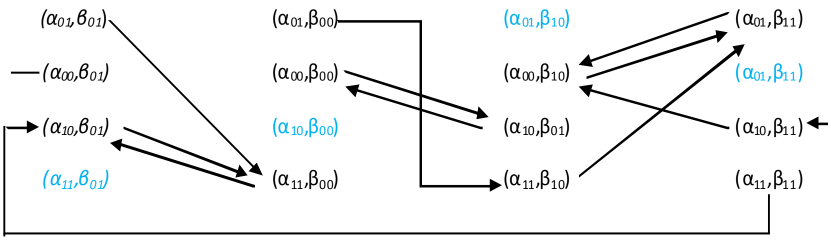

Fig. 4

5. Conclusions

This paper comes from the idea to investigate how the new tools provided by the Complexity Theory may offer interpretative solutions to the optimization of an evolutionary economic complex system

Complexity Theory is presented, in fact, as an opportunity to better understand reality not to neglect a priori all the phenomena that cannot be pigeonholed and explained according to preconceived thesis.

Because complex, the economy is a system that "evolves". Normally we're used to thinking about evolution in a biological context, but the modern evolutionary theory, as branch of Complexity Theory, sees evolution as something much more general. Evolution is an algorithm, is a formula that, through its special brand of trial and error, creating new projects and solve difficult problems. The evolution concerns not only the DNA "substrate", but each system has a feature to process and collect information. In short, the simple recipe of the evolution of "to differentiate, select and amplify" creates news, knowledge and growth. An economic system, then, is how an ecological niche, with different "species" of players, agents, engaged in a struggle aimed at the "survival of the fittest". Paul Krugman calls this metaphorical comparison of the economic and biological systems "biobabble".

Efforts to understand modern economics as an evolutionary system avoid such metaphors, and instead, focus on understanding how the universal algorithm of evolution is literally and specifically implemented in the substrate of information processing of human economic activities.

Having shown that as fiscal federalism is to be understood as a dense network of economic-financial relationships between different coevolving complex and adaptive systems (the central and local government), linked by strong interdependencies clarifies even more the goal of the paper aimed to study fiscal federalism from a dynamic and evolutionary point of view, seeking solutions to problems posed by traditional economic theory with new analysis tools of Complexity.

The solution of a problem built on an adaptive complex system, cannot be searched as if we were solving a simple problem without interconnections. To identify in fact, a single optimal solution while it is possible for a simple system, it is not for a complex system.

In this case it is only possible on the basis of its numerous connections, to determine the process by which different solutions may emerge, more or less favorable to the resolution of the problem. Taking into account the existence of multiple solutions, the same research can be done through a searching algorithm on a *fitness landscape*, a dynamic landscape in which complex adaptive systems are moving in search of optimum conditions. The configuration of this landscape is strongly conditioned by the presence of co-evolution and interdependencies. Also the jurisdictions as result of fiscal decentralization can be regarded as evolving complex systems, although smaller. From these assumptions and on the basis of evolutionary dynamics, we analyzed the behavior of jurisdictions to develop a model to identify their optimal fiscal configurations by using *NK*-model.

The *NK*-model can be considered as composed of two distinct components: a specific problem and a searching algorithm in the space of possible solutions. As we have said, the problem is a set of possible solutions represented as binary strings, each associated with a fitness value, which is in fact the pay-off of that solution. The searching algorithm consists of repeated mechanism in order to scan the solution space from a (usually randomly chosen) initial string, or a binary N-dimensional space. The ongoing research is defined in terms of rules on how to move from one point to another. For example, the typical search, originally proposed (Kauffman, 1993) is to choose randomly a string in which changing one bit, in the case that the changed string has fitness higher than the current one, the new string is accepted, and otherwise it is rejected.

The repeated application of the algorithm generates a pattern in the space of possible solutions. The pattern ends when the rule reaches a string from which all possible strings within the space of solutions were rejected. Two aspects make particularly attractive the *NK*-model. First, you can determine the whole must be the solution space, or the fitness landscape. Building a landscape with few or no interactions (represented by the value of *K*) means to generate the equivalent of simple problems, increasing *K* generates a complex problem.

The second aspect is the representation of the *NK*-model searching algorithm. The *NK*-model assumes a local search. Local, because the research involves the inability to observe the space beyond the near immediate focusing on the goal of improving their present condition. The two aspects, complexity through interaction and local search, paradoxically leads to a simplified and then manageable of many real situations.

It's an interesting tool because it provides the opportunity for the researcher to represent and control the two aspects of problem solving: the complexity of the problem and the degree of expertise for finding a solution. It' s possible to use NK model to generate and to evaluate the space formed by the two dimensions of the complexity of the problems and skills in the resolution strategy in order to represent both aspects of a real-world in small-scale. The use of the *NK*-model arises from the possibility of establishing a sort of relationship between the skills of decision makers and the related difficulty of intervention in economic policy. In this case, it is no more relevant that the modeled problem is much simpler than the real one, since even the solution strategies are modeled in a much less sophisticated way. By controlling both aspects we can be expected that the properties of the set that includes the solutions generated in the model are similar to the set of real solutions generated in real systems with an equivalent ratio of task and skills difficulty in finding a solution.

Starting with Kauffman's work, and introducing some constraints to the full causality, it was built a model whose theoretical results show that a change in the tax planning of a jurisdiction produce new and different opportunities or disadvantages for other jurisdictions. In other words, the results show that a movement of a jurisdiction along the fitness landscape can deform the fitness landscape of other jurisdictions. If the effects of interdependence between jurisdictions are strong enough, the results of co-evolution in each jurisdiction are disturbed by the systematic deformation-induced by simultaneous evolution of the fitness landscape in other jurisdictions.

In this scenario, interdependence constraints have played a selective role influencing the adaptability of systems. In this way, the interdependence constraints have contributed to reduce the uncertainty and disorder in a system, considered as a set of evolving complex systems.

Moreover, according to Kauffman' result, the model shows the complex nature of each system, justified by its ability to associate order and disorder. The disorder is represented by the random choice of fitness contributions fitness, order, however, by a coherent structure, produced by the model and made of configurations, adaptive walk, fitness landscape and fitness value.

To take a systematic relationship between interaction and fitness contributions means that each fitness landscape is drawn from a distribution such that the degree of interaction of an element is correlated with its fitness contribution. A stronger interaction leads to stronger constraints of complementarity. On this premise, the more closely integrated fitness landscape is even more rugged in the average. Moreover, as evidenced by the results of Kauffman, in a landscape more rugged local optima are more numerous, although their average fitness value may be lower. In addition, walks to the local optima involve fewer steps. These properties were used to demonstrate that, at every stage of a co-evolutionary process, the systems evolving on a rugged landscape are more likely to be simultaneously on a peak of fitness, and then to move towards a local optimum. If the systems have a sufficiently large number of N elements, there is a trade-off between the probability that a process co-evolve towards a stable local peak and the average fitness value of a peak. Thus, it confirms that systems with an intermediate degree of interaction have a selective advantage against competitors, characterized by very high or very low complementarity constraints.

These properties are always true, no matter if evolution proceeds by random exploration of such trial and error (as most assumed by Kauffman), or by choosing to impose constraints, as in the proposed model, to help identify optimal choices within a set of local choices. Once the systems are simultaneously on a peak of fitness, co-evolution tends to decrease.

Finally it stressed that the decision to introduce constraints to randomness rises from on the assumption that the introduction, in a totally random model, of some qualitatively and quantitatively important information, without falling into over-simplifications, increases the effectiveness of using of complexity tools.

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